

# Finance Concepts II: Term Structure of Interest Rates

Federal Reserve Bank of New York

Central Banking Seminar  
Preparatory Workshop in Financial Markets,  
Instruments and Institutions

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## Overview

- Principal and Interest (Quick Review)
- Term Structure: Zero Coupon Bonds
- Expectations Hypothesis
- Term Structure: Coupon Bonds
- Influences on the Yield Curve
- Predictive Power of the Yield Curve □

## Principal and Interest

- -\$100 now
- +\$105 in one year (= \$100 principal + \$5 interest/coupon)
- Rate of return or yield is  $5\% = 105/100 - 1$
- Discount factor is  $100/105 = .9524$   
i.e., present value of \$1 in one year is \$.9524

## Principal and Interest II

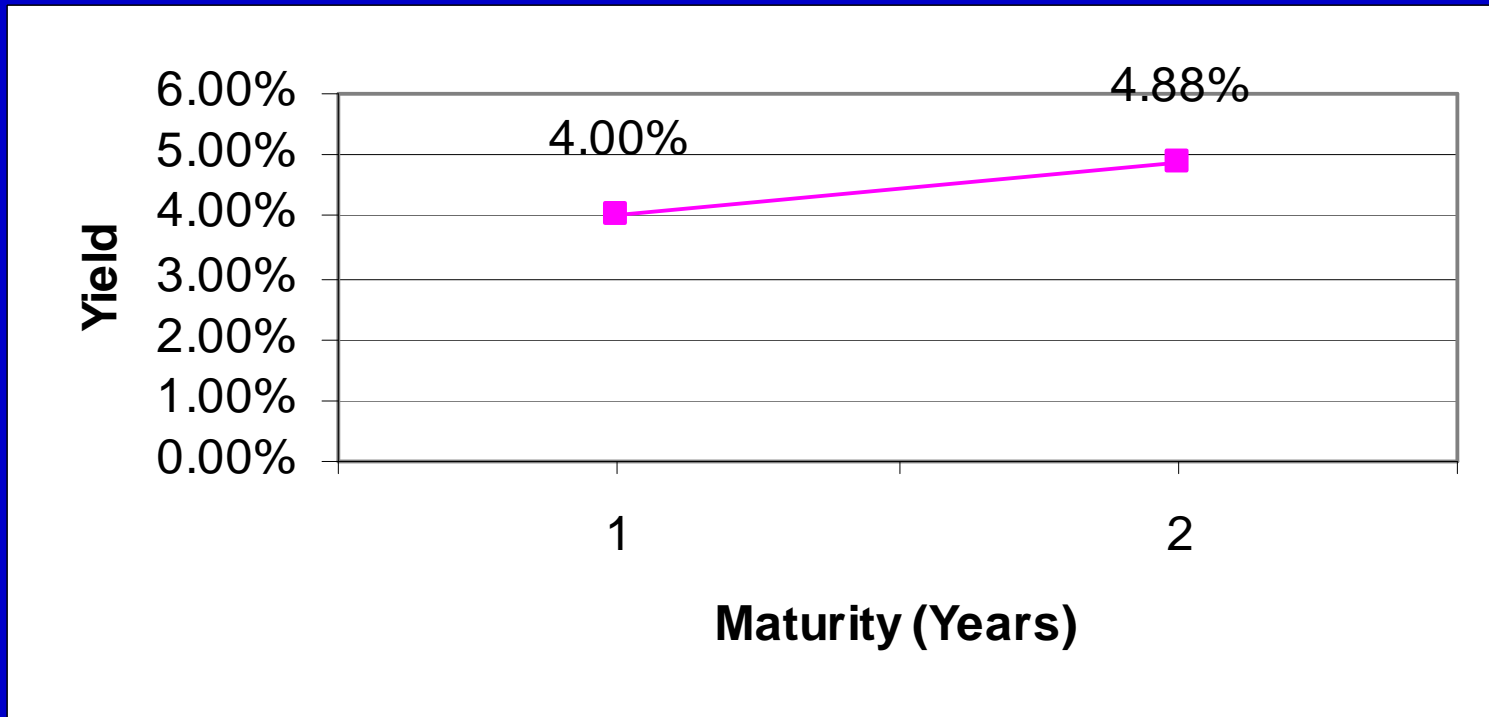
- -\$99 now
- +\$105 in one year (= \$100 principal + \$5 interest/coupon)
- Rate of return or yield is  $6.06\% = 105/99 - 1$
- Discount factor is  $99/105 = .9429$

## The Term Structure: Zero Coupon Bonds

- One-year bond  
-100 now, +104 in one year  
Yield is 4% =  $104/100 - 1$
- Two-year bond  
-100 now, +110 in two years  
Yield (annual) is 4.88% =  $(110/100)^{(1/2)} - 1$   
*(Note exponent!)*

# Zero Coupon Yield Curve

(A very simple one)



## Expectations Hypothesis

- The yield on a two-year bond is the same as the expected yield on two consecutive one-year bonds.
- So, -100 now, +110 in two years  
is the same as  
-100 now, +104 in one year  
plus  
-104 in one year, 110 in two years

## Expectations Hypothesis II

- Yield on the second one-year bond is calculated as  
 $110/104 - 1 = 5.77\%$
- This yield is known as a “forward rate”
- The pure expectations hypothesis says that the expected yield on a one-year bond, one year from now, is 5.77%

## Coupon Bonds

		Cash flow in:		
	Price	Year 1	Year 2	Year 3
<b>Bond 1</b> (Par)	-100	106		
<b>Bond 2</b> (Discount)	-92	2	102	
<b>Bond 3</b> (Premium)	-106	9	9	109

## Yield to Maturity: Coupon Bonds

- Same as internal rate of return
- Bond by bond:

$$100 = (1+y_1)^{-1} * 106$$

- $y_1 = 6\%$

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- $y_1 = 6\%$ ,  $y_2 = 6.39\%$

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$$100 = (1+y_1)^{-1} * 106$$

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$$106 = (1+y_3)^{-1} * 9 + (1+y_3)^{-2} * 9 + \\ (1+y_3)^{-3} * 109$$

- $y_1 = 6\%$ ,  $y_2 = 6.39\%$ ,  $y_3 = 6.73\%$

## Zero Coupon Yields from Coupon Bonds

- Calculate from all bonds simultaneously

$$100 = (df1) * 106$$

$$92 = (df1) * 2 + (df2) * 102$$

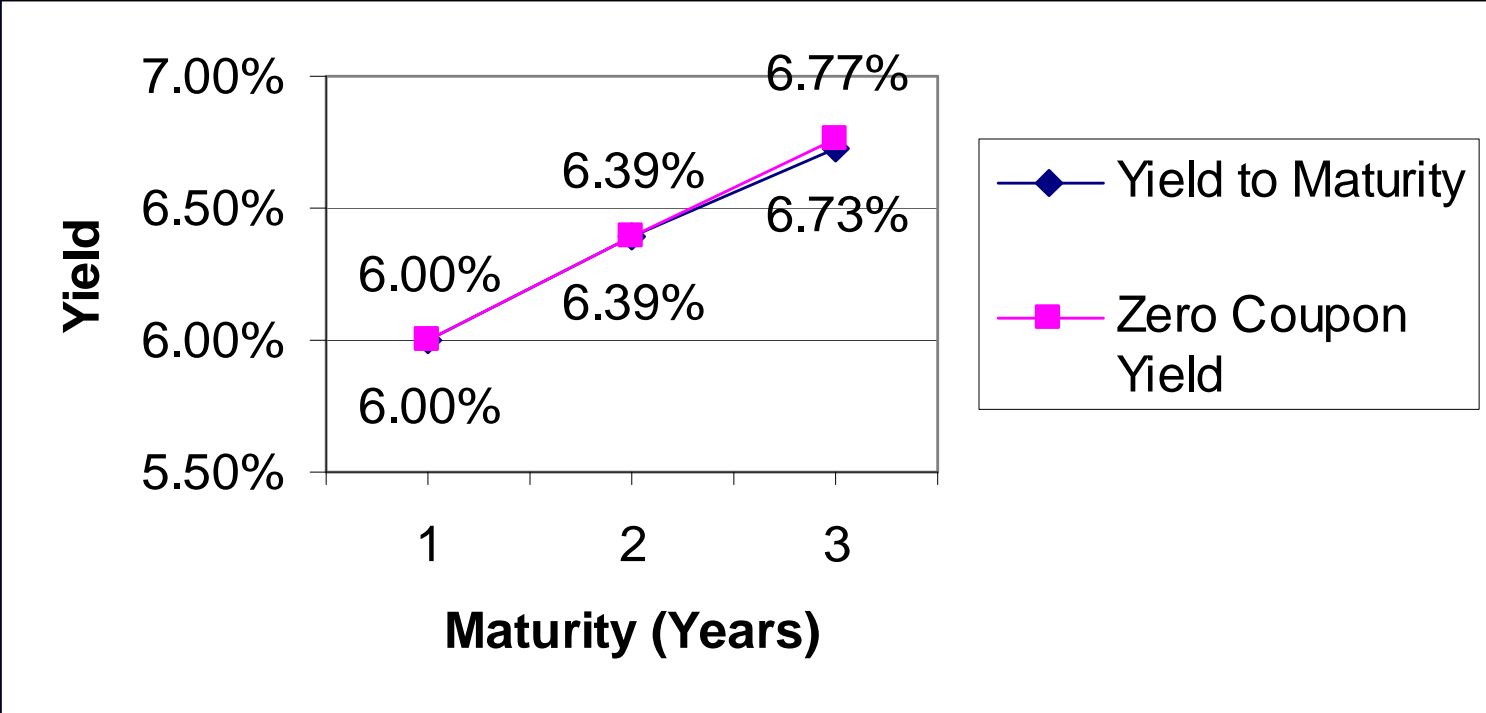
$$106 = (df1) * 9 + (df2) * 9 + (df3) * 109$$

- $df1 = .9434$
- $df2 = .8789$
- $df3 = .8241$

## Zero Coupon Yields

- $\text{zcy1} = \text{zero coupon yield for one year, etc.}$
- $\text{zcy1} = (\text{df1})^{(-1/1)} - 1 = 6\%$
- $\text{zcy2} = (\text{df2})^{(-1/2)} - 1 = 6.39\%$
- $\text{zcy3} = (\text{df3})^{(-1/3)} - 1 = 6.77\%$

# Comparison of Coupon and Zero-Coupon Yield Curves



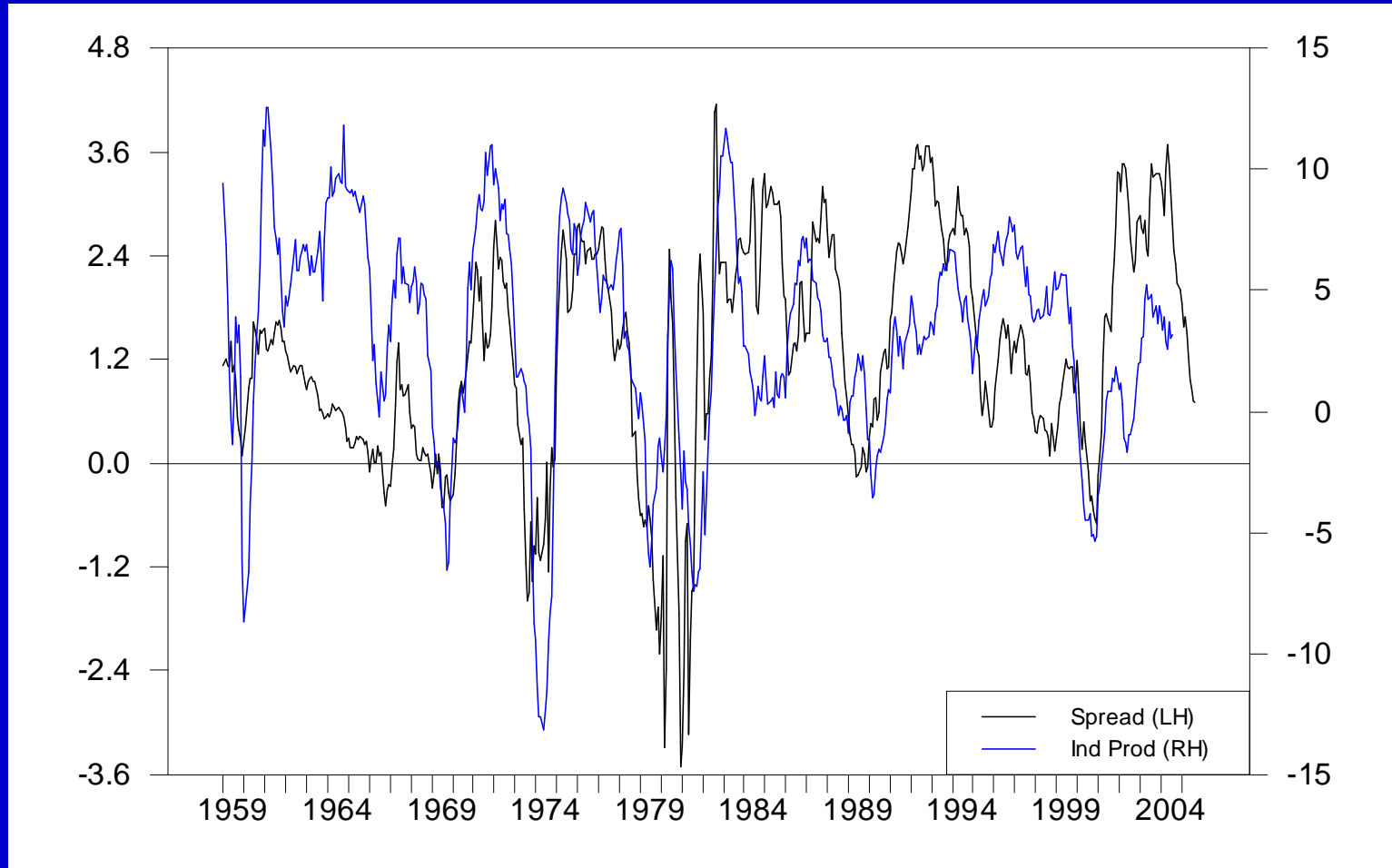
## Changes in the Slope of the Yield Curve: Monetary Tightening

- Short-term rate goes up
- Long-term rate does not go up as much, may even go down
- Expected future real activity slower
- Expected future real interest rates lower
- Expected future inflation lower □

## Changes in the Slope of the Yield Curve: Inflation Expectations

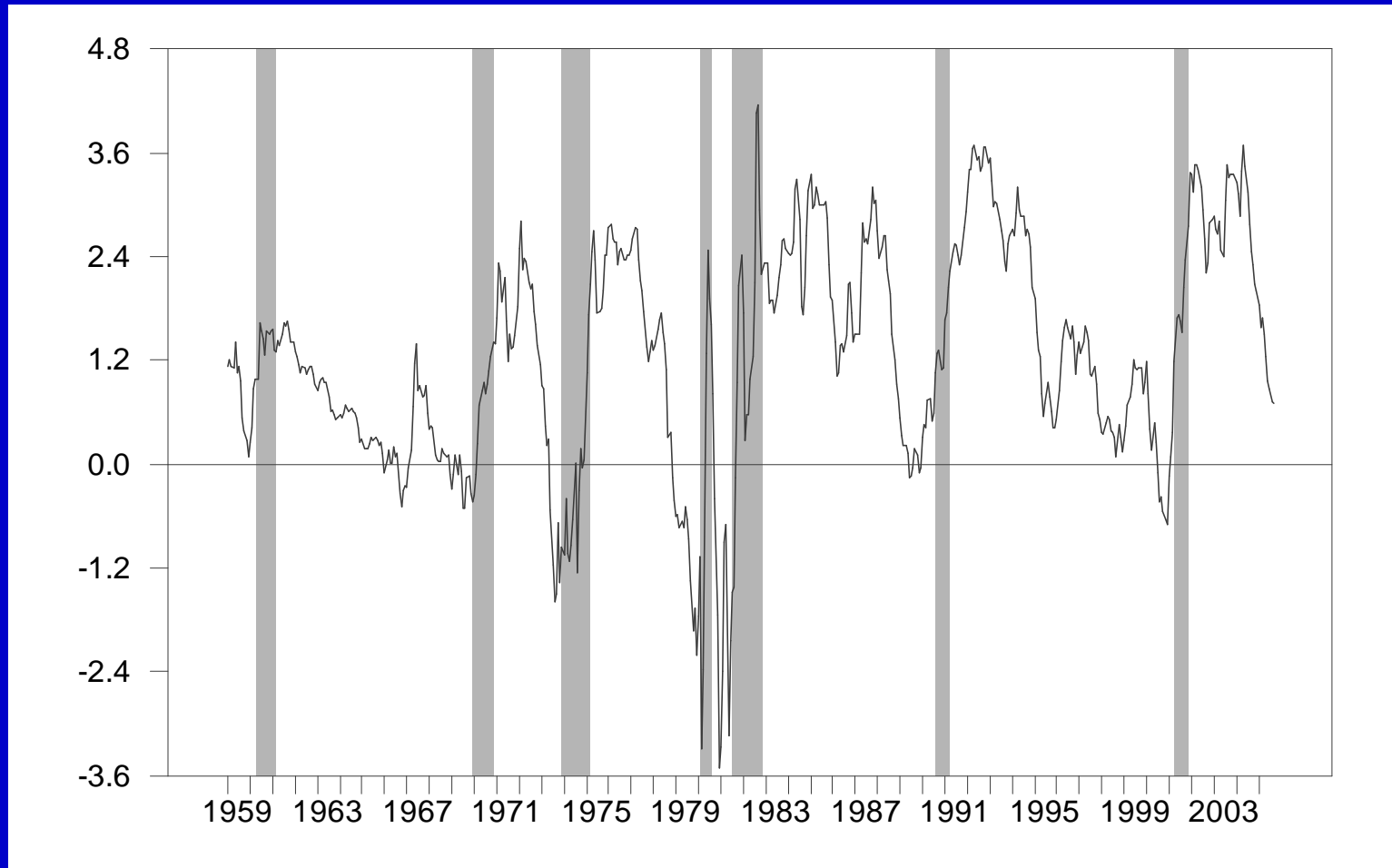
- Rise in expected inflation
- If temporary, only short-term yield goes up and yield curve flattens
- If permanent, yield curve slope is the same, but yield curve is higher
- If accelerating, yield curve slope increases □

## Yield Curve Slope and Future Real Activity



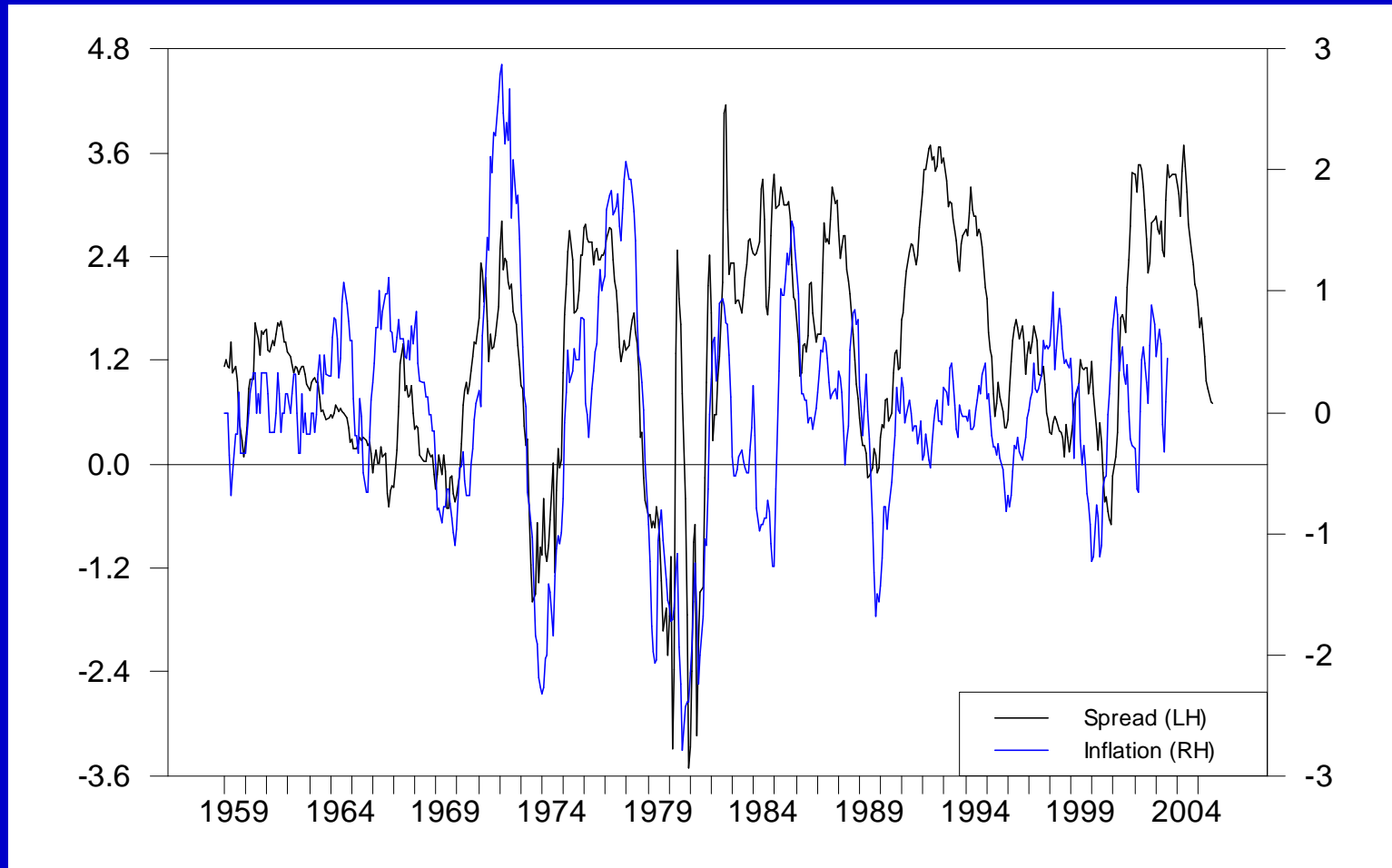
12-month ahead growth in U.S. industrial production and current slope (10-year minus 3-month U.S. Treasury yields)

# Yield Curve Slope and Future Recessions



NBER-dated U.S. recessions 1 year ahead and current slope

# Yield Curve Slope and Future Changes in Inflation



Difference between inflation 12 and 24 months ahead and current slope

# Current Yield Curve October 5, 2005

