Cyclical Risk Aversion, Precautionary Saving and Monetary Policy

Bianca De Paoli*[,a], Pawel Zabczyk[b]

[a] *Federal Reserve Bank of New York and Centre for Economic Performance, LSE*
[b] *European Central Bank and Bank of England*

Abstract

This paper analyzes the conduct of monetary policy in an environment in which cyclical swings in risk appetite affect households' propensity to save. It uses a New-Keynesian model featuring external habit formation to show that taking note of precautionary saving motives justifies an accommodative policy bias in the face of persistent, adverse disturbances. Equally, policy should be more restrictive – that is 'lean against the wind' – following positive shocks. Under sufficiently persistent habits it is, in fact, optimal to increase interest rates following a rise in productivity.

Key words: precautionary saving, monetary policy, cyclical risk aversion, macro-finance, non-linearities, habits.

JEL classification: E32, G12

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*Corresponding author. Address: International Research Function, Federal Reserve Bank of New York, 33 Liberty Street, New York NY 10045. Email: *bianca.depaoli@ny.frb.org*.

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> [T]he reduction in risk associated with an easing of monetary policy and the resulting reduction in precautionary saving may amplify the short-run impact of policy [...]
>
> GOVERNOR BEN S. BERNANKE
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> *Remarks at the London School of Economics Public Lecture*
>
> *London, England, October 9, 2003*

1. Introduction

A lot of modern policy analysis is conducted using linear, or linearized, models. While these may be able to replicate salient features of macroeconomic dynamics, there are important areas where their ability to 'match data' is less satisfactory. In particular, all such models ignore the impact of uncertainty on the transmission mechanism of shocks.[1] Agents in these models do not require compensation for holding risky assets nor do they save for precautionary reasons. To the extent that risk premia are a clear feature of asset price data and uncertainty an important determinant of saving dynamics, using models so badly misspecified along these dimensions could result in systematically biased policy recommendations. In what follows we investigate this issue in more depth.

The impact of nonlinearities and risk on economic dynamics has recently been the subject of considerable attention (e.g. Rubio-Ramirez and Fernández-Villaverde (2005); Andreasen (2008); Rudebusch and Swanson (2008); Hordahl, Tristani, and Vestin (2008); Ravenna and Seppala (2006)). In our model, the presence of uncertainty introduces a desire to save for precautionary reasons.[2] And since saving decisions directly affect equilibrium interest rates, this desire

[1] All linear models are 'certainty-equivalent' and coefficients of their policy functions are independent of uncertainty (shock volatility).

[2] There has been some ambiguity as to what exactly precautionary savings are – see also Floden (2008). Our usage of the term is closest to that in Kimball (1990) and implies that,

will have direct implications for monetary policy. Given that the importance of precautionary motives has long been recognized (with some estimates suggesting that they account for 40% of all wealth accumulation[3]), assessing the policy implications of such motives appears of first-order relevance.

We use a model's ability to match the dynamics of risk-premia as a diagnostic of whether it accounts for risk correctly. Since the benchmark macro model fails to do so, we extend it by introducing persistent external habits – whose appeal in the asset-pricing context was demonstrated by Campbell and Cochrane (1999). In our framework, external habits generate cyclical swings in risk aversion, which translate into fluctuations in the desire to save for precautionary reasons. Crucially, to make this channel relevant, we consider a non-linear approximation to the consumption-Euler equation – explicitly allowing for a state-dependent precautionary-saving motive.

Our first contribution is to characterize factors determining the cyclical properties of precautionary savings. We show that these factors match those driving the dynamics of risk premia. Accordingly, a model in which risk premia vary in line with the data is likely to generate countercyclical precautionary saving motives. We also find that a countercyclical coefficient of risk aversion, which is a standard feature of all habit models, might not be sufficient to generate such dynamics. What is necessary, is that the persistence of shocks and habits is sufficiently high – i.e. agents must expect a fall in living conditions to persist in

absent uncertainty, there would be no precautionary savings. Our model assumes complete financial markets and precautionary savings do not arise from borrowing or liquidity constraints as in Deaton (1991) or Huggett and Ospina (2001).

[3]See also Carroll and Samwick (1998). Other papers highlighting the importance of precautionary savings include Leland (1968), Sandmo (1970), Carroll (1992), Kazarosian (1997) or Ludvigson and Michaelides (2001). Relatedly, Carroll, Slacalek, and Sommer (2012) argue that fluctuations in uncertainty and net wealth capture the bulk of the business-cycle variation in the US saving rate.

order for higher risk aversion to translate into a greater desire to save.[4]

We then analyze the policy implications of such swings in precautionary saving motives. First, we derive expressions for the 'natural' rate of interest – i.e. the one that would prevail if prices were fully flexible – both in a linear, 'certainty-equivalent' framework and in a world in which agents save for precautionary reasons. In doing so, we characterize monetary policy consistent with price stability in both setups.

We find that properly accounting for swings in risk appetite and the desire to save reduces the appropriate monetary policy response to supply shocks. Following a positive supply shock, coming from either an increase in productivity or a fall in the monopolistic mark-up, central bankers striving to maintain price stability cut rates to boost demand and prevent falls in the price level. However, since a persistent positive supply shock also reduces agents' desire to save for precautionary reasons, the cut in rates required to boost demand is smaller – i.e. the intertemporal substitution effect is partially offset by swings in the precautionary motive. Conversely, given that a positive demand shock merits interest rate hikes – and since associated falls in precautionary motives exacerbate the increases in demand – policy needs to respond more strongly once changes in precautionary savings are accounted for. Overall, the precautionary channel introduces a contractionary bias during booms, and an accommodative slant during downturns.

Our analytical expressions show that the size of the 'precautionary correction' is increasing in the degree of shock volatility. The implication is that ignoring the impact of swings in risk appetite and precautionary behavior would tend to lead to larger systematic policy mistakes in highly turbulent times, when shock

[4] These conditions closely mirror those for risk premium countercyclicality derived in De Paoli and Zabczyk (2012). As a result, and as discussed subsequently, our model exhibits desirable asset pricing properties.

volatility is large. In this sense, a period of good luck – i.e. low macroeconomic volatility – translates into good policy, even if policymakers rely on misspecified, linear models.

The final contribution of the paper is to characterize optimal policy under persistent habits in a non-linear setting. As in the standard New Keynesian model, the presence of nominal rigidities introduces relative price distortions that justify stabilizing inflation, while monopolistic competition implies that firms under-produce relative to the social optimum. But the presence of external habits leads to over-consumption, as households do not internalize the effect of their consumption on the economy-wide habit level. So in our model, whether steady-state output is inefficiently high or low depends on the strength of monopoly power vis-a-vis the degree of habit persistence. Besides, our derivation of the optimal monetary policy plan shows that the presence of habits may warrant a smaller interest rate cut in the face of productivity shocks – so as to curb the adverse effect of consumption today on tomorrow's marginal utility. In fact, if the effect of an increase in consumption today on future utility is long lasting – i.e. if habits are persistent – we find that it is optimal to raise, rather than cut, interest rates following an increase in productivity.

Finally, we compute the welfare performance – in our non-linear setting – of a policy designed to maximize welfare in a linear setting. We find that economic dynamics and the welfare performance of the underlying "optimal linear-quadratic policy"are similar to those in the flexible price allocation. This result suggests that the presence of external habits – and the associated welfare costs – are mostly felt in a non-linear world – or in world in which habits introduce time-varying fluctuations in risk taking, risk premia and precautionary savings.

The remainder of the paper is structured as follows. In the next section we present the model. We also characterize the linearized system of equilibrium conditions and the corresponding natural rate of interest. In Section 3, we in-

corporate the precautionary savings channel and analyze (both in analytical and numerical terms) its implications for the natural rate of interest and thus monetary policy. We then proceed by analyzing welfare and optimal policy before summarizing and highlighting possible extensions.

2. Model

Our model economy is inhabited by a continuum of consumer-producers living on the unit interval (and indexed by $j \in [0,1]$). Agents are assumed to maximize expected utility, which is given by

$$U^j = E \sum_{t=0}^{\infty} \beta^t \left(\frac{\xi_{d,t} \left(C_t^j - hX_t \right)^{1-\rho} - 1}{1-\rho} - \frac{N_t^{1+\eta} - 1}{1+\eta} \right) \tag{1}$$

where C_t^j denotes agent j's consumption, X_t is the level of habits and $\xi_{d,t}$ is a preference shock. The second term in the large bracket captures the disutility of labor N_t.

We define the coefficient of relative risk aversion as[5]

$$\vartheta(C_t, X_t) := -C_t \cdot \frac{U_{cc}(C_t, X_t)}{U_c(C_t, X_t)} = \frac{\rho}{S_t} \tag{2}$$

where surplus consumption S_t is given by

$$S_t := \frac{C_t - hX_t}{C_t}. \tag{3}$$

and $U_y(\cdot,\cdot)$ denotes the partial derivative of utility function $U(\cdot,\cdot)$ with respect to y. Since this coefficient measures agents' willingness to enter pure consumption

[5] As noted in Campbell, Lo, and MacKinlay (1997) 'risk aversion may also be measured by the normalized curvature of the value function [...] or by the volatility of the stochastic discount factor [...] While these measures of risk aversion are different from each other in this model, they all move inversely with S_t.'

gambles, given habits equal to X_t, it can be referred to as consumption risk aversion. It is easy to show that $\vartheta(C_t, X_t)$ is countercyclical, when – as in Campbell and Cochrane (1999) – S_t is used as a measure of cyclical stance.

We assume that habits X_t are 'external' – i.e. individual agents treat them as exogenous. We adopt a slow-moving habit specification under which

$$X_t = (1-\phi)C_{t-1} + \phi X_{t-1} \tag{4}$$

where ϕ controls the persistence of the habit process and small letters denote log-deviations from steady state. We further assume that preference shocks follow an autoregressive process given by

$$\varepsilon_{d,\,t+1} = \gamma_d \varepsilon_{d,\,t} + \epsilon_{d,\,t+1}$$

with $\varepsilon_{d,t} \equiv \log(\xi_{d,t})$ and the disturbances $\epsilon_{d,\,t+1}$ being mean zero, uncorrelated *i.i.d.* random variables with variance given by σ_d^2. Hereafter, all lower case variables denote log-deviations from steady-state ($x \equiv \log(X) - \log(\bar{X})$).

2.1. Household's choices

Households' period-by-period budget constraint is given by

$$C_t + \frac{V_t}{P_t} B_t^n + V_t^r B_t^r + V_t^{eq} S_t = \frac{W_t}{P_t} N_t + \frac{1}{P_t} B_{t-1}^n + B_{t-1}^r + \left(V_t^{eq} + D_t\right) S_{t-1} \tag{5}$$

On the right hand side, we have labor income and the current values of financial assets held over from the previous period. In particular, households supply N units of labor, which is remunerated at the nominal market wage W. We shall posit complete markets but include in the budget constraint those assets, whose prices are necessary to define the key returns. Those include a one-period, zero-coupon nominal bond B^n, with a face value of a unit of money, a one-period, zero-coupon real bond B^r, which pays a unit of consumption at maturity, as well as a share in a real equity index, which is a claim on the stream of firms' profits,

S. We shall adopt the convention that prices of real assets are real (i.e. denominated in units of the consumption good), while prices of nominal assets will be denominated in units of money. We denote the prices of the real and nominal bond by V^r and V^n respectively, while the (real) price of the equity share paying (real) dividends D is denoted by V^{eq}. The left hand side of the budget constraint (5) captures expenditures on consumption C, and a new portfolio of assets.

Household's choice variables are consumption C; labor supply N; as well as bond and equity holdings B^r, B^n and S respectively.[6] Defining the gross inflation rate as $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and the Lagrange multiplier on the budget constraint Λ_t, the aggregate first order conditions can be written as

$$\xi_{d,t}\left(C_t - hX_t\right)^{-\rho} = \Lambda_t, \tag{6}$$

$$(N_t)^{\eta} = \Lambda_t \frac{W_t}{P_t} \tag{7}$$

$$\Lambda_t V_t^n = E_t\left[\beta \frac{\Lambda_{t+1}}{\Pi_{t+1}}\right] \tag{8}$$

$$\Lambda_t V_t^r = E_t\left[\beta \Lambda_{t+1}\right] \tag{9}$$

$$\Lambda_t V_t^{eq} = E_t\left[\beta \Lambda_{t+1}\left(V_{t+1}^{eq} + D_{t+1}\right)\right] \tag{10}$$

$$C_t = \frac{W_t}{P_t} N_t + D_t. \tag{11}$$

where the expressions above are, respectively, those for marginal utility (6), labor supply (7), asset prices (8) - (10) and the budget constraint (11). In what follows we shall also refer to the nominal value of firms' profits $Q_t \equiv D_t P_t$. We can also define the rate of return on real and nominal bonds as $R_{t+1} \equiv (V_t)^{-1}$ and $R_{t+1}^n \equiv (V_t^n)^{-1}$, and the return on equities as $R_t^{eq} \equiv (V_t^{eq} + D_t)/V_{t-1}^{eq}$. So,

[6] The assumption of complete asset markets and focus on a symmetric equilibrium eliminate all aggregation related issues and allow us to replace individual choice variables with economy wide averages (or aggregates, as the mass of households equals 1). In this symmetric equilibrium all bonds are in zero net supply $B^r = B^n = 0$ and equity prices have to be such that households choose to own the entire stock of equity $S = 1$.

alternatively, we could rewrite equation (9) as a consumption Euler equation

$$1 = R_t E_t \mathcal{M}_{t+1} \tag{12}$$

where the stochastic discount factor $\mathcal{M}_{t+1}$ is defined as

$$\mathcal{M}_{t+1} \equiv \beta \frac{\xi_{d,t+1}(C_{t+1} - hX_{t+1})^{-\rho}}{\xi_{d,t}\left(C_t - hX_t\right)^{-\rho}}. \tag{13}$$

2.2. Firms

We assume a continuum of monopolistically competitive intermediate goods firms and a perfectly competitive final good sector, where the intermediate varieties are "repackaged"into a single consumption bundle. Monopolistic competition in the intermediary sector allows us to have firms that are price-setters, which facilitates the introduction of nominal rigidities, as in Calvo (1983).

2.2.1. The final goods sector

The final good Y_t is produced by bundling together a range of intermediate goods $Y_t\left(j\right)$ using the Dixit-Stiglitz technology $Y_t = \left[\int_0^1 y_t\left(j\right)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}$, where σ is the elasticity of substitution between the differentiated goods. Cost minimization in the final goods sector implies the following demand for each individual intermediate variety

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\sigma} Y_t \tag{14}$$

as well as an aggregate price index (equal to the price of the composite bundle) given by

$$P_t = \left[\int_0^1 p\left(j\right)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}$$

where $P_t\left(j\right)$ is the price of intermediate variety j.

2.2.2. Intermediate goods sector

There is a continuum of intermediate goods firms, indexed by j, that maximize profits, which are paid out as dividends to households holding shares. These

monopolistically competitive firms know the form of their individual demand functions, given by (14), and produce intermediate variety $y(j)$, using hired labor according to the following technology

$$y_t(j) = \xi_{y,t}^{\frac{\eta}{\eta+1}} N_t(j),^{7}$$

where total factor productivity $\xi_{y,t}^{\frac{\eta}{\eta+1}}$ is stochastic and where $\varepsilon_{y,t} = \log(\xi_{y,t})$ follows an AR(1) process of the form

$$\varepsilon_{y,t} = \gamma_y \varepsilon_{y,t-1} + \epsilon_{y,t},\ \epsilon_y \sim i.i.d.N\left(0, \sigma_y^2\right). \tag{15}$$

In each period, a fraction $\alpha \in [0,1)$ of randomly chosen producers is not allowed to change the nominal price of their output. The remaining fraction of firms, given by $(1-\alpha)$, chooses prices optimally by maximizing the expected discounted value of profits. As described in Walsh (2003), all firms adjusting prices in period t face the same problem and will thus be choosing the same price. Letting P_t^* denote the optimal price chosen by all firms that can reset their prices, and exploiting the fact that firms are owned by households, the optimal price setting of firms can be written as

$$\frac{P_t^*}{P_t} = \xi_{\mu,t} \frac{\sigma}{(\sigma-1)} \frac{PB_t}{PA_t} \tag{16}$$

where the auxiliary variables PA_t and PB_t satisfy

$$PA_t = \Lambda_t Y_t + \alpha\beta E_t \left[PA_{t+1} (\Pi_{t+1})^{\sigma-1}\right] \tag{17}$$

$$PB_t = \Lambda_t \Upsilon_t Y_t + \alpha\beta E_t \left[PB_{t+1} (\Pi_{t+1})^{\sigma}\right] \tag{18}$$

and where Υ denotes the Lagrange multiplier on the market clearing condition. Note that monopolistic competition in production leads to a wedge between the

[7] The production function has the power $\frac{\eta}{\eta+1}$ on productivity A for consistency with a Yeoman-farmer version of the model.

marginal utility of consumption and the marginal disutility of production, represented by $\xi_{\mu,t}\frac{\sigma}{(\sigma-1)}$. We allow for fluctuations in this wedge by assuming that $\varepsilon_{\mu,t} = \log\left(\xi_{\mu,t}\right)$ follows an exogenous process given by[8]

$$\varepsilon_{\mu,t+1} = \gamma_\mu \varepsilon_{\mu,t} + \epsilon_{\mu,t+1},\ \epsilon_\mu \sim i.i.d.N\left(0, \sigma_\mu^2\right). \tag{19}$$

Moreover, if we define a price dispersion term as $PD_t \equiv \int_0^1 \left(P_t\left(j\right)/P_t\right)^{-\sigma} dj$, the Calvo price setting structure implies that average prices satisfy

$$PD_t = (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\sigma} + \alpha\left(\Pi_t\right)^\sigma PD_{t-1} \tag{20}$$

and *aggregate* production can be written as

$$Y_t = \frac{\xi_{y,t}^{\frac{\eta}{\eta+1}} N_t}{PD_t}.$$

Finally, the first order conditions with respect to N_t yield the following labor demand equation

$$\frac{W_t}{P_t} = \Upsilon_t \xi_{y,t}^{\frac{\eta}{\eta+1}}. \tag{21}$$

2.3. Monetary Policy

We close the model with a description of monetary policy. In the first part of our analysis we assume that the central bank follows a targeting rule that ensures price stability – i.e. it sets $\pi_t \equiv \log(P_t/P_{t-1}) = 0$ for every t in order to replicate the flexible price allocation. In subsequent sections of the paper we consider the case in which the monetary authority uses different Taylor-type rules, before finally considering the case in which the central bank follows a policy that maximizes social welfare.

[8] Under one interpretation, these exogenous fluctuations reflect taxes rebated lump-sum to households, meaning that $\xi_{\mu,t}$ has no further fiscal consequences.

2.4. Summary: Log-linear equilibrium conditions

To keep the model parsimonious and allow for an analytical representation of the results, our framework abstracts from capital, there is no storage technology, and agents are homogenous – as in the canonical New Keynesian model. Effectively, this means that in equilibrium there are no savings. Nevertheless, as we will demonstrate shortly, agents' willingness to save does affect dynamics – as reflected by the non-linear approximation to the Euler equation.

In order to understand the equilibrium dynamics in a world where risk plays no role, we now summarize the log-linearized system of equilibrium conditions. After imposing market clearing $C_t = Y_t$, we can write

$$\begin{cases} r_{l,t} = & E_t(\rho\,(1-h)^{-1}\,(\Delta y_{l,t+1} - h\Delta x_{l,t+1}) - \Delta\varepsilon_{d,t+1}) \\ \pi_{l,t} = & k\left(\kappa_0\,(1-h)^{-1}\,y_{l,t} - \rho\,(1-h)^{-1}\,hx_{l,t} - \eta\varepsilon_{y,t} + \varepsilon_{\mu,t} - \varepsilon_{d,t}\right) + \beta E_t\pi_{l,t+1} \\ x_{l,t} = & (1-\phi)y_{l,t-1} + \phi\,x_{l,t-1}. \end{cases}$$

where $k = (1-\alpha\beta)(1-\alpha)/(\alpha(1+\sigma\eta))$ and the subscript l denotes variables in the linearized system.[9]

By setting $\pi_{l,t} = 0$, we can derive the equilibrium interest rate consistent with price stability in a *linear world*

$$r^*_{l,t} = \kappa_1 E_t(\Delta\varepsilon_{y,t+1} - \eta^{-1}\Delta\varepsilon_{\mu,t+1} - (1-h)\Delta\varepsilon_{d,t+1} - h\Delta x^*_{l,t+1}) \tag{22}$$

where $\kappa_0 = (1-h)\eta + \rho$ and $\kappa_1 = \rho\,\eta\,\kappa_0^{-1}$. Note that we could also write the expression for the linear natural rate in terms of demand and supply shocks by defining $\varepsilon_{s,t} = \varepsilon_{y,t} - \eta^{-1}\varepsilon_{\mu,t}$ as a composite supply shock. In this case, we would have

$$r^*_{l,t} = \kappa_1 E_t(\Delta\varepsilon_{s,t+1} - (1-h)\Delta\varepsilon_{d,t+1} - h\Delta x^*_{l,t+1}). \tag{23}$$

[9] In the system above supply is written in terms of output rather than the output gap – or a measure of output relative to its flexible price allocation. Alternatively, we could also write the Phillips curve as $\pi_{l,t} = k\,(1-h)^{-1}\left(\kappa_0(y_{l,t} - y^*_{l,t}) - \rho h(x_{l,t} - x^*_{l,t})\right) + \beta E_t\pi_{l,t+1}$ where the subscript $*$ denotes the flexible-price allocation of the corresponding variable.

Expression (23) shows that the interest rate consistent with full price stability falls [rises] following a positive supply [demand] shock – with the magnitude of the response, on impact, given by κ_1 $[(1-h)\kappa_1]$.

For future reference, we can also characterize the flexible price allocation of output as

$$y^*_{l,t} = \frac{(1-h)}{\kappa_0}(\rho(1-h)^{-1}hx^*_{l,t} + \varepsilon_{d,t} + \eta\varepsilon_{y,t} - \varepsilon_{\mu,t}) \tag{24}$$

or, in terms of the composite supply shock

$$y^*_{l,t} = \frac{(1-h)}{\kappa_0}(\rho(1-h)^{-1}hx^*_{l,t} + \varepsilon_{d,t} + \eta\varepsilon_{s,t}). \tag{25}$$

3. Cyclical Risk Aversion and Precautionary Saving

We now consider the minimum departure from a linear model, in which we can analyze the impact of cyclical swings in risk aversion and precautionary saving motives on the natural rate of interest. As a first step, we retain the linear specification of equilibrium conditions other than the Euler equation (12) – an approach that is similar to the one commonly used in the macro-finance literature.[10] This allows us to single out the effect of precautionary savings in the model's dynamics. Also, under this assumption we can obtain analytical solutions for the determinants of precautionary behavior. Nevertheless, we subsequently relax it and assess the implications of having a non-linear approximation to the entire model.

So to capture the precautionary savings motive we exploit the fact that under

[10] Many macro-finance papers consider a linearized macro model while Euler equations for asset prices, such as the risk-free rate, are determined recursively and approximated non-linearly (see, for example, Jermann (1998)).

conditional log-normality of $\mathcal{M}_t$ the Euler condition (12) becomes[11]

$$-r_t^* = \underbrace{E_t(m_{t+1}^*)}_{\text{Intertemporal substitution effect}} + \frac{1}{2} \underbrace{var_t(m_{t+1}^*)}_{\text{Precautionary savings effect}} \tag{26}$$

where $m_{t+1}^* \equiv \log(\mathcal{M}_{t+1}^*/\bar{\mathcal{M}})$ and the upper bar denotes steady state values. So, although in our framework there are no actual savings, the interest rate that clears the bond market is affected by agents' willingness to save both for precautionary and intertemporal smoothing reasons.

While linear models capture the intertemporal substitution effect, they ignore the "precautionary"term $var_t(m_{t+1}^*)$. Accordingly, to analyze how the precautionary savings channel affects the transmission mechanism of shocks, we need to understand the determinants of $var_t(m_{t+1})$. In particular, we would like to evaluate how such precautionary motives change over the cycle. Defining

$$\widetilde{\mathcal{M}}_{t+1}^* = \beta \frac{(C_{t+1}^* - hX_{t+1}^*)^{-\rho}}{(C_t^* - hX_t^*)^{-\rho}} \tag{27}$$

we can write

$$var_t(m_{t+1}^*) = var_t(\tilde{m}_{t+1}^*) + cov_t(\tilde{m}_{t+1}^*, \Delta\varepsilon_{d,t+1}) + \sigma_d^2. \tag{28}$$

As shown in the Appendix, the covariance term in Equation (28) is necessarily countercyclical (and equal to zero, absent demand shocks). And so, given that shocks are homoskedastic (i.e. σ_d^2 is constant), the countercyclicality of $var_t(\tilde{m}_{t+1}^*)$ suffices to establish countercyclicality of $var_t(m_{t+1}^*)$. We therefore approximate the former term to third order, the lowest which allows for time

[11] This equation holds up to second order without any distributional assumptions on the stochastic discount factor.

variation[12]

$$var_t(\tilde{m}^*_{t+1}) = \kappa_1^2(\eta^{-2}\sigma_d^2 + \sigma_y^2 + \sigma_\mu^2)(1 - \kappa_y\eta\varepsilon_{y,t} + \kappa_\mu\varepsilon_{\mu,t} - \kappa_d\varepsilon_{d,t} + \kappa_x x_t^*) \quad (29)$$

with

$$\kappa_y = \frac{2h\eta(h\gamma(1-\phi) + \kappa_0(\gamma_y + \phi - 1))}{\kappa_0^2} \quad (30)$$

$$\kappa_d = \frac{2h\eta(h\gamma(1-\phi) + \kappa_0(\gamma_d + \phi - 1))}{\kappa_0^2} \quad (31)$$

$$\kappa_\mu = \frac{2h\eta(h\gamma(1-\phi) + \kappa_0(\gamma_\mu + \phi - 1))}{\kappa_0^2} \quad (32)$$

$$\kappa_x = \frac{2h(\rho+\eta)(\kappa_0 + \rho h(1-\phi))}{\kappa_0^2} \quad (33)$$

The above condition for $var_t(\tilde{m}^*_{t+1})$ highlights three channels through which uncertainty affects investors' behavior: the overall level of macroeconomic volatility – given by σ_d^2, σ_y^2 and σ_μ^2; investors' risk aversion – given by ρ, which in turn determines κ_1; current and past economic conditions – as summarized by the state variable x_t^* and shocks $\varepsilon_{d,t}$, $\varepsilon_{y,t}$ and $\varepsilon_{\mu,t}$.

Note that if, for simplicity, one were to assume that mark-up and productivity shocks have the same degree of persistence, then equation (29) could also be written in terms of the composite supply shock $\varepsilon_{s,t}$ as

$$var_t(\tilde{m}^*_{t+1}) = \kappa_1^2(\eta^{-2}\sigma_d^2 + \sigma_y^2 + \sigma_\mu^2)(1 - \kappa_s\eta\varepsilon_{s,t} - \kappa_d\varepsilon_{d,t} + \kappa_x x_t^*) \quad (34)$$

where $\kappa_s = \kappa_y$.

Equation (29) demonstrates that as long as investors are risk averse ($rho > 0$), uncertainty affects their consumption decisions ($\Rightarrow \kappa_1 > 0$). It also illustrates

[12] Note that rather than take the full third order approximation to the Euler equation here we only approximate the variance term. This restriction – which permits us to single out the precautionary savings effect and allows us to derive tractable analytical results – is relaxed in our numerical exercises.

the fact, that without habit formation ($h = 0$) the strength of the precautionary saving motive would not vary over the cycle ($\kappa_y = \kappa_\mu = \kappa_d = \kappa_x = 0 \Rightarrow var_t(\tilde{m}^*_{t+1})$ is constant).[13] Furthermore, inspecting expressions (30) – (31) reveals that

$$\gamma_d + \phi \quad > \quad 1 \Rightarrow \kappa_d > 0 \tag{35}$$

$$\gamma_y + \phi \quad > \quad 1 \Rightarrow \kappa_y > 0 \tag{36}$$

$$\gamma_\mu + \phi \quad > \quad 1 \Rightarrow \kappa_\mu > 0 \tag{37}$$

which means that if shocks affecting economic activity are sufficiently persistent and habits adjust slowly, then $var_t(\tilde{m}^*_{t+1})$ will increase after a fall in productivity, a fall in demand or an increase in firms' mark-up – i.e. precautionary savings will vary countercyclically.[14] In other words, this result implies that investors will increase their willingness to engage in precautionary saving following bad shocks if they expect future economic conditions to remain poor (consumption to remain persistently close to the habit level).

If, on the other hand, the expectation is for an improvement in economic prospects, then negative shocks might not translate into higher precautionary savings – even if the coefficient of risk aversion given by (2) increases. This is because if habits are fast moving and consumption recovers quickly, agents faced with the bad shock will rapidly get used to lower levels of consumption, while at the same time the latter quickly recovers. This means that investors actually expect consumption to be far above its habit level in the future and therefore might be less inclined to engage in precautionary savings. As discussed in De Paoli and Zabczyk (2012), similar conditions are necessary to ensure that risk premia are countercyclical.

[13] Section A.1 in the Appendix shows that, in this case, not only is $var_t(\tilde{m}_{t+1})$ constant, but $var_t(m_{t+1})$ is also time invariant.

[14] As shown in (24), such shocks have a negative effect on economic activity.

4. Precautionary Saving and the Natural Rate of Interest

The implications of precautionary saving for interest rates will, therefore, depend on the structural characteristics of the economy. Absent consumption habits, with time invariant risk aversion, the presence of uncertainty will affect the average level of the natural interest rate, but not its dynamics. In this case, the response of the natural interest rate to shocks would not be affected by buffer-stock saving motives. In the general case, however, changes in perceived uncertainty (captured by changes in $var_t(m_{t+1})$) would generate fluctuations in the equilibrium interest rate – with ramifications for the conduct of monetary policy.

In what follows, we consider the case in which the central bank's goal is to maintain price stability. Crucially, if the monetary authority uses interest rates as an instrument to achieve this goal, then knowing the behavior of the natural rate of interest would be key – as policy rates that ensure price stability track this rate. But how does precautionary behavior affect the natural rate and, thus, the appropriate policy response to shocks?

Equation (23) implies that the magnitude of responses of the natural rate to a supply shock in a 'linear' world is given by κ_1. When accounting for uncertainty, the size of these responses also depends on the cyclicality of precautionary savings. If shocks and habits are persistent, and thus precautionary behavior is countercyclical, then the response to shocks is dampened. That is, the precautionary savings effect (captured by $\kappa_s > 0$ in Equation (34)) counterbalances the intertemporal substitution effect (captured in Equation (23)). A negative supply shock, coming from either a fall in productivity or an increase in the monopolistic mark-up, increases the perceived riskiness of the economic environment, which raises investors' willingness to save and puts downward pressure on interest rates. As a result, the equilibrium interest rate that is consistent with stable prices will

be lower than in a linear economy. These results thus suggest that interest rates should respond less to supply shocks when precautionary savings are taken into account.[15]

We now consider the case of preference, or demand, shocks. Condition (35) shows that when uncertainty is introduced in a model that features persistent shocks and habits, negative preference shocks also lead to higher precautionary savings. But in this case incorporating uncertainty magnifies the impact of the shock. That is, the precautionary savings effect (captured by $\kappa_d > 0$ in Equation (34)) reinforces the intertemporal substitution effect (captured in Equation (23)). Accordingly, in such settings, policymakers striving for price stability should respond more aggressively to demand shocks.

Higher precautionary motives can be thought of as introducing an extra negative demand shock – both following negative productivity and preference shocks. Since productivity and demand shocks call for opposite interest rate reactions (when policymakers' aim is to maintain price stability) these results suggest that depending on the source of the shock, policy that ignores precautionary savings will either undershoot or overshoot its' appropriate level. But the general prescription is that following expansionary shocks (i.e. a positive demand or supply shock) monetary policy should be more restrictive than in a certainty equivalent world, while when the shocks are contractionary monetary policy should be more accommodative.

[15] Note that our analysis evaluates the implications of allowing for higher order approximations (so as to capture the precautionary savings effect) in a model with habits. This is different than the analysis of Amato and Laubach (2004) who assess the differences in the dynamics of interest rates in a linear model with and without habits. Their findings suggest that habits increase the size of interest rate fluctuations because they increase the intertemporal substitution effect, with similar findings reported in Campbell and Cochrane (1999). Our analysis, on the other hand, focuses on the fact that, in a model *with* habits, allowing time-varying risk aversion to affect agents willingness to save reduces the response of the natural rate to supply shocks.

4.1. Some numerical simulations

The analysis developed so far offered an analytical representation of the monetary policy transmission mechanism. However, a numerical illustration of differences in policy responses is of independent interest since it sheds some light on the quantitative relevance of our results. The numerical simulations also allow us to assess the implications of relaxing the assumption of linearity in the supply condition.

For our calibration we define one period as a quarter and set $\beta = 0.99$ to yield a 4% annualized steady-state real interest rate. As in Campbell and Cochrane (1999) the parameter ρ is equal to 2.37 and the degree of habit persistence ϕ is set to 0.97. Following Canzoneri, Cumby, and Diba (2007) we assume a value of 6 for the inverse of the elasticity of labor supply η and set $\alpha = 0.783$ to obtain an average length of price contracts of 4.6 quarters. The elasticity of substitution between differentiated goods σ is assumed to take the value of 10 in line with Benigno and Woodford (2005). Similar to Juillard, Karam, Laxton, and Pesenti (2006) and Banerjee and Batini (2003) we calibrate the habit size parameter to $h = 0.85$. As in Smets and Wouters (2003, 2007), the persistence of supply and preference shocks is set to 0.997 and 0.9 respectively, and the variance of productivity shocks is 3 times higher than that of preference shocks. Finally, we calibrate the overall level of shock volatility to match the variance of US non-durable log-consumption growth in the Campbell and Cochrane (1999) data set (spanning 1946 Q2 to 1996 Q4). The values of all parameters are summarized in Table 4.1.[16]

[16] Some of the values used in our calibration are based on micro-data while others come from linear, general equilibrium models. Arguably, the fact that we allow for non-linearity could justify amending some of these parameters. For example, while a β of 0.99 implies a deterministic-steady state value of the interest rate equal to 4%, the stochastic mean would be below that. For this reason we verified that our results continue to fold for lower values of beta

Noticeably, our benchmark calibration follows closely the ones chosen in models that try to match risk premium dynamics. We believe that a model that correctly captures agents' attitude towards risk and uncertainty should reproduce basic risk premia observations. So, following the insights of Campbell and Cochrane (1999) and De Paoli and Zabczyk (2012), we assume a high level for the habit parameter and a very slow-moving process for the habits. The model then generates average excess equity returns of around 6% per annum, as well as variances and covariances of log-consumption growth and bond and equity returns close to those in post-war US data.[17]

We begin the quantitative part of our investigation by comparing the level of the natural rate of interest in a linear world with the one that would prevail if the precautionary savings channel was additionally taken into account. Differently from the theoretical part, we do not maintain a linearized version of the Phillips curve. Instead we consider a non-linear (or third order) approximation of the full model. This is done to preserve the accuracy of the numerical solution.[18]

Figure 1 illustrates the response of the natural rate of interest to a one standard deviation productivity shock in a linear model (black line) and in a non-linear model (dashed line). The chart shows that the fall in the natural rate is smaller once the precautionary saving motive is incorporated. More specifically, once the decreased desire to save is taken into account, the reduction in interest

– including β of 0.982, which yields an ergodic mean of the interest rate equal to four percent. A sensitivity analysis for other parameters is presented below.

[17] The exact values of these model implied moments will, of course, vary somewhat depending on the assumptions made about monetary policy.

[18] We compute a third order approximation of the model using perturbation methods as implemented in *Dynare++*. After computing Euler equation residuals corresponding to different model specifications, we established that using a "partially linear" model, as proposed in the theoretical part, produces significant *numerical approximation errors*, which is why those results are not reported subsequently.

Parameter	Value	Notes:
β	0.99	To yield a 4% steady-state real interest rate
η	6	As in Canzoneri, Cumby, and Diba (2007)
ρ	2.37	Following Campbell and Cochrane (1999)
α	0.78	Length of average price contract 4.6 quarters
σ	10	Following Benigno and Woodford (2005)
h	0.85	Juillard, Karam, Laxton, and Pesenti (2006) and Banerjee and Batini (2003)
ϕ	0.97	Following Campbell and Cochrane (1999)
γ_d	0.9	Following Smets and Wouters (2003)
γ_μ	0.997	Following de Walque, Smets, and Wouters (2005)
γ_p	0.997	Following de Walque, Smets, and Wouters (2005)
$\sigma_y^2/\ \sigma_d^2$	2	Following Smets and Wouters (2003)
$\sigma_y^2/\ \sigma_\mu^2$	3	Following Smets and Wouters (2003)
$\sigma_{\Delta c}^2$	0.008%	US non-durable Δc data (1946Q2 - 1996 Q4)

Table 1: Parameter values used in the quantitative analysis

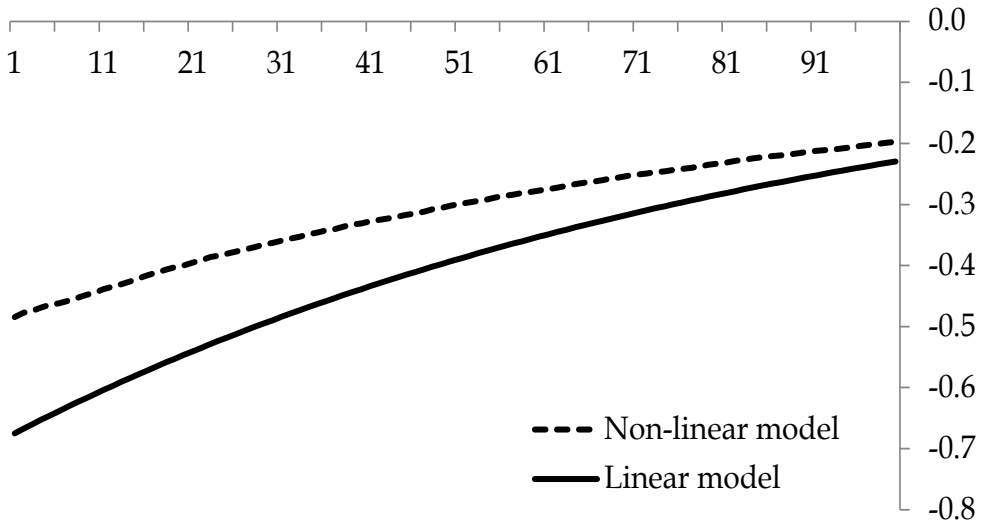

Figure 1: Natural rate of interest following a one standard deviation positive productivity shock (annualized, in percentage points)

rates required to boost demand sufficiently to prevent drops in prices changes from approximately 70bp (annualized) to less than 50bp on impact, under our benchmark calibration. Figures 2 and 3 further illustrate, that the differences in responses are smaller when we reduce the habit parameter (h) and bigger when we increase the volatility of shocks (increasing $\sigma_{\Delta c}$). Additionally, Figure 4 shows that when we reduce the persistence of the shock to a value of $\rho_y = 0.95$, the response of interest rates is still 10bp smaller in the nonlinear model. However, while these differences quickly vanish as ρ_y decreases further, we also note that the corresponding specifications imply unrealistically high interest rate volatility, suggesting the models would fail to match the dynamics of asset prices (this can also be seen in Figure 4, where interest rates on impact fall by around 400bp).

In the simulations above, when going beyond the linear approximation, our solution method accounts for nonlinearities in the entire model – i.e. for those coming from both the supply equation (or the Phillips curve) as well as the

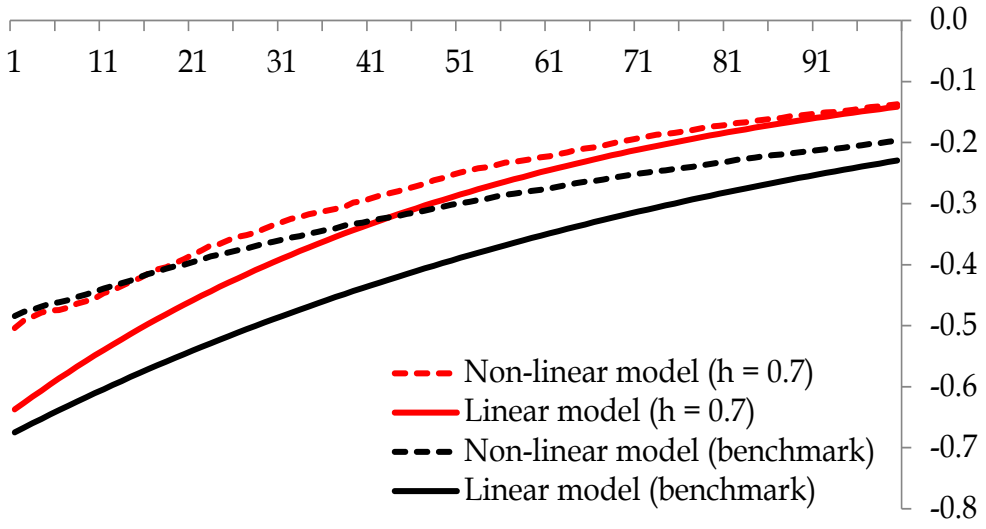

Figure 2: Sensitivity of results to changes in habit parameter (annualized, in percentage points)

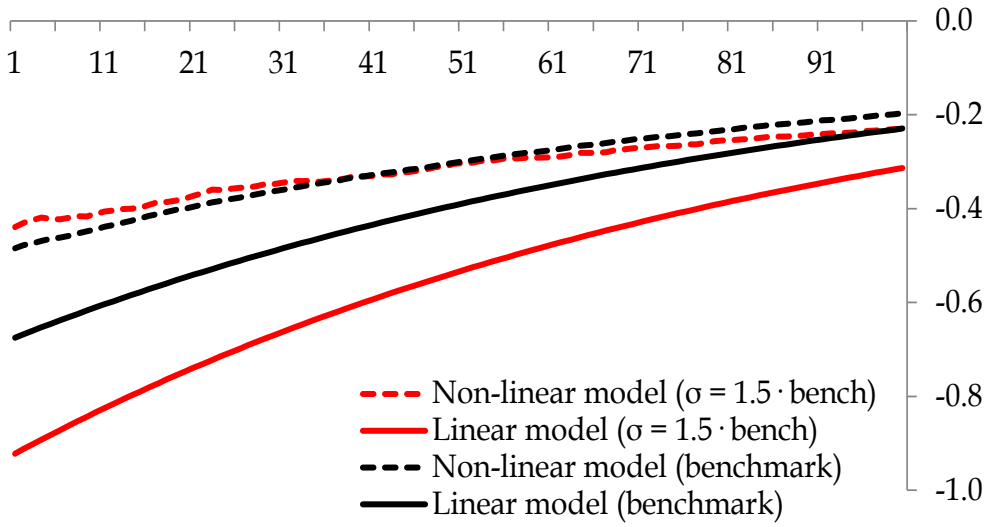

Figure 3: Sensitivity of results to changes in shock volatility (annualized, in percentage points)

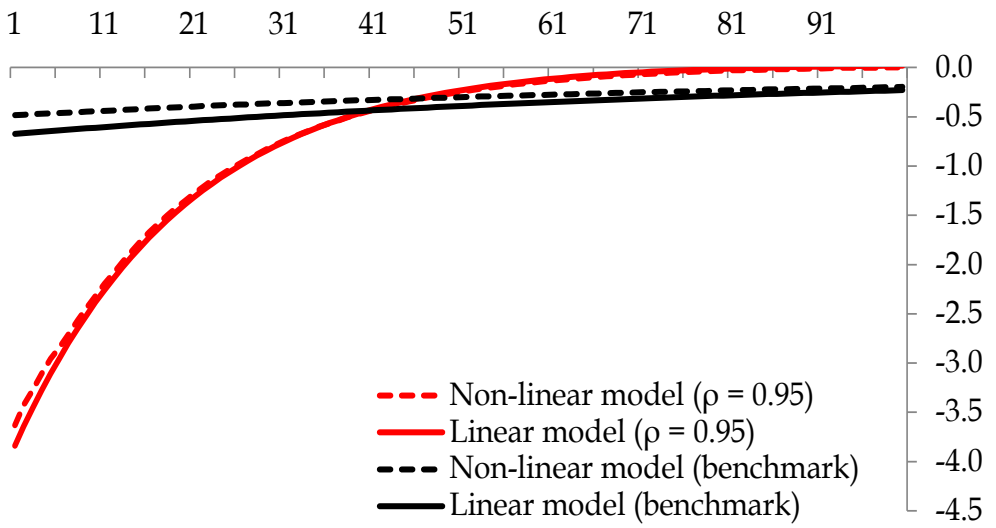

Figure 4: Sensitivity of results to changes in shock persistence (annualized, in percentage points)

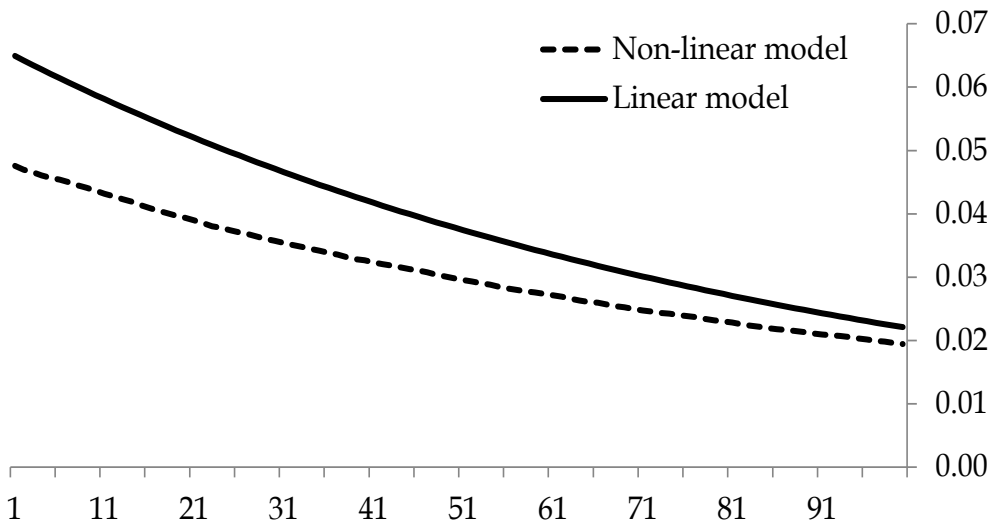

Figure 5: Natural rate of interest following a one standard deviation positive mark-up shock (annualized, in percentage points)

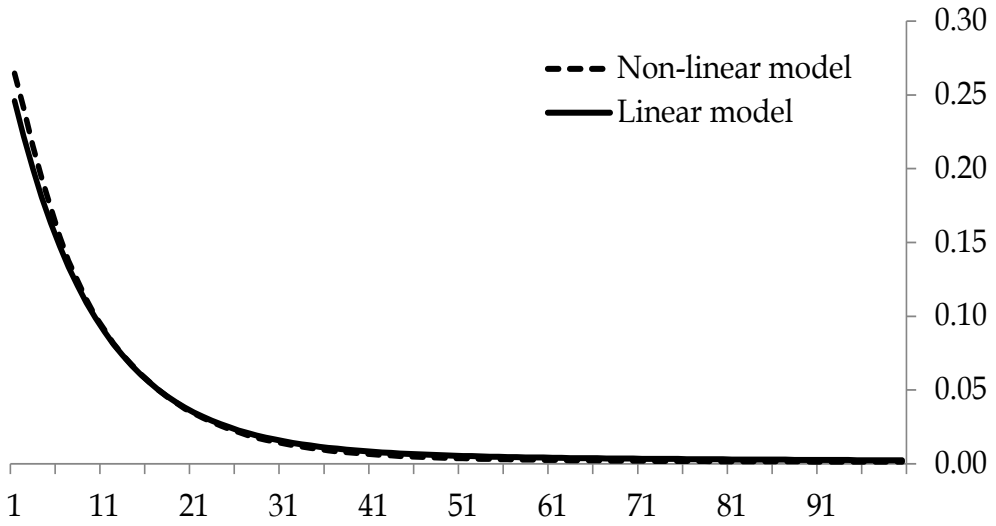

Figure 6: Natural rate of interest following a one standard deviation negative demand shock (annualized, in percentage points)

Euler equation. But we should note that the supply condition when expressed in terms of excess consumption is perfectly log-linear (see also the appendix for a derivation). We also verify numerically that the impulse response function of consumption in a linear model is almost identical to that in a non-linear setup, and that only the behavior of interest rates changes. These two observations suggest that we can attribute the difference in responses in Figures 1 – 4 solely to the effect of precautionary savings appearing via a non-linear approximation of the Euler equation. So, in line with the analytical results, the simulations suggest that a central bank following an interest rate rule should be less aggressive in the face of productivity shocks, with Figure 5 confirming that the results are the same if the shock to supply comes from mark-up fluctuations. If one believes that the benchmark calibration correctly accounts for uncertainty, our exercise would additionally suggest that the effects of precautionary savings can be quantitatively relevant.

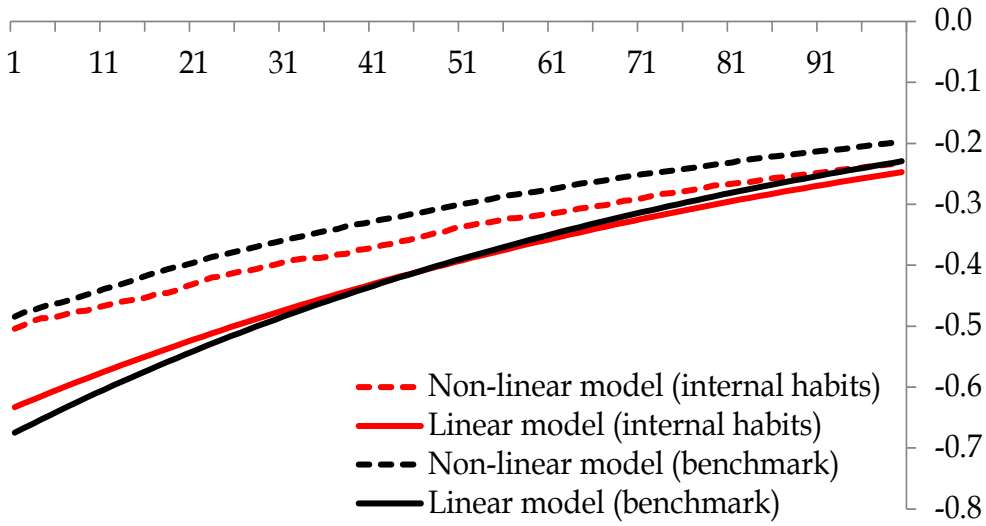

Figure 7: Interest Rate Response to a Productivity Shock Under Internal and External Habits

Finally, Figure 6 demonstrates numerically that the response of the natural rate to a negative preference shock is magnified when the precautionary savings channel is taken into account. Agents' increased desire to save exacerbates the initial shock and calls for more accommodative policy – i.e. bigger cuts in rates. But the quantitative impact of the precautionary saving effect on the natural rate is smaller than in the case of productivity shocks. This is a reflection of the lower persistence of such shocks relative to productivity or mark-up shocks.

4.1.1. Internal Habits

We now investigate how a move from external to internal habits would affect our results. We note that under internal habits agents internalize the effect of their consumption choice on the habit level. Accordingly, in this case the first order condition with respect to consumption would be given by

$$\xi_{d,t}\left(C_t - hX_t\right)^{-\rho} - \Lambda_t - \beta(1-\phi)E_t\Lambda^x_{t+1} = 0 \tag{38}$$

where Λ^x_{t+1} is the Lagrange multiplier on the equation specifying the evolution of habits (4). Moreover, the first order condition with respect to X_t would be

$$-h\xi_{d,t}\left(C_t - hX_t\right)^{-\rho} + \Lambda^x_t - \beta\phi E_t \Lambda^x_{t+1} = 0. \tag{39}$$

So in Figure 7 we contrast the two habit specifications. After an increase in productivity, agents' desire to increase consumption is relatively smaller under internal habits than under external habits. This happens because agents with internal habits anticipate that higher consumption today reduces their utility tomorrow. In a linear world, the smaller desire to increase consumption thus implies that a smaller fall in interest rates is necessary to ensure price stability. But when moving from a linear to a non-linear environment, these results are reversed, that is the model under internal habits features a relatively more volatile natural rate. This occurs because risk aversion is more volatile under external habits, meaning that agents with such preferences will reduce their desire to engage in precautionary savings by more. Put differently, marginal utility under internal habits is forward looking, implying that agents anticipate that the increase in consumption will dissipate and thus their precautionary incentives are less affected.

So, overall, the result that following productivity shocks the natural interest rate falls by less in a non-linear world holds regardless of the habit specification. But with internal habits the difference between the linear and non-linear allocation is smaller.

4.2. Assessing the performance of a misspecified Taylor rule

So far we have focused on the case in which the central bank had a strict inflation targeting mandate. We now consider the case in which the central bank follows alternative policy rules. In particular, we consider the following Taylor type rules

$$r^n_t = \phi_r r^*_{l,t} + \phi_\pi \pi_t + \phi_{ygap}(y_t - y^*_{l,t}) \tag{TR.1}$$

$$r_t^n = \phi_r r_{l,t}^* + \phi_\pi \pi_t + \phi_{\Delta y}(y_t - y_{t-1}) \tag{TR.2}$$

where r_t^n is the nominal interest rate, $r_{l,t}^*$ and $y_{l,t}^*$ are defined in Equations (22) and (24) – i.e. they denote variables consistent with price stability in a linear world. In such an environment, the assumption of $\phi_r = 1$ ensures that rule TR.1 delivers full price stability – i.e. $\forall t \pi_t = 0$.[19] But if the world is non-linear – and risk and uncertainty do affect the transmission mechanism of shocks – then the natural interest rate will incorporate the precautionary motive as shown in equation (26), and so following such a rule will not guarantee price stability.

Table 2 shows the implications of this policy for inflation and the output gap.[20] We see that the version of Taylor rule (TR.1) with $\phi_r = 1$ ensures zero inflation and output gap volatility in a linear world, where the natural rate is driven purely by the 'intertemporal-substitution' channel. But this is no longer the case when uncertainty influences agents' behavior. More specifically, under our benchmark calibration, the misspecified rule (or "wrong policy") increases the standard deviation of the output gap and inflation to 0.5pp and 2.2pp, respectively.

This result can be related to an extensive literature investigating the reasons behind the stability in economic conditions during the period of the so-called "great moderation"(i.e. from the mid 1980's to the beginning of the 2000s – as discussed in Bernanke (2004)). Broadly, while some studies have attributed the stability to improved performance of macroeconomic policy, others have argued in favor of smaller and less frequent shocks. That is, there has been a wide debate on whether the fall in macroeconomic volatility was due to respectively good policy or good luck (see also Stock and Watson (2003) for a discussion).

[19] The statement is conditional on the Taylor principle being satisfied and there being a determinate equilibrium.

[20] The benchmark parameters values for $\phi_\pi = 2$ and $\phi_{ygap} = 0.5$ were chosen in line with Smets and Wouters (2003) and de Walque, Smets, and Wouters (2005).

But clearly, these two explanations are not mutually exclusive – i.e. in times of stable economic conditions it may be much easier to conduct good economic policy. Our exercise provides an example of this argument: if policy mistakes come from the fact that central banks use misspecified models that ignore risk-related non-linearities, then these mistakes would only ever materialize in periods of high uncertainty. A related implication of Table 2 is that decreasing the level of uncertainty would lower the size of inflation and output gap volatility. That is, lower uncertainty would decrease the size of policy mistakes. Thus, in these settings, good luck (meaning central banks confront a stable economic environment) will also translate into good policy (i.e. low policy mistakes). This conclusion, which is consistent with Figure 3, can be illustrated by comparing rows "TR1"and "TR1.b"in Table 2. In the latter case, the volatility of shocks was scaled up, so as to double the standard deviation of consumption growth. Crucially, the higher economic uncertainty more than doubles the size of the "policy mistakes".

The overall results shown in Table 2 suggest that responding to a misspecified measure of the output gap can lead to significant policy errors (or large volatility in inflation and the true measure of the output gap). Rules that respond only to inflation or include output growth as proxy for the output gap lead to smaller variances in the volatility of these variables (see Table 2, rows TR1.c and TR2). But using a simple rule that responds only to observables and does not attempt to track a measure of the natural rate can lead to high volatility in inflation and output gap (as shown in row TR2.b). Clearly, if the objective of the central bank is full price stabilization, then the best rule would be one with a high coefficient on inflation. This can be inferred by comparing the moments reported for rules in rows "TR1"and "TR1.d".

Finally, Table 2 also considers the case in which the central bank follows a rule that responds to changes in the price level. That is, it considers a Wicksellian

Taylor rule	Parameter values				1st order		3rd order	
					σ_π	$\sigma_{y_{gap}}$	σ_π	$\sigma_{y_{gap}}$
TR1	$\phi_r = 1$	$\phi_\pi = 2$	$\phi_{y_{gap}} = 0.5$		0.00	0.00	2.20	0.54
TR1.b	$\phi_r = 1$	$\phi_\pi = 2$	$\phi_{y_{gap}} = 0.5$	$\sigma_{y_{gr}} = 1.5\%$	0.00	0.00	5.26	1.50
TR1.c	$\phi_r = 1$	$\phi_\pi = 2$	$\phi_{y_{gap}} = 0$		0.00	0.00	0.87	0.10
TR1.d	$\phi_r = 1$	$\phi_\pi = 3$	$\phi_{y_{gap}} = 0.5$		0.00	0.00	1.00	0.20
TR1.e	$\phi_r = 0.75$	$\phi_\pi = 2$	$\phi_{y_{gap}} = 0.5$		0.79	0.00	2.07	0.77
TR2	$\phi_r = 1$	$\phi_\pi = 2$	$\phi_{y_{gr}} = 0.5$		0.17	0.00	0.75	0.10
TR2.b	$\phi_r = 0$	$\phi_\pi = 2$	$\phi_{y_{gr}} = 0.5$		0.17	0.10	2.79	2.27
TR2.c	$\phi_r = 0$	$\phi_\pi = 3$	$\phi_{y_{gr}} = 0.5$		1.66	0.86	1.41	0.36
TR3	$\phi_r = 1$	$\phi_p = 2$	$\phi_{y_{gap}} = 0.5$		0.00	0.00	0.10	0.00
TR3.b	$\phi_r = 0$	$\phi_p = 2$	$\phi_{y_{gap}} = 0.5$		0.00	0.00	0.10	0.10

Table 2: Policy Exercise (Values annualized and in percentage points)

rule of the following form

$$r_t^n = \phi_r r_{l,t}^* + \phi_p p_t + \phi_{ygap}(y_t - y_{l,t}^*). \tag{TR.3}$$

Such rules introduce history-dependence in policy (Woodford, 2003). That is, a central bank targeting the price level does not ignore the effect of deviations of inflation from its targets that occurred in the past. And because of the forward looking nature of economic dynamics, correcting for policy mistakes in the past will improve policy performance today. Table 2 shows that this is clearly the case in our setting, as the standard deviation of inflation and the output gap are no larger than 0.1pp.

5. Efficiency, optimal monetary policy and welfare

So far we have focused on the case in which the central bank's only goal is to stabilize inflation or follow a simple policy rule. But such policies do not restore

efficiency in the economy. In particular, the presence of habit persistence (which is the driving force of precautionary savings in our model) introduces further policy incentives. So in this section we derive the optimal policy plan in the fully non-linear model and evaluate the welfare performance of alternative monetary policy rules.

5.1. Economic efficiency

In order to demonstrate the effect of the different distortions in the model, we characterize the efficient allocation i.e. the one that would prevail if there were no economic inefficiencies or externalities. To do so, we consider the case in which habits are internal (i.e. there is no habit externality), prices are perfectly flexible and firms have no monopoly power. In this case the labor-leisure decision would reduce to

$$(C_t)^{\eta} = \Lambda_t A_t^{\frac{\eta}{\eta+1}} \tag{40}$$

where Λ_t is given by the conditions (38) and (39). So, together with the equation defining habits (4), equations (38), (39) and (40) characterize the efficient allocation for consumption – or output, given that market clearing implies $C_t = Y_t$.

We can also the derive the efficient *steady state* and compare it with the one that would arise from the competitive equilibrium in our model. It follows that, in steady state

$$\frac{\overline{Y}^*}{\overline{Y}} = (1-k)^{\frac{1}{\rho+\eta}} \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{\rho+\eta}}$$

where $\overline{Y}^*$ denotes the efficient level of steady-state output and $k \equiv \frac{\beta(1-\phi)h}{1-\beta\phi}$. So whether the competitive equilibrium level is above or below the efficient one depends on

$$\frac{\sigma}{\sigma-1} \gtreqless \frac{1}{1-k}$$

In other words, the strength of monopoly power vis-à-vis the degree of habit persistence determines whether there is an inefficiently high or low level of output

(and consumption) in steady state.[21]

5.2. Optimal policy

We now characterize the optimal plan of a policymaker that maximizes agents' utility, subject to the constraints coming from the competitive equilibrium conditions. The maximization problem and the set of first order conditions are provided in the appendix.

Using a third order approximation of the equilibrium conditions (summarized in Table A) and the policy plan (equations (57) to (66) in the appendix) we now illustrate the behavior of economic variables under optimal policy. We compare the optimal paths with those under the flexible price allocation for interest rates (or the natural rate of interest). In this exercise we assume (in line with footnote 21) that the steady state is efficient, owing to the introduction of a static tax or subsidy. In this setup, absent habits, setting the policy rate equal to the natural rate would guarantee efficiency. Accordingly, the figure below illustrates how optimal monetary policy changes with the introduction of habits and when additionally allowing precautionary motives to vary with economic conditions.

As shown in the Figure 1, the natural rate of interest would fall after a positive productivity shock. As higher productivity tends to reduce inflation,

[21] One can further examine, as is commonly done in the literature (e.g. Woodford (2003)) whether it is possible to design a fiscal instrument that, when imposed in the competitive equilibrium, ensures an efficient steady state. Specifically, if we assumed a labor subsidy that changes firms profits to

$$Q_{t+i}(z) = P_t(z) Y_{t+i}(z) - \frac{W_{t+i}}{sub} N_{t+i}(z)$$

then in order for it to guarantee an efficient steady state, it would have to hold that

$$sub = \frac{(1-k)\sigma}{\sigma - 1}.$$

Hence, whether the optimal fiscal tool is a subsidy (as opposed to a tax – i.e. $sub > 1$) and, thus, whether it increases output in equilibrium, depends on whether $\frac{\sigma}{\sigma-1} > \frac{1}{1-k}$.

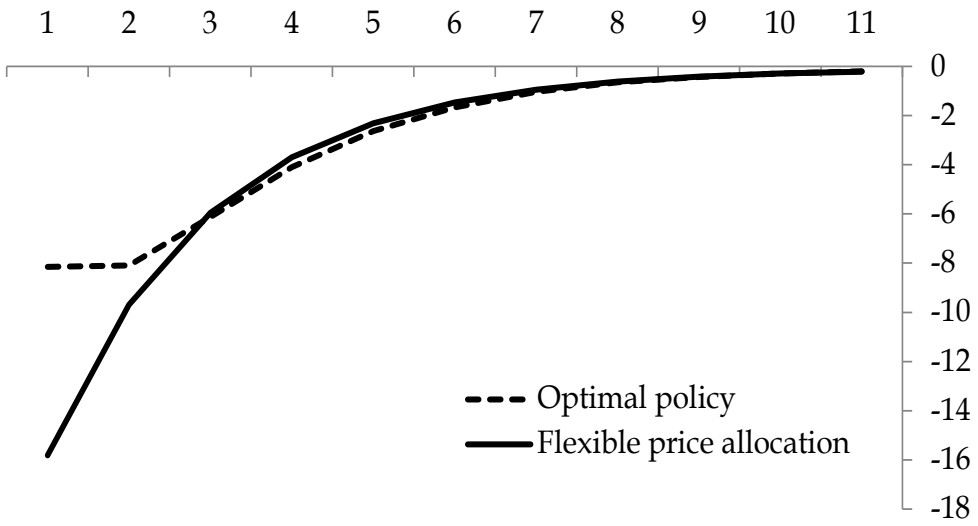

Figure 8: Natural interest rate following a productivity shock, no habit persistence, non-linear model

the central bank striving for price stability should cut interest rates. But with external habits, consumers underestimate the negative impact of a current rise in consumption on future welfare. So there is a policy incentive to curb any associated surge in consumption, which counteracts the incentive to cut rates to avoid deflation.

If habits are not persistent, optimal policy would also imply an interest rate cut, but a smaller one (see Figure 8). In this case, the incentive to offset the fall in inflation on impact is bigger than the one to offset the habit externality. Arguably, this is the standard case analyzed in the macro literature – see e.g. Smets and Wouters (2007), Levine, Pearlman, and Pierse (2008) or Leith, Moldovan, and Rossi (2009)). But, as shown in De Paoli and Zabczyk (2012), the underlying non-persistent habit specifications are likely to generate counterfactual risk premium dynamics (and in our setting, they would also imply an interest rate that is too volatile, as suggested by Figure 8). Accordingly, in Figure 9 we present our benchmark habit specification. Because in this case the effect of an increase

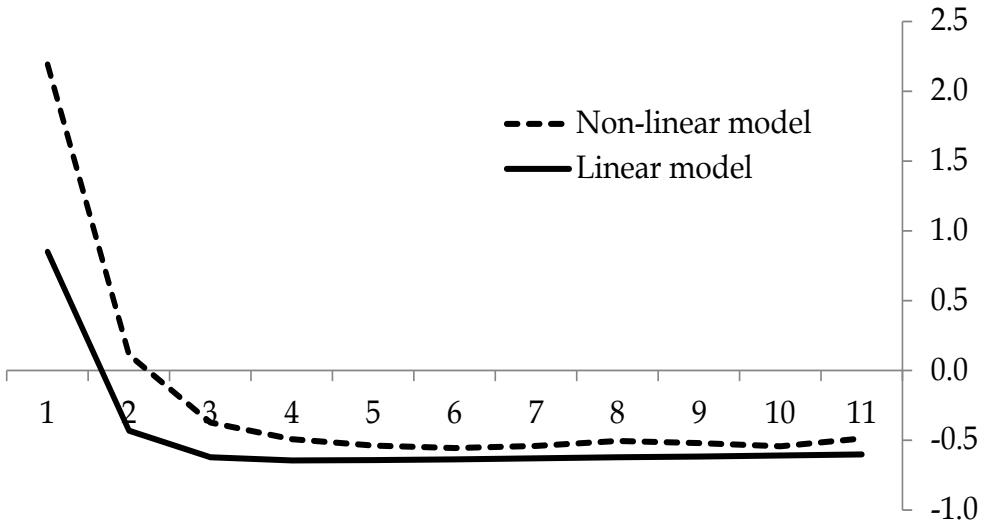

Figure 9: Optimal policy following a productivity shock - linear and non-linear model

in consumption today has an adverse and long lasting effect on utility, optimal policy has an incentive to reduce consumption immediately. As a result, in a model where a positive productivity shock is associated with a compression in risk premia, monetary policy should actually react by increasing interest rates after the shock.

Furthermore, the presence of persistent habits not only changes the policy incentives, but it also affects the transmission mechanism of shocks. In particular, persistent habits imply a fall in precautionary motives following the positive shock. So, incorporating the effect of precautionary savings increases equilibrium interest rates. Figure 9 illustrates this argument by comparing the optimal interest rate path when the model is approximated to first and third order. The latter incorporates the increase in risk appetite and prescribes a tighter policy.

5.3. An evaluation of welfare under alternative policy rules

We now evaluate the welfare performance of different policy rules. In particular, we compute unconditional expected welfare $\mathbf{E}\mathcal{W}$, with welfare satisfying the

Taylor rule	Parameter values	Relative Welfare
TR1	$\phi_r = 1 \quad \phi_\pi = 2 \quad \phi_{y_{gap}} = 0.5$	-8.60%
TR2	$\phi_r = 1 \quad \phi_\pi = 2 \quad \phi_{y_{gr}} = 0.5$	-8.22%
TR2.b	$\phi_r = 0 \quad \phi_\pi = 2 \quad \phi_{y_{gr}} = 0.5$	-10.04%
TR3	$\phi_r = 1 \quad \phi_p = 2 \quad \phi_{y_{gap}} = 0.5$	-8.61%
Opt Linear-Quadratic		-8.32%
Flexible Price Allocation		-8.33%

Table 3: Policy rule performance: welfare relative to the efficient allocation

recursive definition

$$\mathcal{W}_t \equiv \mathbf{E}_t \left\{ \frac{\left(C_t - hX_t\right)^{1-\rho} - 1}{1-\rho} - \frac{(N_t)^{1+\eta} - 1}{1+\eta} + \beta \mathcal{W}_{t+1} \right\} \tag{41}$$

and habits X_t evaluated under the assumption that $C_0 = C_{-1} = \ldots = \bar{C}$. The table below presents the relative performance of the rules considered in Section 4.2. We also compared such rules to one that fully stabilizes inflation.

The table shows that amongst the rules that track a "misspecified"natural rate, the one that responds to consumption growth outperforms the one that responds to the level of output in deviations from its flexible price allocation. This is due to the fact that responding to growth indirectly incorporates the desire to curb the overconsumption coming from the habit externality. Nevertheless, absent a measure of the natural rate – which indicates how close to the flexible price equilibrium interest rates are – it is better to focus on stabilizing inflation (or the price level).

Finally, we compute the welfare performance in our non-linear setting of a policy designed to maximize welfare in a linear setting. In particular, as shown in the appendix, we assume that the central bank follows a linear-quadratic approach to optimal policy. In other words, the central bank minimizes a quadratic loss function (derived from the utility of the household), subject to linear constraints.

As shown in Table 3, the optimal linear-quadratic policy plan in a non-linear setting achieves welfare which is close to that under the flexible price allocation. Arguably, this is because such plan is very close to a policy that targets inflation – with the implied standard deviation of inflation lower than 0.1pp. This result suggests that the presence of external habits – and the associated welfare costs – are mostly felt in a non-linear world – or in world in which habits introduce time-varying fluctuations in risk taking, risk premia and precautionary savings.

6. Conclusion

Our results show that, following persistent, adverse shocks policy-makers might be well advised to aim off predictions of linear models and conduct more accommodative policy – particularly in highly turbulent periods. Equally, when demand and supply conditions are improving, taking note of the precautionary saving motives justifies 'leaning against the wind'. Since the size of the precautionary correction is increasing in the degree of volatility, mistakenly ignoring this channel would be most costly during highly turbulent periods. Moreover, if the central bank is following the socially optimal rule, nominal interest rates rise after an increase in productivity.

In order to obtain intuitive results and single out the precautionary savings channel, our analysis proceeded in a stylized model. An investigation of the impact of other risk channels in a fully-fledged DSGE model might also be of interest. Furthermore, formally accounting for stochastic volatility in a model with investment (in light of the analysis in Bloom (2009)) and enriching the framework by considering Epstein-Zin preferences (as in Bansal and Yaron (2004)) would both make for interesting extensions.[22]

[22] Epstein-Zin preferences would be of particular interest since, with this specification it is possible to calibrate separately the precautionary and the intertemporal smoothing motives for savings.

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APPENDIX – DERIVATIONS & TABLES

The logarithm of the stochastic discount factor is given by

$$\log(\widetilde{\mathcal{M}}_{t+1}) = \log\left(\frac{(C_{t+1}-X_{t+1})^{-\rho}}{(C_t-X_t)^{-\rho}}\right)$$
$$= \log\left(\frac{(C_{t+1}-X_{t+1})^{-\rho}}{C_{t+1}^{-\rho}}\frac{C_t^{-\rho}}{(C_t-X_t)^{-\rho}}\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}\right) = \log\left(S_{t+1}^{-\rho}S_t^{\rho}C_{t+1}^{-\rho}C_t^{\rho}\right)$$
$$= -\rho\big(\log C_{t+1} - \log C_t + \log S_{t+1} - \log S_t\big) \quad (42)$$

and so

$$\widetilde{m}_{t+1} = -\rho\big(c_{t+1} - c_t + s_{t+1} - s_t\big).$$

It thus follows that

$$\mathrm{var}_t\widetilde{m}_{t+1} = \mathbf{E}_t\left(\widetilde{m}_{t+1} - \mathbf{E}_t\widetilde{m}_{t+1}\right)^2 = \rho^2\mathbf{E}_t\Big((c_{t+1}-\mathbf{E}_tc_{t+1}) + (s_{t+1}-\mathbf{E}_ts_{t+1})\Big)^2$$
$$= \rho^2\Big(\mathrm{var}_tc_{t+1} + 2\mathrm{cov}_t(c_{t+1},s_{t+1}) + \mathrm{var}_ts_{+1}\Big) \quad (43)$$

as the conditional expectations of all t-dated variables drop out.

Up to a second order approximation (which is all we need to compute a third-order accurate expression for $\mathrm{var}_t\widetilde{m}_{t+1}$) we get

$$s_{t+1} = \Psi_1\big(c_{t+1} - \frac{1}{2}(1-h)^{-1}c_{t+1}^2 - \widetilde{x}_t + c_{t+1}\widetilde{x}_t(1-h)^{-1} - \frac{1}{2}(1-h)^{-1}\widetilde{x}_t^2\big) - \log(1-h)$$

where we used the fact that the habit at time $t+1$ depends only on variables known at time t and so we changed the notation of x_{t+1} to $\widetilde{x}_t$. We also defined $\Psi_1 := h/(1-h)$.

We can now compute a third order approximation to var_ts_{t+1} and to that of the $\mathrm{cov}_t(c_{t+1},s_{t+1})$ term. From the definition

$$\mathrm{var}_ts_{t+1} = \Psi_1^2\mathrm{var}_t\big(c_{t+1} - \frac{1}{2}(1-h)^{-1}c_{t+1}^2 + c_{t+1}\widetilde{x}_t(1-h)^{-1}\big) \quad (44)$$

where again expectations of all t-dated variables were eliminated. It is easy to see that

$$\mathrm{var}_t s_{t+1} = \frac{\Psi_1^2}{(1-h)^2}\Big((1-h)^2\mathrm{var}_t\big(c_{t+1}\big) + \frac{1}{4}\mathrm{var}_t(c_{t+1}^2) + \widetilde{x}_t^2\mathrm{var}_t\big(c_{t+1}\big)$$
$$- (1-h)cov_t(c_{t+1}, c_{t+1}^2) + 2\widetilde{x}_t(1-h)\mathrm{var}_t c_{t+1} - \widetilde{x}_t\mathrm{cov}_t(c_{t+1}, c_{t+1}^2)\Big). \quad (45)$$

Similarly

$$\mathrm{cov}_t(c_{t+1}, s_{t+1}) = \Psi_1\mathrm{cov}_t\big(c_{t+1}, c_{t+1} - \frac{1}{2}(1-h)^{-1}c_{t+1}^2 + c_{t+1}\widetilde{x}_t(1-h)^{-1}\big)$$
$$= \Psi_1\big(\mathrm{var}_t c_{t+1} - \frac{1}{2}(1-h)^{-1}\mathrm{cov}_t\big(c_{t+1}, c_{t+1}^2\big) + \widetilde{x}_t(1-h)^{-1}\mathrm{var}_t c_{t+1}\big). \quad (46)$$

Consider the case in which shocks follow an AR(1) process and are independent (cross-sectionally and inter-temporally). As shown above, to compute the variance of $\widetilde{m}_{t+1}$ we need expressions for $\mathrm{var}_t c_{t+1}$ and $cov_t(c_{t+1}, c_{t+1}^2)$. We can compute the flexible price allocation of consumption (or output)

$$c_t^* = y_t^* = \big(\rho(1-h)^{-1} + \eta\big)^{-1}\big(\rho(1-h)^{-1}h\widetilde{x}_{t-1}^* + \varepsilon_{d,t} + \eta\varepsilon_{y,t} - \varepsilon_{\mu,t}\big) \quad (47)$$

and so

$$\mathrm{var}_t c_{t+1}^* = \mathrm{var}_t\big(\rho(1-h)^{-1}h\widetilde{x}_t^* + \epsilon_{d,t+1} + \gamma_d\varepsilon_{d,t} + \eta\epsilon_{y,t+1} + \eta\gamma_y\varepsilon_{y,t} - \epsilon_{\mu,t+1} - \eta\gamma_\mu\varepsilon_{\mu,t}\big)$$
$$\cdot\big(\rho(1-h)^{-1} + \eta\big)^{-2} = \Psi_2^{-2}\big(\sigma_d^2 + \eta^2\sigma_y^2 + \sigma_\mu^2\big) \quad (48)$$

where $\Psi_2 = \big(\rho(1-h)^{-1} + \eta\big)$ and where the shocks are assumed to be uncorrelated with one-another.

By a similar token

$$\mathrm{cov}_t(c_{t+1}^*, c_{t+1}^{*2}) = \mathrm{cov}_t\Big(\big(\rho\,\Psi_1\,\widetilde{x}_t^* + \epsilon_{d,t+1} + \gamma_d\varepsilon_{d,t} + \eta\epsilon_{y,t+1} + \eta\gamma_y\varepsilon_{y,t} - \epsilon_{\mu,t+1} - \gamma_\mu\varepsilon_{\mu,t}\big),$$
$$\big(\rho\,\Psi_1\,\widetilde{x}_t^* + \epsilon_{d,t+1} + \gamma_d\varepsilon_{d,t} + \eta\epsilon_{y,t+1} + \eta\gamma_y\varepsilon_{y,t} - \epsilon_{\mu,t+1} - \gamma_\mu\varepsilon_{\mu,t}\big)^2\Big) \cdot \Psi_2^{-3}.$$

Since the shocks are assumed Gaussian and uncorrelated, we can write

$$\mathrm{cov}_t(c^*_{t+1}, c^{*2}_{t+1}) = 2\Psi_2^{-3}\Big(\rho\,\Psi_1\,\widetilde{x}^*_t + \gamma_d\varepsilon_{d,t} + \eta\gamma_y\varepsilon_{y,t} - \gamma_\mu\varepsilon_{\mu,t}\Big)\Big(\sigma_d^2 + \eta^2\sigma_y^2 + \sigma_\mu^2\Big). \quad (49)$$

Using equalities (48) and (49) in equation (45) yields

$$\begin{aligned}\mathrm{var}_t s^*_{t+1} &= \Psi_1^2\Big(\Psi_2^{-2}\big(\sigma_d^2 + \eta^2\sigma_y^2 + \sigma_\mu^2\big) - (1-h)^{-1}2\Psi_2^{-3}\big(\sigma_d^2 + \eta^2\sigma_y^2 + \sigma_\mu^2\big)\\ &\quad\cdot\big(\rho\,\Psi_1\,\widetilde{x}^*_t + \gamma_d\varepsilon_{d,t} + \eta\gamma_y\varepsilon_{y,t} - \gamma_\mu\varepsilon_{\mu,t}\big) + 2\widetilde{x}_t(1-h)^{-1}\Psi_2^{-2}\big(\sigma_d^2 + \eta^2\sigma_y^2 + \sigma_\mu^2\big)\Big)\\ &= \frac{2\Psi_3^2\big(\sigma_d^2 + \eta^2\sigma_y^2 + \sigma_\mu^2\big)}{(1-h)}\left(\frac{(1-h)}{2} + \big(1-\rho\Psi_3\big)\widetilde{x}_t - \frac{\big(\rho\,\Psi_1\,\widetilde{x}^*_t + \gamma_d\varepsilon_{d,t} + \eta\gamma_y\varepsilon_{y,t} - \gamma_\mu\varepsilon_{\mu,t}\big)}{\Psi_2}\right)\end{aligned} \quad (50)$$

where $\Psi^3 := \Psi_1\Psi_2^{-1} = h/\big(\rho + (1-h)\eta\big)$. Similarly, plugging (48) and (49) into equation (46) and denoting $\Psi_4 := \Psi_1\Psi_2^{-2} = \big(h(1-h)\big)/\big(\rho + \eta(1-h)\big)^2$ we can write down

$$\begin{aligned}\mathrm{cov}_t(c^*_{t+1}, s^*_{t+1}) &= \frac{\Psi_4\big(\sigma_d^2 + \eta^2\sigma_y^2 + \sigma_\mu^2\big)}{(1-h)}\\ &\quad\cdot\left(1 - h + (1-\rho\,\Psi_3)\widetilde{x}^*_t - \Psi_2^{-1}\Big(\gamma_d\varepsilon_{d,t} + \eta\gamma_y\varepsilon_{y,t} - \gamma_\mu\varepsilon_{\mu,t}\Big)\right).\end{aligned} \quad (51)$$

We can then use equations (48), (50) and (51) in (43) to obtain

$$\mathrm{var}_t\widetilde{m}^*_{t+1} = \frac{\rho^2\big(\sigma_d^2 + \eta^2\sigma_y^2 + \sigma_\mu^2\big)}{(\rho + \eta(1-h))^2}\left(1 + \frac{2h(\rho+\eta)}{(\rho + \eta(1-h))}\widetilde{x}^*_t - \frac{2h\big(\gamma_d\varepsilon_{d,t} + \eta\gamma_y\varepsilon_{y,t} - \gamma_\mu\varepsilon_{\mu,t}\big)}{(\rho + \eta(1-h))}\right).$$

Given that $\widetilde{x}_t = x_{t+1}$ and so, $\widetilde{x}_t = c_t(1-\phi) + \phi x_t$ we get

$$\begin{aligned}\mathrm{var}_t\widetilde{m}_{t+1} = \frac{\rho^2\big(\sigma_d^2 + \eta^2\sigma_y^2 + \sigma_\mu^2\big)}{(\rho + \eta(1-h))^2}\Bigg(1 &- \frac{2h}{(\rho + \eta(1-h))}(\gamma_d\varepsilon_{d,t} + \eta\gamma_y\varepsilon_{y,t} - \gamma_\mu\varepsilon_{\mu,t})\\ &+ \frac{2h(\rho+\eta)(1-\phi)}{(\rho + \eta(1-h))}c^*_t + \frac{2h(\rho+\eta)\phi}{(\rho + \eta(1-h))}x^*_t\Bigg).\end{aligned}$$

Recalling the definition of c^*_t - equation (47) and plugging it into the expression

derived above yields, after simplifying

$$\begin{aligned}\mathrm{var}_t\widetilde{m}_{t+1} = \Bigg(1 &- \frac{2h((1-h)(\rho+\eta)(\phi-1)+(\rho+\eta(1-h))\gamma_d)}{(\rho+\eta(1-h))^2}\varepsilon_{d,t} \\ &- \frac{2h\eta((1-h)(\rho+\eta)(\phi-1)+(\rho+\eta(1-h))\gamma_y)}{(\rho+\eta(1-h))^2}\varepsilon_{y,t} \\ &+ \frac{2h((1-h)(\rho+\eta)(\phi-1)+(\rho+\eta(1-h))\gamma_y)}{(\rho+\eta(1-h))^2}\varepsilon_{\mu,t} \\ &+ \frac{2h(\rho+\eta)((1-h)\eta+\rho(1-h(\phi-1)))}{(\rho+\eta(1-h))^2}x_t^*\Bigg)\cdot\frac{\rho^2\left(\sigma_d^2+\eta^2\sigma_y^2+\sigma_\mu^2\right)}{(\rho+\eta(1-h))^2}.\end{aligned}$$

which is the expression reported in the body of the text.

A.1. The covariance term $cov_t(\widetilde{m}_{t+1}, \Delta\varepsilon_{d,t+1})$

In line with the argument in the previous section, we can write

$$\begin{aligned}\mathrm{cov}_t(\widetilde{m}_{t+1}, \Delta\varepsilon_{d,t+1}) = -\frac{\rho\sigma_d^2}{(\rho+\eta(1-h))}\Bigg(1 &+ \frac{h(1-h)(\rho+\eta)(1-\phi)}{(\rho+\eta(1-h))^2}\cdot\varepsilon_{d,t} \\ + \frac{h(1-h)(\rho+\eta)(1-\phi)\eta}{(\rho+\eta(1-h))^2}\cdot\varepsilon_{s,t} &+ \frac{h(\rho+\eta)\big((\rho+\eta)\phi+h(\rho^2(1-\phi)-\eta\phi)\big)}{(\rho+\eta(1-h))^2}\cdot x_t\Bigg).\end{aligned}$$

Note that the coefficients on $\varepsilon_{s,t}$ and $\varepsilon_{d,t}$ are negative, so the covariance term always moves countercyclically. The coefficient multiplying x_t is negative when $\rho^2(1-\phi) > \eta\phi$, but given that x_t is predetermined, this would not affect the countercyclicality of the covariance term.

A.2. Approximating the Labor-Leisure decision

Under flexible prices, or if the central bank successfully stabilizes inflation, Equation (9) can be written as

$$\xi_{d,T}\left(C_T - hX_T\right)^{-\rho} = \xi_{\mu,T}\left(y_{t,T}(j)/\xi_{y,T}\right)^{\eta} \tag{52}$$

or, given that $Y_T = C_T$ and defining $C_T^e := C_T - hX_T$

$$\xi_{d,T}\left(C_T^e\right)^{-\rho} = \xi_{\mu,T}\left(C_T/\xi_{y,T}\right)^{\eta} \tag{53}$$

Simply taking logs yields

$$\log(\xi_{d,T}) - \rho \log\left(C_T^e\right) = \log\left(\xi_{\mu,T}\right) + \eta \log C_T - \eta \log(\xi_{y,T}). \tag{54}$$

Given the steady state condition for C_T and C_T^e, this simplifies to

$$\varepsilon_{d,t} - \rho \log\left(C_T^e / \bar{C}^e\right) = \eta \log(C_t/\bar{C}) - \eta \varepsilon_{s,t} \tag{55}$$

or

$$\varepsilon_{d,t} - \rho c_t^e = \eta c_t - \eta \varepsilon_{s,t}. \tag{56}$$

The labor-leisure when written in terms of excess consumption is perfectly log-linear – i.e. there are no higher order terms.

A.3. The optimal policy plan

If a policymaker maximizes (1) subject to the system of private sector's equilibrium conditions summarized in the table below:

Table A: Private Sector Equilibrium conditions and Policy Multipliers

Constraint	Multiplier
$\xi_{d,t}\left(C_t - hX_t\right)^{-\rho} - \Lambda_t = 0$	Φ_t^1
$X_t - \phi X_{t-1} - (1-\phi)C_{t-1} = 0$	Φ_t^2
$\theta^N \left(N_t\right)^{\eta} - \Lambda_t \Upsilon_t \xi_{y,t}^{\frac{\eta}{\eta+1}} = 0$	Φ_t^3
$PB_t - \Lambda_t \Upsilon_t C_t - \alpha\beta E_t\left[PB_{t+1}\left(\Pi_{t+1}\right)^{\sigma}\right] = 0$	Φ_t^4
$PA_t - \Lambda_t C_t - \alpha\beta E_t\left[PA_{t+1}\left(\Pi_{t+1}\right)^{\sigma-1}\right] = 0$	Φ_t^5
$\frac{P_t^*}{P_t} - \xi_{\mu,t}\frac{PB_t}{PA_t} = 0$	Φ_t^6
$1 - (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{1-\sigma} - \alpha\left(\Pi_t\right)^{\sigma-1} = 0$	Φ_t^7
$PD_t - (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\sigma} - \alpha\left(\Pi_t\right)^{\sigma} PD_{t-1} = 0$	Φ_t^8
$C_t - \frac{\xi_{y,t}^{\frac{\eta}{\eta+1}} N_t}{PD_t} = 0$	Φ_t^9

,

which imposes the market clearing condition $Y_t = C_t.$, the resulting first order conditions are

$$C_t{:}\xi_{d,t}\left(C_t-hX_t\right)^{-\rho}-\rho\Phi_t^1\xi_{d,t}\left(C_t-hX_t\right)^{-\rho-1}$$
$$-(1-\phi)\beta E_t\Phi_{t+1}^2-\Lambda_t\Upsilon_t\Phi_t^4-\Lambda_t\Phi_t^5+\Phi_t^9=0 \quad (57)$$

$$X_t{:}-h\xi_{d,t}\left(C_t-hX_t\right)^{-\rho}+\rho h\Phi_t^1\xi_{d,t}\left(C_t-hX_t\right)^{-\rho-1}$$
$$+\Phi_t^2-\phi\beta E_t\Phi_{t+1}^2=0 \quad (58)$$

$$N_t{:}\theta^N\left(N_t\right)^{\eta}+\eta\Phi_t^3\theta^N\left(N_t\right)^{\eta-1}-\frac{\xi_{y,t}^{\frac{\eta}{\eta+1}}}{PD_t}\Phi_t^9=0 \quad (59)$$

$$\Lambda_t{:}\ -\Phi_t^1-\Upsilon_t\xi_{y,t}^{\frac{\eta}{\eta+1}}\Phi_t^3-\Upsilon_tC_t\Phi_t^4-C_t\Phi_t^5+\Phi_t^9=0 \quad (60)$$

$$\Upsilon_t{:}\ -\Lambda_t\xi_{y,t}^{\frac{\eta}{\eta+1}}\Phi_t^3-\Lambda_tC_t\Phi_t^4=0 \quad (61)$$

$$PB_{t+1}{:}\Phi_{t+1}^4-\alpha\Phi_t^4E_t\left(\Pi_{t+1}\right)^{\sigma}-\xi_{\mu,t}\frac{1}{PA_t}\Phi_{t+1}^6=0 \quad (62)$$

$$PA_{t+1}{:}\Phi_{t+1}^5-\alpha\Phi_t^5E_t\left(\Pi_{t+1}\right)^{\sigma-1}+\xi_{\mu,t}\frac{1}{PA_t}\frac{PB_t}{PA_t}\Phi_{t+1}^6=0 \quad (63)$$

$$\Pi_{t+1}{:}-\alpha\sigma\Phi_t^5E_t\left[PB_{t+1}\left(\Pi_{t+1}\right)^{\sigma-1}\right]-\alpha(\sigma-1)\Phi_t^5E_t\left[PA_{t+1}\left(\Pi_{t+1}\right)^{\sigma-2}\right]$$
$$-\alpha(\sigma-1)\Phi_{t+1}^7\left(\Pi_{t+1}\right)^{\sigma-2}-\alpha\sigma\Phi_{t+1}^8\left(\Pi_{t+1}\right)^{\sigma-1}PD_t=0 \quad (64)$$

$$\frac{P_t^*}{P_t}{:}\ \Phi_t^6-(1-\alpha)(1-\sigma)\left(\frac{P_t^*}{P_t}\right)^{-\sigma}\Phi_t^7+(1-\alpha)\sigma\left(\frac{P_t^*}{P_t}\right)^{-\sigma-1}\Phi_t^8=0 \quad (65)$$

$$PD_t{:}\ \Phi_t^8-\alpha\beta E_t\left[\left(\Pi_{t+1}\right)^{\sigma}\Phi_{t+1}^8\right]+\frac{\xi_{y,t}^{\frac{\eta}{\eta+1}}N_t}{PD_t}\frac{1}{PD_t}\Phi_t^9=0 \quad (66)$$

Above we show some first order condition for variables with a lead (namely, PB_{t+1}, PA_{t+1} and Π_{t+1}). This is done in order to better illustrate the evolution of variables, but one should note that analogous conditions hold for PB_t, PA_t and Π_t.

A.3.1. Steady state

Here we derive the steady state relationships of our model. In steady state, the equation for the evolution of the aggregate price level implies

$$\overline{\left(\frac{P^*}{P}\right)} = \left(\frac{1-\alpha\left(\overline{\Pi}\right)^{\sigma-1}}{(1-\alpha)}\right)^{\frac{1}{1-\sigma}}$$

and, from the steady state version of equation (20) we can assert that $\overline{PD} = 1$. Equation (17) gives

$$\overline{PA} = \frac{\overline{\Lambda Y}}{\left(1-\alpha\beta\overline{\Pi}^{\sigma-1}\right)},$$

and thus, equation (16) implies

$$\overline{PB} = \overline{PA}\overline{\left(\frac{P^*}{P}\right)}\left(\frac{\sigma-1}{\sigma}\right).$$

From equation (21) we can obtain the steady state mark-up

$$\overline{\Upsilon} = \left(\frac{\sigma-1}{\sigma}\right).$$

Turning to the real variables, the labor leisure decision can be written as

$$\left(\bar{N}\right)^{\eta} = \left(\bar{C} - h\bar{X}\right)^{-\rho}\bar{\Upsilon}\overline{A}^{\frac{\eta}{\eta+1}},$$

and market clearing implies

$$\overline{X} = \overline{C} = \overline{Y} = \frac{\overline{A}_t^{\frac{\eta}{\eta+1}}\overline{N}}{\overline{PD}} = \overline{A}_t^{\frac{\eta}{\eta+1}}\overline{N}.$$

So

$$\bar{N} = \left(\left[(1-h)\,\overline{A}_t^{\frac{\eta}{\eta+1}}\right]^{-\rho}\bar{\Upsilon}\overline{A}_t^{\frac{\eta}{\eta+1}}\right)^{\frac{1}{\rho+\eta}}.$$

Assuming $\overline{A} = 1$

$$\overline{Y} = \bar{N} = (1-h)^{\frac{-\rho}{\rho+\eta}}\left(\bar{\Upsilon}\right)^{\frac{1}{\rho+\eta}}.$$

A.4. Optimal policy: the linear quadratic approach

A.4.1. The social loss function: A second order approximation

The utility function of the household is given by

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{\xi_{d,t} \left(C_{t+i} - hX_{t+i}\right)^{1-\rho} - 1}{1-\rho} - \frac{\theta^N \left(N_{t+i}\right)^{1+\eta} - 1}{1+\eta} \right].$$

We now approximate each term in turn. First, following Leith, Moldovan, and Rossi (2009) and noting our more general specification of habits,[23] we have

$$\frac{\xi_{d,t} \left(C_{t+i} - hX_{t+i}\right)^{1-\rho} - 1}{1-\rho}$$
$$= \left(\overline{C}^e\right)^{1-\rho} \left\{ \frac{1}{1-h} \left[c_t + \frac{1}{2} c_t^2 \right] - h \frac{1}{1-h} \left[x_t + \frac{1}{2} x_t^2 \right] - \frac{1}{2} \rho (c_t^e - \varepsilon_d/\rho)^2 \right\} + tip + o[2].$$

But, given the definition of habits

$$\begin{aligned} x_t + \frac{1}{2} x_t^2 &= \phi \left[x_{t-1} + \frac{1}{2} x_{t-1}^2 \right] + (1-\phi) \left[c_{t-1} + \frac{1}{2} c_{t-1}^2 \right] \\ &= (1-\phi) \sum_{1}^{t} \phi^{s-1} \left[c_{t-s} + \frac{1}{2} c_{t-s}^2 \right]. \end{aligned}$$

Summing up to the future, we can write the utility of consumption as

$$E_t \sum_{t=0}^{\infty} \beta^t \frac{\xi_{d,t} \left(C_t - hX_t\right)^{1-\rho} - 1}{1-\rho}$$
$$= \left(\overline{C}^e\right)^{1-\rho} \sum_{0}^{\infty} \beta^t \left\{ \frac{1-k}{1-h} \left[c_t + \frac{1}{2} c_t^2 \right] - \frac{1}{2} \rho (c_t^e - \varepsilon_d/\rho)^2 \right\} + tip + o[2],$$

where $k = \frac{\beta(1-\phi)h}{1-\beta\phi}$.

Our approximation of the disutility of labor is also based on Leith, Moldovan, and Rossi (2009). But noting that our specification of the production function implies that $Y_t = \frac{A_t^{\frac{\eta}{\eta+1}} N_t}{PD_t}$, we have

$$\frac{\theta^N \left(N_{t+i}\right)^{1+\eta} - 1}{1+\eta} = \theta^N \overline{N}^{1+\eta} \left[y_t + \frac{1}{2}(1+\eta)(y_t)^2 - \eta y_t a_t + \frac{\sigma}{2} var(p_i) \right] + tip + o[2],$$

[23] The habit specification of Leith, Moldovan, and Rossi (2009) is a special case of the one proposed in this paper in which $\phi = 0$.

and, thus, overall welfare can be written as

$$
\begin{aligned}
& E_t \sum_{t=0}^{\infty} \beta^t \frac{\xi_{d,t} \left(C_t - hX_t\right)^{1-\rho} - 1}{1-\rho} - \frac{\theta^N \left(N_{t+i}\right)^{1+\eta} - 1}{1+\eta} \\
= \; & \left(\overline{C}^e\right)^{1-\rho} \sum_{0}^{\infty} \beta^t \left\{ \frac{1-k}{1-h} \left[c_t + \frac{1}{2} c_t^2 \right] - \frac{1}{2} \rho (c_t^e - \varepsilon_d/\rho)^2 \right\} \\
& -\theta^N \overline{N}^{1+\eta} \sum_{0}^{\infty} \beta^t \left[y_t + \frac{1}{2}(1+\eta)(y_t)^2 - \eta y_t a_t + \frac{\sigma}{2} var(p_i) \right] + tip + o[2].
\end{aligned}
$$

From the steady state derivation we know that, if we assume an efficient subsidy, we have

$$
\frac{1-k}{1-h} \left(\overline{C}^e\right)^{1-\rho} = \theta^N \overline{N}^{1+\eta} = \left(\theta^N\right)^{\frac{\rho-1}{\rho+\eta}} (1-h)^{\frac{-\rho(\eta+1)}{\rho+\eta}} (1-k)^{\frac{1+\eta}{\rho+\eta}}.
$$

Defining $\overline{\varkappa} = \frac{1-k}{1-h} \left(\overline{C}^e\right)^{1-\rho} = \theta^N \overline{N}^{1+\eta}$and noting that $c_t = y_t$, we can write welfare W_0 as

$$
\begin{aligned}
W_0 = & E_0 \sum_{t=0}^{\infty} \beta^t \frac{\xi_{d,t} \left(C_t - hX_t\right)^{1-\rho} - 1}{1-\rho} - \frac{\theta^N \left(N_{t+i}\right)^{1+\eta} - 1}{1+\eta} \\
= \; & -\frac{1}{2} \overline{\varkappa} E_0 \sum_{0}^{\infty} \beta^t \left[\frac{1-h}{1-k} \rho (c_t^e - \varepsilon_d/\rho)^2 + \eta (y_t - a_t)^2 + \sigma var(p_i) \right] + tip + o[2]
\end{aligned}
$$

Or following Woodford (2003)

$$
L_0 = -W_0 = \frac{1}{2} \overline{\varkappa} \sum_{0}^{\infty} \beta^t \left[\frac{1-h}{1-k} \rho (c_t^e - \varepsilon_d/\rho)^2 + \eta (y_t - a_t)^2 + \frac{\sigma}{\kappa} \pi_t^2 \right] + tip + o[2] \quad (67)
$$

where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$ is the slope of the Phillips curve.

A.4.2. Optimal policy

Following the linear quadratic approach, the policymaker maximizes (67) subject to the linear constraints of the problem. Denoting the respective multipliers by

$$
\begin{aligned}
\gamma_{1t}: \quad c_t^e = & \left(y_t - hx_t\right)(1-h)^{-1} \\
\gamma_{2t}: \quad x_t = & (1-\phi) y_{t-1} + \phi \, x_{t-1}. \\
\gamma_{3t}: \quad \pi_t = & \; k \left(\kappa_0 (1-h)^{-1} y_t - \rho (1-h)^{-1} hx_t - \eta \varepsilon_{y,t} + \varepsilon_{\mu,t} - \varepsilon_{d,t} \right) + \beta E_t \pi_{t+1}
\end{aligned}
$$

we arrive at the following system of first order conditions with respect to c_t^e, y_t, $x_t$and π_t

$$\begin{aligned}
0 = & \ \tfrac{1-h}{1-k}\rho c_t^e - \tfrac{1-h}{1-k}\varepsilon_{d,t} - \gamma_{1t} \\
0 = & \ \eta(y_t - a_t) + (1-h)^{-1}\gamma_{1t} + k\kappa_0 (1-h)^{-1}\gamma_{2t} + \beta(1-\phi) E_t\gamma_{3t+1} \\
0 = & \ -(1-h)^{-1} h\gamma_{1t} - k\rho(1-h)^{-1}\gamma_{2t} - \gamma_{3t} + \beta\phi E_t\gamma_{3t+1} \\
0 = & \ \tfrac{\sigma}{\kappa}\pi_t - (\gamma_{2t} - \gamma_{2t-1})
\end{aligned}$$

yielding a system of 7 equations determining the evolution of c_t^e, y_t, x_t, π_t, γ_{1t}, γ_{2t} and γ_{3t}.