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Abstract

This paper provides a model of systemic panic among financial institutions with heterogeneous fragilities. Concerns about potential spillovers from each other generate strategic interaction among institutions, triggering a preemption game in which one tries to exit the market before the others to avoid spillovers. Although financial contagion originates in weaker institutions, systemic risk depends critically on the financial health of stronger institutions in the contagion chain. This analysis suggests that when concerns about spillovers prevail, then 1) increasing heterogeneity of institutions promotes systemic stability and 2) bolstering the strong institutions in the contagion chain, rather than the weak, more effectively enhances systemic stability.

Key words: financial spillovers, panic, financial crises

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1 Introduction

Panics in financial markets are contagious by nature. Financial spillovers, which spread distress among institutions, can arise through a variety of channels.[1] When market participants worry about this domino effect, the fear itself sometimes leads to self-fulfilling panics: If some become concerned and try to exit the market before the mess falls on, others follow and run for the exit in a panic to avoid being left behind (Pedersen (2009)). In such circumstances, coordination problems among market participants, which stem from concerns about spillovers, become critical.

This paper seeks to tackle a new question: How does systemic panic occur when financial institutions are heterogeneous (some are financially stronger than others)? Understanding this mechanism is important since it enables us to address the following policy question: What should we do to contain systemic crises in the future? Our model, by incorporating systemic concerns among heterogeneous institutions, presents answers to these questions with novel implications.

First thoughts suggest that financial distress simply spreads from financially weaker institutions to stronger ones through financial spillovers (as in Diamond and Rajan (2005)). However, the underlying causal relationship is the opposite when strategic interactions from concerns about spillovers are taken into account. Indeed, measures taken by the stronger institutions to protect themselves from spillovers can exacerbate uneasiness among the weaker ones and ultimately destabilize the whole financial system. Here, systemic stability critically depends on the health of the stronger, rather than the weaker institutions.

In our model, strategic considerations about coordination concerns are present not only among the ex-ante identical institutions but also across those with differing financial health

[1] For example, fire-sale externalities (Caballero and Krishnamurthy (2001), Diamond and Rajan (2005), and Brunnermeier and Pedersen (2009)), informational spillovers (Lee (1998), Aghion, Bolton, and Dewatripont (2000), and Kurlat (2010)) and direct/indirect exposures (Rochet and Tirole (1996), Allen and Gale (2000), Freixas, Parigi, and Rochet (2000), and Dasgupta (2004)). See Brunnermeier (2009) for a description of the 2007-09 crisis.

levels.[2] This implies that the stronger institutions are not passive. They do not simply sit and wait, worrying if the financial distress at weaker institutions will spill over to them. Rather, they try to run preemptively for the exit so as to avoid being dragged down. This, however, prompts the weaker ones to act the same, and run even faster. Even though not running is collectively better, such "pre-emption game", in which one tries to exit the market before others, may induce a coordination failure among heterogeneous institutions and undermine systemic stability.

What is remarkable here is that as economic fundamentals deteriorate, a systemic crisis materializes when the *stronger* institutions in the contagion chain lose their confidence and consider dropping out of the chain (exiting the market) to avoid the spillovers. This concern of the stronger eventually prompts the weaker to run preemptively, which self-fulfills the stronger's initial concern and leads them to run as well. Thus, what is critical in the systemic context is not the contagion trigger event itself (distress at the weaker institutions), but the level of the stronger's confidence facing the spillovers following that event. Therefore, systemic risk, or ex-ante likelihood of a systemic crisis, is related to the financial health of the *stronger* institutions which directly affects their levels of confidence.

This argument highlights a striking contrast between our systemic (strategic) approach and the benchmark non-systemic (non-strategic) approach in which coordination problems among institutions are absent. In the benchmark case, the best way to contain contagious distress is to focus on the weakest link in the contagion chain: bolstering the weakest. The following quotation from *Lombard Street* (Bagehot (1873)) represents this view, which is consistent with conventional wisdom:

> ...In wild periods of alarm, one failure makes many, and the best way to prevent

[2] Panic models in the literature have mainly focused on coordination concerns among homogeneous participants (e.g., Diamond and Dybvig (1983), Rochet and Vives (2004), and Goldstein and Pauzner (2005)). Goldstein (2004) studies a coordination problem among investors in different markets, and Corsetti, Dasgupta, Morris, and Shin (2004) consider the problem among small and large traders. Sákovics and Steiner (2012) study policy effectiveness facing a coordination problem among heterogeneous agents with heterogeneous payoffs but homogeneous strength.

the derivative failures is to arrest the primary failure which causes them.

Our analysis suggests that incorporating strategic considerations can reverse this conventional wisdom. Unlike financial distress, which propagates from the weaker to the stronger institutions, the loss of confidence propagates from the stronger to the weaker eventually resulting in a self-fulfilling crisis. Thus, we suggest an alternative recommendation: Bolster the strongest in the contagion chain, not the weakest, to contain systemic panic (destabilizing loss of confidence) and enhance systemic stability effectively.[3] Our approach also yields novel implications for heterogeneity and financial system stability, suggesting that systemic risk is lower in more heterogeneous financial systems when concerns about spillover prevail. This is because coordination problems are less severe in more heterogeneous systems and thus externalities from the coordination failure are weaker. Since systemic soundness critically depends on the health of stronger institutions, it can be enhanced by separating the strong from the weak.

While our mechanism can be applied more generally when coordination concerns exist among heterogeneous agents with differing degrees of exposure to strategic risk (i.e. coordination concerns), this paper specifically considers heterogeneously leveraged institutions holding an illiquid asset subject to a collateral constraint. Our explicit focus is on their optimal market-exit timing facing the following tradeoff:[4]

- Institutions prefer to keep this high-yield but illiquid asset ("stay" in the market) rather than to liquidate immediately ("exit" the market) at a discounted price.
- At the same time, they wish to avoid financial distress (forced liquidation of their assets) that occurs when their collateral constraint is violated.

This implies that the first best strategy is to delay immediate liquidation and exit the market right before the collateral constraint is binding, or funding liquidity evaporation.

[3]Acharya and Yorulmazer (2008) also suggest to bolster the stronger, but in a different context.

[4]The trader's tradeoff in Morris and Shin (2004b) is the closest to that of our model. Investors of Bernardo and Welch (2004, 2012) also face similar tradeoff but fundamental uncertainty exists in their case.

What complicates the choice of this exit timing is coordination problems (strategic risk) among the institutions holding the illiquid assets, whose liquidation value (i.e. collateral value) becomes lower as more of them are liquidated.[5] Thus, collateral value becomes depressed as other institutions exit, which may result in a collateral constraint violation that could have been avoided if the institutions had coordinated. In other words, funding liquidity is provided unless market liquidity is depleted, but the amount of remaining market liquidity depends on collective action of other institutions.

This dependence becomes the source of multiple self-fulfilling equilibria in the manner of Diamond and Dybvig (1983). We adopt the global game technique (Morris and Shin (2003) for the overview, and Toxvaerd (2008) for the exit game setup) to derive a unique equilibrium of our model. In equilibrium, systemic crises are triggered when deteriorating fundamentals cause institutions to run for limited market liquidity. The runs are self-fulfilling because they depress the market value of the assets to the point that collateral constraints are violated and funding liquidity evaporates.

A new source of externality arises in our model that results from the coordination failure among heterogeneously leveraged institutions. This externality precipitates the trigger of systemic panics and increases the systemic risk ex ante, and it becomes stronger if the institutions are more homogeneous. With a continuous time approximation for our dynamic model, we then apply a structural debt pricing framework (see Merton (1974), and Leland (1994)) to calculate credit spreads and systemic risks in a closed form. We explicitly examine the relationship between heterogeneity and credit spread dynamics, and how a microprudential analysis underestimates both credit spreads and true systemic risks by ignoring the externality from concerns about the spillovers in the system. The errors are negligible during normal times, but surge rapidly in a market downturn.

[5] Theoretical models of this price impact are provided by Grossman and Stiglitz (1980), Kyle (1985), Grossman and Miller (1988), Shleifer and Vishny (1992), Allen and Gale (1994), and Brunnermeier and Pedersen (2009).

This paper is related to several strands of literature. We study liquidity crises in financial markets as in Holmström and Tirole (1998), and Allen and Gale (2004) where financial spillovers arise as in Allen and Gale (2000), Diamond and Rajan (2005), and Brunnermeier and Pedersen (2009). Our approach is also related to the literature on coordination problems among market participants. Diamond and Dybvig (1983) provide a classic model of coordination problems that generate self-fulfilling multiple equilibria. Financial panic models with a unique equilibrium are developed using the global game technique by Rochet and Vives (2004), Goldstein and Pauzner (2005), and Morris and Shin (2004b). Goldstein (2004), Corsetti, Dasgupta, Morris, and Shin (2004), and Sákovics and Steiner (2012) extend them to the asymmetric global game setup. Dynamic extensions of a global game with an exit option are considered by Toxvaerd (2008), and Chassang (2010). Without adopting the global game setup, He and Xiong (2011a) study a coordination problem among creditors across different maturity dates. Coordination concerns about the asset liquidation timing in this paper are also studied by Brunnermeier and Pedersen (2005), Carlin, Lobo, and Vishwanathan (2007), and Oehmke (2010) in different contexts. This paper also employs the structural debt pricing approach originally proposed by Merton (1974). Leland (1994), and Leland and Toft (1996) provide solutions for debt valuation with endogenous default thresholds chosen by the equity holder whereas in our case debt contracts can also be terminated when the collateral constraint is violated. Our focus on the effect of coordination failure (thus higher rollover risk, whose effect is also considered in He and Xiong (2011b)) on credit costs is in a similar spirit to Morris and Shin (2001, 2004a). Bruche (2011) also studies this problem by employing a global game with a continuous time approximation, as in this paper.

The paper is organized as follows. Section 2 describes the model setup. Section 3 characterizes the equilibrium of the model and Section 4 discusses the main result in more detail. Section 5 analyzes how coordination failure and heterogeneity affect asset pricing dynamics. Section 6 concludes.

2 Model Setup

We focus on how strategic interactions among heterogeneous institutions affect the systemic risk of a panic run for limited market liquidity (panic run for the exit), while taking their balance sheet structures and financial constraints as given. Consider an infinite horizon economy where time is discrete and advances by increments of $\triangle$, indexed by $0, \triangle, \cdots, t-\triangle, t, t+\triangle, \cdots$. There are two groups of differently leveraged financial institutions (referred to as "institutions" hereafter). In each group, there is a continuum $[0,1]$ of ex-ante identical institutions.

Each institution is endowed with one unit of an asset simultaneously used as collateral for the exogenous initial debt position (financed and purchased at the ex-ante period $t=0$), with the debt principal value P.[6] Since P reflects the scale of the initial debt on the liability side for one unit of asset holding on the asset side, we interpret P as a measure of the initial leverage where higher P implies a higher leverage level. Initial leverage levels are different between the two (H and L) groups.[7] H-group institutions are more highly leveraged with $P=P_H$, than L-group (low-leverage) institutions with $P=P_L$, where $P_H > P_L$.[8]

We focus on the interim market-exit decision of the institutions. At the beginning of each period, the institutions choose either to "stay" in the market (keep the asset to the next period) or to "exit" the market immediately[9] (close their position and pay back the

[6] We can consider that the asset position is financed ex ante partly by some debt using this particular asset as collateral and partly by its own capital, as with repo or ABCP.

[7] Although adopting a dynamic setup, we focus on the short-term (near-crisis) question of what drives systemic panic *given* heterogeneity in the system, without answering the question of ex-ante endogenous optimal leverage studied in Leland (1994), or real-time balance sheet adjustments along the fundamental fluctuations studied in Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Brunnermeier and Pedersen (2009). In the near-crisis situation, it is very costly to issue new equity and institutions mainly consider a deleveraging problem *given* their balance sheet, which is what our model is analyzing. Note that this heterogeneous leverage setup is for expositional simplicity, and we can still get the same results with other forms of heterogenous fragilities (exposures) to financial spillovers. See Appendix C for the generalized setup.

[8] Technically, there exists $\epsilon > 0$ such that $P_H - P_L > \epsilon$.

[9] Partial liquidation can be incorporated within our setup but will not be observed in equilibrium since coordination concerns among the institutions (desire for preemptive run) drive the equilibrium outcome.

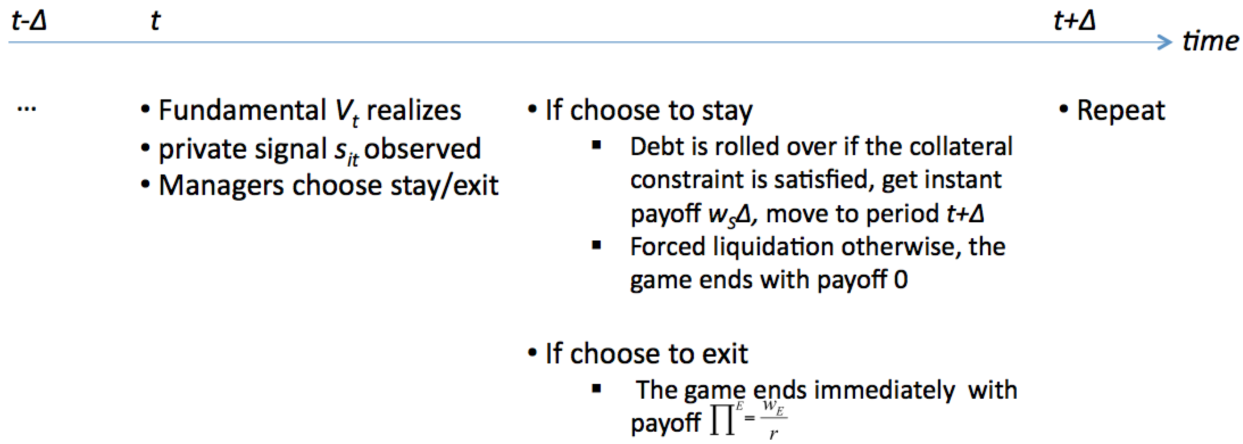

Figure 1: Timeline of the model
The institutions repeatedly choose either to stay in the market or exit the market after observing the private signal each period. When choosing to stay, the debt needs to satisfy a collateral constraint to be rolled over into the next period. The game ends if the institution chooses to exit or its debt rollover is refused.

debt principal). When choosing to keep its leveraged position, that institution has to roll over its debt to move on to the next period, but the debt rollover is allowed only if certain collateral constraint is satisfied as will be discussed in Section 2.2.

2.1 Asset

The unlevered value of one unit of the asset V, which is non-verifiable, follows the stochastic process

$$V_{t+\triangle} = V_t + (r - \delta)V_t\triangle + \sigma V_t z_{t+\triangle}$$

where the innovations $\{z_{t+\triangle}\}$ are i.i.d. $N(0, \triangle)$. Here, r is the risk-free rate and δ is the cash payout ratio.[10] V_t is referred to as the "fundamental" value of the asset at period t.

The assets are illiquid in a sense that their interim liquidation price deviates from the

[10]As $\triangle \to 0$, $\{V_t\}$ converges to a geometric Brownian motion $dV = (r-\delta)V dt + \sigma V dW$, commonly used in the structural debt pricing literature. We assume that probabilities are measured under the risk-neutral measure.

fundamental value as more of them are liquidated in the market. Denote the liquidation price of one unit of the asset at the end of period t as L_t, and let f_t be the amount of the assets that have been liquidated previously. Thus, $f_t(\in [0,2])$ is simply the mass of the institutions that have chosen to exit the asset market up to the beginning of period t. Given the fundamental V_t, the liquidation price L_t at the end of period t follows

$$L_t = V_t - \lambda f_t \tag{1}$$

where λ is the measure of (il)liquidity for this asset.[11] Since the asset is also used as collateral for the debt, we will use the terms "liquidation price" and "collateral value" interchangeably, both indicating L_t. Equation (1) implies that the collateral value becomes lower as more institutions liquidate their asset holdings to exit the market, thereby reducing market liquidity. The decrease is larger when the asset is more illiquid.

2.2 Debt contract and collateral constraints

The institution uses its one unit of the asset holding as collateral for its debt position. Since the fundamental V is non-verifiable, the debt (with the principal $P = P_H, P_L$) is subject to a collateral constraint (Hart and Moore (1998), Kiyotaki and Moore (1997)) that requires the perceived value of the collateral asset (marked to market asset value) L_t to exceed certain threshold proportional to the debt size P. For simplicity, we assume that the debt needs to be fully collateralized. At the end of each period t, a j-group institution ($j = H, L$) can roll

[11] λ reflects market depth in Kyle (1985), market liquidity in Brunnermeier and Pedersen (2009), liquidity premium in Allen and Gale (1994), risk premium in Grossman and Stiglitz (1980), and dislocation cost in Shleifer and Vishny (1992). We use a linear asset pricing curve for simplicity while the price impact term can take more general forms. What's critical for our mechanism is strategic complementarity among the institutions, and strategic complementarity arises as long as the price decrease from asset liquidation is increasing in the number of institutions exiting the market.

over its debt if and only if the collateral value L_t exceeds its debt principal P_j:

$$L_t > P_j \tag{2}$$

If the collateral value is below this threshold, the institution is under "financial distress" facing rollover refusal, which leads to forced liquidation of its asset holding.[12]

The debt pays a constant coupon payment $C_j\triangle$ each period until its termination, which is triggered by either the rollover refusal (puttable option) or the institution's voluntary exit (callable option), paying the principal P_j at that point.[13]

2.3 Market-exit game

We solve for the optimal timing of the institutions' market-exit as the fundamentals deteriorate. At the beginning of each period, each institution (indexed by $i \in [0, 2]$) decides either to stay in the market ($a_{it} = 0$) or to exit immediately ($a_{it} = 1$). We consider the following economic tradeoff microfounded in Appendix B: (i) the institutions prefer to stay in the market rather than to liquidate the asset immediately for a discounted price, but (ii) the institutions at the same time wish to avoid financial distress (following rollover refusal) since voluntary liquidation with early exiting is preferred over forced liquidation. Facing the risk of potential rollover refusal, each institution chooses an optimal action of the two given the state variables. No re-entry is allowed once exiting the market.

Since the collateral value (thus, the debt rollover) depends on collective action of the other institutions (characterized by f_t), the optimal exit decision also depends on what others do. We adopt a dynamic global game setup (as in Toxvaerd (2008)) by introducing a noisy private signal about the fundamental, such that a unique equilibrium can be pinned down.

[12]We assume that the institutions can't raise new capital once in financial distress. We also rule out debt renegotiation. Diamond and Rajan (2001) provides a theoretical model in which a coordination problem among lenders prevents debt renegotiation.

[13]The full-collateral requirement (2) guarantees the full payment of principal P_j. This setup is not critical.

At the beginning of the typical period t, the fundamental V_t is realized but is not common knowledge to the institutions. Instead, a typical institution i receives an idiosyncratic signal s_{it} that follows $s_{it} = V_t + \epsilon_{it}$, where ϵ_{it} is independently uniform over $[-\epsilon, \epsilon]$ with $\epsilon = o(\triangle^{\frac{1}{2}})$.[14] Let $h_t = \{V_s, f_s | s < t\}$ be the past history up to period t, which is common knowledge.

In each period, the institutions make an optimal decision (stay/exit) repeatedly based on these state variables. We suppose a reduced-form setup (as in Rochet and Vives (2004)) in which the optimal decision is delegated to a manager with a simple payoff structure while preserving the tradeoff described previously. This simplifies our analysis to focus explicitly on the strategic interactions without affecting the model's implications.[15]

Managers are risk neutral and discount with the risk-free rate r. At the beginning of period t, a typical manager (indexed by $i \in [0, 2]$, irrespective of the groups they belong to) chooses either to stay in the illiquid market or to exit after observing the private signal s_{it} and the past history h_t. At any interim period, a manager basically prefers keeping the illiquid asset to immediate liquidation; he obtains a high wage of $w_S\triangle$ at the end of each period as long as he keeps the asset and the debt gets rolled over, but only obtains a low wage of $w_E\triangle$ each period (with $0 < w_E < w_S$) once liquidating his position and exiting the market.[16] Therefore, "staying" is strictly better than "exiting" at any given period, as long as it is certain that the debt will be rolled over in that period (funding liquidity is secured).

The downside of staying in the market is that the institution may fall into financial distress at the end of that period if the collateral constraint (2) is violated. The managers of

[14] Using uniform distribution is for simplicity and without loss of generality with $\epsilon \to 0$ as $\triangle \to 0$. If the order of convergence is larger, multiple equilibria exist with too informative public signals (here, past fundamentals). See Angeletos and Werning (2006).

[15] This delegation assumption is for simplicity. We can get essentially the same results by directly analyzing the equity holder's payoff who collects a dividend payment $(\delta V_t - C_j)\triangle$ every period, with an extra penalty for going under forced liquidation which could result from (i) reputational reasons (stigma effect), (ii) lower liquidation price with others' preemptive liquidation (as in Morris and Shin (2004b) and Bernardo and Welch (2005)), or (iii) predatory trading (Brunnermeier and Pedersen (2005)). See Appendix B.

[16] Although we arbitrarily pick w_S and w_E further assuming that these values are identical across the managers belonging to different groups, the specific parametrization of the salary schedule are irrelevant in the limit. Our results require only the minimum assumptions that voluntary preemptive liquidation is preferred over forced liquidation for any institution.

distressed institutions will then be penalized for making the "wrong" decision of remaining in the market, receiving 0 afterwards in that case. This tradeoff between (i) the higher wage for staying over exiting and (ii) the heavy penalty for staying mistakenly when funding liquidity dries up, is the driving force behind the interim exit decision. The managers (institutions) wish to stay in the illiquid market to enjoy high salaries (high yields) while funding liquidity is provided, but do not wish to stay too long so as to avoid the financial mess following funding liquidity evaporation.

When ignoring other institutions, the institutions thus try to delay their market exit until the collateral constraint is surely violated. However, they fail to achieve this outcome since coordination concerns arise among them; one's collateral value (thereby one's debt rollover) depends on collective action of other institutions, thus one may also have to exit when sufficiently many other institutions are exiting. The global game technique enables us to solve for the unique equilibrium exit strategy that takes this strategic uncertainty into account.

We focus on the Markov threshold strategies characterized by respective "(panic) exit thresholds" $s_H^*(h_t)$ and $s_L^*(h_t)$ for the two groups; given the history h_t, the manager i of group $j(=H,L)$ chooses to stay in that period if his signal about the fundamentals is high enough exceeding j-group's exit threshold ($a_{it}(s_{it},h_t)=0$ if $s_{it}>s_j^*(h_t)$), but exit otherwise ($a_{it}(s_{it},h_t)=1$ if $s_{it}\leq s_j^*(h_t)$). In the next section, we define and derive a unique Markovian Perfect Bayesian equilibrium in threshold strategies.

The timeline of the model is summarized in Figure 1. At the beginning of a typical period t, the fundamental V_t is realized and the managers who remain in the market receive their private signals $\{s_{it}\}_{i\in[0,2]}$. Each manager then chooses either to stay or to exit according to their strategy profiles. At the end of period t, the debt rollover is allowed for the institution staying in the market if and only if its collateral constraint (2) is satisfied, which from (1) depends on the aggregate size of the past asset liquidation f_t and the current fundamental

V_t. The wages are then paid according to the salary schedule. If the manager chooses to stay and the debt rollover is allowed, it moves on to the next period $t + \triangle$ and the same game is repeated. The game ends otherwise, either by exiting voluntarily or by a rollover refusal.

3 Bayesian Equilibrium

A single-group global game with a unique equilibrium (Morris and Shin (2003) for the overview) can be easily extended to a multiple-group setup as shown in Frankel, Morris, and Pauzner (2003). For now, we assume that the difference in initial leverages (i.e., debt principal values P_j) between the two groups is not very large while the assets they hold are illiquid, satisfying $P_H - P_L < \lambda$. As will subsequently be discussed, this condition implies a domino effect between the two groups when one of them becomes distressed.

Given the past history h_t, two exit thresholds characterize an equilibrium of the model, $s^*_H(h_t)$ for H-group institutions and $s^*_L(h_t)$ for L-group, with a signal below which a manager loses confidence and chooses to exit the market. A Bayesian equilibrium is defined such that one's strategy in the profile maximizes his conditional expected payoff when all others are following the equilibrium strategy profiles. We take a continuous time approximation of our discrete time model with $\triangle \to 0$ so that (i) closed-form solutions can be derived, and (ii) we can employ the structural debt pricing framework to define and calculate the dynamics of credit spreads. However, this is not a critical condition as discussed in Section 4.6.

3.1 Benchmark case with perfect coordination

Prior to the equilibrium analysis with strategic interactions, consider the first-best benchmark case in which the institutions can perfectly coordinate. As depicted in Figure 2, we verify that all (both H and L group) institutions delay their market exit until the fundamental V eventually hits P_H.

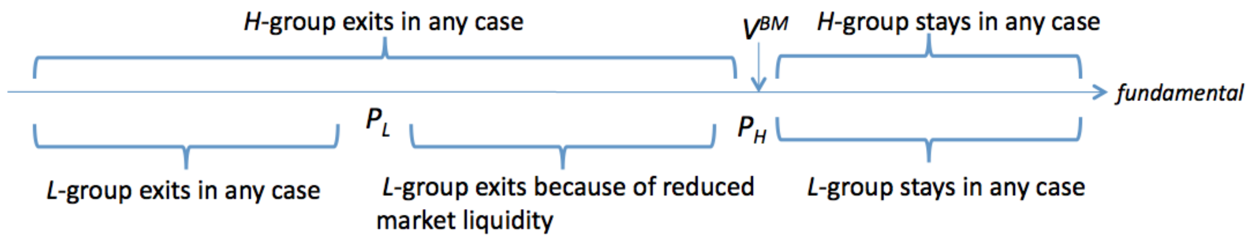

Figure 2: Crisis threshold V^{BM} in the benchmark case
When the institutions can perfectly coordinate, all (both H and L group) institutions exit the market when the fundamental V hits $V^{BM} = P_H$ from above. Here, the crisis is simply triggered when H-group institutions are distressed.

In the benchmark case with perfect coordination, the institutions delay their liquidation until their collateral constraints (2) are surely violated. H-group institutions thus exit the market when the fundamental V hits P_H, at which point depressing the collateral value by $\lambda \times 1$ to $P_H - \lambda$, through mass 1 of H-group's liquidation. This generates financial spillovers (fire-sale externality) to L-group, since L-group's collateral constraint will then be violated with reduced market liquidity; the collateral value L_t is now lower than L-group's debt principal value P_L under our assumption of $P_H - P_L < \lambda$. In anticipation of this domino effect (H-group's liquidation drags down L-group into financial distress contagiously), L-group institutions also choose to exit immediately at this point, setting their exit thresholds at P_H. Therefore, all institutions in both groups choose to exit simultaneously when the fundamentals V eventually deteriorate to $V^{BM} = P_H$, where V^{BM} is referred to as the crisis threshold of the fundamental under the benchmark setup. Here, a "crisis" refers to a systemic event in which all institutions in the system (contagion chain) choose to liquidate their asset simultaneously to exit the market, and systemic risk refers to the risk of this systemic event. Note that the crisis simply happens when *weaker* H-group get distressed at $V = P_H$ in this case, and the crisis threshold (thus the systemic risk) depends on H-group's financial health (leverage level) P_H.

3.2 Equilibrium with coordination concerns

When strategic considerations about coordination concerns (concerns about the spillover) are incorporated, the institutions don't wait passively until others exit, but instead, worry about what others will do. The fear itself can generate a self-fulfilling panic in this case, which increases the systemic risk ex ante. Our contribution is to demonstrate novel implications on systemic stability arising from strategic interactions among heterogeneous institutions. A "pre-emption game" starts between the two groups, and on the margin, it is in effect *stronger* L-group that is critical in initiating the systemic crisis as opposed to the benchmark case.

We begin the equilibrium analysis by defining the critical level of liquidation pressure for each group given V_t, denoted as $f_H^*(V_t)$ and $f_L^*(V_t)$. Let the critical pressure $f_j^*(V_t)$ for group j be such that $V_t - \lambda f_j^*(V_t) = P_j$, then we get

$$f_j^*(V_t) = \frac{V_t - P_j}{\lambda}. \tag{3}$$

This condition along with (1) and (2) implies that given the fundamental V_t, the debt rollover for the remaining j-group institutions is allowed at that period if the total mass of exited institutions (liquidation pressure) turns out to be lower than this threshold ($f_t < f_j^*(V_t)$) but is refused otherwise ($f_t \geq f_j^*(V_t)$). This characterizes the source of coordination concerns among the institutions; the rollover of one's debt depends on collective action of others which may deplete limited market liquidity for the asset. Note that $f_H^*(V_t) < f_L^*(V_t)$ with $P_H > P_L$, implying highly leveraged institutions are more vulnerable to liquidation pressure than less leveraged institutions. Thus, strategic risk (concerns about the spillovers) is more critical for H-group institutions. H-group institutions can become distressed even when L-group institutions are not, yet the opposite cannot happen. H-group is thus financially "weaker" and L-group is "stronger".

We derive a unique equilibrium using the global game technique. As shown in Toxvaerd (2008), this dynamic global game with an exit option can be solved as a sequence of one-shot games, with appropriately defined value functions. We take the continuous time approximation ($\triangle \to 0$) and derive the closed-form solution of our exit game.

We now define the value functions for the respective actions (stay or exit) as of the beginning of period t given the signal s_{it} and the history h_t. When choosing to exit, the manager receives $w_E \triangle$ constantly afterwards and under the continuous time approximation, the value function Π^E is simply defined by

$$\Pi^E = \int_0^\infty e^{-rt} w_E dt = \frac{w_E}{r} \tag{4}$$

for both groups, independent of the private signal. This represents the option value of an immediate exit, or the outside option value for the manager.

When choosing to stay, the value function Π_j^S of j-group given information about the state variables can be defined as

$$\begin{aligned}\Pi_j^S(s_{it}, h_t) = E\Big[\Big(w_S \triangle + e^{-r\triangle} \max\{\Pi_j^S(s_{it+\triangle}, h_{t+\triangle}), \Pi^E\}\Big) \times \mathbf{1}_{[f_t < f_j^*(V_t)]} \\ + 0 \times \mathbf{1}_{[f_t \geq f_j^*(V_t)]}\Big| s_{it}, h_t\Big].\end{aligned} \tag{5}$$

The righthand side of (5) consists of two parts. If the debt contract is rolled over in that period (with $f_t < f_j^*(V_t)$), the manager receives an instant high wage of $w_S \triangle$ and in the next period (at $t + \triangle$) again gets to choose either to stay or to exit, captured by the continuation value $e^{-r\triangle} \max\{\Pi_j^S(s_{it+\triangle}, h_{t+\triangle}), \Pi^E\}$. If the debt is not rolled over (with $f_t \geq f_j^*(V_t)$), he gets fired (getting 0) and the game ends immediately.

Decomposing the total mass of the exited institutions up to period t as $f_t = f_{H,t} + f_{L,t}$, where $f_{j,t} \in [0, 1]$ is the mass of the exited institutions in group j, it is straightforward that strategic complementarities exist not only within one group but also across the different groups (Π_j^S is decreasing both in $f_{j,t}$ and in $f_{-j,t}$); one has to care not only about its own group institutions' run for limited market liquidity but also about the other group's run. Note that $\Pi_j^S(s_{it}, h_t)$ is increasing in s_{it}—"staying" is more attractive with higher signals about the fundamentals since rollover becomes more likely—which enables us to pin down the indifference thresholds $s_j^*(h_t)$ on which switching of the actions occurs ($\Pi_j^S(s_{it}, h_t)$ is greater (less) than Π^E with s_{it} right (left) to that threshold).

As shown in the appendix (Lemma A1), we can interpret our dynamic exit game as a sequence of the identical history-independent one-shot games with $\triangle \to 0$. $s_j^*(h_t)$ and $\Pi_j^S(s_{it}, h_t)$ can thus be denoted as s_j^* and $\Pi_j^S(s_{it})$ which are history independent, where

$$\Pi_j^S(s_{it}) = E\Big[\Big(w_S\triangle + e^{-r\triangle}\max\{\Pi_j^S(s_{it+\triangle}), \Pi^E\}\Big) \times \mathbf{1}_{[f_t < f_j^*(V_t)]} \\ + 0 \times \mathbf{1}_{[f_t \geq f_j^*(V_t)]}\Big| s_{it}\Big]. \quad (6)$$

We take three steps in solving for the equilibrium threshold (s_H^*, s_L^*). We first derive the optimal exit threshold of each group ignoring the other group, which becomes the lower bound of the equilibrium threshold. We next derive a best response of one group given the other group's exit threshold. We then derive the equilibrium thresholds incorporating full strategic interactions, which are best responses of one another.

3.2.1 Optimal exit threshold ignoring the other group

As a first step, we focus on the coordination problem within one group ignoring the existence of the other group. Let $\underline{s}_j^*$ be the optimal exit threshold of j-group when $-j$-group does not exit. Thus $f_{-j,t} = 0$ in this case, and given the signal s_{it}, the option value of staying can be defined by

$$\Pi_j^S(s_{it}) = E\Big[\Big(w_S\triangle + e^{-r\triangle}\max\{\Pi_j^S(s_{it+\triangle}), \Pi^E\}\Big) \times \mathbf{1}_{[f_{j,t} < f_j^*(V_t)]} \\ + 0 \times \mathbf{1}_{[f_{j,t} \geq f_j^*(V_t)]}\Big| s_{it}\Big]. \quad (7)$$

in which only within j-group strategic uncertainty is taken into consideration. The switching threshold $\underline{s}_j^*$ can then be derived from the indifference condition $\Pi_j^S(\underline{s}_j^*) = \Pi^E$.

Lemma 1. (Exit threshold ignoring the other group)

When $-j$-group is ignored, a j-group institution chooses to exit if and only if its signal about the fundamentals is below

$$\underline{s}_j^* = P_j + \lambda + \epsilon. \tag{8}$$

Before adding the between-group strategic interaction, we also derive the upper bound of s_j^*. Let $\overline{s}_j^*$ be the optimal exit threshold of j-group when they take $f_{-j,t} = 1$ as given (all the other group institutions are exiting). Following the derivation of Lemma 1, it is easy to verify that $\overline{s}_j^* = P_j + 2\lambda + \epsilon$. It is obvious that the equilibrium exit threshold (with full strategic interactions) s_j^* should be bounded by these two extreme thresholds, such that

$$\underline{s}_j^* \leq s_j^* \leq \overline{s}_j^* \tag{9}$$

holds for both $j = H, L$.

3.2.2 Equilibrium exit thresholds with full strategic interactions

We now derive the equilibrium thresholds (s_H^*, s_L^*) when institutions of different groups are acting strategically, anticipating the effect of one's action on the others and vice versa. Since strategic complementarities exist both within one group and between different groups, one's incentive to exit increases not only in the number of exiting institutions in its own group, but also in the number of those in the other group. These additional coordination concerns generate a novel externality through a spiral of growing concerns between the two groups; the concern about the other group's panic makes my group more concerned, which in turn makes the other group's concern grow, generating a feedback loop. The spiral is described as a "pre-emption game" between the two groups in which one group raises its exit threshold in response to the other's raise in a vicious cycle, so as to run for the exit faster than the other. In the end, the spiral only stops when stronger L-group drops out, and the equilibrium exit thresholds get pushed up to L-group's upper bound $\overline{s}_L^*$ via this new source of the externality (Figure 3).

The mechanism of this pre-emption game can be best described using the following best response functions, the optimal exit threshold of one group given the other group's exit threshold. Let $s_H^{BR}(s_L^*)$ refer to H-group institution's best response given s_L^*, and define $s_L^{BR}(s_H^*)$ analogously. Note that in equilibrium these two optimal thresholds have to be the best responses of one another, that the system of equations $s_H^* = s_H^{BR}(s_L^*)$ and $s_L^* = s_L^{BR}(s_H^*)$ have to hold. We first derive the following lemma shown in the appendix.

Lemma 2. (Pre-emption game between the two groups)

- *For L-group, if $s_H^* < \overline{s}_L^*$, then $s_L^{BR}(s_H^*) > s_H^*$.*
- *For H-group, if $s_L^* < \overline{s}_H^*$, then $s_H^{BR}(s_L^*) > s_L^*$.*

Lemma 2 characterize the process of pre-emption game between the two groups (iterative elimination of dominated strategies). It would be mutually beneficial for all institutions to delay their exit and keep their exit thresholds as low as possible, but coordination failure prevents them from achieving this. The institutions in one group have an incentive to avoid the spillovers from the other group by acting preemptively, and try to raise their exit threshold slightly higher than that of the other group so that they can run for the exit faster before market liquidity evaporates.

Combining Lemma 2 and (9), we get $s_L^* = \overline{s}_L^*$ (Figure 3); the pre-emption game continues until L-group institutions drop out of the spiral at their upper bound $\overline{s}_L^*$, beyond which they are confident enough to stay in the market with strong enough fundamentals irrespective

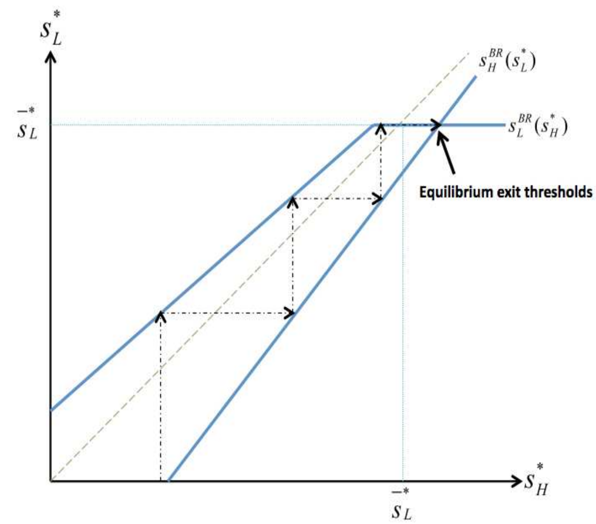

Figure 3: Pre-emption game between the two groups
One group raises its exit threshold in response to the other group's raise to run faster (iterative elimination of dominated strategies). The spiral only stops when L-group drops out at $\overline{s}_L^*$ beyond which the fundamental is high enough such that the stronger institutions don't get panicked even if all of the weaker institutions are exiting. The difference between the two groups' equilibrium exit thresholds becomes negligible in the limit case and both exit at the same threshold $\overline{s}_L^*$.

of what H-group institutions do. We can subsequently derive $s_H^* = s_H^{BR}(s_L^*)$ from their indifference condition given $s_L^* = \overline{s}_L^*$, which can be shown to be converging to s_L^* as the noise becomes small. Intuitively, H-group have no reason to exit "too early" when they know the timing of L-group's exit. They try to exit "right before" L-group do, and the difference becomes negligible in the limit.

In equilibrium, panic runs of H-group simultaneously lead to contagious runs of L-group. Consequently, the institutions all exit together when the fundamental V eventually hits a "crisis threshold" $V^* = P_L + 2\lambda$, while all stay in when V is higher than V^*, as summarized in the following Proposition.

Proposition 1. (Systemic panic run)

Systemic panic run for market liquidity is triggered when the fundamental V hits the

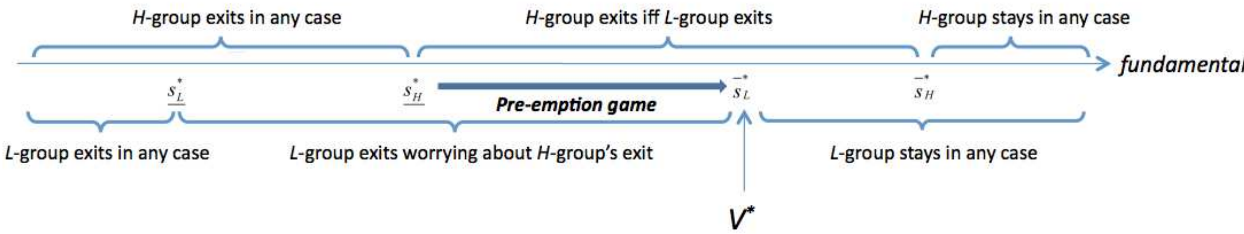

Figure 4: Crisis threshold V^* with concerns about spillovers
H-group exits at $\underline{s}_H^*$ and L-group at $\underline{s}_L^*$ if between-group coordination failure is absent. The crisis threshold gets pushed up through the pre-emption game to $\overline{s}_L^*$ and systemic panic is triggered (both H and L group institutions exit all together) when the fundamental hits $V^* = P_L + 2\lambda$.

crisis threshold $V^* = P_L + 2\lambda$ from above, at which point all institutions exit the asset market simultaneously.

Two points should be remarked on. First, ex-ante systemic risk increases with the coordination failure. The crisis threshold of the fundamental V^* is higher than that under the benchmark case $V^{BM} = P_H$—concerns about the spillovers lead to a self-fulfilling crisis that would not take place if coordination failure were absent. This implies a discrepancy between systemic risk (from macroprudential perspective) and individual institution-wise risk (from microprudential perspective) that will further be analyzed in Section 5.

Second, more importantly, a novel implication on systemic stability arises; the crisis materialization (thus the systemic risk) depends critically on *stronger* L-group on the margin. Notice that the pre-emption game stops eventually (equivalently, systemic panic materializes) at L-group's upper bound $\overline{s}_L^*$ that is independent of P_H as highlighted in Figure 3. Contrast this with the crisis threshold under the benchmark approach $V^{BM} = P_H$. When coordination concerns are absent (passive domino effect), what is critical in initiating the crisis is the materialization of the triggering event—H-group's distress—itself which directly depends on the financial health (debt level) of the weaker group P_H. As the fundamental deteriorates, H-group eventually liquidates when V hits P_H, which consequently prompts the contagious liquidation of L-group through the spillovers as described in Section 3.1.

When strategic interactions are involved (systemic panic), the crisis initiates in a self-fulfilling way from the strategic concerns about the spillovers at $V^* = P_L + 2\lambda$. What essentially causes the crisis on the margin is not the triggering event itself, but the loss of confidence among the institutions about staying in the market. Here, an asymmetry between weaker H-group and stronger L-group exists in terms of whose loss of confidence matters more (Figure 4).

Weaker H-group institutions act as a second mover when choosing their optimal exit timing. What is critical for them is to conjecture when stronger L-group lose confidence and consider exiting, such that they can exit right before that happens. H-group's confidence level is thus subject to L-group's confidence level. Their own financial health (characterized by the debt level P_H) is of secondary importance on the margin, since H-group have to exit any way if L-group exit, to avoid spillovers.

Stronger L-group institutions, on the other hand, anticipate that weaker H-group will always try to exit preemptively, generating financial spillovers to them in equilibrium. Thus, when choosing their own exit threshold (i.e., when to exit the market), L-group take the spillovers from H-group *as given* following the conservative presumption.[17] Hence, on the margin, they only care about whether other L-group institutions will endure the anticipated spillovers from H-group without panicking, which depends critically on L-group's financial health P_L. This L-group's optimal choice is independent of the H-group's health P_H on the margin, since spillovers from H-group will arise to L-group in any case and the scale of the spillovers is irrelevant with the H-group's health. It is when this stronger L-group lose confidence at $\overline{s}_L^*$ that eventually prompts the preemptive run of H-group, which self-fulfills the concern of L-group and leads them to join the run contagiously. Note that in this region,

[17]Strictly speaking, an L-group institution anticipates that all H-group institutions will exit faster than herself if she turns out to be the one with the smallest signal within L-group. See Appendix C for detailed discussion.

H-group institutions have no reason to panic unless L-group do since the fundamental is higher than $\underline{s}_H^*$ as in Figure 4.

In sum, the crisis materialization on the margin depends critically not on the weaker but on the stronger institutions in the contagion chain, and the crisis threshold $V^* = P_L + 2\lambda$ depends on P_L but not on P_H.

4 Discussions

4.1 Heterogeneity and systemic stability

We first discuss the relationship between financial institution heterogeneity and financial system stability. Concretely, we consider whether the system in which institutions are different (in terms of degrees of their individual financial health) is more sound, by comparing different systems based on their crisis thresholds of the fundamental V^* where lower V^* implies higher systemic stability (or lower systemic risks). The answers are quite the contrary depending on whether the strategic interactions are taken into account. Financial systems with more homogeneous institutions are more sound when concerns about spillovers are absent, but the opposite is true when concern about the panic prevails—systemic stability can be enhanced by making the system more heterogeneous.

To illustrate the mechanism, we follow the setup of Section 2 with two groups (H and L) and let the respective endowed debt levels be $P_L = P - u$ and $P_H = P + u$. The "heterogeneity parameter" u is positive and not too large ($u < \frac{\lambda}{2}$) such that a domino effect is anticipated as before. Here, higher u implies a more heterogeneous system, while fixing the average degrees of financial strength for the entire system (total debt outstanding in our case, $\frac{P_H+P_L}{2} = P$ for all u).[18] We now compare the crisis thresholds of different systems

[18] It can also be interpreted as controlling the aggregate bank capital in the system since the aggregate asset size is also fixed (each institution holds one unit of the asset). We can also consider it as separation of good banks and bad banks.

$V^*(u)$ as the dispersion u is varied.

Corollary 1. (Financial institution heterogeneity and systemic stability)

When the average degrees of financial strength are fixed across the systems,

- *Heterogeneous system is more robust when concerns about spillover prevail.*
- *Homogeneous system is more robust when concerns about spillover are absent.*

Recall that in the benchmark case without coordination concerns (Section 3.1), the crisis threshold of the fundamentals is $V^*(u) = V^{BM} = P_H \equiv P + u$ which is increasing in u. Here, the systemic risk can be reduced by making the system more homogeneous with smaller u since it directly suppresses the liquidation triggering event (distress at the weaker institutions) by making the weaker less fragile. This observation is reversed in our approach with the strategic interaction. From Proposition 1, the crisis threshold is now $V^*(u) = P_L + 2\lambda \equiv P - u + 2\lambda$ which is decreasing in u. Here, increasing heterogeneity reduces the systemic risk since it further bolsters stronger L-group institutions to induce them to stay in when exposed to the spillovers, such that a self-fulfilling panic can be contained.

4.2 Recapitalization

We now consider recapitalization and analyze the best way of allocating a fixed amount of capital to enhance systemic stability. Our result suggests that incorporating the strategic considerations, recapitalizing the weaker institutions in the contagion chain may not be as effective in reducing systemic risks as bolstering the stronger.

We consider a capital injection lowering the debt level of an institution (debt/equity swap). Note that a higher level of capital is equivalent to a lower level of leverage, which is captured by lower P in our setup. Denote k_j $(j = H, L)$ as the amount of capital injection into j-group, and let $k = k_H + k_L$ be the aggregate capital injection into the system. Now,

$P'_j = P_j - k_j$ is the new level of j-group's debt after recapitalization and we consider very small k so that we can focus on the marginal effect.[19] Denote $V^*(k)$ as the crisis threshold of this system after capital injection totaling k, and we consider a policy maker who wishes to contain systemic crises by minimizing $V^*(k)$ subject to $P'_H = P_H - k_H, P'_L = P_L - k_L$, and $k_H + k_L = k$.[20] For simplicity, we assume that the policy maker only focuses on the short-term objective of lowering the crisis threshold (ex-ante systemic risk) subject to the resource constraint, ignoring other aspects such as moral hazard problems.

Proposition 1 suggests that systemic risks cannot be reduced on the margin if capital is injected into weaker H-group institutions (lowering P_H)—the panic run still arises at the same crisis threshold when coordination concerns exist ($\frac{\partial V^*}{\partial P_H} = 0$). The system becomes more sound if capital is injected to stronger L-group institutions instead (lower P_L implies lower V^*), since it makes them more confident and induces them to stay in (preventing from dropping out of the chain) such that self-fulfilling panics can be contained. Note that the opposite is true if the coordination problem is absent since V^{BM} depends only on P_H at the margin.

Corollary 2. (Capital injections and systemic stability)

- *When concerns about the spillover prevail, stronger L-group institutions should be recapitalized first in order to enhance systemic stability effectively.*
- *If there's no coordination concern, recapitalizing weaker H-group institutions first enhances stability more effectively.*

[19] As an alternative measure, the government can prevent systemic panic through asset-purchasing program or lending facility instead of recapitalization. However, unless the government has enough liquidity to implement these interventions swiftly, there exists a credibility/commitment problem and systemic panic cannot be contained. We consider a case in which the government only has small amounts of liquidity, thus consider the case with small k.

[20] Although not explicitly modeled, we implicitly assume that there's a welfare loss when the assets are liquidated in the secondary market (i.e., dislocation cost), thus lower $V^*(= V^*(k))$ is more desirable.

4.3 More groups

So far, we have focused on a simple setup ("main model") in order to focus on the mechanism of the pre-emption game. We now explore the predictions of our model under more general setups. We will argue that our main result of the crisis threshold's "(stronger) L-group dependency" prevails with more groups, and is not an artifact of the extreme form of preemption coming from the very short time intervals. However, sizes of institutions do make differences—our L-group dependency result changes if the mass of L-group is very small, or if there's a very large institution in H-group.[21]

In the previous sections, we analyzed strategic interactions between two groups for simplicity. Our result, however, is without loss of generality and can be extended to setups with more groups, keeping the same pre-emption mechanism—the pre-emption game (iterative elimination of dominated strategies as in Lemma 2) lasts until the strongest group in the contagion chain drops out, and the crisis threshold depends on financial health of the strongest group but not others.

Suppose that there exist another group of (a continuum $[0,1]$) institutions, M-group, with $P = P_M$ which satisfies $P_L < P_M < P_H$ while all the other setups are the same as in Section 2. We now have more number of institutions, thus $0 \leq f_t \leq 3$ and $\overline{s}_j^* = P_j + 3\lambda + \epsilon$. We can show the following result by following exactly the same steps as in Section 3.

Proposition 2. (Crisis threshold with three groups)

> *When $P_L < P_M < P_H$, systemic panic run is triggered when V hits the crisis threshold $V^* = P_L + 3\lambda$ from above. This threshold depends on the strongest L-group's financial health, but is independent of the other weaker groups' financial health.*

Following the same argument, an extension to n-group setup with $P_1 < P_2 < \ldots < P_n$

[21] In other words, "too big to fail" problem can still arise even when concerns about the spillovers are incorporated.

and each group with a unit measure is straightforward when $P_j - P_{j-1} < 1$ for $j = 2, 3, \ldots n$. We next relax this assumption and consider cases in which different groups have different sizes.

4.4 Generalized group sizes

We convert to our main model with two groups. Instead of assuming a unit measure of institutions for each group, we now consider a case in which the two groups have different sizes such that there is a continuum $[0, \omega_H]$ of institutions in H-group and $[0, \omega_L]$ in L-group. All the other setups are the same as those of Section 2. Depending on the value of ω_L, we now have the following result.

Proposition 3. (Crisis thresholds with generalized group sizes)

- *If $\omega_L \lambda > P_H - P_L$, we get essentially the same result as Proposition 1. Systemic panic run is triggered when V hits $V^* = P_L + (\omega_H + \omega_L)\lambda$ which depends on financial health of L-group but not H-group.*

- *If $\omega_L \lambda < P_H - P_L$, the exit thresholds for the two groups do not converge to the same limit. H-group institutions exit when V hits $P_H + \omega_H \lambda$ and L-group institutions exit later when V eventually hits $P_L + (\omega_H + \omega_L)\lambda$.*

When L-group is large enough compared to the difference in financial strengths, the pre-emption mechanism is the same as in Section 3—there arise strategic interactions between the two groups, and the pre-emption game stops only when the stronger group drops out at its upper bound $\overline{s}_L^*$. When L-group is small, however, this between-group interaction disappears. Notice that $\overline{s}_L^* = P_L + (\omega_H + \omega_L)\lambda + \epsilon$ is now smaller than $\underline{s}_H^* = P_H + \omega_H \lambda + \epsilon$. Thus H-group institutions care only about their own group institutions, and spillovers from H-group's panic exit (and asset liquidations) are not strong enough to drag down L-group simultaneously. The stronger L-group should not be too small in order to have our previous result of L-group dependency.

4.5 Effect of a large institution

We again convert to our main model of two groups with a unit measure each, and $P_H - P_L < \lambda$. We now consider a case in which a large institution with positive mass co-exists with a continuum of atomistic institutions.[22] An interesting case is when the weaker H-group consists of a single large institution with mass 1 while L-group consists of a continuum [0,1] of institutions. Although this is the only change from the setup of Section 2, we get a completely different result from Proposition 1; with a large weaker institution, systemic stability depends on financial health of the weaker, not the stronger.

Proposition 4. (Crisis thresholds with a large weaker institution)

> *When H-group consists of one large institution with a unit mass while L-group institutions are atomistic, systemic panic run is triggered when V hits $V^* = P_H + \lambda$. The crisis threshold depends on financial health of H-group, but not on L-group.*

Considering the same pre-emption mechanism, now it's H-group that drops out first; there is no within-group spillovers for H-group, but L-group institutions take both within and between-group spillovers into consideration. With less concern about spillovers for H-group and our assumption of $P_H - P_L < \lambda$, we have $\overline{s}^*_H < \overline{s}^*_L$. Again, the sizes of institutions have implications on systemic stability even with concerns about spillovers.

[22] Corsetti, Dasgupta, Morris, and Shin (2004) study a global game model where a single large agent and a continuum of small agents coexist, but within a single group.

4.6 Role of strategic risk aversion

In our dynamic setup with continuous time approximation, aversion to strategic risk (concern about coordination failure from spillovers) becomes very high inducing the extreme form of preemption—an institution tries to exit the market immediately with a slight chance of coordination failure. However, our result is not an artifact of this extreme preemption. In Appendix C, we define strategic risk aversion within a simplified but generalized setup, and show that our main result (stronger L-group dependency) still holds as long as this strategic risk aversion is high enough compared to the difference in fragilities between the two groups. As discussed there, strategic risk aversion (denoted by α) can be defined as the ratio of the payoff of exiting (Π^E in our case) to the payoff of successful roll over when choosing staying ($w_S\triangle + e^{-r\triangle}E[\max\{\Pi_j^S(s_{it+\triangle}), \Pi^E\}|s_{it}]$) evaluated at the switching threshold s_j^*, where $\alpha \in [0, 1]$.[23] Note that this ratio becomes larger and converges to 1 as $\triangle$ becomes smaller. Thus, our limit case is that of $\alpha \to 1$ within the setup of Appendix C, which implies extremely high strategic risk aversion and corresponding extreme form of preemption.

As shown in Proposition 5 of Appendix C, there are 3 types of equilibria depending on the value of α, and our result (Case (i) of Proposition 5) still holds as long as α is large enough. It is reasonable to assume large α when we consider an exit game during financial crises, in which the penalty for coordination failure outcomes are usually high. However, α could be small for some other coordination games. For instance, we could think of a participation game with low participation cost, but high payoffs for the successfully coordinated outcome.

Note that the dynamic setup provides an additional rationale for our exclusive focus on Case (i) with large α among the three cases of Proposition 5. Even if agents have somehow coordinated successfully with others and survived in the current period, their high payoff from the successful survival lasts only for a short period and they could be in trouble again in the next period. As a result, α here becomes larger endogenously with shorter time

[23] To be general, α is the ratio of (payoff of exiting – payoff of roll over failure) to (payoff of successful roll over – payoff of roll over failure).

interval $\triangle$, compared to a one shot game in which the high payoff lasts forever once agents successfully coordinate with others.

5 Credit Spreads, Rollover Risks, and Coordination Failure

In this section, we explicitly analyze how heterogeneity and coordination failure within the system affect the asset pricing dynamics using the structural debt pricing approach. We specifically focus on a single institution (belonging to H-group) and examine its funding costs (credit spreads) and funding risks (rollover risks) as the heterogeneity in the system varies. We start from a very heterogeneous system with no between-group coordination problem, then make the system less heterogeneous such that between-group interactions arise.

Very heterogeneous system We follow the setup of Section 2, but first assume high enough between-group heterogeneity, $P_H - P_L > \lambda$. As shown in Proposition 3, this condition rules out strategic interactions between the groups, such that only within-group coordination problems exist. Financial health of L-group has no effect on H-group's risk in this case.

Note that panics are self-fulfilling; concerns about potential rollover refusal trigger panic run for limited market liquidity when V hits the crisis threshold, followed by the actual violation of the collateral constraint (2) at that point. Denote V^{**} as H-group's crisis threshold of the fundamental in this heterogeneous system, V^{**} can thus alternatively be interpreted as H-group's rollover threshold of the debt on which the debt rollover is anticipated to be refused. Proposition 3 implies that the debt rollover is allowed for H-group as long as V is beyond the rollover threshold $V^{**} = P_H + \lambda$, but is refused as soon as V hits V^{**}.

When all institutions can perfectly coordinate (or an individual institution is analyzed in isolation from microprudential perspective), they delay their market-exit until the fundamental hits $V^{BM} = P_H$, right before the collateral constraint (2) binds. Externality from

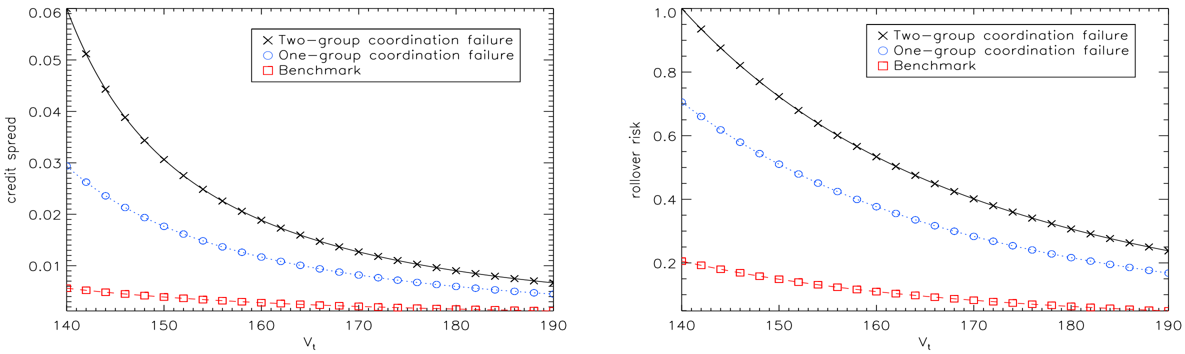

Figure 5: Credit spreads and rollover risks of an H-group institution with different degrees of coordination failure
Both credit spreads and rollover risks increase as the heterogeneity decreases and the coordination problems become more severe. The difference between credit spreads is "coordination failure premium" and that between rollover risks is "additional systemic risk". The parameters in this example are $r = 0.04$, $\sigma = 0.1$, $\delta = 0.02$, $P_H = 100$, $P_L = 60$, $C_H = 10$, $\lambda = 30$. In the two-group coordination failure case of Section 5.4, the difference in two group's initial leverage levels is smaller with $P_L = 80$.

coordination concerns thus precipitates the market exit and funding liquidity evaporation, implying a discrepancy between macroprudential systemic risk and microprudential risk.

Different asset pricing dynamics thus result from different levels of the rollover thresholds with or without coordination concerns ($V^R = V^{**}$ or V^{BM}). In Appendix D, we define and calculate credit spreads and rollover risks. Since higher V^R implies lower debt value, thereby higher credit spreads and rollover risks, coordination failure results in additional spreads along with additional risks in the systemic context compared to microprudential perspective. Figure 5 compares how credit spreads and rollover risks under our systemic (one-group coordination failure) approach vary differently from those under the benchmark microprudential approach. The difference in credit spreads can be interpreted as the "coordination failure premium", and that in rollover risks as "additional systemic risk". Both credit spreads and funding risks are underestimated in the microprudential analysis, and the discrepancies widen as the fundamentals deteriorate.

Flight to quality Note that the rollover threshold $V^{**} = P_H + \lambda$ is increasing in asset

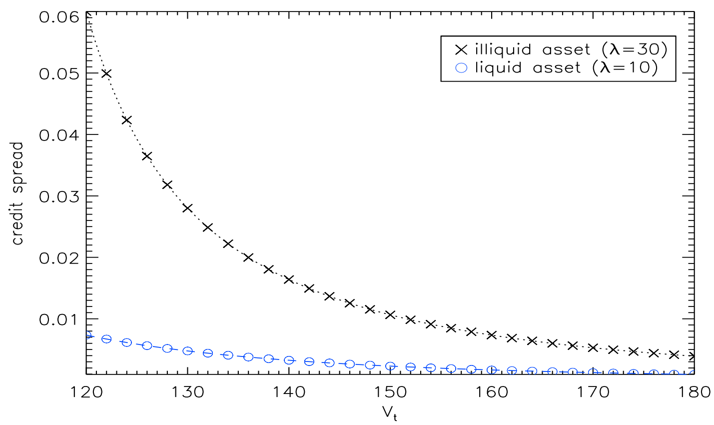

Figure 6: Flight to quality
Flight to quality emerges as the fundamentals deteriorate from more severe coordination concerns for illiquid collateral assets. Other parameters in this example are the same as in Figure 5.

illiquidity λ. Coordination concerns about limited market liquidity are more severe with more illiquid assets, thus funding evaporation is triggered at a higher threshold. Reflecting this additional risk, credit spreads are higher for more illiquid collateral assets. As described in Figure 6, credit spreads rise faster in a downturn for a more illiquid collateral asset, while the differences are very small when the fundamentals are robust.[24]

More homogeneous system with between-group interactions Strategic interactions between heterogeneous groups arise with reduced heterogeneity. Given fixed P_H, suppose that L-group become less robust (higher P_L) such that $P_H - P_L < \lambda$. As discussed in Proposition 3, H-group's risk is now affected by stronger L-group's degree of financial health, and the debt rollover is allowed for H-group as long as V is beyond their rollover threshold $V^* = P_L + 2\lambda$, but is refused as soon as V hits V^*, which is higher than V^{**}.

This indicates the emergence of additional systemic risks with reduced heterogeneity. We observe in Figure 5 that although its own financial health is unchanged, credit spreads and rollover risks for an H-group institution become higher with reduced heterogeneity

[24] Gorton and Metrick (2012), and Krishnamurthy, Nagel, and Orlov (2011) provide empirical evidences for this prediction.

compared to the previous case, and they become even higher simply when the stronger L-group institutions' financial health deteriorates. This additional premium and additional risks result from the strategic concern among heterogeneous institutions.

6 Conclusion

This paper provides a framework for studying systemic panic in financial markets among heterogeneous participants. When we anticipate a contagious chain reaction, conventional wisdom dictates that we ought to focus on the weakest link: Bolster the weakest since it all starts from the distress of the weakest. Our analysis, incorporating concerns about spillovers and corresponding strategic interactions, suggests an alternative approach: Bolster the strongest in the contagion chain since it actually starts when the strongest loses confidence in the market. When the strongest begins evaluating the possibility of an exit, this prompts the weaker to run before the mess materializes and, in turn, a systemic crisis occurs in a self-fulfilling manner.

Although we analyzed a special case with heterogeneity given by different leverage levels and spillovers by fire-sale externalities, our result is not an artifact of these assumptions. The framework of this paper can be applied more generally to cases in which coordination problems arise among agents with differing degrees of fragility to strategic risk, as discussed in Appendix C.

Appendix A: Proofs

Given the history h_s, let $V_j^*(h_s)$ be the threshold of the fundamental on which j-group institutions' collateral constraint binds and the exit game is terminated for that group. We first claim that $V_H^*(h_s) \geq V_L^*(h_s)$. Suppose $V_H^*(h_s) < V_L^*(h_s)$. Note that at $V_L^*(h_s)$, L-group's collateral constraint just binds such that f_s satisfies $V_L^*(h_s) - \lambda f_s = P_L$, so $f_s = \frac{V_L^*(h_s) - P_L}{\lambda}$. Given this f_s and the fundamental $V_L^*(h_s)$, however, H-group's collateral constraint is violated since $L_s = V_L^*(h_s) - \lambda f_s = P_L < P_H$. This implies that $V_H^*(h_s) > V_L^*(h_s)$ which is contradiction. Thus, $V_H^*(h_s) \geq V_L^*(h_s)$, which implies that H-group institutions will always get distressed when L-group are distressed, but not vice versa. We first prove the following lemma.

Lemma A1. (History independence in the limit case $\triangle \to 0$)

(i) If the exit game has not terminated at period t (i.e., $V_s > V_j^*(h_s)$ for all $s < t$), then $f_s = 0$ a.s. for all $s < t$.

(ii) s_{it} is a sufficient statistic for V_t

Proof of Lemma A1

(i) Suppose $V_s > V_H^*(h_s) \geq V_L^*(h_s)$. Note that $V_t - V_{t-1} = O_p(\triangle)\ \forall t$, thus $V_s - V_H^*(h_s) = O_p(\triangle)$ and $V_s - V_L^*(h_s) = O_p(\triangle)$. Note that we assumed $s_{is} - V_s = o_p(\triangle)\ \forall i$, thus $s_H^*(h_s) - V_H^*(h_s) = o_p(\triangle)$ and $s_L^*(h_s) - V_L^*(h_s) = o_p(\triangle)$. These imply that $s_{is} > s_j^*(h_s)$ a.s. $\forall i$ for both $j = H, L$, as $\triangle \to 0$. Thus $f_{j,s} = 0$ a.s. for both $j = H, L$. We get $f_s = 0$ a.s. for all $s < t$.

(ii) This is straightforward from $s_{it} - V_t = o_p(\triangle^{\frac{1}{2}})$ and $V_t - E[V_t|V_s] = O_p(\triangle^{\frac{1}{2}})\ \forall s < t$. ■

This Lemma implies that equilibrium strategy profile $s_j^*(h_t)$ (thus the exit game) is history-independent and V_t is the only state variable of the exit game (there is no partial-exit in our limit case, thus we can ignore history of past f_s and the fundamentals). Since s_{it} is a sufficient statistic for V_t in our limit case, it contains all the relevant information for the optimal decision making and $V_j^*(h_t)$, $s_j^*(h_t)$, $\Pi_j^S(s_{it}, h_t)$ can hence be denoted as V_j^*, s_j^*, and $\Pi_j^S(s_{it})$. Value function of staying is thus simplified as

$$\Pi_j^S(s_{it}) = E\Big[\Big(w_S \triangle + e^{-r\triangle} \max\{\Pi_j^S(s_{it+\triangle}), \Pi^E\}\Big) \times \mathbf{1}_{[f_t < f_j^*(V_t)]} + 0 \times \mathbf{1}_{[f_t \geq f_j^*(V_t)]} \Big| s_{it}\Big] \qquad (10)$$

which is history independent. Thus we can interpret our dynamic exit game as a sequence of the identical one-shot games.

Proof of Lemma 1

When $f_{-j,t} = 0$ is given, the option value of staying for a j-group institution given the private signal s_{it} can be defined as

$$\Pi^S_j(s_{it}) = E\Big[\Big(w_S\triangle + e^{-r\triangle}\max\{\Pi^S_j(s_{it+\triangle}),\Pi^E\}\Big)\times \mathbf{1}_{[f_{j,t}<f^*_j(V_t)]} + 0\times \mathbf{1}_{[f_{j,t}\geq f^*_j(V_t)]}\Big| s_{it}\Big]. \tag{11}$$

We can solve for the optimal switching threshold $\underline{s}^*_j$ from the indifference condition $\Pi^S_j(\underline{s}^*_j) = \Pi^E$. Let $\underline{V}^*_j$ be the crisis threshold of the fundamental V on which the collateral constraint of j-group is breached in this case. We now solve for $\underline{V}^*_j$ and $\underline{s}^*_j$ from the following two equations. First, the actual mass of exit f_j has to be equal to the critical liquidation pressure given the fundamental $\underline{V}^*_j$, defined as $f^*_j(\underline{V}^*_j)$. Following the definition, $f^*_j(\underline{V}^*_j) = \frac{\underline{V}^*_j - P_j}{\lambda}$. Since j-group institutions with signals below $\underline{s}^*_j$ exit, from uniform distribution, $f_{j,t} = \Pr[s_{it} \leq \underline{s}^*_j | \underline{V}^*_j] = \frac{\underline{s}^*_j - (\underline{V}^*_j - \epsilon)}{2\epsilon}$. Equating the two, we get

$$\underline{s}^*_j = \frac{2\epsilon}{\lambda}(\underline{V}^*_j - P_j) + \underline{V}^*_j - \epsilon. \tag{12}$$

Next, the two actions have to be indifferent at the switching threshold $\underline{s}^*_j$, so $\Pi^S_j(\underline{s}^*_j) = \Pi^E$. Rewriting this condition using (11), we get

$$Pr(V_t \geq \underline{V}^*_j | s_{it} = \underline{s}^*_j) \times \Big[w_S\triangle + e^{-r\triangle}E\Big(\max\{\Pi^S_j(s_{it+\triangle}),\Pi^E\}\Big| s_{it} = \underline{s}^*_j\Big)\Big] = \Pi^E \tag{13}$$

Note that optimal switching occurs at $\underline{s}^*_j$ with $\Pi^S_j(\underline{s}^*_j) = \Pi^E$. From this, we get

$$E[\max\{\Pi^S_j(s_{it+\triangle}),\Pi^E\}|s_{it} = \underline{s}^*_j] \to \Pi^E$$

as $\triangle \to 0$.

Plug this in (13), we get $Pr(V_t \geq \underline{V}^*_j | s_{it} = \underline{s}^*_j) = 1$ as $\triangle \to 0$, which can be rewritten as

$$\frac{\underline{s}^*_j + \epsilon - \underline{V}^*_j}{2\epsilon} = 1 \tag{14}$$

since $V_t|(s_{it} = \underline{s}_j^*)$ is uniformly distributed over $[\underline{s}_j^* - \epsilon, \underline{s}_j^* + \epsilon]$. From (12) and (14), we get $\underline{s}_j^* = P_j + \lambda + \epsilon$ and $\underline{V}_j^* = P_j + \lambda$ ■

Proof of Lemma 2

As in the proof of Lemma 1, we define two crisis thresholds of the fundamental V_j^* with $j = H, L$, on which j-group's collateral constraint is breached.

We first solve for H-group's optimal threshold s_H^* given s_L^* (best response $s_H^{BR}(s_L^*)$). On the fundamental threshold of V_H^*, note that

$$f_{H,t} = \Pr[s_{it} \leq s_H^*|V_H^*] = \frac{s_H^* - (V_H^* - \epsilon)}{2\epsilon}$$

and

$$f_{L,t} = \Pr[s_{it} \leq s_L^*|V_H^*] = \frac{s_L^* - (V_H^* - \epsilon)}{2\epsilon}$$

thus

$$f_t = f_{H,t} + f_{L,t} = \frac{s_H^* + s_L^* - 2(V_H^* - \epsilon)}{2\epsilon}.$$

Since this actually coincides with $f_H^*(V_H^*) = \frac{V_H^* - P_H}{\lambda}$, we get

$$s_H^* = \frac{2\epsilon}{\lambda}(V_H^* - P_H) + 2(V_H^* - \epsilon) - s_L^*. \tag{15}$$

The indifference condition at s_H^* implies $\Pi_H^S(s_H^*) = \Pi^E$, and with (6) we get

$$Pr(V_t \geq V_H^*|s_{it} = s_H^*) \times \Big[w_S\triangle + e^{-r\triangle}E\Big(\max\{\Pi_H^S(s_{it+\triangle}), \Pi^E\}\Big|s_{it} = s_H^*\Big)\Big] = \Pi^E. \tag{16}$$

Note that optimal switching occurs at s_H^* with $\Pi_H^S(s_H^*) = \Pi^E$, we get

$$E[\max\{\Pi_H^S(s_{it+\triangle}), \Pi^E\}|s_{it} = s_H^*] \to \Pi^E$$

as $\triangle \to 0$.

Plug this in (16), we get $Pr(V_t \geq V_H^*|s_{it} = s_H^*) = 1$ as $\triangle \to 0$, which can be rewritten as

$$\frac{s_H^* + \epsilon - V_H^*}{2\epsilon} = 1 \tag{17}$$

since $V_t|(s_{it} = s_H^*)$ is uniformly distributed over $[s_H^* - \epsilon, s_H^* + \epsilon]$.

From (15) and (17), solving for s_H^*, we get

$$s_H^* = \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1}\right] \times [s_L^* + \frac{2\epsilon}{\lambda} P_H + 3\epsilon] + \epsilon. \tag{18}$$

We now show that $s_H^* > s_L^*$ (i.e., $s_H^{BR}(s_L^*) > s_L^*$) if $s_L^* < \overline{s}_H^* (= P_H + 2\lambda + \epsilon)$. From the above,

$$\begin{aligned} s_H^* - s_L^* &= \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1}\right] \times [s_L^* + \frac{2\epsilon}{\lambda} P_H + 3\epsilon] + \epsilon - s_L^* \\ &= \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1}\right] \times [3\epsilon + \frac{2\epsilon}{\lambda}(P_H - s_L^*)] + \epsilon \\ &> \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1}\right] \times [3\epsilon - \frac{2\epsilon}{\lambda}(2\lambda + \epsilon)] + \epsilon \\ &= \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1}\right] \times [-\epsilon - \frac{2}{\lambda}\epsilon^2] + \epsilon = -\epsilon + \epsilon = 0 \end{aligned}$$

where the inequality comes from $s_L^* < P_H + 2\lambda + \epsilon$. We thus get $s_H^{BR}(s_L^*) > s_L^*$ if $s_L^* < \overline{s}_H^*$.

Repeating the same steps for the L-group, we get

$$s_L^* = \left[\frac{1}{\frac{2\epsilon}{\lambda} + 1}\right] \times [s_H^* + \frac{2\epsilon}{\lambda} P_L + 3\epsilon] + \epsilon$$

given s_H^*, and $s_L^{BR}(s_H^*) > s_H^*$ if $s_H^* < \overline{s}_L^*$. ■

Proof of Proposition 1

By definition, $\overline{s}_L^* < \overline{s}_H^*$. This then implies that $s_L^* < \overline{s}_H^*$ as $s_L^* \leq \overline{s}_L^*$. This implies that $s_H^* > s_L^*$ always has to hold from Lemma 2. Also, notice that $s_H^* \geq \overline{s}_L^*$ since $s_H^* < \overline{s}_L^*$ implies $s_L^* > s_H^*$ from Lemma 2, which contradicts with the above.

Now, suppose $s_L^* < \overline{s}_L^*$ and $s_L^* - \overline{s}_L^* = O(\triangle)$ as $\triangle \to 0$ (thus $\epsilon \to 0$). Note that from (18) $s_H^* \to s_L^*$ in this case, implying $s_H^* < \overline{s}_L^*$. But then this implies $s_L^* > s_H^*$ from Lemma 2, contradicting with $s_H^* > s_L^*$. Combining with $s_L^* \leq \overline{s}_L^*$, we thus get $s_L^* \to \overline{s}_L^*$ in the limit. $s_H^* \to s_L^*$ implies that $s_H^* \to \overline{s}_L^*$. Note that, in the limit, $\overline{s}_L^* \to P_L + 2\lambda$, and thus both H and L group institutions exit the market altogether when the fundamental V hits $P_L + 2\lambda$. ■

Proof of Proposition 2

Note that $\underline{s}_j^*$ is the same as in Lemma 1 and $\overline{s}_j^* = P_j + 3\lambda + \epsilon$. For each s_H^*, s_M^*, s_L^*, we consider a best response $s_j^{BR}(s_{-j}^*)$, and following the same steps as in the proof of Lemma 2, we have $s_j^{BR}(s_{-j}^*) > \max(\{s_{-j}^*\})$ if $\max(\{s_{-j}^*\}) < \overline{s}_j^*$. Then following the same steps of the proof for Proposition 1, we can show that Proposition 2 holds. ■

Proof of Proposition 3

Following the proof for Lemma 1, $\overline{s}_j^* = P_j + (\omega_j + \omega_{-j})\lambda + \epsilon$ and $\underline{s}_j^* = P_j + \omega_j\lambda + \epsilon$. If $\omega_L\lambda > P_H - P_L$, then $\overline{s}_L^* > \underline{s}_H^*$. We can then show that $V^* = P_L + (\omega_H + \omega_L)\lambda$ following the proofs of Lemma 2 and Proposition 1. If $\omega_L\lambda < P_H - P_L$, then $\overline{s}_L^* < \underline{s}_H^*$. Note that $\underline{s}_j^* \leq s_j^* \leq \overline{s}_j^*$, thus $s_L^* \leq \overline{s}_L^* < \underline{s}_H^* \leq s_H^*$. We then get $s_L^* < s_H^*$ and $s_H^* - s_L^* = O(\epsilon)$. Thus, no L-group institution should exist when V_t hits s_H^* for the first time, but all H-group institutions should have exited when V_t hits s_L^*. Thus $s_H^* = \underline{s}_H^*$ and $s_L^* = \overline{s}_L^*$ by definition. ■

Proof of Proposition 4

Note that we now have $\overline{s}_H^* = P_H + \lambda + \epsilon, \underline{s}_H^* = P_H + \epsilon, \overline{s}_L^* = P_L + 2\lambda + \epsilon, \underline{s}_L^* = P_L + \lambda + \epsilon$, thus $\overline{s}_H^* < \overline{s}_L^*$. With the same pre-emption mechanism as in Lemma 2, we can show that s_H^* and s_L^* both converge to $\overline{s}_H^*$ following the proof of Proposition 1. ■

Appendix B: Non reduced-form setup

We examine the non reduced-form setup without the delegation assumption by analyzing the equity holder's payoff directly. Following the same setup, we first focus only on one-group in isolation as in Section 3.2.1. We claim that this non reduced-form result is the same as Lemma 1 based on the reduced-form setup. Given this, deriving the same Proposition 1 is straightforward following the same steps of Section 3.2.2. We take the following two steps: (i) Show that the equity holder prefers keeping his position rather than liquidating immediately when the collateral constraint is satisfied, and (ii) incorporating coordination concerns, show that the solutions are the same as in Lemma 1. We focus on an H-group institution without loss of generality.

We impose a restriction on the coupon rate such that $\frac{C_H}{P_H} < \frac{r}{1-\frac{1}{1-X}}$ where X is the negative root of $X(X-1)\frac{\sigma^2}{2} + X(r-\sigma) = r$. This condition holds for most of the realistic parametric assumptions. We here rule out the extreme cases in which the coupon rate $\frac{C_H}{P_H}$ is so high that the collateral constraint never binds in equilibrium because of the early default.

First, suppose that there exists a fundamental threshold V^D on which the institution (equity holder) wishes to close its position paying back its debt principal P_H, even though the debt can be surely rolled over (i.e., $f_{H,t} < f_H^*(V^D)$ a.s.). Given this threshold, calculation of the debt value $D(V_t; V^D)$ is straightforward as in Appendix D, and then we can calculate the equity value at V_t, denoted by $\Pi_H^S(V_t) = V_t - D(V_t; V^D)$, as follows

$$\Pi_H^S(V_t) = V_t - \frac{C_H}{r} + \left[\frac{C_H}{r} - P_H\right] \times \left[\frac{V_t}{V^D}\right]^X.$$

However, given our condition on the coupon rate, we can show that $\frac{\partial \Pi_H^S(V_t)}{\partial V_t}\Big|_{V_t=V^D} > 0$ in this case which violates the smooth pasting condition. This implies that when rollover is certainly allowed, the exit decision at V^D will be "too early" since the option value of staying is higher than that of immediate liquidation $V^D - P_H$. Therefore, no such V^D exists and the institutions prefer to stay if funding liquidity is surely provided. This implies that the equity value should be higher than the payoff from immediate liquidation in that region, so $\Pi_H^S(V_t) > V_t - P_H$ if $V_t > P_H + \lambda$ (where $f_{H,t} < f_H^*(V_t)$ with probability 1).

Now we incorporate (coordination) concerns about the rollover refusal. We impose a non-zero "penalty" c to the equity holder in the case of forced liquidation. It can be any positive number, and we can interpret this as a loss in the franchise value or reputational costs. We use the history-independent property of Lemma A1.

When choosing to exit given the signal s_{it}, the equity holder simply expects to get

$$\Pi_H^E(s_{it}) = \max\{s_{it} - P_H, 0\}.$$

When choosing to stay, the equity value incorporating the potential rollover refusal is

$$\Pi_H^S(s_{it}) = E\Big[\Big((\delta V_t - C_H)\triangle + e^{-r\triangle}\max\{\Pi_H^S(s_{it+\triangle}), \Pi_H^E(s_{it+\triangle})\}\Big)\times\mathbf{1}_{[f_{H,t}<f_H^*(V_t)]} - c\times\mathbf{1}_{[f_{H,t}\geq f_H^*(V_t)]}\Big| s_{it}\Big]$$

similar to (7) in Section 3.2. As discussed above, note that the equity value is higher than the outside option when $Pr(f_{H,t} < f_H^*(V_t)|s_{it}) = 1$, so $\Pi_H^S(s_{it}) > \Pi_H^E(s_{it})$ if $s_{it} > P_H + \lambda + \epsilon$. It is straightforward that upper/lower dominance regions exist, strategic complementarities and state monotonicity hold. Thus by Toxvaerd (2008), there exists a unique s^* such that $\Pi_H^E(s^*) = \Pi_H^S(s^*)$. Given $s_{it} = s^*$, as $\triangle \to 0$, $\Pi_H^S(s_{it+\triangle}) \to \Pi_H^S(s^*)$ a.s. and $\Pi_H^E(s_{it+\triangle}) \to \Pi_H^E(s^*)$ a.s., thus we get $Pr(f_{H,t} < f_H^*(V_t)|s^*) = 1$. The rest of the proof is the same as that of Lemma 1, and we can get $s^* = P_H + \lambda + \epsilon$ which is the same as in Lemma 1.

Appendix C: Static setup with generalized parameters

In the setup of Section 2, we studied a dynamic model with continuous time approximation ("main model") which can be represented as a sequence of one-shot static games with appropriately defined value functions as their payoffs. In this section, we present a simplified static version of our main model, with generalized payoff and heterogeneity parameters, in order to discuss what drives our result in more detail.

Setup We again consider two groups of agents, S and W ("Strong" and "Weak") with a continuum $[0, 1]$ of identical agents in each group. An agent $i(\in [0, 2])$ chooses either to "stay" ($a_i = 0$) or "exit" ($a_i = 1$) after observing a private signal $s_i = V + \epsilon_i$. Here, V is the "fundamental" with an improper prior, and i.i.d. noise follows $\epsilon_i \sim U[-\epsilon, \epsilon]$.[25] We focus on the limit case with $\epsilon \to 0$ as in the main model.

[25] We can also use other noise distributions with finite support to obtain similar results.

When choosing to stay, an agent will either "survive" or "fail" depending on the fundamental and the aggregate action of other agents, denoted by f. Here, $f(= f_s + f_w = \int_0^2 a_i di)$ is the total number of agents (both in S and W-group) who have chosen to exit. "Failure" can occur even with the higher fundamental when f is larger.

The two groups are different in terms of their "fragilities". Given V, stronger S-group is less prone to fail than weaker W-group. To be specific, a staying S-group agent fails if $f > V - \delta_s$ and survives otherwise, while a staying W-group agent fails if $f > V - \delta_w$ and survives otherwise, where $\delta_w - \delta_s > 0$ and δ_j is the "fragility" parameter for j-group ($j = S, W$). We now define a "heterogeneity" parameter $\delta \equiv \delta_w - \delta_s$, then larger δ implies more heterogeneity (in fragilities) between the two groups. As in the main model, we focus on the case with $\delta < 1$ such that strategic interactions arise between the groups.

The payoff values are identical across all agents. When choosing to stay and survives, an agent receives 1 but 0 if fails. He instead receives $\alpha(\in [0, 1])$ when choosing to exit. The payoff structure of j-group is summarized in the table below.[26]

	$f \leq V - \delta_j$	$f > V - \delta_j$
Stay	1	0
Exit	α	α

Table 1: Payoff Structure for j-group agents

Equilibrium As in the main model, a Bayesian equilibrium can be characterized by the two exit (switching) thresholds of the private signal s_s^* and s_w^*, with corresponding fundamental (failure) thresholds V_s^* and V_w^* below which a failure occurs, and under our assumption of $\delta < 1$, all these four thresholds converge to some V^* as $\epsilon \to 0$. Our main focus is on whether this fundamental failure threshold V^* depends on δ_s (stronger's fragility), δ_w (weaker's fragility), or both. As will be discussed below, this critically depends on the value of α (which embodies aversion to strategic risk) and δ (heterogeneity in fragility). See Figure 7.

[26] Sákovics and Steiner (2012) also study a global game model with heterogeneous groups, but their focus is on different payoffs (different α in our context) across groups. Our focus is on the case with different fragilities ($\delta > 0$) in which one group can be more fragile to coordination failure than the others (fail with lower f).

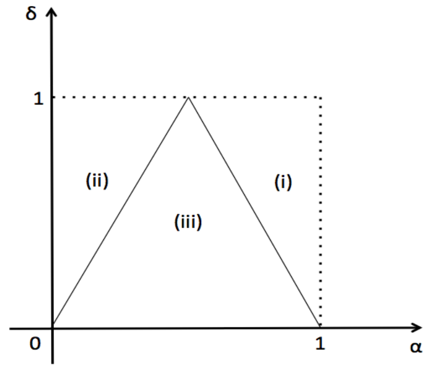

Figure 7: 3 Cases depending on (α, δ).
In Case (i), V^* depends on δ_s; In Case (ii), V^* depends on δ_w; In Case (iii), V^* depends on both δ_s and δ_w.

Proposition 5. *When $\epsilon \to 0$, all agents (S and W) choose to exit if and only if the fundamental V is below V^*, where V^* is equal to:*

(i) $1 + \delta_s + \alpha$ *when* $2\alpha + \delta > 2$ *(depending only on the stronger's fragility).*

(ii) $\delta_w + \alpha$ *when* $\delta > 2\alpha$ *(depending only on the weaker's fragility).*

(iii) $\frac{\delta_s+\delta_w}{2} + 2\alpha$ *when* $\delta < 2\alpha$ *and* $2\alpha + \delta < 2$ *(depending on both).*

Note that the 4 equilibrium thresholds s_s^*, V_s^*, s_w^*, V_w^* can be solved from the following 4 equations as in the main model:

$$\alpha = 1 \times Pr[V > V_s^* | s_i = s_s^*] + 0 \times Pr[V < V_s^* | s_i = s_s^*] \tag{19}$$

$$Pr[s_i < s_s^* | V = V_s^*] + Pr[s_i < s_w^* | V = V_s^*] = V_s^* - \delta_s \tag{20}$$

$$\alpha = 1 \times Pr[V > V_w^* | s_i = s_w^*] + 0 \times Pr[V < V_w^* | s_i = s_w^*] \tag{21}$$

$$Pr[s_i < s_s^* | V = V_w^*] + Pr[s_i < s_w^* | V = V_w^*] = V_w^* - \delta_w \tag{22}$$

We first analyze Case (i) with $2\alpha + \delta > 2$ (both α and δ are large), and discuss why V^* depends only on δ_s, but not δ_w. Focusing on an S-group agent, we can write (19) as:

$$1 - \alpha = Pr[V < V_s^* | s_i = s_s^*] \tag{23}$$

We now define $p \equiv 1 - \alpha$, then (23) becomes $Pr[V < V_s^* | s_i = s_s^*] = p$. This implies that V_s^* is the bottom $100p$ percentile outcome for the posterior distribution of V given $s_i = s_s^*$. Thus, we can now describe an S-group agent's problem (equation (19) and (20)) in the following way:

> *Let's consider the most conservative/pessimistic* $100p$ *percentile scenario given my signal (i.e.* $V = V_s^*$ *such that* $Pr[V < V_s^* | s_i = s_s^*] = p$*). How many of others will be exiting in that scenario (what would be the conjecture about* $f = f_s + f_w$*)? Will I be able to just survive facing that many exits (i.e.* $Pr[s_i < s_s^* | V = V_s^*] + Pr[s_i < s_w^* | V = V_s^*] \leq V_s^* - \delta_s$ *holds with equality)?*

Notice that with larger α (smaller p), an agent considers a more pessimistic scenario and becomes conservative trying to survive even in that scenario facing strategic risks (avoid coordination failure). Thus, we can interpret α is as a "strategic risk aversion" parameter.[27] [28] Now suppose that α is large (p is small). An S-group agent considers a bottom $100p$ percentile event V_s^* in which case his conjecture regarding the number of exiting agents follows equation (20). As just discussed, with larger α he becomes more risk averse and consider a more conservative case (lower V_s^* given the signal)[29] in which more of his own S-group agents are exiting—large α here implies that conjectured $f_s (= Pr[s_i < s_s^* | V = V_s^*])$ is also large.[30]

We now for expository purpose define $\triangle^* > 0$ such that $\triangle^* \equiv s_w^* - s_s^*$. It is easy to verify that these equilibrium thresholds satisfy $s_w^* > s_s^*$ and the difference $\triangle^*$ is increasing in δ,[31] which is intuitively plausible since δ measures heterogeneity in fragilities between the groups. Given these

[27] Alternatively, notice that s_j^* satisfies $\alpha = Pr[f < V - \delta_j | s_i = s_j^*]$. With larger α, an agent tries to expose himself to lower risk of coordination failure in equilibrium.

[28] To be general, what matters is the ratio of (payoff of exit – payoff of failure) to (payoff of success – payoff of failure). This ratio is equal to α in our example since we normalized other payoffs. Goldstein and Pauzner (2004) study a global game model in which different strategic risk aversion comes from wealth effect through a DARA utility function, while our definition of strategic risk aversion is different from theirs.

[29] To be specific, $V_s^* = s_s^* - \epsilon + 2p\epsilon = s_s^* - \epsilon + 2(1 - \alpha)\epsilon$.

[30] To be specific, $Pr[s_i < s_j^* | V = V_j^*] = \alpha$.

[31] $\triangle^*$ also depends on ϵ and $\triangle^* \to 0$ as ϵ goes to 0.

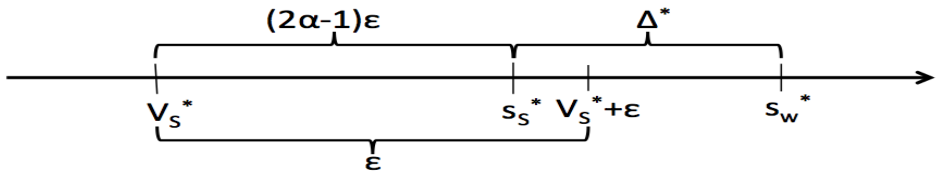

Figure 8: Case (i), S-group's conjecture given s_s^*
For S-group, $V_s^* = (s_s^* - \epsilon + 2p\epsilon)$ is small when $\alpha(= 1-p)$ is large, which is independent of δ_w. If $\triangle^*$ (that is, δ) is large enough such that $V_s^* + \epsilon < s_w^*(= s_s^* + \triangle^*)$, then S-group's conjecture about f_w is equal to 1 ($Pr[s_i < s_w^* | V = V_s^*]) = 1$), independent of δ_w.

characteristics, notice that when f_s is large (that is, α is large as just discussed) and $\triangle^*$ is large (that is, δ is large), we will have $f_w(= Pr[s_i < s_w^* | V = V_s^*]) = 1$ which is the upper bound of f_w, as described in Figure 8. In this case, we can solve for s_s^* from (19) and (20) independently of δ_w. Intuitively, S-group agents are considering a conservative scenario in which even many of the stronger agents are exiting. The number of exiting agents in W-group should be larger than that of S-group since the weaker W-group should care more about the coordination problem thus trying to act more preemptively, and the difference should become larger when the difference in fragilities between the groups is larger. Thus, as agents become more conservative (or heterogeneity becomes larger), eventually all of the weaker agents should be exiting in this stronger agent's pessimistic scenario, and the stronger can ignore the exact fragility of the weaker when solving for his exit threshold. In other words, S-group agents now think that all of the weaker will be exiting any way, and their equilibrium exit threshold becomes $\overline{s}_s^*$ as in the main model where $\overline{s}_s^*$ is the optimal exit threshold of S-group when they take $f_w = 1$ as given, as defined in Section 3.2. Therefore, the stronger's decision making now depends on how fragile his own group agents are, independent of the weaker group's fragility. The weaker's exit threshold, on the other hand, depends critically on the stronger's threshold (and thus stronger's fragility) as in the main model.

Following the similar argument, the opposite holds when strategic risk aversion is low (low α) and δ is not too small (Case (ii). See Figure 9). With high p, agents consider an optimistic scenario. Now it's the weaker that becomes critical—W-group agents consider an optimistic case in which

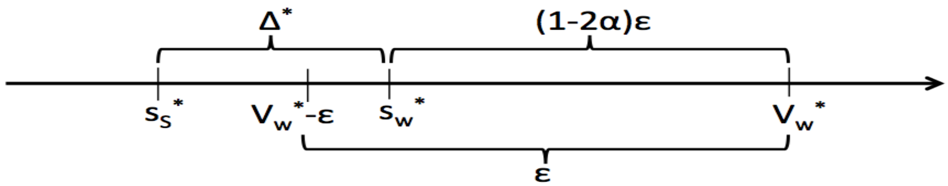

Figure 9: Case (ii), W-group's conjecture given s_w^*
For W-group, V_w^* is large when $\alpha(=1-p)$ is small, independent of δ_s. If $\triangle^*$ (that is, δ) is large enough such that $V_w^* - \epsilon > s_s^*(= s_w^* - \triangle^*)$, then W-group's conjecture about f_s is equal to 0 $(Pr[s_i < s_s^* | V = V_w^*]) = 0)$, independent of δ_s.

even most of the weaker agents themselves are staying, and the number of staying S-group agents should be larger. Indeed, when α is small enough given δ, W-group conjectures that all of S-group agents should be staying in that optimistic scenario. Thus, W-group simply ignores S-group when solving for his optimal exit threshold (equation (21) and (22)) in this case. Therefore, W-group's exit threshold will be $\underline{s}_w^*$ (an optimal exit threshold ignoring S-group) following the definition of Section 3.2, which only depends on δ_w and is independent of δ_s. The stronger's exit threshold, on the other hand, depends critically on this weaker's threshold and thus in the limit V^* depends only on δ_w.

For intermediate α and low δ (Case (iii)), now this one-group dependency disappears; when a j-group agent considers a $100p$ percentile event, his conjecture regarding the number of exiting agents in the other group is also intermediate ($f_{-j} \in (0,1)$ for both $j = S, W$). He thus needs to take the other group's fragility into account which affects the exact value of f_{-j}. Now the exit thresholds depend both on δ_s and δ_w.

Appendix D: Credit spreads and rollover risk

In Section 5, we can apply the structural debt pricing framework to our setup to examine how the externality from coordination concerns affects credit spreads and rollover risks in times of crises, where the rollover risk refers to the risk of the debt rollover refusal.

In terms of asset pricing, the debt contract under consideration can be interpreted as a perpetual coupon debt with both callable and puttable options. Notice that there exists an endogenous threshold of the fundamental (rollover threshold) V^R determined in the model, and the pre-determined coupon $C_H\triangle$ is paid each period until the fundamentals V hits that threshold, at which point the contract is terminated paying the principal. One distinction from the standard debt contract is that the creditor can also terminate this contract (puttable, by refusing to roll over) upon the violation of the collateral constraint (2), not passively waiting for the borrower's default decision (as in Leland (1994), and Leland and Toft (1996)).

Note that coordination failure leads the debt to be terminated at a higher fundamental threshold, at V^* or V^{**} with coordination concerns rather than $V^{BM} = P_H$ without coordination concerns. We now calculate the debt value with these endogenous thresholds and the payoffs on those thresholds. Under continuous time approximation, the debt contract pays the coupon C_H continuously until the fundamentals V hit the rollover threshold $V^R (= V^*, V^{**}$ or $V^{BM})$ where the rollover is refused and the principal P_H is paid.[32] [33]

The fundamentals follow $dV = (r-\delta)Vdt + \sigma VdW$ in the limit and the debt termination paying the principal P_H is triggered at $V_{\tau(V^R)} = V^R$, where the stopping time is defined by $\tau(V^R) = \inf\{t|V_t \leq V^R\}$. The debt value given the current state V_t, denoted as $D(V_t;V^R)$, can be derived from the Bellman equation

$$D(V_t;V^R) = E[\int_t^{\tau(V^R)-t} e^{-r(s-t)}C_H ds + e^{-r(\tau(V^R)-t)}P_H|V_t]. \tag{24}$$

Using Ito's formula, we get an ODE

$$C_H + \frac{1}{2}\sigma^2V^2D_{VV} + (r-\delta)VD_V - rD = 0,$$

[32]To be precise, the institutions exit from the contract voluntarily (exercise the callable option) in anticipation of this rollover refusal, but still paying P_H at $V_{\tau(V^R)} = V^R$.

[33]With this reduced-form setup, we try to capture the qualitative effect of the coordination failure on credit spreads, rather than the quantitative effect which is the focus of the credit spread puzzle literature (Huang and Huang (2003) for the overview). Our simplifying assumptions rule out partial recovery, guaranteeing the full payment of the principal. We can consider cases with partial recovery upon termination but the qualitative implications remain the same since what drives our results is the changes in the endogenous termination (rollover) threshold through coordination concerns among the institutions.

with the boundary conditions

$$\lim_{V \to \infty} D(V) = \frac{C_H}{r},$$

$$D(V^R; V^R) = P_H.$$

Solving this, the debt value can be calculated as

$$D(V_t; V^R) = \frac{C_H}{r} + [P_H - \frac{C_H}{r}] \times [\frac{V_t}{V^R}]^X \tag{25}$$

where X is the negative root of $X(X-1)\frac{\sigma^2}{2} + X(r - \delta) = r$.

We define the credit spread as the difference between the yield and riskfree rate following the standard definition:

$$CS(V_t; V^R) = \frac{C_H}{D(V_t; V^R)} - r. \tag{26}$$

In addition, consider the following measure of the rollover risk $RR(V_t; V^R)$ ranging from 0 to 1:

$$RR(V_t; V^R) \equiv E[e^{-r(\tau(V^R)-t)}|V_t], \tag{27}$$

which is a normalized distance to the rollover threshold V^R given the current fundamental V_t, reflecting how likely the rollover refusal will occur in the near future. It is decreasing in the fundamentals V_t, converging to 1 as V_t approaches to the rollover (crisis) threshold V^R. We can calculate this rollover risk in a closed form, such that $RR(V_t; V^R) = [\frac{V_t}{V^R}]^X$.

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