

Targeting in a Dynamic Model

Support for monetary targeting is eroding. Many economists and government officials express increasing concern that monetary targeting destabilizes both the financial and real sectors of the economy. Unnecessary volatility in the money supply, interest rates, and the levels of income and employment, they argue, comes from attempting to target money too rigidly. The sharp swings in the economy during the three-year period following the change in the Federal Reserve's operating procedures in October 1979 are cited frequently, although the second oil price shock and the credit control program certainly contributed to the increased volatility. Moreover, they view the much smoother performance of the economy since late 1982 as a telling development. Around that time, the Federal Open Market Committee (FOMC) reduced its emphasis on M-1 relative to the broader monetary aggregates; and in view of rapid institutional change, it adopted a more flexible approach to achieving the objectives for the aggregates.

As support for monetary targeting wanes, the search for an alternative approach to policymaking intensifies. Many agree that the Federal Reserve should have targets or numerical objectives of some kind. Targets communicate to the public the long-run direction of monetary policy and provide Congress with some basis to assess FOMC decisions. Against this background, some economists (among them James Tobin and Robert Gordon) have advocated nominal GNP targeting.¹ They claim adopting this strategy will lead to better achievement of the ultimate objectives of monetary policy.

¹James Tobin, "Monetary Policy Rules, Targets, and Shocks", *Journal of Money, Credit, and Banking* (November 1983), pages 506-18, and Robert J. Gordon, "Using Monetary Control to Dampen the Business Cycle: A New Set of First Principles", National Bureau of Economic Research Working Paper, Number 1210

This paper investigates the properties of both monetary targeting and nominal GNP targeting. The first issue to be covered is whether adhering rigidly to monetary targets does indeed lead to unnecessary volatility in the financial and real sectors of the economy. Perhaps the monetary targets and the ultimate objectives of policy are actually best achieved by attempting less rigid control. Currently, these matters are particularly relevant. With the implementation of contemporaneous reserve requirements and the more normal behavior of M-1's velocity recently, some may feel that this year offers an opportune time to return to tighter monetary targeting.

The second issue is the effectiveness of nominal GNP targeting versus monetary targeting. How well do the two policy strategies stabilize nominal income around desired levels? If GNP targeting can be shown to be more effective, then the strategy advocated by Tobin and Gordon has a firmer foundation.

Throughout the article, the issues are examined by using the most compact model of the economy possible. Nevertheless, the model's framework is kept versatile enough to study the consequences of alternative approaches to monetary policy in a dynamic setting—the primary purpose of this article. But many of the practical and institutional constraints that surround both monetary targeting and nominal GNP targeting are left aside.²

The first section shows that a monetary target is best achieved over time by gradually offsetting deviations from

²For a discussion of some of the practical problems with monetary targeting and nominal GNP targeting, see the articles by John Wenninger (page 1) and Douglas M. Woodham (page 16) in this issue. Also see Anthony M. Solomon, "Unresolved Issues in Monetary Policy", *this Quarterly Review* (Spring 1984), pages 1-6, and John B. Carlson, "Nominal Income Targeting", Federal Reserve Bank of Cleveland *Economic Commentary* (May 21, 1984).

target. And the second section goes on to show that moderation in pursuing a monetary target will also contribute to lower volatility in interest rates *and* nominal income. But this raises a question about the usefulness of monetary targeting in stabilizing income in the longer run. In the last section, the strategy of reacting to movements in income emerges as more effective than monetary targeting in stabilizing the level of nominal income. This turns out to be the case for this particular model, even when the demand for money is stable and no financial innovation or deregulation is occurring.³

The volatility of money and the interest rate under monetary targeting

To begin, consider a simple version of the monetary sector, separate from the rest of the macroeconomy. Two relationships comprise it: the demand for money and a policy formula. The demand for money is based on a transactions motive for holding money. The total volume of money demanded by households and firms is determined primarily by the levels of income and the interest rate. Furthermore, it is assumed that households and firms take some time to adjust their holdings to changes in income and the interest rate. Simple one-period lags are specified in the demand-for-money function to reflect this. (Longer and more complex lag patterns could be used, but at the cost of greatly complicating the mathematics underlying the analysis.) This means that the current values of income and the interest rate and their previous period's values jointly determine the quantity of money demanded. Because the relationship is presumed not to hold exactly, a random disturbance term is included, representing all other factors in the demand for money. The disturbance term satisfies all the usual assumptions. So, the demand for money can be written as:

$$(1) M(t) = a - br(t) - cr(t-1) + dY(t) + eY(t-1) + v(t)$$

where:

- M = actual money supply,
 - r = the interest rate,
 - Y = nominal income,
 - v = a random disturbance term, and
 - t denotes the time period.
- (b, c, d, and e > 0)

The policy formula attempts to succinctly represent the essence of decision-making while following a strategy of monetary targeting. The formula used here states that in order to achieve the target the interest rate is moved upward when the money supply is above target, and downward when it is below target. The movement in the interest rate is in proportion to the deviation of the actual

money supply from the target.⁴ The deviation is measured over the interval from the previous FOMC meeting to the present one.⁵ The new level of the interest rate is then to be maintained until the time of the next meeting. So, the policy formula can be written as:

$$(2) [r(t) - r(t-1)] = \lambda[M(t-1) - M^*]$$

where

M^* = the targeted level of the money supply.

The change in the level of the interest rate, $[r(t) - r(t-1)]$, is related to the discrepancy between actual and targeted money, $[M(t-1) - M^*]$, by the coefficient λ . It measures the strength of the response to deviations from target. For example, suppose the FOMC sets the value of λ at 2.00 (and the money supply is measured in billions of dollars). Then the interest rate is moved 2 percentage points for every billion dollars the money supply is away from target.

Implicit in this representation of policymaking is the assumption that the FOMC—with its 12 members, each possessing his or her own views on monetary economics—can reach a consensus on how strong the response to deviations from the monetary target should be. The value of λ reflects this consensus. Now, assuming that the true value of the coefficient b in equation 1 (the short-run response of money demand to the interest rate) is 0.10, the FOMC, by setting λ equal to 2.00, would actually be attempting to correct 20 percent of a deviation from target immediately. Each FOMC member, however, may not estimate the coefficient b to be exactly 0.10. So, each FOMC member could believe that setting λ at 2.00 means that something other than 20 percent immediate correction is being sought.

In general, setting λ equal to $(1/b)$, whatever the value of b is, implies that, in reality, immediate correction of deviations is sought, although individual FOMC members may think differently. A value of λ between zero and $(1/b)$ means that partial correction is attempted in the upcoming period. In other words, the time horizon over which the money supply is to be brought back to target is somewhere

⁴A similar representation of monetary targeting was used by Jared Enzler and Lewis Johnson, "Cycles Resulting from Monetary Targeting", in *New Monetary Control Procedures*, Federal Reserve staff study, Volume 1, Board of Governors of the Federal Reserve System Washington, D C (1981), page 3.

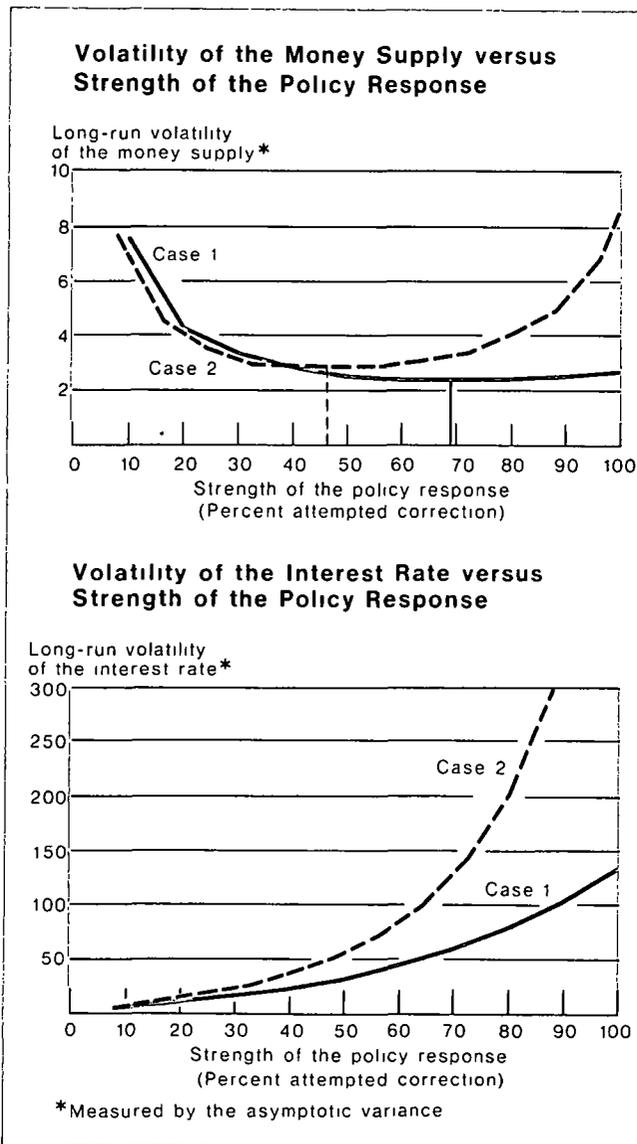
⁵The policy formula contains the previous period's, and not the current period's observation on the money supply. The assumption then is that the FOMC reacts to observed, rather than anticipated changes in money at the time of each meeting. The erratic nature of the monetary data, as well as the difficulty of accurately projecting the money stock, suggests that this is a reasonable assumption. If the current period's observation is incorporated instead, many of the results obtained in the analysis are reversed. Furthermore, equation 2 should not be construed as a representation of monetary targeting with a nonborrowed reserve target at the tactics level. In that setting, there is some immediate response to a deviation from target.

³The article by John Wenninger in this issue addresses the problems for monetary policy created by financial innovation and deregulation.

beyond the upcoming period, the smaller the value of λ , the longer the horizon

If these two equations are combined, they can be used to derive the long-run volatility (or asymptotic variance) of the money supply about its target, and correspondingly, the interest rate about its level that is consistent with the monetary target⁶. These values, of course, depend on (1) the interest rate elasticity and lag structure of money demand and (2) the particular value selected for the coefficient λ .

⁶The use of the asymptotic variance to measure the effectiveness of stabilization policies was developed in E. Phillip Howery, "Stabilization Policy in Linear Stochastic Systems", *Review of Economics and Statistics* (August 1967), pages 404-11



There will be, though, a value of λ that minimizes the variance of the money supply for a prespecified demand for money. This minimum variance implies that the money supply is being kept, on average, as close to the target as possible.

Rather than presenting at this point the algebraic solution for the value of λ that produces the tightest effective control over money, two specific examples will be drawn. The two examples differ in terms of the speed with which individuals and firms adjust their money holdings to interest rate changes. In the first example, two-thirds of the total adjustment in money holdings to a change in the interest rate occurs immediately, that is, in the same time period that the interest rate changes (Or, $b = 0.10$ and $c = 0.05$ in the money-demand equation 1.) In the second example, the adjustment takes place less rapidly, occurring almost equally in the current and the following period ($b = 0.08$ and $c = 0.07$)⁷.

The two examples are used to show the relationships between, on the one side, the strength of the policy response, and on the other, the long-run variances of the money supply and the interest rate. The relationships are plotted in the diagram, the strength of the policy response is varied from 0 to 100 percent attempted correction in the upcoming period. In the upper panel of the chart, monetary targeting appears most accurate in the first example when the response is to attempt to eliminate 69 percent of the deviation in the upcoming period, in the second example, 46 percent. Clearly, in neither case is the money supply kept closest on average to target by attempting to eliminate deviations from target entirely in the next period.

In general terms, the presence of lagged interest rate effects on the volume of money demanded explains why attempts to immediately correct deviations from target do not produce the greatest effective control. If the interest rate is moved upward to correct an overshoot in money completely in the upcoming period, the lagged effect of the interest rate change would push the money supply below target in the following period⁸. Thus, attempting to correct deviations too quickly only increases the volatility of the money supply and is ultimately counterproductive. Conversely, attempting to correct deviations from target gradually—in other words, over an appropriately long horizon—can lead to greater success on average in keeping the money supply close to target. In fact, as long as there is partial adjustment in the same period in which the interest rate moves, complete immediate correction will be less than optimal.

The lower panel in the chart shows that the asymptotic variance of the interest rate increases exponentially as the

⁷If $b < c$ the model exhibits undamped (explosive) cycles.

⁸That is, unless the disturbance term takes on a large positive value in the upcoming period.

strength of the policy response increases.⁹ This agrees with the widespread view that in order to smooth the course of the money supply, the interest rate must fluctuate widely. In this situation, uncertainty over the relative size of the current and lagged interest rate effects on the demand for money would lead policymakers to respond conservatively to money supply deviations. To illustrate with the second example, the money supply is kept closest to target by attempting to eliminate 46 percent of the observed deviation in the upcoming period. If this is done, the asymptotic variance of the interest rate is about 48 times the variance of the disturbance term attached to the demand for money. If the policy response is strengthened with the intention of eliminating 64 percent of the observed deviation immediately, interest rate volatility roughly doubles. Conversely, if the policy response is weakened to 32 percent, interest rate volatility drops off to 24 times the variance attached to money demand.

With these figures in mind, suppose it is somewhat uncertain how quickly the public adjusts its money holdings. Policymakers are then unsure as to how quickly to bring money back to target. Obviously, it would be advantageous for them to err on the low side in determining the speed with which the money supply is brought back to target.

The effect of adding the real sector to the model

The model used in the previous section represented just the monetary sector. The real sector was omitted, as mentioned earlier, to keep the mathematics relatively simple. But to understand the basis for nominal GNP targeting and then compare its performance with that of monetary targeting, the real sector must also be part of the model. Incorporating the real sector modifies the two major results regarding monetary targeting. The results now reflect the impact of the monetary sector on the real sector and vice versa, but they are not fundamentally changed.

Let us add to the monetary sector a version of the widely used multiplier-accelerator model. There are two behavioral relationships in this model. First, consumption spending depends solely on income. Second, investment spending responds to the interest rate and to changes in the level of output. Neither relationship is assumed to hold exactly, random disturbance terms are included. Thus, the real sector can be represented as:

$$(3) C(t) = f + gY(t) + u_1(t)$$

$$(4) I(t) = h - jr(t) + k[Y(t-1) - Y(t-2)] + u_2(t)$$

⁹This result appears to be the analogue in a stochastic model to the problem of "instrument instability" in a deterministic model, identified first by Holbrook. See Robert S. Holbrook, "Optimal Economic Policy and the Problem of Instrument Instability", *American Economic Review* (March 1972), pages 57-65.

where.

C = nominal consumption,

I = nominal investment, and

u_1 and u_2 are disturbance terms.

($0 < g$ and $k < 1$, $j > 0$)

By combining these two equations with the income identity, we obtain the "IS curve", equation 5:

$$(5) (1-g)Y(t) - kY(t-1) + kY(t-2) = (f+h) - jr(t) + u(t)$$

Adding the real sector expands the model from two equations to three: the demand for money and the policy formula, plus the IS equation. Now the monetary sector affects the real side of the economy, which in turn feeds back on the monetary side. So, a change in the interest rate affects the demand for money directly by changing the opportunity cost of holding money. But a change in the interest rate also affects money demand indirectly by its impact on spending and income, the other key element in money demand.

Note also that the lagged effects present in the model have been increased substantially. Earlier, only the lagged effect of the interest rate on money demand was relevant; now the lagged income effect on money demand is also relevant. Furthermore, the real sector has an important lagged effect. Aggregate demand is in part determined by the level of income one and two periods earlier, due to the accelerator mechanism. All together, the dynamic structure of the expanded model is much more complex than that of the original model.

Including the real sector, however, does not materially change either the analysis or the thrust of the results. This is demonstrated algebraically in the appendix. The two principal results, however, must be modified to reflect the connections between the monetary and real sectors. The results are changed to the following:

- Keeping the money supply as close, on average, as possible to target still requires seeking partial, and not complete, correction in the current period. But, the percentage correction that accomplishes this is now determined by the elasticities and the lag structures in both the demand for money and the aggregate demand for goods and services.

- As the strength of the policy response to deviations from target increases, the long-run volatility of both the interest rate and income rises exponentially. This is because they play parallel roles. Income and the interest rate jointly determine the quantity of money demanded. And appropriate movements in the levels of income and the interest rate are the means employed to keep the money supply close to its target.

The second result, modified now to reflect the real sector, has important implications. As the policy response is increased until the greatest attainable stability in the money supply is achieved, the volatility of income and the interest rate increases. This means, as it did earlier, that uncertainty over the structure of the economy would make policymakers prefer bringing the money supply back to target too slowly rather than too quickly. But the motive here is concern about unnecessary fluctuations in income and employment, not just fluctuations in interest rates.

Moreover, adopting strict monetary targeting could, in fact, be self-defeating if the intent were to stabilize income. To hit a money supply target, the interest rate and income must be maneuvered so that random disturbances shocking the system have a minimal impact on the money supply. That is, the process of targeting money insulates the money supply from shocks to the economy, with income and the interest rate bearing the brunt of the shocks. Taking this line of thinking one step further, it may be that, in order to stabilize income, policy should respond to deviations of income, not money, from target. The money supply, in other words, should be the "shock absorber" instead of income.

Monetary targeting versus focusing directly on income

Now bearing the larger model in mind, let us compare how well targeting money stabilizes income with how well focusing on income itself stabilizes income. To simulate monetary targeting, the system must contain three equations: the IS curve, the demand for money, and a policy formula oriented to deviations of the money supply from its target. To simulate nominal GNP targeting, the system must consist of the IS curve and a policy formula relating changes in the interest rate to movements in income itself. The interest rate changes can be made relative to the deviation of income (Y) from its target (Y^*), which would be represented as:¹⁰

$$(6a) [r(t) - r(t-1)] = \beta[Y(t-1) - Y^*]$$

Or instead, the interest rate changes could be made relative to the observed changes in income, which would mean the formula would be written as:¹¹

$$(6b) [r(t) - r(t-1)] = \gamma[Y(t-1) - Y(t-2)]$$

To simplify the comparisons between nominal GNP targeting and monetary targeting, consider two cases. First, all disturbances are in the monetary sector; second, all disturbances are in the real sector.¹² In the first case, focusing on income itself must be a superior strategy to monetary targeting. If all shocks originate in the monetary sector, aggregate demand is perfectly stable. Therefore, if policy focuses on income, it will be left undisturbed after coming to rest at the target level. But focusing on the money supply requires movements in the interest rate to keep it on target. These movements in the interest rate will in turn cause income to fluctuate, at times moving far away from the target. Thus, GNP targeting is preferred to monetary targeting.

This finding could have been expected on the basis of Poole's work with a static model.¹³ He shows that the money supply is inferior to the interest rate as an intermediate target when the monetary sector (or LM curve) is the source of instability in the economy, not the real sector (IS curve).

In the other polar case, where all disturbances originate in the real sector, a simulation exercise must be conducted to compare the long-run volatility of income under the three policy formulas. Starting from equilibrium, the model was simulated for 250 periods. Under GNP targeting, a search was conducted for the value of the policy parameters (β or γ in equations 6a and 6b) that minimized the variance of income. Under monetary targeting, the search was for the value of the policy parameter (λ in equation 2) that minimized the variance of the money supply, and the corresponding variance of income was noted.

The simulations show, surprisingly, that monetary targeting was less effective, although only slightly less so, than either version of GNP targeting. The fact that monetary targeting is ranked below the other two strategies, even by a slight amount, is decisive. This polar case, in which all shocks originate in the economy's real sector, is where monetary targeting is supposed to be most effective. Moreover, the differences in the ranking of the strategies could probably be substantially widened either by altering the model's structure or by selecting different values for some key parameters. In any event, the two polar cases together indicate that over the entire spectrum a monetary targeting strategy is outperformed by a strategy of concentrating on the economy itself.

¹⁰National income accounts data are compiled quarterly, whereas the time period in the model is one half of a quarter. So, if the model were made operational, GNP data would have to be interpolated. Considering the volume of data on the real economy released monthly, and the fact that monthly estimates of GNP are made in the private sector, the task could be performed.

¹¹The policy formulas are examples of proportional and derivative control in the Phillips framework for stabilization policy. See A. W. Phillips, "Stabilization Policy in a Closed Economy"; *Economic Journal* (June 1954), pages 290-323.

¹²Because the policy strategies are evaluated on their ability to stabilize nominal GNP, "supply-side" or "price" shocks are not considered. If the strategies are evaluated in terms of real output and inflation, the aggregate supply of goods and services must be incorporated in the model. See Gordon H. Sellon, Jr. and Ronald L. Teigen, "The Choice of Short-Run Targets for Monetary Policy"; *Federal Reserve Bank of Kansas City Economic Review* (April 1981), pages 3-16.

¹³William Poole, "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model"; *Quarterly Journal of Economics* (May 1970), pages 197-216.

Identifying the source of inefficiency in monetary targeting
Considering the results of the simulations, there must be a source of inefficiency intrinsic to monetary targeting in addition to the one Benjamin Friedman identified. He argued that "the intermediate-target procedure for monetary policy, based on the money stock, is in general an inefficient means of processing the information contained in observations on the money stock."¹⁴ More specifically, monetary targeting implicitly attributes all money supply surprises to the disturbance term in the real sector, which is generally not believed to be true. But in the case of the simulations conducted here, it is true. All disturbances in the simulations are by design in the real sector and monetary targeting is still inefficient.

The extra source of inefficiency apparently lies in the lagged interest rate and income effects in the demand for money. With GNP targeting, the demand for money does not enter into the determination of income. But naturally, with monetary targeting it does, and consequently its presence introduces more lagged effects into the system. Since lagged effects create substantial difficulties in stabilizing any system, monetary targeting turns out to be inefficient relative to GNP targeting even when all disturbances are on the real side. If there are no lagged effects in the demand for money, this particular source of inefficiency in monetary targeting disappears, the inefficiency Friedman identified remains, however.

While the simulations point to GNP targeting's relative efficiency, they conceal a complication. Economists believe that interest rate movements have virtually no impact on

aggregate demand in the very short run. The effect first becomes noticeable perhaps three months later. In contrast, interest rate changes do have a discernible impact on money supply growth in the very short run. Thus, policymakers would see their actions having an effect sooner on the money supply than on nominal GNP.

Summary

This paper examined the properties of monetary targeting in a compact model of the macroeconomy. The first conclusion is that a monetary target is most effectively achieved by returning the money supply gradually to its target following a deviation. Attempts to bring the money supply back to target too rapidly cause unnecessary volatility in the money supply, interest rates, and the level of income. Hence, it is not just concern about volatility in rates that argues for a gradualistic approach, but unnecessary volatility in GNP itself.

The second conclusion is that, in principle, monetary policy could more effectively stabilize nominal income by focusing on the economy directly instead of on a monetary aggregate. Moreover, this is true even when the demand for money is stable and no financial innovation is occurring. This finding lends support to the argument for shifting the focus of monetary policy from the monetary aggregates to the performance of the economy. But, of course, the results of this analysis are limited by the particular model used, which does not incorporate some potentially significant factors, such as expectations. In other words, different models can yield different results. But even more importantly, a comprehensive evaluation of monetary targeting and GNP targeting would also take into account several practical and institutional considerations.

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¹⁴Benjamin M. Friedman, "The Inefficiency of Short-Run Monetary Targets for Monetary Policy", *Brookings Papers on Economic Activity II* (1977), page 318.

Asymptotic Variance

In the first section of the paper, the monetary sector is separated from the rest of the macroeconomy. Equations 1 and 2 can be solved to obtain the final-form equations for the money supply and the interest rate.

$$(1A) \quad M(t) + (\lambda b - 1)M(t-1) + \lambda cM(t-2) = (b+c)M^* + v(t) - v(t-1)$$

$$(2A) \quad r(t) + (\lambda b - 1)r(t-1) + \lambda cr(t-2) = (a - M^*) + \lambda v(t-1)$$

The asymptotic variances of the money supply, s_M^2 , and the interest rate, s_r^2 , are shown in equations 3A and 4A

$$(3A) \quad s_M^2 = \frac{(1 + \lambda c) 2s_v^2}{(1 - \lambda c) [(1 + \lambda c)^2 - (\lambda b - 1)^2]}$$

$$(4A) \quad s_r^2 = \frac{(1 + \lambda c) \lambda^2 s_v^2}{(1 - \lambda c) [(1 + \lambda c)^2 - (\lambda b - 1)^2]}$$

In the second section of the paper, the real sector, represented by equation 5, is joined to the monetary sector. Combining this IS curve with equations 1 and 2 yields final-form equations for the money supply, income, and the interest rate, equations 5A, 6A, and 7A, respectively

$$(5A) \quad AM(t) + BM(t-1) + CM(t-2) + DM(t-3) + EM(t-4) = F_M + G_M(t)$$

$$(6A) \quad AY(t) + BY(t-1) + CY(t-2) + DY(t-3) + EY(t-4) = F_Y + G_Y(t)$$

$$(7A) \quad Ar(t) + Br(t-1) + Cr(t-2) + Dr(t-3) + Er(t-4) = F_r + G_r(t)$$

where:

$$A = 1 - g$$

$$B = [(1-g)(\lambda b - 1) - k + \lambda dj]$$

$$C = [(1-g)\lambda c - k(\lambda b - 1) + k + \lambda ej]$$

$$D = [k(\lambda b - 1) - \lambda ck]$$

$$E = \lambda ck$$

$$F_M = \lambda[(1-g)(b+c) + j(d+e)]M^*$$

$$F_Y = \lambda[(b+c)(f+h) - j(a-M^*)]$$

$$F_r = \lambda[(d+e)(f+h) + (1-g)(a-M^*)]$$

$$G_M(t) = dw(t) + (e-d)w(t-1) - ew(t-2) + (1-g)v(t) - (1-g+k)v(t-1) + 2kv(t-2) - kv(t-3)$$

$$G_Y(t) = w(t) - (1-\lambda b)w(t-1) + \lambda cw(t-2) - \lambda jv(t)$$

$$G_r(t) = \lambda dw(t-1) + \lambda ew(t-2) - \lambda(1-g)v(t-1) + \lambda kv(t-2) - \lambda kv(t-3)$$

There are two features of these equations to be noted all three equations, by necessity, have the same autoregressive structure, and the three equations differ in their composite disturbances

It is anticipated that, if the expressions for the asymptotic variance were available, essentially the same results would be found for this fourth-order system of three variables as were found for the second-order system of two variables. These results, modified to take account of the addition of the real sector, would be that

- There is a value of λ between zero and $(1-g)/[b(1-g) + dj]$ that minimizes s_M^2 , the asymptotic variance of the money supply. In the expanded system, $[b(1-g) + dj]/(1-g)$ is equal to the contemporaneous impact on the money supply brought about by a fall in the interest rate of one percentage point. It is the combined effect that the interest rate has on the demand for money directly and indirectly, via a change in income. The value $(1-g)/[b(1-g) + dj]$ sets the value of λ corresponding to 100 percent elimination of deviations in the upcoming period.
- As λ increases, the asymptotic variance of both the interest rate and income rises exponentially. This is because the parameter λ appears in the composite disturbance terms of these two variables (See $G_Y(t)$ and $G_r(t)$ in equations 6A and 7A.) Thus, the solutions for the asymptotic variance will have λ^2 in the numerator, just as it appeared in the solution for the asymptotic variance of the interest rate in the smaller system, equation 4A.

The last point has an important implication. As the policy response is increased until the greatest attainable stability in the money supply is achieved, the volatility of the other endogenous variables increases at an explosive rate. In other words, as λ is increased until s_M^2 is minimized, s_Y^2 and s_r^2 are growing exponentially.