MODELING THE INSTABILITY OF MORTGAGE-BACKED PREPAYMENTS

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ABSTRACT

Prepayment plays a critical role in the valuation and performance of mortgage-backed securities. For this reason, market participants have devoted substantial resources to developing formal mathematical models of mortgage prepayment. This paper investigates the structure of the prepayment function. We demonstrate that the prepayment function is nonlinear and heteroskedastic, that is, prepayments are increasingly more volatile at higher interest rate spreads. Our analysis suggests that these unusual properties of pool prepayments are inherently caused by statistical aggregation.

JEL Classification: G13

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1. INTRODUCTION

The U.S. mortgage market is the largest debt market in the world. As of the second quarter 2002, residential mortgage debt outstanding was close to $6.5 trillion. Today, almost half of the debt is securitized and resold as mortgage-backed securities (MBS) by governmental, quasi-governmental institutions and private mortgage originators. Mortgage securitization has greatly enhanced liquidity in the mortgage market. The mortgage market has also benefited from an expanding secondary market facilitated by the increased participation of loan brokers and private insurance mortgage companies. These entities provide a wide variety of services that allow mortgage originators and other investors to trade large portfolios of conforming or nonconforming loans in the secondary market (whole loan market).

In many respects, a mortgage security is similar to an ordinary bond. Like bonds, mortgage securities promise their holders a stream of payments over a number of periods. Mortgage passthroughs, however, are different from a typical government bond because the promised cash payments depend on prepayment. Mortgage borrowers in United States are given the right to prepay part or the entire principal without penalty. This embedded option can change drastically the expected cash flows from a mortgage security. The adverse effect of prepayment is particularly exaggerated in more exotic mortgage-backed derivative products such as collateralized mortgage obligations (CMOs) and stripped MBSs.

The prepayment experience of mortgage securities in the 1980s and 1990s has been quite bumpy. During this period, the mortgage market experienced several intense refinancing cycles, which were prompted by sharp declines in interest rates. Most market participants were not

surprised by the changes in interest rates. What took Street forecasters by surprise, however, was the precipitous rise (or fall) and extremely volatile nature of pool prepayments. Some have attributed the changing intensity of prepayment cycles to the evolving character of the mortgage market. Mortgage banks and brokers increasingly dominated housing finance in the 1990s. These financial institutions are more agile than banks and thrifts because they face fewer restrictions and regulations. Aided by advances in information technology mortgage bankers and brokers have expanded geographically by offering more attractive loan products.

The increased competition among lenders has lowered transaction costs for mortgage borrowers, which in turn has led to a rise in the propensity to refinance. Bennett et al. (2001) report that 12 percent of the borrowers in the early 1990s prepaid their loan after 5 years. Under the same economic conditions, the prepayment rate in 1980s would have only been around 7 percent. A study by Lekkas (1994) provides further support to the changing nature of refinancings. The author argues that the high-rate borrowers who reduced their monthly payments by refinancing into lower-interest loans dominated the 1986-87 experience. By contrast, during 1992-93 borrowers elected to shift into shorter-maturity mortgages.

Although borrowers appear to be more responsive to interest rate movements in the 1990s, it is hard to rationalize how such a moderate shift in the incentive to refinance would have generated so much volatility in mortgage prepayments. Why are MBSs prepayment speeds so erratic and unpredictable? In investigating this question, this paper takes a somewhat unorthodox approach. Most studies in the literature have analyzed prepayment speeds at the aggregate (pool) level. We focus instead on the microstructure of the MBS. The prepayment experience of the passthrough security is simply the sum of all individual prepayment decisions in the pool. We will argue that this process of aggregation makes the prepayment function

inherently unstable.

2. BACKGROUND

To understand the instability in MBS prepayment rates, consider a simple scatter diagram of prepayment speeds and the relative interest rate differential between the weighted average coupon (WAC) and the prevailing mortgage rate (Figure 1). The prepayment rates describe the experience of 30-year conventional Federal National Mortgage Association (FNMA) passthroughs with coupon rates between 7 and 12 percent from 1982 to 1994.[1] The solid curve in the scatter plot represents an in-sample forecast of PSA prepayment rates.[2] The S-shape configuration of the in-sample prediction illustrates the nonlinear nature of prepayments. Homeowners are reluctant to refinance when spreads are negative because their mortgage option is out of the money. In this negative range, residual prepayments are small, resulting mostly from life events or other idiosyncratic factors.

The scatter plot shows that prepayment rates accelerate as interest rate spreads become more positive. The rising incentive to prepay at higher coupon spreads is best seen by the steepening slope of the forecast curve. Note, however, that at the same time prepayments become more dispersed at higher interest rate spreads. Put another way, prepayments are *heteroskedastic* with respect to the coupon spread. For example, FNMA prepayment rates range from 100 PSA to

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[1]The Public Securities Administration (PSA) convention assumes pool prepayments rise 0.02 percent per month for the first 30 months of the life of the pool, and then remain constant at 6 percent (per year) from the thirtieth month until maturity.

[2]We use the penalized least squares (PLS) method to estimate prepayment forecasts. This smoothing spline approximation technique is based solely on the coupon spread. Our objective here is to simply demonstrate the nonlinear nature of prepayments. In a later section, however, we will show that a polynomial specification is also a good approximation for the prepayment function.

300 PSA when the interest rate spread is zero. By contrast, prepayments are more dispersed when the spread is 200 basis points, ranging from 200 PSA to 900 PSA. This large disparity in prepayment rates is found in individual FNMA coupon cohorts and is also observed in single pools. Thus, the presence of heteroskedasticity cannot be attributed to the fact that the scatter plot portrays the prepayment experience of wide class of FNMA securities.

Why is the relationship between prepayment rates and coupon spreads heteroskedastic? One possible explanation for this paradox is that it may be the result of "path dependency". By construction, a MBS pool is made up of a finite number of mortgages. When a mortgage is prepaid, the servicer returns the principal to investors. Subsequent cash flows of the security are paid out of the remaining mortgages in the pool. Prepayment is therefore equivalent to sampling without replacement. This process introduces path dependency because it changes the composition of the pool. Consider, for example, an unseasoned mortgage pool that experiences consecutively two identical interest rate cycles. In both cycles, assume that the interest rate spread first rises by 200 basis points but eventually returns back to its original level. In the first episode, rate-sensitive homeowners will rush to take advantage of favorable interest rates exiting the pool, pushing prepayments higher. As mortgage rates decline for the second time around, however, prepayment rates will be lower because the pool now consists of constrained mortgagors who are less able to take advantage of the favorable interest rate environment.

This paper will offer a somewhat different interpretation for the heteroskedastic traits of prepayment. We will argue that the unusual dispersion in prepayments is not necessarily caused by path dependency. Rather, we will show that this phenomenon is simply a statistical artifact of aggregation. In fact, we will demonstrate that pool-level prepayment rates continue to be heteroskedastic, even though an exactly identical individual replaces a prepaying mortgage

holder in the pool. To be sure, "burn out" is important. But its role is more critical in shaping the average propensity to prepay, which accounts for the nonlinear S-shape of the prepayment function.

3. A STATISTICAL MODEL FOR INDIVIDUAL PREPAYMENTS

3.1 *Mortgage Mathematics*

A traditional mortgage loan is an amortized contract that requires the borrower to pay interest and repay the principal in equal installments. At the same time, the mortgagor is given the right to prepay part or the entire principal before maturity without penalty. Like any contract with standardized payment streams, a mortgage loan obeys a well-developed mathematical framework (for more details, see Hayre and Mohebbi (1992)). Assume that the $(i-th)$ homeowner takes out a conventional fixed-rate mortgage loan at month $(t=0)$. The mortgage rate is r_i and the loan is amortized over T periods (typically, T equals 360 months). Let B_{ti} represent the remaining balance on the loan at month t (thus, B_{0i} represents the original balance of the loan). The remaining balanced B_{ti} includes all partial (unscheduled) payments. When B_{ti} reaches zero, the loan is fully repaid at month t. In the absence of any prepayment, the remaining balance of a mortgage is given by

$$B_{\mathrm{ti}} = B_{0\mathrm{i}} \frac{(1+r_{\mathrm{i}})^{\mathrm{T}} - (1-r_{\mathrm{i}})^{\mathrm{t}}}{(1+r_{\mathrm{i}})^{\mathrm{T-1}}} = B_{0\mathrm{i}} \times \alpha_{\mathrm{ti}}, \tag{1}$$

The term α_{ti} is known as the amortization factor. It follows that the proportion of the scheduled loan balance outstanding in any month is defined by:

$$q_{\mathrm{ti}} = \frac{B_{\mathrm{ti}}}{\bar{B}_{ti}}. \tag{2}$$

A mortgage loan that does not incur any partial payments will always have a q_{ti} ratio equal to 100 percent. The variable q_{ti} is useful for defining the standard measures of prepayment. The fraction of the outstanding loan balance that is prepaid each month is simply given by

$$p_{\mathrm{ti}} = \frac{\Delta\, q_{\mathrm{ti}}}{q_{\mathrm{t\text{-}1,i}}}. \tag{3}$$

In the monthly context, p_{ti} becomes the single monthly mortality rate (SMM). The SMM rate represents the proportion of the outstanding balance of mortgage i prepaid at month t. Typically, SMM, or its annualized version conditional prepayment rate (CPR), are used to measure pool-level prepayments. However, these prepayment measures are also applicable to a single mortgage, although at this micro level prepayment rates are lumpy.

3.2 *A Simple Econometric Model for Mortgage Prepayment*

Mortgage prepayments can occur because of three basic reasons: (1) refinancings, (2) property sale, and (3) default. Refinancings represent prepayment by nonmover occupants. The rational prepayment literature stipulates a mortgagor would refinance when the intrinsic value of the loan, defined as the immediate benefit from refinancing measured in present value terms, is greater than the benefit from waiting to refinance in a subsequent period (the "time value" of the option plus transaction costs). The decision to terminate a mortgage by moving or defaulting also depends on the moneyness of the mortgage option, however, one would expect that personal characteristics (income, education) and other idiosyncratic events (job loss, death, divorce) to play also an important role.

Recent studies have taken a more direct approach to modeling the cross sectional heterogeneity in prepayment behavior. The rational prepayment model is characterized by an empirical specification that employs loan-level information on mortgage terminations (see

Cunningham and Capone (1990), Caplin et al. (1996), and Peristiani et al. (1997)). These empirical studies find strong evidence that prepayments are driven by two particular factors: post-origination home equity and homeowner creditworthiness.

The empirical methodology is also useful for defining a general stochastic model of individual prepayments. The decision to prepay can be simply expressed as

$$p_{\mathrm{ti}}^{*} = \beta_{0i} + \mathrm{x}_{\mathrm{t}}\beta_{1\mathrm{i}} + \varepsilon_{\mathrm{ti}}, \tag{4.1}$$

where

$$\begin{array}{lll} p_{\mathrm{ti}} = 100 & \text{if} & p_{ti}^{*} \geq 100 \\ p_{\mathrm{ti}} = p_{ti}^{*} & \text{if} & 0 < p_{ti}^{*} < 100; \\ p_{\mathrm{ti}} = 0 & \text{if} & \text{otherwise.} \end{array} \tag{4.2}$$

As before, the variable p_{ti} denotes a broad measure of actual prepayment (e.g., the SMM rate or the annualized conditional prepayment rate). For simplicity, we assume that is bounded above by 100 percent (full prepayment) and below by zero (no prepayment). The variable p_{ti}^{*} represents the unobservable notional desire to prepay. In contrast to actual prepayment, the notional desire is a continuous variable that can be negative or exceed 100 percent. If the notional desire to prepay is positive but less than 100 percent, the homeowner will partially prepay the loan.[3]

The willingness to prepay is determined by a systematic factor x_t, representing market

[3]The contribution of partial prepayments (or curtailments) to overall prepayment is generally quite small. For fixed-rate mortgages, partial prepayments contribute on average around 0.2 percent to conditional prepayment rates. As with partial prepayments, defaults make up a small portion of total prepayment. Usually, the homeowner default on fixed-rate mortgages is less than 0.5 percent per year. Since most passthroughs are insured against credit risk, we do not consider this option in the censored regression model. However, one can explicitly include this outcome by using a more generalized version of the censored model.

conditions. For simplicity, here we let x_t be a scalar factor representing the spread between the coupon rate and the prevailing market rate. The parameters, β_{1i} and β_{0i}, capture homeowner or loan characteristics. Credit- or collateral-constrained borrowers, on average, are expected to have small positive slope coefficients β_{1i} because they are less sensitive to economic conditions. In other words, a constrained borrower is less responsive to a rise in the interest rate spread x_t.[4]

The term $\varepsilon_{\mathrm{ti}}$ denotes the random error that accounts for all unexplained variation in the decision to prepay. We assume that the error of the model is drawn from a normal distribution with zero mean and variance σ^2. Equation (4) defines a two-limit censored regression model (see Maddala (1983)). In contrast to the ordinary regression model, the distribution of monthly prepayment in the two-limit model is determined by a mixture of discrete (unobserved) and continuous (observed) variables. The probabilities of the three distinct outcomes of prepayment are given by

$$\begin{aligned} &\text{P(i-th homeowner prepays fully in month t)} &&= 1 - \Phi(\lambda_{ti}^{u}), \\ &\text{P(i-th homeowner partially prepays in month t)} &&= \Phi(\lambda_{\mathrm{ti}}^{\mathrm{u}}) - \Phi(-\lambda_{ti}), \\ &\text{P(i-th homeowner does not prepay in month t)} &&= 1 - \Phi(\lambda_{ti}), \end{aligned} \tag{5}$$

where $\lambda_{\mathrm{ti}} = (\beta_{0i} + x_t \beta_{1\mathrm{i}})/\sigma$ and $\lambda_{\mathrm{ti}}^{\mathrm{u}} = (100/\sigma) - \lambda_{\mathrm{ti}}$. The function $\Phi(\lambda)$ is the standard normal cumulative distribution integrated between λ and ∞. Note that all three probability outcomes in (5) sum to one.

In the censored regression model, the likelihood of prepayment is still determined by the

[4]Peristiani et al. (1997) and Caplin et al. (1997) find evidence that strongly supports this premise. Using a large sample of homeowners, these studies estimate a qualitative model for the decision to refinance. Their empirical findings suggest that credit quality and collateral value have a significant effect on the probability of refinancing.

homeowner's characteristics and interest rate conditions. However, the censored nature of individual prepayments complicates the error structure. We can show that:

$$\mathrm{E}(\varepsilon_{\mathrm{ti}}) = \sigma \frac{\varphi(\lambda_{\mathrm{ti}}) - \varphi(\lambda_{\mathrm{ti}}^{\mathrm{u}})}{\Phi(\lambda_{\mathrm{ti}}^{\mathrm{u}}) - \Phi(\lambda_{ti})} = \sigma\, h(\beta_{0\mathrm{i}}, \beta_{1i}, x_t) = \sigma\, h(\lambda_{\mathrm{ti}}), \tag{6.1}$$

$$\mathrm{Var}(\varepsilon_{\mathrm{ti}}) = \sigma^2\ [1\text{-h}(\lambda_{\mathrm{ti}})^2 + 100\ \frac{\varphi(-\lambda_{\mathrm{ti}}^{\mathrm{u}})}{\Phi(\lambda_{\mathrm{ti}}^{\mathrm{u}}) - \Phi(\lambda_{\mathrm{ti}})}\ ] = \sigma^2 \upsilon(\beta_{0\mathrm{i}}, \beta_{1\mathrm{i}}, x_t) = \sigma^2 \upsilon(\lambda_{\mathrm{ti}}). \tag{6.2}$$

The error in the mortgage prepayment model has therefore a nonzero mean and its variance is heteroskedastic (that is, $\mathrm{Var}(\varepsilon_{\mathrm{ti}})$ is a function of x_t). The significance of these statistical properties of individual prepayments would become more apparent in the next section. Nonetheless, it is not difficult to understand why these unusual characteristics of individual prepayments are extremely important. A MBS comprises a finite number of borrowers. Since a borrower's decision function is heteroskedastic, then this property would also transfer to the MBS prepayment function.

4. THE MBS PREPAYMENT FUNCTION

Consider a typical mortgage passthrough security consisting of number of conventional mortgage loans. At origination $(t = 0)$, the mortgage pool contains n_0 fixed-rate mortgages with maturity T. After the MBS is issued, the number of mortgages in the pool may decline, (e.g., $n_{\mathrm{t}+\ell} \le n_t \le n_0$). The overall size of the pool at origination equals B_0. Each loan in the pool contributes B_{0i}, such that $B_0 = \sum B_{0i}$. At origination, the WAC of the MBS is $\bar{r} = \sum \omega_{0\mathrm{i}} r_i$, and the weighted average maturity (WAM) is T months. The scaling factor ω_{0i} represents the relative weight of each mortgage loan at $(t = 0)$ (or more generally, $\omega_{ti} = B_{\mathrm{ti}} / B_{\mathrm{t}}$).

At its inception, the steam of payments of the MBS is the cash flows of n_0 mortgage

loans. Ultimately, the cash flows of the security are determined by the prepayment experience of the pool. To complete the model, we assume that individual prepayment process is defined by equation (4). The prepayment experience of the pool at any given month is the sum of all individual prepayments. Algebraically, we can express this aggregate pool prepayment rate as

$$P_t = \beta_{0t} + \beta_{1t}\, x_t + \varepsilon_t \tag{7}$$

such that $P_t = \sum \omega_{ti} p_{ti}^*$, $\beta_{kt} = \sum \omega_{ti} \beta_{ik}$ $(k = 0,1)$, and $\varepsilon_t = \sum \omega_{ti} \varepsilon_{ti}$. We should note the parameters β_{0t} and β_{1t} are time varying, meaning that the slope and intercept of the prepayment function change over time. Since mortgagors are not replaced when they exit the pool, the composition of pool changes over time with prepayment. In our simple framework, where prepayment rates are determined by a single factor x_t (the coupon spread), the slope of the prepayment function will be fairly flat at negative values of the spread. In this range, we observe small residual prepayments resulting from idiosyncratic events. The slope of prepayment function steepens as coupon differentials become positive and widen. Large positive spreads trigger rapid refinancing as borrowers with a higher propensity to prepay (those with high positive β_{1i}) are now in the money. Eventually, the slope of prepayment function flattens at very high values of spread because the pool is "burned out", meaning that the pool now contains mostly constrained borrowers (low-beta homeowners) who are unable to refinance at any rate.[5]

Since individual prepayments are heteroskedastic and have a nonzero mean, pool-level

[5]Another way to look at path-dependency in prepayments is by examining the stochastic properties of n_t the number of mortgagors remaining in the pool at time t. Because loans are not replaced in the pool, the conditional expectation of n_t depends on n_{t-1}. In turn, this means that the conditional expectation of n_t depends on lagged values of x_t.

prepayments have also a similar structure. We can show that

$$\varepsilon_t = \sigma \sum_{i=1}^{n_t} \omega_{ti} h(\lambda_{ti}) \tag{8}$$

$$\mathrm{Var}(\varepsilon_t) = \sigma^2 \sum_{i=1}^{n_t} \omega_{ti} \; \upsilon(\lambda_{ti}). \tag{9}$$

The error structure of pool prepayments is again heteroskedastic in the sense that the variance depends on the level of coupon spread x_t. Equations (8)-(9) can be simplified by linearizing the functions $h^*(\omega_{ti}, \lambda_{ti}) = \omega_{ti} h(\lambda_{ti})$ and $\upsilon^*(\omega_{ti}, \lambda_{ti}) = \omega_{ti} \upsilon(\lambda_{ti})$. Using a multivariate Taylor approximation rule, we can modify these functions to

$$E(\varepsilon_t) \cong \sigma[\alpha_0 + \alpha_1 x_t + \alpha_2 x_t^2 + \ldots + \alpha_k x_t^k] = \sigma \, h(x_t), \tag{10}$$

$$\mathrm{Var}(\varepsilon_t) \cong \sigma^2[\gamma_0 + \gamma_t x_t + \gamma_2 x_t^2 + \ldots + \gamma_k x_t^k] = \sigma^2 \upsilon(x_t), \tag{11}$$

where k represents the polynomial order of the Taylor expansion. Thus, an additive form of heteroskedasticity, which depends on the scalar exogenous factor, can approximate the error structure of the prepayment function x_t.

Equation (11) reveals an interesting finding. The variance of the prepayment errors also depends on x_t. This relationship suggests that statistical inference is more uncertain at larger values of coupon spread as the confidence interval for the prepayment forecast is wider. We can also use the Taylor approximation in same manner to specify the theoretical structure of the prepayment function. The aggregate prepayment rate can be expressed as

$$E(P_t) \cong [\beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \ldots + \beta_k x_t^k]. \tag{12}$$

The aggregate prepayment function is therefore nonlinear. But more important, this nonlinear function can be easily approximated by polynomial regression model, assuming that x_t is fully

known.

These findings can be easily generalized to the case where a borrower's decision to prepay is influenced by several variables represented by the row vector $x_{t\bullet}=(1, x_{t1}, x_{t2}, \ldots, x_{tp})$. In this multivariate case, we can show that prepayment errors would still be heteroskedastic, albeit the functional form of additive heteroskedasticity is more complicated.

4.1 *Simulation Examples*

An alternative way of illustrating the effects of aggregation in prepayments is through simulation. The premise of the simulation examples presented in this section is straightforward. We construct artificial pools of mortgages. The decision to prepay is determined by the interest rate spread defined by equation (4). For simplicity, however, we assume that borrowers do not partially prepay their loans. In each period, mortgage holders are exposed to a different interest rate spread plus a random shock. The simulation example asserts that individuals in the pool have completely different prepayment functions. Each mortgagor has a different propensity to prepay (that is, they have unique β_{0i} and β_{1i}).

We perform two distinct simulation experiments. The first simulation experiment assumes that the mortgage holder exits the pool when the willingness to prepay p_{ti}^{*} is greater than zero; otherwise, the borrower does not prepay (remember, there are no curtailments). The second simulation experiment assumes again that a borrower would prepay when $p_{ti}^{*} > 0$; however, now the prepaying mortgage holder is replaced in the pool by an exactly identical borrower. In this way, we maintain the size of the pool constant.[6] The results of the two

[6]Borrowers are heterogeneous in the sense that $\beta_{\ell i} = \overline{\beta}_{\ell i}(1+\rho\zeta_i)$, ℓ=0,1 , where $\overline{\beta}_{\ell i}$ is the predetermined value for the intercept and slope, ρ is a small constant (usually, 0.05) and ζ_i is a

simulation examples are graphically presented in Figure 2. Essentially, we observe two distinct scatter plots in the figure. Observations marked by the symbol (×) represent the prepayment experience of pools that prepay with replacement. The symbol (□) denotes pools that prepay without replacement. The solid curves in the figure represent again in-sample predictions for aggregate prepayments, which were estimated from a simple polynomial regression. Prepayment rates are unstable for large values of interest rate spread, indicating the heteroskedastic nature of the prepayment function. Note, however, that prepayments are heteroskedastic in both cases. This is an important finding because it helps to demonstrate that burnout (prepayment without replacement) alone cannot explain the heteroskedastic nature of prepayments. The pattern in prepayments is still heteroskedastic, even though individuals that prepay are replaced in the pool.[7]

What clearly distinguishes these two simulation examples is the shape of the average prepayment function. Pool prepayment rates are, on average, much larger when prepaying borrowers are replaced in the pool. This outcome is not surprising because in this case the composition of the pool is unchanged. At negative spreads, only a small fraction of these individuals wish to prepay. But as spreads become positive and widen, an increasing number of mortgage holders are willing to prepay because the pool does not burn out. In contrast, when borrowers are not replaced in the pool, aggregate prepayments tend to level off after a point, giving rise to the distinct S-shape. In summary, our simulation findings suggest that pool burnout does not necessarily account for the phenomenon of heteroskedasticity in MBS

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random shock generated from a standard normal distribution.

[7]A simple F-test shows that the error sum of squares for the two experiments is not statistically significantly different from each other.

prepayment rates. However, burnout is solely responsible for the nonlinear S-shape structure found in most pool prepayment functions.

5. IMPLICATIONS

Our analysis provides a compelling theoretical argument that the MBS prepayment function is inherently heteroskedastic. As shown above, prepayments are more likely to be scattered at large positive coupon spreads. The unusual nature of prepayments raises a number of interesting questions. Should we be concerned with the heteroskedastic structure of the prepayment function? Can this distinctive error structure in prepayments distort pricing?

Broadly speaking, heteroskedastic errors diminish the power of statistical inference because the least-squares regression estimator is inefficient. The impact of heteroskedasticity is quite evident in the wide discrepancy of published forecasts available from Bloomberg. Figure 3 summarizes the prepayment forecasts made by 8 firms for new FNMA 8s 30-year conventional passthroughs. The figure clearly shows that forecast uncertainty (here measured by the range of the PSA forecasts) is significantly higher for large interest rate shifts.

5.1 *The Effect on Pricing*

The prepayment function is an indispensable part of any MBS pricing methodology. Prepayment assumptions allow investors to figure out cash flows and determine the price of the security. In theory, we expect that the value of a MBS would be influenced by interest rate dynamics and prepayment behavior. We can formally define the price of a mortgage security (j) at time (t) as

$$V_{tj} = V[\Omega_j, \Re_t, P(\beta_j, x_{t\bullet}, \varepsilon_t)], \tag{14}$$

where $\Re_t$ represents the interest rate process at time (t) and Ω_j is a vector of security-specific

attributes. We also assume that the exogenous vector $x_{t\bullet}$ and individual characteristics β_j drive prepayments. As shown previously, prepayment errors are not identically distributed but are instead heteroskedastic (e.g., $\mathrm{Var}(\varepsilon_t)=\sigma^2 \upsilon(x_{t\bullet})$).

The large dispersion in prepayment errors introduces the potential for greater disparity in MBS prices. Thus, two MBSs may end up having very different price realizations, although ex ante the securities were fundamentally similar. The magnitude of the price distortion depends essentially on the importance of the underlying prepayment assumptions. In some instances, the value of mortgage securities is extremely vulnerable to changes in prepayment projections. To illustrate the sensitivity of prices to prepayment assumptions consider again the prepayment forecasts in Figure 3 and assume that interest rate increase by 50 basis. For this shift, prepayment forecasts range from a low of 434 PSA (forecast by First Boston) to a high of 868 PSA (forecast by Salomon). Using a Bloomberg pricing algorithm, we computed the OAS cost of a January-1998 TBA comparable passthrough under the different prepayment scenarios. When interest rates are unchanged (zero interest shift), the median OAS costs for the FNMA 8 percent passthrough is 75 basis points. Given the wide confidence bounds on prepayments, however, OAS values can range from 23 basis points to 112 basis points. Thus, a mere 50 basis points shift in interest rates has produced a large disparity in prices.

6. CONCLUSION

This study has shown that mortgage prepayments become extremely unstable when the spread between the WAC and the mortgage rate prevailing in the market is large and positive. The customary view attributes this trait to path dependency or burnout. According to this premise, prepayments are more dispersed (heteroskedastic) because often after a few bouts of

refinancing the pool is made up of mostly constrained mortgagors. In this paper we provide an alternative interpretation for this phenomenon. Although burnout is an important determinant of prepayment, its role is more evident in the nonlinear shape of the prepayment function. We demonstrate that the large dispersion in prepayments is not necessarily related to burnout, but is caused instead by statistical aggregation.

Our findings highlight the unstable nature of the prepayment function. We observe little dispersion in prepayments for negative or for small positive interest spreads. Prepayments, however, become increasingly more volatile when interest rate spreads cross a certain threshold. Consequently, the task of forecasting MBS prepayments becomes more arduous in an economic environment marred by unanticipated interest rate movements. Even a moderate shift in interest rates could alter cash flows in a way that would adversely affect the value of the mortgage security.

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Figure 1. Prepayment Experience of FNMA 6s-12s 30-Year Passthroughs,1982-1994

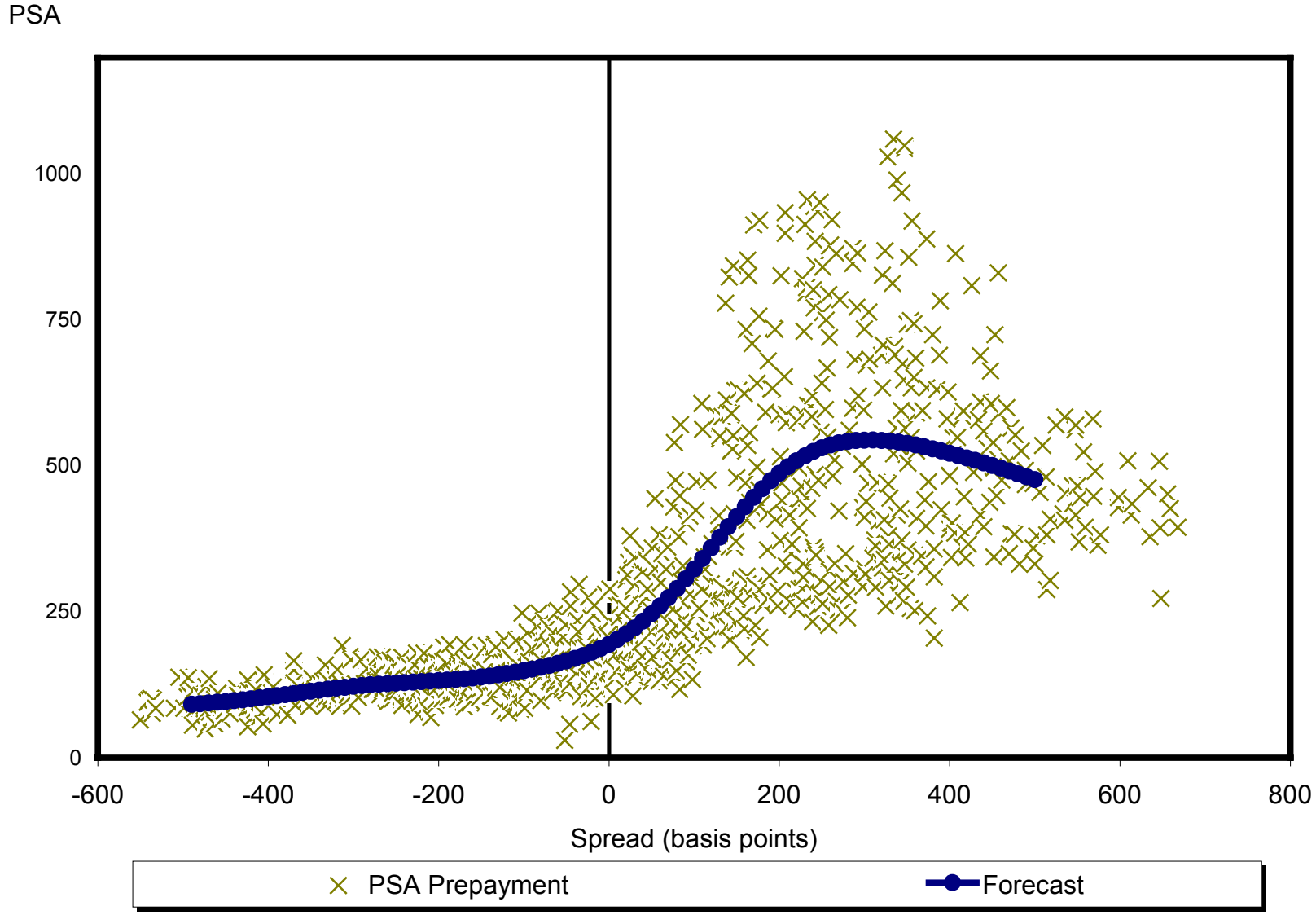


Figure 2. Simulated Pool Prepayments With and Without Replacement

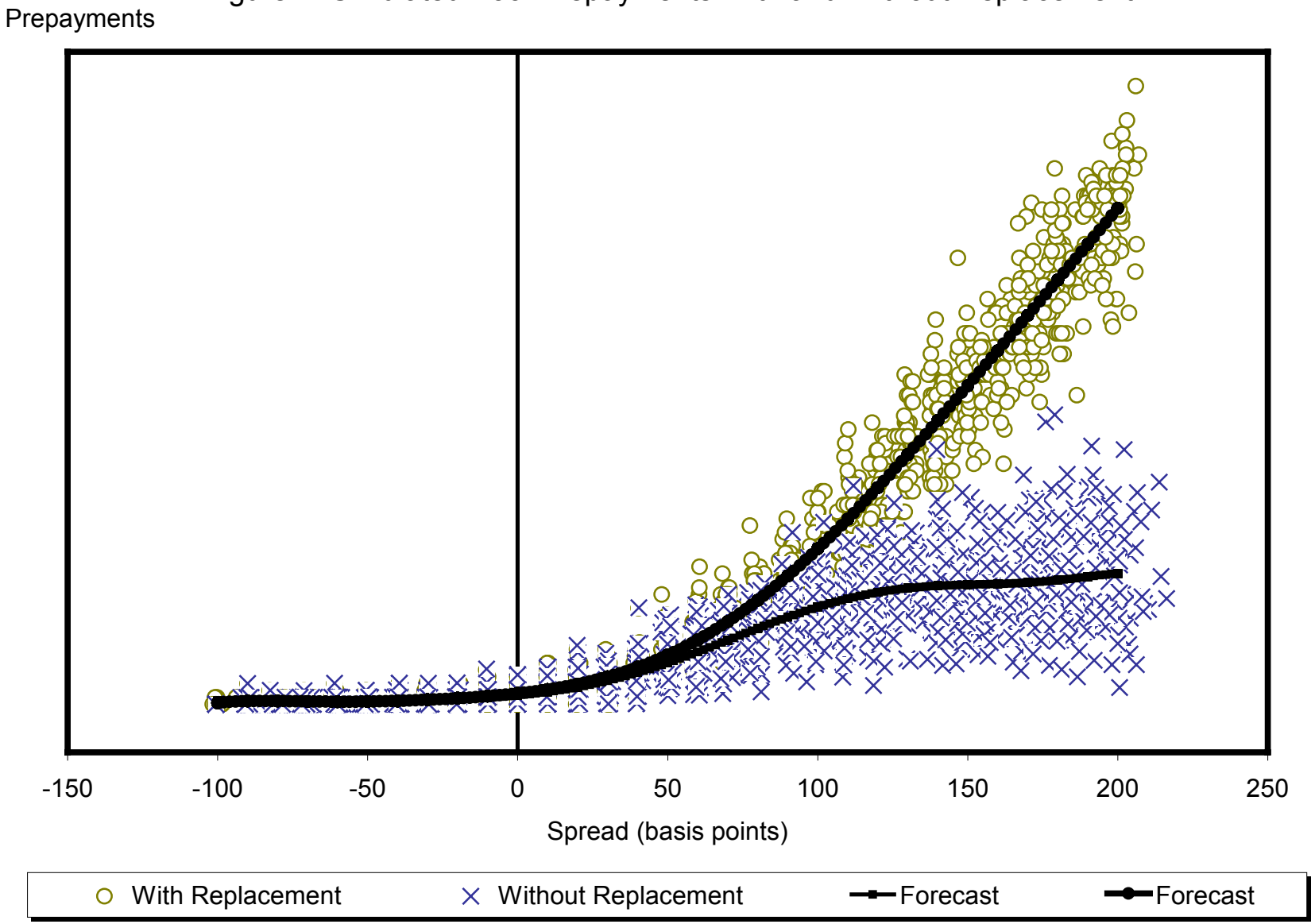
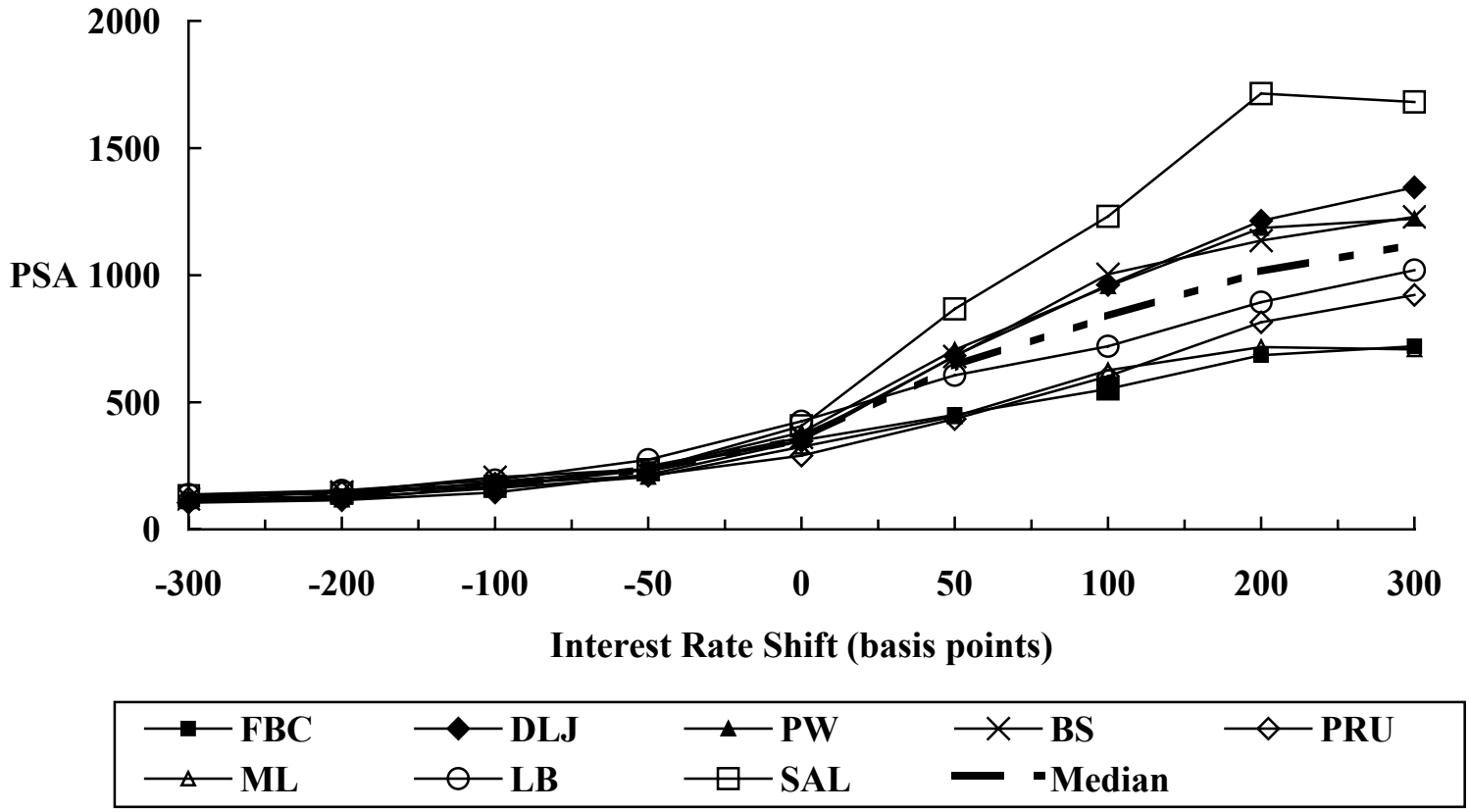


Figure 3. Dealer Prepayment Forecasts for FNMA 8s,(as of January 1998)



Source: Bloomberg Financial.
Note: FBC = First Boston Corporation, DLJ = Donalson Lufkin Jenrette, PW = Paine Webber, BS = Bear Stearns, PRU = Prudential, ML = Merrill Lynch, LB = Lehman Brothers, SAL = Salomon.