Firms and Flexibility

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Abstract

We study the effect of labor market rigidities and frictions on firm-size distributions and dynamics. We introduce a joint model of endogenous entrepreneurship, labor market frictions, and firm size dynamics with many types of rigidities: firing costs, search frictions with vacancy costs, unemployment benefits, entry costs, and a tax wedge between wages and labor costs. We use our model to analyze the role of each rigidity in explaining firm-size differentials between the U.S. and France. In particular, we find that our model with all rigidities and frictions, except hiring costs and search frictions, goes a long way in accounting for firm-size differentials between the U.S. and France. The addition of search frictions with vacancy costs generates implausibly large differentials in firm-size distributions.

Keywords: Labor markets, productivity, real rigidities, search theory

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1 Introduction

Differences in labor market outcomes between the U.S. and Europe have been well-documented.[1] One popular explanation of these observed differences has been the lack of *flexibility* in European countries.[2] Labor market outcomes, however, are tightly connected to firm size dynamics[3] through firm entry and exit, and hiring and firing decisions. Moreover, the allocation of workers across firms, reflected by these dynamics, is an important determinant of average labor productivity. This has an important implication. Any theory that explains differences in labor market outcomes between the U.S. and Europe through a lack of flexibility should also be able to account for differences in firm-size distributions, dynamics, and average labor productivity.

While differences between the U.S. and Europe in labor market outcomes have been widely studied, differences in firm-size distributions and dynamics have not been extensively documented. To set the stage, we present evidence from Bartelsman et. al. (2003, 2004)[4] on the distributions of firms and workers over five firm-size bins for the U.S. and France in Figure 1.

As Figure 1 shows, over 80% of firms employ 20 workers or less and about half a percent employ more than 500 workers, both in France and the U.S. At 23.2 workers per firm, the average firm size in the U.S. is smaller than that in France, which is 27.4. One notable difference between the U.S. and France is the higher concentration of workers in large firms (500 employees or more) in the U.S., reflected in the worker distribution in the left panel of Figure 1. This reflects a *squeeze* in the firm-size distribution in France relative to the U.S.; midsize firms make up a larger fraction of the firm population and employ a higher fraction of workers in France than in the U.S.

Entry and exit rates of firms are higher in France. The annual firm entry rates in the U.S. and France are 10.4% and 15.9%, respectively, while the exit rates are 9.1% and 11.6%. Average labor

[1] See, for example, Blanchard and Summers (1986), Blanchard and Wolfers (2000), Layard, Nickell, and Jackman (1991), Ljungqvist and Sargent (1998), Machin and Manning (1999), and Freeman (2007).

[2] Nickell (1997) and Botero et. al. (2004) find that the U.S. has less stringent labor market regulations than do European countries. Blanchard and Portugal (2001) emphasize the effect of employment protection. Djankov, McLiesh, and Shleifer (2006), Pissarides (2003) and Djankov et. al. (2002) argue that start-up costs and entry barriers are higher in Europe than in the US.

[3] Throughout, we define firm size as number of employees and firms as production units, i.e. establishments.

[4] We use this dataset primarily because it is based on a harmonized firm classification across countries. The data are taken from business registers and compiled to conform to a firm being "an organisational unit producing goods or services which benefits from a certain degree of autonomy in decision-making, especially for the allocation of its current resources" (Eurostat, 1995). The data presented are averages for 1990-1995.

productivity in France is 101% of that in the U.S.

To reconcile these observations with documented differences in labor market frictions and rigidities between the U.S. and France, we introduce a model that explicitly combines a theory of labor market frictions with a theory of the firm-size distribution. Our model consists of four main theoretical components.

The first component, based on Lucas (1978), is that each firm is managed by an entrepreneur. Whether or not people become an entrepreneur in Lucas (1978) depends on their innate entrepreneurial ability. In contrast, members of the labor force in our model are ex-ante identical. People who do not work develop one business idea in every period. Depending on the quality of their idea, they decide whether to start a business or look for a job instead. Hence, just like in Lucas (1978), the marginal entrepreneur that starts a business equates the expected value of starting a business to the expected value of her labor market opportunities.

The second component, similar to Hopenhayn and Rogerson (1993), is firm-size dynamics under labor adjustment costs. In addition to the firing costs considered in Hopenhayn and Rogerson, we also consider hiring costs, as in Pissarides (2000). The latter are determined by the cost of posting vacancies. In Hopenhayn and Rogerson, low productivity firms exit because they face a fixed operating cost; however, firms in our economy do not incur such a fixed cost. What drives firms to exit in our model is that their managers will close shop if their outside labor market opportunities exceed the expected value of continuing their business.

The third component is labor market search frictions, in the spirit of Mortensen and Pissarides (1994). That is, not all workers who look for a job will necessarily get a job offer. Moreover, not all vacancies that are posted by firms will necessarily be filled. The probability of an unemployed person finding a job depends on the aggregate ratio of number of vacancies to unemployed people.

Fourth, adjustment costs to labor, as well as search frictions, imply that each job match embodies a surplus for both the firm and the worker. This leaves the potential for bargaining between firms and workers. In this paper, we follow Cooper, Haltiwanger, and Willis (2007) and consider the case in which workers have no bargaining power, and all the surplus flows to the entrepreneurs.

The combination of these four components implies that firm-size dynamics and labor market conditions are closely intertwined in three important ways: (*i*) labor market conditions determine the search cost of hiring a worker and thus the firm-size decision, (*ii*) labor market conditions

determine the outside value for entrepreneurs and thus affect their entry and exit decisions,[5] and (*iii*) labor market conditions affect wages.

We use our model to consider the joint effects of hiring and firing costs, firm entry costs, unemployment benefits, a tax wedge between wages and labor costs, and labor matching frictions on firm-size dynamics and labor markets. We do so in the following way. First, we calibrate the preference and technology parameters in a version of the model without rigidities to match the main summary statistics of U.S. firm-size dynamics. This is our *flexible benchmark* model. Next, we add labor market rigidities to the model. We calibrate these rigidities to a magnitude that is consistent with those documented for France. This is our *rigid comparison* model. We then compare the flexible and rigid equilibria and identify the joint effect of labor market rigidities on both labor market outcomes and firm-size dynamics. We also consider the effect of individual ridigities to determine which ones contribute most to the joint effect. Because we compare equilibria under different rigidities with a flexible benchmark, our results can be interpreted as a numerical comparative statics exercise, in which we keep preferences and technology fixed and only change the degree of rigidities.

Our model implies two main effects of rigidities on the firm-size distribution: a *productivity selection effect* and a *reallocation effect*. The productivity selection effect is caused by the impact of the rigidities on entrepreneurs' entry and exit decisions. The reallocation effect reflects how the rigidities impede on the firms' hiring and firing decisions, reducing the reallocation of labor across firms.

We find that the rigid comparison model, which includes all the rigidities, has an unrealistically high reallocation effect. This is mainly due to the high vacancy cost that is needed to match the job-finding rate in France. This high vacancy cost makes productive firms so reluctant to hire and unproductive firms so reluctant to exit that, in equilibrium, there is an unrealistically high fraction of unproductive mid-size firms and large firms do not grow as big as observed in France. Without search frictions, our model is quite successful in matching the main differentials between the U.S. and France: higher firm entry and exit in France, *squeezed* firm-size and worker distributions, and similar average labor productivity levels.

We find that our model with all rigidities and frictions, except hiring costs and search frictions,

[5] Because non-working members of the labor force get the opportunity to start a new business, the labor market conditions in our model also reflect the option value of becoming an entrepreneur.

goes a long way in accounting for firm-size differentials between the U.S. and France. In particular, firing costs, which we calibrate relatively low compared to other studies, seem to go a long way in explaining these differentials. We conclude that labor market search frictions are not necessary to account for firm-size differentials between the U.S. and France. One interpretation of this result is that a large part of the unemployment differential between the U.S. and France is not due to labor market frictions, but instead reflects voluntary unemployment in response to different policy measures.

2 Model

In our model, members of the labor force can either be an employee at a firm, run a firm themselves, or not be employed. What is different from other models of labor market search, based on Mortensen and Pissarides (1994), is that each firm here can have more than one employee and is run by a member of the labor force. In this sense, our model is similar to Lucas (1978). Lucas' model is static and managers self-select based on their entrepreneurial abilities. Our model is dynamic and we assume that all workers are ex-ante identical. Instead of assuming that workers have a particular innate entrepreneurial ability, we assume that in each period all members of the labor force that are not employed, who we call (tongue-in-cheek) couch potatoes, will get a business idea. Given this business idea, they then make the occupational choice either to start a firm or to enter the labor market and look for a job. This setup means that the outside opportunity for entrepreneurs is to close their business and become a couch potato again. Hence, our model has both endogenous entry and exit of firms based on the labor market opportunities of the entrepreneurs that run the firms.

Each time period in our model can be divided into three phases. In the first phase, nature deals: all firms get hit by a productivity shock and all couch potatoes get a business idea. In the second phase, the couch potatoes make a career choice: they either decide to become an entrepreneur and start a new business (with zero employees) or they look for a job in the last part of the period. In the final phase, the labor market clears: some of the unemployed find a job, others do not. Firms change their size and some of them exit. Finally, some workers get laid off, while the rest remains employed with the firm they were with before. Because of the productivity shock firms get hit with, firms are not likely to be of the same productivity level and size at the end of the period as

at the beginning of the period.

The reason that we split up each period in three phases is that this allows us to separately focus on the career choice of couch potatoes. It is exactly this endogenous occupational choice that enables us to combine the labor market search model with a model of endogenous entrepreneurship and firm-size distributions in one encompassing framework. Moreover, it also facilitates our discussion of the types of labor market frictions that we consider in our model.

2.1 Members of the labor force

We start by defining the state space, both at the beginning as well as in the middle of the period. We abstract from time subscripts, since we focus on steady state outcomes in which the relevant variables are constant over time.

At the beginning of the period, each person in the labor force is either a couch potato, which we denote by CP, an entrepreneur, denoted by E, operating a firm of size n at productivity level z, or a worker, denoted by W, employed at a firm of size n and operated at productivity level z. We denote the expected present discounted values of future income associated with these states by V^{CP}, $V^{E}(z,n)$, and $V^{W}(z,n)$, respectively. The fraction of the labor force that is in each of these states at the beginning of the period is given by Φ^{CP}, $\Phi^{E}(z,n)$, and $\Phi^{W}(z,n)$.

In the middle of the period, at the end of the second phase, each person is either looking for a job, which we will denote as U, running a firm of size n at the new productivity level z', denoted by $\widetilde{E}$, or working at a firm of size n at the new productivity level z', which we denote by $\widetilde{W}$. The expected present discounted values of future income associated with these states are V^{U}, $V^{\widetilde{E}}(z',n)$, and $V^{\widetilde{W}}(z',n)$, respectively. The associated distribution of the labor force over these states is given by Φ^{U}, $\Phi^{\widetilde{E}}(z',n)$, and $\Phi^{\widetilde{W}}(z',n)$. Figure 2 illustrates the flows of members of the labor force, the choices they make, and their associated states for a representative period.

The labor market outcome in the third phase consists of the change in the size of the firms from n to n', as well as the exit of firms from the market. Labor market interactions are subject to search frictions. Just like in Mortensen and Pissarides (1994), the severity of these frictions depends on a matching technology that relates the probability of workers and employers finding a suitable counterpart in the labor market to the ratio of vacancies to unemployed members of the labor force. Following Mortensen and Pissarides (1994), we denote this ratio by θ. In addition to the labor market tightness ratio, θ, the distribution of job offers, which we denote by $F^{O}(z',n')$, is

also endogenously determined.

We describe our model in two steps. In the first step we consider how the decisions of the individuals and the associated value functions depend on the overall state of the economy, i.e. the distribution of the labor force over the different states, the job offer distribution, and the degree of labor market tightness. In the second step we consider how the overall state of the economy is the aggregate outcome of the decisions taken by the individuals.

2.2 Decisions of individuals

It is most transparent to consider the decisions for the last phase of the period separately from the first two phases.

<u>Phases I and II: Nature deals and career decision</u>

Entrepreneurs and workers

Both entrepreneurs and workers do not make any decision in the first two phases of the period. The only thing that happens to them is that the firms that they manage or at which they are employed are hit by a productivity shock that changes their productivity level.

The productivity shocks are independent across firms and follow a first order Markov process where the distribution of the new productivity level, z', conditional on the current productivity level z, is given by $Q\left(z'\,|z\right)$.

If the productivity shock that hits an entrepreneur that runs a firm at productivity level z with n employees moves the productivity level to z', then the expected continuation value at the beginning of the second phase of the period is $V^{\widetilde{E}}\left(z',n\right)$. Consequently, the value of being an entrepreneur that runs a firm of size n at productivity level, z at the beginning of a period, i.e. $V^{E}\left(z,n\right)$, equals

$$V^{E}\left(z,n\right)=\int_{z'\in Z}V^{\widetilde{E}}\left(z',n\right)dQ\left(z'\,|z\right). \tag{1}$$

Similarly, if a worker is employed at a firm of productivity z and size n and this firm's productivity level shifts to z', then the expected continuation value at the beginning of the second phase of the period for this worker is $V^{\widetilde{W}}\left(z',n\right)$. The resulting value of being employed, at the beginning of a period, at a firm of size n and operated at productivity level z satisfies

$$V^{W}\left(z,n\right)=\int_{z'\in Z}V^{\widetilde{W}}\left(z',n\right)dQ\left(z'\,|z\right). \tag{2}$$

Couch potatoes and business plans

At the beginning of a period, a couch potato knows that she will get one idea for a business plan that period. That idea would allow that person to start a business at a particular productivity level z'. Conditional on that business idea, the occupational choice of the couch potato consists of either choosing to start a business or not.

Starting a new business involves a start up cost equal to η. After paying this entry cost, the newly started business is operated at productivity level, z', but has no employees yet, i.e. $n = 0$. The workers are only hired in the third phase of the period. Hence, after paying the start-up cost, the new entrepreneur faces an expected continuation value equal to $V^{\widetilde{E}}(z', 0)$.

If a couch potato decides to forego the opportunity to become an entrepreneur, she or he will look for a job in the last part of the period. The expected continuation value of this job search is V^U.

Given the quality of the business idea, z', the couch potato will thus decide to become an entrepreneur when

$$V^{\widetilde{E}}\left(z', 0\right) - \eta \geq V^U \tag{3}$$

and will look for a job otherwise.

Let the business ideas be drawn from the distribution $F(z')$, then the expected value of being a couch potato equals

$$V^{CP} = \int_{z' \in Z} \max\left\{V^{\widetilde{E}}\left(z', 0\right) - \eta, V^U\right\} dF\left(z'\right). \tag{4}$$

The resulting optimal choice is that couch potatoes only decide to become an entrepreneur when the quality of their business idea is equal to or exceeds a cut-off level. We denote this cut-off level by $\underline{z}$.

<u>Phase III: Labor market outcome</u>

We first consider the optimal decisions of the unemployed, firms, and workers conditional on the wage setting mechanism. We then present the bargaining set up and the resulting wage schedule.

Unemployed individuals

The way we determine the value of being unemployed, V^U, is as follows. If the unemployed individual does not get an offer or does not accept an offer, at the beginning of the next period she or he will get inspired by another business plan.

For each unemployed person, there are two possible outcomes of the labor market equilibrium. The first possibility is that a worker finds a job at a firm of type (z', n'), gets paid the wage $w(z', n')$, and ends up, at the beginning of the next period, as a worker employed at a firm of the same type (z', n'). This possibility results in the value

$$w\left(z', n'\right) + \frac{1}{1+r} V^W\left(z', n'\right). \tag{5}$$

The second possible outcome is that a worker is not hired, receives unemployment benefits b,[6] and ends up, at the beginning of next period, as a couch potato. That is, the reservation value of someone looking for a job is determined by the value of being a couch potato rather than of being unemployed in the next period. This outcome results in the value

$$b + \frac{1}{1+r} V^{CP} \tag{6}$$

when a worker gets an offer from a firm of type (z', n'), then he or she decides to accept the offer if (5) exceeds (6). Otherwise, the offer is declined. The wage setting scheme that we consider is such that for any firm in business, (5) exceeds (6) and thus all job offers made will be accepted.

The search frictions on the supply side of the labor market are thus not that unemployed individuals get offers that they do not accept but rather that not all unemployed find a firm that makes them an offer.

Just like in Mortensen and Pissarides (1994), we assume that the probability of getting a job offer is $\theta q(\theta)$, where $q(\theta)$ is the matching function. Since, in equilibrium, all job offers are accepted, this is known as the job finding rate. Let $F^O(z, n)$ be the distribution of job offers over firm types. This distribution is endogenously determined in equilibrium. The probability of getting an offer of type (z', n') is then given by $\theta q(\theta) F^O(z', n')$.

[6] We interpret b here as unemployment benefits. Alternatively, one can think of it as the value of leisure/home production.

Given these probabilities and decision rules, the value of being unemployed can be written as

$$(7) \qquad V^U = \theta q(\theta) \int_{Z\times N} \left(w(z',n') + \frac{1}{1+r} V^W(z',n') \right) dF^O(z',n') + (1-\theta q(\theta)) \left(b + \frac{1}{1+r} V^{CP} \right).$$

Entrepreneurs

The optimal firm-size and exit decisions of the firm are as follows. If the entrepreneur decides to continue with the firm, the optimal new firm size, n', satisfies

$$(8) \qquad \nu^*(z',n) = \arg\max_{n'} \left\{ \pi(z',n') - g(n,n';\theta) + \frac{1}{1+r} V^E(z',n') \right\}.$$

Here $\pi(z',n')$ represents the flow profits obtained from employing n' workers in the period at productivity level z'. The function $g(n,n';\theta)$ is the adjustment cost function of labor. It depends on θ to allow for hiring and firing costs to depend on labor market tightness.

Given that all output is homogenous and sold at the, normalized, price of one, we write flow profits as

$$(9) \qquad \pi(z',n') = f(z',n') - (1+\Delta) w(z',n') n'$$

where $f(.)$ denotes the value added generated by the firm's labor inputs and Δ denotes the tax wedge between employment costs and wages received by the workers.

Entrepreneurs that decide to close shop have to pay the adjustment cost of labor to lay off their current employees.[7] They then receive benefits b and become a couch potato in the next period. Entrepreneurs will exit, whenever they would prefer to leave rather than to keep workers employed. This is the case whenever

$$(10) \qquad b - g(n,0;\theta) + \frac{1}{1+r} V^{CP} \geq \max_{n'} \left\{ \pi(z',n') - g(n,n';\theta) + \frac{1}{1+r} V^E(z',n') \right\}.$$

Let $X(z',n)$ be the indicator function that equals 1 if this condition holds and 0 if not. Then, a firm of size n with a new productivity level z' will choose a new size $n' = \nu'(z',n)$, where

$$(11) \qquad \nu'(z',n) = \begin{cases} \nu^*(z',n) & \text{if} \quad X(z',n) = 0 \\ 0 & \text{if} \quad X(z',n) = 1 \end{cases}.$$

[7] As described in Samaniego (2006), the assumption that firms have to pay the severance pay when they shut down the firm means that only small firms will exit. If this assumption is dropped, large firms might also want to exit to avoid the obligation to incur severance costs.

Given this decision rule, the value of being an entrepreneur that runs a business at productivity level z' and size n at the onset of the third phase of the period equals the maximum of the value of exit and the value of continuing at the new optimal firm size. Mathematically, this can be written as

$$(12) \qquad V^{\widetilde{E}}\left(z',n\right) = \max\left\{b - g\left(n,0;\theta\right) + \frac{1}{1+r}V^{CP}, \max_{n'}\left\{\pi\left(z',n';w\left(z',n'\right)\right) - g\left(n,n';\theta\right) + \frac{1}{1+r}V^{E}\left(z',n'\right)\right\}\right\}.$$

Workers

Finally, as we explain below, no matter what job workers have, they will always get a wage above their reservation level. As a consequence, the situation for workers is the reverse of that for the unemployed. That is, instead of facing the probability of being hired, workers face the potential probability of being laid off and getting paid the benefit, b. This might happen in case the firm at which the worker is employed shrinks. If a firm downsizes, then the workers that are laid off are randomly drawn from its workforce.

We denote the probability of being laid off for a worker in a firm of type (z',n) in the middle of the period by $P^F\left(z',n\right)$. This probability can be written as

$$(13) \qquad P^F\left(z',n\right) = \max\left\{0, \frac{n-\nu'\left(z',n\right)}{n}\right\}.$$

Given this probability, we can write the value of being a worker in a firm of type (z',n) in the last phase of the period as the probability of being fired times the value obtained from receiving unemployment benefits and continuing as a couch potato and the probability of not being laid off times the wage paid plus the discounted continuation value of remaining a worker with the same firm.

Mathematically, this equals

$$(14) \qquad V^{\widetilde{W}}\left(z',n\right) = P^F\left(z',n\right)\left[b + \frac{1}{1+r}V^{CP}\right] + \left(1 - P^F\left(z',n\right)\right)\left[w\left(z',\nu'\left(z',n\right)\right) + \frac{1}{1+r}V^{W}\left(z,\nu'\left(z',n\right)\right)\right].$$

In the above value function, as well as those of the unemployed and the firm, we have taken the production function, $f\left(z,n\right)$, the matching technology, $q\left(\theta\right)$, the adjustment cost function, $g\left(n,n';\theta\right)$, as well as the wage schedule, $w\left(z,n\right)$, as given. We fill in these gaps in the next two subsections.

2.3 Technology and adjustment costs

Production Technology

We assume that the production function is of the form

$$f(z,n) = zn^a, \text{ where } 0 < a < 1. \tag{15}$$

This is the same production function as used by Hopenhayn and Rogerson (1993). As we show in Appendix A, this production function can also be interpreted as the value added of labor that results from a constant returns to scale Cobb-Douglas production function in capital and labor subject to economies of scope, as in Lucas (1978), and with flexible capital inputs. We use this interpretation for our calibration.

Labor adjustment costs

The labor adjustment cost function that we assume is a hybrid of recruitment costs in case the firm expands, as in Pissarides (2000, Chapter 3), and separation costs in case the firm downsizes, as in Hopenhayn and Rogerson (1993). We explain each of these two components separately and then combine them to define the labor adjustment cost function $g(n,n';\theta)$.

Increasing the size of the firm from n to n', given a degree of labor market tightness, θ, involves the posting of a given number of vacancies. Let $q(\theta)$ be the probability that a suitable candidate can be found to fill a particular vacancy. Then, by the law of large numbers the firm will have to post $(n'-n)/q(\theta)$ vacancies to increase its size from n to $n' > n$.

Let $\upsilon(n,n';\theta)$ be the number of vacancies a firm needs to post to change its workforce from n to n', then

$$\upsilon(n,n';\theta) = \begin{cases} 0 & n' \leq n \\ (n'-n)/q(\theta) & n' > n \end{cases}. \tag{16}$$

Following Pissarides (2000), we assume that there is a per period posting cost of a vacancy. This cost equals c.

Following Hopenhayn and Rogerson (1993), we assume that when the firm decides to downsize, i.e. when $n' < n$, employment protection of its employees requires the firm to pay a separation penalty equal to τ per worker that is laid off. This separation penalty can be interpreted as capturing both a penalty to be paid to the government as well as the cost, in terms of output, incurred by the firm to complete all the procedures involved in the dismissal process.

Combining the labor adjustment costs for the posting of vacancies in case the firm expands and the lay off costs in case the firm downsizes gives us the labor adjustment cost function

$$g\left(n,n';\theta\right)=\begin{cases}\tau\left(n-n'\right) & n'\leq n\\ \left(c/q\left(\theta\right)\right)\left(n'-n\right) & n'>n\end{cases}. \tag{17}$$

It is depicted in Figure 3. This particular adjustment cost function has the property that the marginal adjustment cost of labor only depends on whether the firm is expanding, in which case it is $(c/q(\theta))$, or downsizing, when it is τ, and not on the size of the firm. Cooper, Haltiwanger, and Willis (2007) provide empirical evidence in favor of such linear adjustment costs.

For the firms that stay in business, the optimal firm size choice is given by (8), which has the associated first order necessary condition that the marginal adjustment cost of labor equals the marginal value of labor, mvl. That is,

$$\frac{\partial}{\partial n'}g\left(n,n';\theta\right)=\frac{\partial}{\partial n'}\pi\left(z',n'\right)+\frac{1}{1+r}\frac{\partial}{\partial n'}V^{E}\left(z',n'\right)=mvl(z',n'), \tag{18}$$

where the marginal value of labor consists of the marginal flow profits plus the discounted marginal continuation value of labor.

Because the left hand size of the above equation only depends on whether the firm expands or downsizes and not on the size of the firm and the right hand side only depends on n' and z', the resulting marginal condition can be depicted as in Figure 4. Of the firms that got a new productivity level z', firms that started at $n<\underline{n}'(z')$ will increase their size to $\underline{n}'(z')$, firms that started at $n\in(\underline{n}'(z'),\overline{n}'(z'))$ will stay of the same size, while firms that started at $n>\underline{n}'(z')$ will downsize to $\overline{n}'(z')$. Hence, at the end of the period all firms of productivity level z' will be of size $n'\in[\underline{n}'(z'),\overline{n}'(z')]$. Bentolila and Bertola (1990) refer to the interval $[\underline{n}'(z'),\overline{n}'(z')]$ the *corridor*.

Matching technology

Similar to Mortensen and Pissarides (1994), the matching technology can be represented by the following isoelastic matching function

$$q\left(\theta\right)=\psi\theta^{-\xi},\text{ where }\psi>0\text{ and }0<\xi<1 \tag{19}$$

such that the probability of getting a job offer is increasing in the degree of labor market tightness, while the probability of find a suitable job candidate for an open vacancy is decreasing in θ.

Wage determination

In this paper, we limit ourselves to the case in which the entrepreneur has all the bargaining power. This is similar to the take-it or leave-it wage offer setup in Cooper, Haltiwanger, and Willis (2007).[8] The firm always makes the wage offer such that the worker is indifferent between staying at the firm or leaving the firm. If the worker stays at the firm she receives the wage $w(z', n')$ and the continuation value $V^W(z', n')$. If the worker leaves she gets the benefit b and the continuation value V^{CP}. As a result, the wage $w(z', n')$ is such that the surplus for the worker of its employment relationship with the firm equals zero. That is,

$$S^W(z', n') = \left[w(z', n') + \frac{1}{1+r} V^W(z', n')\right] - \left[b + \frac{1}{1+r} V^{CP}\right] = 0 \tag{20}$$

Because this happens in every period and at all types of firms, we find that for all z, n, and n'

$$V^W(z, n) = V^{\widetilde{W}}(z', n') = V^U = \left[b + \frac{1}{1+r} V^{CP}\right] \tag{21}$$

and that all workers get paid the same reservation wage

$$w(z', n') = \frac{r}{1+r}\left[b + \frac{1}{1+r} V^{CP}\right] = w. \tag{22}$$

3 State of the economy

The individual decision rules and the wage bargaining schedule above imply a particular path of the state of the economy. In the following we derive this path as a function of the individual decisions under the assumption that the economy is in steady state.

Labor market equilibrium

In equilibrium, the number of workers employed at firms of type (z, n) is the number of firms of that type, times the size of each firm. That is,

$$\Phi^W(z, n) = \Phi^E(z, n)\, n \text{ for all } (z, n). \tag{23}$$

[8] In a companion paper, we focus on the case where the wage level in each firm is determined through intra-firm bargaining with non-binding contracts, as in Stole and Zwiebel (1996a,b).

This concept of wage bargaining has been applied in a broad range of recent papers on labor market frictions and large firms, like Ebell and Haefke (2004, 2006), Rotemberg (2006), Acemoglu and Hawkins (2006), and Elsby and Michaels (2007). The case we focus on here is nested in that case in the sense that it is the case in which workers have no bargaining power.

Similarly, at the beginning of the third stage of the period,

$$\Phi^{\widetilde{W}}\left(z',n\right)=\Phi^{\widetilde{E}}\left(z',n\right)n \text{ for all } \left(z',n\right). \tag{24}$$

These two identities imply that the total amount of labor supplied to each firm is the total amount employed.

The equilibrium in our model is in large part determined by the search frictions. These frictions are a function of the ratio of vacancies to the number of unemployed individuals. The unemployed are made up of those who, at the beginning of the period, were couch potatoes and did not receive a business idea that was worth pursuing. That is, in equilibrium

$$\Phi^{U}=\Phi^{CP}F\left(\underline{z}\right). \tag{25}$$

The aggregate number of vacancies, υ, is obtained by integrating the number of vacancies that firms post. That is,

$$\upsilon=\int_{Z\times N}\Phi^{\widetilde{E}}\left(z',n\right)\upsilon\left(n,n';\theta\right)dz'dn \tag{26}$$

such that the ratio of vacancies to the number of unemployed workers equals

$$\theta=\upsilon/\Phi^{U} \tag{27}$$

which determines labor market tightness.

Note that in the take-it or leave-it offer setting that we consider all jobs pay the same wage and, thus, the job offer distribution, $F^{O}\left(z',n'\right)$, is no longer required to compute the value of being unemployed.

Stationary firm-size distribution

Since the number of firms is constant in steady state, the number of firms that enter equals the number of firms that exit at each point in time.

The number of firms that enter is the number of couch potatoes that draw a business idea that induces them to start a business, i.e. an idea of quality $z'\geq\underline{z}$. Hence, the number of entrants, denoted by μ_E, equals

$$\mu_E=\Phi^{CP}\left(1-F\left(\underline{z}\right)\right). \tag{28}$$

In order to consider the number of exiting firms, let us first define the types of firms that exit. Note that no firm enters and exits in the same period, because this involves incurring an entry cost without ever generating any revenue.

The above argument implies that all firms that exit in a given period are firms that existed in the previous period and got a bad productivity shock that made their manager to decide to exit. Let us define the set of such firm types as

$$F_x = \left\{ \left(z', n\right) \in Z \times N \left| X\left(z', n\right) = 1 \right. \right\}. \tag{29}$$

The number of exiting firms, which we denote by μ_X, is

$$\mu_X = \int_{F_x} \Phi^{\widetilde{E}}\left(z', n\right) d\left(z', n\right). \tag{30}$$

Hence, in steady state (28) equals (30).

Besides the number of firms being constant, the final steady state condition is that the firm-size dynamics imply a stationary firm-size distribution.

We can write the dynamics of firm sizes in the last phase of the period as

$$\Phi^{E}\left(z', n'\right) = \int_{F_n(z', n')} \Phi^{\widetilde{E}}\left(z', n\right) d\left(z', n\right). \tag{31}$$

where $F_n\left(z', n'\right)$ is the set of firms that will end up the period being of type $\left(z', n'\right)$. This set equals

$$F_n\left(z', n'\right) = \left\{ \left(z', n\right) \in Z \times N \left| X\left(z', n\right) = 0 \wedge \nu'\left(z', n\right) = n' \right. \right\}. \tag{32}$$

Moreover, the firms of type $\left(z', n\right)$ in the middle of the period either are entrants or they existed in the previous period. Entrants are of size $n = 0$ in the middle of the period. We can thus write the transitional dynamics in the first two phases of the period as

$$\Phi^{\widetilde{E}}\left(z', n\right) = \Phi^{CP} f\left(z'\right) I\left[n = 0\right] I\left[z' \geq \underline{z}\right] + \int_{Z} \Phi^{E}\left(z, n\right) dQ\left(z' \left| z\right.\right), \tag{33}$$

where $I\left[.\right]$ is an indicator function that equals one if the condition in the argument holds and zero otherwise.[9]

Because of (23) and (24), a stationary distribution of firms also implies a stationary distribution of workers.

Labor force resource constraint

Finally, we normalize the total size of the labor force to equal one, such that the labor market resource constraint is that the total mass of couch potatoes, workers, and entrepreneurs at the

[9] In a measure theoretic sense, this is actually a Dirac δ function.

beginning of the period equals one. The same is true for the mass of unemployed, workers, and entrepreneurs in the middle of the period. That is

$$
\begin{aligned}
(34) \qquad 1 &= \Phi^{CP} + \int_{Z\times N} \left\{ \Phi^{E}(z,n) + \Phi^{W}(z,n) \right\} d(z,n) \\
&= \Phi^{U} + \int_{Z\times N} \left\{ \Phi^{E}(z',n) + \Phi^{W}(z',n) \right\} d(z',n) .
\end{aligned}
$$

This resource constraint allows us to pin down the number of couch potatoes in equilibrium.

This completes our description of the equilibrium dynamics of the firm-size distribution. The solution method we follow is described in Appendix B.

4 Equilibrium

We compute the equilibrium of our model by using numerical methods. We present our numerical results in three parts.

In the first part, we focus on our benchmark case in which there are no rigidities and frictions. In that case, the equilibrium is such that all firms hire workers up to the point that their marginal product equals the wage. We use this benchmark case for two purposes. Most importantly, we use it to show what margins will be affected by the introduction of rigidities. In addition, we use it to calibrate the technology parameters in our model such that the model matches the main properties of the U.S. firm-size distribution. In this sense, we use the U.S. as our *flexible* benchmark.

The following observations suggest that it is reasonable to consider the U.S. as a *flexible* benchmark. First, as described in Hobijn and Şahin (2007), in the U.S. about 73% of the unemployed find a job within three months after becoming unemployed. Since these unemployed also contain the structurally unemployed to whom our model is not applicable, the actual quarterly job finding rate for the frictionally unemployed in the U.S. is probably close to 80%. Second, Bertola, Boeri, and Cazes (1999) report that severance pay in the U.S. is very small as compared to other countries in the OECD. Finally, Djankov et. al. (2002) as well as Fonseca, Garcia-Lopez, Pissarides (2001) report that the number of procedures and time required to start up a business is much lower in the U.S. than in other OECD countries. These observations suggest that the level of rigidities in the U.S. is considerably lower than the level of rigidities in the OECD countries. Thus, for our comparative analysis of U.S. versus France, it is reasonable to assume that the U.S. economy can be modelled as a *flexible* benchmark.

In the second part, we add the rigidities to our model. We calibrate them to be of approximately the same magnitude as documented for France. This is our *rigid* comparison model. We illustrate how rigidities affect the equilibrium outcome.

In the third part, we isolate the effect of each of the individual rigidities on the equilibrium and consider a third comparison model that includes all but the search frictions, which we call our *all-but-search* case.

Since we compare equilibrium under different rigidities with the flexible benchmark, our results can be interpreted as a numerical comparative statics exercise, in which we keep preferences and technology fixed and only change the degree of rigidities. This is consistent with a view that preferences and technologies are similar in France and the U.S. and all observed differences are due to rigidities.

4.1 Flexible benchmark

The benchmark model that we consider is one in which there are no rigidities that directly affect the decisions of firms. This means that we consider the case in which there are: (i) no vacancy costs, $c = 0$; (ii) no firing costs, $\tau = 0$; (iii) no entry costs, $\eta = 0$, and (iv) no tax-wedge $\Delta = 0$.

Calibration of preference and technology parameters

Given these restrictions, the benchmark model contains the following parameters: (i) the real interest rate r, that determines the discount factor $\frac{1}{1+r}$; (ii) the benefit or home production level, b; (iii) the production function parameter, a; and (iv) the parameterization of the stochastic process that drives productivity shocks, i.e. the parameters that determine $Q(z'|z)$ and $F(z)$. We calibrate our model for a quarterly frequency of observation.

The real interest rate, r, is set to match a 5.5% annualized real rate of interest. We set the benefit level, b, to equal 0.70.[10]

We choose $a = 0.92$, which, according to the derivation in Appendix A, is consistent with a 68% labor share and with 0.97 returns to scale. The 68% labor share matches the average labor share of the U.S. non-farm business sector over the postwar period, while the 0.97 returns to scale is within the range of estimates presented in Bailey et. al. (1992) for the U.S. manufacturing sector.

[10] This normalization can be done without any loss of generality, since it can be offset by a change of the mean of the distribution of the productivity levels, z.

We use the same functional form for the transitional distribution of the productivity shocks, $Q\left(z'|\,z\right)$, as Hopenhayn and Rogerson (1993). That is, we assume that the logarithm of the firm specific level of productivity follows an AR(1)-process with mean $\ln\overline{z}$,

$$\left(\ln z' - \ln\overline{z}\right) = \rho\left(\ln z - \ln\overline{z}\right) + \varepsilon, \text{ where } \varepsilon \sim N\left(0,\sigma\right) \text{ and } -1 < \rho < 1. \tag{35}$$

In addition, the business ideas of the couch potatoes are drawn from the stationary distribution of this process. Hence

$$F\left(z'\right) = \Phi\left(\sqrt{1-\rho^2}\left(\frac{\ln z' - \ln\overline{z}}{\sigma}\right)\right) \tag{36}$$

where $\Phi\left(.\right)$ is the standard normal cumulative density function. This choice of functional form implies that the stochastic process that drives productivity shocks is determined by the mean, $\ln\overline{z}$, the standard deviation, σ, and the persistence parameter, ρ.

We use the benchmark model to pin down the parameterization of the process for the productivity shocks. The reason we use the flexible benchmark model is that it is computationally feasible to solve the model for many different combinations of the technology parameters. The addition of search frictions increases the computational burden to such an extent that searching over this parameter space is infeasible.

Conditional on our choices of r, b, and a, we choose these three productivity shock parameters such that our benchmark model matches the main properties of the U.S. firm-size distribution.[11] Average firm size, the firm-size distribution, as well as the overall entry/exit rates get high weights in our calibration.[12] The distribution of workers over firm sizes gets a medium weight, while the size distribution of entrants and exiters as well as the size dependent entry and exit rates get low weight. In all, we choose the three parameters that determine the productivity shock process to approximate 43 summary statistics of the U.S. firm-size distribution and U.S. firm-size dynamics.[13] The calibrated values of the productivity parameters are $\overline{z} = 1.686$, $\rho = 0.966$, and $\sigma = 0.054$. For our numerical solution we approximate the distribution of z on a 50 point grid, equally distributed in logs over the interval $[0.5, 5.0]$, such that z increases in steps of 4.7% on it.

[11] In particular, we minimize a weighted sum of squared deviations of the summary statistics of the firm size distribution from the same summary statistics in U.S. data. The data we use are averages over the 1990-1995 period.

[12] Because we consider the model in steady state without growth, the entry and exit rates implied by our model are the same.

[13] This calibration method of the stochastic productivity process is similar to the one used by Veracierto (2008).

Before we consider the equilibrium outcome of the benchmark model for these parameters, we first describe the main properties of the equilibrium.

Properties of the flexible equilibrium

Because there are no adjustment costs to labor, firms can costlessly hire workers. The result is that firms choose their labor demand such that the marginal worker generates no marginal value, i.e.

$$mvl\left(z,n'\right)=\frac{\partial}{\partial n'}\pi\left(z',n';w\right)+\frac{1}{1+r}\frac{\partial}{\partial n'}V^{E}\left(z',n'\right)=0. \tag{37}$$

This condition does not depend on whether a firm is expanding or downsizing. All firms of the same productivity level z' choose the same size.

In the absence of adjustment costs to labor inputs, future labor input decisions do not depend on the ones currently made. This means that the marginal value of an additional worker at the end of the period is zero, such that

$$\frac{\partial}{\partial n'}V^{E}\left(z',n'\right)=0. \tag{38}$$

Hence, the firm chooses the labor input that maximizes current flow profits

$$\frac{\partial}{\partial n'}\pi\left(z',n';w\right)=0 \tag{39}$$

which is the labor input level that equates the marginal product of labor to the wage rate; the efficient labor input choice. Hence, the allocation of labor across firms in this economy mimics that of a flexible competitive labor market.

Flexible equilibrium for calibrated parameter values

The equilibrium outcome for the particular calibrated parameter values is depicted in the $(\ln z', \ln n)$-space in Figure 5.[14] This space allows us to show which types of firms, in the middle of the period, will exit, downsize, or expand. It also allows us to depict the set of productivity levels at which firms enter.

In this case, all firms below $\underline{z}=1.54$ ($\ln \underline{z}=0.43$) exit and all firms above it remain in business. Moreover, all couch potatoes that get a business idea of quality $\underline{z}$ or better start a business. All firms that start the period at size n and draw a new productivity level, z', such that $n<n'\left(z'\right)$ expand to $n'\left(z'\right)$. Firms that start the period at size n and that get a negative productivity shock,

[14] We solved our model requiring a minimum firm size of $n=1$ for all firms.

such that $n > n'(z')$, downsize from n to $n'(z')$. The optimal labor demand function, $n'(z')$, is determined by the level of the equilibrium wage, which is $w = 1.67$.[15]

This is essentially a dynamic version of Lucas' (1978) model of firm-size distributions. Similar to Lucas (1978), all firms decide on their labor demand by equating the marginal product of labor to the real wage. The real wage is, in turn, determined by the outside option of the marginal entrepeneur. That is, entry and exit of firms in our model are determined by V^{CP} and the benefit level, b, because this is what determines the outside opportunities in the labor market in case an entrepreneur decides to change career and become a worker. The dynamic nature of our model implies that, contrary to Lucas (1978), firms face an option value that reflects the possibility of getting a positive productivity shock in the future.[16]

As can be seen from the 'benchmark' column of Table 1, at the beginning of each period, 94.3% of the members of the labor force choose to be workers and 3.9% choose to be entrepreneurs. The remaining 1.8% are couch potatoes. In each period, 7.4% of couch potatoes start a business, while the other 92.6% decide to look for a job. This means that the fraction of the labor force that looks for a job in each period is 1.65%.

Since vacancy posting is free, an infinite number of vacancies is posted in each period, i.e. $\theta = \infty$, such that all jobseekers find a job with certainty. Hence, the couch potatoes at the beginning of the next period consist of the entrepreneurs that decide to close their business in the current period as well as the workers affiliated with the firms that downsize or shut down.

Average labor productivity (ALP), which is defined as output divided by the number of entrepreneurs and employees equals 1.74. If entrepreneurs are not necessary for a firm to operate, then ALP under flexible labor inputs and optimal labor demand equals $w/a = 1.82$. Since one entrepreneur per firm is necessary for production in our model,

$$\lim_{z' \longrightarrow \infty} \frac{z' \left[n'(z')\right]^a}{n'(z') + 1} = \frac{w}{a}$$

and ALP limits towards 1.82 in large firms and is substantially lower in small firms due to entre-

[15] This suggests that, if one interprets b solely as unemployment benefits, the benefit replacement rate in the frictionless benchmark is about 41%. This is higher than the 27% replacement rate documented for the U.S. for 1991-1995 by Nickell and Nunziata (2001). However, a broader interpretation of b also includes some of the benefits of not working through the disutility of working and home production.

[16] In equilibrium, the marginal firms, that operate at level $\underline{z}$, make negative flow profits in anticipation of high potential future profits. This is due to the option value of remaining in business.

preneurial overhead. This means that the entrepreneurial overhead requirement reduces ALP by 4.1% as compared to the ALP that would have been realized at the same wage without overhead.[17]

To what extent our model replicates the facts on the U.S. firm-size dynamics, that we chose our parameters to match, can be seen by comparing the 'benchmark' and 'U.S.' columns of Tables 1 and 2. Every period (quarter) in our model 11.3% of the firms that existed four quarters ago have been replaced. Hence, the annual entry and exit rates in our model, which have to be equal at the steady state, are 11.3% as compared to U.S. entry and exit rates of 10.4% and 9.1%, respectively. The average firm size in our benchmark equilibrium is 23.9, while that in the U.S. is 23.2. The model also closely replicates average firm sizes conditional on the firm-size bins. Small firms, i.e. smaller than 20 employees, are somewhat smaller than in the data: 3.0 versus 4.7.[18] The firm-size distribution over size bins generated by the model is remarkably close to that in the data. As for the distribution of workers over firms of specific sizes, the model puts too much weight on mid-size firms. Where in the U.S. 18.2% of workers work in firms with less than twenty employees and 50.6% work in firms with more than five hundred employees, in our model these fractions are 11.2% and 47.0% respectively. Our model does not capture the size-dependent pattern of exits and entries very well. Entry and exit in our model is almost fully attributable to firms with less than twenty employees while in the U.S. entry and exit rates are more than 1.5%, even among large firms.[19]

4.2 Rigid comparison

Keeping the preference and technology parameters the same as in the flexible benchmark, we now add rigidities to our model. In particular, we consider (i) vacancy costs, $c > 0$; (ii) firing costs, $\tau > 0$; (iii) entry costs, $\eta > 0$; (iv) a tax wedge between labor costs and wages received, $\Delta > 0$, and (v) higher unemployment benefits, b. We calibrate these rigidities to match their magnitude in France relative to the U.S. and then compare the equilibrium outcome with rigidities to our results for the flexible benchmark.

[17] The entrepreneurial overhead effect here is similar to the one that amplifies the productivity effects of financial frictions in Amaral and Quintin (2007).

[18] We do not count the entrepreneur as an employee in our results. In most firms in the data, however, the entrepreneur/manager would be included in the headcount that determines establishment size.

[19] Some of these entries and exits of large firms might be spurious; many of them reflect mergers and acquisitions as well as linking problems in the underlying microdata sources.

Calibration of rigidities

We calibrate the firing cost such that τ equals about 4 months of wages. This is in line with the evidence on the sum of maximum notice and severance pay in France in 1993, reported in Bertola, Boeri, and Cazes (1999, Table 1). We choose the magnitude of the entry cost, η, to be approximately 6 weeks (half a quarter) of wages. Six weeks is, according to Fonseca, Lopez-Garcia, and Pissarides (2001), the average time it takes to set up a business in France. We increase b relative to the benchmark model to be consistent with a higher benefit replacement ratio in France. Similar to our benchmark calibration, the benefit replacement rate that we match is higher than in the data because b reflects more than solely unemployment benefits. In particular, we choose b to be 85% of the equilibrium wage.[20] Nickell and Nunziata (2001) report that the average tax wedge in France is 17% higher than in the U.S. To match the difference in average tax wedge for our calibration, we choose $\Delta = 0.17$ in our rigid comparison model.

The calibration of the vacancy cost parameter is more cumbersome. This is because the unit of measurement of a vacancy is not well defined in the data. According to Hobijn and Şahin (2007), the quarterly job finding probability in France is 24%. Consistent with this observation and the arbitrary units of measurement of vacancies, we choose the constant in the matching technology function, ψ, to equal 0.24. We then choose the vacancy cost parameter, c, to match an equilibrium ratio of vacancies to unemployed workers, θ, to equal 1. Since we calibrate the rigid comparison model to match $\theta = 1$, its equilibrium does not depend on the elasticity of the matching technology (19) with respect to θ, i.e. ξ.[21]

These moment conditions allow us to pin down the five parameters related to rigidities in our model. The calibrated parameter values are $\tau = 1.30$, $\eta = 0.65$, $b = 0.90$, $c = 3.33$, and $\Delta = 0.17$.

Effect of rigidities on equilibrium

We explain the effect of rigidities on the equilibrium outcome of our model in three parts. First, we examine the consequences of these rigidities on the optimal firm-size decision. Secondly, we describe

[20] Nickell and Nunziata (2001) report a benefit replacement rate of 58% for France for 1991-1995. The implied benefit replecament rate in our calibration is higher than the one reported by Nickell and Nunziata (2001) reflecting the broader interpretation of b.

Under this more general interpretation of b, this replacement rate is similar to the one suggested by Hagedorn and Manovskii (2007).

[21] In the off-steady analysis of our model, this parameter would determine the slope of the Beveridge curve.

their effect on the exit decisions of the entrepreneurs. Finally, we consider the couch potatoes' entry decisions.

For each productivity level there exist $\underline{n}(z') < \overline{n}(z')$, such that if a firm stays in business then: (i) if the firm is of size $n < \underline{n}(z')$ when it gets the new productivity level, z', it will expand to $\underline{n}(z')$; (ii) if the firm is bigger than $\overline{n}(z')$ when it draws z', it will downsize to $\overline{n}(z')$; and (iii) if the firm's size is in between $\underline{n}(z')$ and $\overline{n}(z')$ then it will not change its size at all. As a consequence, the firm's optimal size decision involves a range of inertia, a *corridor*, within which it does not adjust its size in response to changes in its productivity level.

The shape of the corridor is also indirectly affected by the firm's entry costs, since, in the case of hiring and firing costs, the marginal value of labor, $mvl(z', n')$, depends on the expected path of future productivity and thus on the probability of the firm going out of business and the entrepreneur becoming a couch potato again. The entry cost directly affects the value of these margins.

In terms of the exit decisions we obtain the following. For each productivity level, z', we find that firms exit whenever their size is below a certain threshold level, which we denote by $n^X(z')$. The function $n^X(z')$ is discontinuous in the sense that if $n^X(z') \geq \overline{n}(z')$ then all firms of productivity level z' will exit. We denote the cut-off productivity level for which this is the case by $\underline{z}^X$.

Moreover, there exists a $\overline{z}^X$ for which all firms with productivity $z' \geq \overline{z}^X$ stay in business, even those that are of size zero in the middle of a period. It is important that such a productivity level exists, because entrants become firms of size zero in the middle of a period. So for entering firms not to exit immediately, and thus to enter in the first place, there must at least be some productivity levels for which firms of size zero do not exit. If there are no hiring and firing costs, then the value of a firm does not depend on its size and thus $\underline{z}^X = \overline{z}^X$.

Similar to our benchmark model, there exists a threshold productivity level, $\underline{z}$, such that all couch potatoes that draw a business idea of level $z' \geq \underline{z}$ will start a business. It must be the case that $\underline{z} \geq \overline{z}^X$.

Rigid comparison equilibrium for calibrated parameter values

We compute the equilibrium for the calibrated parameter values using the algorithm described in Appendix B. The results are listed in the second column, labeled 'rigid', of Tables 1 and 2. Figure 6 depicts the resulting $\ln \underline{n}(z')$ and $\ln \overline{n}(z')$ in the $(\ln z', \ln n)$-space. In addition to $\underline{z}^X$, $\overline{z}^X$, $\underline{z}$, and

$n^X(z')$, the figure also contains $\ln n'(z')$, which is the optimal labor demand of the firm under no labor adjustment costs at the prevailing equilibrium wage rate.

The rigidities make couch potatoes more hesitant to become entrepreneurs in the sense that the threshold productivity level, $\underline{z}$, increases from 1.54 to 1.86. Where in the flexible benchmark, in each period, 7.4% of couch potatoes start a business, in this case only 1.0% of couch potatoes do so.

The introduction of rigidities reduces the value of being an entrepreneur, which, in turn, diminishes the value of being a couch potato. This is the option value that determines the equilibrium wage, through (22). The career choice part of our model implies an important feedback from the wage on firm entry and exit, not captured by other models of firm-size dynamics. In those models, the free entry condition equates the expected present discounted value of flow profits to a fixed entry cost of the firm. Here, the expected present discounted value of the flow profits of the marginal entrant is equated to a fixed entry cost plus a value that reflects the entrepreneur's outside options in the labor market. As a result, the equilibrium wage in the rigid comparison case is 37% lower than in the flexible equilibrium.

Under flexible labor inputs, a decrease of the equilibrium wage would result in larger firms. Not so in the presence of the ridigities. The rigidities reduce the average firm size from 23.9 in the benchmark to 16.6. To see what prevents firms from growing bigger in this economy, consider Figure 6. In order for a firm to become of size n in this economy, it must, at least once during its existence, have had a productivity level z', such that $\ln \underline{n}(z') = \ln n$. So, what is important for firm growth in this model is the shape of $\ln \underline{n}(z')$ in equilibrium. As can be seen from the figure, for each productivity level the corridor of inertia is very wide and, in particular, there is a big gap between $\ln \underline{n}(z')$ and $\ln n'(z')$.

The main cause of this gap are the vacancy costs. The calibrated level of vacancy costs needed to match the low job-finding rate in France, is about three quarters of an annual wage. It is much higher than any of the other calibrated adjustment costs, although in line with the estimates reported in Hamermesh and Pfann (1996). This high cost of hiring workers directly reduces firms' willingness to do so.

As Figure 6 shows, the corridor of inertia is asymmetric around the flexible labor demand $\ln n'(z')$ and the asymmetry is different for high and low productive firms. The gap between $\ln n'(z')$ and $\ln \underline{n}(z')$ is wider for more productive firms. This is due to the mean reverting nature

of the productivity process. A productive firm is more likely to have to fire a worker in the near future, which increases the firm's reluctance to hire a worker in the first place. Conversely, the gap between $\ln \overline{n}(z')$ and $\ln n'(z')$ is higher for less productive firms. Low productivity firms expect an increase in their productivity and to hire workers in the near future, making them less willing to let go of their workers in the current period.

As a consequence, in the rigid equilibrium firms have to be much more productive to grow big than in the flexible one. This is consistent with Bartelsman et. al. (2003) who provide evidence that the main difference between the U.S. and Europe in firm-size dynamics is that U.S. firms grow much faster after entry (conditional on survival).

In sum, there are two main effects of rigidities on firm-size dynamics. First of all, the rigidities drive up the threshold productivity level for entrants and reduce the exit productivity level for exiters. This is what we refer to as the *productivity selection effect* of firm entry and exit.[22] The second effect is the *reallocation effect* of rigidities that depresses the reallocation of labor across firms and, in particular, prevents firms from growing as fast under rigidities as in the flexible benchmark. The combined result of these effects is that low productivity firms are bigger under rigidities than in the flexible benchmark, while high productivity firms are smaller.

Consistent with the evidence on the U.S. and France, also presented in Tables 1 and 2, small firms, of size twenty or less, are bigger under rigidities. However, the reallocation effect implied by our calibrated model seems to be far bigger than that observed in the data. In France, rigidities especially hamper growth of large firms, of more than 500 workers, resulting in a lower average firm size for large firms; 1667 in France as compared to 2856 in the U.S. In our rigid comparison model the reallocation effect is so strong that it not only reduces the average firm size of large firms, but of all firms larger than 50 employees. Moreover, since low productivity firms do not downsize as much and high productivity firms do not expand as much, it *squeezes* the firm-size distribution towards the average firm size. In the flexible benchmark 87.8% of firms employ less than 20 workers while in the rigid comparison this is only 71.5%.

This *squeezing* of the firm-size distribution implies that the worker distribution across firms under rigidities is very different from the benchmark model. In the benchmark model 71.8% of workers are employed at firms of size 100 or more, while in the rigid comparison 76.6% is employed at firms of size 49 or less. This is not consistent with the worker distribution observed in France,

[22] This effect is similar to the mechanism that generates TFP differentials in Lagos (2006).

where 27% of workers are employed at firms of size 49 or less.

The reluctance of firms to change size is also reflected by their reluctance to enter and exit. Under rigidities $\underline{z}$ is not only higher than in the flexible benchmark, but also $\underline{z}^X$ and $\overline{z}^X$ are lower than $\underline{z}$ in the benchmark. There are low productivity firms that hang on to their workers because the option value of remaining in business exceeds the firing cost they have to incur when they close shop. As a result, the equilibrium annual entry and exit rate under rigidities is 1.5%, which is much lower than in the benchmark and also goes in the opposite direction of what we observe in the data; France exhibits a slightly higher entry and exit rate of firms than the U.S.

The low 1.5% entry and exit rates also manifest themselves in the distribution of the labor force. In each period, 92.2% of the labor force is employed, while 5.6% is an entrepreneur and runs a firm. The ratio of this is the 16.6 average firm size. The other 2.2% are couch potatoes. 1.0% of these couch potatoes start a business, while 24% of the remaining 99% find a job. This means that, in each period, 1.7% of the labor force are jobseekers that do not get a job offer, which is what we report as the unemployment rate.

Contrary to most search models of unemployment, this unemployment rate is solely the result of endogenous separations initiated by firms due to downsizing and exits and the search friction that determines the job finding rate. We abstract from exogenous separations, which are often used to calibrate the unemployment rate in other search models of the labor market.

The wage decreases substantially once we introduce rigidities into our framework. In particular, in the rigid comparison case, the wage is 1.06 while in the benchmark model it is 1.67. The introduction of rigidities lowers the value of being an entrepreneur. However, since the option value of being an entrepreneur also affects the value of being a worker, the wage goes down. Recall that the entrepreneur makes take-it-or-leave-it offers to the workers. Since the outside value of the worker goes down as a result of a decline in the value of being an entrepreneur, the firm lowers the wage to the point which makes the worker indifferent. This feedback effect allows the firm to lower the wage and make workers, who have no bargaining power, absorb the burden of the rigidities. As a result, the vacancy creation behavior and the job finding probability do not respond much to the change in the vacancy cost.

This mechanism requires us to set the vacancy cost parameter at a relatively high 3 months of wages to match our calibration target of a quarterly job finding probability of 24%. However, if the worker would have bargaining power then this feedback effect would be lower. In other words, the

firm will not be able to lower the wage as much as in the case of zero worker-bargaining power in response to vacancy costs. Consequently, the level of vacancy costs required to satisfy the targeted job-finding rate would not be as high as in the current case.

ALP is 17.8% lower in the rigid comparison case than in the flexible benchmark (1.43 versus 1.74). Even though entrants are, on average, more productive under rigidities, this is offset by the innefficient allocation of labor due to adjustment costs as well as due to unproductive firms staying in business. Moreover, the smaller average firm size reduces ALP because of the entrepreneurial overhead effect. In total, this is a substantially bigger labor productivity effect of rigidities than has been found in other studies, like Hopenhayn and Rogerson (1993) and Alvarez and Veracierto (2001).

The calibrated rigidities in our model seem to have too much of a distortionary effect on the equilibrium firm-size distribution. In order to understand which particular rigidities or frictions are driving this excess effect, we consider two experiments. First of all, we introduce each of the rigidities, except the search frictions, individually in our benchmark model. Second, we consider them all together to isolate the effect of the search frictions on the equilibrium. We denote the result of the latter experiment as the *all-but-search* comparison model.

4.3 Individual rigidities and all-but-search

Effect of individual rigidities

At the calibrated level of about half a quarterly wage, entry costs, η, alone do not substantially affect the equilibrium outcome of the benchmark model. This can be seen from the column labeled 'η' in Tables 1 and 2. Entry costs do not distort the marginal conditions that determine labor demand. Thus, just like in the benchmark model, the labor allocation mimics that of a flexible competitive labor market. In principle, the entry cost η drives a wedge between the entry cut-off $\underline{z}$ and the exit cut-off levels $\underline{z}^X = \overline{z}^X$. However, the wedge implied by the calibrated value of η is smaller than 4.7% and does not result in a change in the entry and exit threshold values in our approximation on the productivity grid. We find that entry costs of the calibrated magnitude do not have any significant effect on ALP.[23] Additional, unreported, calculations suggest that, for

[23] This is contrary to Barseghyan (2006) who provides evidence of a cross-country correlation between productivity and entry costs using instrumental variable regressions. This difference in results is probably due to a correlation between entry costs and other rigidities and factors that affect productivity.

our calibration, the effect of entry costs is still insignificant for η of the order of magnitude of two-and-a-half years of wages.

An increase in the tax-wedge, Δ, results for which are in the column labeled 'Δ' in Tables 1 and 2, amounts to an increase in the employment cost of labor. This increase in the marginal cost of labor is partly offset by a reduction of the equilibrium wage from 1.67 to 1.45. The new cost of labor is $(1+\Delta)\,w = 1.70$. The net effect is an increase in the cost of labor from the equilibrium wage in flexible benchmark of 1.67 to 1.70. In the absence of any distortions on the marginal conditions underlying labor demand, the increase in the cost of labor results in an across-the-board reduction in firm sizes for all productivity levels. In this sense, this rigidity has a *symmetric* effect on all types of firms and does not produce the type of *squeeze* of the firm-size and worker distributions that we observe in France relative to the U.S.

An increase in b from 0.7 to 0.9 is similar to an increase in the tax-wedge: it does not affect the marginal labor demand conditions but only affects the equilibrium cost of labor. In this case the wage increases because of the higher reservation wage implied by (22). The 0.2 increase in b is partly absorbed by a reduction in the value of being a couch potato, due to the reduced value of entrepreneurship under higher wages. As a consequence, a 0.2 increase in b only increases the wage by 0.02. Similar to the tax wedge case, the increase in labor costs due to an increase in b has a *symmetric* effect on all types of firms.

Finally, the individual rigidity, besides the search friction, with the biggest impact on the equilibrium in our model is the firing cost τ. A fixed firing cost generates a two sided *corridor* of inertia in labor demand around the flexible labor demand decision $n'(z')$ at the prevailing equilibrium wage. It is the existence of this corridor that causes less productive firms to shrink less and more productive firms to grow less than in the flexible benchmark, which is the main mechanism underlying the reallocation effect. The equilibrium results with only the firing costs are listed in column 'τ' of Tables 1 and 2.

The reallocation effect is apparent in these results from the relatively large size of small firms, of size 20 or less, and the smaller size of large firms. The fraction of firms of size 20-49 increases at the cost of the share of small firms. The changes in the firm-size and worker distributions induced by the firing cost are very similar to the difference between the U.S. and France. However, there are three main aspects of the data that this version of our model with only firing costs does not seem to capture well. The first is the average firm size, which is higher in France but lower under

firing costs in our model. The second is the higher entry and exit rates in France relative to the U.S. The third is that France has a 1% higher level of ALP than the U.S., while the model with firing costs has an ALP level that is 2% lower than the flexible benchmark.

Hopenhayn and Rogerson (1993) and Alvarez and Veracierto (2001), obtain productivity effects of a similar magnitude under firing costs. However, their postulated firing costs, of one year of wages, are four times higher than the ones we calibrated. This is because, in addition to the marginal distortions that affect ALP in Hopenhayn and Rogerson (1993) and Alvarez and Veracierto (2001), ALP in our model is also affected by the entrepreneurial overhead requirement. Therefore, our model tends to generate higher effects of firing costs on ALP.

All-but-search comparison equilibrium

To isolate the effect of the search frictions, we consider the equilibrium of our model with all the four rigidities above but not with vacancy costs, i.e. $c = 0$. This is the *all-but-search* comparison model tabulated in the third column of Tables 1 and 2. The associated equilibrium optimal firm-size decisions are depicted in $(\ln z', \ln n)$-space in Figure 7.

The joint effect of the four rigidities makes the entry productivity threshold move up.[24] This change in the entry and exit margin results in higher entry and exit in equilibrium relative to the benchmark model and consistent with the data. Moreover, the productivity selection effect increases ALP relative to the case in which there were only firings costs. The overall effect is that ALP is almost the same in the all-but-search model as in the benchmark model. This finding is consistent with the empirical evidence that ALP levels are very similar for the U.S. and France. [25]

In terms of the firm-size distribution, entry and exit rates, and ALP, the all-but search model matches data on France better than all the other cases we consider.[26]

Why the all-but-search and the firing cost model seem to match the French firm-size and worker distributions better than the rigid comparison model can be best seen by comparing the 'corridors' of Figures 6 and 7. In the rigid model, the search friction results in a very wide and asymmetric corridor around $n'(z')$ which stifles growth of productive firms to a degree that is counterfactual. The firing costs, along with the other rigidities, in the all-but-search model do not generate such

[24] It does not, however, drive a substantial wedge between entrants and exiters, such that $\underline{z} = \underline{z}^X = \overline{z}^X$ in our numerical solution.

[25] See Conference Board and Groningen Growth and Development Centre's "Total Economy Database (2007).

[26] Our model exhibits too little unemployment since we do not allow for exogenous separations.

an asymmetry. At the same productivity level, $\ln \underline{n}(z')$ is bigger, resulting in higher growth of productive firms. In addition, the number of workers employed at low productivity mid-size firms is lower, resulting in higher ALP.

The main message that we take away from this finding is that the search frictions needed to match job finding probabilities in this model imply much larger effects on the firm-size distribution than the differentials between the U.S. and France observed in reality. The search frictions generate counterfactually large differentials in outcomes for firms. Conversely, they generate a smaller differential in unemployment rates than observed in the data. The difference in unemployment rates in the flexible and rigid models is 1.7% while the difference between the unemployment rate in the U.S. and that in France is 4.1%. This is mainly due to the fact that unemployment in our model is solely the result of endogenous separations initiated by firms due to downsizing and exits and the search friction that determines the job finding rate. We abstract from exogenous separations, which are often used to calibrate the unemployment rate in other search models of the labor market.

The fact our model overpredicts the effect of labor market frictions on the firm-size distribution and underpredicts their effect on the unemployment rate leads us to conclude a large part of the unemployment differential between the U.S. and France is not due to labor market frictions, but instead reflects voluntary unemployment in response to different policy measures.

5 Conclusion

We develop a model that combines a theory of labor market frictions with one of the firm-size distribution. Ours is a model of firm-size dynamics with a broad set of rigidities to explain differences in firm-size distributions between the U.S. and Europe. Its main components are based on Lucas' (1978) model of entrepreneurship and the firm-size distribution, Hopenhayn and Rogerson's (1993) model of firm-size dynamics with firing costs, and Mortensen and Pissarides' (1994) labor market setup with search frictions.

We use our model to consider the joint effects of hiring and firing costs, firm entry costs, unemployment benefits, a tax wedge between wages and labor costs, and labor matching frictions on firm-size dynamics and labor markets. We do so in the following way. First, we calibrate the preference and technology parameters in a version of the model without rigidities, which we use as a *flexible* benchmark, to match the main summary statistics of U.S. firm-size dynamics. Second,

we add labor market rigidities and calibrate them to match those documented for France. We also consider a third case where we shut down the search frictions in the labor market.

Our model implies two main effects of rigidities on the firm-size distribution: a *productivity selection effect* and a *reallocation effect*. The productivity selection effect is caused by the rigidities affecting the entry and exit decisions of entrepreneurs. The reallocation effect reflects how the rigidities impede on the firms' hiring and firing decisions, reducing the reallocation of labor across firms.

We find that the model that includes all the rigidities has an unrealistically high reallocation effect. This is mainly due to the high vacancy cost that is needed to match the job-finding rate in France. This high vacancy cost makes productive firms so reluctant to hire and unproductive firms so reluctant to downsize or exit that, in equilibrium, there is an excessively high fraction of unproductive mid-size firms and large firms do not grow as big as observed in the data. Without search frictions, our model is quite successful in matching the main differentials between the U.S. and France; higher firm entry and exit in France, more compressed firm-size and worker distributions, and a similar average labor productivity level.

We conclude that labor market search frictions are not necessary to account for firm-size differentials between the U.S. and France. Instead, firing costs, which we calibrate relatively low compared to other studies, seem to go a long way in explaining these differentials. One interpretation of this result is that a large part of the unemployment differential between the U.S. and France is not due to labor market frictions, but instead reflects voluntary unemployment in response to different policy measures. This effect is not captured in our current framework, since we only consider frictional unemployment and take the overall labor supply as given. We thus ignore the potential labor supply effect of taxation as an alternative explanation of different labor market outcomes, as in Prescott (2004) and Ohanian, Raffo, and Rogerson (2006).

An extension of our model framework along this line is one direction of future research. Another direction is the inclusion of alternative wage bargaining schemes, for example, the collective bargaining considered in Stole and Zwiebel (1996a,b) and Ebell and Haefke (2004, 2006).

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A Mathematical details

Derivation of production function as reduced form of Cobb-Douglas with economies of scope

The production function that we consider

$$f(z,n) = zn^a, \text{ where } 0 < a < 1 \tag{40}$$

can be interpreted as the value added of labor that results from a constant returns to scale Cobb-Douglas production technology, with an output elasticity of capital equal to $0 < \alpha < 1$, subject to a Lucas (1978) economies of scope parameter $0 < \eta < 1$[27] and flexible capital inputs.

To show this, consider the entrepreneur that decides on renting the capital input, k, at the rental rate R to maximize profits

$$\widetilde{z}\left[k^{\alpha}n^{1-\alpha}\right]^{\eta} - Rk \tag{41}$$

The resulting optimal capital rental condition is

$$Rk = \alpha\eta\widetilde{z}\left[k^{\alpha}n^{1-\alpha}\right]^{\eta} \tag{42}$$

such that the capital rental cost share equals $\eta\alpha$ and the optimal capital input level is given by

$$k = \widetilde{z}\left(\frac{\alpha\eta}{R}\right)^{\frac{1}{1-\alpha\eta}} n^{\frac{(1-\alpha)\eta}{1-\alpha\eta}}$$

and the profits, net of the rental costs, equal

$$\pi^*(n) = [1-\alpha\eta]\,\widetilde{z}\left[k^{\alpha}n^{1-\alpha}\right]^{\eta} = [1-\alpha\eta]\left[\frac{\alpha\eta}{R}\right]^{\frac{\alpha\eta}{1-\alpha\eta}} \widetilde{z}^{\frac{1}{1-\alpha\eta}} n^{\frac{(1-\alpha)\eta}{1-\alpha\eta}} \tag{43}$$

Now define

$$z = [1-\alpha\eta]\left[\frac{\alpha\eta}{R}\right]^{\frac{\alpha\eta}{1-\alpha\eta}} \widetilde{z}^{\frac{1}{1-\alpha\eta}} \text{ and } a = \frac{(1-\alpha)\eta}{1-\alpha\eta} < 1 \tag{44}$$

then we can write these profits as

$$\pi^*(z,n) = zn^a = f(z,n) \tag{45}$$

Hence, the production function that we consider can be interpreted as the value added generated by the labor inputs for a constant returns to scale Cobb-Douglas production technology with flexible capital inputs which is subject to the manager's economies of scope or decreasing returns to scale η.

[27] This is equivalent to the assumption that it is a Cobb-Douglas production technology with decreasing returns to scale, where α is the gross returns to scale parameter.

B Solution method

This appendix contains an explanation of the solution method that we used to find the steady state equilibrium of our model. We first explain the broad steps we use to solve the model and then proceed by explaining the underlying details and refering to particular equations in the main text.

Outline of the solution method

We solve our model by iterating over four nested loops until convergence. We describe the basic purpose of these loops here. The details are explained in separate subsections below. The nested loop structure is as follows

Initialization:

Start with initial values for the level of labor market tightness, θ, the distribution of the labor force over the state space, Φ^{CP}, $\Phi^{E}(z,n)$, and $\Phi^{W}(z,n)$,[28] the wage w, and the value functions V^{CP}, $V^{E}(z,n)$, and $V^{W}(z,n)$. Start with the innerloop, i.e. loop 3, as defined below.

Nested loops:

Loop 0: Labor market tightness

Update: θ.

Conditional on: (i) the distributions of the labor force over the state space Φ^{CP}, $\Phi^{E}(z,n)$, and $\Phi^{W}(z,n)$, (ii) the wage w, and (iii) the value functions V^{CP}, $V^{E}(z,n)$, and $V^{W}(z,n)$.

Using: Equations (26) and (27).

Loop 1: Distributions

Update: Φ^{CP}, $\Phi^{E}(z,n)$, and $\Phi^{W}(z,n)$.

Conditional on: (i) the degree of labor market tightness, θ, (ii) the wage , w, and (iii) the value functions V^{CP}, $V^{E}(z,n)$, and $V^{W}(z,n)$ and the implied optimal decisions.

Using: Solution method explained in 'Loop 1 details: Solving for the equilibrium distribution of the labor force' below.

Loop 2: Wage

Update: w.

Conditional on: (i) the degree of labor market tightness, θ, (ii) the distributions of the labor force over the state space Φ^{CP}, $\Phi^{E}(z,n)$, and $\Phi^{W}(z,n)$, and (iii) the value functions V^{CP}, $V^{E}(z,n)$, and $V^{W}(z,n)$.

Using: Equation (22).

Loop 3: Value functions

Update: V^{CP}, $V^{E}(z,n)$, and $V^{W}(z,n)$.

Conditional on: (i) the degree of labor market tightness, θ, (ii) the wage, w, and (iii) the distribution of the labor force over the state space, Φ^{CP}, $\Phi^{E}(z,n)$, and $\Phi^{W}(z,n)$.

[28] The within period distributions, Φ^{U}, $\Phi^{\tilde{E}}(z',n)$, and $\Phi^{\widetilde{W}}(z',n)$, are implied by the other initialized variables.

Using: Solution method explained in 'Loop 3 details: Value function iterations and implied optimal choices and choice sets' below.

We solve the model by iteratively looping through loop 3, then loop 2, then loop 1, and, finally, loop 0.

Loop 1 details: Solving for the equilibrium distribution of the labor force

We consider here how to solve for the equilibrium distribution of the labor force, conditional on (i) the degree of labor market tightness, θ, (ii) the wage, w, and (iii) the value functions V^{CP}, $V^{E}(z,n)$, and $V^{W}(z,n)$ and the implied optimal decisions, which determine the sets F_x and $F_n(z',n')$.

In order to solve for the equilibrium distribution of the labor force, we first solve for the steady state firm-size distribution and determine, up to a constant, $\Phi^{E}(z,n)$ and $\Phi^{\widetilde{E}}(z',n)$. The missing constant is the number of firms and we solve it using the labor market resource constraint (34).

First of all, because firms do not exit and enter in the same period, we know that for $(z',n) \in F_x$ the transitional equation (33) simplifies to

$$\Phi^{\widetilde{E}}(z',n) = \int_{Z} \Phi^{E}(z,n)\, dQ(z'|z) \tag{46}$$

This allows us to write the number of exiting firms μ_X as

$$\mu_X = \int_{F_x}\int_{Z} \Phi^{E}(z,n)\, dQ(z'|z)\, d(z',n) \tag{47}$$

Because, in equilibrium, $\mu_E = \mu_X$ we can use this to solve for the implied number of couch potatoes

$$\Phi^{CP} = \frac{1}{1-F(\underline{z})}\int_{F_x}\int_{Z} \Phi^{E}(z,n)\, dQ(z'|z)\, d(z',n) \tag{48}$$

Substitution of this result into the transitional equation for the number of firms in the middle of the period, i.e. (33), yields

$$\begin{aligned}\Phi^{\widetilde{E}}(z',n) &= \frac{f(z')\, I[z' \geq \underline{z}]}{1-F(\underline{z})} I[n=0] \int_{F_x}\int_{Z} \Phi^{E}(z,n)\, dQ(z'|z)\, d(z',n) \\ &\quad + \int_{Z} \Phi^{E}(z,n)\, dQ(z'|z)\end{aligned} \tag{49}$$

By applying (31), we find that

$$\begin{aligned}\Phi^{E}(z',n') &= \frac{f(z')\, I[z' \geq \underline{z}]}{1-F(\underline{z})} \left[\int_{F_x}\int_{Z} \Phi^{E}(z,n)\, dQ(z'|z)\, d(z',n)\right] \int_{F_n(z',n')} I[n=0]\, dn \\ &\quad + \int_{F_n(z',n')}\int_{Z} \Phi^{E}(z,n)\, dQ(z'|z)\, dn\end{aligned} \tag{50}$$

which is a contraction mapping in $\Phi^{E}(z,n)$. This contraction mapping allows us to solve $\Phi^{E}(z,n)$ up to a scaling constant. This scaling constant essentially determines the number of firms in equilibrium.

We obtain it by combining (24), (34), and (48) to solve

$$1 = \frac{1}{1-F(\underline{z})}\int_{F_x}\int_{Z} \Phi^{E}(z,n)\, dQ(z'|z)\, d(z',n) + \int_{Z\times N} \Phi^{E}(z,n)(1+n)\, d(z,n) \tag{51}$$

which then fully determines the equilibrium distribution of the labor force over the states of couch potato, workers, and entrepreneurs.

This defines a contraction mapping in Φ^{CP}, $\Phi^{E}(z,n)$, and $\Phi^{W}(z,n)$ over which we iterate until convergence to find the implied steady state distribution of the labor force over the state space.

Loop 3 details: Value function iterations and implied optimal choices and choice sets

We update the value functions by iterating over the following contraction mapping. Each iteration of this contraction mapping starts with particular solutions for the value functions V^{CP}, $V^{E}(z,n)$, and $V^{W}(z,n)$.

Given these end of period continuation values, we can calculate the optimal labor market decisions of firms. The decisions are: (i) the optimal firm-size decision, $\nu^{*}(z',n)$, determined by (8); and (ii) the exit decision, which yields $X(z',n)$, based on (10). These two decisions yield the combined optimal firm-size and exit decision, $\nu'(z',n)$, from (11).

The choice sets associated with these optimal firm-size choices are: (i) F_x, given by (29); (ii) $F_n(z',n')$, given by (32).

The optimal firm-size and exit decisions determine the within period value of the firms, i.e. $V^{\widetilde{E}}(z',n)$, through the Bellman equations (12).

These decisions of the firms also affect the expected labor market outcomes of the workers. They do so because they determine the probability of getting fired being employed in a given job, i.e. $P^{F}(z',n)$, through (13).

The expected labor market outcomes, in turn, translated into the within period value functions of workers and the unemployed. That is, they allow us to update $V^{\widetilde{W}}(z',n)$, through (14), and V^{U}, through (7).

The within period value functions can then be used to solve for the optimal career decision made by a couch potatoes, i.e. $\underline{z}$. This is the minimum z' that satisfies (3).

This now allows us, in turn, to update the beginning of period value functions V^{CP}, $V^{E}(z,n)$, and $V^{W}(z,n)$, through (4), (1), and (2) respectively.

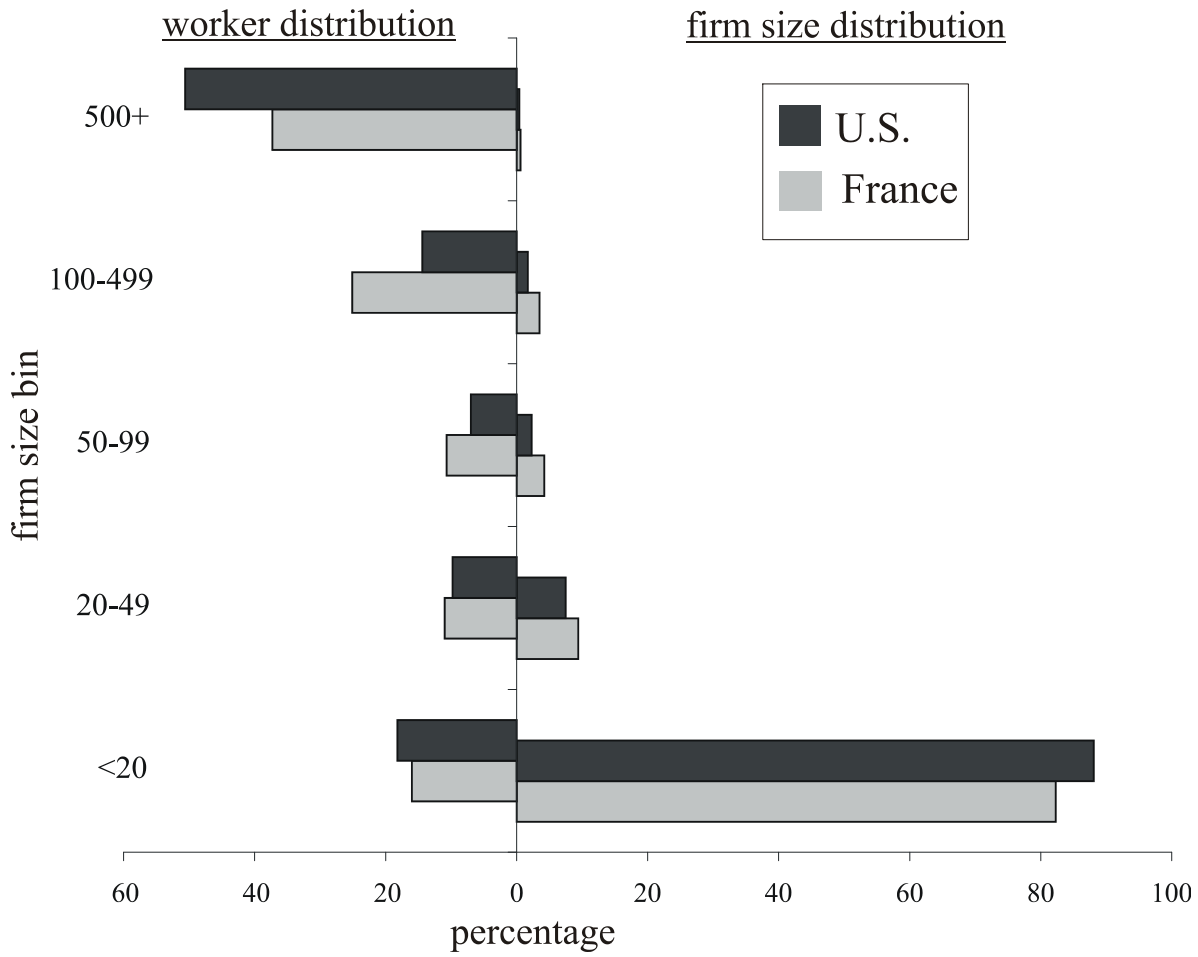

Figure 1: Firm size and worker distributions for U.S. and France

Table 1: Equilibrium under different parameter combinations (part 1)

Result	Comparison models			Individual rigidities				Data	
	Bench-mark	Rigid	All-but-search	η	Δ	τ	b	U.S.	France
				0.65	0.17	1.30	0.90	90-95	90,92-95
$\underline{z}$	1.54	1.86	1.62	1.54	1.54	1.54	1.54	-	-
Wage	1.67	1.06	1.37	1.68	1.45	1.56	1.69	100	104
ALP	1.74	1.43	1.73	1.74	1.76	1.70	1.74	100	101
			Distribution of the labor force (in percentages)						
CPs	1.8	2.2	4.1	1.8	2.1	1.9	1.9	-	-
Entrepreneurs	3.9	5.6	4.6	3.9	4.7	4.2	4.2	-	-
Workers	94.3	92.2	91.3	94.3	93.2	94.0	94.0	-	-
Unemployed	1.7	2.2	3.9	1.7	2.0	1.7	1.7	-	-
Unemployment rate	0.0	1.7	0.0	0.0	0.0	0.0	0.0	6.6	10.7
				Search frictions					
θ	∞	1.0	∞	∞	∞	∞	∞	-	-
Job finding prob.	1.00	0.24	1.00	1.00	1.00	1.00	1.00	0.73	0.24
			Overall entry/exit rates (annual, percentage of firms)						
Entry	11.3	1.5	13.9	11.3	11.3	11.3	11.3	10.4	15.9
Exit	11.3	1.5	13.9	11.3	11.3	11.3	11.3	9.1	11.6
Size bin			Average firm size (overall and for size bins)						
all	23.9	16.6	19.9	23.9	19.8	22.6	22.6	23.2	27.4
<20	3.0	9.4	4.3	2.9	2.7	4.2	2.9	4.7	5.3
20-49	33.7	25.8	28.6	34.6	28.0	32.6	31.8	30.1	31.8
50-99	82.1	61.6	67.8	84.0	69.0	71.7	77.5	68.5	70.4
100-499	243	168	217	247	200	188	229	197	197
500+	2351	707	1621	2332	1919	1343	2218	2856	1667
			Firm size distribution (percentage)						
<20	87.8	71.5	86.0	87.8	87.8	84.1	87.8	88.1	82.3
20-49	7.2	23.4	6.6	7.3	7.3	9.4	7.2	7.5	9.4
50-99	1.9	4.4	4.5	2.0	2.0	2.6	2.0	2.3	4.2
100-499	2.4	0.6	2.6	2.4	2.4	3.2	2.4	1.7	3.5
500+	0.5	0.0	0.3	0.5	0.5	0.6	0.5	0.4	0.6

Note: Firm size distribution and dynamics data from Bartelsman et. al. (2003, 2004). *Entrants* are firms that did not exist a year ago. *Exiters* are firms that do not exist a year later. Reported firm size excludes the entrepreneur. *ALP* is average labor productivity from Conference Board and Groningen Growth and Development Centre (2007). Data on *wages* reflect average 1991-1995 hourly compensation costs for production workers in manufacturing from Bureau of Labor Statistics (2006a). Data on *unemployment rates* are 1991-1995 averages from Bureau of Labor Statistics (2006b).

Table 2: Equilibrium under different parameter combinations (part 2)

Size bin	Comparison models			Individual rigidities				Data	
	Bench-mark	Rigid	All-but-search	η	Δ	τ	b	U.S.	France
				0.65	0.17	1.30	0.90	90-95	90,92-95
			Distribution of workers over firm size (percentage)						
<20	11.2	40.4	18.8	10.7	11.9	15.6	11.3	18.2	16.0
20-49	10.2	36.2	9.5	10.5	10.2	13.6	10.2	9.8	11.0
50-99	6.8	16.4	15.1	7.0	6.9	8.3	6.8	7.0	10.7
100-499	24.8	6.2	28.7	25.2	24.6	26.6	24.8	14.4	25.1
500+	47.0	0.7	27.9	46.6	46.3	35.8	46.9	50.6	37.3
			Fraction of firms that entered in the last year (percentage)						
<20	12.8	2.1	16.2	12.8	12.8	13.4	12.8	11.1	18.3
20-49	0.8	0.0	0.3	0.8	0.8	0.3	0.8	5.5	6.0
50-99	0.4	0.0	0.3	0.4	0.4	0.1	0.4	4.5	3.8
100-499	0.2	0.0	0.1	0.2	0.2	0.1	0.2	3.6	3.0
500+	0.1	0.0	0.0	0.1	0.1	0.0	0.0	1.8	2.5
			Fraction of firms that exit in a year (percentage)						
<20	12.8	2.1	16.2	12.9	12.9	13.4	12.8	9.1	11.6
20-49	0.8	0.0	0.0	0.0	0.0	0.0	0.8	4.7	5.9
50-99	0.4	0.0	0.0	0.0	0.0	0.0	0.4	3.8	4.3
100-499	0.2	0.0	0.0	0.0	0.0	0.0	0.2	3.1	3.7
500+	0.1	0.0	0.0	0.0	0.0	0.0	0.0	1.9	2.8
			Firm size distribution of entrants (percentage)						
<20	99.3	99.6	99.8	99.3	99.3	99.6	99.3	94.4	94.7
20-49	0.5	0.4	0.1	0.5	0.5	0.3	0.5	3.9	3.5
50-99	0.1	0.0	0.1	0.2	0.1	0.1	0.1	1.0	1.0
100-499	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.6	0.6
500+	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
			Firm size distribution of exiters (percentage)						
<20	100.0	100.0	100.0	100.0	100.0	100.0	100.0	94.2	91.5
20-49	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.1	5.3
50-99	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.7
100-499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	1.3
500+	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2

Note: *Entrants* are firms that did not exist a year ago. *Exiters* are firms that do not exist a year later. Reported firm size excludes the entrepreneur.

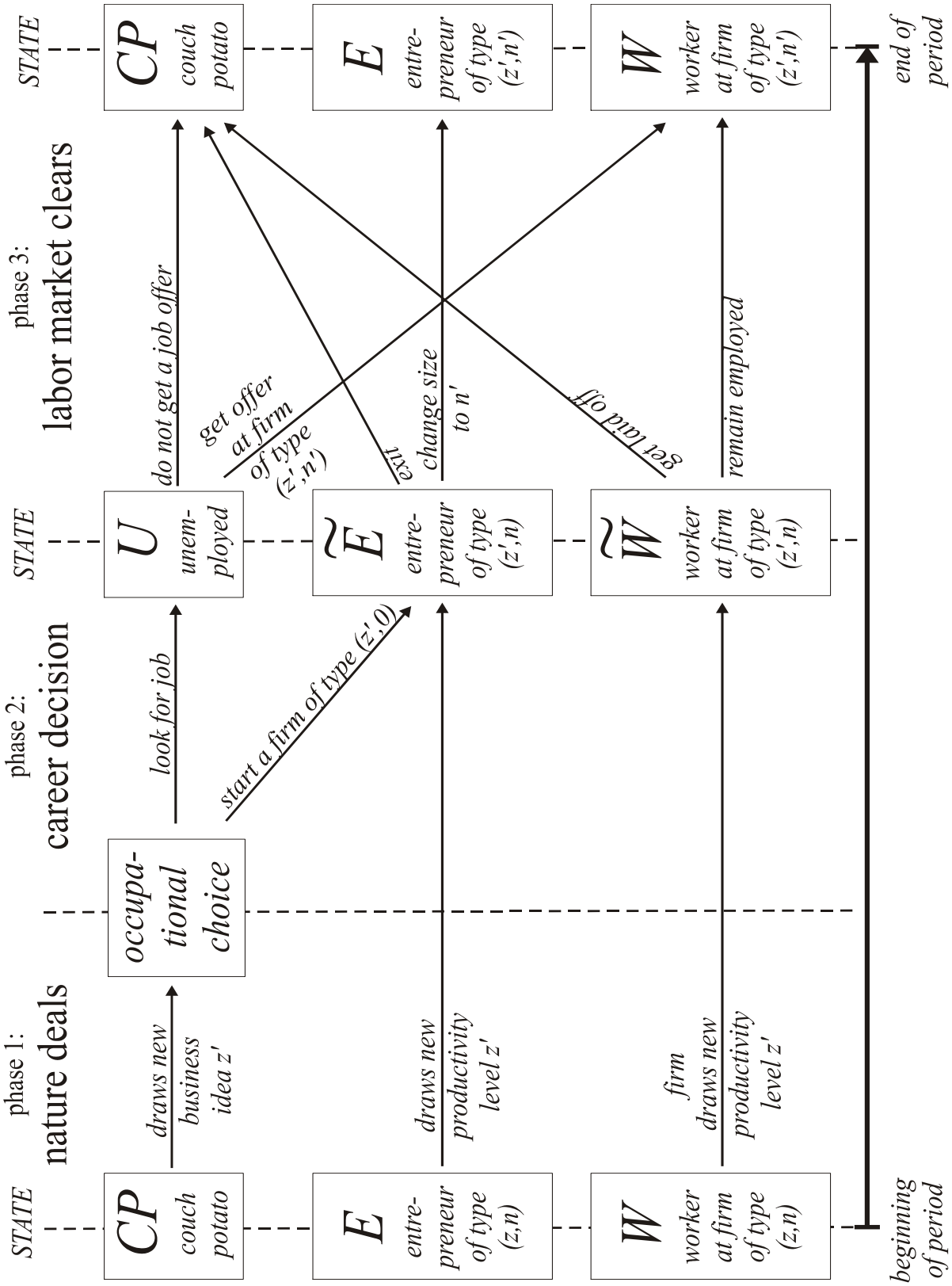

Figure 2: Diagram of within period choices and states of members of labor force

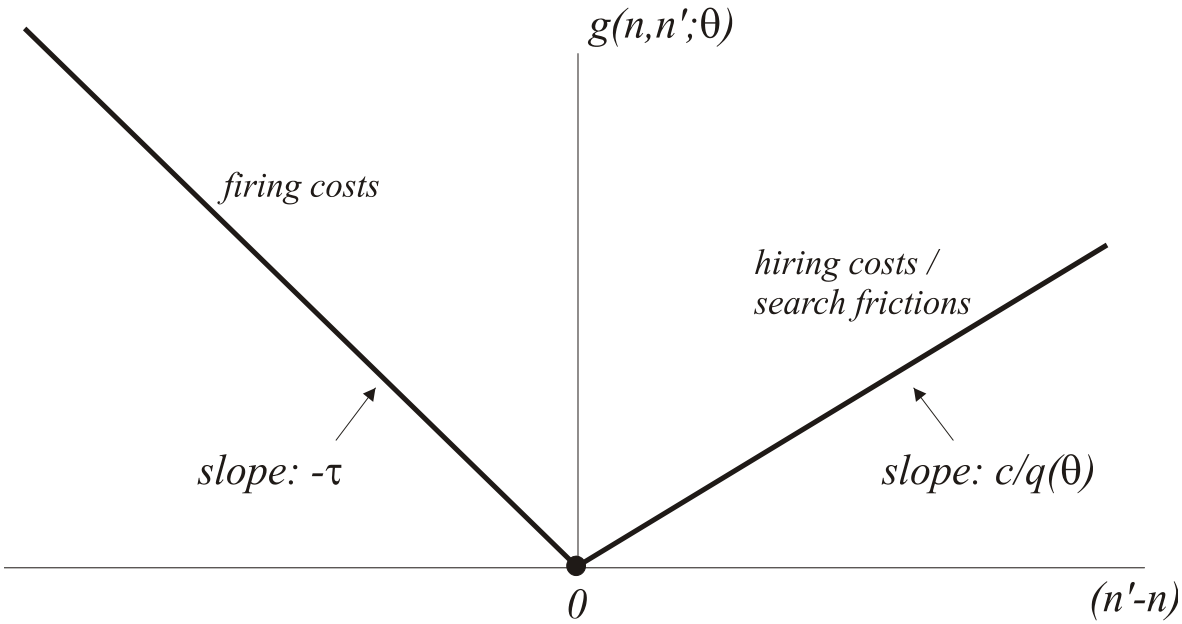

Figure 3: Labor adjustment costs

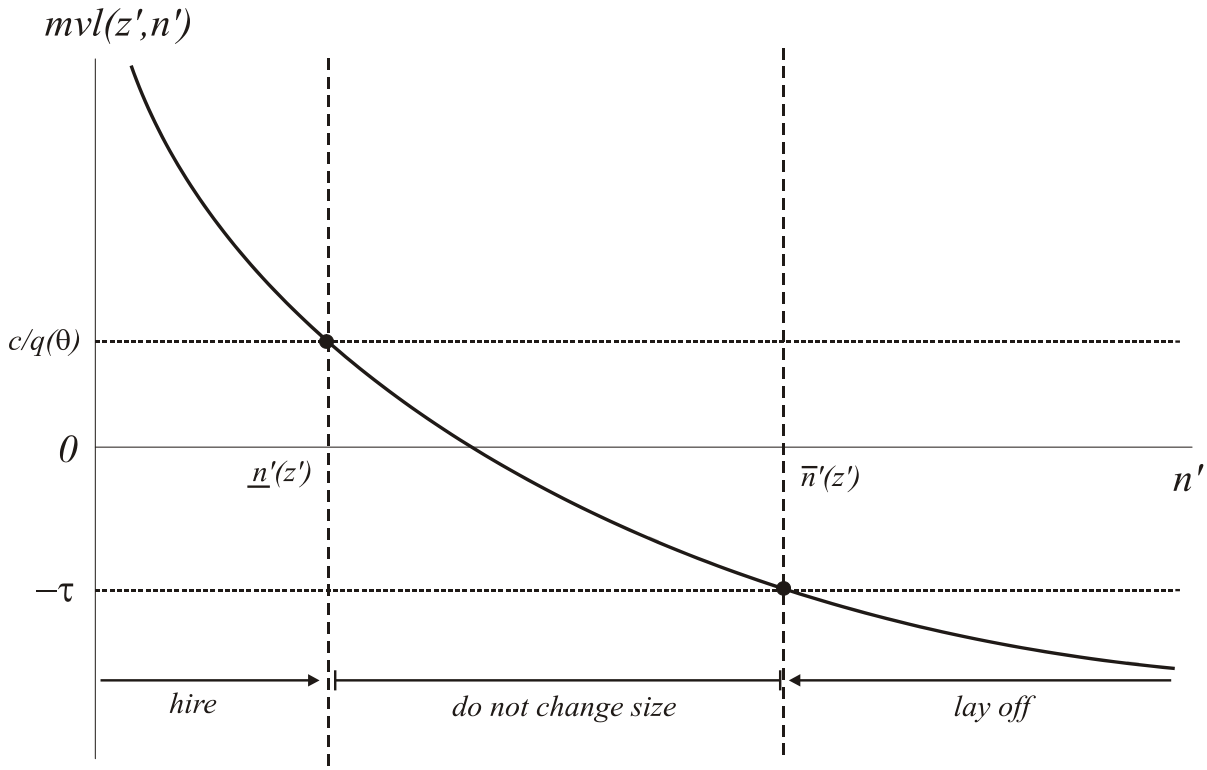

Figure 4: Marginal condition for hiring and lay off decisions

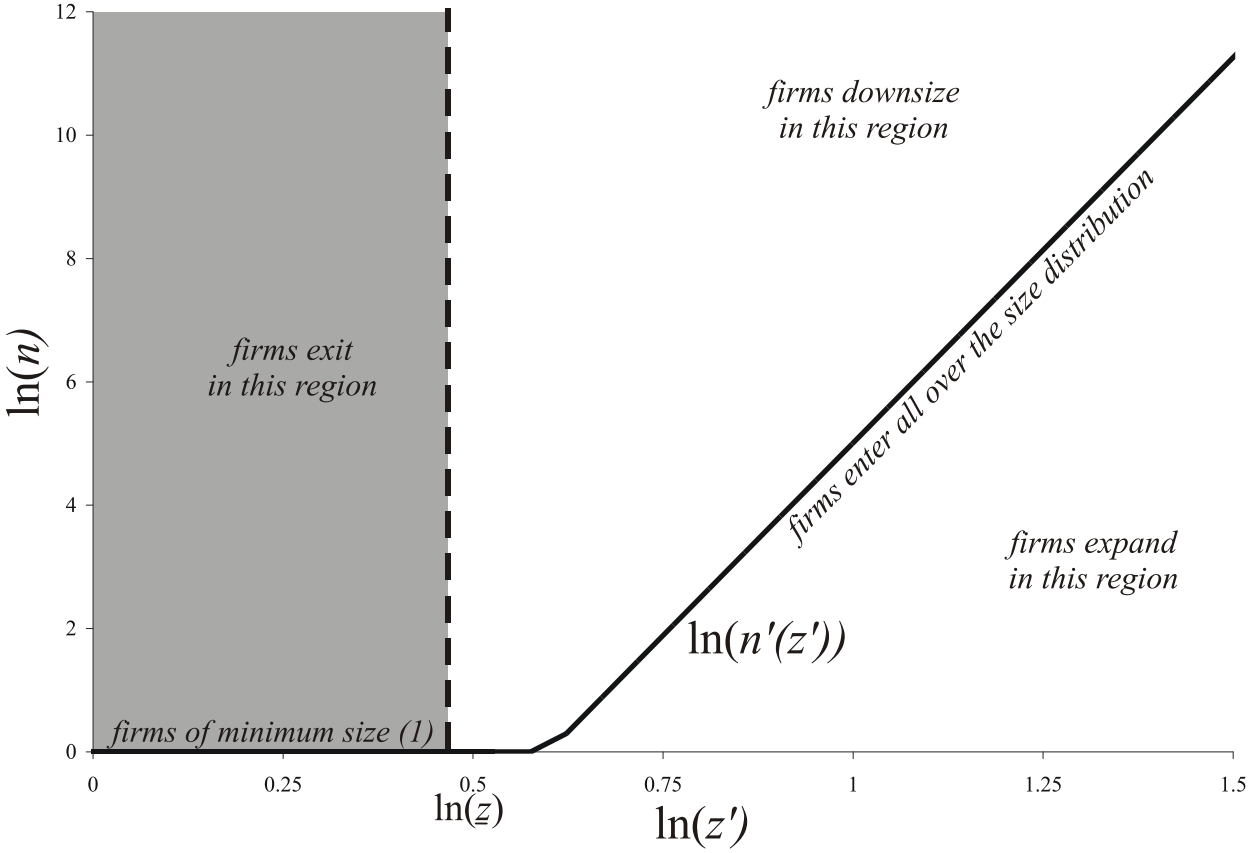

Figure 5: Firm size dynamics choice sets in benchmark case without rigidities

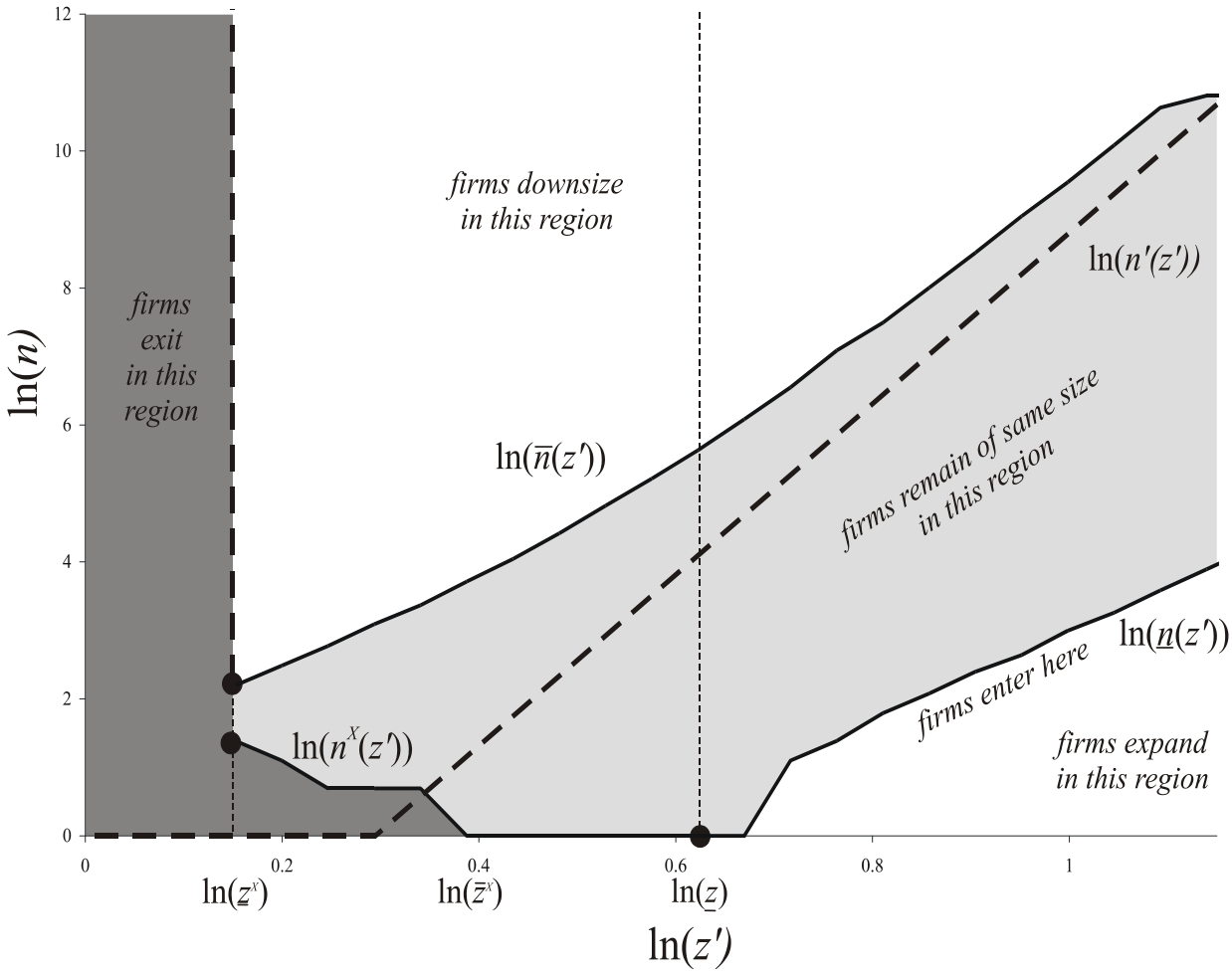

Figure 6: Firm size dynamics choice regions in rigid comparison case

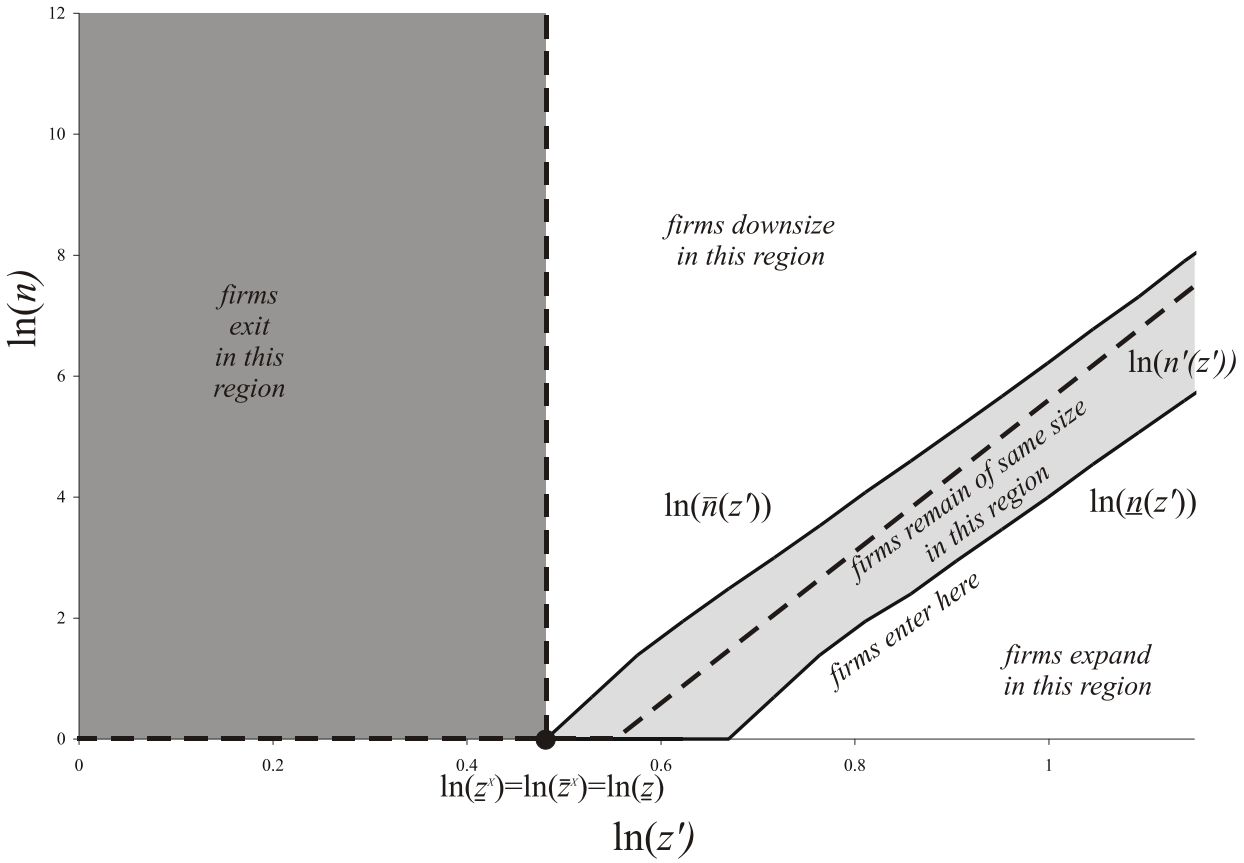

Figure 7: Firm size dynamics choice regions with all-but-search frictions