

Debt, Deleveraging, and the Liquidity Trap:

A Fisher-Minsky-Koo approach

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In this paper we present a simple New Keynesian-style model of debt-driven slumps – that is, situations in which an overhang of debt on the part of some agents, who are forced into rapid deleveraging, is depressing aggregate demand. Making some agents debt-constrained is a surprisingly powerful assumption: Fisherian debt deflation, the possibility of a liquidity trap, the paradox of thrift and toil, a Keynesian-type multiplier, and a rationale for expansionary fiscal policy all emerge naturally from the model. We argue that this approach sheds considerable light both on current economic difficulties and on historical episodes, including Japan’s lost decade (now in its 18th year) and the Great Depression itself.

The views expressed in the paper are those of the authors and are not necessarily reflective of views at the Federal Reserve Bank of New York or the Federal Reserve System. We thank the editor and the referees for many helpful comments, as well as discussants and participants at various universities and conferences.

Introduction

If there is a single word that appears most frequently in discussions of the economic problems now afflicting both the United States and Europe, that word is surely “debt.” As Table 1 shows, there was a rapid increase in gross household debt in a number of countries in the years leading up to the 2008 crisis; this debt, it is widely argued, set the stage for the crisis, and the overhang of debt continues to act as a drag on recovery. Debt is also invoked – wrongly, we will argue – as a reason to dismiss calls for expansionary fiscal policy as a response to unemployment: you cannot solve a problem created by debt by running up even more debt, say the critics.

The current preoccupation with debt harks back to a long tradition in economic analysis. Irving Fisher (1933) famously argued that the Great Depression was caused by a vicious circle in which falling prices increased the real burden of debt, which led in turn to further deflation. The late Hyman Minsky (1986), whose work is back in vogue thanks to recent events, argued for a recurring cycle of instability, in which calm periods for the economy lead to complacency about debt and hence to rising leverage, which in turn paves the way for crisis. More recently, Richard Koo (2008) has long argued that both Japan’s “lost decade” and the Great Depression were essentially caused by balance-sheet distress, with large parts of the economy unable to spend thanks to excessive debt. And finally since the onset of the crisis, Hall (2011) has argued that the tightening of household borrowing constraint is essential to understand the crisis in the US, while Mian and Sufi (2011a,b) have argued that differences in the debt overhang of households across US counties is helpful to explain why unemployment is higher in some regions than others.

There is also a strand of thinking in international monetary economics that stresses the importance of debt, especially debt denominated in foreign currency. Krugman (1999), Aghion et. al (2001) and others have suggested that “third-generation” currency crises – the devastating combinations of drastic

currency depreciation and severe real contraction that struck such economies as Indonesia in 1998 and Argentina in 2002 – are largely the result of private-sector indebtedness in foreign currency. Such indebtedness, it is argued, exposes economies to a vicious circle closely related to Fisherian debt deflation: a falling currency causes the domestic-currency value of debts to soar, leading to economic weakness that in turn causes further depreciation.

Given both the prominence of debt in popular discussion of our current economic difficulties and the long tradition of invoking debt as a key factor in major economic contractions, one might have expected debt to be at the heart of most mainstream macroeconomic models— especially the analysis of monetary and fiscal policy. Perhaps somewhat surprisingly, however, it is quite common to abstract altogether from this feature of the economy.¹ Even economists trying to analyze the problems of monetary and fiscal policy at the zero lower bound – and yes, that includes the authors (see e.g. Krugman 1998, Eggertsson and Woodford 2003) -- have often adopted representative-agent models in which everyone is alike, and in which the shock that pushes the economy into a situation in which even a zero interest rate is not low enough takes the form of a shift in everyone’s preferences. Now, this assumed preference shift can be viewed as a proxy for a more realistic but harder-to-model shock involving debt and forced deleveraging. But as we will see, a model that is explicit about the distinction between debtors and creditors is much more useful than a representative-agent model when it comes to making sense of current policy debates.

Consider, for example, the anti-fiscal policy argument we have already mentioned, which is that you cannot cure a problem created by too much debt by piling on even more debt. Households borrowed too much, say many people; now you want the government to borrow even more?

¹ Important exceptions include Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Considerable literature has sprung from these papers, for a comprehensive review see Gertler and Kiyotaki (2010). For another recent contribution that takes financial factors explicitly into account see, e.g., Christiano, Motto and Rostagno (2009).

What is wrong with that argument? It assumes, implicitly, that debt is debt -- that it does not matter who owes the money. Yet that cannot be right; if it were, debt would not be a problem in the first place. After all, to a first approximation debt is money we owe to ourselves -- yes, the US has debt to China etc., but that is not at the heart of the problem. Ignoring the foreign component, or looking at the world as a whole, the overall level of debt makes no difference to aggregate net worth -- one person's liability is another person's asset.

It follows that the level of debt matters only if the distribution of that debt matters, if highly indebted players face different constraints from players with low debt. And this means that all debt is not created equal -- which is why borrowing by some actors now can help cure problems created by excess borrowing by other actors in the past. In particular, deficit-financed government spending can, at least in principle, allow the economy to avoid unemployment and deflation while highly indebted private-sector agents repair their balance sheets.

This is, as we will see, just one example of the insights we can gain by explicitly putting private debt in our model.

In what follows, we begin by setting out a flexible-price endowment model in which “impatient” agents borrow from “patient” agents, but are subject to a debt limit. If this debt limit is, for some reason, suddenly reduced, the impatient agents are forced to cut spending; if the required deleveraging is large enough, the result can easily be to push the economy up against the zero lower bound. If debt takes the form of nominal obligations, Fisherian debt deflation magnifies the effect of the initial shock. The type of liquidity constraints we consider are relatively standard, see for example Bewley (1977), Aiyagari (1994) and Huggett (1993). In these papers, however, borrowing is motivated by uninsurable idiosyncratic shocks, while we motivate it from difference in impatience to consume which simplifies the analytics considerably. We next turn to a sticky-price model in which the deleveraging shock affects

output instead of, or as well as, prices. In this model, a shock large enough to push the economy up against the zero lower bound also lands us in a world of topsy-turvy, in which many of the usual rules of macroeconomics are stood on their head. The familiar but long-neglected paradox of thrift emerges immediately; but there are other perverse results as well, including both the “paradox of toil” (Eggertsson 2010b) – increasing potential output may reduce actual output – and the proposition that increasing price flexibility makes the real effect of a debt shock worse, not better.

Finally, we turn to the role of monetary and fiscal policy, where we find, as already indicated, that more debt *can* be the solution to a debt-induced slump. We also point out a possibly surprising implication of any story that attributes the slump to excess debt: precisely because some agents are debt-constrained, Ricardian equivalence breaks down, and old-fashioned Keynesian-type multipliers in which current consumption depends on current income reemerge. At the end of the paper we summarize the results of various extensions of the model (mostly relegated to the appendixes) to suggest further empirical prediction of the model. We show, for example, that a deleveraging shock predicts largest drop in investment and durable consumption, as was observed during the crisis. We also illustrate that our basic findings are not affected by various modeling details and finally to show how the deleveraging mechanism we propose can be incorporated into much richer models such as the medium scale DSGE models now common in the literature.

The paper relates to several strands of the literature that has started emerging following the current crisis. We have already mentioned work by Hall (2011) and Mian and Sufi (2011a,b) which we view as complementary to our findings. Curdia and Woodford (2009, 2010, 2011) and Del Negro, Eggertsson, Ferrero and Kiyotaki (2011) also draw the connections that disturbances in the financial markets can trigger a drop in the natural rate of interest and focus on the policy response of the Federal Reserve to these disturbances. The main difference relative to our model is that in our case there is a sudden

reduction in the quantity of debt the household can borrow, which we argue corresponds well with a “Minsky moment,” while in these papers the source of the shock is a drop in resellability of financial assets in secondary markets (Del Negro et al (2010)) or various types of disturbances that directly affect the financial intermediaries (Curdia and Woodford (2009)). Independently, Guerrieri and Lorenzoni (2010) study a Bewley-Aiyagari-Hugget model and subject the households to a reduction in the borrowing limit as we do,² while Goldberg (2010) conducts a similar exercise with firms that are also subject to idiosyncratic shocks. One important difference is that these authors do not incorporate the nominal debt deflation mechanism which is central to our analysis. Another difference is that in these papers borrowing and lending is motivated by idiosyncratic shocks, rather than differences in preferences, which precludes a simple closed form solution.³

1. *Debt and interest in an endowment economy*

Imagine a pure endowment economy in which no aggregate saving or investment is possible, but in which individuals can lend to or borrow from each other. Suppose, also, that while individuals all receive the same endowments, they differ in their rates of time preference. In that case, “impatient” individuals will borrow from “patient” individuals. We will assume, however, that there is a limit on the amount of debt any individual can run up. Implicitly, we think of this limit as being the result of some kind of incentive constraint; however, for the purposes of this paper we take the debt limit as exogenous.

Specifically, assume for simplicity that there are only two representative agents, each of whom gets a constant endowment $(1/2)Y$ each period. They have log utility functions:

² One interesting difference in Guerrieri and Lorenzoni (2010) relative to this paper is that the borrowing limit is not strictly binding for most consumers due to precautionary savings. Yet a tightening in the borrowing limit has qualitatively the same effect in their model as in our framework.

³ Our closed form solution is what allows us to show several of the paradoxes and key results via simple AS-AD diagrams. Our assumption allows us to show how deleveraging can be put into a medium scale DSGE model and facilitates various extensions of the model (see section 7).

$$E_t \sum_{t=0}^{\infty} \beta(i)^t \log C_t(i) \text{ with } i = s \text{ or } b$$

where $\beta(s) = \beta > \beta(b)$ – that is, the two types of individuals differ only in their rates of time preference. We assume initially that borrowing and lending take the form of risk-free bonds denominated in the consumption good. In that case the budget constraint of each agent is

$$D_t(i) = (1 + r_{t-1})D_{t-1}(i) - \frac{1}{2}Y + C_t(i) \text{ with } i = s \text{ or } b$$

using the notation that a positive D means debt, and a negative D means a positive asset holding. As for example in Ayagari (1993) and Huggett (1994) agents need to respect a borrowing limit D^{high} so that at any date t (here we include the next period interest rate payments for analytic simplicity)

$$(1 + r_t)D_t(i) \leq D^{high} > 0$$

We assume that this bound is at least strictly lower than the present discounted value of output of each agent, i.e. $D^{high} < (1/2)(\beta/(1-\beta))Y$. Because one agent (b) is more impatient than the other (s), the steady state solution of this model is one in which the impatient agent will borrow up to his borrowing limit so that

$$C^b = \frac{1}{2}Y - \frac{r}{1+r}D^{high}$$

where r is the steady state real interest rate. All production is consumed so that

$$Y = C^s + C^b$$

Implying

$$C^s = \frac{1}{2}Y + \frac{r}{1+r}D^{high}$$

Consumption of the saver satisfies a consumption Euler equation in each period:

$$\frac{1}{C_t^s} = (1 + r_t)\beta E_t \frac{1}{C_{t+1}^s}$$

implying that in the steady state the real interest rate is given by the discount factor of the patient consumer so that

$$r = \frac{1 - \beta}{\beta}.$$

2. *The effects of a deleveraging shock*

We have not tried to model the sources of the debt limit, nor will we try to in this paper. Clearly, however, we should think of this limit as a proxy for general views about what level of leverage on the part of borrowers is “safe,” posing an acceptable risk either of unintentional default or of creating some kind of moral hazard.

The central idea of debt-centered accounts of economic instability, however, is that views about safe levels of leverage are subject to change over time. An extended period of steady economic growth and/or rising asset prices will encourage relaxed attitudes toward leverage. But at some point this attitude is likely to change, perhaps abruptly – an event known variously as the Wile E. Coyote moment or the Minsky moment.⁴

In our model, we can represent a Minsky moment as a fall in the debt limit from D^{high} to some lower level D^{low} , which we can think of as corresponding to a sudden realization that assets were overvalued and that peoples’ collateral constraints were too lax. In our flexible-price economy, this downward revision of the debt limit will lead to a temporary fall in the real interest rate, which corresponds to the natural rate of interest in the more general economy we will consider shortly. As we will now see, a large enough fall in the debt limit will temporarily make the natural rate of interest *negative*, an observation that goes to the heart of the economic problems we currently face.

⁴ For those not familiar with the classics, a recurrent event in Road Runner cartoons is the point when Wile E. Coyote, having run several steps off a cliff, looks down. According to the laws of cartoon physics, it’s only when he realizes that nothing is supporting him that he falls. The phrase “Minsky moment” actually comes not from Minsky himself but from Paul McCulley of Pimco, who also coined the term “shadow banking.”

Suppose, then, that the debt limit falls unexpectedly from D^{high} to D^{low} .⁵ Suppose furthermore that the debtor must move quickly to bring debt within the new, lower, limit, and must therefore "deleverage" to the new borrowing constraint. What happens?

To simplify, divide periods into "short run" and "long run." Denote short run with S and long run with L. Again, as in steady state, in the long run we have for the borrower

$$C_L^b = \frac{1}{2}Y - \frac{r}{1+r}D^{low} = \frac{1}{2}Y - (1-\beta)D^{low}$$

where we substituted for the long-run equilibrium real interest rate. In the short run, however, the borrower needs to deleverage to satisfy the new borrowing limit. Hence his budget constraint in the short run is

$$D_S = D^{high} - \frac{1}{2}Y + C_S^b$$

Let us assume that he must deleverage to the new debt limit within a single period. We are well aware that this assumption sweeps a number of potentially important complications under the rug, and will return to these complications at the end of the paper. For now, however, assuming that the borrower must deleverage within a single period to the new debt limit, we have $D_S = \frac{D^{low}}{1+r_S}$ so his consumption is given by

$$C_S^b = \frac{1}{2}Y + \frac{D^{low}}{1+r_S} - D^{high}$$

The long run consumption of the saver is

$$C_L^s = \frac{1}{2}Y + \frac{r}{1+r}D^{low} = \frac{1}{2}Y + (1-\beta)D^{low}$$

⁵ We do not address here the possibility that consumers may anticipate this reduction which may be quantitatively important, see e.g. Kiyotaki, Michaelides and Nikolov (2010).

Again recall that all production in the short run is consumed so that

$$C_S^s + C_S^b = Y$$

Substituting for the consumption of the borrower we get

$$C_S^s = \frac{1}{2}Y - \frac{D^{low}}{1 + r_S} + D^{high}$$

The optimal consumption decision of the saver satisfies the consumption Euler equation

$$C_L^s = (1 + r_S)\beta C_S^s$$

Substitute the short and long run consumption of the saver into this expression and solve for $1+r_S$ to obtain

$$1 + r_S = \frac{\frac{1}{2}Y + D^{low}}{\beta \frac{1}{2}Y + \beta D^{high}}$$

Now all we need for a deleveraging shock to produce a potentially nasty liquidity trap is for the natural rate of interest r_S to go negative, i.e.

$$(C1) \quad \frac{\frac{1}{2}Y + D^{low}}{\beta \frac{1}{2}Y + \beta D^{high}} < 1 \text{ or}$$

$$\beta D^{high} - D^{low} > \frac{1}{2}(1 - \beta)Y.$$

This condition will apply if $\beta D^{\text{high}} - D^{\text{low}}$ is big enough, i.e. if the "debt overhang" is big enough. The intuition is straightforward: the saver must be induced to make up for the reduction in consumption by the borrower. For this to happen the real interest rate must fall, and in the face of a large deleveraging shock it must go negative to induce the saver to spend sufficiently more.

3. Determining the price level, with and without debt deflation

We have said nothing about the nominal price level so far. To make the price level determinate, let us assume that there is a nominal government debt traded in zero supply so that we also have an arbitrage equation that needs to be satisfied by the savers:

$$\frac{1}{C_t^s} = (1 + i_t)\beta E_t \frac{1}{C_{t+1}^s} \frac{P_t}{P_{t+1}}$$

where P_t is the price level and i_t is the nominal interest rate. We need not explicitly introduce the money supply; the results that follow will hold for a variety of approaches, including the "cashless limit" as in Woodford (2001), a cash-in-advance constraint as in Krugman (1998), and a money in the utility function approach as in Eggertsson and Woodford (2003)).

We impose the zero bound

$$i_t \geq 0.$$

Let us now follow Krugman (1998) and fix $P_L = P^*$, i.e. assume that after the deleveraging shock has passed the zero bound will no longer be binding, and the price level will be stable; we can think of this long-run price level as being determined either by monetary policy, as explained below, or by an exogenously given money supply, as in Krugman (1998). Then we can see that in the short run,

$$1 + r_s = (1 + i_s) \frac{P_s}{P^*}$$

If the zero bound was not a problem, it would be possible to set $P_S=P^*$. But if we solve for the nominal interest rate under the assumption that $P_S=P^*$, we get

$$1 + i_S = (1 + r_S) \frac{P^*}{P_S} = \frac{\frac{1}{2}Y + D^{low}}{\beta \frac{1}{2}Y + \beta D^{high}} < 1$$

That is, maintaining a constant price level would require a negative nominal interest rate if condition C1 is satisfied. This cannot happen; so if we substitute $i_S = 0$ instead and solve for the price level, we get

$$\frac{P_S}{P^*} = \frac{\frac{1}{2}Y + D^{low}}{\beta \frac{1}{2}Y + \beta D^{high}} < 1.$$

As pointed out in Krugman (1998), then, if a shock pushes the natural rate of interest below zero, the price level must drop now so that it can rise in the future, creating the inflation necessary to achieve a negative real interest rate.

This analysis has assumed, however, that the debt behind the deleveraging shock is indexed, i.e., denominated in terms of the consumption good. But suppose instead that the debt is in nominal terms, with a monetary value B_t . In that case, deflation in the short run will increase the real value of the existing debt. Meanwhile, the debt limit is presumably defined in real terms, since it is ultimately motivated by the ability of the borrower to pay in the future out of his endowment. So a fall in the price level will increase the burden of deleveraging. Specifically, if debt is denominated in dollars, then $D^{high} = B^{high}/P_S$, and the indebted agent must make short-run repayments of

$$\frac{B^{high}}{P_S} - \frac{D^{low}}{1 + r_S}$$

to satisfy the debt limit. Hence, as the price level drops he must pay more. Thus the natural rate of interest becomes

$$1 + r_s = \frac{\frac{1}{2}Y + D^{low}}{\beta \frac{1}{2}Y + \beta \frac{B^{high}}{P_s}}$$

What this tells us is that the natural rate of interest is now *endogenous*: as the price level drops, the natural rate of interest becomes more negative, thus making the price level drop even more, and so on. This is simply the classic "Fisherian" debt deflation story.

4. *Endogenous output*

We now move to an economy with production. To do this we assume that C_t now refers not to a single good, but instead is a Dixit-Stiglitz aggregate of a continuum of goods giving the producer of each good market power with elasticity of demand given by θ . Our representative consumers thus have the following utility function

$$\sum_{t=0}^{\infty} \beta(i)^t [u^i(C_t(i)) - v^i(h_t(i))] \text{ with } i = s \text{ or } b$$

where consumption now refers to $C_t = \left[\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$ and P_t is the corresponding price index

$P_t = \left[\int_0^1 p_t(i)^{(\theta-1)} \right]^{\frac{1}{1-\theta}}$ while h_t is the number of hours the agents work. We also make a slight generalization of our previous setup. We assume that there is a continuum of consumers of measure 1, and that an arbitrary fraction χ_s of these consumers are savers and a fraction $\chi_b=1-\chi_s$ are borrowers.

Aggregate consumption is thus

$$C_t = \chi_s C_t^s + \chi_b C_t^b$$

where C_t has the interpretation of being per capita consumption in the economy, while C_t^s is per capita savers' consumption, and C_t^b per capita borrowers' consumption. Instead of receiving an endowment income, the agents now receive their income through wage, W_t , paid for each hour they work and

through the profits of the firms in the economy. Aside from this different source of income, they face the same budget constraint as before.

There is a continuum of firms of measure one, each of which produce one type of the varieties the consumers like. We assume all firms have a production function that is linear in labor. Suppose a fraction $1-\lambda$ of these monopolistically competitive firms keep their prices fixed for a certain planning period while the λ fraction of the firms can change their prices all the time. We assume that the firms are committed to sell whatever is demanded at the price they set and thus have to hire labor to satisfy this demand.

In the Appendix we put all the pieces of this simple general equilibrium model together. After deriving all the equilibrium conditions, we approximate this system by a linear approximation around the steady state (denoted by a bar) of the model where $D_t = D^{low} = \bar{D}$ and report the linearized equations of the model below.⁶

There are two main new elements. The first is that since production is endogenous agents are not only choosing consumption but they are also choosing how much to work. This gives rise to an optimal work and consumption choice, so that each type satisfies a new first order condition given by (in a linearized form)

$$\widehat{W}_t = \omega \widehat{h}_t^i + \sigma^{-1} \widehat{C}_t^i \text{ where } i = b \text{ or } s$$

$$\text{where } \sigma \equiv -\frac{\bar{u}_c^b}{\bar{u}_{cc}^b \bar{Y}} \equiv -\frac{\bar{u}_c^s}{\bar{u}_{cc}^s \bar{Y}} > 0, \omega \equiv \frac{\bar{v}_{hh}^b \bar{h}}{\bar{v}_h^b} \equiv \frac{\bar{v}_{hh}^s \bar{h}}{\bar{v}_h^s} > 0, \widehat{C}_t^i \equiv \frac{C_t^i - \bar{C}^i}{\bar{Y}} \text{ and } \widehat{W}_t \equiv \log \frac{W_t}{\bar{W}}.$$

Note that because the borrower can get more income by working he can now deleverage by either cutting consumption or increasing hours worked. In equilibrium he will do both. As production is linear in labor, we have

⁶ See Eggertsson and Woodford (2003) for a discussion of the conditions under which the linearization is valid.

$$\hat{Y}_t = \chi_s \hat{h}_t^s + \chi_b \hat{h}_t^b.$$

where $\hat{Y}_t \equiv \log \frac{Y_t}{\bar{Y}}$.

The second main new element comes from the fact that production is endogenous and that some prices are rigid. This implies a "New Classical Phillips curve", or AS equation, of the following form:

$$\pi_t = \kappa \hat{Y}_t + E_{t-1} \pi_t$$

where $\kappa \equiv \left(\frac{\lambda}{1-\lambda} \right) (\omega + \sigma^{-1})$, and $\pi_t \equiv \log \frac{P_t}{P_{t-1}}$. The key point is that if inflation is higher than expected in the short run than will be above potential.

We are now also a bit more specific about how monetary policy is set. In particular we assume that the central bank follows a Taylor rule of the following form:

$$i_t = \max(0, r_t^n + \phi_\pi \pi_t)$$

where $\phi_\pi > 1$ and r_t^n is the natural rate of interest (defined below) and i_t now refers to $\log(1+i_t)$ in terms of our previous notation. The rest of the model is the same as we have already studied, with minor adjustments due to the way in which we have normalized our economy in terms of per capita consumption of each group and the different sources of income (which are no longer derived from an endowment but through wages and profits). Linearizing the consumption Euler equation of savers gives

$$\hat{C}_t^s = E_t \hat{C}_{t+1}^s - \sigma(i_t - E_t \pi_{t+1} - \bar{r})$$

where $\sigma \equiv -\frac{\bar{u}_c^s}{\bar{u}_{cc}^s \bar{Y}}$, $\hat{C}_t^s \equiv \frac{C_t^s - \bar{C}^s}{\bar{Y}}$, and $\bar{r} \equiv \log \beta^{-1}$. Linearizing the resource constraint yields

$$\hat{Y}_t = \chi_s \hat{C}_t^s + \chi_b \hat{C}_t^b.$$

To close the model, it now remains to determine the consumption behavior of the borrowers, which is again at the heart of the action. To simplify exposition, again, let us split the model into "short run" and "long run" with an unexpected shock occurring in the short run. We can then see immediately from the AS equation that $\hat{Y}_L = 0$ so that the economy will revert back to its "flexible price" equilibrium in the

long run as this model has long run neutrality. The model will then, with one caveat, behave exactly like the flexible price model we just analyzed. We have already seen that in the long run $\hat{C}_L^b = \hat{C}_L^s = 0$ in the model with flexible prices. Also note that away from the zero bound the policy rule implies a unique bounded solution for the long run in which $i_L = r_L^n = \bar{r}$ and $\pi_L = 0$.⁷ Again, then, all the action is in the short run. The caveat here involves the determination of the long-run price level. Given the Taylor rule we have just specified, prices will *not* revert to some exogenously given P^* . Instead, they will be stabilized after the initial shock, so that prices will remain permanently at the short-run equilibrium level P_S . It would be possible to write a different Taylor rule that implies price level reversion; as we will see shortly, the absence of price level reversion matters for the slope of the aggregate demand curve.

Back to the model: in the short run, the borrower once again needs to deleverage to satisfy his borrowing limit. His consumption is now given by

$$\hat{C}_S^b = \hat{I}_S^b - \hat{D} + \gamma_D \pi_S - \gamma_D \beta (i_S - \pi_L - \bar{r})$$

where $\gamma_D \equiv \frac{\bar{D}}{\bar{Y}}$, $\hat{D} \equiv \frac{D^{high} - \bar{D}}{\bar{Y}}$ and \hat{I}_S^b refers to the income of the borrower (but before we had the endowment in its place). Note that this is a “consumption function” in which current consumption is in part determined by current income (recall that in our current notation \hat{I}^b is output per capita in percentage deviation from steady state)– not, as has become standard in theoretical macroeconomics, solely by expectations of future income. The explanation is simple: by assumption, the borrower is liquidity-constrained, unable to borrow and paying down no more debt than he must. In fact, the marginal propensity to consume out of current income on the part of borrowers is 1.

Meanwhile, the saver’s consumption is given by

$$\hat{C}_S^s = \hat{C}_L^s - \sigma (i_S - \pi_L - \bar{r}).$$

⁷ See Woodford (2003) but the key condition is that the Taylor principle is satisfied according to our policy rule. Here we abstract from the possibility that the zero bound can be binding due to self-fulfilling expectations, see e.g. discussion in Eggertsson and Woodford (2003) about how this sort of equilibrium can be excluded in the long run.

By using the aggregate resource constraint, the consumption of the saver, and the optimal labor decisions of each household and $\hat{C}_L^b = \hat{C}_L^s = \pi_L = 0$, we now can solve for the income of the borrower as

$$\hat{l}_S^b = \hat{W}_S + \hat{h}_S^b = \mu \hat{Y}_S - \chi_s \omega^{-1} \chi_b^{-1} (i_S - \bar{r})$$

where $\mu \equiv (1 + \omega^{-1})(\omega + \sigma^{-1}) - \sigma^{-1} \omega^{-1} \chi_b^{-1}$. Now substitute the two consumption functions into the resource constraint to obtain

$$\hat{Y}_S = \chi_s \{-\sigma(i_S - \bar{r})\} + \chi_b \{\hat{l}_S^b - \hat{D} + \gamma_D \pi_S - \gamma_D \beta (i_S - \bar{r})\}$$

and substituting for \hat{l}_S^b with the expression from above and rearranging to obtain

$$(2) \quad \hat{Y}_S = -\frac{\chi_s(\omega^{-1} + \sigma) + \chi_b \gamma_D \beta}{1 - \chi_b \mu} (i_S - \bar{r}) - \frac{\chi_b}{1 - \chi_b \mu} \hat{D} + \frac{\chi_b \gamma_D}{1 - \chi_b \mu} \pi_S \quad or$$

$$(3) \quad \hat{Y}_S = -\frac{\chi_s(\omega^{-1} + \sigma) + \chi_b \gamma_D \beta}{1 - \chi_b \mu} (i_S - r_S^n)$$

where in the last line we have used the definition of the natural rate of interest (i.e. the real interest rate if prices were fully flexible) given by

$$r_S^n \equiv \bar{r} - \frac{\chi_b}{\chi_s(\omega^{-1} + \sigma) + \chi_b \gamma_D \beta} \hat{D} + \frac{\chi_b \gamma_D}{\chi_s(\omega^{-1} + \sigma) + \chi_b \gamma_D \beta} \pi_S$$

Here once again, the nature rate of interest depends directly on the amount of deleveraging just as in the endowment economy. As the borrower cuts back on his spending, then the real interest rate needs to decline for the saver to pick up the slack. Observe that the strength of this effect depends upon by how much the borrower delevers by increasing work instead of cutting consumption. In the extreme where labor is perfectly elastic, i.e. $\omega \rightarrow 0$, this effect disappears, while the endowment economy we just studied corresponded to the other extreme where $\omega \rightarrow \infty$. In what follows, we only consider the

empirically relevant case in which ω is positive but finite, and also impose the restriction that $1 - \chi_b \mu > 0$ in equation (2).⁸

What does the equation (3) mean? It is an IS curve, a relationship between the interest rate and total demand for goods. And the underlying logic is very similar to that of the old-fashioned Keynesian IS curve.⁹ Consider what happens if i_S falls, other things equal. First, savers are induced to consume more than they otherwise would. Second, this higher consumption leads to higher income for both borrowers and savers. And because borrowers are liquidity-constrained, they spend their additional income, which leads to a second round of income expansion, and so on.

Once we combine this derived IS curve (3) with the assumed Taylor rule we obtain aggregate demand relationship or an AD curve. It is immediately clear that there are two possible regimes following a deleveraging shock. If the shock is relatively small, so that the natural rate of interest remains positive, the actual interest rate will fall to offset any impact on output. If the shock is sufficiently large, however, the zero lower bound will be binding, and output will fall below potential.

The extent of this fall depends on the aggregate supply response, because any fall in output will also be associated with a fall in the price level, and the natural rate of interest is endogenous thanks to the Fisher effect. Since the deleveraging shock is assumed to be unanticipated, so that $E_t \pi_S = 0$, the aggregate supply curve may be written as

$$\pi_S = \kappa \hat{Y}_S$$

Substituting this into equation (2), and assuming the shock to D is large enough so that the zero bound is binding, we obtain

⁸ If this condition were not satisfied, demand would fall as interest rates are cut. Note that this condition is always satisfied for small enough χ_b . Hence this condition requires that there is a sufficient number of agents which are unconstrained and thus respond directly to interest rate variations through the standard consumption Euler equation, i.e., the number of constrained agents cannot be “too big.” More precisely the condition is satisfied if $\chi_b < \frac{1}{1+\omega}$.

⁹ Provided $\mu > 0$ which is the case as long as there are enough constrained players, or more specifically provided $\chi_b >$

$$\frac{\omega^{-1}\sigma^{-1}}{1+\omega^{-1}+\sigma^{-1}+\omega^{-1}\sigma^{-1}}$$

$$\hat{Y}_S = \Gamma - \frac{\chi_b}{1 - \chi_b(\mu + \kappa\gamma_D)} \hat{D} < 0$$

$$\pi_S = \kappa \Gamma - \frac{\chi_b \kappa}{1 - \chi_b(\mu + \kappa\gamma_D)} \hat{D} < 0$$

where $\Gamma > 0$.¹⁰ So the larger the debt shock, the larger the fall in both output and the price level. But the really striking implications of this model come when one recasts it in terms of a familiar framework, that of aggregate supply and aggregate demand. The basic picture is shown in Figure 1. The short-run aggregate supply curve is, as we've already seen, upward sloping. The surprise, however, is the aggregate demand curve: in the aftermath of a large deleveraging shock, which puts the economy up against the zero lower bound, it is *also* upward sloping – or, if you prefer, backward bending.¹¹ The reason for this seemingly perverse slope should be obvious from the preceding exposition: because a lower price level increases the real value of debt, it forces borrowers to consume less; meanwhile, savers have no incentive to consume more, because the interest rate is stuck at zero.

We next turn to the seemingly paradoxical implications of a backward-sloping AD curve for some key macroeconomic issues.

5. *Topsy-turvy: Paradoxes of thrift, toil, and flexibility*

The paradox of thrift is a familiar proposition from old-fashioned Keynesian economics: if interest rates are up against the zero lower bound, a collective attempt to save more will simply depress the economy, leading to lower investment and hence (through the accounting identity) to lower savings.

¹⁰ $\Gamma = \frac{\chi_s(\omega^{-1} + \sigma) + \chi_b \gamma_D \beta}{1 - \chi_b(\mu + \kappa\gamma_D)} \bar{r}$. The denominator has to be positive due the restriction discussed in footnote 7 and the first term has to be less than the second under the assumption that the natural rate of interest is negative.

¹¹ We assume that the AD curve, while backward-sloping, remains steeper than the AS curve. Otherwise the short-run equilibrium will be unstable under any plausible adjustment process. This amounts to the assumption that $1 - \chi_b \mu > \kappa \chi_b \gamma_D$. Note that if $\chi_b = 0$ then the AD is vertical. As we increase the number of constrained people it starts sloping backwards, eventually so far that the AS and AD become closed to parallel, the model explodes, and our approximation is no longer valid. Our assumption guarantees that this is not the case. In the absence of a deleveraging shock, then the zero bound is not binding, and we have a regular looking AD curve that is downward sloping and a well behaved unique equilibrium.

Strictly speaking, our model cannot reproduce this paradox, since it is a pure consumption model without investment. However, it does give a plausible mechanism through which the economy can find itself up against the zero lower bound. So this model is, in spirit if not precisely in letter, a model of a paradox-of-thrift type world.¹²

Beyond this, there are two less familiar paradoxes that pop up thanks to the backward-sloping AD curve.

First is the “paradox of toil,” first identified by Eggertsson (2010b) but appearing here in a starker, simpler form than in the original exposition, where it depended on expectation effects. Suppose that aggregate supply shifts out, for whatever reason – a rise in willingness to work, a change in tax rates inducing more work effort, a rise in productivity, etc.¹³ As shown in Figure 2, this shifts the aggregate supply curve AS to the right, which would ordinarily translate into higher actual output. But the rise in aggregate supply leads to a fall in prices – and in the face of a backward-sloping AD curve, this price decline is contractionary via the Fisher effect. So more willingness and/or ability to work ends up reducing the amount of work being done.

Second, and of considerable relevance to the ongoing economic debate, is what we will term the “paradox of flexibility.” It is commonly argued that price and wage flexibility helps minimize the losses from adverse demand shocks. Thus Hamilton (2007), discussing the Great Depression, argues that “What is supposed to help the economy recover is that a substantial pool of unemployed workers should result in a fall in wages and prices that would restore equilibrium in the labor market, as long as the

¹² See Eggertsson (2010b) for an example of how the paradox occurs with endogenous investment but through preference shocks which also show up as a decline in the natural rate of interest.

¹³ For simplicity we are considering a shock which only shifts out the AS curve without any effect on the AD schedule. Most shocks, however, shift both curves at the same time since they also affect the labor income of the borrower (positively or negatively). It can be shown, for example, that a uniform increase in willingness to work across the two types will not only shift out the AS curve, it will also shift the AD curve *backwards* because it lowers real wages which translates into lower income for the borrowers. This type of shock, thus, makes the paradox of toil even more severe in the model than what we consider in the figure.

government just keeps the money supply from falling.” The usual criticism of New Deal policies is that they inhibited wage and price flexibility, thus blocking recovery.

Our model suggests, however, that when the economy is faced by a large deleveraging shock, increased price flexibility – which we can represent as a steeper aggregate supply curve – actually makes things worse, not better. Figure 3 illustrates the point. The shock is represented as a leftward shift in the AD curve from AD_1 to AD_2 ; we compare the effects of this shock in the face of a flat AS curve AS_{sticky} , corresponding to inflexible wages and prices, and a steeper AS curve AS_{flexible} , corresponding to more responsive wages and prices. The output decline in the latter case is larger, not smaller, than in the former. Why? Because falling prices do not help increase demand, they simply intensify the Fisher effect, raising the real value of debt and depressing spending by debtors.¹⁴

6. *Monetary and fiscal policy*

What can policy do to avoid or limit output loss in the face of a deleveraging crisis? Our model has little new to say on the monetary front, but it offers some new insights into fiscal policy.

On monetary policy: as pointed out by Krugman (1998) and reiterated in part 1 of this paper, expected inflation is the “natural” solution to a deleveraging shock, in the sense that it is how the economy can achieve the negative natural real interest rate even though nominal rates are bounded at zero. In a world of perfect price flexibility, deflation would “work” under liquidity trap conditions, if it does, only by reducing the current price level relative to the expected future price level, thereby generating expected inflation. It is therefore natural, in multiple senses, to think that monetary policy can

¹⁴ A similar paradox is documented in a general equilibrium model in Eggertsson (2010b) but unlike here, there it relies on an expectation channel. Earlier literature documenting that increased price flexibility can increase output volatility includes Tobin (1975) and De Long and Summers (1986), see Bhattarai, Eggertsson and Schoenle (2011) for a more recent exposition.

deal with a deleveraging shock by generating the necessary rise in expected inflation directly, without the need to go through deflation first.

In the context of the model, this rise in expected inflation could be accomplished by changing the Taylor rule; this would amount to the central bank adopting, at least temporarily, a higher inflation target. As is well understood, however, this would only work if the higher target is credible – that is, if agents expect the central bank to follow through with promises of higher inflation even after the deleveraging crisis has passed. Achieving such credibility is not easy, since central bankers normally see themselves as defenders against rather than promoters of inflation, and might reasonably be expected to revert to type at the first opportunity. So there is a time consistency problem.¹⁵

Where this model adds something to previous analysis on monetary policy is what it has to say about an incomplete expansion – that is, one that reduces the real interest rate, but not enough to restore full employment. The lesson of this model is that even such an incomplete response will do more good than a model without debt suggests, because even a limited expansion leads to a higher price level than would happen otherwise, and therefore to a lower real debt burden.

Where the model really suggests new insights, however, is on fiscal policy. It is a familiar proposition, albeit one that is strangely controversial even within the macroeconomics community, that a temporary rise in government purchases of goods and services will increase output when the economy is up against the zero lower bound; Woodford (2010) offers a comprehensive account of what representative-agent models have to say on the subject. Contrary to widely held belief, Ricardian equivalence, in which consumers take into account the future tax liabilities created by current spending, does not undermine this proposition. In fact, if the spending rise is limited to the period when the zero lower bound is binding, the rise in income created by that spending fully offsets the rise in future taxes; the multiplier on government spending in a simple one period liquidity trap consumption-only model

¹⁵ See Eggertsson (2006) for a formalization of this dynamic inconsistency problem, building on Krugman (1998).

like the one considered here, but without debt, ends up being exactly one (once multiple periods are studied, and expectations taken into account, this number can be much larger, especially at the zero bound as for example shown in Christiano et al (2009) and Eggertsson (2010a))

What does modeling the liquidity trap as the result of a deleveraging shock add? First, it gives us a reason to view the liquidity trap as temporary, with normal conditions returning once debt has been paid down to the new maximum. This in turn explains why more (public) debt can be a solution to a problem caused by too much (private) debt. The purpose of fiscal expansion is to sustain output and employment while private balance sheets are repaired, and the government can pay down its own debt after the deleveraging period has come to an end.

Beyond this, viewing the shock as a case of forced deleveraging suggests that fiscal policy will, in fact, be more effective than standard models suggest – because Ricardian equivalence will not, in fact, hold. The essence of the problem is that debtors are liquidity-constrained, forced to pay down debt; this means, as we have already seen, that their spending depends at the margin on current income, not expected future income, and this means that something resembling old-fashioned Keynesian multiplier analysis reemerges even in the face of forward-looking behavior on the part of consumers.¹⁶

Let us revise the model slightly to incorporate government purchases of goods and services, on one side, and taxes, on the other. For now, let us assume that future taxes on the borrowers stay constant across our policy experiments; so that any fiscal adjustment is offset by current or future taxes for the saver (we relax this assumption in the next section).¹⁷ We assume that the government purchases the same composite good consumed by individuals, but uses that good in a way that, while it may provide

¹⁶ The closest parallel to our debt-constrained consumers in studies of fiscal policy in New Keynesian models are the “rule-of-thumb” consumers in Gali, Lopez-Salido and Valles (2007). In their work a fraction of workers spend all their income (because of rules of thumb or because they do not have access to financial markets). This gives rise to a multiplier of a similar form as we study here since in their model aggregate spending also depends in part directly on income as in old Keynesian models.

¹⁷ Because the saver is un-constrained, Ricardian equivalence applies, so it does not matter if the fiscal adjustment takes place by taxes in the short or the long run.

utility to consumers, is separable from private consumption and therefore does not affect intertemporal choices of the savers. We also assume that taxation takes a lump-sum form. The budget constraint for borrowers may now be written

$$\hat{C}_S^b = \hat{I}_S^b - \hat{D} + \gamma_D \pi_S - \gamma_D \beta (i_S - \bar{r}) - \hat{T}_S^b$$

while the savers' consumption Euler equation remains the same. The income of the borrower is now given by

$$\hat{I}_S^b = \mu \hat{Y}_S - \omega^{-1} \chi_b^{-1} \chi_s (i_S - \bar{r}) + \sigma^{-1} [\omega^{-1} \chi_b^{-1} \chi_s - 1] \hat{G}_S.$$

The AS equation is now

$$\pi_S = \kappa \hat{Y}_S - \varphi \kappa \hat{G}_S.$$

The resource constraint is now given by

$$\hat{Y}_S = \chi_s \hat{C}_S^s + (1 - \chi_s) \hat{C}_S^b + \hat{G}_S.$$

Substituting the AS equation into the consumption function of the borrower, and substituting the resulting solution into the resource constraint, together with the consumption of the saver, and solving for output, we obtain an expression for output as a function of the fiscal instruments:

$$(4) \quad \hat{Y}_S = \Gamma - \frac{\chi_b}{1 - \chi_b(\mu + \kappa \gamma_D \kappa)} \hat{D} - \frac{\chi_b}{1 - \chi_b(\mu + \kappa \gamma_D \kappa)} \hat{T}_S^b + \frac{1 + \omega^{-1} \sigma^{-1} \chi_s - \sigma^{-1} \chi_b - \chi_b \kappa \gamma_D \varphi}{1 - \chi_b(\mu + \kappa \gamma_D \kappa)} \hat{G}_S$$

To understand this result, it is helpful to focus first on a special case, that of a horizontal short-run aggregate supply curve, i.e., $\kappa=0$.¹⁸ In that case the third term simplifies to

$$\left(\frac{1 + \omega^{-1} \sigma^{-1} \chi_s - \sigma^{-1} \chi_b}{1 - \chi_b \mu} \right) \hat{G}_S.$$

¹⁸ In this case we abstract from the "Fisher effect" of inflation reducing real debt and thus creating more expansion, but we also abstract from the fact that an increase in government spending increases AS which works in the opposite direction due to the paradox of toil.

With just a little bit of algebra, using the expression for μ it is easy to show that a temporary rise in government spending always has a multiplier greater than one (the condition is that $\omega \geq -1$ which is always satisfied since this parameter must be positive), with the size of that multiplier depending positively on the share of debt-constrained borrowers in the economy. Consider for example the commonly used parameter values $\sigma = 2$ and $\omega = \frac{1}{2}$. Then the multiplier is $\frac{2 - \frac{3}{2}\chi_b}{2 - 3\chi_b}$ so if constrained borrowers receive one-third of income, for example, the multiplier would be 1.5; if they receive half of income, it would be 2.5, and so on.

If we now reintroduce an upward-sloping aggregate supply curve, so that $\kappa > 0$, the multiplier is affected by two forces. First, the fiscal expansion has the additional effect of raising the price level above what it would have been otherwise, and hence reducing the real debt burden. Second, the increase in spending increases aggregate supply¹⁹ which now has a price effect, and works in the opposite direction due to the paradox of toil. By taking a partial derivative of the multiplier with respect to κ we can see that the first effect will always dominate, so that the multiplier is increasing in κ (because $\varphi < 1$). Overall this model suggests a relatively favorable view of the effectiveness of fiscal policy after a deleveraging shock.

Also note the middle term: in this model tax cuts and transfer payments are effective in raising aggregate demand, as long as they fall on debt-constrained agents. In practice, of course, it is presumably impossible to target such cuts entirely on the debt-constrained, so the old-fashioned notion that government spending gets more bang for the buck than taxes or transfers survives. And the model also suggests that if tax cuts are the tool chosen, it matters greatly who receives them.

The bottom line, then, is that if we view liquidity-trap conditions as being the result of a deleveraging shock, the case for expansionary policies, especially expansionary fiscal policies, is substantially

¹⁹ It makes people work more due to an increase in the marginal utility of private consumption.

reinforced. In particular, a strong fiscal response not only limits the output loss from a deleveraging shock; it also, by staving off Fisherian debt deflation, limits the size of the shock itself.

7. Extensions

7.1. Deleveraging, endogenous debt limits and long-run taxes

How important is it that we assumed that it was current or future taxes on the saver that financed the fiscal policy experiments in the last section? And how important is it for our conclusion about fiscal policy that the debt limit is exogenous?²⁰ Here we show that fiscal policy can be even more expansionary if it is financed by taxes on future borrower instead of the savers. Yet, while the results from the last section did not depend on the debt limit being exogenous, we show that this new conclusion depends critically on the debt limit being fixed.

Consider first the possibility that a tax cut on the borrower is not met by current or future tax increases on the saver, but instead by higher long-run taxes on the borrower. This makes the tax cuts even more expansionary under our assumption about an exogenous debt limit (as the reader can confirm in equation (5) below). The reason for this is that while the borrower will spend every additional dollar of income in the short-run, the savers spending depends upon his current and expected future income. Since an increase in taxes on the borrower in the future (relative to our previous policy experiment) moves money from the borrower to the saver in the long-run, the savers short-run consumption increases. In fact, the deleveraging shock can in principle be completely undone by a tax cut targeted at the borrower in the short-run that is large enough to fully offset the amount of deleveraging needed, as long as this tax cut is financed by a corresponding increase in long-run taxes on the borrower.

²⁰ The formal way of verifying this is to observe that the set of equations that pin down the equilibrium in the short run do not depend on the tax rates of the saver, see Appendix A (extended version).

Intuitively, the governments “circumvent” the change in the private borrowing limit this way by using its own ability to borrow.

That shifting the future tax burden from the saver to the borrower is expansionary due to higher short-run consumption of the saver is a rather special result. It relies heavily on the assumption that the borrowing limit is exogenous.²¹ Consider a slight generalization of our previous debt limit so that

$$D_t^b = (1 - \gamma^l)D^l + (1 + r_t)\gamma^l E_t \sum_{j=1}^{\infty} R_{t,t+j} (I_{t+j}^b - T_{t+j}^b),$$

where $R_{t,t+j} \equiv \frac{1}{1+r_t} \frac{1}{1+r_{t+1}} \dots \frac{1}{1+r_{t+j}}$ is the real discount factor and $0 \leq \gamma^l \leq 1$. Here the debt limit does not only depend on an exogenous constant, it also depends on the net present value of future income net of taxes ($\gamma^l = 1$ corresponds to the “natural borrowing limit” while $\gamma^l = 0$ is our previous case). We now think of a reduction in D_t^b as being due to an unexpected drop in D^l (as before), a drop in expected future disposable income or a change in the level of future interest rates. Importantly, a change in expectation about the future tax can also reduce the debt limit, as higher future taxes reduce the borrower ability to pay back.

Denote taxes on the borrower in the short run by \hat{T}_S^b and long run taxes by \hat{T}_L^b , while the (now endogenous) debt limit is \hat{D}_S^b .²² Going through exactly the same steps as when deriving (4) we obtain a solution for output in the short-run as

$$(5) \quad \hat{Y}_S = \Gamma - \frac{\chi_b}{1 - \chi_b(\mu + \kappa\gamma_D\kappa)} \hat{D} - \frac{\chi_b}{1 - \chi_b(\mu + \kappa\gamma_D\kappa)} \hat{T}_S^b + \frac{1 + \omega^{-1}\sigma^{-1}\chi_s - \sigma^{-1}\chi_b - \chi_b\kappa\gamma_D\varphi}{1 - \chi_b(\mu + \kappa\gamma_D\kappa)} \hat{G}_S \\ + \frac{\chi_b\beta}{1 - \chi_b(\mu + \kappa\gamma_D\kappa)} \hat{D}_S + \frac{\chi_b + \omega^{-1}\sigma^{-1}}{1 - \chi_b(\mu + \kappa\gamma_D\kappa)} \hat{C}_L^S,$$

²¹ Which is one reason why we assumed long-run taxes on the borrower were constant in our previous experiment, i.e. we did not want to exaggerate the effect of fiscal policy.

²² Again we assume that any shortfall or surplus in the government budget is made up by current or future lump sum taxes on the saver.

where the last two terms are new relative to expression (4) and are determined by long-term taxes on the borrower as follows

$$\hat{C}_L^s = \frac{\chi_b}{\chi_s} \frac{1 - \gamma^l}{1 + \omega^{-1} \sigma^{-1} (1 - \gamma^l)} \hat{T}_L^b$$

and

$$\hat{D}_S^b = - \frac{(1 - \beta)^{-1} \gamma^l}{1 + \omega^{-1} \sigma^{-1} (1 - \gamma^l)} \hat{T}_L^b.$$

Observe first that under this more general specification of the borrowing limit the results from the last section are unchanged as long as the government spending or tax cuts on borrowers are financed by current or future taxes on the saver (since in this case that $\hat{T}_L^b = 0$ and the new terms drop out). It now becomes clear, however, that it is no longer the case that an increase in future taxes of the borrower necessarily increases demand in the short run: An increase in future taxes of the borrower reduces the debt limit of the borrower thus requiring an ever bigger reduction in his spending due to deleveraging. We can see that in the limit as γ^l gets closer to 1 there is a Ricardian equivalence result of sorts that enters the picture: if a short-run tax cut on the borrower is financed by a higher long run taxes on the borrower, it has no effect due to the endogenous effect on the borrowing limit. But note that even in this limiting case, our result from last section survives, i.e. the tax cut on the borrower is still expansionary (because it was being financed by current and future taxes on the saver), but in this limiting case it is *only* expansionary due to the fact that it reflects a redistribution of the tax burden from the borrower to the saver.

Before concluding this section it is worth noting that there are several quite reasonable cases in which an endogenous debt limit amplifies the effect of fiscal policy beyond what we have already studied. Consider, for example, a debt limit that depends also on the current level of income of the borrower, for example $D_S^b = f(Y_S)$ with $f'(Y_S) > 0$. In this case a tax cut that increases demand will have a “second

round effect” due to the fact that it increases aggregate income and hence increases his debt limit, thus increasing demand even further, and so on. There are many reasons to expect the borrowing limit to depend to some extent on current condition, for example if the collateral value of the borrowers assets depend on current market conditions (such as the price of houses), or if current income of the borrower is used by banks to predict his future income.

7.2 Deleveraging, the trade-off between inflation and output and inefficient policy responses

Here we outline how a deleveraging cycle can matter even when the interest rates are positive. We also clarify that our theory does not imply that there is *always* deflation at that zero bound. To show this we need to abandon a simplifying assumption we made in our earlier sections, which was that the deleveraging shock had no effect on marginal costs of firms, hence it did not shift the aggregate supply relationship between inflation and output in the short run. More generally, however, this short-run relationship is given by

$$\pi_S = \kappa_1 \hat{Y}_S + \kappa_2 (i_S - \bar{r})$$

where κ_1 and κ_2 are defined in the footnote.²³ The new term comes about due to the fact that a variation in the interest rate can increase/decrease firms’ marginal costs through the effect it has on labor supply and thus the real wage rate. Hence a deleveraging shock can trigger both a variation in the natural rate of interest and at the same time create a tradeoff between output and inflation. This effect can be positive or negative, depending on how the model is parameterized, although we have not found this to be quantitatively important in numerical experiments. It is not difficult to imagine, however, other

²³Here $\kappa_1 \equiv \left(\frac{\lambda}{1-\lambda}\right) \frac{\omega^b + \sigma^{b-1}}{1 - \chi_s \left(1 - \frac{\omega^b}{\omega^s}\right)}$ and $\kappa_2 \equiv \left(\frac{\lambda}{1-\lambda}\right) \frac{\chi_s \left(1 - \frac{\omega^b}{\omega^s}\right) - \chi_s \left(1 - \frac{\sigma^s}{\sigma^b}\right)}{1 - \chi_s \left(1 - \frac{\omega^b}{\omega^s}\right)}$. If $\omega^b = \omega^s$ and $\sigma^s = \sigma^b$ we obtain our previous equation.

quantitatively relevant channels through which a deleveraging shock can affect firms' marginal costs²⁴ and hence we think the specification above can illuminate at least two general lessons that go beyond the specifics of the current model and apply more broadly to deleveraging cycles.

First, a “cost push” term of this kind suggests that even in the absence of the zero bound, a deleveraging shock could be relevant for the policy stabilization problem of the government, to the extent it creates not only a need to reduce the nominal interest rate, but also a trade-off between inflation and output. Second, this specification illustrates that our theory does not necessarily imply that there needs to be deflation in equilibrium once the economy is up against the zero bound or even any reduction in current or expected inflation: All that is needed is for the natural rate of interest to be below what the central bank wants it to be (leading to an output slack). If there are strong enough “cost push” shocks triggered by a deleveraging disturbance then a liquidity trap²⁵ can very well happen at positive inflation. Nevertheless, at positive or negative inflation, the slope of the AD and AS curves we have derived is exactly the same, and all the results we have derived remain unchanged (i.e. the Fisher effect of inflation reducing real value of debt remains there, the paradox of thrift and toil, large multipliers and so on).

One other mechanism through which deleveraging cycles can matter for business cycle fluctuation away from the zero bound is if deleveraging shocks are not fully offset by monetary policy. Above we noted that this will happen if deleveraging shock triggers simultaneous cost-push pressures even if the central bank offsets the drop in the natural rate of interest. It is straight forward to think of many other reasons monetary policy does not fully offset a deleveraging cycle, e.g., due to suboptimal monetary policy, additional constraints on what policy can do (e.g. due to concerns about un-anchoring inflation

²⁴ If the firms need to deleverage, for example, this would show up in a cost push shock under some plausible specifications. Within the context of the current crisis, of course, we have seen other sources of increases in marginal costs due to disturbances in the commodity markets.

²⁵ Which we define, simply, as the central bank policy to be constrained by the zero bound.

expectations). Finally, consider a large currency area in which the deleveraging cycle hits some regions of the currency union harder than others. In this case policy aimed at offsetting the cycle in one region will not offset it in the other.

7.3 Deleveraging, durable goods and investment

So far we assumed only one type of private spending, a good that was consumed within one period. One interesting aspect of a deleveraging shock, compared to the reduced form preference shock common in the literature, is that it also has predictions about how the deleveraging takes place, i.e., through cut backs of which *component* of aggregate spending, such as perishable-consumption, durable goods, or productive investment. We now briefly summarize what our model has to say about this (although most of the derivation and numerical examples are left to Appendix C).

Broadly speaking, our analysis suggests that in response to a deleveraging shock, the main adjustment should take place via cuts in spending on investment and/or durable goods rather than on perishable consumption goods. The reason for this is simple: If agents need to cut their spending in the short-run to satisfy a debt limit, it makes sense to do so by cutting those spending components that yield benefit over a long period of time (if a borrower is trying to deleverage, for example, that would not be a good time to upgrade his car at the expense of buying food). In the case of investment in capital that enters as a factor of production there is the additional effect that a short-run recession reduces the incentives of people to invest since the factors of production already in existence are being underutilized.

Consider first durable goods. The utility function now not only involves one period consumption but also durable consumption that enters utility additively separately: $u^i(C_t(i)) + d^i(K_t(i))$, where $K_t(i) = I_t(i) + (1 - \delta)K_{t-1}(i)$. The household needs to decide between purchasing the perishable consumption good $C_t(i)$ and the durable consumption good $I_t(i)$ that yields a flow utility over time through the capital

stock of the durable good $K_t(i)$.²⁶ Both the saver and the borrower will now satisfy an additional first order condition

$$(6) \quad \hat{C}_t^i = (1 - \beta_i(1 - \delta)) \frac{\sigma_C}{\sigma_K} \hat{K}_t^i + \beta_i(1 - \delta) E_t \hat{C}_{t+1}^i \text{ with } i=s,b$$

Our characterization nests our previous specification: If $\delta=1$ then the two goods are identical so that the model is the same as before and the deleveraging consumers cut down on each consumption item by the same amount. As δ decreases then the deleveraging consumers start cutting back more and more on durables relatively to the perishable goods. This is illustrated by a numerical example in table 2. With $\delta=1$ the cutback in consumption of the two goods is the same, but with $\delta=0.25$ the borrower cuts spending on durables 2-3 times than his spending on perishable consumption (Appendix B describes in detail the numerical example and some variations). Under flexible prices this is offset in the aggregate by an increase in the savers' spending on durables (as in the case of non-durables as we established earlier in the paper), leaving aggregate spending unchanged as seen in the column labeled "flexible prices." Once nominal frictions are added, however, the fall in the spending by the borrower carries over to the aggregate, since a substantial reduction in the real interest rate is need to stimulate spending by the saver as can be seen in the column labeled "sticky". That cannot be achieved due to the zero bound. Overall these examples suggest that the results are even stronger once durable goods enter the picture.

Moving to productive investment, the same basic insight applies although the details differ. We outlined the main differences below. Now capital does not add anything to utility but is instead a factor of production according to the standard Cobb-Douglas function $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$. There is an adjustment cost of investment given by $\gamma(\frac{I_t}{I})$ so that the capital owners pay a convex cost of investing differently from steady state \bar{I} . This cost allows us to approximate our previous specification. If the cost of adjusting the capital stock is infinite

²⁶ Here we assume as in the neoclassical literature that the consumer can decide to use the final good as capital or consumption but with the difference that the capital represents a durable good that contributes only to utility rather than being a factor of production.

this variation of the model approximates the one without capital, since then all the deleveraging takes place through the consumption and labor margin. As the cost of adjusting is reduced, then more and more of the deleveraging takes place via drop in investment.

A simple way to motivate lending is to assume that the borrower has access to “investment opportunities” while the saver can only invest in the risk-free bond.²⁷ The extension thus makes clear that we should think of “borrowers” more generally than common interpretation of these agents as “liquidity constrained poor”. More generally, the borrowers correspond to those that – for whatever reason – are in need of funding, for example, due to investment opportunities, but their lending is constrained by a debt limit.²⁹ The saver’s problem is the same as before, but relative to our previous model the borrower satisfies an additional Euler equation, analogous to (6), given by

$$(7) \quad \hat{C}_t^b = -\sigma^b \chi_q \hat{q}_t + E_t \hat{C}_{t+1}^b + \frac{\sigma^b \gamma_{II}}{\beta_b(1-\delta)} \hat{I}_t - \sigma^b \gamma_{II} E_t \hat{I}_{t+1}$$

where $\chi_q \equiv \frac{1-\delta}{\delta} \frac{\beta}{\beta}$, \hat{q}_t is the rental rate of capital in deviation from steady state and γ_{II} reflects the investment adjustment cost. Table 3 shows a numerical example (assuming rigid prices) comparing the model with and without variable investment. The first column corresponding to the case in which the adjustment cost is infinite, it is an approximation to the model with fixed capital stock (similar to our previous example in Table 2 with $\delta=0$). Here the deleveraging happens via a drop in aggregate consumption driven by the deleveraging of the borrower. The next column shows how the model behaves once the cost of adjusting investment is relaxed. We now see that the deleveraging is no longer achieved by a drop in consumption to the same extent but instead by an even larger drop in investment. Relative to the model with durable goods, the drop in aggregate demand is bigger for a given

²⁷ One could also include age dynamics and housing to motivate lending.

²⁹ An important implication of our assumption that only borrowers have investment opportunities is that q_t , the rental rate of capital the borrower obtains, can be different from the risk-free borrowing rate. The link between the risk-free real interest rate and marginal productivity of capital is thus broken and a negative real interest rate in the bond market is perfectly consistent with positive marginal rate of return of capital (even in the absence of adjustment costs).

deleveraging shock, because investment responds even more aggressively than durable consumption, leading to an ever bigger contraction in demand. When investment is productive, there is even less reason to spend on capital goods than on regular consumption since production is already well below capacity, giving rise to even greater demand instability. In Appendix B we show the full underlying model, discuss the extension in more detail, and illustrate more numerical examples.

7.4 Dynamic Deleveraging

How important was it that we assumed the deleveraging happened only in one condensed time period that we labeled as “short-run.” We now briefly outline how the model can be extended so that the period of deleveraging stretches out over several time periods. In this case the duration of the liquidity trap becomes an endogenous variable. This, then, may provide even stronger case for policy intervention, as policy can now not only alleviate the effects of the crisis itself, it can also reduce its duration.

The borrower now faces a borrowing rate that is a declining function of the “safe” level of debt (thus the higher the “safe” level of debt, the better terms the borrower faces for any given level of debt). Hence the borrower is no longer at a corner and needs to make a decision about his level of outstanding debt in each period. The basic structure of a loan contract is adopted from Curdia and Woodford (2009) with the main difference being that the borrowing rate is increasing in the borrowers *own* level of debt, in line with our earlier assumption, rather than aggregate debt³⁰ In particular we assume that the borrower faces the borrowing rate $1 + i_t^b = (1 + i_t^d)(1 + \omega_t)$, where i_t^b is the interest rate faced by the borrower, i_t^d is the interest rate faced by the saver (“depositor”) and $1 + \omega_t$ is the spread between the two. We assume the spread is given by the function $\omega_t = \omega\left(\frac{B_t}{P_t}, b^j\right)$ where the spread is increasing in both the real value

³⁰ We also keep the same structure of heterogeneity we have earlier in the paper, but Curdia and Woodford (2010) instead consider an economy where the type of the consumers vary stochastically over time. The main advantage of our modeling device is simplicity but the disadvantage is that it is less suitable for normative analysis of the type found in their paper since our model does not have a natural welfare criterion. Note also that our “Minsky moment” shock represents a permanent shock that Curdia and Woodford (2003) do not consider in their model.

of the debt $\frac{B_t}{P_t}$ and declining in the “safe value of debt” b^j .³¹ We now assume that b^j shifts unexpectedly at time zero from b^{high} to b^{low} , i.e., now the save level of debt each borrower faces has been unexpectedly reduced. The economy, in other words, experiences a Minsky moment. As a consequence of the Minsky moment the borrowing rate increases and the borrower has a strong incentive to “deleverage”.³² Crucially, with this more general structure, the borrower will choose an optimal deleveraging path over time rather than being forced exogenously to cut down spending within one “short-run.”

Using the same structure of heterogeneity as in the baseline model and assuming that prices are set at a staggered interval as in Calvo (1982) we show in Appendix C that the model (abstracting from taxes and government spending) can be summarized as

$$\begin{aligned}\hat{Y}_t &= E_t \hat{Y}_{t+1} - \sigma(i_t^d - E_t \pi_{t+1} - r_t^e) \\ \pi_t &= \kappa \hat{Y}_t + \beta E_t \pi_{t+1},\end{aligned}$$

where r_t^e is the natural rate of interest given by $r_t^e = -\chi_b(1 + \lambda)\hat{\omega}_t = -\chi_b(1 + \lambda)\vartheta\hat{b}_t$ where σ, β are defined as in previous sections, while $\kappa, \lambda, \vartheta$ are defined in the appendix, \hat{b}_t is the real value of the debt of the borrower and the other hatted variables are defined as before. These two equation are exactly the same as in the canonical New Keynesian model, explored for example in Eggertsson and Woodford (2003) at the zero bound, with the key difference being that the natural rate of interest is now no longer exogenous. Instead it is an endogenous variable that is a function of the level of private debt in the economy, \hat{b}_t , that is optimally chosen by the borrowers. In order to solve for \hat{b}_t we need to solve an 11 equation dynamic system reported in Appendix C. Rather than going into the details, what we want to stress here is two points. First, this extension naturally nests the standard New Keynesian model but with an explicit way of thinking about how long the zero bound may be binding due to a

³¹ We assume that government debt does not affect this relationship, to the extend it does, this may affect the results as we saw in section 7.1.

³² In this variation of the model the incentive to deleverage is driven only by the increase in spreads. A more general environment could allow banks to reduce aggregate borrowing not only via higher spreads but also through margins such as higher “lending standard”, i.e., such as an increase in “down-payments” for house purchases, increased requirements for “collaterals,” and so on. All these margins have been exploited by banks following the crisis of 2008.

negative natural rate of interest. This makes clear that it is relatively straight forward to incorporate a deleveraging mechanism into a larger DSGE model. Second, the dynamic deleveraging mechanism illustrated above can lead to meaningful economic contraction in a calibrated version of the model and the dynamic deleveraging assumption amplifies the effect of government spending. This is illustrated via numerical example in Table 4, which is discussed in more detail in Appendix C. As we see in the table a debt overhang of 30 percent, leads to a drop in output of about 7 percent on impact, and the zero bound is binding for 10 quarters. A small increase in government spending almost eliminates the recession. The effect of government spending becomes even larger than in our previous examples; because in a dynamic setting it also has an effect by shortening the expected duration of the zero bound (the duration is reduced from 10 quarters to 3 as shown in Table 4). This effect of government spending comes on top of the effect spending has on inflation expectation during the period in which the zero bound, a major theme of recent work by Eggertsson (2010a) and Christiano et al (2009). Taken together the case for fiscal intervention is thus even stronger once deleveraging is modeled in a more dynamic way.

8. Relation to empirical findings

One stark prediction of our theory is relevant for fiscal policy: The model suggests that government spending should have a large effect on output at zero interest rates – since then it will not be offset by monetary policy – and this number should be greater than one. Meanwhile, at positive interest rates the effects should be much weaker (depending on the policy reaction function of the central bank). It is difficult, therefore, to draw any conclusion about the government spending multipliers in the current crisis in the US based upon cross country studies (e.g. Alesina and Ardagna (2010) and Guarjardo et al (2011)). Most of the sample in these papers is from periods with positive interest rates. This also applies to some studies of US fiscal policy using post WWII data (e.g. Blanchard and Perotti (2002) and Romer and Romer (2010)).

Rather than a cross-country study, a better benchmark may be the US during the Great Depression or Japan during the Great Recession. An obvious challenge is data limitation and the fact that both episodes

had several moving parts. Nevertheless, there is some literature on both episodes that is largely consistent with our bottom-line and can be summarized as: “Fiscal policy worked when it was tried,” see e.g. Brown (1956) for the Great Depression [and Eggertsson and Pugsley (2006) and Eggertsson (2008)] and Posen (2010) for the case of the Great Recession in Japan that (as well as Kuttner and Posen (2002)).

There is also a relatively recent empirical literature that focuses on the effect of variations in defense spending in the US, where WWII features prominently. During this period short-term nominal interest rates were close to zero. Hence fiscal policy should have had an effect larger than one according to our theory. Barro and Redlick (2009), Hall (2009) and Ramey (2010) estimate the multiplier of government spending in the range of 0.5-1.0 during this period.³³ Meanwhile, Gordon and Krenn (2010) find that the early part of WWII spending had a large impact but then this effect faded as the economy reached capacity constraints in key industries. Overall the US time series evidence seems mixed with respect to the effect of government spending at zero interest rates. Evidence based upon US cross-state evidence, however, is better consistent with larger multiplier in the range of 1.5-2.2³⁴ (see e.g. Steinsson and Nakamura (2010) and Shoag (2010)).

Another interesting prediction of our model relates to deleveraging. The basic message of our model was that a debt overhang will increase the amount of interest cut needed for output to stay at potential, and once those interest rates are no longer feasible, the larger the debt overhang, the larger the contraction and the slower the recovery. While we are not aware of cross-country evidence on this issue

³³ There is not a consensus on whether the special circumstances of war would lead to over or an underestimate. Patriotism would tend to lead to an overestimate (see Barro and Redlick (2009)), while various war time price controls and rationing to an underestimate (see Hall (2009)).

³⁴ Our theory does not explicitly allow us to make prediction across states, but Steinsson and Nakamura (2010) suggest that their “open economy” multiplier is a reasonable approximation to a zero bound multiplier due to the unresponsiveness of monetary policy to the state level shocks they consider.

there does exist interesting empirical work on the effect of deleveraging on employment in the US counties (where arguably monetary and fiscal policy have been largely the same), see Mian and Sufi (2011a,b). They find that drop in demand has been more pronounced in high household debt counties. Similarly, in those counties, the unemployment rate has remained higher post crisis (see Philippon and Midrigan (2011) for an alternative model to rationalize these finding).

Conclusions

In this paper we have sought to formalize the notion of a deleveraging crisis, in which there is an abrupt downward revision of views about how much debt is safe for individual agents to have, and in which this revision of views forces highly indebted agents to reduce their spending sharply. Such a sudden shift to deleveraging can, if it is large enough, create major problems of macroeconomic management. For if a slump is to be avoided, someone must spend more to compensate for the fact that debtors are spending less; yet even a zero nominal interest rate may not be low enough to induce the needed spending.

Formalizing this concept integrates several important strands in economic thought. Fisher's famous idea of debt deflation emerges naturally, while the deleveraging shock can be seen as our version of the increasingly popular notion of a "Minsky moment." And the process of recovery, which depends on debtors paying down their liabilities, corresponds quite closely to Koo's notion of a protracted "balance sheet recession."

One thing that is especially clear from the analysis is the likelihood that policy discussion in the aftermath of a deleveraging shock will be even more confused than usual, at least viewed through the lens of the model. Why? Because the shock pushes us into a world of topsy-turvy, in which saving is a vice, increased productivity can reduce output, and flexible wages increase unemployment. However, expansionary fiscal policy should be effective, in part because the macroeconomic effects of a

deleveraging shock are inherently temporary, so the fiscal response need be only temporary as well. And not only does the model suggests that a temporary rise in government spending will not crowd out private spending, it will lead to increased spending on the part of liquidity-constrained debtors.

The major limitation of this analysis, as we see it, is its reliance on strategically crude dynamics. To simplify the analysis, all the action takes place within a single, aggregated short run, with debt paid down to sustainable levels and prices returned to full ex ante flexibility by the time the next period begins. In the last section of the paper we extend the model to incorporate dynamic deleveraging and find that the model can then naturally nest the canonical New Keynesian model. We see further exploration on this variation as one of the major areas of improvements. Moreover, we hope that our simple extension makes clear that incorporating a deleveraging mechanism into medium scale estimated DSGE models now common in the literature is relatively simple.

We do believe, however, that even the present version of the model sheds considerable light on the problems presently faced by major advanced economies. And yes, it does suggest that the current conventional wisdom about what policy makers should be doing now is almost completely wrong.

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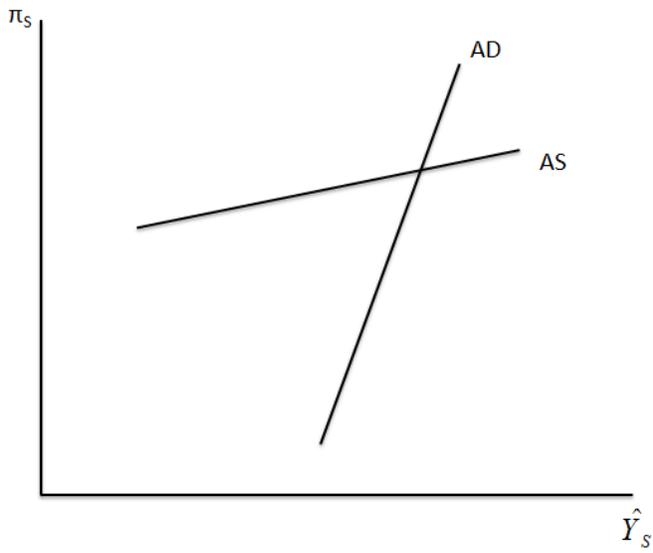


Figure 1 : Topsy-turvy economics

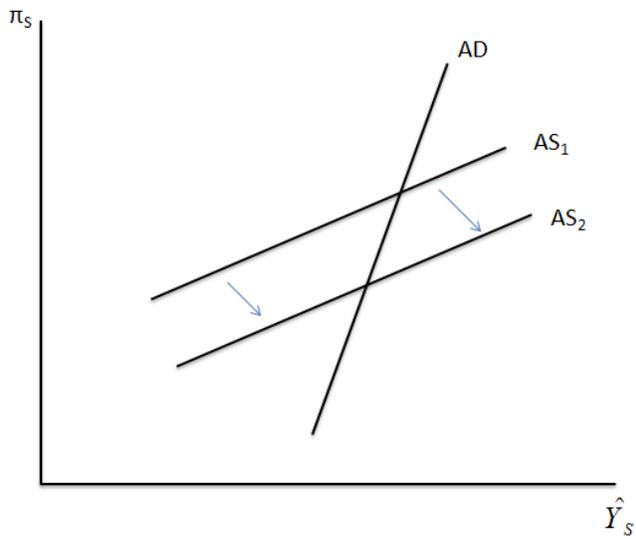


Figure 2: The paradox of toil

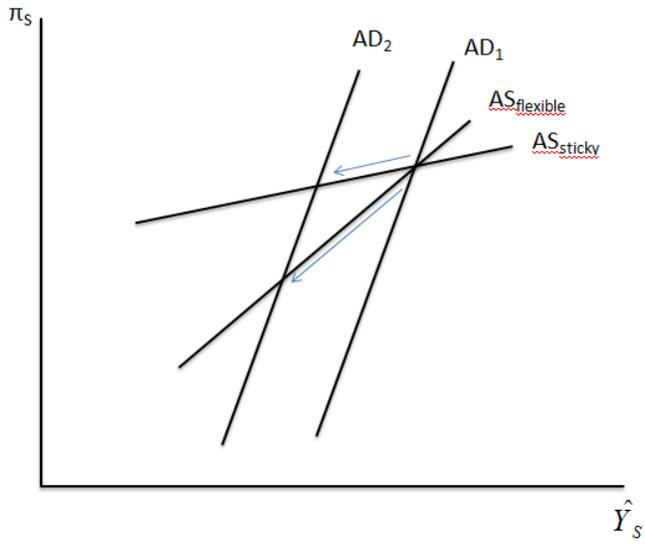


Figure 3: The paradox of flexibility

Table 1: Household gross debt as % of personal income

	2000	2008
US	96	128
UK	105	160
Spain	69	130

Source: McKinsey Global Institute (2010)

Table 2: Durable goods extension³⁵

	$\delta = 1$	$\delta = 1$	$\delta = 0.25$	$\delta = 0.25$
	Flexible Prices	Sticky	Flexible prices	Sticky
$\hat{Y}_S (= \hat{C}_S)$	0	-5.4	0	-8.2
π_S	0	-5.2	0	-8.5
r_S	-1.7	0	-1.7	0
\hat{C}_S^b	-6.7	-20.5	-6.2	-23.3
\hat{I}_S^b	-6.7	-20.5	-20.5	-78
\hat{C}_S^s	3.4	2	3	0.3
\hat{I}_S^s	3.4	2	12.2	4.5

Table 3: Investment dynamics (zerobound)³⁶

	$\gamma_{II} = \infty,$ $\chi_s = 0.7$	$\gamma_{II} = 0.5,$ $\chi_s = 0.9$
\hat{Y}_S	-7.6	-8.6
\hat{C}_S	-10.3	-4
\hat{I}_S	0	-21.7
π_S	-8.1	-3.3
r_S	0	0

³⁵ See Appendix B for parameter values assumed and further discussion. All variables are expressed in annual percentage terms.

³⁶ See Appendix B for parameter values assumed and further discussion. All variables are expressed in annual percentage terms.

Table 4: Dynamic deleveraging³⁷

	No \hat{G}_0	With \hat{G}_0
\hat{Y}_0	-7	-0.7
π_0	-2	-0.2
i_0^d	0	0
i_0^b	11.4	10.3
b_{-1}	30	30
G_0	0	1
Duration of ZLB	10	3

³⁷ See Appendix C for parameter values assumed and further discussion. All variables are expressed in annual percentage terms.

Appendix A

This appendix summarizes the micro foundations of the simple general equilibrium model studied in the paper and shows how we obtain the log-linear approximations stated in the text (a textbook treatment of a similar model with the same pricing frictions is found in Woodford (2003), Chapter 3).

A.1. Households

There is a continuum of households of mass 1 with χ^s of type s and $1 - \chi^s$ of type b . Their problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta(i)^t [u^i(C_t(i)) - v^i(h_t(i))]$$

where $i = s$ or b , s.t.

$$B_t(i) = (1 + i_{t-1})B_{t-1}(i) - W_t P_t h_t(i) - \int_0^1 \Pi_t(i) + P_t C_t(i) + T_t(i)$$

$$(1 + r_t) \frac{B_t(i)}{P_t} \leq D_t(i) < 0,$$

where i_t is the nominal interest rate that is the return on one period riskfree nominal bond, while r_t is the riskfree real interest rate on a one period real bond. We derive the first order conditions of this problem by maximizing the Lagrangian

$$\mathcal{L}_0(i) = E_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{l} \beta(i)^t \{u^i(C_t(i)) - v^i(h_t(i))\} \\ + \phi_{1t}(i) [B_t(i) - (1 + i_{t-1})B_{t-1}(i) + W_t P_t h_t(i) + \int_0^1 \Pi_t(i) - P_t C_t(i) - T_t(i)] \\ + \phi_{2t}(i) [D_t - (1 + r_t) \frac{B_t(i)}{P_t}] \end{array} \right\}$$

First order conditions

$$\frac{\partial \mathcal{L}_t(i)}{\partial C_t(i)} = u_c^i(C_t(i)) - \phi_{1t}(i) P_t = 0$$

$$\frac{\partial \mathcal{L}_t(i)}{\partial h_t(i)} = -v_h^i(h_t(i)) + P_t W_t \phi_{1t}(i) = 0$$

$$\frac{\partial \mathcal{L}_t(i)}{\partial B_t(i)} = \phi_{1t}(i) - \beta(i) E_t \phi_{1t+1}(i) (1 + i_t) - \phi_{2t}(i) \frac{(1 + r_t)}{P_t} = 0$$

Complementary slackness condition

$$\phi_{2t}(i) \geq 0, D_t(i) \geq (1 + r_t) \frac{B_t(i)}{P_t}, \phi_{2t}(i) \left[D_t(i) - (1 + r_t) \frac{B_t(i)}{P_t} \right] = 0.$$

The $C_t(i)$ above refers to the Dixit-Stiglitz aggregator

$$C_t(i) = \left[\int_0^1 c_t(i, j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}$$

and P_t to the corresponding price index

$$P_t = \left[\int_0^1 p_t(j)^{(1-\theta)} di \right]^{1/(\theta-1)}$$

The household maximization problem implies an aggregate demand function of good j given by

$$c_t(j) = C_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta}$$

A.2 Firms

There is a continuum of firms of measure one with a fraction λ that sets prices freely at all times and a fraction $(1 - \lambda)$ that set their prices one period in advance. $y_t(i) = h_t(i)$. We define the average marginal utility of income as $\widetilde{\phi}_t = \chi^s \phi_{1t}^s + (1 - \chi^s) \phi_{2t}^s$. Firms maximize profits over the infinite horizon using $\widetilde{\phi}_t$ to discount profits (this assumption plays no role in our log-linear economy but is stated for completeness):

$$E_t \sum_{t=0}^{\infty} \widetilde{\phi}_t [(1 - \tau) p_t(j) y_t(j) - W_t P_t h_t(j)],$$

s.t.

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta}$$

$$y_t(j) = h_t(j)$$

Where τ is a subsidy we have introduced for notational convenience (see in linearization). From this problem, we can see that the λ fraction of firms that set their price freely at all times they set their price so that

$$(1 - \tau) \frac{p_t(1)}{P_t} = \frac{\theta}{\theta - 1} W_t$$

and each charging the same price $p_t(1)$. Those that set their price one period in advance, however, satisfy

$$E_{t-1} \widetilde{\phi}_t P_t^{1+\theta} Y_t p_t(2)^{-\theta-1} \left(\left(\frac{\theta-1}{\theta} \right) (1-\tau) \frac{p_t(2)}{P_t} - W_t \right) = 0$$

A.3 Government

Fiscal policy is the purchase of G_t of the Dixit-Stiglitz aggregate and the collects taxes T_t^s and T_t^b .

For any variations in T_t^b or G_t we assume that current or future T_t^s will be adjusted to satisfy the government budget constraint (see extended Appendix A for more details). Monetary policy is the choice of i_t . We assume it follows the Taylor rule specified in the text.

A.4 Log linear approximation

Aggregate consumption is

$$C_t = \chi^s C_t^s + (1 - \chi^s) C_t^b,$$

where C_t^s and C_t^b is the of the consumption levels of each type. Similarly aggregate hours are

$$h_t = \chi^s h_t^s + (1 - \chi^s) h_t^b,$$

while aggregate output is given

$$Y_t = C_t + G_t.$$

We consider a steady state of the model in which b borrows up to its limit, while the s does not,

Let us start with linearizing the demand side. Assuming type b is up against his borrowing constraint and aggregating over all types we obtain

$$C_t^b = - \left(\frac{1 + i_{t-1}}{1 + r_{t-1}} \right) \frac{P_{t-1}}{P_t} D_{t-1} + \frac{D_t}{1 + r_t} + I_t^b - T_t^b$$

where I_t^b is wage income of the borrower given by

$$I_t^b = W_t h_t^b$$

Log-linearizing this around $D_t = \bar{D}$ (see extended Appendix A for detailed description of the steady state) we obtain

$$\hat{C}_t^b = \hat{I}_t^b + \beta \hat{D}_t - \hat{D}_{t-1} + \gamma_D \pi_t - \gamma_D \beta (i_t - E_t \pi_{t+1} - \bar{r}) - \hat{T}_t^b.$$

where $\hat{C}_t^b \equiv \frac{c_t^b - \bar{c}^b}{\bar{Y}}$, $Y_t = \log Y_t / \bar{Y}$, $\hat{D}_t = \frac{D_t - \bar{D}}{\bar{Y}}$, i_t is now $\log(1 + i_t)$ in our previous notation, $\bar{r} \equiv \log \beta^{-1}$, $\pi_t \equiv \log P_t / P_{t-1}$, $\hat{T}_t^b \equiv \frac{T_t^b - \bar{T}}{\bar{Y}}$, $\gamma_D = (\bar{D} / \bar{Y})$ and $I_t^b = \frac{i_t^b - I}{\bar{Y}}$ and

$$\hat{I}_t^b = \hat{W}_t + \hat{h}_t^b$$

For type s we obtain

$$u_c^s(C_t^s) = \beta(1 + i_t) E_t u_c^s(C_{t+1}^s) \left(\frac{P_t}{P_{t+1}} \right)$$

and log-linearizing this around steady state yields

$$\hat{C}_t^s = E_t \hat{C}_{t+1}^s - \sigma(i_t - E_t \pi_{t+1} - \bar{r}),$$

where $\hat{C}_t^s \equiv \frac{C_t^s - \bar{C}^s}{\bar{Y}}$ and $\sigma \equiv -\left(\frac{u_c^s}{u_{cc}^s \bar{Y}} \right)$. Aggregate consumption is then

$$\hat{C}_t = \chi^s \hat{C}_t^s + (1 - \chi^s) \hat{C}_t^b,$$

where $C_t \equiv \frac{C_t - \bar{C}}{\bar{Y}}$ and

$$\hat{Y}_t = \hat{C}_t + \hat{G}_t,$$

where $\hat{Y}_t \equiv \log Y_t / \bar{Y}$ and $\hat{G}_t \equiv \frac{G_t - \bar{G}}{\bar{Y}}$.

Let us now turn to the production side. The pricing equations of the firms imply

$$\log p_t(1) = \log P_t + \hat{W}_t$$

$$\log p_t(2) = E_{t-1} [\log P_t + \hat{W}_t]$$

where $\hat{W}_t = \log W_t / \bar{W}$. This implies that

$$\log p_t(2) = E_{t-1} \log p_t(1)$$

Log-linearizing the aggregate price index, implies

$$\log P_t = \lambda \log p_t(1) + (1 - \lambda) \log p_t(2)$$

so it follows (with a few manipulations using the equations above) that

$$\pi_t - E_{t-1} \pi_t = \log P_t - E_{t-1} \log P_t = \left(\frac{\lambda}{1 - \lambda} \right) [\log p_t(1) - \log P_t] = \left(\frac{\lambda}{1 - \lambda} \right) \hat{W}_t$$

To solve for \hat{W}_t we linearize each of the optimal labor supply first order condition for each type to yield

$$\widehat{W}_t = \omega^b \widehat{h}_t^b(i) + \sigma^{b-1} \widehat{C}_t^b$$

$$\widehat{W}_t = \omega^s \widehat{h}_t^s(i) + \sigma^{s-1} \widehat{C}_t^s$$

where $\omega^b \equiv \left(\frac{\bar{v}_{hh}^b h}{\bar{v}_h^b}\right)$, $\omega^s \equiv \left(\frac{\bar{v}_{hh}^s \bar{Y}}{\bar{v}_h^s}\right)$ and $\sigma^b \equiv -\left(\frac{\bar{u}_c^b}{\bar{u}_{cc}^b \bar{Y}}\right)$, $\sigma^s \equiv -\left(\frac{\bar{u}_c^s}{\bar{u}_{cc}^s \bar{Y}}\right)$, and $\widehat{h}_t^b(i) \equiv \frac{h_t^b - \bar{h}^b}{\bar{Y}}$ and $\widehat{C}_t^b \equiv \frac{c_t^b - \bar{c}}{\bar{Y}}$.

Observe that $\widehat{h}_t = \widehat{Y}_t$. We now assume that $\omega^b = \omega^s = \omega$ and that $\sigma^b = \sigma^s = \sigma$. Using this we can combine the labor supply of the two types to yield

$$\widehat{W}_t = \omega \widehat{Y}_t + \sigma^{-1} \widehat{C}_t$$

Combine this with our previous result, together with $\widehat{Y}_t = \widehat{G}_t + \widehat{C}_t$ to yield

$$\pi_t = \left(\frac{\lambda}{1-\lambda}\right) (\omega + \sigma^{-1}) \widehat{Y}_t - \left(\frac{\lambda}{1-\lambda}\right) \sigma^{-1} \widehat{G}_t + E_{t-1} \pi_t$$

$$\pi_t = \kappa \widehat{Y}_t - \kappa \psi \widehat{G}_t + E_{t-1} \pi_t$$

where $\kappa \equiv \left(\frac{\lambda}{1-\lambda}\right) (\omega + \sigma^{-1})$, $\psi \equiv \left(\frac{\sigma^{-1}}{\sigma^{-1} + \omega}\right)$.

Appendix B-C

Available at the authors website: