A Structural Decomposition of the U.S. Trade Balance: Productivity, Demographics and Fiscal Policy

APPENDIX - NOT FOR PUBLICATION

Andrea Ferrero*

Federal Reserve Bank of New York

April 6, 2010

* *Address:* 33 Liberty Street, New York, NY 10045. *E-mail:* andrea.ferrero@ny.frb.org.
All errors are exclusively my own responsibility. The views expressed herein do not necessarily reflect the position of the Federal Reserve Bank of New York or of the Federal Reserve System.

1 The Model in Details

This section presents the derivation of the individual problems (retirees and workers), the main aggregation result, the problem of the firm and a characterization of the steady state.

1.1 Problem of a Retiree

The consumption-saving allocation for a retiree r, born in period j and retired in period i, consists of choosing consumption $C_t^{jr}(i)$ and assets $A_t^{jr}(i)$ to solve

$$V_t^{jr}(i) = \max \left[\left(C_t^{jr}(i) \right)^{\rho} + \beta \gamma_{t,t+1} \left(V_{t+1}^{jr}(i) \right)^{\rho} \right]^{\frac{1}{\rho}}, \tag{1}$$

subject to

$$A_{t+1}^{jr}(i) = \frac{R_{W,t} A_t^{jr}(i)}{\gamma_{t-1,t}} - C_t^{jr}(i). \tag{2}$$

The first order condition with respect to asset accumulation is

$$\left(C_t^{jr}(i) \right)^{\rho-1} = \beta \gamma_{t,t+1} \left(V_{t+1}^{jr}(i) \right)^{\rho-1} \frac{\partial V_{t+1}^{jr}(i)}{\partial A_{t+1}^{jr}(i)}.$$

The envelope condition is

$$\frac{\partial V_t^{jr}(i)}{\partial A_t^{jr}(i)} = \left(V_t^{jr}(i) \right)^{1-\rho} \left(C_t^{jr}(i) \right)^{\rho-1} \frac{R_{W,t}}{\gamma_{t-1,t}}.$$

The resulting Euler equation takes the standard form

$$C_{t+1}^{jr}(i) = (\beta R_{W,t+1})^{\sigma} C_t^{jr}(i), \tag{3}$$

where $\sigma \equiv (1-\rho)^{-1}$ is the elasticity of intertermporal substitution.

Guess that consumption is a fraction of total wealth

$$C_t^{jr}(i) = \xi_t \left(\frac{R_{W,t} A_t^{jr}(i)}{\gamma_{t-1,t}} \right). \tag{4}$$

Substitution into the Euler equation (3) yields a law of motion for the marginal propensity

to consume of a retiree ξ_t

$$\xi_{t+1}\left(\frac{R_{W,t+1}A_{t+1}^{jr}(i)}{\gamma_{t,t+1}}\right)=\left(\beta R_{W,t+1}\right)^{\sigma}\xi_t\left(\frac{R_{W,t}A_t^{jr}(i)}{\gamma_{t-1,t}}\right).$$

Substitution of the guess (4) into the budget constraint of a retiree (2) leads to express assets as

$$A_{t+1}^{jr}(i)=(1-\xi_t)\frac{R_{W,t}A_t^{jr}(i)}{\gamma_{t-1,t}}.$$

Combining the last expression with the Euler equation above gives a non-linear first order difference equation for the marginal propensity to consume of the form

$$\xi_t=1-\gamma_{t,t+1}\beta^{\sigma}R_{W,t+1}^{\sigma-1}\frac{\xi_t}{\xi_{t+1}}. \tag{5}$$

Moreover, conjecture that the value function is linear in consumption according to

$$V_t^{jr}(i)=\Delta_t^r C_t^{jr}(i). \tag{6}$$

Then, from (1), it must be the case that

$$\left(\Delta_t^r C_t^{jr}(i)\right)^{\rho}=\left(C_t^{jr}(i)\right)^{\rho}+\beta\gamma_{t,t+1}\left(\Delta_{t+1}^r C_{t+1}^{jr}(i)\right)^{\rho}.$$

Substituting for consumption at $t+1$ from (3) and simplifying yields

$$(\Delta_t^r)^{\rho}=1+\gamma_{t,t+1}\beta^{\sigma}R_{W,t+1}^{\sigma-1}\left(\Delta_{t+1}^r\right)^{\rho}.$$

From (5), it then follows that the proportionality term in (6) is

$$\Delta_t^r=\xi_t^{-\frac{1}{\rho}}.$$

1.2 Problem of a Worker

The consumption-saving allocation of worker w, born in period j, consists of choosing consumption C_t^{jw} and assets A_{t+1}^{jw} to solve

$$V_t^w(j)=\max\ \left\{(C_t^w(j))^{\rho}+\beta\left[\omega_{t,t+1}V_{t+1}^w(j)+(1-\omega_{t,t+1})V_{t+1}^r(j)\right]^{\rho}\right\}^{\frac{1}{\rho}}, \tag{7}$$

subject to

$$A_{t+1}^{w}(j) = R_{W,t}A_{t}^{w}(j) + W_t - T_t^{w} - C_t^{w}(j). \tag{8}$$

The first order condition for asset holding is

$$(C_t^{w}(j))^{\rho-1} = \beta\left[\omega_{t,t+1}V_{t+1}^{w}(j) + (1-\omega_{t,t+1})V_{t+1}^{r}(j)\right]^{\rho-1}\left[\omega_{t,t+1}\frac{\partial V_{t+1}^{w}(j)}{\partial A_{t+1}^{w}(j)} + (1-\omega_{t,t+1})\frac{\partial V_{t+1}^{r}(j)}{\partial A_{t+1}^{w}(j)}\right].$$

The envelope conditions are

$$\frac{\partial V_t^{w}(j)}{\partial A_t^{w}(j)} = (V_t^{w}(j))^{1-\rho}(C_t^{w}(j))^{\rho-1}R_{W,t}$$

and

$$\frac{\partial V_t^{r}(j)}{\partial A_t^{w}(j)} = \frac{\partial V_t^{r}(j)}{\partial A_t^{r}(j)}\frac{\partial A_t^{r}(j)}{\partial A_t^{w}(j)} = \frac{\partial V_t^{r}(j)}{\partial A_t^{r}(j)},$$

where the last result depends on the fact that individuals are risk neutral with respect to labor income fluctuations and hence choose the same asset profile independently of their employment status. Workers do not turn their wealth over to the mutual fund and hence do not receive the additional return that compensates for the probability of death. Combining this assumption with the envelope condition for retirees yields an Euler equation for worker j

$$\begin{aligned}(C_t^{w}(j))^{\rho-1} &= \beta R_{W,t+1}\left[\omega_{t,t+1}V_{t+1}^{w}(j) + (1-\omega_{t,t+1})V_{t+1}^{r}(j)\right]^{\rho-1} \\ &\quad \left[\omega_{t,t+1}\left(V_{t+1}^{w}(j)\right)^{1-\rho}\left(C_{t+1}^{w}(j)\right)^{\rho-1} + (1-\omega_{t,t+1})\left(V_{t+1}^{r}(j)\right)^{1-\rho}\left(C_{t+1}^{r}(j)\right)^{\rho-1}\right].\end{aligned} \tag{9}$$

Conjecture that the worker's value function has the same form as (6)

$$V_t^{w}(j) = \Delta_t^{w}C_t^{w}(j). \tag{10}$$

Substituting the guess back into the Euler equation (9) together with (6) leads to

$$\begin{aligned}(C_t^{w}(j))^{\rho-1} = \beta R_{W,t+1}&\left[\omega_{t,t+1}\Delta_{t+1}^{w}C_{t+1}^{w}(j) + (1-\omega_{t,t+1})\Delta_{t+1}^{r}C_{t+1}^{r}(j)\right]^{\rho-1} \\ &\left[\omega_{t,t+1}\left(\Delta_{t+1}^{w}\right)^{1-\rho} + (1-\omega_{t,t+1})\left(\Delta_{t+1}^{r}\right)^{1-\rho}\right].\end{aligned}$$

Define the adjustment term Ω_t as

$$\Omega_t \equiv \omega_{t-1,t} + (1-\omega_{t-1,t})\left(\frac{\Delta_t^{r}}{\Delta_t^{w}}\right)^{1-\rho}.$$

The Euler equation then becomes

$$\omega_{t,t+1}C_{t+1}^{w}(j)+(1-\omega_{t,t+1})\left(\frac{\Delta_{t+1}^{r}}{\Delta_{t+1}^{w}}\right)C_{t+1}^{r}(j)=(\beta\Omega_{t+1}R_{W,t+1})^{\sigma}C_{t}^{w}(j). \tag{11}$$

The guess for the decision rule of a worker is

$$C_{t}^{w}(j)=\pi_{t}\left(R_{W,t}A_{t}^{w}(j)+H_{t}^{w}\right), \tag{12}$$

where H_t^w represents the present discounted value of current and future human wealth net of taxation and is independent of individual-specific characteristics

$$H_{t}^{w}\equiv\sum_{v=0}^{\infty}\frac{W_{t+v}-T_{t+v}^{w}}{\prod_{s=1}^{v}\left(\Omega_{t+s}R_{W,t+s}/\omega_{t+s-1,t+s}\right)}=W_{t}-T_{t}^{w}+\frac{\omega_{t,t+1}H_{t+1}^{w}}{\Omega_{t+1}R_{W,t+1}}. \tag{13}$$

The term $\Omega_{t+1}R_{W,t+1}/\omega_{t,t+1}$ constitutes the effective discount rate for a worker. The first component of the higher discounting captures the effect of the finite lifetime horizon (less value attached to the future). The term $\omega_{t,t+1}$ augments the actual discount factor because workers need to finance consumption during the retirement period (positive probability of retiring).

From (4), the decision rule for a retiree born in period j who just abandoned the labor force is

$$C_{t}^{r}(j)=\xi_{t}R_{W,t}A_{t}^{w}(j).$$

Substituting the guess into the Euler equation gives

$$\begin{aligned}\omega_{t,t+1}\pi_{t+1}\left(R_{W,t+1}A_{t+1}^{w}(j)+H_{t+1}^{w}\right)+(1-\omega_{t,t+1})\left(\frac{\Delta_{t+1}^{r}}{\Delta_{t+1}^{w}}\right)\xi_{t+1}R_{W,t+1}A_{t+1}^{w}(j)\\ =(\beta\Omega_{t+1}R_{W,t+1})^{\sigma}\pi_{t}\left(R_{W,t}A_{t}^{w}(j)+H_{t}^{w}\right).\end{aligned}$$

Rearranging the last expression yields

$$\begin{aligned}\omega_{t,t+1}\left(A_{t+1}^{w}(j)+\frac{H_{t+1}^{w}}{R_{W,t+1}}\right)+(1-\omega_{t,t+1})\left(\frac{\Delta_{t+1}^{r}}{\Delta_{t+1}^{w}}\right)\epsilon_{t+1}A_{t+1}^{w}(j)\\ =(\beta\Omega_{t+1})^{\sigma}R_{W,t+1}^{\sigma-1}\frac{\pi_{t}}{\pi_{t+1}}\left(R_{W,t}A_{t}^{w}(j)+H_{t}^{w}\right),\end{aligned}$$

where $\epsilon_t \equiv \xi_t/\pi_t$. Using the definition of Ω_t, the last expression becomes

$$A_{t+1}^w(j) + \frac{\omega_{t,t+1} H_{t+1}^w}{\Omega_{t+1} R_{W,t+1}} = \beta^\sigma \left(\Omega_{t+1} R_{W,t+1}\right)^{\sigma-1} \frac{\pi_t}{\pi_{t+1}} \left(R_{W,t} A_t^w(j) + H_t^w\right).$$

Moreover, from the budget constraint of a worker and the guess (12), it is possible to see that

$$A_{t+1}^w(j) + \frac{\omega_{t,t+1} H_{t+1}^w}{\Omega_{t+1} R_{W,t+1}} = (1-\pi_t)\left(R_{W,t} A_t^w(j) + H_t^w\right).$$

Substituting this result back into the Euler equation above, it follows that the marginal propensity to consume for a worker evolves according to

$$\pi_t = 1 - \beta^\sigma \left(\Omega_{t+1} R_{W,t+1}\right)^{\sigma-1} \frac{\pi_t}{\pi_{t+1}}. \tag{14}$$

Finally, the original guess of the value function (10) is valid if

$$\left(\Delta_t^w C_t^w(j)\right)^\rho = \left(C_t^w(j)\right)^\rho + \beta\left[\omega_{t,t+1}\Delta_{t+1}^w C_{t+1}^w(j) + (1-\omega_{t,t+1})\Delta_{t+1}^r C_{t+1}^r(j)\right]^\rho.$$

The last equation, combined with (11), yields

$$\left(\Delta_t^w\right)^\rho = 1 + \beta^\sigma \left(\Omega_{t+1} R_{W,t+1}\right)^{\sigma-1} \left(\Delta_{t+1}^w\right)^\rho.$$

Expression (14) then implies that

$$\Delta_t^w = \pi_t^{-\frac{1}{\rho}}.$$

Hence, the ratio of the proportionality factors in the value function is $\Delta_{t+1}^r/\Delta_{t+1}^w = \epsilon_t^{-(1/\rho)}$ and

$$\Omega_t = \omega_{t-1,t} + (1-\omega_{t-1,t})\,\epsilon_t^{\frac{1}{1-\sigma}}.$$

1.3 Aggregation

The marginal propensity to consume of workers and retirees is independent of individual characteristics. Hence, the total consumption of workers and retirees simply equals the sum of individual consumption, irrespective of age and retirement period.[1]

Aggregate consumption for retirees is

$$C_t^r = \xi_t R_{W,t} A_t^r, \tag{15}$$

[1] An aggregate variable Q_t^z for cohort $z = \{w, r\}$ takes the form $Q_t^z \equiv \int_0^{N_t^z} Q_t^z(i)\,dk$, where the aggregation is independent of the birth date i.

where A_t^r is total wealth that retirees carry from period $t-1$ into period t. Similarly, aggregate consumption for workers is

$$C_t^w = \pi_t \left(R_{W,t} A_t^w + H_t \right), \tag{16}$$

where A_t^w is workers' total aggregate financial wealth. The aggregate value of human wealth evolves according to

$$H_t = N_t^w W_t - T_t + \frac{\omega_{t,t+1} H_{t+1}}{(1 + n_{t,t+1}) \Omega_{t+1} R_{W,t+1}}. \tag{17}$$

The aggregate consumption function C_t is the sum of (15) and (16). If $\lambda_t \equiv A_t^r / A_t$ denotes the share of total assets A_t held by retirees, the aggregate consumption function is

$$C_t = \pi_t \left[(1 - \lambda_t) R_{W,t} A_t + H_t + \epsilon_t \lambda_t R_{W,t} A_t \right]. \tag{18}$$

Relative to the standard neoclassical growth model, the distribution of assets across cohorts is an additional state variable which keeps track of the heterogeneity in wealth accumulation due to the life-cycle structure.

Aggregate assets for retirees depend on the total savings of those who are retired in period t as well as on the total savings of the fraction of workers who retire between t and $t+1$

$$A_{t+1}^r = R_{W,t} A_t^r - C_t^r + (1 - \omega_{t,t+1}) \left(R_{W,t} A_t^w + N_t^w W_t - T_t - C_t^w \right). \tag{19}$$

Aggregate assets for workers depend only the savings of the fraction of workers who remain in the labor force

$$A_{t+1}^w = \omega_{t,t+1} \left(R_{W,t} A_t^w + N_t^w W_t - T_t - C_t^w \right). \tag{20}$$

The law of motion for the distribution of wealth across cohorts derives from substituting expressions (15) and (20) into (19)

$$\lambda_{t+1} A_{t+1} = (1 - \omega_{t,t+1}) A_{t+1} + \omega_{t,t+1} (1 - \xi_t) R_{W,t} \lambda_t A_t. \tag{21}$$

Expression (21) relates the evolution of the distribution of wealth λ_t to the aggregate asset position A_t.

1.4 Supply Side

Firms hire labor, accumulate capital and make investment decisions to maximize the present discounted value of profits subject to the technology constraint given by the production

function

$$Y_t = (X_t N_t^w)^\alpha K_t^{1-\alpha}, \tag{22}$$

where labor-augmenting productivity grows over time according to

$$X_{t+1} = (1 + x_{t,t+1}) X_t.$$

The presence of adjustment costs makes the firms' problem dynamic. Denote by $V(I_{t-1}, K_t)$ the present discounted value of profits. Then, firms choose N_t^w, I_t and K_{t+1} to solve

$$V(I_{t-1}, K_t) = \max \left[(X_t N_t^w)^\alpha K_t^{1-\alpha} - W_t N_t^w - I_t + \frac{V(I_t, K_{t+1})}{R_{W,t+1}} \right]$$

subject to

$$K_{t+1} = (1-\delta) K_t + \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - \mu_t \right)^2 \right] I_t. \tag{23}$$

Denote by q_t the lagrange multiplier on the constraint (23). The first order condition for labor yields the standard equalization of the real wage to the marginal product of an additional worker

$$W_t = \alpha \frac{Y_t}{N_t^w}.$$

The first order condition for investment is

$$-1 + \frac{V_I(I_t, K_{t+1})}{R_{W,t+1}} + q_t \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - \mu_t \right)^2 \right] - \phi q_t \left(\frac{I_t}{I_{t-1}} - \mu_t \right) \frac{I_t}{I_{t-1}} = 0,$$

where V_I is the partial derivative of the value function with respect to investment. The first order condition for capital is

$$\frac{V_K(I_t, K_{t+1})}{R_{W,t+1}} - q_t = 0,$$

where V_K is the partial derivative of the value function with respect to capital. The envelope conditions are

$$V_I(I_{t-1}, K_t) = \phi \left(\frac{I_t}{I_{t-1}} - \mu_t \right) \left(\frac{I_t}{I_{t-1}} \right)^2 q_t$$

and

$$V_K(I_{t-1}, K_t) = (1-\alpha) \frac{Y_t}{K_t} + (1-\delta) q_t.$$

Substituting the two envelope conditions into the first order conditions for capital and in-

vestment yields

$$q_t = \frac{1}{R_{W,t+1}} \left[(1-\alpha) \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) q_{t+1} \right]$$

and

$$q_t \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - \mu_t \right)^2 - \phi \left(\frac{I_t}{I_{t-1}} - \mu_t \right) \frac{I_t}{I_{t-1}} \right] = 1 - \frac{\phi q_{t+1}}{R_{W,t+1}} \left(\frac{I_{t+1}}{I_t} - \mu_{t+1} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 .$$

The last two equations show that if $\phi = 0$ (no adjustment costs), $q_t = 1$ and $R_{W,t}$ equals the marginal product of capital net of depreciation.

2 The Steady State

This section characterizes the stationary (i.e. after detrending) symmetric steady state of the model. Symmetry implies that the stock of net foreign assets and the trade balance are zero. Additionally, the term μ ensures that steady state adjustment costs are zero too, so that $q = 1$.

The steady state marginal propensities to consume for retirees and workers are sufficient statistics to characterize the two cohorts. For a given level of the world interest rate R_W, the marginal propensity to consume of each cohort is given by

$$\xi = 1 - \gamma \beta^\sigma R_W^{\sigma-1} \tag{24}$$

and

$$\pi = 1 - \beta^\sigma (\Omega R_W)^{\sigma-1} , \tag{25}$$

where

$$\Omega = \omega + (1-\omega) \epsilon^{\frac{1}{1-\sigma}} \tag{26}$$

and

$$\epsilon = \xi / \pi . \tag{27}$$

In steady state, retirees have a higher marginal propensity to consume than workers, i.e. $\epsilon > 1$. If the contrary were true ($\epsilon \leq 1$), the ratio of (24) and (25) gives

$$\gamma \geq \Omega^{\sigma-1} = \left[\omega + (1-\omega) \epsilon^{\frac{1}{1-\sigma}} \right]^{\sigma-1} .$$

Since $\gamma < 1$, the last condition holds if only if $\epsilon > 1$, which constitutes a contradiction. For

a given level of the world interest rate, the system of equations (24) − (27) determines the solution of the individual marginal propensities to consume and for the variables Ω and ϵ.

Furthermore, for a given interest rate, the probability of surviving has a negative effect on the workers' marginal propensity to consume. The text alludes to this result. The proof makes use of the implicit function theorem applied to the function

$$F(\gamma, \pi) = \pi - 1 + \beta^{\sigma} \left[\Omega(\gamma, \pi) R_W\right]^{\sigma - 1},$$

where

$$\Omega(\gamma, \pi) = \omega + (1 - \omega) \left(\frac{1 - \gamma \beta^{\sigma} R_W^{\sigma - 1}}{\pi} \right)^{\frac{1}{1-\sigma}}.$$

The effect of γ on π depends on

$$\frac{\partial \pi}{\partial \gamma} = -\frac{\partial F(\gamma, \pi) / \partial \gamma}{\partial F(\gamma, \pi) / \partial \pi}.$$

The two partial derivatives are

$$\frac{\partial F}{\partial \gamma} = (1 - \omega) \beta^{2\sigma} \Omega^{\sigma - 2} R_W^{\sigma - 1} \epsilon^{\frac{\sigma}{1-\sigma}} / \pi > 0$$

and

$$\frac{\partial F}{\partial \pi} = 1 + (1 - \omega) \beta^{\sigma} \Omega^{\sigma - 2} R_W^{\sigma - 1} \epsilon^{\frac{1}{1-\sigma}} / \pi > 0,$$

so that

$$\frac{\partial \pi}{\partial \gamma} < 0.$$

Life expectancy negatively affects the workers' marginal propensity to consume for workers, for a given world interest rate.

The steady state supply side exactly corresponds to a standard neoclassical growth model. The production function is

$$y = k^{1-\alpha}. \tag{28}$$

The firm first order condition for capital pins down the capital-ouput ratio for a given level of the world return

$$R_W = (1 - \alpha) \frac{y}{k} + (1 - \delta).$$

The last two expressions combined deliver a solution for capital per unit of efficient labor

$$k = \left[\frac{1-\alpha}{R_W - (1-\delta)}\right]^{\frac{1}{\alpha}}. \tag{29}$$

Investment is a constant share of the capital stock to compensate for depreciation and population growth

$$i = (x + n + \delta)\, k. \tag{30}$$

The equation for aggregate consumption (18) yields

$$c = \pi \left\{ [1 + (\epsilon - 1)\, \lambda]\, R_W a + h \right\}. \tag{31}$$

In steady state, the distribution of wealth across cohorts λ is a function of the dependency ratio ψ

$$\lambda = \frac{(1-\omega)}{(1+x+n) - \omega\,(1-\xi)\, R_W} = \frac{(1+n-\gamma)\,\psi}{(1+x+n) - \gamma\omega\,(\beta R_W)^{\sigma}}, \tag{32}$$

where

$$\psi = \frac{1-\omega}{1+n-\gamma}. \tag{33}$$

The steady state distribution of wealth only depends on the level of the world interest rate. The capitalized value of human wealth that enters the steady state consumption function is

$$h = \frac{(\alpha - \tau)\, y}{1 - \omega\,(1+x)\,/\,(\Omega R_W)}. \tag{34}$$

In a symmetric steady state, net foreign assets are equal to zero so that total assets are given by

$$a = k + by. \tag{35}$$

From the government budget constraint, taxes equal expenditure plus the net interest rate payment on the stock of steady state debt

$$\tau = g + [R_W - (1 + x + n)]\, b. \tag{36}$$

The system of equations $(31) - (36)$ determines the demand side of the model in steady state for a given interest rate. The resource constraint determines the equilibrium real interest rate

$$(1-g)\, y = c + i, \tag{37}$$

for given steady state value of the stock of government debt b and of government spending g.

3 Data Sources

This section provides the details about the data used in the paper and the methodology to construct aggregates for the G6.

The series for US GDP and the trade balance versus the rest of the world and versus the G6 (figure 1 in the text) are from the Bureau of Economic Analysis (NIPA Tables and International Transactions, respectively). Bilateral trade data for France, Germany and Italy are available only starting in 1986. Western Europe plus Canada and Japan proxy for the G6 between 1960 and 1966. EU 6 (Belgium, France, Germany, Italy, Luxembourg and Netherlands) plus United Kingdom replace Western Europe between 1966 and 1985. All series are in current US dollars.

Data on real GDP per capita (top left panel of figure 2 in the text) are from the Groningen Growth and Development Centre (GGCD). All values are in PPP at 2002 US dollars. The series for the G6 GDP is the sum of GDP across countries. GGDC is also the data source for average annual hours (figure 1 below).

The data on historical and projected demographic trends (right panels of figure 2 in the text) are from United Nations World Population Prospects: The 2004 Revision. G6 population growth rates are the growth rates of the sum of total population across G6 countries. The years of life expectancy are a population-weighted average of the national counterparts. The data for life expectancy by age for selected years (table A.1 below) are from the United Nations Demographic Yearbook.

Data for total government balance (bottom left panel of figure 2 in the text) and net government debt are from the International Monetary Fund (World Economic Outlook Database). PPP factors from the same source convert national currencies in 2000 US dollars. The same methodology applies to the G6 GDP series which serves as denominator.

Total factor productivity (TFP) is the Solow residual of the production function of the model (with $\alpha = 2/3$). Labor is civilian employment from GGDC. The perpetual inventory method yields a series for the capital stock, given the depreciation rate ($\delta = 0.1$) and the average GDP growth rate over the sample period. The data on output and investment are from the World Bank (World Development Indicators).

Finally, the data for real interest rates (figure 2 below) are from the International Monetary Fund (International Financial Statistics). For each country, the available long term (generally 10-year) government bond yield minus realized CPI inflation yields the ex-post real interest

rate. The "world" real interest rate (figure 6 in the text) averages the country-specific rates using population weights. Aggregation with GDP weights yields almost identical results.

4 Extension to a Three-Country World

This section considers an extension of the model to a three-country world. The labels for the three countries, which have the same economic structure, are US, G6 and CH (think of China and other emerging economies). The only modification relative to the model in the text is that the equilibrium condition in the foreign asset market becomes

$$F_t^{US} + F_t^{G6} + F_t^{CH} = 0.$$

The three countries differ only in their relative economic size, measured as in the paper by the terms $\Phi_t^{US} \equiv X_t^{US} N_t^{wUS} / \left(X_t^{CH} N_t^{wCH} \right)$ and $\Phi_t^{G6} \equiv X_t^{G6} N_t^{wG6} / \left(X_t^{CH} N_t^{wCH} \right)$.

The quantitative analysis of the three-country model differs from the main text only in few dimensions. First, this section uses labor productivity from GGDC, instead of total factor productivity, because of the lack of internationally comparable investment series to obtain reliable estimates of the capital stock.[2] For similar reasons, the simulation also abstracts from fiscal policy. Second, the simulation starts in 1990, instead of 1970, because China was essentially a closed economy before then. The values of the parameters are as in the text.

The simulation of this model yields counterfactual implications for the trade balance. The US runs trade surpluses for most part of the periods, except for modest deficits early in the sample and in the second part of the 1990s. The average US surplus is above 1% while the maximum deficit is -1% in 1998. The G6 runs trade surpluses for the entire sample, with an average of almost 4%. Most strikingly, China experiences huge trade deficits over the entire sample, averaging about -11%.

This counterfactual findings are perhaps not very surprising in light of the main results in the paper and the pattern of productivity in China (see figure 4). Chinese productivity averaged almost 8% in the period $1990 - 2007$, with only a short-lived slowdown between 1996 and 1998. This period, in the simulation, corresponds to the mild US trade deficits discussed above.

Very high investment rates are responsible for the simulated Chinese trade deficits. This feature of the model is, at least qualitatively, consistent with the data. The counterfactual results stem from the failure of the the model in predicting the right level of the saving rate,

[2] Figure 4 plots the series. The correlation between the TFP series calculated for the simulations in the text and the labor productivity series used in this section is 0.84 for the US and 0.83 for the G6.

which is too low in the simulations.[3] China finances high investment levels with external borrowing, hence not sacrificing too much consumption. To the contrary, in the data, Chinese savings finance not only high domestic investment but also US consumption.

This analysis suggests that the real determinants of external imbalances that work well for industrialized countries have troubles in accounting for the Chinese trade and current account surpluses. Two potential explanations may reconcile the theory with the data. First, less developed domestic capital market relative to advanced economies may explain why China has accumulated a large stock of US securities and hence financed the US external deficits in recent years. Bernanke (2005) labeled this the global saving glut hypothesis, as several other emerging markets economies have followed similar patterns.[4] Second, Chinese household behavior may display a much stronger precautionary saving motive than typical in advanced economies, perhaps due to the absence of a formal social security net in China. Further research in both directions should prove useful to shed more light on this very important dimension of the global imbalances debate.

[3] The simulation does capture one aspect of Chinese saving behavior. As labor productivity in China converges to G7 levels (by assumption), the Chinese saving rate increases over time.

[4] See Caballero, Fahri and Gourinchas (2008) and Mendoza, Quadrini and Rios-Rull (2009) for models consistent with the saving glut idea.

5 Additional Tables and Figures

Table A.1. Life expectancy at birth and at 20 in G7 countries (selected years).

Country	**Year**	**At birth**		**At 20**	
		Male	**Female**	**Male**	**Female**
United States	1975	68.7	76.5	50.8	58.1
	1983	71	78.3	52.6	59.5
	1998	73.8	79.5	55	60.3
	2003	74.8	80.1	55.8	60.9
Canada	1975-1977	70.19	77.48	52.09	58.95
	1985	73.04	79.73	54.27	60.65
	1992	74.6	80.9	55.6	61.7
	2003	77.4	82.4	58.2	63
France	1974	69	76.9	50.8	58.3
	1988	72.33	80.46	53.53	61.36
	1993-1995	73.64	81.7	54.57	62.39
	2003	75.9	82.9	56.6	63.4
Germany	1973-1975	68.04	74.54	50.59	56.49
	1999	74.7	80.7	55.5	61.3
	2002-2004	75.9	81.5	56.6	62.1
Italy	1974-1977	69.69	75.91	51.99	57.75
	1995	74.6	81	55.6	61.7
	2002	77.1	83	57.8	63.5
Japan	1975	71.76	76.95	53.31	58.11
	1985	74.84	80.46	55.8	61.18
	1995	76.36	82.86	57.15	63.46
	2004	78.6	85.6	59.2	66
United Kingdom	1987-1989	72.42	78.03	53.63	58.94
	2000	75.3	80.1	56.1	60.8

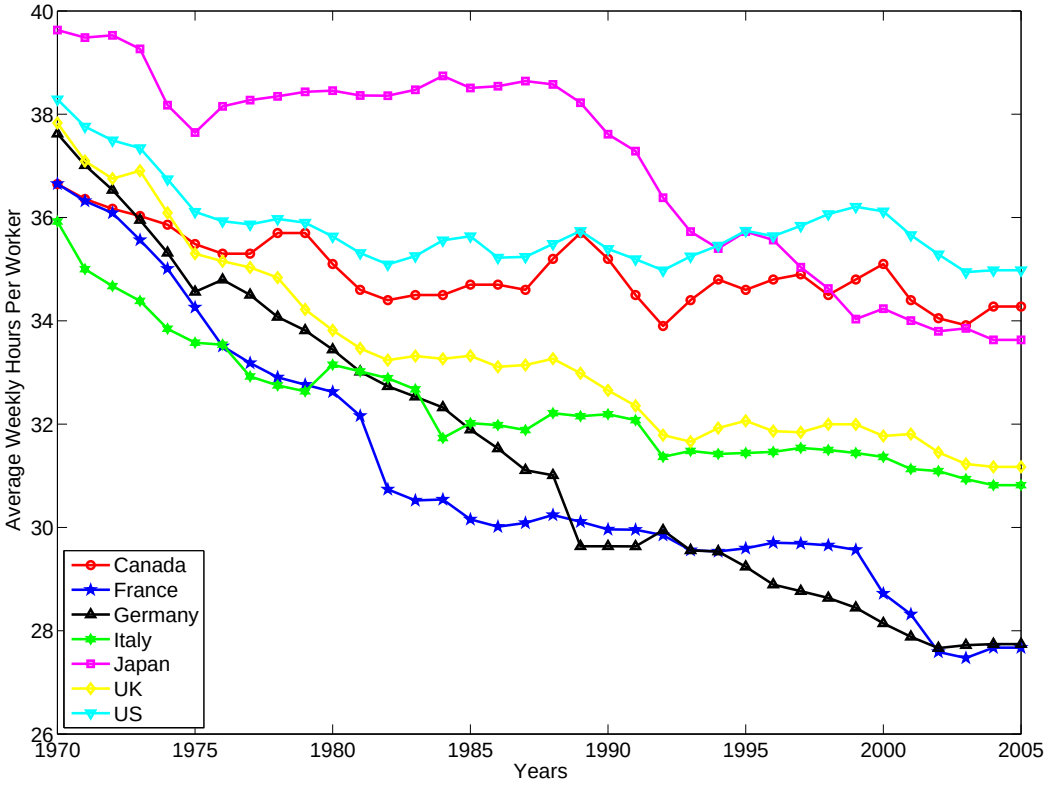

Figure 1: Average weekly hours per worker in G7 countries.

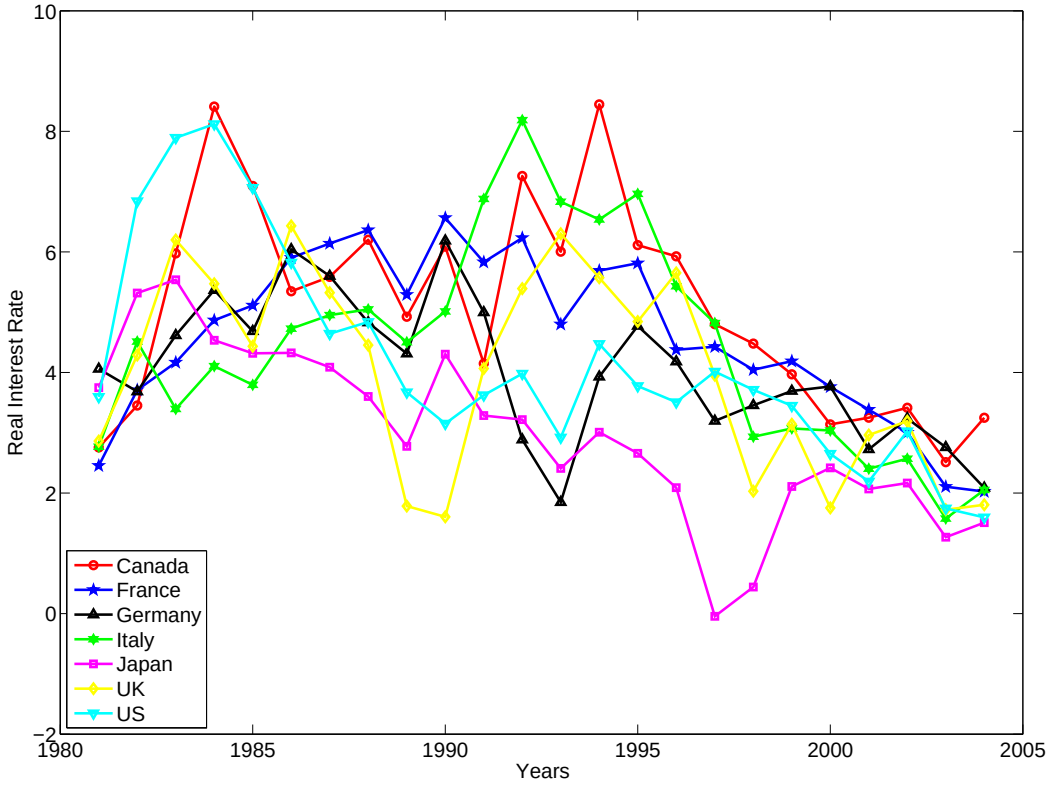

Figure 2: Real interest rates in G7 countries.

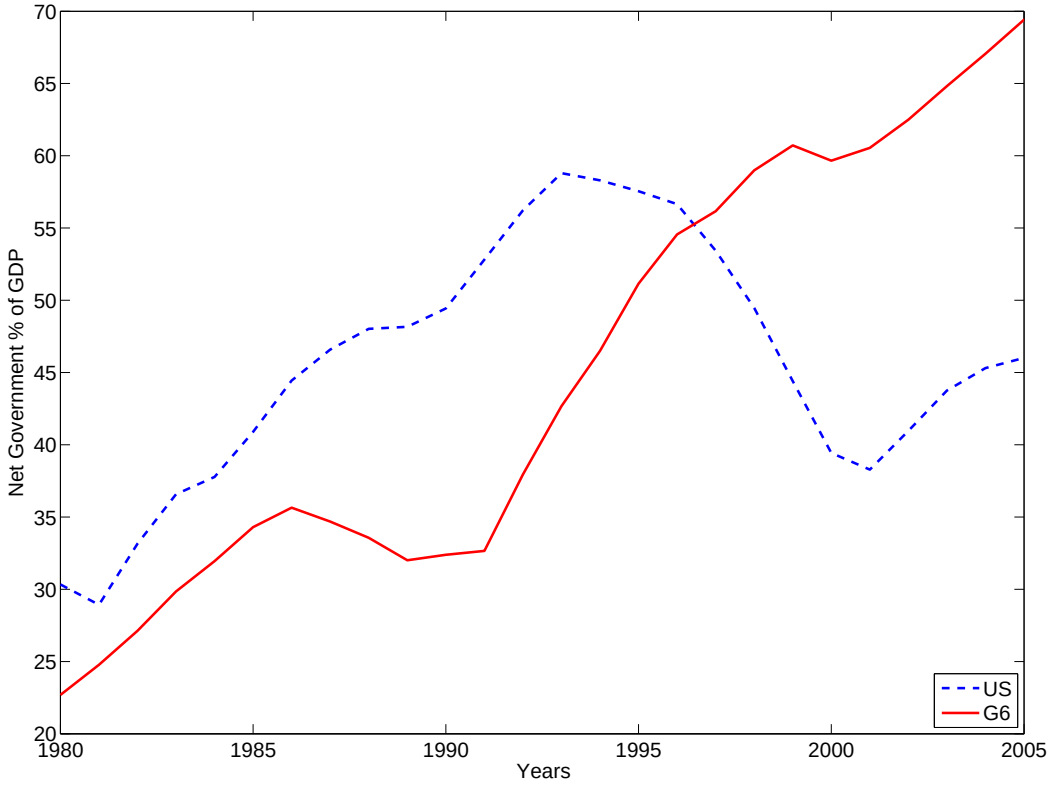

Figure 3: Net government debt % of GDP.

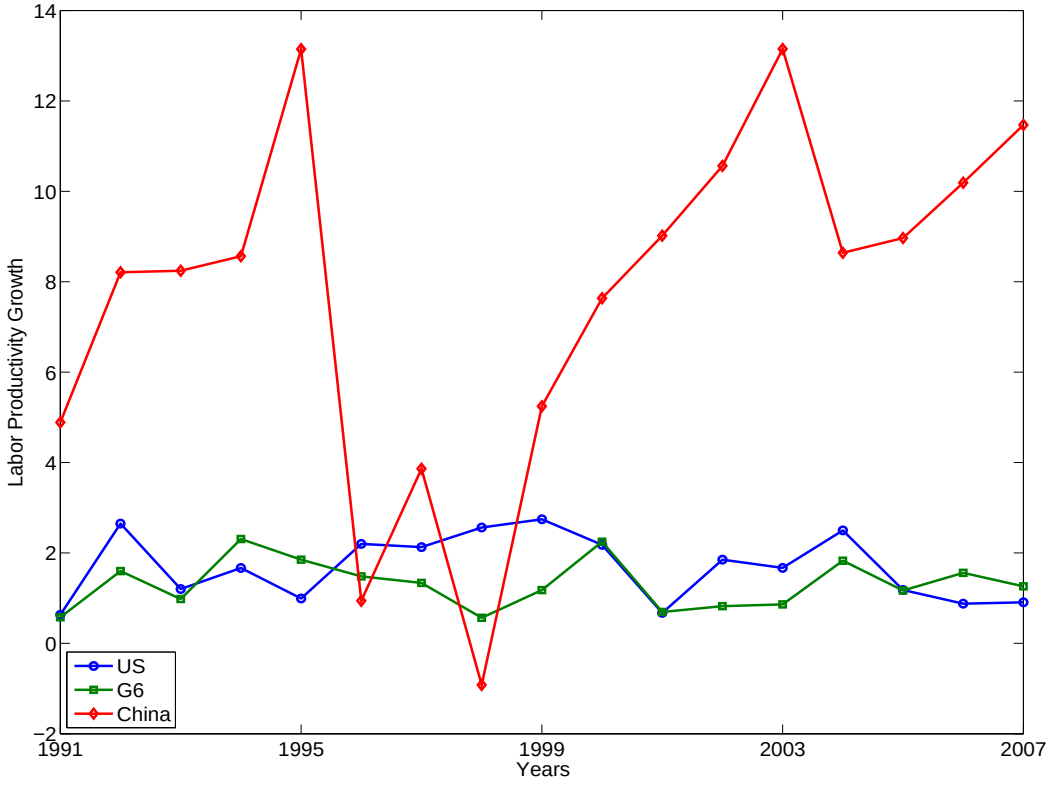

Figure 4: Labor productivity growth in US, G6 and China.