Dealers' Hedging of Interest Rate Options in the U.S. Dollar Fixed-Income Market

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As derivatives markets have grown, the scope of financial intermediation has evolved beyond credit intermediation to cover a wide variety of risks. Financial derivatives allow dealers to intermediate the risk management needs of their customers by unbundling customer exposures and reallocating them through the derivatives markets. In this way, a customer's unwanted risks can be traded away or hedged, while other exposures are retained. For example, borrowers and lenders can separate a loan's interest rate risk from its credit risk by using an interest rate swap to pass the interest rate risk to a third party. In another example of unbundling, an option allows an investor to acquire exposure to a change in asset prices in one direction without incurring exposure to a move in asset prices in the opposite direction.

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The derivatives markets' rapid growth has been driven by a number of developments. In addition to advances in finance and computing technology, the rough balance of customer needs on the buy and sell sides of the market has contributed to this expansion. This balance allows dealers to intermediate customer demands by passing exposures from some customers to others without assuming excessive risk themselves. Without this ability to pass exposures back into the market, the markets' growth would be constrained by dealers' limited ability to absorb customers' unwanted risks.

The balance between customer needs on both sides of the market is most apparent in the swaps market, the largest of the derivatives markets, where only a small amount of residual risk remains with dealers.[1] In the over-the-counter U.S. dollar interest rate options market, however, significant residual risks are concentrated among dealers, who have sold 50 percent more options to customers than they have purchased (Table 1, top panel). This imbalance has left dealers with significant net exposure to price risk that must be hedged in the underlying fixed-income markets.

Until now, the scale of hedging across all dealers in the over-the-counter interest rate options market has not been studied in the literature. The concentration of sold options among dealers, however, makes it an ideal place to explore how dealers' hedging of options affects underlying markets. Using data from a global survey of derivatives dealers and other sources, this article estimates the volume and potential impact of such hedging by U.S. dollar interest rate options dealers. In our analysis, we address two questions: First, are dealers' hedge adjustments large enough to affect trading volume and liquidity in the most common hedging instruments? Second, what effects might potential hedging difficulties have on risk premia in options prices and the structure of the market for over-the-counter interest rate options? In addressing these questions, we also consider whether dealers' dynamic hedging transactions have the potential to amplify price shocks.

We find that, on the whole, transaction volume in the underlying fixed-income markets is large enough to enable dealers to manage the risks incurred through their intermediation of price risk in the interest rate options market. Indeed, at shorter maturities, turnover volume in the most liquid hedging instruments is more than large enough to absorb the transaction volume generated by dealers' dynamic hedging. For medium-term maturities, however, an unusually large interest rate shock could cause the hedging of exposures in this segment of the yield curve to generate trading demand that is high relative to turnover volume in the more liquid trading instruments. Dealers then face a risk management trade-off between reducing price risk or incurring the liquidity costs of immediately rebalancing their hedge positions. However, only very large interest rate shocks, such as those occurring during a currency crisis or a period of high inflation, are likely to present dealers with this hedging problem.

In addition to analyzing hedging volume, we examine the term structure of options premia to assess whether option prices show any sign of potential hedging difficulties. We find an apparent risk premium in options prices at the medium-term segment of the yield curve that corresponds to the maturity range where our analysis of trading volume suggests that hedging difficulties might occur. This pattern in the term structure of options premia suggests that the liquidity risk in dynamic hedging may influence options pricing.

Table 1
OVER-THE-COUNTER INTEREST RATE OPTIONS DATA

NOTIONAL AMOUNTS REPORTED BY DEALERS, IN BILLIONS OF U.S. DOLLARS

	Bought Options			Sold Options		
Contracts with	U.S. Dollar Interest Rates	Other Interest Rates	Total	U.S. Dollar Interest Rates	Other Interest Rates	Total
Other dealers	529.4	726.5	1,255.9	576.1	681.9	1,258.1
Customers	431.6	340.6	772.2	690.4	398.1	1,088.4
Total	961.1	1,067.1	2,028.1	1,266.5	1,080.0	2,346.5

MARKET VALUES REPORTED BY DEALERS, IN BILLIONS OF U.S. DOLLARS

	Bought Options			Sold Options		
Contracts with	U.S. Dollar Interest Rates	Other Interest Rates	Total	U.S. Dollar Interest Rates	Other Interest Rates	Total
Other dealers	—	—	22.4	—	—	21.6
Customers	—	—	15.2	—	—	14.6
Total	20.8	16.7	37.6	19.4	16.8	36.2

MATURITY DISTRIBUTION OF U.S. DOLLAR INTEREST RATE OPTIONS, IN PERCENT

	Bought Options	Sold Options
Up to one year	30	29
More than one year and up to five years	58	56
More than five years	12	15

Source: Bank for International Settlements (1996).

Dynamic Hedging, Volatility of Financial Asset Prices, and Market Liquidity

An important question in any discussion of options hedging is whether the dynamic hedging of options in response to a price shock can introduce transactions large enough to amplify the initial price shock or to affect market liquidity. In asset price dynamics, such "positive feedback" occurs when an initial price change causes a shift in investor or trader demand that leads to a further change in price in the same direction. For example, a shift in investor sentiment in response to a sharp price decline can cause the sell-off of assets or widespread hedging of open positions—outcomes that would drive prices down further. The hedging of options also has the potential to cause positive feedback because dealers typically adjust their hedge positions by selling (buying) the underlying asset after its price falls (rises). These dynamic hedge adjustments in response to a fall in prices could introduce further downward pressure on prices.

Some observers cite the stock market crash of 1987—which occurred in the absence of any significant change in economic fundamentals—as an example of positive feedback dynamics. These observers suggest that the sharp fall in stock prices was intensified by portfolio insurance trading strategies that prescribe the sale (purchase) of stocks when prices fall (rise).[2] Although no empirical proof exists that positive feedback affects market prices, a number of papers (for example, Bank for International Settlements [1986], Grossman [1988], Gennotte and Leland [1990], and Pritsker [1997]) have suggested that dynamic hedging can cause positive feedback. In addition, Fernald, Keane, and Mosser (1994) discuss a possible example of positive feedback in the behavior of the term structure of interest rates.

If positive feedback is more than a theoretical possibility, then dynamic hedging would have the potential to amplify the volatility of asset prices when prices fall abruptly. Higher price volatility can in turn introduce other problems in financial markets. Most significantly, volatility can heighten uncertainty about credit risks and disrupt the intermediation of credit. For example, during the 1987 stock market crash, the increase of credit exposures in securities and margin settlement caused liquidity and funding problems for securities firms (see Bernanke [1990]). The potential for such financial market disruptions makes it worthwhile to consider the relationship between dynamic hedging and positive feedback in asset prices.

Dynamic hedging can also have implications for market liquidity. The financial innovations that have broadened the scope of financial intermediation to include the intermediation of price risks are positive developments that might be expected to lower risk premia in asset prices. Some of these forms of intermediation, however, rely on the ability of dealers to manage their risks dynamically. In the absence of market liquidity—which makes dynamic risk management possible—dealers would exact higher premia for their intermediation services. Some investors and fund managers may also rely on market liquidity in their investment and risk management strategies. If significant numbers of economic agents are relying on the liquidity of the core trading markets, either directly or indirectly, then part of the risk premia in financial asset prices might depend on assumptions about the robustness of that market liquidity. A sudden realization by investors and dealers that expectations of market liquidity were overly optimistic could lead to a sharp adjustment in asset prices. For this reason, assessments of the potential impact of dynamic hedging and risk management strategies on market liquidity are particularly useful. A related question is whether such dynamic risk management strategies by individual risk managers would be feasible in the aggregate during periods of extreme price volatility.[3]

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A Brief Description of the Estimation

The analysis in this paper is based on data from the 1995 Central Bank Survey of Foreign Exchange and Derivatives Market Activity (Bank for International Settlements 1996), market growth data from the surveys of the International Swaps and Derivatives Association (ISDA), and historical interest rate data. The central bank survey reports the global market totals of outstanding derivatives contracts at the end of March 1995. The over-the-counter options data in the survey include notional amounts and market values of outstanding contracts, broken down by bought and sold options. A key part of our analysis is the derivation of strike prices that are consistent with the notional amount and market value data from the survey.[4]

Our estimation of dealers' hedging transactions has three principal steps. First, using the notional amount and rough maturity data from the central bank survey and market growth rates from the ISDA surveys, we estimate the distribution of notional amounts over maturities and origination dates. Next, we combine the estimated notional amounts at each origination date with historical interest rate data to estimate options strike prices that are consistent with the market values reported in the central bank survey and with historical interest rates. Finally, we use these strike prices to estimate the price sensitivity of a portfolio consisting of all dealers' interest rate options. Specifically, given the estimated strike prices, we calculate the delta of the global portfolio, that is, the change in the portfolio's value relative to changes in forward interest rates. The delta and its sensitivity to interest rate changes give us an estimate of dealers' hedge demands and dealers' hedge adjustments to interest rate shocks. (For a detailed description of the data and estimation, see the appendix.)

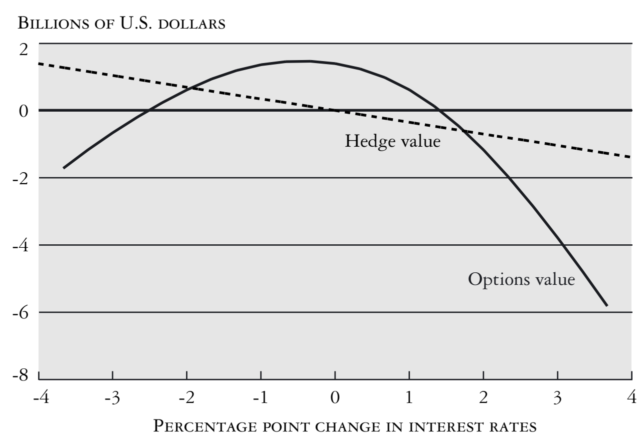
Estimated Price Risk in the Global Dealer Portfolio

We begin our analysis by using the estimated strike prices to derive the value of the global dealer portfolio of options at different interest rates (the solid line in Chart 1). The value of the options portfolio at the prevailing interest rates is the net market value reported in the central bank survey (Table 1, middle panel). The values at the indicated changes in interest rates are the option values calculated from the estimated strike prices. Chart 1 also shows, as a mirror image, the value of a hedge position that provides a delta-neutral hedge of the options at the initial interest rates (the dashed line). The hedge position is derived by using the estimated strike prices to calculate the price sensitivity (the delta) of the options portfolio. The estimated price sensitivity is used to construct a hedge position in fixed-income securities whose gain or loss in value offsets the change in value of the options portfolio for small changes in interest rates in either direction. The chart reveals a number of interesting facts about the dealers' portfolio of options.

First, at prevailing interest rates, the net value of the dealers' portfolio is positive. Although in notional amounts dealers sell more options than they purchase, at prevailing interest rates the bought options have higher market values than the sold options (Table 1, top and middle panels). This relationship between the notional amounts and the market values implies that the options sold to customers have a lower degree of moneyness than options purchased from customers (for definitions of terms, see box). The strike prices we estimate show the same relationship: relative to

Chart 1

Options and Hedge Values as a Function of Interest Rate Changes



Source: Author's caculations.

Notes: The hedge value is the mirror image of the value of a hedge position that provides the dealer with a delta-neutral position at the initial interest rate. The hedged portfolio has a positive value when the option value (the solid line) is above the hedge value (the dashed line).

swap interest rates at origination, sold options have estimated strikes that are out-of-the-money, while options purchased from customers are slightly in-the-money (see appendix). Because dealers are net sellers of options, however, large interest rate shocks will drive the sold options into-the-money, causing them to gain value, and as a result, the total value of the sold options will exceed the bought options' value. Hence, if the portfolio is not hedged, the aggregate dealers' portfolio value becomes negative when interest rates rise more than 125 basis points.

Options Terms and Concepts

Moneyness

An option's moneyness is a measure of its payoff at expiration. An option's payoff is defined relative to a specified level of the underlying asset's price called the *strike price*. For a call (put) option, the option is *in-the-money* at expiration when the asset price is above (below) the strike price, and the option pays the difference. When the asset price is below (above) the strike price at expiration, the call (put) option pays nothing, and the option is said to be *out-of-the-money*. An option's varying sensitivity to price risk is a result of the asymmetry in an option's payoff. An option is *at-the-money* when the underlying asset's price is equal to the strike price, and a call (put) option's moneyness is higher when the underlying asset's price is higher (lower).

Interest Rate Caps

Caps and floors are options on interest rates. In an interest rate cap (floor), if the interest rate at expiration of the contract is above (below) the strike rate specified in the contract, the buyer receives the difference, and nothing otherwise. Caps and floors are variations of call and put options. In terms of fixed-income securities, a cap is equivalent to a put option on a bond; in terms of interest rates, a cap is equivalent to a call option on interest rates. A cap (put option on a bond) gains value when interest rates rise (bond prices fall).

Varying Sensitivity to Price Risk and Positive Feedback

A call option's value increases by an amount smaller than the increase in the value of the underlying asset because there is always some probability that the price of the underlying asset will fall below the strike price at expiration, rendering the option worthless. As the underlying asset's price rises, this probability becomes smaller, and the value of the option becomes more sensitive to changes in the underlying asset's price. To compensate for this increase in the price sensitivity of a call option, a hedge position in the underlying asset must be made larger after the price of the underlying asset rises. This adjustment in the hedge position introduces the potential for positive feedback in price dynamics because the hedge adjustment is to buy (sell) the underlying asset after its price rises (falls).

Hedge Adjustments and Options Prices

As the value of the underlying asset rises, the writer of a call option must make the hedge position larger to ensure that its value is sufficient to cover the rising option exposure. As the value of the underlying asset falls, the hedge position must be reduced in size to ensure that the writer of the option is not left holding the underlying asset when the option expires out-of-the-money. Thus, the hedge adjustments in dynamic hedging involve buying the underlying asset after the price goes up and selling it after the price goes down. The cumulative cost of these "buy high, sell low" hedge adjustments equals the value of the option (for further discussion of option hedging, see Hull [1993]).

Volatility and Options Risk

The path-breaking option-pricing models developed more than two decades ago rely on continuous hedge adjustments to construct a dynamically hedged portfolio of underlying assets that perfectly replicates the payoff of an option (under the assumption that volatility remains constant). This ability to replicate the option means that the option does not contain unique risks, and, therefore, its value can be derived straightforwardly from the probability distribution of the underlying assets by using a risk-neutral expected value calculation. In practice, however, continuous hedge adjustments are not possible, and the difficulty in constructing a hedge portfolio that would perfectly replicate an option leaves the writer of an option with a unique and unhedgeable volatility risk.

Implied Volatility and Volatility Smiles

Market prices of options differ in characteristic ways from theoretical prices derived from benchmark pricing models, depending on the options' moneyness. These differences are manifested as differences in the implied volatility of the underlying asset when the benchmark model is used to infer the volatility of the underlying asset from the observed market price of the option. Options that are either deep out-of-the-money or deep in-the-money typically are priced in the market as if they had higher volatility in the log-normal distribution embedded in the benchmark pricing model. (This implied volatility pattern is called the volatility smile.) By incorporating these implied volatility differences in the benchmark pricing model, analysts can use the model to generate observed market prices.

Second, Chart 1 shows that a static hedge can only protect against small interest rate shocks. For a small change in interest rates, the change in the value of the options portfolio is offset by a corresponding change in the value of the hedge position. For a large interest rate change, however, the change in the value of the hedge position cannot offset the change in value of the options portfolio. Indeed, the hedged portfolio value turns negative where the options value and the hedge value intersect. Thus, with only a static hedge in place, the value of the hedged portfolio will turn negative after a large interest rate shock—specifically, an interest rate increase of more than 175 basis points. Dynamic adjustments to the hedge position as interest rates change, however, can prevent such an adverse outcome.

Third, to hedge against interest rate changes fully, dealers must adjust their hedge position after an interest rate shock. This adjustment compensates for the fact that the option portfolio's value falls at an increasing rate as interest rates rise.[5] As Chart 1 shows, without the hedge adjustment, the gain in value of the initial hedge position will no longer compensate for the decline in value of the option portfolio if interest rates continue to rise. This need to adjust the hedge position dynamically as interest rates change introduces the potential for positive feedback. Because the required hedge is a short position in fixed-income securities, the hedge adjustment to an increase in interest rates will introduce additional sales into the fixed-income market and may contribute further upward pressure on interest rates (by driving bond prices lower).

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Finally, Chart 1 suggests that not all dealers can hedge their options exposures with offsetting exposures within their firms. The conventional view of financial institutions' interest rate risk profiles holds that these firms have a structural long position in the fixed-income market. That is, they have a firmwide exposure to rising rates. The negative slope of the options value curve at the prevailing forward rates, however, shows that the aggregate dealer portfolio of options has an exposure to rising interest rates as well. Thus, because the options portfolio and the other portfolios are exposed to rising rates, dealers as a group cannot hedge their net options exposures with offsetting structural positions in other parts of their firms. Although some dealers may rely on offsetting exposures elsewhere in their firms to hedge their options position, Chart 1 suggests that dealers as a group cannot hedge internally.

Estimated Scale of Dealers' Dynamic Hedging

A comparison of the size of dealers' hedge adjustments and transaction volume in the most common hedging instruments enables us to assess the market impact of dealers' hedging. As an option's moneyness increases after a price shock, the sensitivity of its value to further changes in prices increases. Thus, to maintain an option portfolio's exposure to price risk within a given limit, a dealer must adjust the hedge position after a price shock to allow for the change in the options' price sensitivity. For a given interest rate shock, we estimate the change in the hedge position required to restore the portfolio's price sensitivity (the delta) to its initial level. This hedge adjustment is the incremental demand of dealers for hedge instruments after an interest rate shock, if we assume that dealers maintain their exposure to price risk at some initial comfort level.

In our analysis of dealers' dynamic hedging, we make a number of assumptions. First, we assume that *customers* do not dynamically hedge their options positions because doing so would negate the investment or hedging objective that motivated the purchase of the option. Thus, we need consider only dealers' hedging demands. Second, we assume that interdealer options do not result in a net increase in dealers' hedge demands because they create only offsetting exposures among dealers.[6] Thus, we calculate dealers' net hedge requirements from dealers' contracts with customers. Finally, to calculate our benchmarks of dealers' hedging demands, we assume that dealers match the maturity of an option's interest rate exposure with the interest rate maturity of its hedge. For this reason, our benchmark hedge estimates are exact hedges that do not have yield curve or correlation risk. (For further discussion of these assumptions, see the appendix.)

The interest rate shocks we use in our estimates are increases in forward interest rates of 25 and 75 basis points. These interest rate changes are consistent with historical experience. For example, consider forward interest rates in the four-to-seven-year segment of the yield curve during the period 1991-95. For that period, a change of 25 basis points is slightly less than the largest daily change, and a change of 75 basis points is slightly less than the largest two-week change. During the last two decades, the ten largest daily changes in forward rates in the medium-term segment of the yield curve ranged from 60 to 100 basis points. At the short-term end of the yield curve, the ten largest daily changes in the three-month Treasury bill rate ranged from 80 to 130 basis points. These episodes of extreme volatility occurred between 1979 and 1981.

Estimates for the Most Common Hedging Instruments

In the U.S. dollar fixed-income market, options dealers executing their hedges can choose from a wide range of fixed-income instruments such as futures contracts, forward rate agreements (FRAs), interest rate swaps, interbank deposits, and Treasury and other bonds. These instruments, however, are imperfect substitutes because they have different credit risks, liquidity, and transaction costs. These differences create a need and an opportunity for intermediation. Dealers who provide risk management services to the markets take on and manage the risks and costs resulting from holding portfolios of such imperfect substitutes. When dealers have enough time to hedge a position or replace an initial hedge with a cheaper or better instrument, they can usually keep their exposure to price risk within manageable limits while still earning a profit from intermediation. When an immediate hedge adjustment in large volume is needed, however, dealers' hedging alternatives are more limited. For example, although the market for interest rate swaps is very large and becoming increasingly liquid, the daily turnover volume of swaps is still very small relative to outstanding contracts. The turnover of swaps is also small compared with turnover in the Eurodollar futures markets.[7] This difference in turnover volume suggests that swaps are more likely used to hedge structural or longer term exposures than to hedge positions that require frequent adjustment. Consequently, for dynamic hedge adjustments, dealers are likely to use the most liquid instruments as hedging vehicles. In the U.S. dollar fixed-income market, these instruments are *Eurodollar futures, Treasury securities,* and *Treasury futures*.[8]

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Hedging with Eurodollar Futures

Our estimate of dealers' hedging demands suggests that at shorter maturities the Eurodollar futures market is more than large enough to accommodate dealers' hedging—even when large interest rate shocks occur. For the hedging of longer maturity exposures, however, the Eurodollar futures market appears able to accommodate only the hedging of residual exposures, that is, marginal hedge adjustments and exposures that remain after the use of other hedging instruments.

As Tables 2 and 3 show, at maturities of up to one year, daily turnover volume exceeds the estimated hedge adjustment even when forward rates increase by as much as 75 basis points.[9] At longer maturities, however, the estimated hedge adjustments are sometimes larger than turnover volume. For a 25-basis-point change in forward rates, the largest daily turnover volume of Eurodollar futures contracts exceeds the estimated hedge adjustments for maturities out to five years and for maturities between eight and ten years (Table 2). For a 75-basis-point change in forward rates, however, the largest daily trading volume exceeds the estimated hedge adjustment out to only two years' maturity (Table 3). For small interest rate changes of, say, 10 basis points, daily turnover volume exceeds the estimated hedge adjustment at all maturities.

A comparison of the hedge estimates to another benchmark—the difference between the market's largest daily volume and its average volume—yields a similar conclusion. For maturities of up to two or three years, the surge in the largest daily volume exceeds the estimated hedge adjustment for a 25-basis-point change in forward rates (Table 2). For a 75-basis-point change in forward rates, however, the surge in volume exceeds the estimated hedge adjustment only out to maturities of a year and a half (Table 3). Thus, in response to a large interest rate shock, hedging volume at maturities beyond two years would be larger than daily turnover volume.

The stock of outstanding Eurodollar futures contracts also suggests that the market can support dealers' hedge adjustments. Our estimated hedge adjustments are smaller than the stock of outstanding futures contracts at all maturities. Even in the case of hedge adjustments in response to a 75-basis-point change in forward rates, the estimated hedge adjustment at most maturities is much less than half the outstanding futures contracts (Table 4). This result, along with our analysis of turnover volume, suggests that difficulties executing hedge adjustments are likely to be liquidity problems. That is, at the medium-term maturities, the Eurodollar futures market would have difficulty accommodating the entire hedging volume immediately, but the hedge adjustments could be absorbed over time.

So far, we have considered whether turnover volume is large enough to absorb transactions from *adjustments* to hedge positions in response to a price shock. Another consideration, however, is how large the hedge *position* is relative to outstanding contracts in the market. For the estimated

Table 2
CHANGE IN HEDGE POSITION FROM 25-BASIS-POINT INCREASE IN FORWARD RATES AND THE DAILY TURNOVER VOLUME OF EURODOLLAR FUTURES
Billions of U.S. Dollars

Maturity (Years)	Change in Hedge Position	Largest Daily Volume		Average Daily Volume		Difference between Largest and Average Daily Volume	
		First Contract	Second Contract	First Contract	Second Contract	First Contract	Second Contract
0.5	-6.3	**374.0**	**334.1**	**115.7**	**148.4**	**258.2**	**185.7**
1	-9.2	**260.9**	**135.2**	**92.1**	**35.8**	**168.8**	**99.4**
1.5	-7.7	**55.1**	**39.7**	**20.0**	**14.0**	**35.1**	**25.7**
2	-5.7	**26.9**	**18.9**	**9.4**	**6.0**	**17.5**	**13.0**
2.5	-4.6	**9.2**	**7.5**	4.0	3.3	**5.2**	4.3
3	-3.7	**7.3**	**4.5**	2.7	1.9	**4.6**	2.5
3.5	-3.1	**3.9**	2.6	1.5	1.3	2.4	1.3
4	-2.6	**2.7**	**3.3**	1.2	1.1	1.5	2.2
4.5	-2.1	**2.4**	**2.3**	0.9	0.8	1.5	1.5
5	-1.9	**2.0**	1.4	0.8	0.5	1.2	1.0
5.5	-1.6	1.3	**2.4**	0.2	0.2	1.1	**2.2**
6	-1.4	1.3	1.3	0.2	0.2	1.0	1.1
6.5	-1.2	1.0	1.2	0.2	0.2	0.9	1.1
7	-1.0	**3.3**	0.7	0.2	0.1	**3.1**	0.6
7.5	-0.9	0.6	**1.2**	0.1	0.1	0.5	**1.1**
8	-0.6	**0.8**	**3.7**	0.1	0.1	**0.7**	**3.5**
8.5	-0.4	**1.2**	**1.2**	0.1	0.1	**1.1**	**1.1**
9	-0.3	**1.2**	**1.7**	0.1	0.1	**1.1**	**1.6**
9.5	-0.1	**1.0**	**0.7**	0.1	0.1	**0.9**	**0.6**
10	—	1.2	1.0	0.1	0.0	1.1	1.0

Source: Author's calculations.

Notes: Hedge estimates are based on data as of the end of March 1995. Contract volume is for the first half of 1995. Bold type indicates that the contract volume exceeds the change in hedge position. Negative values indicate a short position. Because the futures contracts are contracts on a three-month interest rate, the hedge for each six-month exposure requires two back-to-back contracts ("first contract" and "second contract" in the table).

hedge position at longer maturities, the Eurodollar futures market is not large enough to accommodate all of the hedge demands that would be generated by a fully delta-neutral hedging strategy, particularly for exposures beyond four or five years (Table 4). Rather, at longer maturities, the Eurodollar futures market can accommodate only marginal hedge adjustments and the hedging of exposures that remain after the use of other hedging instruments.

Hedging with Treasury Securities and Treasury Futures

The Treasury securities market and the market for futures contracts on Treasuries—which are both large and highly liquid—are ideal complements to the Eurodollar futures market for dealers' hedging needs. To estimate the hedge of an exposure to forward rates between five and ten years' maturity, we assume that dealers' hedges consist of a short position (the sale of a borrowed security) in the ten-year note and a long position in the five-year note. This hedge can be executed with either cash market securities or futures contracts. The position is a hedge of the exposure between five and ten years' maturity because the long and short positions extinguish exposures to forward rates below five years, leaving only the exposure to forward rates beyond five years.

The Treasury cash and futures markets are generally large enough to accommodate dealers' dynamic hedging (Table 5). The estimated hedge adjustment is less than the combined daily turnover volume of on-the-run securities[10] and Treasury futures even for large interest rate shocks. However, dealers' hedging demand could be significant relative to the size of the markets. For example, the estimated hedge adjustment to a 75-basis-point shock could be as large as 21 percent of the combined average daily turnover in the Treasury futures and interdealer on-the-run cash markets, and almost 10 percent of the outstanding stocks of the on-the-run securities and futures contracts. Moreover, the estimated hedge position could be as large as a third of total outstanding contracts in the two markets. In sum, the Treasury cash and futures markets significantly expand the pool of fixed-income

Table 3
CHANGE IN HEDGE POSITION FROM 75-BASIS-POINT INCREASE IN FORWARD RATES AND THE DAILY TURNOVER VOLUME OF EURODOLLAR FUTURES
Billions of U.S. Dollars

		Largest Daily Volume		Average Daily Volume		Difference between Largest and Average Daily Volume	
Maturity (Years)	Change in Hedge Position	First Contract	Second Contract	First Contract	Second Contract	First Contract	Second Contract
0.5	-31.9	**374.0**	**334.1**	**115.7**	**148.4**	**258.2**	**185.7**
1	-31.2	**260.9**	**135.2**	**92.1**	**35.8**	**168.8**	**99.4**
1.5	-23.7	**55.1**	**39.7**	20.0	14.0	**35.1**	**25.7**
2	-17.2	**26.9**	**18.9**	9.4	6.0	**17.5**	13.0
2.5	-13.6	9.2	7.5	4.0	3.3	5.2	4.3
3	-11.0	7.3	4.5	2.7	1.9	4.6	2.5
3.5	-9.0	3.9	2.6	1.5	1.3	2.4	1.3
4	-7.6	2.7	3.3	1.2	1.1	1.5	2.2
4.5	-6.2	2.4	2.3	0.9	0.8	1.5	1.5
5	-5.5	2.0	1.4	0.8	0.5	1.2	1.0
5.5	-4.7	1.3	2.4	0.2	0.2	1.1	2.2
6	-4.1	1.3	1.3	0.2	0.2	1.0	1.1
6.5	-3.5	1.0	1.2	0.2	0.2	0.9	1.1
7	-3.0	**3.3**	0.7	0.2	0.1	**3.1**	0.6
7.5	-2.4	0.6	1.2	0.1	0.1	0.5	1.1
8	-1.9	0.8	**3.7**	0.1	0.1	0.7	**3.5**
8.5	-1.3	1.2	1.2	0.1	0.1	1.1	1.1
9	-0.7	**1.2**	**1.7**	0.1	0.1	**1.1**	**1.6**
9.5	-0.3	**1.0**	**0.7**	0.1	0.1	**0.9**	**0.6**
10	—	1.2	1.0	0.1	0.0	1.1	1.0

Source: Author's calculations.

Notes: Hedge estimates are based on data as of the end of March 1995. Contract volume is for the first half of 1995. Bold type indicates that the contract volume exceeds the change in hedge position. Negative values indicate a short position. Because the futures contracts are contracts on a three-month interest rate, the hedge for each six-month exposure requires two back-to-back contracts ("first contract" and "second contract" in the table).

instruments available to options dealers for their hedging needs. However, dealers' hedge demands could amount to a significant share of turnover volume and outstanding contracts in the Treasury on-the-run cash and futures markets.[11]

Alternative Hedging Estimates

The hedge estimates above do not account for changes in the volatility of interest rates. An option's hedge position against changes in the underlying asset's price, however, depends on the underlying asset's price volatility as well as on the asset's price level. Moreover, large changes in asset prices are often associated with higher implied volatilities in options. For this reason, alternative hedge adjustments were estimated assuming simultaneous volatility and interest rate level shocks (Table 6). Although the estimated hedge adjustment is larger, the difference does not appreciably change the conclusions because the difference from the base case is small relative to the turnover volume in the hedge instruments.

Table 4
DELTA-NEUTRAL HEDGE POSITION IN EURODOLLAR FUTURES CONTRACTS AND EURODOLLAR FUTURES CONTRACTS OUTSTANDING
Billions of U.S. Dollars

		Open Interest		
Maturity (Years)	Hedge Position	First Contract	Second Contract	Change in Hedge from a 75-Basis-Point Shock
0.5	38.3	**561.9**	**366.4**	-31.9
1	23.9	**279.7**	**222.0**	-31.2
1.5	2.8	**174.0**	**145.4**	-23.7
2	-4.0	**114.2**	**96.3**	-17.2
2.5	-9.8	**84.9**	**68.6**	-13.6
3	-13.4	**60.3**	**54.8**	-11.0
3.5	-16.4	**49.5**	**38.8**	-9.0
4	-17.9	**34.4**	**27.2**	-7.6
4.5	-20.2	**22.6**	14.5	-6.2
5	-18.9	12.9	9.5	-5.5
5.5	-18.8	7.5	7.7	-4.7
6	-18.4	6.2	5.9	-4.1
6.5	-17.5	6.7	6.8	-3.5
7	-15.1	6.8	4.5	-3.0
7.5	-12.6	3.8	2.5	-2.4
8	-9.6	1.6	2.2	-1.9
8.5	-6.2	1.8	1.8	-1.3
9	-3.4	1.7	2.0	-0.7
9.5	-1.4	0.8	0.9	-0.3
10	—	0.8	0.0	—

Source: Author's calculations.

Notes: The table reports delta-neutral hedge estimates and open interest as of the end of March 1995. Bold type indicates that the contract volume exceeds the change in hedge position. Negative values indicate a short position. Because the futures contracts are contracts on a three-month interest rate, the hedge for each six-month exposure requires two back-to-back contracts ("first contract" and "second contract" in the table).

DEALERS' HEDGE ADJUSTMENTS AND MARKET LIQUIDITY

Our estimate of dealers' hedging demands suggests that dealers might encounter hedging difficulties only for exposures beyond three or five years' maturity when large interest rate shocks occur. Together, the Eurodollar futures, on-the-run Treasury securities, and Treasury futures markets can absorb hedge adjustments to interest rate shocks as large as 25 basis points along the entire term structure. For example, for exposures between five and ten years' maturity, the estimated hedge adjustment to a 25-basis-point shock is only 7 percent of the combined turnover in the Treasury futures and interdealer on-the-run cash markets (Table 5).

For a large interest rate shock, however, such as a 75-basis-point shock to forward rates, dealers' dynamic hedge adjustments in the medium-term segment of the yield curve would generate significant demand relative to turnover in these hedging instruments. This demand would amount to 21 percent of the combined turnover in the Treasury futures and interdealer on-the-run cash markets (Table 5). In addition, the hedge adjustment in the three-

Table 5
HEDGE POSITION IN TREASURY SECURITIES AND FUTURES
Billions of U.S. Dollars

		Change in Hedge Position from an Interest Rate Shock of			On-the-Run Treasury Securities[a]		Treasury Futures[b]	
	Hedge Position	10 Basis Points	25 Basis Points	75 Basis Points	Outstanding	Daily Volume	Outstanding	Daily Volume
Five-year	13.0	0.4	1.0	2.9	13.2	9.0	19.7	5.1
Ten-year	-13.0	-0.4	-1.1	-3.3	13.8	6.0	25.8	9.2

Source: Author's calculations.

Notes: Hedge estimates are based on data as of the end of March 1995. Negative values indicate a short position.

[a]Outstanding amount as of the end of March 1995. Daily volume is estimated from interdealer trading volume (Fleming 1997).

[b]Five- and ten-year note contracts. Outstanding contracts are as of the end of March 1995. Daily volume is for the first half of 1995.

to-five-year maturity segment would be large relative to Eurodollar futures turnover volume (Table 3). If all dealers executed their hedge adjustments simultaneously, these transactions could have an impact on turnover volume and affect market liquidity. Moreover, in the presence of a large interest rate shock, other traders and investors might undertake transactions in the same direction as options dealers' hedge adjustments. All these demands together suggest that dealers wishing to adjust their hedge positions immediately could indeed encounter market liquidity problems.

Turnover volume in standard hedging instruments appears large enough to accommodate dealers' dynamic hedging in all but the most extreme periods of interest rate volatility.

Dealers can manage the impact on market liquidity by trading off price risk against the cost of immediacy or liquidity. For example, only part of the exposure opened up by a large interest rate shock might be hedged initially, with the remainder hedged over time. Dealers can spread the hedge adjustment over a number of days by executing a series of transactions that are small relative to daily turnover in the hedge instruments. This strategy reduces the market impact of the hedge adjustment but leaves the dealer exposed to some price risk until the hedging transactions are completed. Alternatively, by assuming some correlation risk, a dealer could also hedge the longer maturity exposures with the first three near-term futures contracts. The volume of these shortest maturity contracts is large enough to accommodate the hedging of longer maturity exposures easily, but returns on these contracts are less than perfectly correlated with longer maturity interest rates.

In another alternative, dealers could use an interest rate term structure model to design a hedge that avoids concentrated transactions at yield curve sectors with liquidity problems. For example, using a two-factor interest rate term structure model, a dealer could construct a hedge of exposures between five and ten years using a position in one-year bills and thirty-year bonds that replicates the exposure to the term structure factors that drive forward rates between five and ten years. Such hedges, however, are vulnerable to atypical price shocks not accounted for by the correlations in the term structure model.

Regardless of how the trade-off between price risk and the cost of immediacy or liquidity is executed, the terms of the trade-off depend on the volatility of interest rates. If volatility rises at the same time that liquidity is most impaired, then these hedging strategies could leave the firm exposed to higher than usual price risk.

These results suggest that transaction volume in the underlying fixed-income markets is large enough to enable dealers to manage the risks acquired from the intermediation of price risk in the interest rate options market. Turnover volume in standard hedging instruments appears large enough to accommodate dealers' dynamic hedging in all but the most extreme periods of interest rate volatility. For very large interest rate shocks, however, the hedging of exposures in the medium-term segment of the yield curve could lead to trading demand that is large relative to turnover volume in the more liquid trading instruments. Thus, for large interest

Table 6
CHANGE IN REQUIRED HEDGE POSITION FROM SIMULTANEOUS FORWARD AND VOLATILITY RATE SHOCKS
Billions of U.S. Dollars

Maturity (Years)	Interest Rate Shock	Volatility Shock	Interest Rate and Volatility Shocks
EURODOLLAR FUTURES			
0.5	-31.9	-6.0	-40.7
1	-31.2	-9.7	-38.7
1.5	-23.7	-8.7	-29.4
2	-17.2	-7.7	-22.6
2.5	-13.6	-6.2	-17.9
3	-11.0	-4.9	-14.4
3.5	-9.0	-3.9	-11.6
4	-7.6	-3.0	-9.5
4.5	-6.2	-2.2	-7.5
TREASURY SECURITIES AND FUTURES			
5	2.9	0.8	3.4
10	-3.3	-0.8	-3.8

Source: Author's calculations.

Notes: Hedge estimates are based on data as of the end of March 1995. The table assumes a 75-basis-point increase in forward rates. Volatility is assumed to increase by 25 percent relative to initial volatility levels at six months' maturity and by 8 percent at ten years' maturity. Negative values indicate a short position.

rate shocks, dealers' risk management decisions appear to be driven by a trade-off between price risk and the liquidity costs of immediate hedge rebalancing. Even so, for interest rate shocks typical of a low-inflation environment, the trade-off would need to be managed only for a short period of time.

The Effect of Dynamic Hedging on the Prices of Underlying Assets

Our results on the impact of dealers' dynamic hedging on prices in the fixed-income market are less clear. Any comprehensive assessment would need to account for the demands of other market participants as well. For example, investors whose demands are driven by macroeconomic fundamentals might undertake transactions in the opposite direction of dealers' dynamic hedging flows if interest rates were driven to unreasonable levels. If these investors constitute a sufficiently large part of the market, then their transactions could stabilize prices and reduce or even eliminate positive feedback dynamics (Pritsker 1997). These stabilizing investors, however, are not the only players. Traders who follow short-term market trends in "technical trading" strategies and speculators who anticipate the impact of positive feedback trading also participate in the market. These short-term traders could amplify the price impact of dealers' dynamic hedging because they would trade in the same direction as dealers' hedging transactions (see DeLong et al. [1990]). The ultimate impact of dealers' dynamic hedging would depend on the relative size of different types of market participants. For this reason, our analysis of the volume of dealers' hedging demands provides only a preliminary assessment of the potential for positive feedback because we have data on the hedging demands of dealers exclusively.

At shorter maturities, both the transaction volume and the outstanding stock of the most liquid trading instruments are much larger than dealers' dynamic hedging flows, so that the occurrence of positive feedback from dealers' dynamic hedging seems unlikely, even for very large interest rate shocks. At maturities beyond three years, however, if dealers fully rebalance their hedge positions, dynamic hedging in response to a large interest rate shock could be of significant volume relative to transaction volume and outstanding contracts in the most liquid trading instruments. At this segment of the yield curve, the potential for positive feedback when a very large interest rate shock occurs cannot be dismissed. The volume of dynamic hedging in response to an unusually large interest rate shock could be large enough to affect order flows and might temporarily affect the medium-term segment of the yield curve.

The Effects of Hedging Difficulties on Options Pricing and Market Structure

Our results suggest that the hedge adjustments of dealers in aggregate could be large relative to the size of the market for hedge instruments at the medium-term segment of the yield curve. Given this potential market impact, we consider whether market prices of interest rate options with medium-term maturities contain a premium to cover potential hedging difficulties. To look for evidence of a premium, we compare the implied volatility of interest rates derived from the market prices of options to historical interest rate volatility. An option's *implied volatility* is a volatility parameter in an options-pricing model that causes the model's option price to equal the option's actual price. If the market pricing of options contains a premium, we would expect the implied volatility to be large relative to other measures of the underlying asset's price volatility. Although by no means a comprehensive test, the simple comparison of the term structures of implied volatilities and historical volatilities provides a quick assessment of the possible existence of such a premium.

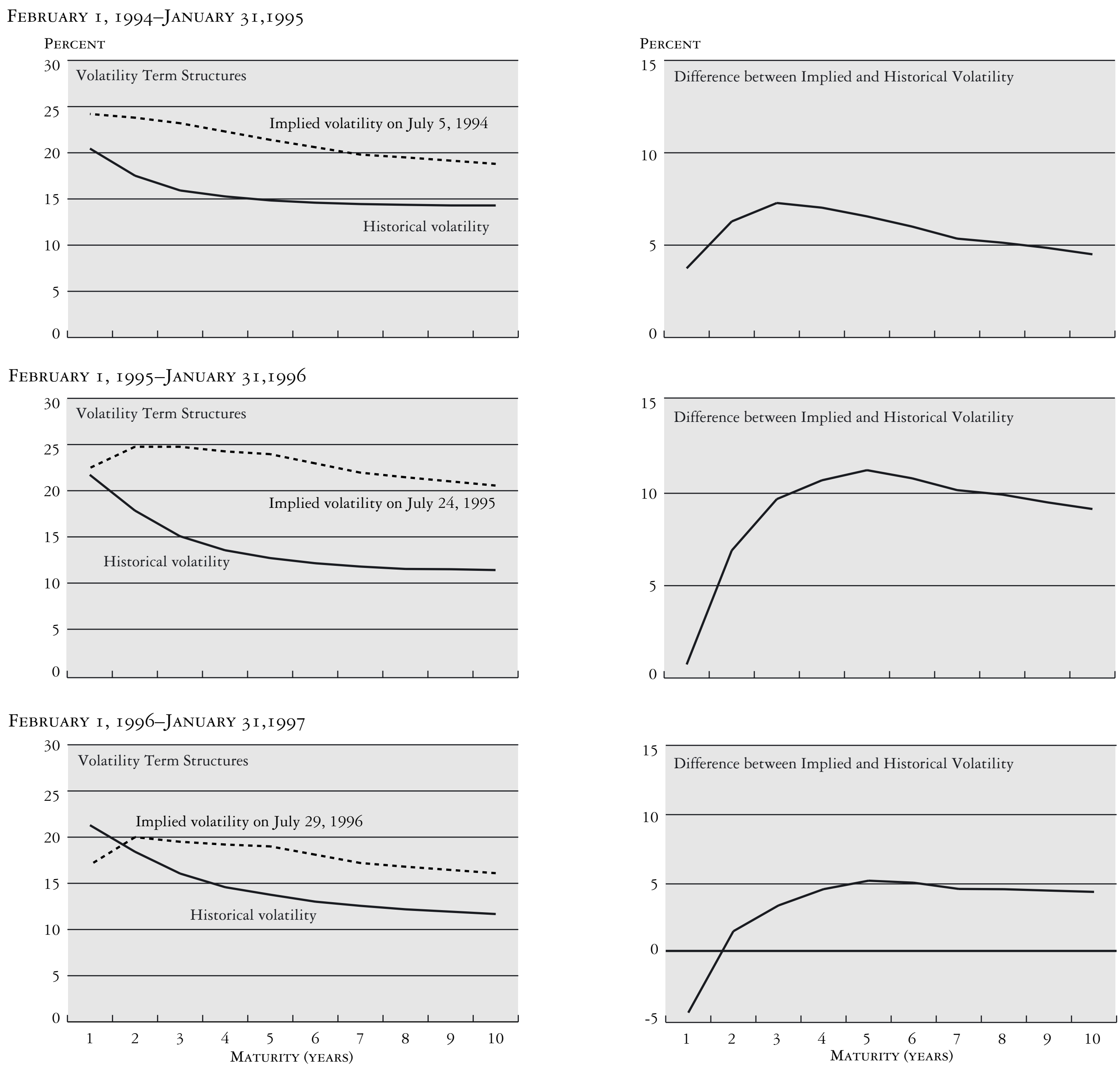
The difference between the term structure of implied volatility and the term structure of the historical volatility of forward Eurodollar interest rates is shown in Chart 2. Notably, the difference is greatest at the medium-term maturities (three to seven years), where the estimated hedge adjustments are large relative to the transaction volume of the hedge instruments. By contrast, the difference between implied volatility and historical volatility is smallest at short-term maturities (under two years). The estimated hedge adjustment relative to transaction volume in hedge instruments is also smallest at this maturity range. Although the difference between the historical and implied volatility term structures could reflect uncertainty about interest rate volatility in the medium-term segment of the yield curve, its shape is consistent with the existence of a premium for hedging difficulties. This apparent consistency between the term structure of options premia and our analysis of hedging volumes suggests a need for further research on how potential hedging difficulties may affect the term structure of interest rate options prices.

Volatility Risk and Hedging Costs

The change in the cost of hedging when interest rate volatility changes also affects the value of an option. Although the

Chart 2

Term Structures of Forward Interest Rate Volatility



Sources: Historical volatilities were derived using the yields on Eurodollar futures contracts as reported by DRI/McGraw-Hill. Implied volatilities are from *Derivatives Week*. In each panel, the historical volatility is for the period indicated.

exposure of an option's value to changes in the level of the underlying asset's price can be hedged, exposure to changes in the volatility of the underlying asset is not hedgeable with a linear fixed-income instrument such as a bond or a futures contract. Rather, an option's volatility risk can be hedged fully only with another option. Given that dealers as a group are net writers of options, their exposure to volatility risk is significant. If most customer options in the over-the-counter market are held to maturity, changes in volatility would affect dealers through changes in the cost of hedging over the life of an option. Higher volatility would raise these hedging costs because it amplifies the costs of adjusting hedge ratios.[12] When volatility changes, the change in an option's value is equal to the expected change in hedging costs over the life of the option.

To look for evidence of a premium, we compare the implied volatility of interest rates derived from the market prices of options to historical interest rate volatility.

An estimate of the sensitivity to volatility shocks of the global dealer portfolio of interest rate options is shown in Chart 3. In the chart, the estimated strike prices are used to revalue the dealers' options portfolio for the indicated changes in volatility. An increase in volatility of approximately 35 to 40 percent causes the portfolio value to turn negative.[13] This change in value of the dealers' options portfolio is a measure of the volatility risk incurred by options dealers.

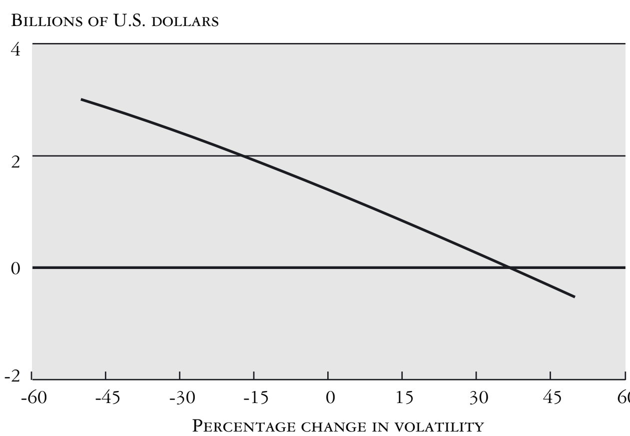
Hedging Difficulties, Volatility Risk, and Market Structure

Volatility risk and potential hedging difficulties may also affect the structure of the interest rate options market. In other derivatives markets, end-user needs are roughly balanced across buyers and sellers,[14] but in the over-the-counter interest rate options market, end-users are mostly buyers. As we noted earlier, dealers of U.S. dollar interest rate options have sold about 50 percent more options to customers than they have bought from customers. Thus, dealers are more willing than other investors to take on the volatility risk in selling an option. Given the wide range of financial assets and risks that investors are willing to acquire, why do they leave interest rate option exposure to dealers?

The concentration of interest rate option exposure among dealers implies that sellers of these options bear unique risks that are not present in the returns of underlying assets and that dealers are more willing to bear those risks. Volatility risk is one risk that is unique to options. Another is the difficulty in adjusting hedge positions as rapidly as required for the accurate hedging of an option's price risk.[15] The fact that dealers are more willing than other investors to sell interest rate options suggests that dealers are in a better position to bear the options' volatility and hedging risks. Dealers have two possible advantages in this area. First, they may be able to execute hedging transactions faster and at lower costs than other investors. Second, they may have other sources of income that offset the volatility risk in an option position. If dealers' income from market making in products other than options rises during periods of higher volatility, then that income will offset the increase in volatility risk from selling options. While some of that higher income would be compensation for the higher risk that dealers incur in making prices in volatile markets, any

Chart 3

Options Values as a Function of Volatility Changes



Source: Author's calculations.

remaining excess returns would offset the higher volatility risk from the options book. Such offsetting risks may explain why large interest rate options dealers also are market makers in a broad array of fixed-income products, and, for that reason, are willing to bear volatility risk at a smaller premium than other investors.

The evidence that market making in options *and* other products provides dealers with offsetting exposures to changes in volatility is not strong, however. For instance, even though the turnover volume of derivatives grew rapidly during 1994, dealers' trading income suffered from the bond market turbulence that occurred in that year. It has been reported that a significant part of the 1994 earnings decline occurred in dealers' bond and proprietary trading positions and not in their market-making activity.[16] Nevertheless, we lack detailed data on market-making income that would enable us to resolve with any certainty the question of offsetting exposures to volatility.

CONCLUSION

Our analysis suggests that transaction volume in underlying markets is large enough for dealers to manage the price and liquidity risks they incur through the intermediation of price risk in selling interest rate options. With the possible exception of the medium-term segment of the term structure, turnover volume in the most liquid hedging instruments is large enough to absorb dealers' dynamic hedging.

In the case of an unusually large interest rate shock at the medium-term segment of the term structure, the full rebalancing of hedge positions would generate hedging transactions that would be large relative to daily transaction volume in the most liquid medium-term instruments. In this case, dealers' risk management decisions would appear to be driven by a trade-off between price risk and the liquidity costs of immediate hedge rebalancing. For interest rate shocks of the size experienced in the last five years, dealers' hedge adjustments would be a small proportion of only a few days' worth of turnover volume, and dealers would need to manage the trade-off between liquidity and price risks only for a short period of time. For large interest rate shocks, however, such as those experienced by a country in the midst of a currency crisis or a period of high inflation, the hedging of exposures in the medium-term segment of the yield curve could lead to trading demand that is large relative to turnover volume in the more liquid trading instruments.

The ratios of estimated hedge adjustments to transaction volume in trading instruments at different maturities are consistent with the pattern we find in the term structure of option premia. The term structure of implied volatility shows an apparent risk premium for options at the medium-term segment of the yield curve, a segment that corresponds to the maturity range where hedging difficulties might occur. The structure of the over-the-counter interest rate options market is also consistent with the hypothesis that such hedging problems may exist. Despite investors' willingness to hold a wide variety of financial assets and risks, they choose to leave interest rate options exposures in the hands of dealers. This preference suggests that interest rate options sellers are exposed to risks that are not present in the returns of underlying assets. These risks are likely volatility and hedging-related risks, which may be managed more effectively by dealers than by other market participants.

The results presented in this article provide a preliminary assessment of the impact of dynamic hedging on market liquidity and price dynamics in the fixed-income market. As the appendix makes clear, limitations of the data make further investigation worthwhile. In addition, an estimate of the market excess demand function and the relationship between prices and quantities would be useful. Such an analysis, however, would require data that do not currently exist on investors' demands in addition to dealers' hedging demands. Nonetheless, comparing potential hedging demand with transaction volume in typical hedging instruments is useful in assessing the likelihood of positive feedback.

THE DATA

Our primary source of data is the 1995 Central Bank Survey of Foreign Exchange and Derivatives Market Activity (Bank for International Settlements 1996). This survey of derivatives dealers worldwide reports global market totals of outstanding contracts as of the end of March 1995. The over-the-counter options market is a dealer market where all options contracts involve a dealer on at least one side of the contract. An option contract can either be a transaction between two dealers or a contract between a dealer and a customer. The central bank survey captured the entire over-the-counter options market by collecting data from the dealers that executed all contracts.[17] The options data in the survey include notional amounts, market values, and maturity data, broken down by bought and sold options, as shown in Table 1.[18] The options were also broken down by the survey's three counterparty categories: interdealer options, options bought from customers by dealers, and options sold to customers by dealers. Because reporters in the survey were derivatives dealers, interdealer transactions appear as both bought and sold options. In other words, an option bought by one dealer from another was reported as a bought option by one dealer and as a sold option by the other.[19]

OPTION VALUATION

All options in the estimation are caps and floors on six-month interest rates. In accordance with the data from the International Swaps and Derivatives Association (ISDA), 73 percent of the options in the estimation are assumed to be caps, and the remainder are assumed to be floors. Although a small proportion of interest rate options are swaptions (19 percent at year-end 1994 in the ISDA data), for simplicity, we treat all options as either caps or floors.[20] Option values are calculated using Black's forward contract option model, the benchmark model used for implied volatility quotes for interest rate options (see Hull [1993]). The valuation uses the term structure of forward rates and the term structure of implied volatilities coinciding with the central bank survey data (end of March 1995).[21] To test the valuations, we also calculate the option values using different volatility structures. The baseline case assumes that caps and floors have identical implied volatilities that do not vary with moneyness. Alternative valuations using a volatility smile for options with different degrees of moneyness and higher implied volatilities for floors relative to caps did not affect our conclusions (see table below).

MATURITY DISTRIBUTION

To estimate the distribution of notional amounts over the remaining maturity and origination dates, we fit a quadratic function defined over original maturities to the remaining maturity data from the central bank survey and the ISDA

VOLATILITY ASSUMPTIONS: CHANGE IN HEDGE POSITION FROM 75-BASIS-POINT INCREASE IN FORWARD RATES
Billions of U.S. Dollars

	Assumption			
Maturity (Years)	Base	Cap/Floor Volatility	Volatility Smile	Cap/Floor Volatility and Smile
EURODOLLAR FUTURES				
0.5	-31.9	-31.5	-27.2	-26.8
1	-31.2	-31.2	-27.8	-27.7
1.5	-23.7	-23.7	-21.1	-21.0
2	-17.2	-17.2	-14.6	-14.5
2.5	-13.6	-13.6	-11.4	-11.3
3	-11.0	-10.9	-9.1	-9.0
3.5	-9.0	-9.0	-7.4	-7.4
4	-7.6	-7.5	-6.2	-6.2
4.5	-6.2	-6.2	-5.3	-5.3
TREASURY SECURITIES AND FUTURES				
5	2.9	2.9	2.6	2.6
10	-3.3	-3.3	-2.9	-2.9

Source: Author's calculations.

Notes: Hedge estimates are based on data at the end of March 1995. Negative values indicate an increase in a short position. The base is the estimate using the assumptions in the text. For the cap/floor volatility assumption, option values were calculated with higher volatility for floors using cap and floor implied volatility differences reported by DRI/McGraw-Hill. For the volatility smile assumption, option values were calculated using a volatility smile derived from Eurodollar futures options prices.

market growth data. The three maturity categories in the central bank survey provide the three equations required to estimate the three parameters of the quadratic function. In the estimation, the market growth rates between origination dates are applied as a scaling factor to the quadratic function. We estimate separate maturity distributions for options purchased from customers and options sold to customers.[22]

In the estimated distribution, most outstanding contracts have less than four years' remaining maturity and have origination dates that fall within three years of the central bank survey date. The estimated distribution has a trough along the diagonal for caps with maturities at origination of between five and ten years. This feature of the distribution suggests that long maturity caps are originated at discrete maturities, specifically at the ten-year maturity.[23]

STRIKE PRICES

Strike prices are derived from historical yield curves and assigned to the options using the estimated distribution of notional amounts over origination dates. Because separate market values are not available for caps and floors, the estimation requires that a relationship between the strikes of caps and floors be imposed. The relationship assumed is that buyers (or sellers) of caps and floors have similar preferences regarding their options' moneyness. Under this assumption, if buyers of caps desire out-of-the money options because of their cheaper premia, then buyers of floors will also.

We implement the assumed relationship regarding the moneyness of caps and floors in three different ways. In all three approaches, the historical swap term structure at an option's origination date is our starting point. The first method is a proportional displacement of the strike rates from the historical swap term structure (in the same proportion, but opposite directions, for caps and floors). The other two methods are displacements of the strikes from the historical swap term structure with a constraint that caps and floors (of the same maturity and origination date) have equal premia (the second method) or equal deltas (the third method). Under the last two methods, the strikes for caps and floors can have different displacements from the swap term structure.

In these specifications, a cap will be out-of-the-money at origination when a floor is out-of-the-money. In each specification, the restrictions are applied to bought and sold options separately. The figures in the text are derived using the first approach, but similar results followed from the other specifications.[24]

ESTIMATED STRIKE PRICES AND OPTION VALUES

Given the strike price specification relative to historical yield curves, option values are calculated as functions of the displacement of the strike prices from the historical yield curves. The estimation then involves finding the displacement that produces option values equal to the market values observed in the central bank survey.

The objective of the estimation is to find values of the strike price displacement variables (A_b, A_s) such that

$$V_b(A_b) + v_D = v_b$$
$$V_s(A_s) + v_D = v_s,$$

where v_b and v_s are the observed market values of U.S. dollar options bought and sold by dealers (including interdealer options), and v_D is the market value of interdealer options. The functions $V(A)$ are the option values as functions of the displacement (A) of the strike prices from the historical term structures, and the subscripts indicate options bought (b) and sold (s) from customers.[25] In the proportional displacement specification of the strike price, the term A is a single variable. In the other two cases, the term A is a vector with two elements—the displacement for caps and the displacement for floors. In both cases, the additional equation required to solve for the two displacement variables is the equal premia or equal delta restriction in the specification of the strike prices.

In the option value equations above, the market value of U.S. dollar interdealer options is not available directly from the central bank survey data because market values are reported in aggregate—across counterparty types and across currencies (Table 1, middle panel). To find the displacement variables for bought and sold options (A_b and A_s) in the option value equations above, we first estimate the value of U.S. dollar interdealer options.

INTERDEALER OPTIONS

Estimates of the market value of U.S. dollar interdealer options are calculated in four different ways. In the first three methods we make assumptions about the strike rates of interdealer options: (1) interdealer options have strikes equal to swap rates (at-the-money strikes relative to the swap term structure); (2) interdealer options have the same strikes as options bought from customers; and (3) interdealer options have the same strikes as options sold to customers.[26] In the fourth method, we estimate the value of U.S. dollar interdealer options using the data reported in Table 1. In this method, the market value of interdealer options is distributed between U.S. dollar options and options on other currencies so as to minimize the discrepancy between the ratio of market value to notional amount for each currency and counterparty type and the ratio of the market value to the notional amount of the margin totals in the top and middle panels of Table 1.

The first and last alternatives produce comparable values for U.S. dollar interdealer options, while the other two do not. The estimation using the at-the-money assumption produces a value of interdealer options of $11.3 billion, while the last method results in a value of interdealer options of $10.9 billion. Strike prices that produce a value of $10.9 billion would be very slightly out-of-the-money at origination. The comparability of the estimates in methods one and four implies that interdealer options have strikes closer to at-the-money than do customer options. This result is plausible because dealers using the interdealer market to hedge their short volatility and negative gamma position would obtain more hedging benefit from at-the-money options. Such options have larger gamma and provide the most hedging benefit relative to their premia. The results reported in the text are derived using the fourth method. Despite the different estimates of interdealer option values, similar hedge estimates follow from all four alternatives.

CUSTOMER OPTIONS

For options sold to customers, estimated strikes consistent with observed market values are deep out-of-the money (relative to swap rates of comparable maturity) at origination. This result is plausible—customers buying options to hedge can acquire inexpensive protection against large interest rate shocks with deep out-of-the money options. For caps sold to customers, the estimated strike rates are 18 percent higher than swap rates of the same maturity at origination. The figure of 18 percent is comparable to the standard deviation of annual changes in interest rates, or two standard deviations of quarterly interest rate changes (six-month LIBOR rates from January 1991 to December 1995).

For options bought from customers, strike prices consistent with the observed market values are slightly in-the-money (relative to swap rates of comparable maturity) at origination. This relationship is the opposite of the relationship found for options sold to customers. Although this result might appear counterintuitive and could point to a problem in the estimation, it is consistent with market commentary in the early 1990s. Customers looking for "yield enhancement" during the low-interest-rate regime of the early 1990s acquired higher premia by selling interest rate caps with a higher degree of moneyness. While this higher yield is the market price or compensation for the expected payout of the option, investors speculating on the path of interest rates by selling options would obtain higher investment returns (or losses) per option by selling in-the-money options. In addition, investors who believed that the forward curve was an overestimate of the future path of spot rates would have sold options that were in-the-money relative to swap rates.

HEDGING ASSUMPTIONS

The final step in the estimation of dealers' hedge adjustments is the calculation of the delta and the change in delta of the global dealers' portfolio using the estimated distribution of notional amounts and the estimated strike prices. The analysis of dealers' hedging behavior relies on the following assumptions:

1. After an interest rate shock, dealers restore the net delta of their position to its initial level. Dealers may or may not fully hedge the initial delta of the options book, and whatever hedging is done initially may be accomplished either internally, with offsetting positions in the firm, or externally, with hedging transactions. These initial offsetting positions, either internal or external, are assumed to have a small gamma, so that changes in interest rates—and thus the options' delta—make additional hedging transactions necessary to return the portfolio's net delta to its original level.

2. An option exposure to a period t interest rate is hedged with an instrument that also has exposure to the period t interest rate—there is no basis risk in hedged positions. Using this assumption, we calculate a separate hedge ratio for each maturity's exposure.

3. Customers do not hedge their options positions. Customers who have bought or sold options are assumed not to hedge, because doing so would negate whatever investment or hedging objective the options were used for. Customers who have sold options to dealers presumably did so for speculative "yield enhancement" or intertemporal income shifting. The costs of delta-hedging the options would negate that investment objective. Customers who have bought options from dealers for hedging purposes would not hedge the option because doing so would expose the underlying position that the option was hedging. Thus, the impact of dynamic hedging is assessed using the aggregate dealers' position.

   If customers were to hedge their options, perhaps as a result of a reassessment of risks, then the market impact of dealers' hedge adjustments would be smaller because these adjustments would be offset by customer hedges. Because most of our results support the claim that the market impact of dealers' hedging is small relative to the size of the market, dropping the assumption would strengthen our conclusion that the markets for typical hedging instruments are sufficiently large for dealers to manage the price risk acquired from market making in options.

4. Interdealer options have no effect on dealers' net demand for hedge instruments. Using this assumption, interdealer options can be ignored, and the net hedge position and hedge adjustment of dealers in the aggregate can be calculated from dealers' contracts with customers. This assumption is reasonable when interdealer options are executed to reallocate customer exposures among dealers or to take a position in volatility risk but not directional interest rate risk. In the first case, the interdealer option that passes a customer exposure from one dealer to another does not create additional net option exposure for dealers in the aggregate. Thus, dealers' net hedge demands would be unaffected by such interdealer options.

   The second type of interdealer trading that is consistent with this assumption is position taking on changes in interest rate volatility. This trading strategy entails the hedging of directional interest rate risk. If executed by dealers on the two sides of an interdealer trade, such hedges would offset each other in the market, with no impact on the net dealer hedge amount.

An important justification for the presumption that dealers' position taking in options is a position on volatility changes is the fact that dealers wishing to take directional exposures to interest rate risk could do so less expensively with instruments other than options.

ROBUSTNESS CHECKS

To test our results, we derive estimates of dealers' hedging using alternative assumptions about implied volatility, the structure of strike prices, and other restrictions. The variation in hedge demands across these assumptions is small relative to turnover volume in the hedge instruments and does not change our conclusions. The results under these alternative assumptions are available in Kambhu (1997, Tables 5-8).

Although the results are robust to alternative assumptions, they might be influenced by certain features of the central bank survey data. First, dealers might have had options positions that were not reported in the central bank survey. Index amortizing interest rate (IAR) swaps, for example, might have been reported as swaps instead of options. These instruments were popular in the early 1990s, when investors were searching for yield enhancement. The extra yield in this instrument is the premium for a written option embedded in the instrument's payoff structure. Most of the volume of IAR swaps, however, was in contracts of three years' or shorter maturity. By the time of the survey, outstanding volume was likely to have been too small to affect the results.[27]

In addition, the timing of the central bank survey may have caused the survey data to capture patterns in option strike rates, the mix of bought and sold options, or maturity that were unique to 1995. The survey in 1995 followed a period of low interest rates in the early 1990s and a shift to tighter monetary policy in 1994. Data from the ISDA surveys show that the over-the-counter interest rate options market grew rapidly in 1993 and 1995. Growth, however, was lower than usual in 1994. The interest rate swaps market, by contrast, grew rapidly in 1994, especially during the first half of the year. Whether these patterns affected the survey results can best be determined by replicating the study at some future date.[28]

ENDNOTES

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1. For additional discussion of the intermediation of price risk, see Kambhu, Keane, and Benadon (1996).

2. Gennotte and Leland (1990) summarize the debate surrounding the role of portfolio insurance in the 1987 stock market crash.

3. For further discussion of market liquidity and risk management, see Bank for International Settlements (1995, Chap. 2).

4. The ISDA data consist of notional amounts but not market values. As a result, the analysis in this article was not possible until the 1995 central bank survey supplied market value data as well as a breakdown of gross options positions by bought and sold transactions.

5. The value of an interest rate cap becomes more sensitive to changes in interest rates as rates rise (see box). In Chart 1, the dealers' portfolio value falls at an increasing rate because dealers are net sellers of options and thus incur increasing option liability as rates rise.

6. This assumption is reasonable when interdealer options are executed to reallocate customer exposures among dealers, or to take a position in volatility risk but not directional interest rate risk. For further explanation, see the hedging assumptions section in the appendix.

7. Turnover volume for U.S. dollar interest rate swaps at the time of the survey was $17 billion per day (Bank for International Settlements 1996). In contrast, Eurodollar futures turnover volume was $463 billion per day, and turnover of the five- and ten-year Treasury on-the-run securities and futures was $29 billion per day.

8. Exchange-traded options on futures contracts are also a potential hedging instrument. The survey data, however, show that dealers as a group have bought and sold roughly equal amounts of exchange-traded options. Thus, these instruments cannot be providing a net hedge to the aggregate dealer position, and dealers as a group must be relying on other hedging instruments.

9. In Tables 2, 3, and 4, the hedge for each six-month exposure requires two back-to-back futures contracts because the contracts are on a three-month interest rate. For example, in Table 2, in response to a 25-basis-point rise in interest rates, at the two-and-a-half-year maturity the hedge adjustment comprises a sale of $4.6 billion in each of the two back-to-back contracts that span the interval between two-and-a-half and three years. This amount is less than the turnover volume of $9.2 billion and $7.5 billion in the two contracts that match the maturity of the hedge position.

10. On-the-run securities, or the most recently issued securities, are the most liquid Treasury issues. As a security ages, a larger proportion of the issue tends to be held in long-term investment portfolios and thus is traded less frequently.

11. The cash market for the on-the-run Treasury security by itself appears too small to accommodate dealers' hedge demands. If dealers fully hedged their exposures beyond five years using five- and ten-year on-the-run issues, the required hedge position would be approximately equal to the outstanding amount of the on-the-run five- and ten-year notes (Table 5). The Treasury securities market, however, can still accommodate a significant share of this hedging demand in two ways. First, the lending of Treasury securities in the repo market allows a fixed stock of on-the-run Treasury securities to meet trading demands that exceed the size of the on-the-run issue. Through the repo market, a trader who sells a borrowed security to establish a short position enables another trader to establish a long position in the security. As a result, market participants' long positions in the security can be significantly larger than the outstanding stock of the security. Second, off-the-run issues can be used as long as they are available. Fleming (1997) reports that off-the-run securities account for approximately 24 percent of daily turnover in the interdealer market.

12. Dynamic hedging requires a dealer to buy the underlying asset after its price rises and to sell it after the price falls. The cost of implementing this "buy high, sell low" trading strategy is higher when price changes are more volatile.

13. In the five-year period beginning in 1991, the three largest changes in implied volatility for one-year options on Eurodollar futures were between 33 percent and 38 percent for two-week changes in implied volatility.

14. See Kambhu, Keane, and Benadon (1996).

15. For a study of how market prices of options are influenced by volatility and hedging risks, see, for example, Jameson and Wilhelm (1992) for the pricing of exchange-traded stock options.

16. See *Risk* (1994) and *Swaps Monitor* (1996).

17. The interest-rate-related options were predominantly caps and floors. The central bank survey also included data for over-the-counter options on traded interest rate securities (bond options). These options were not included in our analysis, because they amounted to less than 8 percent of options related to interest rates. Moreover, the bought and sold amounts

Note 17 continued
of these options were in rough balance, leaving dealers with very little residual hedging needs from positions in these options.

18. The notional amount of a derivative contract is a reference amount used to calculate the size of the cash flows between the counterparties to the contract. (These cash flows are determined by the price of an underlying asset.) The market value of a derivative contract is the net value of the cash flows to be exchanged between the counterparties over the remaining life of the contract. Notional amounts are a measure of contract size that is independent of the price of the underlying asset, while market values are a measure of contract size that is based on the market value of the transaction. Market values of derivative contracts are almost always a small proportion of the notional amount.

19. Because of reporting error, the bought and sold amounts of interdealer options reported in the survey are slightly different. To account for this reporting error, we average the bought and sold figures to arrive at the interdealer volume used in the estimation. This averaging should reduce the effects of the error.

20. The exclusion of swaptions is not likely to alter the article's conclusions for the following reasons. If a one-year option on a five-year swap was reported as a one-year option, then the swaptions would appear as shorter maturity options in the data. Hence, the true exposures of shorter maturity would be less than estimated, with the result that hedging demand for shorter maturity instruments would be smaller than estimated. This effect would only strengthen the conclusion that shorter maturity hedging volumes are small relative to transaction volume in Eurodollar futures. The swaptions, however, would add to the estimated hedging demand at longer maturities. Nevertheless, because swaptions make up only 19 percent of the market, the net increment to estimated hedging demand would not significantly change our conclusions. Rather, the effect would be to strengthen the conclusion that longer maturity hedging demand could be significant relative to transaction flows in longer maturity hedge instruments, but not so much larger as to overwhelm the market.

21. The implied volatility and forward interest rate data are from *Derivatives Week* (1995). The *Derivatives Week* data on forward rates and implied volatility are consistent with those implied by Eurodollar futures prices and Eurodollar futures options prices.

22. For further details, see Kambhu (1997).

23. To test whether the clustering at the ten-year maturity was a product of the quadratic function in the estimation, we derived an alternative estimate using a linear maturity distribution out to seven years and a separately estimated ten-year share. This alternative produced similar results for both the maturity distribution and the hedging volumes. In a further test, a linear distribution out to nine years with a separate ten-year share produced nonsensical results with negative values at the longer maturities in the linear segment of the distribution.

We also derived results with alternative maturity estimates. These alternatives did not change our conclusions (see Kambhu [1997, Table 7] for further details). The heavy distribution of notional amounts in the one-year remaining maturity range, which constrains the effects of the alternative estimation methods, may explain the robustness of the results.

24. Alternative strike price structures did not produce much variation in the hedge estimates relative to turnover volume in the hedge instruments and thus did not affect the conclusions. See Kambhu (1997, Table 5) for further details.

25. These functions are defined by the strike price specification and the estimated distribution of notional amounts. See Kambhu (1997) for further details.

26. In the first three methods, the estimation of interdealer market values relies on the assumption that the maturity structure of interdealer options is equal to the average of the bought and sold options' maturity distributions.

27. Cumulative volume of IAR swaps originating between 1990 and 1994 was about $100 billion to $150 billion in notional principal (Galaif 1993-94).

28. Beginning in June 1998, global derivatives market data similar to the 1995 survey will be collected on a semiannual basis by the Group of Ten central banks and published by the Bank for International Settlements.

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