A Note on Expectations-Driven Business Cycles in Economies with Production Externalities*

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Abstract

Expectations-driven business cycles (EDBC) are defined as positive co-movement between consumption, investment and hours that result from a change in expectations, holding constant technology, preference and government intervention. This note explores the possibility of expectations-driven business cycles in business cycle models with external effects. It is found that in one-sector models conditions for EDBC and conditions for multiple equilibria are tightly connected. In two-sector models those conditions appear to be mutually exclusive.

Keywords: Expectations-driven business cylces, indeterminacy, externalities.

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1 Introduction

Shifts in expectations are thought to play an important role in determining the observed business cycles fluctuations. However, as first pointed out in Barro and King (1984) a standard RBC model cannot produce expectations-driven business cycles. That is, holding constant technology, preferences and government intervention shifts in expectations alone cannot induce the observed positive co-movement in consumption, investment and hours worked.

Introducing imperfect competition or externalities in simple RBC models can induce indeterminate equilibria and self-fulfilling expectations. As shown in Farmer and Guo (1994) and Wen (1998), among others, sunspot-driven equilibria in this class of models can mimic observed business cycle fluctuations.

More recently, Beaudry and Portier (2004, 2006) have provided general conditions for the existence of expectations-driven business cycles (EDBC) that include models without market imperfections. In particular, multi-sector models with cost complementarities can induce EDBC. Others have shown that EDBC can also be obtained by modifying household preferences, firms's technology or by introducing adjustment costs[1]. In this class of models expectations driven business cycles do not arise from self-fulfilling expectations but as a result of incomplete information about the current state of the economy[2], or assuming that agents receive imperfect signals about *future* changes in productivity[3].

This note explores the relation between conditions for indeterminacy and conditions for EDBC in one and two-sector models with external effects. In particular, two aspects of this class of models are of interest. First, models with external effects might be able to generate EDBC under assumptions about preferences and technology that guarantee a unique equilibrium. Uniqueness of equilibrium eliminates the problems of equilibrium selection and lack of predictive power, a frequent objection to models of indeterminacy[4]. Second, two-sector models have a strong propagation mechanism of shocks but for a wide set of parameters display

[1] See Jaimovich and Rebelo (2006) and Flodèn (2006).

[2] See Lorenzoni (2006).

[3] see Beaudry and Porter (2004) and Jaimovich and Rebelo (2007), for example.

[4] Indeterminate equilibria in models with external effects have been shown to be not robust to learning, see Duffy and Xiao (2007).

negative co-movement between consumption and output[5]. The paper analyzes the parameter restrictions, if any, that allow EDBC in this class of models.

The results show that in one-sector models there exist a tight connection between equilibrium indeterminacy and EDBC. The ability to generate EDBC is a necessary but not sufficient condition for indeterminacy. Therefore, there exist a portion of the model parameter space for which EDBC are possible and the equilibrium is unique. In particular, for sufficiently high external effects, low discount rate or high capital depreciation guarantee a unique equilibrium, while preserving the conditions for EDBC.

On the contrary, in two-sector models the connection between indeterminacy and EDBC breaks down. Indeterminacy can be obtained with small investment-specific externalities, and the required externality is lower the higher the capital intensity of the consumption sector with respect to the investment sector. Instead, EDBC are possible only if, 1) large externalities exist in the consumption sector, 2) the consumption sector is more labor intensive than the capital sector and, 3) the intertemporal elasticity of substitution of consumption is high. Furthermore, in two-sector models indeterminacy and EDBC appear to be mutually exclusive.

The paper is divided in two main sections. The first section discusses the one-sector model and the second section extends the analysis to a two-sector model. The Appendix contains the details of the proofs and different modeling assumptions.

2 One-Sector Models

Consider the standard RBC model augmented with economy-wide external effects. A representative agent chooses consumption, hours worked and investment to maximize

$$E_t \sum_{T=t}^{\infty} \beta^t \left[\ln (C_T) - \phi \frac{h_T^{1+\gamma}}{1+\gamma} \right]$$

subject to the resource constraint

$$C_t + K_{t+1} - (1-\delta) K_t = Y_t = \Psi_t K_t^{\alpha} h_t^{(1-\alpha)}$$

[5] see Schmitt-Grohè (2000).

taking K_t and Ψ_t as given. External effects are described by the term

$$\Psi_t = \bar{K}_t^{\alpha\eta} \bar{h}_t^{(1-\alpha)\eta}$$

where $\bar{K}_t$ and $\bar{h}_t$ represent the aggregate stock of capital and hours worked. The parameter η denotes the size of the externality: with $\eta = 0$ the model becomes a standard RBC model with constant returns to scale in production, while $\eta > 0$ implies increasing returns to scale in the social production function. Finally, β is the discount rate, δ is the depreciation rate of capital, and α denotes the capital share[6].

The problem first order conditions and the resource constraint give the following equations describing the evolution of the economy in equilibrium,

$$\phi h_t^{\gamma} C_t = (1-\alpha) \frac{Y_t}{h_t} \tag{1}$$

$$C_t + K_{t+1} - (1-\delta) K_t = Y_t \tag{2}$$

$$Y_t = A_t K_t^{\alpha(1+\eta)} h_t^{(1-\alpha)(1+\eta)} \tag{3}$$

and

$$C_t^{-1} = E_t \left(C_{t+1}^{-1} \alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) \right). \tag{4}$$

The equilibrium in the labor market is described by (1), while (2) and (3) represent the resource constraint and the aggregate production function. Finally, (4) is the consumption Euler equation. In the next section I first discuss the conditions for existence of EDBC and then compare them with the conditions for indeterminacy. In the paper all the conditions have only local validity, since I consider a local approximation of the model around the unique non-stochastic steady state.

2.1 Expectations-Driven Business Cycles

Following Beaudry and Portier (2006), EDBC "represent a positive co-movement between consumption, investment and employment induced by a change in expectations holding current technology, preferences and government intervention constant". First, a relation between

[6] Notice that the model does not include an exogenous stochastic process for productivity.

consumption and hours is derived. Log-linearizing (1) and substituting for the production function gives

$$\left[\bar{b} - (1+\gamma)\right] \hat{h}_t + \bar{a}\hat{K}_t = \hat{C}_t$$

where

$$\begin{aligned} \bar{b} &= (1-\alpha)(1+\eta); \\ \bar{a} &= \alpha(1+\eta). \end{aligned}$$

EDBC are possible if dC/dh and dK/dh are positive. Since capital at time t is predetermined ($\hat{K}_t = 0$), the relation between consumption and hours becomes

$$\left[\bar{b} - (1+\gamma)\right] \hat{h}_t = \hat{C}_t \tag{5}$$

which is positive provided $\bar{b} - (1+\gamma)$ is positive. External effects need to be sufficiently strong so that

$$(1-\alpha)(1+\eta) - (1+\gamma) = \bar{\psi} > 0, \tag{6}$$

implying, as in Benhabib and Farmer (1994), that the labor demand is upward-sloping and steeper than the labor supply. Finally, log-linearizing the resource constraint and using (5) gives

$$\hat{k}_{t+1} = \left[\delta\bar{b} + \frac{C}{K}(1+\gamma)\right] \hat{h}_t$$

where I use $I/K = \delta$ in steady state. Hence, the relation between hours and investment is positive for every parameter value.

As shown by (6), low external effects cannot generate EDBC. For example, the calibration chosen by Baxter and King (1990) does not allow EDBC: shifts in expectations would lead to a negative correlation between consumption and hours. External effects induce positive co-movement in their model because of highly persistent exogenous demand shocks. To give some intuition for this result, assume an increase in consumption. As the income effect induces an upward shift in the labor supply, hours can increase only if the real wage also increases but with a fixed downward-sloping labor demand this is not possible. If the externality is high enough that the labor demand is upward sloping and steeper than labor supply, the upward shift in labor supply results in higher employment and higher real wages, so that EDBC are possible. I now turn to the determinacy analysis.

2.2 Indeterminacy and EDBC.

Local determinacy can be evaluated by considering the following reduced-form system of difference equations

$$\begin{bmatrix} E_t\hat{h}_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = M \begin{bmatrix} \hat{h}_t \\ \hat{k}_t \end{bmatrix}$$

where the matrix M is shown in the Appendix. The conditions for an indeterminate equilibrium depend on the eigenvalues of the matrix M. Given that the capital stock is the only predetermined variable, indeterminacy obtains if both eigenvalues of M are inside the unit circle. A *necessary* condition for indeterminacy is

$$|\det(M)| < 1. \tag{7}$$

where the determinant can be shown to be

$$\det(M) = \beta^{-1}\left(1 - \frac{(\gamma+1)(\beta\delta+1-\beta)\eta}{(1-\alpha)(1-\delta)\beta(\eta+1)-(\gamma+1)}\right).$$

Given that the numerator inside the brackets is always positive, condition (6) is also a *necessary* condition to have an indeterminate equilibrium, as shown in Benhabib and Farmer (1994). Hence, sunspots-driven fluctuations produce positive co-movement in consumption, investment and hours. The condition is generally not *sufficient* to have indeterminacy. Necessary and sufficient condition for indeterminacy also requires

$$|tr(M)| < |1 + \det(M)|.$$

The following Proposition consider the particular case in which (6) becomes also a sufficient condition.

Proposition 1 *Consider an economy with $\delta = 0$. As $\beta \to 1$ condition (6) becomes necessary and sufficient condition for local indeterminacy.*

Proof. First, it is straightforward to show that in the case of $\delta = 0$, the determinant converges to 1 from below, as β converges to 1. Second, for $\delta = 0$, the trace becomes

$$tr(M) = 2 - \frac{\beta^{-1}-1}{\alpha}\frac{(1-\beta+\alpha\beta(1+\eta))(\gamma+1)}{(1-\alpha)\beta(\eta+1)-(\gamma+1)}$$

which is positive for β close to 1. Assume $(1-\alpha)\beta(\eta+1)-(\gamma+1)>0$. The last step involves showing that for $\beta \to 1$

$$tr(M)-1-\det(M)<0$$

which gives

$$1-\beta^{-1}-\frac{\beta^{-1}-1}{\alpha}\frac{(1-\beta+\alpha\beta(1+\eta))(\gamma+1)}{(1-\alpha)\beta(\eta+1)-(\gamma+1)}-\beta^{-1}\frac{(\gamma+1)(1-\beta)\eta}{(1-\alpha)\beta(\eta+1)-(\gamma+1)}<0,$$

completing the proof. ■

The following intuitive explanation can help understanding the result. As in the simplest dynamic model, indeterminacy occurs if expectations have a strong impact on current decisions. This is the case if the discount rate is sufficiently high but also if capital depreciation is sufficiently low. With full capital depreciation any investment today only affects production tomorrow (because of the time to build assumption). As a consequence, with $\delta \to 1$ indeterminacy cannot arise, independently of the value of β. In contrast, as $\delta \to 0$ current investment decisions have consequences even in the distant future and therefore revisions in expectations have strong impact on current decisions. On the contrary, the parameter restriction for EDBC depends only on *intratemporal equilibrium conditions*, which in this version of the model do not depend on β and δ.

2.3 Capacity utilization. The model of Wen (1998).

In the simple model described above, EDBC and indeterminacy obtain for level of externalities that are judged implausible. Wen (1998) shows that introducing variable capacity utilization lowers the degree of externalities required for indeterminacy and improves the empirical fit of the model. In this version of the model the capital depreciation rate is endogenously determined according to

$$\delta_t=\frac{1}{\theta}u_t^{\theta}$$

where u_t denotes capacity utilization. The first order condition with respect to capacity utilization gives

$$\delta_t=\frac{\alpha}{\theta}\frac{Y_t}{K_t}$$

which, combined with the other equilibrium conditions gives the following aggregate social production function

$$Y_t = \tilde{A} K_t^{\hat{a}} h_t^{\hat{b}}$$

where

$$\hat{a} = \alpha (1+\eta) \frac{\theta - 1}{\theta - \alpha (1+\eta)}$$

and

$$\hat{b} = (1-\alpha)(1+\eta) \frac{\theta}{\theta - \alpha (1+\eta)}.$$

The other equilibrium conditions are the same as the ones derived in the previous section, allowing for time-varying capital depreciation. Following the same steps as for the case of exogenous capital depreciation, the log-linear relationship between consumption and hours can be found to be

$$\hat{c}_t = \left(\hat{b} - (1+\gamma)\right) \hat{h}_t \tag{8}$$

while the relation between investment and hours becomes

$$\hat{k}_{t+1} = \frac{C}{K} (1+\gamma) \hat{h}_t.$$

Condition (8), again, corresponds to the *necessary* condition for indeterminacy. It can also be shown that as $\beta \to 1$ this condition becomes also sufficient[7].

2.4 An example

This section shows the conditions for both EDBC and indeterminacy in a calibrated version of the model. Three different calibrations are considered for monthly, quarterly and annual time intervals. The role of capacity utilization is also explored. In both versions of the model, the capital share, α, is set equal to 0.3. In the monthly calibration I set $\beta = 0.997$ and $\delta = 0.008$, in the quarterly calibration $\beta = 0.99$ and $\delta = 0.025$ and in the annual calibration $\beta = 0.96$ and $\delta = 0.1$. In the version of the model with capacity utilization, the parameter θ is set according to the steady state relationship

$$\theta = \frac{1 - \beta(1-\delta)}{\beta\delta}.$$

[7] For details, see Wen (1998).

For each different calibration, Table 1 shows $\eta_{\min}^{\text{indet}}$ and $\eta_{\min}^{\text{EDBC}}$representing the minimum value of external effects required to obtain indeterminacy and EDBC. In accordance with Proposition 1, in the model calibrated at a monthly frequency, the conditions for indeterminacy and EDBC are nearly equivalent. But in the annual calibration there is a sizable discrepancy. EDBC can occur in a determinate equilibrium. In the model with capacity utilization, the conditions for indeterminacy and EDBC are closer. This because changes in β and δ *do* affect the intratemporal equilibrium conditions in the model. A decrease in δ or an increase in β imply a lower elasticity of capital depreciation with respect to capital utilization, thus lowering the minimum external effect required to have EDBC. At the same time decrease in δ or an increase in β increase the the indeterminacy region. As a result, the set of parameter values for which determinacy and EDBC obtains is smaller.

[TABLE I ABOUT HERE]

Summing up, one-sector models with increasing returns can generate EDBC but for the commonly used quarterly calibration conditions for EDBC are very close to the conditions for local indeterminacy.

2.5 Monopolistic competition and entry. The model of Jaimovich (2007)

The second class of models that produces increasing returns assumes monopolistic competition. Portier (1995) first showed the existence of indeterminate equilibria in a model with entry. Here I consider the model discussed in Jaimovich (2007a). The production side of the economy involves the production of a final good (in a perfectly competitive market), according to a constant returns to scale production function

$$Y_t = \left[\int_0^1 Q_t(j)^{\omega} dj\right]^{\frac{1}{\omega}}, 0 < \omega < 1$$

where $Q_t(j)$ is a sectorial good. In each sector j, there exist $N_t > 1$ firms producing differentiated goods $x_t(j,i)$ where i denote the single firm. The sectorial good is produced according

to the production function

$$Q_t(j) = N_t^{1-\frac{1}{\tau}} \left[\sum_{i=1}^{N_t} x_t(j,i)^{\tau} \right]^{\frac{1}{\tau}}, 0 < \tau < 1$$

and where

$$\frac{1}{1-\omega} < \frac{1}{1-\tau}$$

Finally, each intermediate good is produced according to the following production function

$$x_t(j,i) = k_t^{\alpha} h_t^{1-\alpha} - \phi$$

where ϕ represent a fixed cost. Producers of the intermediate good operate in monopolistically competitive markets. The consumption, saving and labor decision problems are the same as presented in the previous sections. Solving for the model first order conditions and imposing equilibrium conditions give

$$-H_t^{\gamma} + C_t^{-1} \frac{1-\alpha}{\mu(N_t)} K_t^{\alpha} H_t^{-\alpha} = 0 \tag{9}$$

$$Y_t = \frac{1}{\mu(N_t)} K_t^{\alpha} H_t^{1-\alpha} \tag{10}$$

$$\frac{\phi N_t}{Y_t} = \mu(N_t) - 1 \tag{11}$$

$$C_t + K_{t+1} - (1-\delta) K_t = Y_t \tag{12}$$

where $K_t = N_t k_t$ and $H_t = N_t h_t$. Let us consider the conditions for the existence of EDBC. Log-linearizing (9) and (10) we obtain

$$\begin{aligned} \hat{c}_t &= \hat{Y}_t - (1+\gamma)\hat{h}_t \\ &= [(1-\alpha) - (1+\gamma)]\hat{h}_t - \frac{(\mu-1)(\mu^{-1}-\tau)}{1-\tau}\hat{N}_t. \end{aligned}$$

Combining (10)

$$\hat{Y}_t = (1-\alpha)\hat{h}_t - \frac{(\mu-1)(\mu^{-1}-\tau)}{1-\tau}\hat{N}_t$$

and (11)

$$\hat{N}_t = \frac{1-\tau}{\tau(\mu-1)}\hat{Y}_t$$

output can be expressed as

$$\hat{Y}_t = \check{b}\hat{h}_t$$

where

$$\check{b} = \tau\mu\left(1-\alpha\right)$$

Finally, solving in terms of consumption and hours, the condition for the existence of EDBC becomes

$$\tau\mu\left(1-\alpha\right) > 1+\gamma.$$

the derivation of dK/dh is the same as for the model with external effects. This corresponds to the necessary condition for indeterminacy shown in Jaimovich (2007a).

3 Two-Sector Model

Benhabib and Farmer (1996) have shown that in a two-sector model with sector-specific externalities indeterminacy obtains for empirically plausible levels of external effects. Moreover, in Benhabib and Nishimura (1998) sector-specific external effects can lead to indeterminacy, even under constant return returns to scale. Consider a two-sector model with separate production functions for consumption and investment. The representative agents maximizes

$$\max E_t \sum_{T=t}^{\infty} \beta^t \left[\frac{C_T^{1-\sigma}}{1-\sigma} - \phi\frac{h_T^{1+\gamma}}{1+\gamma}\right]$$

subject to

$$I_t = h_{I,t}^{\beta_0} K_{I,t}^{\beta_1} \Psi_{I,t}$$

$$C_t = h_{C,t}^{\alpha_0} K_{C,t}^{\alpha_1} \Psi_{C,t}$$

where

$$\Psi_{I,t} = \bar{K}_{I,t}^{b_1} \bar{h}_{I,t}^{b_0}; \quad \Psi_{C,t} = \bar{K}_{C,t}^{a_1} \bar{h}_{C,t}^{a_0}$$

and

$$K_{t+1} - \left(1-\delta\right) K_t = I_t \tag{13}$$

$$K_t = K_{I,t} + K_{C,t} \tag{14}$$

$$h_t = h_{C,t} + h_{I,t}. \tag{15}$$

I assume no costs in reallocating resources across sectors. Also, I assume a more general form of utility function, where $\sigma \geq 0$ to obtain a richer set of results[8].

The consumption Euler equation can be derived to be

$$E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \beta \frac{r_{t+1} + (1-\delta) P_{I,t+1}}{P_{I,t}} \right] = 1 \tag{16}$$

where $P_{I,t}$ denotes the relative price of investment in consumption units and r_t denotes the rental rate of capital, as defined below. The first order conditions with respect to sector-specific capital and labor give

$$\beta_1 \frac{P_{I,t} I_t}{K_{I,t}} = \alpha_1 \frac{C_t}{K_{C,t}} = r_t \tag{17}$$

and

$$\beta_0 \frac{P_{I,t} I_t}{h_{I,t}} = \alpha_0 \frac{C_t}{h_{C,t}} = w_t. \tag{18}$$

which are obtained by equating marginal product of capital and labor across sectors (w_t denotes the real wage). The total supply of hours is determined by

$$\phi h_t^{\gamma} = \alpha_0 \frac{C_t^{1-\sigma}}{h_{C,t}} = C_t^{-\sigma} w_t \tag{19}$$

and the aggregate production functions for investment and consumption are

$$I_t = h_{I,t}^{\beta_0 + b_0} K_{I,t}^{\beta_1 + b_1} \tag{20}$$

$$C_t = h_{C,t}^{\alpha_0 + a_0} K_{C,t}^{\alpha_1 + a_1}. \tag{21}$$

Equations (13)-(21) describe the equilibrium evolution of the economy.

3.1 Expectations-Driven Business Cycles

As shown in Beaudry and Portier (2006), under constant returns to scale and no external effects EDBC cannot occur. Assume C_t, h_t and $P_{I,t} I_t$ increase at the same time. Equation

[8] For example, Benhabib and Nishimura (1998) show that low values of σ are required to obtain indeterminacy with constant returns to scale. Moreover, Benhabib et al. (2006) show that low values of σ produce a better fit with the data. This restriction on preferences can be abandoned by considering a three sector model with two investment goods, but such analysis is left for further research.

(19) implies that the real wage has to increase as well. From (18) and (21) the ratio $h_{c,t}/K_{C,t}$ decreases, leading to a decrease in the rental rate. But this, from (17) implies that *both* $K_{I,t}$ and $K_{C,t}$ need to increase, leading to a contradiction since K_t is predetermined.

I consider now the general case with external effects. Combining (19) and (21) and log-linearizing gives

$$\hat{C}_t = (\alpha_0 + a_0)\left[(1-\sigma)\,\hat{C}_t - \gamma\hat{h}_t\right] + (\alpha_1 + a_1)\,\hat{k}_{C,t}. \tag{22}$$

Log-linearizing (14) and (15) gives

$$\frac{K_C}{K}\hat{k}_{C,t} = -\frac{K_I}{K}\hat{k}_{I,t}$$

and

$$\left(1 + \frac{h_C}{h_I}\right)\hat{h}_t = \frac{h_C}{h_I}\hat{h}_{C,t} + \hat{h}_{I,t}$$

where, the first expression obtains because aggregate capital is predetermined ($\hat{k}_t = 0$). Combining with (17) and (18) gives

$$\Xi_0^{-1}\hat{k}_{C,t} = (1-\sigma)\,\hat{C}_t - (1+\gamma)\,\hat{h}_t, \tag{23}$$

where

$$\Xi_0 = \left(1 + \frac{K_C}{K_I}\right)^{-1}\left(1 + \frac{h_C}{h_I}\right) > 0.$$

Finally, combining (22) and (23) gives the relation between consumption and aggregate hours

$$A_0\hat{C}_t = A_1\hat{h}_t,$$

where

$$A_0 = 1 - (1-\sigma)\left[(\alpha_0 + a_0) + (\alpha_1 + a_1)\,\Xi_0\right]$$

and

$$A_1 = -\left[(\alpha_0 + a_0)\,\gamma + (\alpha_1 + a_1)\,(1+\gamma)\,\Xi_0\right] < 0.$$

Hence we can state the first result,

Proposition 2 *Assume $\sigma \geq 1$. Then the two-sector model with external effects cannot generate EDBC.*

As the Proposition shows, sector-specific external effect do not generate EDBC, at least for low values of the consumption intertemporal elasticity of substitution. But, as shown in Benhabib and Farmer (1996) indeterminate equilibria exist; the close relationship between conditions for EDBC and conditions for indeterminacy appear to break down in multi-sector models. Let us consider a simplified version of the model where the two-sectors have the same factor intensity ($\alpha_0 = \beta_0 = 1 - \alpha_1 = 1 - \beta_1$), and re-define $a_i = \alpha_i \eta$ for $i = 0, 1$. Following Benhabib and Farmer (1996) the social production possibility frontier can be written as

$$\left[C_t^{\frac{1}{1+\eta}} + I_t^{\frac{1}{1+\eta}}\right]^{1+\eta} = \tilde{\Psi}_t K_t^{\alpha} h_t^{1-\alpha}, \ \tilde{\Psi}_t \ = (K_t h_t)^{1+\eta}. \tag{24}$$

This production function is similar to the one that can be obtained from a model with cost complementarities in the production of consumption and investment, as shown in Beaudry and Portier (2006). They show that such model *does exhibit* EDBC. The main difference is that firms in that model *take into account* the cost complementarity in (24), when taking their production decisions. Thus a firm increasing the production of consumption goods will find optimal also to increase the production of investment goods, expanding hours worked. Conversely, in the model with external effects the private production function does not correspond to the social production function.

More generally, under the assumption of equal factor intensities across sectors, $K_C/K_I = h_C/h_I$, implying $\Xi_0 = 1$. The relation between consumption and hours becomes

$$\hat{C}_t = -\frac{1 - \eta(1-\sigma)}{(a_0 + a_1)\gamma + \alpha_1 + a_1}\hat{h}_t$$

so that provided $\eta < 1$ no EDBC are possible for every value of $\sigma \geq 0$. Importantly, sector-specific externalities does not produce an upward sloping labor demand, making EDBC more difficult to obtain.

Consider the case of an economy with external effects but with constant returns to scale in each sector, that is

$$\alpha_0 + a_0 + \alpha_1 + a_1 = 1 = \beta_0 + b_0 + \beta_1 + b_1$$

Benhabib and Nishimura (1998) show that for σ close to zero indeterminacy can occur. In their example, setting $\sigma = 0$, $\beta_0 = 0.34$, $b_0 = b_1 = 0$, $\alpha_0 = 0.3, \alpha_1 = 0.65$ and $a_0 = 0.05$

gives local indeterminacy for very low external effects. The following Proposition discusses the possibility of EDBC.

Proposition 3 *Consider the economy with $\sigma = 0$ and constant returns to scale. Then the two-sector model cannot generate EDBC.*

Proof. Assuming constant return to scale gives

$$A_0 = 1 - (\alpha_0 + a_0) + (1 - \alpha_0 - a_0)\,\Xi_0 = (\alpha_1 + a_1)\,(1 - \Xi_0)\,.$$

If $0 < \Xi_0 < 1$, $dC/dh < 0$ and EDBC cannot occur. In case $\Xi_0 > 1$ for some parameter values there is a positive relation between consumption and hours worked. We show that this implies a *negative* relation between investment and hours. First,

$$A_0^{-1} A_1 = -\frac{(\alpha_0 + a_0)\,\gamma + (\alpha_1 + a_1)\,(1 + \gamma)\,\Xi_0}{(\alpha_1 + a_1)\,(1 - \Xi_0)}.$$

Second, Log-linearization of (18) gives

$$\hat{P}_{I,t} + \hat{I}_t = \hat{h}_{I,t} + \hat{C}_t - \hat{h}_{C,t}$$

and substituting the hours resource constraint and (19) gives

$$\hat{P}_{I,t} + \hat{I}_t = \left[\left(1 + \frac{h_C}{h_I}\right)(1 + \gamma)\right]\hat{h}_t - \left[\left(1 + \frac{h_C}{h_I}\right)(1 - \sigma) - 1\right]\hat{C}_t.$$

Finally, substituting for consumption and imposing $\sigma = 0$ the expression can be simplified to

$$\begin{aligned}
\hat{P}_{I,t} + \hat{I}_t &= \frac{h_C}{h_I}\left[\frac{\Xi_0}{(1 - \Xi_0)} + \left(\frac{h_C}{h_I}\right)^{-1} + 1\right]\hat{h}_t + \\
&\quad + \frac{h_C}{h_I}\gamma\left[\frac{(\alpha_0 + a_0) + (\alpha_1 + a_1)\,\Xi_0}{(\alpha_1 + a_1)\,(1 - \Xi_0)} + \left(\frac{h_C}{h_I}\right)^{-1} + 1\right] \\
&= \frac{h_C}{h_I}\left[\frac{1}{1 - \Xi_0} - \frac{1}{1 - \Xi_0\,(K_c/K_I + 1)}\right]\hat{h}_t + \\
&\quad + \frac{h_C}{h_I}\gamma\left[\frac{1}{(1 - \alpha_0 - a_0)\,(1 - \Xi_0)} - \frac{1}{1 - \Xi_0\,(K_c/K_I + 1)}\right]\hat{h}_t
\end{aligned}$$

where $\left[\frac{1}{(1-\alpha_0-a_0)(1-\Xi_0)} - \frac{1}{1-\Xi_0(K_c/K_I+1)}\right]$ is negative. This completes the proof. ■

Summing up, the above results suggest that in a two-sector model indeterminacy and EDBC are unrelated. The following example shows what conditions need to be verified for EDBC to occur and suggests that indeterminacy and EDBC are in fact mutually exclusive.

Consider the model with $\sigma = 0.05$, $\alpha_0 = \beta_0 = 0.3$, $\alpha_1 = 1 - \alpha_0$, $\beta_1 = 1 - \beta_0$ and $\eta_I = 0.014$, $\eta_C = 0$, where $a_0 = \alpha_0 \eta_C$, $a_1 = \alpha_1 \eta_C$, $b_0 = \beta_0 \eta_I$ and $b_1 = \beta_1 \eta_I$. Under this calibration[9] the equilibrium is locally indeterminate. It can also be shown that if the consumption sector is more capital intensive than the investment sector, a lower externality is required for indeterminacy. In the extreme case of $\alpha_0 = 0.1$ and $\beta_0 = 0.9$ the required degree of externality is $\eta_I = 0.0006$! At the same time EDBC cannot occur under this parametrization. Consider the alternative calibration with externalities in the consumption sector: $\eta_C = 0.09$ $\eta_I = 0$. Also, assume that the investment good is highly capital intensive ($\beta_0 = 0.1$) and the consumption good is highly labor intensive ($\alpha_0 = 0.9$). In this calibration EDBC are possible. In the calibrated two-sector model with sector-specific external effects three conditions appear to be necessary in order to generate EDBC:

1. high intertemporal elasticity of substitution of consumption, implying small income effects on labor supply;

2. external effects in the consumption sector, in particular on the labor input (external effects in the investment sector are not required). To help intuition, a higher consumption increases the marginal product of its production factors, increasing the relative price of investment and also the labor input re-allocated toward the investment sector, inducing positive co-movement between consumption and investment[10];

3. the investment sector is capital intensive and the consumption sector is labor intensive. Different factor intensities make costly to reallocate factors across sectors (that is add curvature to the private production possibility frontier). This facilitates positive co-movement between consumption and investment.

[9] The discount rate and capital depreciation are calibrated as in the quarterly model.

[10] For sufficiently higher values of the consumption externality both I_t and $P_t I_t$ co-move with consumption (for $\sigma = 0.05$, the minimum value is $\eta = 0.15$).

As shown in the example the parameters' restrictions that lead to indeterminacy are very different than the restrictions required to obtain EDBC. Moreover, EDBC arise for parameter values consistent with a *locally determinate* equilibrium, suggesting that at least in these examples indeterminacy and EDBC are mutually exclusive.

The conditions for EDBC discussed in the example are tight and somewhat extreme. In particular large external effects in the consumption sector are not empirically plausible. The model can be enriched in several dimensions. On the one hand, the inclusion of capacity utilization, small economy-wide externalities, countercyclical markups, might reduce the degree of external effects in the consumption sector. On the other hand, modifying the preferences to reduce income effects as in Jaimovich and Rebelo (2006) or moving to a three-sector model can alleviate the restrictions on σ. The analysis of more complex models is left for further research.

4 Conclusions

This note investigates the connection between the conditions for indeterminacy and the conditions for EDBC. I find that in a one-sector model with economy-wide externalities the connection is tight. Conditions for EDBC are necessary (albeit not sufficient) to obtain indeterminate equilibria. In a two-sector model the connection is broken. It is shown that EDBC do not necessarily imply indeterminacy and vice-versa. Specific examples indicate that conditions for EDBC and indeterminacy are mutually exclusive.

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5 Appendices

5.1 The M matrix

The log-linearized equilibrium conditions can be re-written as

$$M_1 \begin{bmatrix} E_t \hat{h}_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = M_2 \begin{bmatrix} \hat{h}_t \\ \hat{k}_t \end{bmatrix}$$

where

$$M_1 = \begin{bmatrix} -\bar{b}R + \bar{\psi} & \bar{a} - R(\bar{a} - 1) \\ 0 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} \bar{\psi} & \bar{a} \\ \delta\bar{b} + \frac{C}{K}(1+\gamma) & \delta\bar{a} + \frac{C}{K}(1+\gamma) + (1-\delta) \end{bmatrix}$$

and where

$$\begin{aligned} R &= 1 - \beta(1-\delta) \\ \frac{C}{K} &= \frac{\beta^{-1} - (1-\delta)}{\alpha} - \delta. \end{aligned}$$

5.2

TABLES

Table 1.

	Benhabib and Farmer (1994)	Wen (1998)
M.	$\eta_{\min}^{\text{indet}} = 0.450$; $\eta_{\min}^{\text{EDBC}} = 0.429$	$\eta_{\min}^{\text{indet}} = 0.091$; $\eta_{\min}^{\text{EDBC}} = 0.090$
Q.	$\eta_{\min}^{\text{indet}} = 0.494$; $\eta_{\min}^{\text{EDBC}} = 0.429$	$\eta_{\min}^{\text{indet}} = 0.102$; $\eta_{\min}^{\text{EDBC}} = 0.095$
A.	$\eta_{\min}^{\text{indet}} = 0.750$; $\eta_{\min}^{\text{EDBC}} = 0.429$	$\eta_{\min}^{\text{indet}} = 0.121$; $\eta_{\min}^{\text{EDBC}} = 0.097$