Clearing with two-sided limited commitment

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Abstract

This paper analyzes clearing arrangements in the presence of two-sided limited commitment. Risk-averse agents invest in projects that are subject to price uncertainty. To smooth income they trade OTC forward contracts. Clearing arrangements serve as commitment devices. Two-sided limited commitment motivates margins, segregation, novation and mutualization present in diverse clearing arrangements. Bilateral clearing is inferior to third-party clearing as segregation allows to save collateral. Mutualization with central counterparty clearing can improve welfare further if loss-sharing can be diversified. Though, welfare gains from bilateral to third-party clearing can be larger than the welfare gains from third-party clearing to CCP clearing.

Keywords: clearing, segregation, novation, mutualization

JEL Classification: G13, G14, G18, G2, G28, D53, D82

[Very preliminary - do not circulate]

1. Introduction

With the development of complex financial instruments, post-trade arrangements have become increasingly important. After "trading", counterparties must "clear" obligations until final "settlement" – the discharge of obligations – takes place. While we can define trading as the time when terms of trades are agreed, clearing spans the period immediately after

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trading until final settlement occurs. Clearing is meant to secure the "physical" exchange of goods and money. This is important because most trades are settled with a lag. While on spot markets trading, clearing and settlement are most closely aligned, even on spot markets settlement can take place up to a few days after trade date. When it comes to future markets, contracts involve promises that can span years between trade and settlement. For these financial contracts, it is crucial to have effective clearing arrangements, as trading is useless if trades cannot be enforced: I can make empty promises if I know I am not going to be held liable for those.

We analyze trading with two-sided limited commitment and how different clearing arrangements can cope with this friction. Risk-averse agents trade forward contracts on over-the-counter (OTC) markets to insure against consumption risk. Consumption risk results from price uncertainty over agents' returns on their investments. They bilaterally negotiate the terms of the forward contracts under the assumption of Nash bargaining. With perfect commitment, forward contracts allow agents to insure themselves perfectly. However, limited commitment allows agents to default strategically on their future obligations. If price uncertainty resolves to their advantage, agents are better off if they do not fulfill forward contracts. Thus, the future contract negotiated entails the terms of clearing that are meant to secure contract fulfillment. Even though limited commitment is the only friction present in the economy, it is sufficient to explain diverse clearing arrangements.

In order to clear, agents need to reduce investment. Resulting savings serve as collateral that can be pledged at so-called clearing agents. Clearing agents realize collateral pledged by trading agents that do not meet future obligations with respect to the forward contract and compensate parties that meet obligations. If nobody defaults, collateral is redeemed to all counterparties. Thus, collateral pledged serves as an incentive device to induce agents to commit to their obligations. This enables agents to trade and smooth income over different states of the world. Thus, agents face the trade-off between forgone returns on investment and income smoothing. This trade-off is crucially influenced by the clearing arrangements used.

We analyze three clearing arrangements. First, counterparties to a forward contract dispose over a technology that allows them to pledge collateral to their counterparty – often referred to as margins – i.e trading agents themselves act as clearing agents. We name this 'bilateral clearing'. Second, a third agent that is not involved in trading activities runs a technology that permits agents to pledge collateral at its vault. We name this technology segregation and refer to this framework as 'segregated collateral clearing'. Finally, we envis-

age another third-party clearer that – in addition to segregation – runs a technology which allows it to engage in novation and mutualization. We name this particular third-party clearer a central counterparty (CCP).

Posting of collateral in an environment with endogenous default incentivizes commitment as collateral can be seized by clearing agents to set off losses of non-defaulting counterparties. With bilateral clearing, agents can realize collateral pledged by the defaulting counterparty as a protection against strategic default. However, they will lose their own collateral pledged at the defaulting agent because limited commitment allows the defaulting agent to run away from its obligations including the redemption of its counterparty's collateral.

In other words, collateral is not segregated with bilateral clearing. In contrast, the third-party clearer is understood as 'bankruptcy remote' because it does not trade itself. This third-party can be understood as a safety-box that can only be opened with two keys, one in the hand of each trading agent. If one agent defaults, the other can refuse to open the safety box. Thus, with segregated collateral clearing an agent that defaults cannot run away with the collateral of its counterparty or, in other words, collateral posted by trading agents is segregated and cannot be seized by a defaulting counterparty. The same holds true with CCP clearing. In addition to segregation, a CCP offers services that are often referred to as novation and mutualization. Novation allows a CCP to enter a forward contract as the counterparty to the original seller and the original buyer. Due to novation, however, the CCP faces the risk of default. The CCP manages default risk by asking for margins and by introducing mutualization of its losses. Mutualization refers to the technology that enables a CCP to run a loss-sharing arrangement to share potential losses of a trader's default among surviving traders.

With limited commitment perfect insurance as the first-best cannot be achieved. We show bilateral clearing is used if the return is below a threshold determined by the cost of collateral, price uncertainty and agents preferences. If return is above this threshold, agents prefer autarky to any trade related income smoothing. The terms of trade crucially depend on the bargaining power of agents. We further show that symmetric collateralization (both agents pledge the same amount of collateral) is feasible only if agents have identical bargaining power and the return is below some threshold.

Segregated collateral clearing is generally welfare improving in comparison to bilateral clearing as it relaxes incentive constraints. The relaxation results from the ability of the third-party to isolate collateral from the default of a trading agent's counterparty. As a consequence, the defaulting agent cannot run with the collateral of the non-defaulting agent.

This is what we refer to as segregation. Again, segregated collateral clearing is used whenever the return is lower than a certain threshold. In comparison to bilateral clearing, third-party clearing is feasible for higher levels of return. Given the same amount of collateral, we show that with segregated collateral clearing agents' income can be smoothed to a greater extent than with bilateral clearing. Even though segregated collateral clearing shows relaxed incentive constraints, it is not generally the case that collateral savings are higher (more insurance) or lower (more investment) than under bilateral clearing. Compared with bilateral clearing, if the return is below a certain threshold, agents save less and, if return is above this threshold, agents start to put up more collateral with segregated collateral clearing. Again, symmetric collateralization with segregated collateral clearing requires identical bargaining power. In contrast to bilateral clearing, feasibility of symmetric solutions are not restricted by a certain level of return.

We show that the feasibility of CCP clearing and, thus, mutualization depends on the possibility to diversify loss-sharing. Diversification is achievable if price uncertainty is not systemic but probabilistic. If the CCP clears a portfolio entailing systemic risk, it cannot improve upon third-party clearing because systemic risk is uninsurable. However, if the clearing portfolio is diversified, CCP clearing can improve welfare. We further highlight that segregation may be accountable for most of the welfare gains of a CCP. Mutualization increases welfare more than segregation only if diversification and return are at high levels.

We contribute to the literature in several ways. First, to our knowledge this is the first paper analyzing clearing from the perspective of two-sided limited commitment, i.e. both the seller and the buyer to a forward contract can strategically default. While Duffie and Zou (2010) and Maegerle and Nellen (2011) acknowledge two-sided default risk, they focus on multilateral netting, analyzing netting efficiency resulting with different clearing structures. Haene and Sturm (2009) and Nahai-Willisamson et al. (2012) model exogenous default risk. Koeppl and Monnet (2009), Koeppl and Monnet (2010), Koeppl (2011), Acharya and Bisin (2011), Biais, Heider and Hoerova (2012a and b) and Carappella and Mills (2012) assume default risk to be related to the seller exclusively. In contrast to these papers, we model two-sided limited commitment and allow the strategically defaulting counterparty to be endogenously determined by the realizing state of nature.

Second, to our knowledge this is the first paper to discuss a range of clearing arrangements explicitly. While the literature focuses on CCP clearing, we believe that our paper is a first step towards a more comprehensive analysis of clearing. In particular, we add bilateral and segregated collateral clearing to the set of clearing frameworks analyzed and derive

conditions that establish the welfare contribution of each arrangement. In particular, the welfare gains of CCP clearing can be decomposed into the welfare gains a third-party clearer can achieve with segregation and with mutualization. To motivate the different clearing functions economically is policy relevant because it allows to more rigorously determine the optimal clearing structure depending on prevailing circumstances.

Third, Acharya and Bisin (2011) incorporate limited liability of agents that exhibit short positions into a general equilibrium model. CCP clearing is understood as way to eliminate counterparty risk externalities. Because OTC markets are opaque, market participants are unable to find efficient prices. In particular, default risk cannot be priced as it depends on unknown trade positions. This lack of transparency allows agents to build up excessive short positions which provokes inefficient risk sharing and systemic risk. Provided bankruptcy costs are sufficiently large, bilateral clearing may impose efficiency associated with no default. In contrast, we characterize segregated collateral clearing and CCP clearing to be welfare improving. Understanding limited commitment to mirror sufficiently large bankruptcy cost, it is interesting to ask how the differences in results are explained. Acharya and Bisin (2011) use general equilibrium as the contract finding mechanism and apply exogenous collateralization. In contrast, we apply bilateral bargaining and endogenize collateralization. In this framework, segregated collateral and CCP clearing relax incentive constraints and turns out to be more efficient than bilateral clearing. Furthermore, if price uncertainty is probabilistic, the potential for mutualization allows a CCP to improve further upon segregated collateral clearing (and bilateral clearing).

As we understand collateralization as an essential part of a forward contract in OTC markets, in our setup, bargaining power has a role to play. This is important because market power is a real issue in OTC markets. We show two-sided limited commitment to restrict bargaining power. While bilateral and segregated collateral clearing allow agents to exercise some bargaining power, with CCP clearing this possibility is shut down as symmetric collateralization is enforced. Therefore, different forms of clearing affect trading and achieve different market outcomes. Further integrating trading and clearing might, thus, enhance our understanding of OTC markets. However, this goes beyond the focus of our paper and is left for future research. Rather than on trading our focus lies on the efficiency or welfare implications of clearing. Thus, we primarily compare allocations achieved for different clearing frameworks on the basis of symmetric collateralization.

Forth, Carapella and Mills (2012) also investigate an information problem in bilaterally and CCP cleared OTC markets. In contrast to Acharya and Bisin (2011) and this paper they

look at asymmetric information and emphasize its role in the massive dry up seen in OTC markets during the financial crisis starting 2007. They argue that many OTC products are information sensitive. CCP clearing is understood to make securities information insensitive and, as consequence, to prevent market dry up. In contrast, we highlight the fact that markets may simply vanish if the pay-off structure of financial products changes adversely such that it is too costly to put up enough collateral to enforce trades. As a consequence, agents prefer to face their income risk in full instead of smoothing it by means of forward trading. Asymmetric information has certaintly been a key source of the financial disruptions seen, however, we show that it is not a necessary condition for a market breakdown. We go beyond this as we are able to relate potential effects on trading to the prevalent clearing framework. With third-party clearers, a complete market breakdown can be withstood for higher levels of return and price risk for instance. Furthermore, given a diversified clearing portfolio, CCP clearing can prevent a market breakdown for higher levels of return and price risk than segregated collateral clearing. However, third-party clearers too are not able to prevent a market breakdown if conditions become too adverse.

Fifth, Haene and Sturm (2009) analyze the optimal division of a CCP's waterfalls into defaulter-pays resources (usually associated with margins) and survivors-pay instruments (default fund). With exogenous default risk and risk neutral traders, they find that it is always optimal to establish a default fund, and in some cases a sufficiently large default fund is even all it takes. They recommend the use of margins if the opportunity cost of collateral is lower than the probability of a participant's default and if margins are associated with risk-mitigating incentives. In contrast, we let default be endogenously determined - assuming two-sided limited commitment - and find that, if the cost of collateral is not too high, the optimal CCP arrangement is to ask for more margins than default fund contribution. We go beyond this by characterizing the price of collateral as forgone investments. Insofar, the price of collateral does not only depend on the economy's risk-free rate but depends on the price uncertainty of the cleared portfolio, the potential for diversification and traders' risk aversion. This predictions can be tested by analyzing a CCP's waterfall ratio (the sum of default fund contributions divided by the sum of initial margin contributions). For instance, the waterfall ratio increases with the cleared portfolio's price volatility.

The next section provides the basic model setup. Section 3 analyzes bilateral clearing. Segregated collateral clearing is investigated in section 4. Section 5 is devoted to CCP clearing. Section 6 provides empirical evidence on the waterfall ratio and discusses current policy debates in light of the model. Section 7 concludes.

2. Model

We consider a small open economy, that lasts for three periods, $t = 0, 1, 2$. There is a continuum of two types of agents that we denote by 1 and 2. There are three goods, X and Y and Z. Agents are endowed with one perfectly divisible seed. Agents 1 are endowed with a seed of good X while agents 2 are endowed with a seed of good Y.

Both types of agents have access to two technologies that transform seeds into goods. First, agents can use a short term technology that instantaneously gives one unit of good g for each unit of seed $g = X, Y$ invested. Second, agents can use a long-term technology that returns $R > 1$ units of good $g = X, Y$ at $t = 2$ for each unit of seed g invested at $t = 1$.

Agents 1 and 2 consume only good Z. They both have the same preferences represented by an isoelastic (constant coefficient of relative risk aversion α) utility function $u(.)$ which is increasing, concave and satisfy the usual conditions: $u(0) = 0$, $u'(0) = \infty$ and $u'(\infty) = 0$.

Agents meet pairwise at $t = 1$. An agent 1 always meets an agent 2. Pairs of agents can be in two possible states at $t = 1$. A measure σ of pairs of agents 1 and 2 enters a safe environment, while a measure $1 - \sigma$ of pairs of agents 1 and 2 enters a risky environment. In the safe environment one unit of good X can buy $1/2$ units of good Z at $t = 2$ and one unit of good Y can buy $1/2$ units of good Z at $t = 2$. Therefore, there is no risk involved for agents in the safe environment and, as a consequence, there is no scope for writing an insurance contract at $t = 1$.

When the environment is risky, the (world) price of good X or Y in terms of good Z at $t = 2$ is stochastic. There are two possible states at $t = 2$, H and L, that are as likely. In state H one unit of good X can buy $p < 1$ units of good Z, while one unit of good Y can buy $1 - p$ units of good Z. In state L, it is the reverse: one unit of good X can buy $1 - p$ units of good Z, while one unit of good Y can buy p units of good Z. For simplicity, we assume that $1 > p > 1 - p$ and $R > 1$. This implies that there is scope for some insurance for agents 1 and 2 in the risky state at $t = 1$.

We first consider the autarky benchmark where both agents can only trade in the world spot market at $t = 2$. Then in state i, an agent chooses his long-term investment $1 - l$ and his consumption of good z to solve

$$\max_{z,s \in [0,1]} u(z)$$
$$\text{s.t. } z \leq p_i R(1 - \ell) + p_i \ell$$

The budget constraint reads as follows: the agent invests $1-\ell$ in the long term technology, with a return $R(1-\ell)$ of good $g = X$ or Y, and ℓ in the short term technology (savings or storage) and sells the return at world price $p_i \in \{p, 1-p\}$ for good Z. Clearly, the agent maximizes his payoff setting $\ell = 0$. The expected payoff of any agent at $t = 1$ is then

$$V = 0.5u(pR) + 0.5u((1-p)R),$$

and at $t = 0$, the expected payoff is $\sigma u(R/2) + (1-\sigma)V$.

Let us discuss some of our assumptions. First, we assume that pairs of agents 1 and 2 can be in a safe or a risky state at date $t = 1$ to get to the idea that agents may end up trading more or less risky contracts. In the safe state at $t = 1$ agents will only trade safe contracts, i.e. contracts that do not involve any price uncertainty and, as a consequence, any counterparty risk. However, in the risky state agents know that their contracts involve price risk (as this is what they want to insure against). Therefore, they face counterparty risk as their counterparty could choose to renege on their promises if the price agreed in the contracts does not play in the favor of their counterparty.

Second, we assume that agents invests at date $t = 1$ rather than at date $t = 0$. We made this choice in order to obtain a notion of initial margin in the analysis of CCP clearing: generally, CCPs define their margins as a function of price fluctuations in some predetermined period of the past. To get to this idea, we assume that the risky state is one where agents know that price fluctuations are much higher than in the safe state, where there is no price uncertainty at all. Also, when analyzing CCP clearing, date $t = 0$ serves to join the CCP, providing the default contribution with a notion of a membership fee that allows to enter the loss-sharing agreement.

Third, it might appear restrictive to set the price in the safe state to 1/2 and prices in the state H and L to $p > 1/2$ and $1-p$ respectively. However, they can be understood as the result of a normalization. Indeed, we could instead have assumed the following set-up. The return to the productive technology is $\tilde{R}$. In the safe state, the price of good Z in terms of good Xor Y is $q > 0$, so that in the safe state, agents 1 and 2 are able to consume $q\tilde{R}$ for each unit of investment. In the risky state, however, the price of good X is $q + \tilde{x}$ in state H and $q - \tilde{x}$ in state L (and inversely for the price of good Y). Here, we could set q to any positive number while $\tilde{x}$ is restricted to be below q. Then we get our original set-up by setting $R = 2q\tilde{R}$ and $p = \frac{1}{2}(1 + \tilde{x}/q)$. Notice that this bears consequences for some of the statistics of the model. In particular, the variance of prices in the original environment is

$Var(q) = (1-\sigma)\tilde{x}^2$ while it is $Var(p) = (1-\sigma)(p-1/2)^2$ in the normalized environment. Therefore, the normalized variance is at most $Var(p) = (1-\sigma)/4$ which seems at first very small, but the original variance can be as high as $Var(q) = (1-\sigma)q^2$ which can be made large with q. Finally, notice that R could be quite high. In particular, it would be misleading to think of R as the risk free rate. For example, q could be very large and correspondingly increase R. For instance, if X is gold, Y is platinum, and Z is bread and $\tilde{R}$ is the risk free rate, then R could be much higher than 2. Therefore, in what follows, we will not restrict R to be necessarily arbitrarily close to a risk free rate. All this just to stress that our assumption on prices is only a normalization and, thus, is without any loss of generality.

3. Full Commitment Benchmark

We now investigate the allocation in the environment where a pair of agents 1 and 2 writes a forward contract at $t = 1$ given they can fully commit to their promises. As we consider OTC markets, we assume that there is no world market for forward contracts (or that they cannot access it). A forward contract (F_H, F_L) is the promise to deliver F_i units of good Z in state $i = H, L$ at $t = 2$. We assume that agents 1 and 2 bargain over (F_H, F_L). Before specifying the bargaining problem it is useful to consider the budget constraint of each agent. The constraint of agent 1 given contract (F_H, F_L) when the state is $i = H, L$ is

$$z_i^1 \leq p_i R - F_i$$

where z_i^1 is his consumption of good Z. The constraint reads as follows: in state i, agent 1 sells his output of good X for $p_i R$ units of good Z but he has to deliver F_i units of good Z (or receive if $F_i < 0$) to agent 2 in state i. Similarly, the constraint of agent 2 given contract (F_H, F_L) is

$$z_i^2 \leq (1-p_i)R + F_i$$

as he - in addition to his investment in state i - receives F_i units of good Z (or delivers if $F_i < 0$). Adding both constraints, we notice that the resource constraint the pair faces is simply

$$z_i^1 + z_i^2 \leq R.$$

Given agents can commit, there is no need to store goods so that $l_1 = l_2 = 0$. Then, the

forward contract is a solution to the following bargaining problem

$$\max_{z_i^1, z_i^2} \left[\frac{1}{2}u(z_H^1) + \frac{1}{2}u(z_L^1) - V\right]^{\theta} \left[\frac{1}{2}u(z_H^2) + \frac{1}{2}u(z_L^2) - V\right]^{1-\theta}$$

$$\text{s.t.}$$

$$z_i^1 + z_i^2 \leq R$$

The resource constraint will always bind and the first order conditions with respect to z_i^1 give us $z_i^1 = z^1$ and $z_i^2 = z^2$ for $i = H, L$ where $z^2 = R - z^1$ and z^1 solves

$$\frac{u'(z^1)}{u'(R - z^1)} = \frac{1-\theta}{\theta} \frac{u(z^1) - V}{u(R - z_1) - V}$$

Therefore, the forward contract gives full insurance to both agents as their consumption is independent of the realization of the state at $t = 2$.[1]

4. Bilateral Clearing

From now on, we assume agents cannot commit to make good on their promises. Then, the contract with full commitment is not compatible with agents' incentives: agent 1 will refuse to pay in state H as $pR > z^1$ and agent 2 will refuse to pay in state L as $pR > z^2$. Therefore, with no commitment, agents will have to pledge collateral to prevent strategic default. In this section, we concentrate on bilateral clearing where agents pledge collateral with each other. This means that, at date $t = 1$, agent 1 will surrender some amount of good X to agent 2 and agent 2 will surrender some amount of good Y to agent 1. Since agent 1 (respectively agent 2) does not have the technology to invest good Y (respectively good X), the goods serving as collateral will be stored, i.e. collateral will take the form of storage with a return of 1. Thus, storage is costly because it does not earn the return R. In addition, with bilateral clearing, agents will be able to abscond with the collateral pledged by their counterparty. This implies that - in addition to the delivery in each state - the forward contract must also specify the collateral requirements.

Given p_i, the collateral requirement must be such that agents are not better off defaulting. Note that since collateral is pledged ahead of date $t = 2$, collateral is state-independent. If

[1] It is easy to check that $z^1 \geq z^2$ whenever $\theta \geq 1/2$. Also, it must be the case that $pR > z^1$ and $z^2 > (1-p)R$. Indeed, suppose that $z^1 > pR$. Then $z^2 < (1-p)R$. However, this implies that $u(z^2) < V$ which violates the participation constraint of agent 2. Similarly for $z^2 > (1-p)R$.

c_1 and c_2 denote the collateral requirement from agent 1 and 2 respectively (and agents can run with the collateral posted by the other agents), the incentive constraint of agent 1 looks like

$$p_i\left[(1-c_1)R+c_1\right]-F_i \geq p_i(1-c_1)R+(1-p_i)c_2 \tag{1}$$

and the incentive constraint of agent 2 looks like

$$(1-p_i)\left[(1-c_2)R+c_2\right]+F_i \geq (1-p_i)(1-c_2)R+p_ic_1 \tag{2}$$

Thus, F_i is bounded above and below by $p_ic_1-(1-p_i)c_2$ so that

$$F_i = p_ic_1-(1-p_i)c_2 \tag{3}$$

In other words, incentive compatibility requires that the forward contract is exactly equal to the difference in value of collateral pledged.

Feasibility requires

$$\begin{aligned} z_i^1 &\leq p_i\left[(1-c_1)R+c_1\right]-F_i \\ z_i^2 &\leq (1-p_i)\left[(1-c_2)R+c_2\right]+F_i \end{aligned}$$

Using (3), we obtain the following budget constraints

$$\begin{aligned} z_i^1 &\leq p_i(1-c_1)R+(1-p_i)c_2 \\ z_i^2 &\leq (1-p_i)(1-c_2)R+p_ic_1 \end{aligned}$$

It should be clear that these constraints will hold with equality. Therefore, we can write the bargaining problem as

$$\begin{aligned} \max_{c_1,c_2\in[0,1]} \quad & \left[\tfrac{1}{2}\textstyle\sum_{i=H,L} u\left(p_i(1-c_1)R+(1-p_i)c_2\right)-V\right]^{\theta} \\ & \times\left[\tfrac{1}{2}\textstyle\sum_{i=H,L} u\left((1-p_i)(1-c_2)R+p_ic_1\right)-V\right]^{1-\theta} \end{aligned}$$

The first order conditions give us

$$\begin{aligned}
\frac{\theta}{1-\theta}\frac{s_2}{s_1}R\left[\sum_{i=H,L} p_i u'(z_i^1)\right] &\geq \sum_{i=H,L} p_i u'(z_i^2) \ (= \text{ if } c_1 > 0) \\
\frac{\theta}{1-\theta}\frac{s_2}{s_1}\left[\sum_{i=H,L} (1-p_i) u'(z_i^1)\right] &\leq R \sum_{i=H,L} (1-p_i) u'(z_i^2) \ (= \text{ if } c_2 > 0)
\end{aligned}$$

where s_j is the trading surplus for agent $j = 1, 2$. Using these first order conditions, denoting with $\bar{c}_j$ and $\bar{z}_i^j$ the solutions for the levels of collateral and consumption with bilateral clearing, we obtain the following result (omitted proofs are found in the Appendix).

Proposition 1. *With bilateral clearing, there is never full insurance (i.e. we know that $\bar{z}_H^1 > \bar{z}_L^1$ and $\bar{z}_L^2 > \bar{z}_H^2$) unless $R = 1$. There is $\bar{R} \geq 1$ such that $\bar{c}_1 = \bar{c}_2 = 0$ whenever $R > \bar{R}$. For all $R \in (1, \bar{R})$, $\bar{c}_1 > 0$ and $\bar{c}_2 > 0$.*

Even though agents pledge collateral to each other, it may at first appear suspicious that they can trade although they cannot commit to their promises. Indeed, suppose the solution is such that agent 1 pledges a units of collateral to agent 2 and agent 2 also pledges a units of collateral to agent 1. Then it appears that the incentive to make good on promises are not affected. Rather, the only effect is that the amount that can be invested is reduced. However, this reasoning fails to recognize that the value of the collateral differs: agent 1 wants to default in state H, but it is also in state H that the collateral in the hand of agent 2 is more valuable and the collateral in the hand of agent 1 is the less valuable. This is closely related to expression 3, saying that a contract F may not defer from the difference in value of collateral pledged. This represents an essential feature of clearing in the real world, namely the exchange of variation margins that is meant to preserve incentives.

Also we obtain conditions for a symmetric solution to exists, i.e. $c_1 = c_2 = c > 0$.

Proposition 2. *Suppose $\theta = 1/2$ and $p > R(1-p)$. Then a symmetric solution is feasible with $\bar{z}_h^1 = \bar{z}_\ell^2 = \bar{z}_h > z_\ell^1 = z_h^2 = \bar{z}_\ell$ where*

$$u'(\bar{z}_h) = \frac{p - R(1-p)}{Rp - (1-p)} u'(\bar{z}_\ell).$$

In addition, we obtain $\bar{z}_h = pR - [pR - (1-p)]\bar{c}$ and $\bar{z}_\ell = (1-p)R + [p - (1-p)R]\bar{c}$. Therefore, $\bar{z}_h$ is decreasing in $\bar{c}$ while $\bar{z}_\ell$ is increasing in $\bar{c}$. Also, notice that $\bar{z}_h - \bar{z}_\ell = (2p-1)[R(1-\bar{c}) - \bar{c}]$ such that the dispersion in consumption is increasing in R. This means

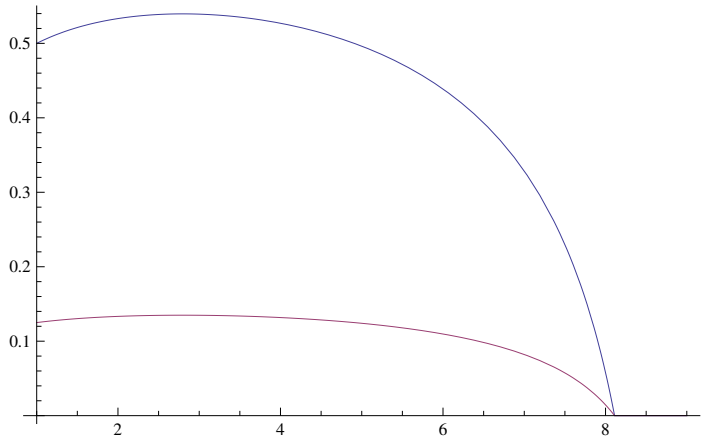

Figure 1: Symmetric solution with bilateral clearing: individual (blue) and total collateral requirement (red) as a function of R

that increasing collateral lowers dispersion and so increases insurance (respectively the level of income smoothing trade). In the economy with bilateral clearing, the total collateral requirement is $(1-\sigma)(\bar{c}_1+\bar{c}_2)/2$. Figure 1 plots it as a function of R for the symmetric case. The red curve is the total posted collateral of the economy (remember, a measure of σ pairs of agents 1 and 2 enter the safe state), while the blue curve is the collateral that a single agent posts (assuming $p = 0.85$ and $\alpha = 2$). These curves are hump-shaped for an intuitive reason. When R is small, the cost from pledging collateral is lower than the gains from insurance and, therefore, collateral is positive and increasing in R. However, as collateral becomes more and more costly to post, agents prefer to limit their insurance-motivated trade and, eventually, trade collapses.[2]

5. Segregated Collateral Clearing

In this section, we study the effect on the optimal contract and welfare of introducing a technology that allows to pledge collateral at a third party. We refer to this technology as "segregation." This technology prevents a defaulter from selling the collateral and to consume it. At the same time, it allows the non-defaulting agent to sell the defaulting agent's collateral and to consume it. One could imagine a third-party clearer that allows agents to store their collateral at its vault. In contrast to agents, the third-party clearer can commit to either give collateral back to the respective collateral provider - given no one

[2]Remember that R is not necessarily the risk-free rate of government bonds eligible as collateral. See section 2 for a discussion of R.

defaults - or to give all collateral to the non-defaulting agent.[3]

A consequence of segregation is to change the incentive constraints of the two agents. For agent 1 incentive compatibility newly requires

$$p_i\left[(1-c_1)R+c_1\right]-F_i \geq p_i(1-c_1)R \tag{4}$$

and for agent 2 incentive compatibility requires

$$(1-p_i)\left[(1-c_2)R+c_2\right]+F_i \geq (1-p_i)(1-c_2)R \tag{5}$$

Indeed, if agents 1 or 2 were to default, they would not pay F_i but they would only be left with the value of their investment. Therefore, it should be clear that incentive constraints do no longer depend on the other agent's collateral. Therefore, incentive constraints are relaxed by the fact that the agent who defaults can not run away with the other agent's collateral. From (4) and (5), the forward contract is bound by the following inequalities

$$F_h \leq pc_1$$

$$F_\ell \geq -pc_2$$

Since collateral is costly, these constraints hold with equality. Therefore, we can simplify the budget constraints for agent 1 as

$$\begin{aligned} z_h^1 &= p(1-c_1)R \\ z_\ell^1 &= (1-p)\left[(1-c_1)R+c_1\right]+pc_2 \end{aligned}$$

Similarly for agent 2,

$$\begin{aligned} z_h^2 &= (1-p)\left[(1-c_2)R+c_2\right]+pc_1 \\ z_\ell^2 &= p(1-c_2)R \end{aligned}$$

Assuming equality, the bargaining problem can be written as

$$\max_{c_1,c_2} \quad \left[\tfrac{1}{2}\textstyle\sum_{i=H,L} u\left(z_i^1\right)-V\right]^{\theta}\left[\tfrac{1}{2}\textstyle\sum_{i=H,L} u\left(z_i^2\right)-V\right]^{1-\theta}$$

[3]Such a clearing form is often found in repo markets where the third party is referred to as the tri-party agent. Theory refers to this technology as the safety-box.

subject to $c_1 \in [0,1]$ and $c_2 \in [0,1]$. The first order condition for c_1 and c_2 look as follows

$$\begin{aligned}
\frac{\theta}{1-\theta}\frac{s_2}{s_1}\left[pRu'(z_h^1)+(1-p)(R-1)u'(z_\ell^1)\right] &\geq pu'(z_h^2)\ (=\text{ if } c_1>0)\\
\frac{1-\theta}{\theta}\frac{s_1}{s_2}\left[pRu'(z_\ell^2)+(1-p)(R-1)u'(z_h^2)\right] &\geq pu'(z_\ell^1)\ (=\text{ if } c_2>0)
\end{aligned}$$

Using these first order conditions and denoting by $\hat{c}_j$ and $\hat{z}_i^j$ the solutions for the collateral and consumption levels with segregated collateral, we obtain

Proposition 3. *With segregated collateral clearing, there is never full insurance unless $R = 1$. However, given the same collateral requirement, i.e. $\hat{c}_j = \bar{c}_j$, there is more insurance with segregated collateral clearing than under bilateral clearing ($\bar{z}_h^1 > \hat{z}_h^1 > \hat{z}_\ell^1 > \bar{z}_\ell^1$* and similarly for agent 2)*. There is $\hat{R} > \bar{R}$ such that $\hat{c}_1 = \hat{c}_2 = 0$ whenever $R > \hat{R}$. For all $R \in (1, \hat{R})$, $\hat{c}_1 > 0$ and $\hat{c}_2 > 0$.*

Again, we can ask how a symmetric solution with segregated collateral looks like with segregation.

Proposition 4. *Suppose $\theta = 1/2$. Then a symmetric solution is feasible with $\hat{z}_h^1 = \hat{z}_\ell^2 = \hat{z}_h > \hat{z}_\ell^1 = \hat{z}_h^2 = \hat{z}_\ell$ where*

$$u'(\hat{z}_h) = \left[1 - \frac{R-1}{Rp}\right] u'(\hat{z}_\ell).$$

Therefore, the insurance (as measured by $\hat{z}_h - \hat{z}_\ell$) is decreasing in R and increasing in p. It is interesting to note that with segregated clearing the symmetric solution is not restricted through $p > R(1-p)$ as with bilateral clearing. Relaxed incentive constraints widen the feasibility of clearing.

We can further compare the optimal collateral requirements with bilateral and segregated collateral clearing. Let $\bar{c} = (\bar{c}_1 + \bar{c}_2)/2$ and $\hat{c} = (\hat{c}_1 + \hat{c}_2)/2$ denote the average collateral with bilateral and segregated collateral clearing. We obtain the following result

Proposition 5. *There is a $\tilde{R} \in \left[1, \bar{R}\right)$ such that for $R \geq \widetilde{R}$ we have $\hat{c}(R) \geq \bar{c}(R)$, while for $R < \tilde{R}$ there is $\hat{c}(R) < \bar{c}(R)$.*

In other words, a move from bilateral clearing to segregated collateral clearing does not necessarily reduce collateralization. Rather, for high values of R, it can be that extending insurance beyond what is feasible with bilateral clearing pays off with segregated collateral clearing even though incentive constraints are relaxed. Figure 2 plots $\bar{c}(R)$ (red curve) and

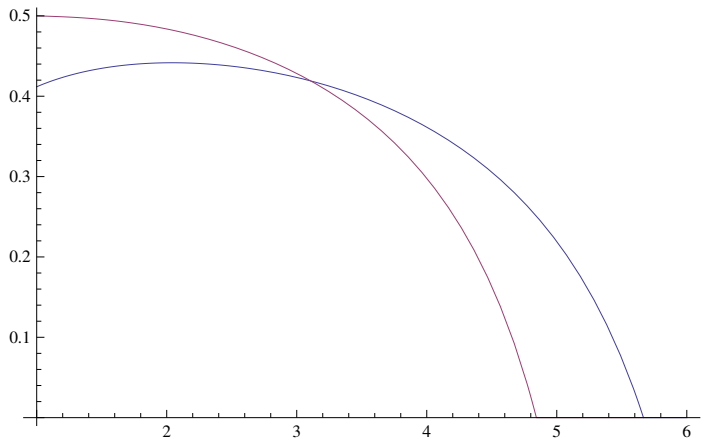

Figure 2: Average collateral with bilateral (red) and segregated collateral clearing (blue) as a function of R ($p = 0.85$ and $\sigma = 2$)

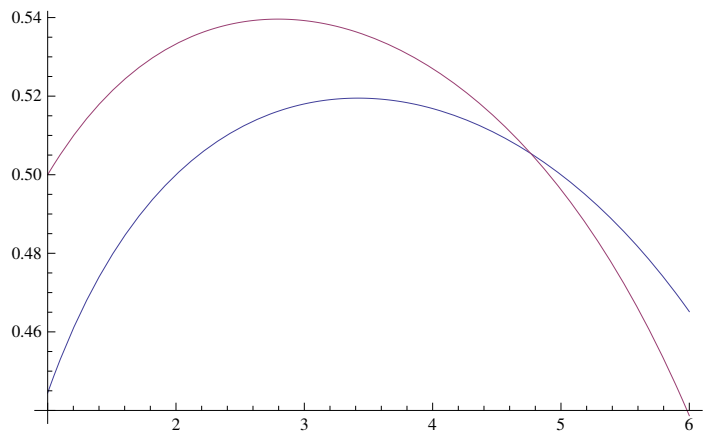

Figure 3: Average collateral with bilateral and segregated collateral clearing as a function of R ($p = 0.9$ and $\sigma = 2$)

$\hat{c}(R)$ (blue curve) for the following parameters: $p = 0.85$ and $\sigma = 2$. Notice that the blue curve is increasing when R is small (this depends on p - for p too close to 0.5 both curves are always decreasing). For p large both curves are first increasing and then decreasing. Figure 3 shows this for $p = 0.9$ and $\sigma = 2$.

Furthermore, it is easy to see that third-party clearing always dominates bilateral clearing.

Proposition 6. *Welfare under segregated clearing is higher than under bilateral clearing.*

Indeed, the constraint set of the bilateral clearing contract is given by (1) and (2) for $i = h, \ell$ and the constraint set of segregated collateral clearing is given by (4) and (5). Clearly, any contract (c_1, c_2, F_i) that satisfies the bilateral clearing constraint set also satisfies the

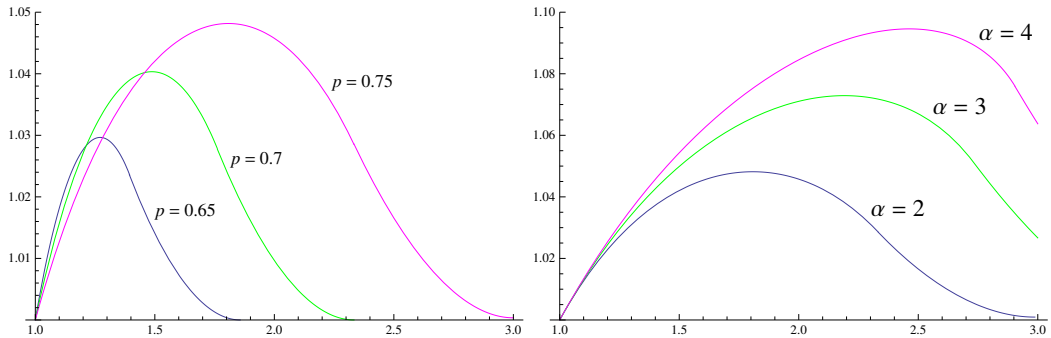

Figure 4: $\lambda - 1$ as a function of R

segregated collateral clearing constraint set. Since the objective function under both clearing arrangement is the same, it must be that the solution with segregation achieves a higher payoff for agents.

Figure 4 shows the consumption equivalent of moving from bilateral to segregated collateral clearing for different sets of parameters, that is it computes λ such that

$$\sigma u(\lambda \frac{R}{2}) + \frac{(1-\sigma)}{2}\left[u(\lambda \bar{z}_h) + u(\lambda \bar{z}_\ell)\right] = \sigma u(\frac{R}{2}) + \frac{(1-\sigma)}{2}\left[u(\hat{z}_h) + u(\hat{z}_\ell)\right]$$

so that an agent is willing to give up $\lambda - 1$ to move from an economy with bilateral clearing to an economy with segregated collateral clearing for different levels of risk aversion α and for different levels of price uncertainty p.

In Figure 4 we set $\sigma = 0.15$ and we plot λ (y-axis) as a function of R (x-axis). When $p = 0.65$ and the risk aversion is $\alpha = 2$, agents are willing to give up around 3 percent of their consumption in order to shift from bilateral to segregated clearing when $R = 1.25$. This can go up to 5 percent of consumption for $p = 0.75$ and $R = 2$. However, risk aversion has a much bigger effect than prices uncertainty. Indeed, from $p = 0.75$ if we raise α to 4 and $R = 2.5$ (magenta curve), agents are now willing to give up around 9 percent of consumption to move to segregated collateral clearing. The bottom line is that there seems to be important welfare gains from segregation. We move to central counterparty clearing next.

6. Central Counterparty Clearing

In this section, we analyze how CCP clearing fares relative to segregated collateral clearing. Next to offering segregation, a CCP is able to offer additional added-value through novation

and mutualization. While novation refers to the entrance of the CCP into the contract between a seller and a buyer of a financial contract, mutualization refers to the loss-sharing agreement implemented by the CCP among all its participants. Through novation traders eliminate bilateral contracts and accept the CCP to step in between the seller and the buyer by assuming the duties and rights of the respective counterparty. In particular, the CCP promises to fulfill the contracts given one of the original counterparties defaults. In so doing, the CCP concentrates counterparty risks on its book. To cope with these risks, a CCP usually collects margins as a defaulter-pays instrument. In addition, a CCP establishes a default fund that serves to cover losses that go beyond a defaulting participant's margin contribution, i.e. it represents a survivors-pay instrument. This default fund represents the loss sharing agreement a CCP imposes on its participants as it is funded by all participants. Essentially, participants insure potential losses created by the default of one of its counterparties among themselves.

In the following, we analyze the benefits of mutualization. We would already presume that there is no added-value to a CCP beyond segregation if the economy consists of just two traders. We refer to such a constellation as a situation of systemic risk because the clearing portfolio cannot be diversified. Diversification becomes feasible as we assume a continuum of agents that either enter a safe state or a risky state. Now agents in the safe state play a crucial role, while their role was rather muted with bilateral and segregated clearing. To the extent that safe agents have an incentive to participate in the CCP, we say that the clearing portfolio is diversifiable.

We take the view that the CCP is an insurance contract among all the participants at $t = 0$.[4] Furthermore, we assume that there is mandatory clearing, i.e. the fallback option is autarky rather than bilateral clearing or segregated clearing. We assume mandatory clearing to focus on each clearing framework's welfare contribution taking autarky as the reference.[5] Therefore, the CCP will maximize the welfare of its participants, subject to the constraint that they should be better off under the CCP contract than in autarky. We assume the CCP is able to commit, in contrast to the agents it clears for. In other words, we require a CCP is able to replace cleared forward contracts if one of the agents defaults strategically. As with segregated collateral clearing, we assume a benevolent third-party clearer, i.e. the CCP does

[4]See Diamond and Dybvig (1983) for a similar analogy.

[5]Leaving agents the choice to choose between clearing arrangement might help to understand why different form of clearing have or have not evolved in different contexts. However, it would complicate our analysis and distract the attention from one of our main messages, namely that different forms of clearing have different welfare implications.

offer its services for free and it does require the least possible amount of collateral possible to survive anticipated default (if any).

The CCP first collects default fund contributions d at $t = 0$ from all traders. These contributions cannot be made state dependent as the information is not available at $t = 0$. In some sense, default fund contributions are like membership fees. Then the CCP asks for margins m from traders that commit to the contract at $t = 1$. There can be two types of margins: m_r denotes margins posted by a measure $1 - \sigma$ of agents that are in the risky state at $t = 1$ and m_s denotes margins posted by a measure σ of agents that are in the safe state. It is worth re-emphasizing that, in fact, CCPs define their initial margins as a function of recent price fluctuations: so, we can view the risky state as one where agents know that price uncertainty is much higher than in the safe state (where there is no price fluctuations at all).

The CCP "delegates" investment, in the sense that it cannot undertake investment on behalf of agents. This is a natural assumption. But let us mention what would happen otherwise: if the CCP could re-organize investments, it could collect all the available resources and invest them with the safe agents with a sure return of $R/2$. Of course, agents in the safe state would have a very strong incentive to default on the CCP once their investment matures. While appealing, this seems a far-fetched schemes relative to current market practices. Hence, we only assume that the CCP cannot invest or direct resources for investment to some agents. Rather, the notion of safe and risky agents allows us to reflect to what degree the clearing portfolio can be diversified.

At $t = 1$, the CCP promises payments z at $t = 2$ to agents in the safe state, and payments (z_h, z_ℓ) to agents in the risky state, where z_h is the payment to agents in the high state at $t = 2$ and z_ℓ is the payment to agents in the low state at $t = 2$. Then the CCP solves the

following problem

$$\max_{d,m,z_i} \sigma u(z) + \frac{(1-\sigma)}{2}[u(z_h) + u(z_\ell)]$$

$$\text{subject to}$$

$$\sigma z + \frac{(1-\sigma)}{2}(z_h + z_\ell) = \quad \text{Feasibility}$$

$$\sigma(1-d-m_s)\frac{R}{2} + (1-\sigma)(1-d-m_r)\frac{R}{2} + \sigma\frac{1}{2}(d+m_s) + (1-\sigma)\frac{1}{2}(m_r+d)$$

$$\sigma u(z) + \frac{(1-\sigma)}{2}[u(z_h) + u(z_\ell)] \geq \sigma u\left(\frac{R}{2}\right) + \frac{(1-\sigma)}{2}[u(pR) + u((1-p)R)] \quad \text{PC}$$

$$u(z) \geq u\left((1-d)\frac{R}{2}\right) \quad \text{Interim PC}$$

$$u(z_h) + u(z_\ell) \geq u(p(1-d)R) + u((1-p)(1-d)R) \quad \text{Interim PC}$$

$$z_h \geq pR(1-d-m_r) \quad \text{IC}$$

$$z_\ell \geq (1-p)R(1-d-m_r) \quad \text{IC}$$

$$1-d-m_i \geq 0 \quad \text{for } i = s, r$$

To understand the feasibility constraint, notice that the CCP receives collateral m_s from a measure σ of agents in the safe state. Each unit of this collateral has value 1/2 in this state. Also, the CCP receives collateral m_r from a measure $1-\sigma$ of agents facing the risky state. By the law of large numbers, a measure 1/2 of this collateral will have value p while a measure 1/2 will have value $1-p$ so that the average value of the collateral posted by risky agents is 1/2.

First, notice that m_s has no impact on incentive constraints, but only reduces the resources available to the CCP. Therefore, the CCP optimally sets $m_s = 0$. To simplify notation, we will now set $m_r = m$.

Then, to understand the impact of d and m notice how they affect the CCP's feasibility constraint. If we denote the right-hand side of the feasibility constraint by F (these are the resources available to the CCP), we have $\partial F/\partial d = \frac{1}{2}(1-R)$. Therefore, increasing the default fund reduces the CCP's resources by the forgone return $1-R$ on investment in the safe and risky projects. Similarly, $\partial F/\partial m = \frac{(1-\sigma)}{2}(1-R)$. Thus, by increasing margins, the CCP gives up the return on the risky investment only. Hence, for the sake of saving resources, margins are preferred to default funds. However, notice that the CCP has an insurance motive, as it wants to have z, z_h and z_ℓ as close as possible to each other. Looking at the interim PC for the safe agents, the CCP may want to increase the default fund in

order to achieve more insurance across agents. So, we guess that the CCP will first increase m leaving $d = 0$ and then increase d.

In the Appendix, we show that the interim participation constraint for agents in the safe state at date $t = 1$ and the incentive constraint for agents in the state h at date $t = 2$ both bind. The intuition is simple. There is full insurance whenever no constraint binds. However, with full insurance, agents in state h have a very strong incentive to default (as with segregated collateral clearing). Therefore, at least their incentive constraint has to bind. If the incentive constraint of the agents in state h is binding, then safe agents will have to be promised the same amount as agents in state l. This results as a consequence of the reduced maximization problem that just takes into account the feasibility constraint and the incentive constraint of agents in state h. However, in that case, agents in the safe state at $t = 1$ will refuse to participate in the CCP as they are better off in their riskless autarky. Therefore, the interim participation constraint of the safe agents also has to bind.

We now turn to the problem where both the incentive constraints for agents in state h and the interim participation constraint for safe agents are binding. Therefore, we impose

$$\begin{aligned} z_h &= pR(1-d-m) \\ z &= (1-d)\frac{R}{2} \end{aligned}$$

Furthermore, using the feasibility constraint, we obtain

$$z_\ell = (1-p)(1-d-m)R + \frac{1}{1-\sigma}d + m$$

Notice that the CCP is redistributing all the available collateral to agents in state ℓ at $t = 2$. This is how the CCP can shift resources from one type of agents to another. Using the resulting expressions in the CCP's objective function, we obtain

$$\max_{d,m} \sigma u\left[(1-d)\frac{R}{2}\right] + \frac{(1-\sigma)}{2}\left\{u\left[pR(1-d-m)\right] + u\left[(1-d-m)(1-p)R + m + \frac{1}{1-\sigma}d\right]\right\}$$

and the first order conditions with respect to d and m give respectively,

$$\frac{\sigma}{2}\left[u'(z_\ell) - Ru'(z)\right] - \frac{(1-\sigma)}{2}\left\{pRu'(z_h) - \left[1-(1-p)R\right]u'(z_\ell)\right\} \leq 0 \qquad (= \text{ if } d > 0) \quad (6)$$

$$-\frac{(1-\sigma)}{2}\left\{pRu'(z_h) - \left[1-(1-p)R\right]u'(z_\ell)\right\} \leq 0 \qquad (= \text{ if } m > 0) \quad (7)$$

Notice that the decision to set $m > 0$ does not depend on the marginal utility of agents in the safe state. Therefore, m is only used to equate the marginal utility of agents in the risky state. We can rewrite (7) as

$$\frac{1}{2}pRu'(z_h) + \frac{1}{2}(1-p)Ru'(z_\ell) \geq \frac{1}{2}u'(z_\ell)$$

The LHS is the expected value of the investment for agents in the risky state at $t = 1$ and captures the negative effect of pledging margins. The RHS is the expected value of an additional unit of return for the agent in the low state at $t = 2$ and captures the positive effect of pledging margins (since the CCP redistributes all the available collateral to agents in state ℓ). If this inequality is reversed, agents at $t = 1$ would be willing to give up on the return on investment by pledging margin $m > 0$, to insure against being in the low state. Similarly, we can rewrite (6) as

$$\frac{\sigma}{2}Ru'(z) + \frac{(1-\sigma)}{2}\left[pRu'(z_h) + (1-p)Ru'(z_\ell)\right] \geq \frac{1}{2}u'(z_\ell)$$

Again, the LHS is the expected value of the investment for agents at $t = 0$ and it captures the negative effect of contributing to the default fund. However, the RHS is the expected benefit of the additional contribution to the default fund, as it measures the expected value of an additional unit of return for the agent in the low state at $t = 2$.

Proposition 7. *With CCP clearing, there is never full insurance unless $R = 1$. The solution is always symmetric so that $z_h^1 = z_\ell^2 = z_h$ and $z_\ell^1 = z_h^2 = z_\ell$. Consider the symmetric solution for bilateral and segregated collateral clearing. Then, given the same collateral requirement $d + m = \hat{c}$, there is more insurance with CCP clearing than with segregated clearing* (i.e. $\hat{z}_h = z_h > z_\ell > \hat{z}_\ell$). *Also, there is $R^C \geq \hat{R} > \bar{R}$ such that $d = m = 0$ whenever $R \geq R^C$.*

This is interesting because it says that the CCP solution will have trades for a larger set of parameters than segregation. However, if R is large enough, then autarky is still better than CCP clearing. The reader should notice that while Proposition 7 states that there is more insurance built in the solution of CCP clearing, this gives a very incomplete characterization of the solution. Indeed, we can find examples where $z_h - z_\ell > \hat{z}_h - \hat{z}_\ell$ so that under the optimal collateral policy rule of the CCP, the difference between what agents in state h and ℓ consume is greater than with segregated collateral clearing. Although they look more insured with segregated collateral clearing they also consume less, as we can show $z_h > \hat{z}_h$ and $z_\ell > \hat{z}_\ell$. Therefore, agents in the risky state at $t = 1$ face more risk (as measured

by $z_h - z_\ell$) than under segregated collateral clearing, but they are also better off in either state. Which effect dominates depends on the level of d. If $d = 0$ then the solution is the same as with segregated collateral. However, if $d > 0$, then agents in state ℓ obtain more collateral back than the amount they actually pledged, which makes them strictly better off than under segregated collateral clearing.

Lemma 1. *Suppose $R = 1$ then $m = 1 - 1/2p$ and $d = 0$. The levels of consumption are the same as under symmetric ($\theta = 1/2$) segregated collateral clearing.*

It should not be surprising that when collateral is costless ($R = 1$), the solution to the CCP problem is the same as the one with segregated collateral clearing. Indeed, in this case there is real benefits from asking safe agents to contribute collateral at $t = 0$. The interesting consequence is that by continuity of the solution in R, we will obtain that margins (m) are higher than contributions to the default fund (d) whenever $R > 1$ although close to 1. However, this does not tell us whether default fund contributions will be used at all, i.e. we still do not know if $m > 0$ and $d > 0$ can be a solution. This is important, because CCPs use both initial margins and default funds contributions as tools to manage their risk. Thus, it is important for a theory of CCP clearing to deliver that $m > 0$ and $d > 0$ as a solution. Indeed, we can show

Lemma 2. *Suppose $R > 1$ is low enough, then $m > 0$ and $d > 0$.*

What is more, we can also characterize regions such that total margin contributions are higher than total default fund contributions or the reverse. Basically, when collateral is not too costly, the optimal CCP arrangement is to ask for a higher margin requirement than default fund contribution, while the CCP arrangement will be to require a higher default than margin contribution if collateral is expensive.

Proposition 8. *There is $\tilde{R}_0 < R^C$ such that for all $R \in (1, \tilde{R}_0)$ then $m > d \geq 0$, and for all $R \in (\tilde{R}_0, R^C)$ then $d > m \geq 0$.*

The proof of Proposition 8 illustrates the importance of how agents in different states value consumption. If the expected value of investment for agents in the safe state at $t = 1$ is lower than the expected value of investment for agents in the risky state at $t = 1$, i.e.

$$Ru'\left(\frac{R}{2}\right) < pRu'(pR) + (1-p)Ru'((1-p)R)$$

then the CCP finds it optimal to set a higher default fund requirement. Indeed, this shifts resources from safe to risky agents in an optimal way. So, if this inequality is satisfied at $\tilde{R}$ then for lower R the CCP will find it optimal to require a higher default fund than margin requirement and we get $d > m$. Otherwise, we obtain $m > d$. Notice that when $d > 0$ and $m > 0$ we have that all agents value investment the same way at $t = 1$, or

$$Ru'(z) = pRu'(z_h) + (1-p)Ru'(z_\ell).$$

Figure 5 illustrates Proposition 8 by showing $m(R)$ and $d(R)$ for $\sigma = 0.15$, $\alpha = 2$ and $p = 0.65$.

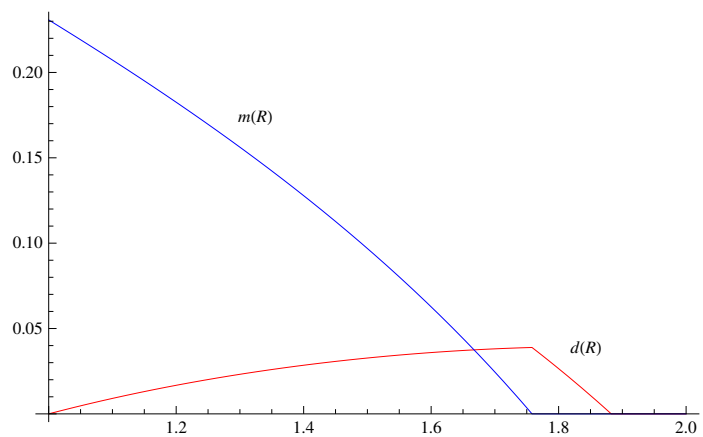

Figure 5: Margin $m(R)$ and default fund $d(R)$ requirement

We now turn to welfare gains.

Proposition 9. *Welfare under CCP-clearing is higher than under segregated clearing.*

Although we show this result in the Appendix formally, it is intuitive: There is nothing that segregated clearing does that CCP-clearing cannot do. In particular, setting $d = 0$, the constraint of the CCP is the same as under segregated clearing. Therefore, the CCP cannot do worse than segregated clearing.

We now compute the welfare gains from moving from bilateral clearing to CCP clearing and from segregated collateral clearing to CCP clearing. In this way, we can decompose the gains from CCP clearing that originate from the CCP being a clearinghouse (third-party/segregated clearing) and the CCP being an insurance. We compute the consumption

equivalent λ_b as follows:

$$\sigma u(\lambda_b \frac{R}{2}) + \frac{(1-\sigma)}{2}\left[u(\lambda_b \bar{z}_h) + u(\lambda_b \bar{z}_\ell)\right] = \sigma u(z) + \frac{(1-\sigma)}{2}\left[u(z_h) + u(z_\ell)\right]$$

so that an agent is willing to give up $\lambda_b - 1$ to move from an economy with bilateral clearing to an economy with CCP clearing. Again, using $u(z) = z^{1-\alpha}/(1-\alpha)$ with $\alpha = 2$ and $\sigma = 0.15$, Figure 6 shows λ_b (y-axis) as a function of R (x-axis) for different values of p.

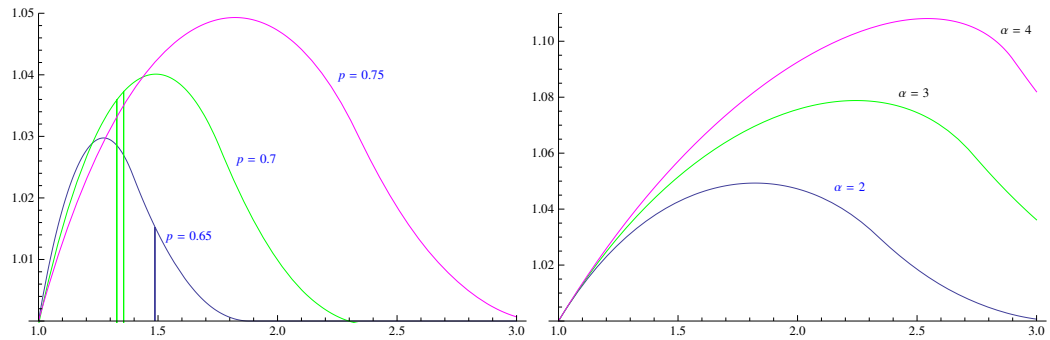

Figure 6: λ_b as a function of R

Similarly, we compute λ_s as

$$\sigma u(\lambda_s \frac{R}{2}) + \frac{(1-\sigma)}{2}\left[u(\lambda_s \hat{z}_h) + u(\lambda_s \hat{z}_\ell)\right] = \sigma u(z) + \frac{(1-\sigma)}{2}\left[u(z_h) + u(z_\ell)\right]$$

where z, z_h and z_ℓ are the solutions to the CCP problem we found above. For the same parameters as before, Figure 7 shows λ_s (y-axis) as a function of R (x-axis) for different values of p.

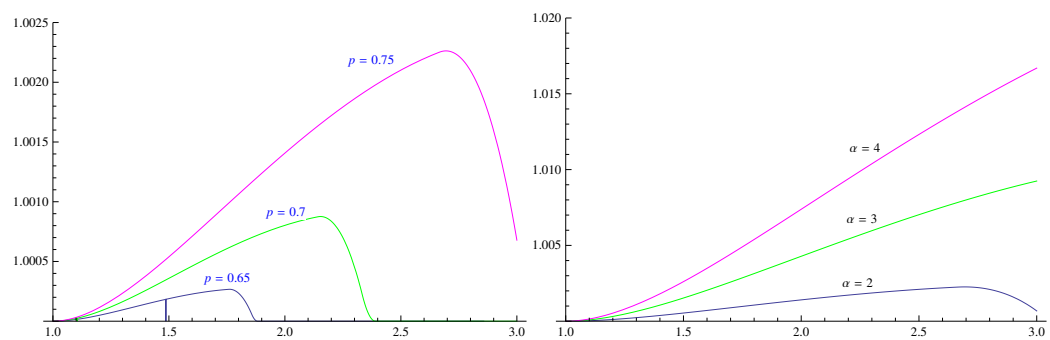

Figure 7: λ_s as a function of R

Comparing (4) with (7), it should be clear that under the parametrization of our example,

agents who are clearing bilaterally would mostly benefit from moving towards segregation. Then, once they clear with segregated collateral, they gain relatively little to move toward CCP-clearing. However, this is is not generally the case. Indeed, the benefit of a CCP is increasing as σ and relative risk aversion increase. To show this, we plot λ, λ_s and λ_b still as functions of R but for $\sigma = 0.1$ and $\sigma = 0.6$ and for $p = 0.9$.

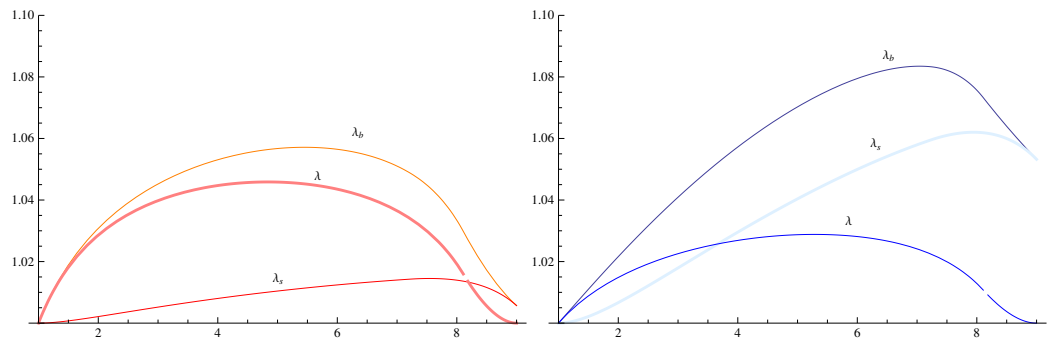

Figure 8: Left-$\sigma = 0.1$;Right-$\sigma = 0.6$

Figure 8 shows the benefit of moving from bilateral clearing to CCP-clearing as measured by λ_b decomposed into the benefit from introducing segregation into bilateral clearing, as measured by λ, and the benefit of introducing mutualization into segregation, as measured by λ_s. For low enough R the benefit of segregation outweighs the benefits of mutualization. So that adding a CCP to segregated clearing would not add much to agents in that case. However, as R increases, mutualization clearly becomes more valuable, and more so as σ increases. However, at some point, the income effect is so strong that the benefit from mutualization starts to decrease.

Finally, we show the aggregate amount of collateral that is posted in the different clearing arrangement for an economy where $p = 0.9$, $\sigma = 0.6$ and $\alpha = 2$. As Figure 9 shows, although CCP clearing achieves higher welfare, it is also much more greedy in terms of collateral.

7. Basic Stylized Facts and Policy Issues

We believe it is essential to recognize two-sided default risk to formulate sound policy advise in the field of clearing. While our model setup is just a first step to analyze clearing with two-sided default risk, model predictions under the assumption of two-sided limited commitment are in line with actual practices, related empirical evidence and experiences gained during the financial crisis. In addition, the model provides a first step to understand the market

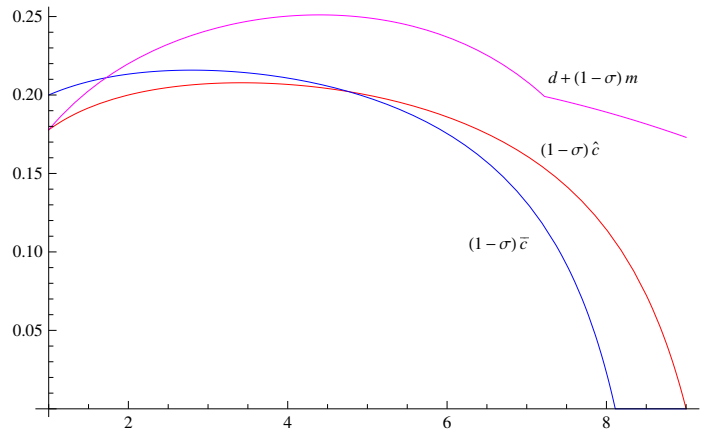

Figure 9: Aggregate collateral under different clearing arrangements

structure of clearing. Thus, the analysis of clearing with two-sided limited commitment allows to draw some initial policy conclusions that are novel or differently motivated than in the existing literature.

7.1. Basic Stylized Facts

CCPs usually rely on two major defense lines (financial resources) to be able to withstand losses caused by the default of a member. First, initial margins as a defaulter-pays instrument and, second, the default fund as a survivors-pay or loss-sharing instrument. We define the ratio of available financial resources to the default fund over total initial margins as the waterfall ratio d/m. While figure 10 reflects the wide variety of waterfall ratios observed among some major internationally active CCPs[6], proposition 8 provides a theoretical notion of the optimal waterfall ratio which is found to depend on the return on investment. Proposition 8 states that there is $\tilde{R}_0 < R^C$ such that for all $R \in (1, \tilde{R}_0)$ it holds true that $m > d \geq 0$, and for all $R \in (\tilde{R}_0, R^C)$ it holds true that $d > m \geq 0$. Thus, if $d, m > 0$, then the waterfall ratio d/m increases with increasing R.[7]

In Section 2 we laid out that R and p are the result of a normalization. R can be decomposed into the risk free rate of return $\tilde{R}$ and the return of investment q such that $R = 2q\tilde{R}$ and $p = \frac{1}{2}(1 + \tilde{x}/q)$. As a result, we find the variance of price to be defined as $Var(p) = (1-\sigma)\tilde{x}^2$ and that the variance reaches its maximum $(1-\sigma)q^2$ at $\tilde{x} = q$. Usually,

[6]Data is taken from annual reports of the year 2011.

[7]Note that if $R > R^C$ then d/m is not defined as $m = 0$; in other words, for $R \to R^C$ we have $d/m \to \infty$; similarly, if $R \to 1$ we have $d/m \to 0$.

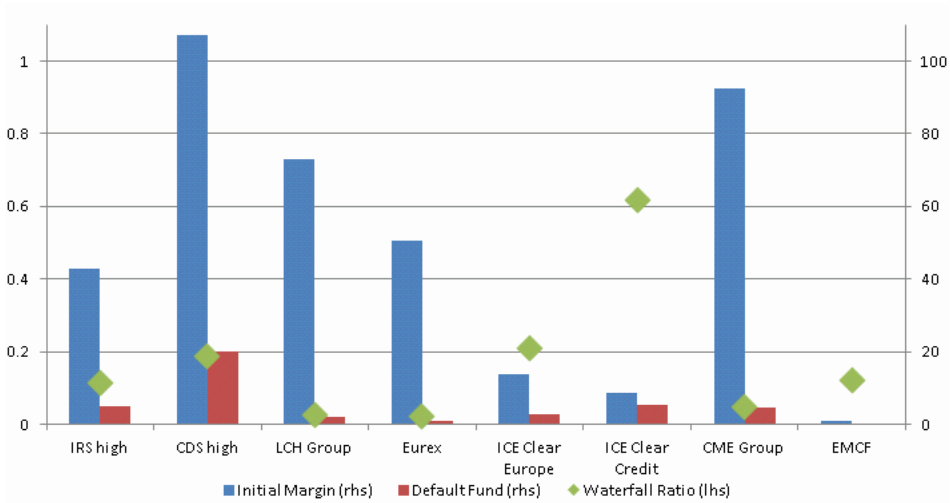

Figure 10: Waterfall ratios (lhs), yearly averages of default fund and initial margins (in USD billion, rhs) of the two artificial global IRS and CDS CCPs and some major CCPs (data can be found in the respective annual reports 2011)

CCPs determine default fund contributions such that they can withstand the default of the largest or the two largest members during severe market stress. Thus, they conduct stress tests considering periods with the highest price volatility observed, say q. In contrast, margins are usually designed to cover potential losses of a default given normal times with a probability of 99% or higher. Thus, margins are determined as a function of actually observed price volatility, say $\tilde{x}$. In light of Proposition 8, this is reflected as follows: first, a single CCP's waterfall ratio $d(q)/m(\tilde{x})$ decreases with increasing volatility $\tilde{x}$ since $d(q)$ is fixed and $m(\tilde{x})$ is increasing in $\tilde{x}$. Second, comparing two CCPs (A and B) with clearing portfolios that are different in their inherent price volatility (saying that $q^A > q^B$ and $\tilde{x}^A > \tilde{x}^B$), we expect that $d(q^A)/m(\tilde{x}^A) > d(q^B)/m(\tilde{x}^B)$.

To test these predictions with real data is difficult for two reasons. On the one hand, publicly available data is limited to a CCP's waterfall (if at all). To the best of our knowledge, data on the price volatility of a CCP's clearing portfolio is not published by any CCP. Also, many CCP's simultaneously clear several product categories. While there may be separate default funds for at least some specific categories of products such as interest rate derivatives (IRS) or credit default swaps (CDS), some CCPs may nevertheless determine margins on the basis of their overall clearing portfolio, exploiting multilateral netting opportunities between different financial contracts traded (such as OTC derivatives and repos, exchange-traded derivatives, cash equities and fixed-income). On the other hand, CCPs do not only differ in

	IRS low	CDS low	IRS medium	CDS medium	IRS high	CDS high
Initial Margin	15	10	29	51	43	107
Default Fund	5	20	5	20	5	20
Waterfall Ratio (Initial Margin)	3	0.5	5.8	2.55	8.6	5.35
Price Volatility (bp)	3.568741	7.1617	5.002501309	17.8780894	8.015211	28.90909

Table 1: Daily averages of initial margins (USD billion), default fund (USD billion), waterfall ratios and price volatilities (bp) for CCPs clearing IRS and CDS during periods of low, medium and high volatility as defined by Heller and Vause (2012)

the price volatility of their clearing portfolio, but may also substantially differ with regard to their membership structure and various risk management characteristics. These factors make it particularly difficult to compare CCPs. For instance, a CCP may establish a comparatively larger default fund because its membership structure is characterized by a few very large participants (which obviously increases the relative default fund size in contrast to a CCP with many large participants).

To circumvent these problems, we contrast model predictions with the results of a simulation exercise by Heller and Vause (2012). They simulate CCPs for the global IRS and CDS markets applying a stylized risk management model that is inspired by actual risk management practices applied by CCPs. To the greatest extent possible, Heller and Vause (2012) rely on real data that is related to the turnover generated by G14 dealers in the respective markets. They estimate initial margins and default fund contributions over the period 1/10/2004 to 30/09/2010. While initial margins depend on actual price volatility, the default fund is estimated to allow a CCP to withstand losses caused by the default of the largest participant given the maximum price volatility observed during the sample considered. In line with theory, we compare waterfall ratios for each of the two CCPs and between the two CCPs considering periods with low, medium and high price volatility (see table 1).

Heller and Vause (2012) provide estimates for a global IRS and a global CDS CCP that are designed to cover losses of default of their largest participant[8]. Table 1 reports average initial margins and average default fund contributions for three periods with low, medium and high price volatility. For each of these periods the average price volatility of the CDS[9]

[8]In 2012, the Committee for Payment and Settlement Systems (CPSS) and the International Organization of Securities Commissions (IOSCO) published new minimum standards. The 'Principles for Financial Markets Infrastructures' (PFMI) - as set out in CPSS/IOSCO (2012) - require the CCP to be able to withstand the default of the two largest members. Heller and Vause (2012) note that - with equal margins - each of two CCPs considered would have to increase its default fund by 50% to be compliant.

[9]As a proxy for overall volatility, we report the price volatility of CDX.NA.IG, a CDS index that offers protection against default losses on a portfolio of 125 North American investment-grade corporate bonds.

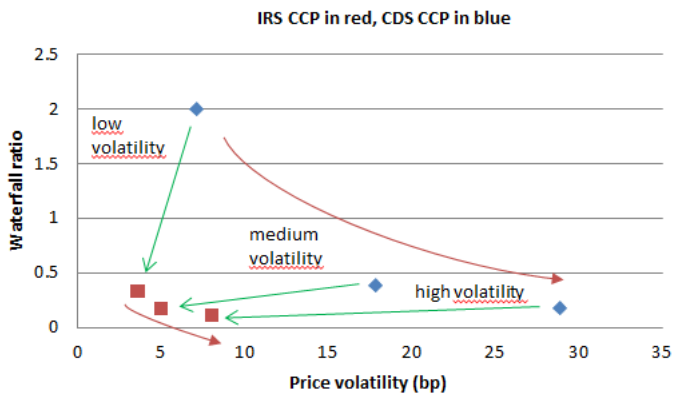

Figure 11: Waterfall ratios in relation to price volatility (bp)

CCP is greater than for the IRS[10] CCP. Figure 11 illustrates model predictions with waterfall ratios plotted against the respective price volatilities for simulated CCPs for IRS (red) and CDS (blue). CCPs' waterfall ratios for the same periods are connected by a green arrow, while the waterfall ratios of the same CCP but for different periods are connected by red arrows. Consistent with predictions, the waterfall ratio of a single CCP falls with increasing price volatility and the waterfall ratio of a CCP clearing a portfolio with a higher price volatility is lower than the one of a CCP clearing a portfolio with a lower price volatility. Thus, our model's predictions on the waterfall ratio seem to be consistent with real practices.

Nevertheless, figure 10 shows that among real CCPs there are huge differences in waterfall ratios observed. Also, figure 11 illustrates that optimal waterfall ratios for CCPs with differing clearing portfolios react structurally similar but with huge differences in scale to changing price volatility. The question is whether these differences can be explained by other relevant variables. For instance, real CCPs' default fund size might crucially depend on the CCP's membership structure. While these questions are left for future research, it is important to note its policy relevance. In particular, comparing real CCPs' waterfall ratios in relation to the price volatility and other relevant variables might be an easy way to check a CCP's efficiency as well as the adequacy of its risk management model.

[10] As a proxy for overall volatility, we report the unweighted average price volatility of the 42 IRS that represent the most traded maturities (7) and currencies (6).

7.2. Policy Issues

Overall, we provide a justification for the G20 commitment for mandatory clearing of OTC derivatives that is absent in the literature. Even though clearing is costly, in a model with two-sided limited commitment more advanced forms of clearing are welfare improving. In particular, third-party clearing is able to improve welfare if compared to a prudent bilateral clearing environment. This is important as estimates of the costs of clearing are shown to be of macroeconomic relevance – see for instance Singh (2010) and Heller and Vause (2012).

However, we also provide a rationale for why mandatory CCP clearing might not be the efficient choice. Depending on the circumstances, mutualization may not add much to welfare (see discussion after Proposition 9). In particular, if the potential for diversification is limited, mutualization may not be able to increase welfare substantially enough to justify the additional costs a CCP involves. To a certain extent, this assertion may be relaxed due to a time inconsistency problem. CCP clearing becomes more valuable as risk aversion increases, i.e. during crisis times.

Next to segregation and mutualization, CCP clearing is able to provide the benefits of multilateral netting as emphasized by Duffie and Zhu (2011) and Maegerle and Nellen (2011). While mutualization alone may not be able to justify the additional cost of a CCP, segregated collateral clearing is not able to provide the benefits of multilateral netting but is limited to provide bilateral netting. Thus, a thorough cost-benefit analysis would have to consider netting efficiency.

Nevertheless, one might ask why G20 has not defined mandatory clearing broader. Even though current policy debates focus on CCP clearing as a response to the G20 commitment for mandatory clearing of OTC derivatives, important parts of the clearing system will, indeed, remain outside of this commitment. For instance, some OTC derivatives were already granted exemption from mandatory CCP clearing, some are just too complex to be cleared through a CCP and other financial instruments such as repurchase agreements (repo) are not at all dealt with by the G20 commitment. In addition, our model may serve as an initial step towards a framework to think about how to structure capital requirements of trade exposures. This is policy relevant as Basle Committee on Banking Supervision (BCBS) plans substantial changes with the new Basle III framework.[11]

Let us further explore these issues considering repo markets. Some of these markets were prone to major turbulences during the financial crisis starting 2007, particularly so the

[11] See for instance BCBS (2012).

USD repo markets. Gorton and Metrick (2012) investigate the instability of the bilaterally cleared USD repo market that suffered from heavy increases in haircuts which they perceive to resemble a run on repo financing. While they point out that this is related to the adverse selection argument brought forward by Dang, Gorton and Holmstrom (2009)[12], we highlight that two-sided limited commitment is sufficient to explain what they refer to as a run phenomenon. In particular, higher margins and lower trade levels go along with a higher cost of collateral that is related to the price volatility of a repo's underlying collateral as well as increased risk aversion. Even though we show that these effects are inherent to clearing with two-sided limited commitment, we also highlight that these effects could be dampened with segregated collateral or CCP clearing.

In contrast to the bilateral repo market, haircuts in the USD tri-party repo market increased only moderately as shown by Copeland, Martin and Walker (2010). Rather than a run on the market as a whole, what could be seen was a run on individual dealers. For instance, the collapses of Bear Sterns and Lehman Brother were triggered by sudden decreases of funding in this market. However, Copeland, Martin and Walker (2010) also point out that the market value decreased from its peak of USD 2.8 trillion in April 2008 to an average USD 1.5 trillion during the period July 2009 to January 2010. While the issues pointed out by Martin, Skeie and von Thadden (2010) and Ennis (2012) are relevant to explain the run on individual dealers, we provide a complementary story for the drastic decline of the market overall that is connected to clearing itself.

That the clearing framework and its design alter allocational efficiency may be further backed by the observation that bilaterally cleared repo markets encountered on average more serious problems than markets that are served by a bankruptcy remote third-party clearing agents (tri-party agent or a CCP). For instance, CCP cleared repo markets in the U.K. and in Germany were reasonably stable or grew substantially during the first phase of the financial crisis, in particular so before unconventional monetary policy flooded markets with reserve balances. The same holds true for the Swiss franc repo market being served by a bankruptcy remote tri-party agent. This is in sharp contrast to the experience in the USA. While more information on different markets can be found in CPSS (2010), these experiences were not left unanswered. For instance, in order to improve the resiliency of the beforehand bilaterally cleared Canadian repo market, Canada decided to introduce a CCP to its repo

[12]The issue of information sensitivity is further discussed in Carapella and Mills (2012) in the context of CCPs.

market by the end of 2012.[13]

While third-party clearing is associated with welfare gains if compared to bilateral clearing, it is not evident that CCP clearing can improve upon segregated collateral clearing. We show that mutualization generally yields low welfare gains. Only if risk-aversion is high and/or loss-sharing is based on a highly diversified clearing portfolio, we find mutualization to increase welfare substantially. While a repo market of the size and with the number of participants as given in the U.S. might actually be thought of offering the necessary level of diversification and multilateral netting, this is not evident per se. Generally, repo markets show a tendency to be rather one-sided, i.e. some participants are always long while others are always short. Essentially, this resembles a structure with just two traders or, in other words, a situation of systemic risk with a limited potential for diversification ($\sigma = 0$). At the same time, a rather one-sided market can profit from multilateral netting to a limited extent. Thus, the added-value of CCP clearing may largely stem from segregated collateral clearing and to a much lower degree from mutualization and multilateral netting. This may explain the dominant position of repo markets with tri-party agents rather than CCPs as third-party clearers.

Duffie and Zhu (2011) show that multilateral netting efficiency suggests concentration of products and counterparties in a single CCP to be the optimal clearing structure. Provided that a clearing portfolio becomes more diversified the more participants a CCP has and the larger its trading volume is (in terms of scale and scope of financial products cleared), we suggest another rationale for a single CCP to be the optimal clearing structure. However, developments observed in the market structure for clearing services suggest the opposite: fragmentation in clearing services increasingly gains momentum. For instance, the future OTC derivatives clearing structure is likely to be substantially fragmented across products and times zones – see for instance Duffie, Lee and Lubke (2010) or Committee for Global Financial Stability (CGFS, 2011). Against the background of increasing fragmentation, Maegerle and Nellen (2011) show that interoperability - i.e. links between CCPs - may allow for the coexistence of CCPs without reducing multilateral netting efficiency. However, linking CCPs might not be able to fully exploit the benefits of mutualization as some potential for diversification of the clearing portfolio might be lost due to it being split between several CCPs. Moreover, recent announcements by CCPs suggest that fragmentation is gaining momentum also within a single CCP. CCPs increasingly start to establish clearing services for individual financial product categories on a fragmented basis, i.e. with at least specifically

[13]See Chatterjee, Embree and Youngman (2012).

dedicated default funds. For instance, major CCPs run distinct clearing processes - in particular separate default funds - for the clearing of OTC derivatives such as CDS and IRS (LCH Group, CME Group, ICE Group). While this may crucially be influenced by political considerations and regulatory requirements, it is hard to reconcile with economic reasoning.

We show that if the clearing portfolio is well diversified and there are large multilateral netting effects, CCP clearing can improve upon bilateral clearing and third-party clearing. As a consequence, we would presume CCP clearing to emerge as an outcome of private initiatives similarly as it has been observed for exchange-traded derivatives. However, we have not touched upon distributional effects as Fontaine, Perez-Saiz and Slive (2012) do in their analysis of the strategic control of CCP membership and corresponding effects on competition in OTC markets. While they presume a world with mandatory CCP clearing, nobody has yet analyzed why more advanced forms of clearing have not evolved in most OTC markets in the first place. Even though this is beyond the scope of our paper, we offer a starting point for a framework to think about the evolution of clearing arrangements. By enforcing symmetric collateralization, CCPs reduce one degree of freedom of financial contracts cleared. As a consequence, CCPs might restrict the bargaining power of some traders. While this may help to explain why CCP clearing has not emerged, the reasoning seems to fall short in explaining why segregated collateral clearing has not emerged. Thus, there might be other important aspects to OTC trading and clearing that are not touched upon in our paper but which hinder more advanced clearing forms to emerge in some OTC markets.

8. Conclusions

Absent perfect commitment, clearing arrangements are essential for trade to take place. This is particularly true so in relation to complex financial products such as traded on future markets for which trade date and settlement date are disconnected. However, this seems to equally apply to exchange traded derivatives and to repo markets. We contribute to the literature by looking at clearing from the perspective of two-sided limited commitment. This is important as clearing of financial contracts takes place between financial institutions that are prone to default, both on the side of the seller and the buyer of a financial contract.

Limited commitment of risk-averse agents is found to be sufficient to motivate different forms of clearing, namely bilateral clearing between trading agents directly, segregated collateral clearing by a third-party clearing agent and CCP clearing. We are further able to

distinguish the welfare that each of these forms of clearing contributes. Insofar, the model presented takes a further step towards policy relevance.

We characterize the welfare gains to be dependent on the return of the financial products traded. The return can be decomposed into a risk free interest rate and the price uncertainty of the respective financial contracts. Both variables mirror the cost of collateralization and, thus, the opportunity cost of clearing as forgone returns of investment. Trading, respectively income smoothing can become too expensive if the return level exceeds certain thresholds. These thresholds become more relaxed the more sophisticated the form of clearing applied is. Thus, welfare gains increase by adding segregation and mutualization.

As a novelty, we identify segregation as a key technology of clearing by a third-party clearer such as a bankruptcy remote tri-party agent for repo markets or a CCP. Surprisingly, segregation accounts for most of the welfare gains, while mutualization is found to be adding little to welfare if the portfolio cleared does not exhibit a high degree of diversification, the financial contract does not entail high price volatility and agents are not overly risk-averse.

However, this is not to say that a CCP cannot be welfare improving. Next to mutualization, a CCP enables multilateral netting that is abstracted from in our model. Because novation and mutualization are a necessary condition to allow for multilateral netting, a CCP is required. Thus, as a rule of thumb policy recommendation, one might argue that a CCP allows to increase welfare if multilateral netting allows to reduce exposures beyond what bilateral netting can achieve.

Overall, investing into more advanced forms of clearing is welfare improving. In particular, even though more advanced forms of clearing may require more collateral and correspondingly reduce productive investment, nevertheless, welfare increases as income smoothing compensates for lost returns of investment.

Furthermore, the model is in line with actual practices and predicts basic statistics in the field of CCP clearing. We find the default fund size to be increasing and initial margins to be decreasing in the cost of collateral or forgone returns of investment. Because we are able to decompose returns of investment into a risk free interest rate and the price volatility of the traded contracts, the former translates into lower waterfall ratios the more volatile the clearing portfolio of a CCP is. While we rely on a simulation exercise based on actual practices of CCPs observed, it would be interesting to observe actual waterfall ratios. In particular, this simple relation may represent a simple test to check the efficiency and adequacy of a CCP's risk management model in relation to other CCPs.

Our analysis is just a first step towards a comprehensive theory of clearing with two-sided

default risk. Future research may sharpen the analysis by introducing more complex trading environments where adverse selection plays a role. This may allow to combine theories of clearing and trading to reach a more comprehensive understanding of market dynamics and to formulate firmer policy recommendations.

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10. Appendix

Proof of Proposition 1

Notice first that full insurance cannot be a solution whenever $R > 1$. For full insurance, we need $z_H^j = z_L^j$ for $j = 1, 2$ so that each agent has the same consumption in both states (although their individual consumption may differ). It is easy to check that full insurance requires $c_1 = c_2 = R/(1+R)$. Therefore, using the first order conditions, we would obtain

$$\frac{\sum_{i=H,L} p_i R}{\sum_{i=H,L}(1-p_i)} = \frac{\sum_{i=H,L} p_i}{\sum_{i=H,L}(1-p_i)R}$$

which clearly cannot be the case unless $R = 1$. Hence, we know that $z_H^1 > z_L^1$ and $z_L^2 > z_H^2$, or

$$\begin{aligned} p(1-c_1)R + (1-p)c_2 &> (1-p)(1-c_1)R + pc_2 \\ p(1-c_2)R + (1-p)c_1 &> (1-p)(1-c_2)R + pc_1 \end{aligned}$$

and arranging both inequalities (recall that $p > 1/2$), we obtain

$$(1-c_1)R > c_2 \tag{8}$$

$$(1-c_2)R > c_1 \tag{9}$$

Also, agents prefer not to trade whenever R is too large. Indeed, suppose the FOCs are holding with strict inequality at $c_1 = c_2 = 0$, then the solution is no trade. In this case, the surplus from trade is equal, i.e. $s_2 = s_1$. Hence this is the case when

$$\frac{\theta}{1-\theta}R > \frac{pu'((1-p)R) + (1-p)u'(pR)}{pu'(pR) + (1-p)u'((1-p)R)} = 1 + \frac{(2p-1)\left[u'((1-p)R) - u'(pR)\right]}{pu'(pR) + (1-p)u'((1-p)R)} \tag{10}$$

$$\frac{1-\theta}{\theta}R > \frac{(1-p)u'(pR) + pu'((1-p)R)}{(1-p)u'((1-p)R) + pu'(pR)} = 1 + \frac{(2p-1)\left[u'((1-p)R) - u'(pR)\right]}{pu'(pR) + (1-p)u'((1-p)R)} \tag{11}$$

and the derivative of the RHS with respect to R is

$$\frac{(2p-1)}{(.)^2}\frac{u'(pR)u'((1-p)R)}{R}\left[\frac{(1-p)Ru''((1-p)R)}{u'((1-p)R)}-\frac{pRu''(pR)}{u'(pR)}\right]=0$$

as we assumed that the utility is isoelastic, with a constant coefficient of relative risk aversion. Therefore there is $\bar{R}$ such that for all $R>\bar{R}$ then $c_1=c_2=0$.

For all $R\in(1,\bar{R})$, a solution will never be such that one agent pledges collateral while the other does not, or $c_{i\neq j}=0<c_j$. Indeed, suppose this is the case, then computing z_H^j and z_L^j it is straightforward to see the that agent would prefer autarky.

Proof of Proposition 2

A symmetric contract is such that $c_1=c_2=c>0$. In this case, notice that $z_h^1=z_\ell^2=\bar{z}=p(1-c)R+(1-p)c$ while $z_\ell^1=z_h^2=\underline{z}=(1-p)(1-c)R+pc$. Clearly, $\bar{z}>\underline{z}$ and $s_1=s_2$. Since $c>0$ both first order conditions must hold at equality, so that

$$\begin{aligned}\frac{\theta}{1-\theta}R\left[pu'(\bar{z})+(1-p)u'(\underline{z})\right] &= pu'(\underline{z})+(1-p)u'(\bar{z})\\ \frac{\theta}{1-\theta}\left[(1-p)u'(\bar{z})+pu'(\underline{z})\right] &= R(1-p)u'(\underline{z})+Rpu'(\bar{z})\end{aligned}$$

This can only hold if $\theta=1/2$. Thus, assuming this is the case, we obtain

$$u'(\bar{z})=\frac{p-R(1-p)}{Rp-(1-p)}u'(\underline{z})$$

This gives us one equation in one unknown. Therefore, symmetric collateralization c is feasible if and only if $p>R(1-p)$. Also notice that in this case $u'(\bar{z})<u'(\underline{z})$ as expected. If $p\leq R(1-p)$ then we can conclude that $c_1=c_2$ is not a solution.

Proof of Proposition 3

First, we can show that full insurance cannot be achieved. Indeed, suppose that $z_\ell^i=z_h^i$ for $i=1,2$. This requires that $c_1>0$ and $c_2>0$ so that the first order conditions hold with equality. Dividing the LHS of the first FOC by the RHS of the second one, we obtain

$$\frac{pRu'(z_h^1)+(1-p)(R-1)u'(z_\ell^1)}{pu'(z_\ell^1)} = \frac{pu'(z_h^2)}{pRu'(z_\ell^2)+(1-p)(R-1)u'(z_h^2)}$$

and after some simplifications,

$$\frac{pR + (1-p)(R-1)}{p} = \frac{p}{pR + (1-p)(R-1)}$$

which can only hold if $R = 1$. Hence we cannot have full insurance with tri-party clearing.

Second, we can show that given the solution (c_1^b, c_2^b) of the bilateral clearing case, there is more insurance built in the third-party clearing case. To show this, it suffices to compare the level of consumption, $(\bar{z}_1, \bar{z}_2)$ in the bilateral and $(\hat{z}_1, \hat{z}_2)$ in the tri-party clearing case for the same amount of collateral, i.e.

$$\begin{aligned} p(1-c_1^b)R + (1-p)c_2^b &= \bar{z}_h^1 > \hat{z}_h^1 = p(1-c_1^b)R \\ (1-p)(1-c_1^b)R + pc_2^b &= \bar{z}_\ell^1 < \hat{z}_\ell^1 = (1-p)[(1-c_1^b)R + c_1^b] + pc_2^b \end{aligned}$$

Therefore agent 1 consumes less in the high state with tri-party clearing, but more in the low state: His consumption is more equal. Similarly for agent 2,

$$\begin{aligned} p(1-c_2^b)R + (1-p)c_1^b &= \bar{z}_\ell^2 > \hat{z}_\ell^2 = p(1-c_2^b)R \\ (1-p)(1-c_2^b)R + pc_1^b &= \bar{z}_h^2 < \hat{z}_h^2 = (1-p)[(1-c_2^b)R + c_2^b] + pc_1^b \end{aligned}$$

so that agent 2 consumes less in the low state (i.e. the good state for him) but more in the high state (the bad state for him): His consumption becomes more equal with tri-party clearing given the same amount of collateral.

Third, we can show that the threshold $\hat{R}$ such that there is no trade with tri-party clearing is higher than the threshold $\bar{R}$ under bilateral clearing. Indeed, suppose the FOCs are holding with strict inequality in tri-party clearing at $c_1 = c_2 = 0$, then the solution is no trade. In this case, $s_2 = s_1$. Hence this is the case when

$$\begin{aligned} \frac{\theta}{1-\theta}R &> \frac{pu'((1-p)R) + (1-p)u'(pR)}{pu'(pR) + (1-p)u'((1-p)R)} + (1-p)\frac{\left[u'((1-p)R)\frac{\theta}{1-\theta} - u'(pR)\right]}{pu'(pR) + (1-p)u'((1-p)R)} \quad (12) \\ \frac{1-\theta}{\theta}R &> \frac{pu'((1-p)R) + (1-p)u'(pR)}{pu'(pR) + (1-p)u'((1-p)R)} + (1-p)\frac{\left[u'((1-p)R)\frac{1-\theta}{\theta} - u'(pR)\right]}{pu'(pR) + (1-p)u'((1-p)R)} \end{aligned}$$

The first term on the LHS is identical to the term with bilateral clearing. Furthermore, the second term on the RHS is larger than zero. We know from before that the first term decreases with increasing R. Taking the derivative of the second term and after some ma-

nipulations we get the following condition for the second term on the RHS to be decreasing in R as well

$$\frac{(1-p)Ru''((1-p)R)}{u'((1-p)R)} \leq \frac{pRu''(pR)}{u'(pR)}$$

Thus, as it is the case for the first term, the second term too decreases with increasing R given agents have utility functions with constant relative risk aversion. Assuming this is the case, the threshold $\hat{R}$ above which $c_1 = c_2 = 0$ is such that $\hat{R} > \bar{R}$.

Finally, looking at the incentive and participation constraints, notice that $c_{i \neq j} = 0 < c_j$ is impossible. Furthermore, as long as $1 < R < \widehat{R}$, there is a solution with positive collateralization, i.e. $c_i \neq 0$ for $i = 1, 2$. Thus, with third-party clearing trading occurs even if $R \geq \bar{R}$.

Proof of Proposition 4

Obvious.

Proof of Proposition 5

We know that under bilateral clearing there is no trade for $R > \bar{R}$, while there is trade with third party clearing, i.e. $c^T(R) > c^B(R) = 0$ for $R > \bar{R}$. Let us consider optimal collateralization for $R = 1$. We know that both with bilateral and with third-party clearing full insurance can be achieved. We further know that full insurance with bilateral clearing results in $c^B/2 = c_1 = c_2 = \frac{R}{R+1} = \frac{1}{2}$. With third-party clearing perfect insurance and $R = 1$ imply $c^T = c_1 + c_2 = \frac{2p-1}{2p}$. Thus, $c^T(R) < c^B(R)$ for $R = 1$. Because the problem is well-behaved, we can deduce that there is a $\tilde{R} \in \left[1, \bar{R}\right)$ such that for $R \geq \widetilde{R}$ there is $c^T(R) > c^B(R)$, for $R < \tilde{R}$ there is $c^T(R) < c^B(R)$ and for $R = \tilde{R}$ the following equality results, $c^T(\tilde{R}) = c^B(\tilde{R})$.

Proof of Proposition 5

Obvious.

Binding constraints in the CCP problem

Let us first consider the case with no commitment problem. Then we know that the solution to the relaxed problem is $z = z_h = z_\ell = R/2$. However, this solution violates the incentive

constraint of agent h. Hence, at least this constraint has to bind. When only this constraint binds, we have

$$z_h = pR(1-d-m)$$

so that agent in state h is just indifferent between defaulting and not. The problem then becomes

$$\max_{d,m,z_i} \sigma u(z) + \frac{(1-\sigma)}{2}\left[u\left[pR(1-d-m)\right] + u(z_\ell)\right]$$

$$\text{subject to}$$

$$\sigma z + \frac{(1-\sigma)}{2} z_\ell = \sigma(1-d)\frac{R}{2} + (1-\sigma)(1-d-m)(1-p)\frac{R}{2} + \frac{1}{2}d + (1-\sigma)\frac{1}{2}m \qquad \text{Feasibility}$$

Using the feasibility constraint to replace z_ℓ in the objective function, we obtain the following FOC:

$$\begin{aligned} u'(z) &= u'(z_\ell) \\ \left[\frac{1-\sigma R}{1-\sigma} - (1-p)R\right] u'(z_\ell) &\leq pRu'(z_h) \ (=\text{if } d>0) && (13) \\ \left[1-(1-p)R\right] u'(z_\ell) &\leq pRu'(z_h) \ (=\text{if } m>0) && (14) \end{aligned}$$

Notice that $m = d = 0$ whenever $R > 1/(1-p)$ since the LHS of both inequalities is negative while the RHS is always positive. Hence, we assume that $R < 1/(1-p)$: This means that the return on the bad state is worse than the return from the storage technology. Given it must be that $m > 0$ or $d > 0$ or both (otherwise agent h defaults), the FOC gives us $m > 0$ and $d = 0$. Indeed, notice that (14) is imposing a stricter condition that (13). Therefore, if (14) holds with equality, (13) cannot hold with equality. Hence, $m > 0$ and $d = 0$ is the only possible solution unless $\sigma = 0$ or $R = 1$. Therefore, the solution when only the IC constraint for agent h binds is

$$z = z_\ell < z_h = pR(1-m),$$

where the inequality follows from the fact that $1-(1-p)R < pR$. From the feasibility constraint setting $d = 0$, we obtain

$$z_\ell = R\left[\frac{1-(1-\sigma)p}{1+\sigma}\right] + m\left[1-(1-p)R\right]\left[\frac{1-\sigma}{1+\sigma}\right]$$

Notice, however, that the maximum revenue of the CCP is $R/2$ (when setting $d = m = 0$). So if $m > 0$, $d = 0$ and $z_h > z = z_\ell$, it must be that $z < R/2$. Which implies that the interim PC constraint of safe agents is violated at the solution with only the IC of agent h binding.

CCP Problem

Lemma 3. *If $d > 0$ then $m > 0$ if $Ru'(\hat{z}) > u'(z^\ell\big|_{m=0})$ and $m_r = 0$ otherwise.*

Proof. Using (6) the fact that $d > 0$ implies

$$-\sigma\frac{R}{2}u'(\hat{z}) - \frac{(1-\sigma)}{2}pRu'(z_h) + \frac{(1-\sigma)}{2}\left[\frac{1}{1-\sigma} - 1 + 1 - (1-p)R\right]u'(z^\ell) \quad = \quad 0$$

or

$$\begin{aligned} -\frac{(1-\sigma)}{2}pRu'(z_h) + \frac{(1-\sigma)}{2}\left[1-(1-p)R\right]u'(z^\ell) &= \sigma\frac{R}{2}u'(\hat{z}) - \frac{(1-\sigma)}{2}\left[\frac{1}{1-\sigma} - 1\right]u'(z^\ell) \\ &= \frac{\sigma}{2}\left[Ru'(\hat{z}) - u'(z^\ell)\right] \end{aligned}$$

Therefore, $m > 0$ whenever $Ru'(\hat{z}) > u'(z^\ell\big|_{m=0})$, and $m_r = 0$ otherwise. Hence, $m_r > 0$ whenever

$$\begin{aligned} Ru'(\hat{z}) &> u'\left[(1-p)(1-d)R + \frac{1}{1-\sigma}d\right] \\ Ru'\left[(1-d)\frac{R}{2}\right] &> u'\left[(1-p)(1-d)R + \frac{1}{1-\sigma}d\right] \end{aligned}$$

where d solves

$$-\frac{\sigma}{1-\sigma}Ru'(\hat{z}) - pRu'(z_h) + \left[\frac{1}{1-\sigma} - (1-p)R\right]u'(z^\ell) \quad = \quad 0$$

or

$$u'(z^\ell) = \sigma Ru'(\hat{z}) + (1-\sigma)\left[pRu'(z_h) + (1-p)Ru'(z^\ell)\right]$$

□

Lemma 4. If $m > 0$ then $d > 0$ *if $Ru'(\hat{z}) < u'(z^\ell\big|_{d=0})$, and $d = 0$ otherwise.*

Proof. Using (16) the fact that $m > 0$ implies

$$-\frac{(1-\sigma)}{2}pRu'(z_h) + \frac{(1-\sigma)}{2}\left[1-(1-p)R\right]u'(z^\ell) = 0$$

while the FOC with respect to d is

$$-\sigma\frac{R}{2}u'(\hat{z}) - \frac{(1-\sigma)}{2}pRu'(z_h) + \frac{(1-\sigma)}{2}\left[\frac{1}{1-\sigma} - 1 + 1 - (1-p)R\right]u'(z_\ell) \quad \leq \quad 0$$

Hence it is

$$-\sigma\frac{R}{2}u'(\hat{z}) + \frac{(1-\sigma)}{2}\left[\frac{1}{1-\sigma} - 1\right]u'(z_\ell) \quad \leq \quad 0$$

or

$$Ru'(\hat{z}) - u'(z_\ell) \geq 0$$

with equality if $d > 0$ and $d = 0$ otherwise. Therefore, $d > 0$ if at $d = 0$ we have

$$Ru'(\hat{z}) < u'(z_\ell|_{d=0}).$$

Suppose $m_r > 0$ then this condition is

$$Ru'(\frac{R}{2}) - u'((1-p)(1-m)R + m) < 0 \tag{15}$$

where m_r solves

$$[1-(1-p)R]\,u'((1-p)(1-m)R + m) = pRu'(pR(1-m)). \tag{16}$$

□

Proof of Proposition 7

It is clear that we cannot have perfect insurance in autarky, i.e. when $m = d = 0$.

Lemma 5. If $m > 0$ and $d > 0$ then $z_h > z > z_\ell$.

Proof. With $m > 0$ and $d > 0$ then the first order conditions give us

$$\begin{aligned} pRu'(z_h) &= [1-(1-p)R]\,Ru'(z) \\ Ru'(z) &= u'(z_\ell) \end{aligned}$$

and perfect insurance requires $R = 1$. Therefore, we cannot have perfect insurance if $m > 0$ and $d > 0$. □

We now want to find conditions such that $m = d = 0$. As in the previous sections, this will be the case if R is large enough. Indeed, the conditions to obtain $m = d = 0$ are

$$\begin{aligned} \sigma R u'\left(\frac{R}{2}\right) + (1-\sigma)pRu'(pR) + (1-\sigma)(1-p)Ru'((1-p)R) &> u'((1-p)R) \\ pRu'(pR) + (1-p)Ru'((1-p)R) &> u'((1-p)R) \end{aligned}$$

Suppose first that $Ru'(R/2) > pRu'(pR) + (1-p)Ru'((1-p)R)$. Then a necessary and sufficient condition for $m = d = 0$ is

$$pRu'(pR) + (1-p)Ru'((1-p)R) > u'((1-p)R) \tag{17}$$

which holds if R is large enough. However, (17) and (12) define the same threshold for R when $\theta = 1/2$. Now, suppose that

$$Ru'(R/2) \leq pRu'(pR) + (1-p)Ru'((1-p)R),$$

then a necessary and sufficient condition for $m = d = 0$ is

$$\sigma R u'\left(\frac{R}{2}\right) + (1-\sigma)\left[pRu'(pR) + (1-p)Ru'((1-p)R)\right] > u'((1-p)R)$$

which then defines a higher threshold for R than (12). Since we already know that $\hat{R} > \bar{R}$, the result follows.

Proof of Lemma 1

When $R = 1$ we have that $m > 0$ and $d = 0$ whenever

$$Ru'(\frac{R}{2}) \geq u'((1-p)(1-m)R + m)$$

where m solves

$$[1-(1-p)R]\, u'((1-p)(1-m)R + m) = pRu'(pR(1-m))$$

and replacing $R = 1$ we obtain that m is

$$m = 1 - \frac{1}{2p}$$

such that at $d = 0$, the FOC for d is satisfied with equality.

Proof of Lemma 2

To show this, we compute $\partial m/\partial R$ evaluated at $R = 1$ using (16). Using this, we compute the total derivative of the LHS of (15) at $R = 1$. Since (15) is zero at $R = 1$, we get that $d > 0$ around $R = 1$ if and only if the LHS of (15) increases with R.

Taking the total derivative of (16), and using $x_1 = (1-p)(1-m)R+m$ and $x_2 = pR(1-m)$, we obtain for $\partial m/\partial R$,

$$\begin{aligned}\{-(1-p)u'(x_1) + [1-(1-p)R]\,(1-p)(1-m)u''(x_1) - pu'(x_2) - pRp(1-m)u''(x_2)\}\,dR \\ + \left\{[1-(1-p)R]^2 u''(x_1) + (pR)^2\, u''(x_2)\right\} dm \quad = 0\end{aligned}$$

Therefore, evaluating at $R = 1$ where $m_r = 1 - 1/2p$ we find

$$\frac{dm}{dR}\,|_{R=1} = \frac{(1-p)u'(x_1) - \frac{(1-p)}{2}u''(x_1) + pu'(x_2) + \frac{p}{2}u''(x_2)}{p^2u''(x_1) + p^2u''(x_2)}$$

and since at $R = 1$ $x_1 = x_2 = 1/2$ we obtain

$$\frac{dm}{dR}\,|_{R=1} = \frac{u'(\frac{1}{2}) - \frac{1}{2}u''(\frac{1}{2}) + pu''(\frac{1}{2})}{2p^2u''(\frac{1}{2})}$$

since we assume that the coefficient of relative risk aversion is greater or equal to one, we obtain that if p is close to 1/2 then $\frac{dm}{dR}\,|_{R=1} < 0$ while if p is close to 1 then $\frac{dm}{dR}\,|_{R=1} \geq 0$. As the coefficient of RRa increases, the threshold p above which $\frac{dm}{dR}\,|_{R=1} \geq 0$ decreases. If the coefficient of RRa is 1 (as in the case of log) then $\frac{dm}{dR}\,|_{R=1} \leq 0$ for all p. We can now turn to the LHS of (15). Taking the total derivative of the LHS, we have

$$u'(\frac{R}{2}) + \frac{R}{2}u''(\frac{R}{2}) - (1-p)(1-m_r)u''(x_1) - [1-(1-p)R]u''(x_1)\frac{dm}{dR}$$

and evaluating at $R = 1$ we

$$\begin{aligned}
u'(\frac{1}{2}) + \frac{1}{2}u''(\frac{1}{2}) - (1-p)\frac{1}{2p}u''(\frac{1}{2}) - pu''(\frac{1}{2})\frac{dm}{dR}\,|_{R=1} &= \\
u'(\frac{1}{2}) + \frac{1}{2}u''(\frac{1}{2}) - (1-p)\frac{1}{2p}u''(\frac{1}{2}) - pu''(\frac{1}{2})\left\{\frac{u'(\frac{1}{2}) - \frac{1}{2}u''(\frac{1}{2}) + pu''(\frac{1}{2})}{2p^2u''(\frac{1}{2})}\right\} &= \\
u'(\frac{1}{2}) + u''(\frac{1}{2}) - \frac{1}{2p}u''(\frac{1}{2}) - \left\{\frac{u'(\frac{1}{2}) - \frac{1}{2}u''(\frac{1}{2}) + pu''(\frac{1}{2})}{2p}\right\} &= \\
u'(\frac{1}{2}) + \frac{1}{2}u''(\frac{1}{2}) - \frac{1}{2p}u''(\frac{1}{2}) - \left\{\frac{u'(\frac{1}{2}) - \frac{1}{2}u''(\frac{1}{2})}{2p}\right\} &= \\
\left(1 - \frac{1}{2p}\right)\left[u'(\frac{1}{2}) + \frac{1}{2}u''(\frac{1}{2})\right] &\leq 0
\end{aligned}$$

where the last inequality follows from the fact that $1-1/2p \geq 0$ and the coefficient of relative risk aversion is greater than one. Therefore, (15) will be satisfied whenever R is close to 1, so that $m > 0$ and $d > 0$. By continuity of the problem, $m > 0$ and $d > 0$ will prevail for R low enough.

Proof of Proposition 8

To show Proposition 8, consider the equation defining $\tilde{R}$. From Proposition 7, we have two cases. In case (1) $\tilde{R}$ satisfies

$$p\tilde{R}u'(p\tilde{R}) + (1-p)\tilde{R}u'((1-p)\tilde{R}) = u'((1-p)\tilde{R}) \tag{18}$$

$$p\tilde{R}u'(p\tilde{R}) + (1-p)\tilde{R}u'((1-p)\tilde{R}) < \tilde{R}u'(\tilde{R}/2) \tag{19}$$

The RHS of (18) is decreasing in R at speed $(1-p)u''((1-p)R)$. The LHS of (18) is also decreasing in R (since the RRa is greater than 1) at speed

$$\begin{aligned}
p\left[u'(pR) + pRu''(pR)\right] + (1-p)\left[u'((1-p)R) + (1-p)Ru''((1-p)R)\right] &= \\
(1-\alpha)\left[pu'(pR) + (1-p)u'((1-p)R)\right]
\end{aligned}$$

where $\alpha > 1$ is the coefficient of relative risk aversion. Evaluated at $\tilde{R}$ this can be simplified using (18) to

$$(1-\alpha)\left[pu'(p\tilde{R}) + (1-p)u'((1-p)\tilde{R})\right] = \frac{(1-\alpha)}{\tilde{R}}u'((1-p)\tilde{R})$$

Now, the RHS is decreasing at speed (evaluated at $R = \tilde{R}$)

$$(1-p)u''((1-p)\tilde{R}) = -\alpha \frac{u'((1-p)\tilde{R})}{\tilde{R}}$$

Evaluated at $\tilde{R}$ it should now be obvious that the RHS is decreasing faster than the LHS. Therefore, for $R < \tilde{R}$ sufficiently close to $\tilde{R}$, we have (R is decreased below $\tilde{R}$):

$$pRu'(pR) + (1-p)Ru'((1-p)R) < u'((1-p)R)$$

Comparing this inequality with the FOC for m, this shows that it is optimal to increase m starting from $m(\tilde{R}) = 0$ as R decreases. However, by continuity (19) will also be satisfied for R close enough to $\tilde{R}$. In this case, $d(R) = 0$ for R close enough but below $\tilde{R}$. Therefore in case (1) we obtain $\tilde{R}_0 = \tilde{R}$. Moving to case (2).

In case (2), $\tilde{R}$ satisfies

$$\sigma \tilde{R} u'\left(\frac{\tilde{R}}{2}\right) + (1-\sigma)\left[p\tilde{R}u'(p\tilde{R}) + (1-p)\tilde{R}u'((1-p)\tilde{R})\right] = u'((1-p)\tilde{R}) \tag{20}$$

$$p\tilde{R}u'(p\tilde{R}) + (1-p)\tilde{R}u'((1-p)\tilde{R}) \geq \tilde{R}u'(\tilde{R}/2) \tag{21}$$

We can now use the same reasoning as before: The RHS of (20) is decreasing at speed (evaluated at $R = \tilde{R}$)

$$(1-p)u''((1-p)\tilde{R}) = -\alpha \frac{u'((1-p)\tilde{R})}{\tilde{R}}$$

while the LHS is also decreasing at rate

$$\begin{aligned} &\sigma\left[u'(\frac{R}{2}) + \frac{R}{2}u''(\frac{R}{2})\right] \\ +(1-\sigma)\left\{p\left[u'(pR) + pRu''(pR)\right] + (1-p)\left[u'((1-p)R) + (1-p)Ru''((1-p)R)\right]\right\} &= \\ (1-\alpha)\left[\sigma u'(\frac{R}{2}) + (1-\sigma)\left[pu'(pR) + (1-p)u'((1-p)R)\right]\right] \end{aligned}$$

and evaluated at $R = \tilde{R}$ this can be simplified using (21) to

$$(1-\alpha)\left[\sigma u'(\frac{\tilde{R}}{2}) + (1-\sigma)\left[pu'(p\tilde{R}) + (1-p)u'((1-p)\tilde{R})\right]\right] = (1-\alpha)\frac{u'((1-p)\tilde{R})}{\tilde{R}}$$

Hence the RHS of (20) is decreasing faster than its LHS. Therefore, for $R < \tilde{R}$ but close enough to $\tilde{R}$ we have

$$\sigma R u'\left(\frac{R}{2}\right) + (1-\sigma)\left[pRu'(pR) + (1-p)Ru'((1-p)R)\right] < u'((1-p)R)$$

Comparing this inequality with the FOC for d, this shows that it is optimal to increase d starting from $d(\tilde{R}) = 0$ as R decreases. However, by continuity (21) will also be satisfied for R close enough to $\tilde{R}$. In this case, $m(R) = 0$ for R close enough but below $\tilde{R}$. Therefore in case (1) we obtain $\tilde{R}_0 < \tilde{R}$.

Proof of Proposition 9

To show that the CCP solution does better than segregation only, we consider the CCP's constraints whenever $d = 0$. This is

$$\begin{aligned}
z_h + z_\ell &= (1-m)R + m && \text{Feasibility} \\
u(z_h) + u(z_\ell) &\geq u(pR) + u((1-p)R) && \text{PC} \\
u(\hat{z}) &= u\left(\frac{R}{2}\right) && \text{Interim PC} \\
z_h &\geq pR(1-m) && \text{IC} \\
z_\ell &\geq (1-p)R(1-m) && \text{IC}
\end{aligned}$$

while with segregation the constraints are (for the symmetric case $\theta = 1/2$):

$$\begin{aligned}
pR(1-c) &= z_h \\
(1-p)(1-c)R + c &= z_\ell
\end{aligned}$$

Clearly, the constraints under segregation are subsumed into the constraint of the CCP's problem when $d = 0$. Therefore, the CCP solution will be at least as good as the solution under segregation.