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Market Rates' Volatility

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## **Cross-Country Differences in Monetary Policy Execution and Money Market Rates' Volatility**

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### **Abstract**

The volatility patterns of overnight interest rates differ across industrial countries in ways that existing models, designed to replicate the features of the U.S. federal funds market, cannot explain. This paper presents an equilibrium model of the overnight interbank market that matches these different patterns by incorporating differences in policy execution by the world's main central banks, including differences in central banks' management of marginal lending and deposit facilities in response to shocks. Our model is consistent with central banks' observed practice of rationing access to marginal facilities when the objective of stabilizing short-term interest rates conflicts with another high-frequency objective, such as the targeting of exchange rates.

Key words: interbank market, interest volatility, central bank procedures

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# 1 Introduction

The overnight market for unsecured interbank loans plays a key role in the financial structure of most industrial countries. It anchors the term structure of interest rates and serves as the channel through which monetary policy is executed and liquid funds are funneled to the non-bank sector. To make models tractable for term-structure analysis and asset-pricing, most models of this market have not allowed for an explicit role of monetary policy in determining short-term interest rates. In fact, the few studies that have taken into explicit account the role of central banks in the interbank market, have done so with the goal of replicating key features of individual markets (typically, the U.S. federal funds market). Cross-country evidence (see, for instance, BIS, 1997, Ball and Torous, 1999, and Prati *et al.*, 2002), however, points to patterns in interest rate behavior in the main industrial countries that call for a more unified framework than available from models tailored to fit individual money markets.

In this paper, after building on previous empirical studies to present more comprehensive evidence on cross-country patterns in short-term interest volatility, we develop an equilibrium model of the overnight interbank market that replicates these patterns. The main ingredient of our analysis is to allow for central banks' varying degree of accommodation of liquidity shocks, specifically: for varying degree of access by banks to official marginal liquidity-management facilities. This feature plays a central role in our analysis as it does in actual policy execution in the industrial world, whereby central banks routinely ration market participants' access to official lending and borrowing when the day-to-day goal of stabilizing interest rates conflicts with another high-frequency objective, such as an exchange rate target.

By incorporating this ingredient, our model provides a unified analytical framework that accounts for key similarities and differences in overnight interest volatility patterns in the main industrial countries. Specifically, like previous models (see, for instance, Spindt and Hoffmeister, 1988, and Griffiths and Winters, 1995), our model can replicate the observed cyclical behavior of interest rate volatility associated with periodic reserve requirements. The model also accommodates the observed *decline* in interest rate volatility as market rates

approach rates on marginal official facilities, which is typical of the United States and other countries whose central banks are committed to stabilizing interest rates at high frequency. Differently from previous studies, however, our model can also reproduce an *increase* in volatility as market rates approach rates on marginal facilities, which is typical of countries where access to these facilities is rationed, for instance, because interest rate targeting is subordinated to exchange rate targeting. We show that this result is also consistent with evidence of rising interest volatility as a currency's exchange rate departs from its target level, when the central bank adheres to a recognizable exchange rate target.

Our work contributes to recent research on the equilibrium determination of short-term interest rates and on the micro-foundations of monetary policy execution. The standard reference for this research is Hamilton (1996), whose model, however, allows only for bank-level uncertainty and deterministic interest rates. As most other models in the literature (for instance, Clouse and Dow, 1999 and 2002, and Hayashi, 2001 and, earlier, Ho and Saunders, 1985, and Campbell, 1987), Hamilton focuses only on the demand side of the money market, leaving no role for central bank intervention.

Models allowing for central bank intervention are offered by Nautz (1998), Bartolini *et al.* (2002), and Valimaki (2003). These models, however, are tailored to the specific features of German, U.S., and Euro Zone markets, respectively, and lack the flexibility needed to explain cross-country differences in interest rate behavior. In particular, they assume that banks can borrow and lend funds freely at official marginal facilities. By contrast, our analysis shows that rationing of marginal borrowing and lending is crucial to capture the cross-country behavior of interest rate volatility.

More than other studies in this area, Farnsworth and Bass (2000) shares our concern with the effects of partial commitment of central banks to stabilize short rates' behavior. Farnsworth and Bass assume, as we do, a central bank to intervene to target very short rates only imperfectly. Aside from technical differences, their model focuses on the dynamics of target rates, while our model focuses on higher-frequency dynamics of market rates for given targets, viewing the latter as determined, at lower frequency, by central banks' response

to changes in inflation, employment, and other macro aggregates. Also, Farnsworth and Bass's interest in a model numerically tractable for bond pricing leads them to abstract from institutional features such as periodic reserve requirements and the distinction between marginal and intra-marginal liquidity provision. We include these details explicitly, and assign to central banks' willingness to provide funds in response to shocks a key role in shaping the behavior of the main industrial countries' interbank markets.

## 2 The Empirical Behavior of Overnight Interest Rates

Although our paper is mostly concerned with explaining the behavior of overnight interest rates in industrial countries, in this section we offer also a more comprehensive empirical account of this behavior than available in previous research. With few exceptions, previous studies of interest rates have focussed on individual-country data (typically, the United States; see Spindt and Hoffmeister, 1988, Rudebusch, 1995, Hamilton, 1996, and Balduzzi *et al.*, 1997, 1998). This one-country/one-model approach is not well suited to analyze the behavior of interest rates across policy regimes, however. Model heterogeneity makes cross-country results essentially impossible to compare, a shortcoming that we tackle by first imposing on all our sample countries a common general specification and then adapting this specification to each sample following a standard model-search methodology.

One of the few studies examining cross-country differences in interest rate behavior is Prati *et al.* (2002). Our work here improves on that study in two main respects. First, we follow a formal general-to-specific model selection procedure, which is absent from that study and which allows us to provide more solid econometric evidence for our subsequent modeling effort. Second, we link the volatility of interest rates to past levels of interest rates (as opposed to interest rate targets, which are rarely available and are often insufficiently variable to allow identifying their effects on interest volatility). These changes increase the estimation effort by an order of magnitude, given the computer-intensive nature of our non-linear estimation. They lead, however, to a description of interest rate behavior that is both

more satisfactory and more coherent with the model that follows.

Another study with an international focus is Ball and Torous (1999) which, however, analyzes *one-month* interbank rates, without controlling for reserve requirements, interest rate corridors, and other institutional features of the interbank market. Despite differences in data and focus, this and other studies (including Brenner *et al.*, 1996, and Andersen and Lund, 1997) document properties of short rates such as fat tails, GARCH effects, and dependence of volatility on interest rate levels, which we must incorporate in our empirical model to provide an accurate characterization of interest rate behavior.

## 2.1 Empirical Model

To study the main high-frequency patterns in overnight interest rate volatility in the seven largest industrial countries and the integrated Euro Zone, we follow Hamilton (1996) and select a model that nests the benchmark “martingale” hypothesis for overnight rates into a more general model that allows for predictable day-to-day rate changes.<sup>1</sup> Specifically, our model allows for effects of reserve requirements and calendar time on both mean and volatility of interest rates, for fat-tailed and asymmetric distribution of errors, and for GARCH effects and dependence of volatility on the rate’s level (measured, as explained below, as the position of the market rate within a corridor defined by rates on official borrowing/deposit facilities).

We specify the dynamics of the overnight rate  $r_t$  as

$$r_t = \mu_t + \sigma_t \nu_t , \tag{1}$$

where  $\nu_t$  is a mean-zero, unit-variance i.i.d. error term;  $\mu_t$  is the conditional mean of  $r_t$ ; and  $\sigma_t$  is the conditional Exponential GARCH (EGARCH) volatility parameter. As we now explain, the evolution of  $\mu_t$  and  $\sigma_t^2$  is governed by Equations (2) and (3) below.

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<sup>1</sup>The “martingale hypothesis” rests on the role of periodic reserve requirements in determining the time-series behavior of mean overnight rates. According to this hypothesis, these rates should not display predictable daily changes within each reserve-averaging period, otherwise banks could profit by selling reserves on “high” rate days and buying them on “low” rate days, while still satisfying reserve requirements.

To model  $\mu_t$  and  $\sigma_t$ , we first control for calendar effects by including fixed-effect dummies  $\delta_{c_t}$  and  $\xi_{c_t}$  in the mean and variance equations, respectively, for each weekday and for holidays, end-months, end-quarters, end-years, and the days before and after them.

Next we incorporate differences in reserve requirements in our eight samples, requiring banks to maintain minimum average reserve balances over reserve periods lasting as long as one month in Japan, Germany, the Euro Zone, France, and Italy, or as short as one day in Canada and the United Kingdom. (The United States is in intermediate position with a reserve period of two weeks.) We capture this periodicity by including fixed-effect dummies for each day of the reserve period,  $\delta_{m_t}$  and  $\xi_{m_t}$ , in the equations for  $\mu_t$  and  $\sigma_t^2$ , respectively, where  $m_t = 0, 1, \dots, T - 1$  counts days left until the end of a reserve period of length  $T$ .

One of the main goals of our work is to study the high-frequency impact on interest volatility of central banks' liquidity-management procedures. Liquidity provision by central banks usually involves *intra-marginal* draining and injection of funds through repo operations at a key policy (or "target") rate  $r_t^*$ , often combined with *marginal* draining and injection of funds at  $r_t^F$  and  $r_t^C$ , the "floor" and "ceiling," respectively, of corridors for market rates around target, with  $r_t^F < r_t^* < r_t^C$ . To study the volatility impact of these arrangements, we first control for level-shift effects of changes in official rates by including series of changes in floor, target, and ceiling rates,  $\Delta_t = \{(r_t^F - r_{t-1}^F), (r_t^* - r_{t-1}^*), (r_t^C - r_{t-1}^C)\}$ , as determinants of mean rates. We also include in the variance equation a set of constant terms  $\Sigma_t$  to control for unusual volatility on days when floor, target, and ceiling rates change (that is, on days when the corresponding term in  $\Delta_t$  is not zero). More important, we include as a determinant of  $\sigma_t^2$  the (centered) position of the overnight rate in its fluctuation corridor,  $h_t = \frac{r_{t-1} - (r_t^F + r_t^C)/2}{r_t^C - r_t^F}$ .<sup>2</sup> (Corridor rates are defined for each country in Appendix A.)

Finally, for countries explicitly targeting exchange rates, we include in the variance equa-

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<sup>2</sup>Our results are essentially unchanged if  $h_t$  is defined using lagged corridor rates in stead of contemporaneous rates, as in  $h_t = \frac{r_{t-1} - (r_{t-1}^F + r_{t-1}^C)/2}{r_{t-1}^C - r_{t-1}^F}$ . This robustness is intuitive, since the two definitions differ only over the few days when corridor rates change. We present results with  $h_t$  defined as in the text, since this is more consistent with the presumed dependence of the conditional volatility of  $r_t$  on its corridor at  $t$ .

tion a variable  $x_t$  measuring the position of the exchange rate in its own target corridor.<sup>3</sup> This variable proxies for circumstances in which a central bank may be reluctant to offset inflows or outflows of liquidity from the interbank market. Our particular proxy is motivated by the fact that central banks targeting exchange rates typically ration interbank liquidity in periods of exchange rate pressure to discourage flows of funds from (occasionally into) the domestic financial sector. This conflict at high frequency between interest rate targeting and another (possibly overriding) target may lead to greater interest rate volatility.

The resulting equations for the mean and the variance of overnight rates are

$$\mu_t = r_{t-1} + \delta_{m_t} + \delta_{c_t} + \gamma \Delta_t, \quad (2)$$

$$\log(\sigma_t^2) = \lambda \log(\sigma_{t-1}^2) + (1 - \lambda L)(\xi_{m_t} + \xi_{c_t} + \omega \Sigma_t + \beta h_t^2 + \eta x_t^2) + \kappa |\nu_{t-1}| + \theta \nu_{t-1}, \quad (3)$$

where  $\lambda$  is the autoregressive EGARCH coefficient,  $L$  is the lag operator, and we allow for asymmetric response of  $\log(\sigma_t^2)$  to positive and negative shocks when  $\theta \neq 0$ . In (3), both  $h_t$  and  $x_t$  are included as *squared* deviations of interest and exchange rates from their mid-corridor levels, since the testable hypothesis is that interest volatility should rise when interest and exchange rates approach either of their corridors' margins.

Since banks' ability to arbitrage over cross-period interest rate gaps by carrying reserve imbalances over to the next reserve period is restricted,<sup>4</sup> a different model should be used to describe the behavior of overnight rates on the first day of each reserve period. To this end, we follow Hamilton (1996) by including a more general auto-regressive term in the equation of the mean when  $m_t = 1$ . In particular, we include changes in  $r_t$  in the previous five days, that is, the term  $\sum_{i=1}^5 \phi_i(r_{t-i} - r_{t-i-1})$ . For the first day of each period, we also include a constant in the variance equation and, for countries publishing an explicit interest rate target, the difference between the overnight rate on the last day of the maintenance period and the target rate on the first day,  $r_{t-1} - r_t^*$ , in the equation of the mean.

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<sup>3</sup>For these countries (i.e., European countries that participated in the Exchange Rate Mechanism [ERM]), we use the official indicator of divergence between a currency's market rate and its central ERM parity.

<sup>4</sup>In our sample, only France and the United States allow some carryover of imbalances across periods.

## 2.2 Results

Figure 1 plots the overnight market rate and marginal rate series used in the empirical analysis documented in detail in Appendix A. We discuss here in detail the results most relevant for the theoretical analysis that follows, namely, those pertaining to the link between interest rate volatility and monetary policy execution frameworks. The remaining results, concerning estimated mean coefficients and statistical properties of the error terms, are reviewed in Appendix A.

A first set of results from our multi-country analysis matches earlier findings for U.S. data (Spindt and Hoffmeister, 1988; Hamilton, 1996; Bartolini *et al.*, 2002). In all countries relying on periodic reserve requirements to stabilize interbank rates (that is, in all our sample countries except Canada and the United Kingdom), settlement-day rates are significantly more volatile than non-settlement-day rates. This finding, formally documented in Table A1, is clearly apparent in Figure 2, which plots estimates of reserve-period effects on volatility (the coefficients  $\xi_{m_t}$  in (3)) for all our countries except the United Kingdom.<sup>5</sup>

In addition, higher settlement-day volatility tends to propagate to the days immediately preceding settlement: the last seven days of the reserve period in Germany, the last five days in the Euro Zone, the last four days in Italy, the last three days in the United States, and the last two days in Japan display interest volatility higher than the average of the previous days in the period. (Statistical significance, at the 5 percent level, reflects Wald tests, details of which are available on request.) Even in France, where we estimate a rather unstable pattern of volatility, there is a clear tendency for volatility to rise from mid-period to end-period.

Our subsequent model shows that propagation of settlement-day volatility to previous days is to be expected in a regime of reserve averaging in which the central bank does not provide liquidity in a fully elastic way in response to shocks: when central banks do not fully

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<sup>5</sup>No reserve averaging was in place throughout our U.K. sample; hence, we estimated no reserve-period effects for this sample. Reserves were averaged over a two-week period in Canada until February 1999. This allowed us to estimate reserve period coefficients from pre-1999 data. Not surprisingly, since Canadian reserve ratios were already zero over this period, these coefficients displayed no reserve-related cycle.

offset aggregate shocks to banks' liquidity, shocks occurring in the middle of a reserve period become correlated with banks' end-period reserve balances. Mid-period rates then tend to fall in response to positive shocks to liquidity and rise in response to negative shocks, with a stronger response in rates as settlement day nears. If, instead, the central bank is expected to offset fully aggregate reserve imbalances before end-period, then mid-period shocks have no predictive power for end-period rates and induce no response in current rates.

There are several reasons why a central bank might not provide liquidity fully elastically in response to shocks. The simplest — and most pragmatic — reason is that many central banks, especially in Europe, have committed to infrequent intervention schedules. This commitment prevents them from offsetting shocks occurring before period-end if no intervention is scheduled before then.<sup>6</sup> More subtle reasons include some central banks' reluctance — or inability — to fully accommodate large shocks (as has been the case in the United States),<sup>7</sup> or their desire to let interest rates partly absorb shocks, to induce banks to manage liquidity prudently (a practice historically followed by the Bundesbank). Still another reason is the existence of conflicts between interest rate targeting and other operational goals for monetary policy, an issue we discuss next.

The next set of results points to diverse effects on volatility exerted by interest rate corridors in our sample's "large" (United States, Euro Zone, Japan, and Germany) and "small" (France, United Kingdom, Italy, and Canada) countries. As shown in Table 1, as

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<sup>6</sup>In this respect, our results parallel those of Harvey and Huang (2001), who document higher volatility of short-rate futures on days without Fed intervention. Harvey and Huang also interpret their finding as reflecting lack of interest rate smoothing by the Fed.

<sup>7</sup>The Federal Reserve has found it difficult to implement unusually large operations, in part because the collateral needed to execute repos is not freely available. Indeed, to mitigate thinness in the repo market at intervention time, the Fed shifted its intervention from 11:30 a.m. to 9:30 a.m. in the late 1990s and expanded the set of securities accepted under repo. Like other central banks, the Fed has also feared that large operations could pressure banks against their daily overdrafts limits: a large reserve drain in a single day may induce many banks to scramble to avoid daily overdrafts; a large injection may soften the market as banks try to avoid excess reserves that may be difficult to unwind before settlement.

the (lagged) interest rate moves to the margins of its fluctuation corridor, interest volatility *falls* in the first group of countries, but *rises* in the second: the coefficient  $\beta$  of the centered, squared position of the overnight rate in its corridor,  $h_t$ , is estimated as significantly negative for the first four countries, but as significantly positive for the latter four.

We propose the following interpretation of this finding. Monetary policy is executed in essentially different ways in our large and small countries. Central banks in the latter group of countries tend to ration the provision of liquid funds in response to shocks, especially at marginal facilities, a practice that causes interest rates to respond more strongly to shocks as they depart from their target level. The opposite is true for our large countries.

The main reason for this difference is that in our four large countries, the operational goal of monetary policy — to stabilize very short interest rates — does not conflict with other high-frequency objectives: The day-to-day goal of the central bank *is* to keep interbank rates near target (which itself stays constant until new information accrues, at lower frequency, on the evolution of macro aggregates such as inflation and unemployment). This policy truncates further movements of market rates as they sway away from target, causing them to become less responsive to shocks as they approach the corridor's margins. This pattern results in a negative coefficient  $\beta$  for the squared position of the interest rate in its corridor, for the United States, the Euro Zone, Japan, and Germany.

Monetary policy operates very differently when interest rate stabilization is subordinated to an alternative high-frequency objective. Such is typically the case with central banks of small, open economies, whose operational goals often include stabilizing the exchange rate. In these countries, deviations of interest rates from target correlate with outflows of funds from the domestic banking sector that pull the exchange rate away from its own target. To stem such flows, central banks ration access to marginal liquidity, curb intra-marginal intervention, and occasionally adjust target rates. These rationing strategies cause interest rates to become *more* responsive to shocks as they depart from target. This is the essence of so-called interest rate defenses of exchange rates (see Drazen, 2001, for a review), which are picked up in Table 1 by positive estimates for  $\beta$  for the textbook cases of Italy and France,

and — to a lesser extent — Canada.<sup>8</sup> In fact, as our later model shows, a U-shaped link between the interest rate’s position in its corridor and interest volatility should be expected whenever marginal liquidity provision is rationed, irrespective of a central bank’s motives for doing so, as exemplified by the case of the Bank of England.<sup>9</sup>

The final set of results that we discuss here relates to another dimension along which a central bank’s commitment to stabilize interest rates affects interest volatility. As noted above, we included in the variance equation the (squared) distance of the exchange rate from its target for countries linked in an explicit exchange rate arrangement (Germany, France, Italy until 1992 and after 1996, and the United Kingdom from 1990 to 1992). Our conjecture is that, for these countries, overnight rates’ volatility should be higher in periods with stronger “exchange rate pressure” (irrespective of the shape of the link between volatility and the rate’s position in its corridor), because at these times a central bank may be more reluctant to provide funds to banks in response to shocks.

Indeed, we find strong evidence that in countries explicitly targeting exchange rates, overnight interest rates were more volatile during periods in which the exchange rate was further away from target than at other times: the coefficient  $\eta$  linking interest volatility to the

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<sup>8</sup>Notably, we focus on *de facto* — rather than formal — cross-country differences in policy execution. For instance, while the Bundesbank was formally committed to an exchange rate target within the ERM, exchange rate targeting played a minor role in its policy execution: it was the task of Germany’s ERM partners to anchor their currencies to the Deutsche mark. Conversely, while not formally committed to an exchange rate target, the Bank of Canada has historically assigned to the exchange rate an important role in policy execution: the exchange rate weighs one-third in the Bank’s monetary condition index, and its movements have often led the Bank to adjust policy. (The textbook case is the rate hike of August 1998 after the sudden weakening of the Canadian dollar in the wake of the Russia/LTCM crisis.)

<sup>9</sup>The Bank of England targeted exchange rates only from October 1990 to September 1992. However, the “late lending facility” designed to limit interest rates’ upward fluctuations has been, historically, tightly rationed. To confirm the role of this arrangement in shaping interest rate dynamics, we tested for a break in interest volatility in June 1998, when the rationing of the late-lending facility was relaxed. We expected, and found, the overall level of volatility to fall significantly at that time. (For comparability with other samples, we omit the level-split dummy from our final specification, but details are available upon request.)

squared ERM divergence indicator was precisely estimated with a positive sign in Germany, France, and Italy (see Table 1); it was also positive, though statistically insignificant, in the U.K. sample, which included only a short ERM period. Explaining these results, along with our findings on the cyclical behavior of interest volatility and on volatility patterns over interest rate corridors, is on the agenda for the next sections.

### 3 Interbank Markets with Central Bank Intervention

#### 3.1 The Model

We now present an equilibrium model of an interbank market that captures the stylized facts documented above. The model is stylized, and simplifies the operation of the interbank market in two main ways. First, it abstracts from cross-bank heterogeneity, including in size and exposure to shocks, that has played a role in previous studies of interbank markets (see, for instance, Allen and Saunders, 1986, and Hamilton, 1996). This simplification allows us to apply a representative-agent solution method that greatly simplifies the analysis. Second, the model assumes a hybrid trading environment that captures the main common features of our eight sample markets, leaving two parameters measuring the commitment of the central bank to interest rate stabilization as the main degrees of freedom to capture cross-country differences in central bank behavior.

We consider a market populated by competitive, identical, risk-neutral banks, whose stocks of liquid funds (“reserves”) are exposed to common shocks. We study this market’s equilibrium over a reserve period lasting  $T$  days. We denote aggregate bank reserves at the end of day  $t$  by  $z_t$  and cumulative holdings of reserves since the beginning of the period by  $c_t \equiv z_1 + \dots + z_t$ . Banks’ reserves are subject to the end-period requirement  $c_T \geq C$ . (We consider the case of an additional daily requirement  $z_t \geq Z$ , which is superfluous for our main results, in Section 4.)

Each day  $t = 1, \dots, T$ , the sequence of events is as follows.

At the beginning of the day (at 9:00 a.m., say), the central bank intervenes by injecting a net amount of reserves  $m_t$  into the banking sector ( $m_t < 0$  for a reserve drain). The central bank chooses  $m_t$  to keep the equilibrium interbank overnight rate for day  $t$ , expected as of its “a.m.” intervention time,  $E_{t^{a.m.}}[r_t]$ , as close as possible to the target  $r^*$ , choosing  $m_t$  from the interval  $[-m, m]$ . Thus,  $m$  parameterizes the central bank’s commitment to  $r^*$  through intra-marginal intervention: the higher is  $m$ , the stronger is the adherence to the target.

At 10:00 a.m., a random shock to reserves  $\nu_t$ , with cumulative distribution  $G(\nu)$ , mean  $\mu_\nu$ , and variance  $\sigma_\nu^2$ , reaches banks from the nonbank sector.

At 12:00 p.m. the interbank market opens, allowing banks to trade reserves through unsecured overnight loans at the rate  $r_t$ .

At 3:00 p.m., after the market has cleared, a second random shock to reserves  $\epsilon_t$  is realized, with cumulative distribution  $F(\epsilon)$ , mean  $\mu_\epsilon$ , and variance  $\sigma_\epsilon^2$ .<sup>10</sup>

At day’s close, banks’ daily reserves  $z_t$  are computed and added to the cumulative balance for the period,  $c_t$ .

This routine is repeated each day  $t = 1, \dots, T$ . At the end of (settlement) day  $T$ , cumulative reserves  $c_T$  are assessed against required reserves  $C$ , and banks can access rationed “marginal” borrowing and deposit facilities: they must fill reserve deficiencies by borrowing up to  $k$  from the central bank at the ceiling rate  $r^C$ , and can dispose of excess reserves by depositing up to  $k$  at the floor rate  $r^F$ . Excess reserves beyond  $k$  are worthless; reserve deficiencies in excess of  $k$  can be filled only at a much higher cost  $R$ .<sup>11</sup> Thus,  $k$  parameterizes the central bank’s commitment to the corridor  $[r^F, r^C]$  (with  $r^F < r^* < r^C$ ) through marginal provision/drain of liquidity: the higher is  $k$ , the stronger is the adherence to the corridor.

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<sup>10</sup>We assume that both  $G(\mu)$  and  $F(\epsilon)$  satisfy standard regularity conditions, in particular, that they are continuous and strictly increasing over an infinite support (e.g., they are distributed as normal variables).

<sup>11</sup> $R$  plays no essential role in the analysis, but must be bounded away from infinity to secure a bounded solution for  $r_t$ . To fix ideas,  $R$  can be thought of as the cost of emergency funds available from the central bank at a very high penalty, as the charge on alternative, very costly, funds available to banks from the non-bank sector, or as the prorated cost of shutting down a bank unable to meet reserve requirements.

### 3.2 Market equilibrium on settlement day

Our assumption of identical, competitive banks allows us to study the market's equilibrium as it results from the behavior of a single representative bank, which competitively takes the rate  $r_t$  as given. To study this equilibrium, we proceed backward from settlement day,  $T$ , to day 1 of the reserve period.

Denote by  $\tilde{c}_T$  the representative bank's cumulative reserve position at the opening of the market on day  $T$ . That is,  $\tilde{c}_T = c_{T-1} + m_T + \nu_T$  equals the cumulative reserve balance inherited from day  $T - 1$ ,  $c_{T-1}$ , plus the reserve flows  $m_T$  and  $\nu_T$  recorded earlier in day  $T$ .

Given  $r_T$  and  $\tilde{c}_T$ , the representative bank chooses how much to borrow in the interbank market,  $b_T$ , to maximize the profits generated by its overnight interbank position, net of penalties and returns associated with borrowing and lending funds from and to the central bank at day's end. That is, its problem is to maximize over  $b_T$ :

$$\begin{aligned}
 V_T = & -r^C \int_{-\infty}^{C-\tilde{c}_T-b_T} [C-\tilde{c}_T-b_T-\varepsilon]dF(\varepsilon) - (R-r^C) \int_{-\infty}^{C-\tilde{c}_T-b_T-k} [C-\tilde{c}_T-b_T-k-\varepsilon]dF(\varepsilon) \\
 & + r^F \int_{C-\tilde{c}_T-b_T}^{C-\tilde{c}_T-b_T+k} [(\tilde{c}_T + b_T + \varepsilon - C)dF(\varepsilon) + r^F \int_{C-\tilde{c}_T-b_T+k}^{\infty} kdF(\varepsilon) - r_T b_T]. \quad (4)
 \end{aligned}$$

Equation (4) sums the representative bank's costs and returns over each possible realization of  $\tilde{c}_T + b_T + \varepsilon$ , the bank's end-period reserve balance, relative to the required level  $C$ . Specifically, the first term in (4) measures the expected cost (at the rate  $r^C$ ) of a reserve deficiency  $C - \tilde{c}_T - b_T - \varepsilon$ . The second term in (4) adds the cost of incurring a deficiency larger than  $k$ , since only up to  $k$  can be borrowed from the central bank at  $r^C$ , while additional deficiencies cost  $(R - r^C)$  per unit. The third term in (4) measures the return, at the rate  $r^F$ , to depositing funds in amount up to  $k$  at the central bank. The fourth term in (4) adds the fixed (per unit) amount  $r^F k$  earned when excess reserves exceed  $k$ . Finally, (4) includes the cost of borrowing  $b_T$  in the interbank market at the rate  $r_T$ .

The first-order condition for (4) yields the representative bank's optimal borrowing,  $b_T^*$ :

$$0 = -r^C \int_{-\infty}^{C-\tilde{c}_T-b_T^*} dF(\varepsilon) - (R-r^C) \int_{-\infty}^{C-\tilde{c}_T-b_T^*-k} dF(\varepsilon) + r^F \int_{C-\tilde{c}_T-b_T^*}^{C-\tilde{c}_T-b_T^*+k} dF(\varepsilon) - r_T. \quad (5)$$

Setting  $b_T^* = 0$ , since the market must clear with no net borrowing or lending,

**Result 1.** *The market-clearing rate on settlement day  $T$  is*

$$r_T = (R - r^C)F(C - \tilde{c}_T - k) + (r^C - r^F)F(C - \tilde{c}_T) + r^F F(C - \tilde{c}_T + k) , \quad (6)$$

*a continuous, decreasing function  $r_T(\tilde{c}_T)$  of  $\tilde{c}_T$ .*

Intuitively,  $r_T$  equals a weighted average of  $R$ ,  $r^C$ , and  $r^F$  (and zero); rises with  $R$ ,  $r^C$ , and  $r^F$ ; and it falls with the representative bank's reserve position at market opening,  $\tilde{c}_T$ .

Consider next the central bank's problem of deciding, on the morning of day  $T$ , how much liquidity to inject into (or drain from) the market, to bring the expected rate for day  $T$  as close as possible to the target rate  $r^*$ .  $m_T$  solves

$$m_T = \min\{m, \max\{-m, m^* : \mathbb{E}_{T \text{ a.m.}}[r_T | m^*] \equiv r^*\}\} , \quad (7)$$

where the expectation is taken over  $\nu_T$ , since it is conditional on information available to the central bank at its 9:00 a.m. intervention time. In (7),  $m^*$  is the amount of intervention that equalizes  $r^*$  to the rate expected for day  $T$ , while the operator  $\min\{m, \max\{-m, m^*\}\}$  truncates  $m^*$  to the interval  $[-m, m]$ . Equation (7) yields  $m_T(c_{T-1})$  as an implicit function of  $c_{T-1}$ , with the following properties (proof is in Appendix B):

**Result 2.** *There is a unique solution  $m_T(c_{T-1})$ , continuous and non-increasing in  $c_{T-1}$ .*

### 3.3 Market equilibrium on non-settlement days

For a generic day  $t < T$ , the representative bank's problem is

$$V_t^* = \max_{b_t} -r_t b_t + \frac{1}{1 + r_t} \mathbb{E}_t[V_{t+1}^*] , \quad (8)$$

where  $V_{t+1}^*$  is the continuation value function, with terminal value given by its value  $V_T^*$  for settlement day; and the expectation  $\mathbb{E}_t[\cdot]$  is taken, conditionally on information available at market opening on day  $t$ , over  $\varepsilon_t$  (the shock realized after market closing on day  $t$ ) and  $\nu_{t+1}$  (the shock realized before market opening on day  $t + 1$ ). The expectation also incorporates the known central bank's intervention policy at  $t + 1$ ,  $m_{t+1}(c_t)$ .

The first-order condition for this standard dynamic programming problem is that the derivative with respect to  $b_t^*$  of the single-period payoff at  $t$ ,  $r_t b_t^*$ , should equal the expected, discounted derivative with respect to  $b_{t+1}^*$  of the single-period payoff at  $t + 1$ ,  $r_{t+1} b_{t+1}^*$ :

$$r_t = \frac{1}{1 + r_t} E_t[r_{t+1}] . \quad (9)$$

Intuitively, (9) expresses the condition that a bank should not expect to profit by borrowing one more dollar in the interbank market at  $t$  and one less dollar at  $t + 1$ . From this condition, we obtain the following result (proof is in Appendix B):

**Result 3.** *There is a unique solution  $r_t(\tilde{c}_t)$ , continuous and decreasing in  $\tilde{c}_t$ .*

Stepping back to the morning of day  $t$ , the central bank's problem is, as in day  $T$ ,

$$m_t = \min\{m, \max\{-m, m^* : E_{t a.m.}[r_t | m^*] \equiv r^*\}\} . \quad (10)$$

Equation (10) defines the intervention policy  $m_t$  as a function  $m_t(c_{t-1})$  of cumulative reserves at the end of day  $t - 1$ . For day  $t$ , the analog of Result 2 also holds (with proof identical to that of Result 2, except for a change in time subscripts from  $T$  to  $t$ ):

**Result 4.** *There is a unique solution  $m_t(c_{t-1})$ , continuous and non-increasing in  $c_{t-1}$ .*

## 4 The Behavior of Interest Rate Volatility

Results 1-4 show that the model exhibits a single daily equilibrium for the market rate  $r_t$  and the intervention policy  $m_t$ , as a function of the state variables  $\tilde{c}_t$  and  $c_t$ . In general, it is difficult to advance analytically beyond these results, since  $m_t(c_{t-1})$  is defined by (10) only in implicit form. Complete analytic solutions are available in limiting cases, such as when  $m = \infty$  or  $m = 0$ , but are of limited practical interest.

Given Results 1-4, however, it is legitimate to seek the model's unique solutions by numerical methods. We do this using a standard backward-recursive grid method (details are

in Appendix C). To illustrate the model’s properties, we then let  $\varepsilon_t$  and  $\nu_t$  be distributed as normal, i.i.d. random variables, with zero mean and variance  $\sigma_\varepsilon = 0.5$  and  $\sigma_\nu = 2$ ; and let  $r^C = 0.06$ ,  $r^* = 0.05$ ,  $r^F = 0.04$ ,  $R = 0.10$ ,  $C = 0$ , and  $m = 1$ . Although these are plausible parameters,<sup>12</sup> we did not choose them to replicate the magnitude of empirical interest volatilities, since this reflects also factors — such as intra-day and cross-bank dispersion of liquidity — that our representative-agent model with instantaneous market clearing is clearly not designed to capture. Also, reality is clearly more complicated than the crisp, symmetric examples discussed below. (For instance, U.S. rates have moved mostly in the upper part of their corridor, while our examples assume target rates in the middle of the corridor.) Our purpose is to illustrate the economic forces at work on average over our eight sample markets, more than to capture accurately the reality of any single market.

**The demand for overnight funds.** Figure 3 plots families of interest rate curves  $r_t(\tilde{c}_t)$  as a function of cumulative reserves at market opening,  $\tilde{c}_t$ , for each of the five days of our hypothetical reserve period. Each panel is parameterized by a different value of  $k$ ,  $k = \{0, 5, 10, 20\}$ , the maximum amount of funds that banks can trade with the central bank at  $r^F$  and  $r^C$ . For our choice of parameters,  $k = 20$  effectively approximates the case  $k = \infty$ .

The first notable property of the curves  $r_t(\tilde{c}_t)$  is their negative slope, which conforms with Results 1 and 3 and reflects the lower likelihood of penalties on reserve overdrafts associated with higher values of  $\tilde{c}_t$ .  $r_t(\tilde{c}_t)$  also becomes flatter as  $t$  goes from settlement day,  $T = 5$ , to day 1. Formally, this feature reflects the iterative application of the expectation operator in (9): at each step  $t = T - 1, \dots, 1$ , Jensen’s inequality makes  $E_t[r_{t+1}]$  smoother than  $r_{t+1}$ . The economic intuition, discussed in Section 2.2, is that shocks to liquidity occurring early

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<sup>12</sup>With these parameters, the interest corridor of 2 percent is in line with those prevailing in several industrial countries;  $\sigma_\varepsilon/\sigma_\nu$ , which parameterizes the ratio of relevant information available daily to the central bank, is set at  $1/4$ , in line with the case of a central bank intervening daily around mid-morning;  $m = 1$ , in turn, yields the case of a central bank that offsets by intra-marginal intervention about  $3/4$  of the shocks experienced by banks over a typical reserve period.

in the period (plotted as movements on the horizontal axis of Figure 3) provide only a very imprecise signal of banks' likely end-period penalties on reserve deficiencies. Hence, they cause only a small response in rates, that is, curves that are flatter for early days of the period than for late days.

Another property displayed in Figure 3 is the inflection of the curves  $r_t(\tilde{c}_t)$  around the target  $r^*$  (here set at 0.05). This inflection illustrates the stabilizing role of *intra-marginal* intervention, which partly offsets the exogenous shocks  $\varepsilon_t$  and  $\nu_t$ . This offset is relatively more complete the smaller are the shocks, that is, the smaller is the departure of  $r_t$  from  $r^*$ . By contrast, larger shocks are more likely to push the central bank against the limit  $m$  that parameterizes its commitment to  $r^*$ . Hence, they cause a proportionally stronger response in rates, that is, curves that initially steepen as  $r_t$  moves away from  $r^*$ .

The behavior of  $r_t(\tilde{c}_t)$  at the corridor's margins,  $r^F$  and  $r^C$ , illustrates the role of *marginal* liquidity provision by the central bank. As a central bank's commitment to the corridor  $[r^F, r^C]$  becomes stronger — that is, as  $k$  rises — the amount of trading between the representative bank and the central bank at marginal rates increases, truncating realizations of  $r_t$  outside the corridor. For high values of  $k$ , the curves  $r_t(\tilde{c}_t)$  approach the corridor's margins asymptotically. Shocks to reserves when  $r_t$  is close to  $r^C$  and  $r^F$  then are almost fully absorbed by injection or drainage of liquidity at corridor rates, causing a negligible response in market rates. The converse holds for low values of  $k$ .

**Volatility over and across reserve periods.** The previous properties can be mapped into predictions matching the empirical results of Section 2.

We begin by plotting in Figure 4 the conditional volatility of interest rates (the standard deviation of  $r_t - r_{t-1}$ ) over a typical reserve period for various values of  $k$ . In accord with Figure 3, it is intuitive that steepening interest rate curves as  $t \rightarrow T$  should cause volatility to rise over the period. This is indeed the case, as shown in Figure 4 for all values of  $k$ . This prediction is consistent with evidence of cyclical volatility behavior provided, for the United States, by Spindt and Hoffmeister (1988), Griffith and Winters (1995), and Bartolini *et al.*

(2002) and others, and — for other industrial countries — in Section 2 above.

Figure 4 also shows that volatility falls in reserve periods when banks can borrow and deposit more funds at the marginal facilities (that is, when  $k$  is higher): flatter interest curves within the corridor and reduced scope for rates to sway outside the corridor dampen volatility over the whole reserve period. This prediction also parallels our results in Section 2. There, we used a measure of the exchange rate’s distance from its target to proxy for a specific cause for a central bank’s reluctance to provide funds in response to shocks. The model shows this prediction to be more general: whenever a central bank is reluctant to drain or inject funds at its marginal facilities in response to shocks, and irrespective of its motives for doing so, very short rates’ volatility should rise.

**Volatility over the corridor.** Figure 5 illustrates one of the most interesting properties of the model. In the figure, volatility is plotted as a function of the position of the interest rate in its target corridor,  $[r^F, r^C]$ , for two regimes: a regime of “low commitment” by the central bank to the corridor,  $k = 5$ , and a regime of “high commitment,”  $k = 20$ . The panels on the left side plot volatility without controlling for the effect of changes in days of the reserve period; in the panels on the right side, these effects are instead controlled for, by plotting the volatility profile separately for each day of the period.

In the figure, the low-commitment regime displays a straight U-shaped pattern in volatility over the corridor, in sharp contrast with the inverted U-shaped pattern displayed in the high-commitment regime. (The pattern in the lower-left panel actually displays a double hump, in good part because it does not control for the larger share of observations drawn, in the middle of the corridor, from early-in-the-period days with low volatility.)

Clearly, these patterns reflect the curvature properties of  $r_t(\tilde{c}_t)$  in Figure 3. In particular, the asymptotic behavior of  $r_t(\tilde{c}_t)$  around  $r^F$  and  $r^C$  for high  $k$  causes volatility to fall to zero as  $r_t$  approaches  $r^F$  and  $r^C$ : near the corridor’s margins, shocks to liquidity are almost-fully absorbed at the marginal facilities if  $k$  is high, causing a negligible response in interest rates. Conversely, the “fanning” behavior of  $r_t(\tilde{c}_t)$  through  $r^F$  and  $r^C$  for low  $k$  causes volatility

to rise as  $r_t$  approaches these rates: given a limit on funds tradable at the facilities, the larger are the shock to banks' liquidity, the stronger is — proportionally — the response of market rates. This split prediction parallels our finding of a qualitatively different behavior of volatility in markets where the central bank is committed to interest rate stability at high frequency, and markets where the central bank — for any reason — rations banks' access to marginal facilities, for instance, because interest rate stability conflicts with an alternative, high-frequency objective (such as for the exchange rate).

**Intra-marginal intervention and volatility.** Finally, how does interest volatility behave for different central bank commitment to the target  $r^*$  by *intra-marginal* intervention, here parameterized by  $m$ ? Our benchmark is given by Figures 3-5, which illustrate the model's predictions when  $0 < m < \infty$ . What happens when  $m$  falls to zero, or rises to infinity?

When  $m = 0$ , the central bank does not intervene *at all* intra-marginally at  $r^*$ : the task of keeping rates within a corridor around  $r^*$  is assigned solely to marginal provision of liquidity at  $r^F$  and  $r^C$ . In this case — we omit details for brevity — the model yields interest rate curves that look very similar to those of Figure 3, except for their lack of inflection around  $r^*$ . This lack of inflection, which reflects the lack of *any* interest stabilization at  $r^*$ , causes interest rate volatility to display always an inverted U-shaped pattern over the corridor, since the curves  $r_t(\tilde{c}_t)$  are steepest in the middle of the corridor and flatter near the corridor's margins. Yet, aside from its analytical properties, the case  $m = 0$  is utterly unrealistic. Intra-marginal intervention is the chief means of stabilizing interest rates in industrial countries. If anything, the opposite case of  $m = \infty$  is closer to reality.

When  $m = \infty$ , the central bank provides funds infinitely elastically at  $r^*$ . From (9) and (10), then,  $r_t = r^*$  for all days before settlement: Banks have no incentive to trade among themselves at rates other than  $r^*$ , if they can count on the central bank to offset all imbalances — and secure funds at the expected rate  $r^*$  — until the morning of settlement day. Hence, there is no interest rate volatility for all  $t < T$ . Only on settlement day is  $r_T$  bid away from  $r^*$  and its volatility becomes positive. Thus, in contrast with the standard reference in the

literature (Spindt and Hoffmeister, 1998), a pattern of gradually rising volatility such as the one displayed in Figure 4 and those estimated in Section 2, is *not* a necessary offspring of average reserve requirements. For volatility to propagate from settlement to pre-settlement days, a central bank should not be expected to intervene, in defense of a known target, to eliminate *all* risk of aggregate liquidity imbalances.

## 5 High-Frequency Liquidity Effects

Recent research (including Furfine, 2000, Pérez Quirós and Rodríguez Mendizábal, 2000, and Clouse and Dow, 2002) has suggested that restrictions on banks' ability to allocate reserve holdings across days may help explain patterns in means and volatilities of overnight rates such as the prevalence of high *and* volatile rates on settlement days, Mondays, and other calendar days. In particular, these studies suggest that penalties on low daily reserve balances — normally imposed on banks — contribute to predictable patterns in mean overnight rates by limiting banks' ability to shift holdings of reserves between days.

To incorporate these effects in our model, assume that banks ending a day with reserves less than  $Z$  face a cost  $r^D$  per unit of overdraft  $Z - z_t$ , in addition to the penalty on end-period *cumulative* overdrafts considered in our basic model.

In this case, when trading in the interbank market, a bank must forecast the effect of its decisions on both end-day and end-period balances. As in Section 3, at the equilibrium rate  $r_t$ , a bank should expect no gain from borrowing one dollar less on day  $t$  and one additional dollar on day  $t + 1$  than on the equilibrium path. On day  $t$  this perturbation would yield lower borrowing costs by  $r_t$ , but also an additional expected penalty  $r^D F_t(Z - \nu_t - m_t)$  on end-day overdrafts. On day  $t + 1$ , the same perturbation would yield additional (expected) borrowing costs  $E_t[r_{t+1}]$ , but also a decline in (expected) end-day penalties by  $r^D \int F_t(Z - \nu - m_{t+1})dG(\nu)$ . Setting total expected profits at zero,

$$r_t - \frac{1}{1 + r_t} E_t[r_{t+1}] = r^D [F_t(Z - \nu_t - m_t) - \frac{1}{1 + r_t} \int F_t(Z - \nu_{t+1} - m_{t+1})dG(\nu_{t+1})] . \quad (11)$$

Equation (11) has two main implications, both of which reflect the failure of the martingale hypothesis  $r_t - \frac{E_t[r_{t+1}]}{1+r_t} = 0$  in the extended model with costs of daily liquidity overdrafts.

First, there is now scope for high-frequency liquidity effects of monetary policy: an injection of funds of size  $\Delta$ , shifting  $m_t(c_{t-1})$  to  $m_t(c_{t-1}) + \Delta$ , lowers  $r_t$  below  $\frac{E_t[r_{t+1}]}{1+r_t}$ . By contrast, in our previous model,  $r_t = \frac{E_t[r_{t+1}]}{1+r_t}$  at all times, as  $r_t$  and  $\frac{E_t[r_{t+1}]}{1+r_t}$  depend only on projected *cumulative* end-period balances.

Second, days with greater variance of shocks should display higher mean rates, since they are associated with a higher probability of end-day overdrafts. For instance, if we allow the distributions  $F_t$  and  $F_{t+1}$  to differ, and substitute  $F_t(\cdot)$  with a mean-preserving spread  $\tilde{F}_t(\cdot)$ , then, everything else constant, the probability  $\tilde{F}_t(Z - \nu_{t+1} - m_{t+1})$  of a daily balance in the lower tail of the distribution rises, for low values of  $Z$ . Days with greater volatility of shocks will then exhibit rates that are both higher ( $r_t > \frac{E_t[r_{t+1}]}{1+r_t}$ ) and more volatile.

How consistent are these predictions with the behavior of very short-term rates in industrial countries' interbank markets?

The first prediction finds considerable support in several studies of high-frequency liquidity effects of monetary policy, including Hamilton (1997) for the United States, Hayashi (2001) for Japan, and Angeloni and Bisagni (2002) for the Euro Zone. These studies document that, in violation of the martingale hypothesis and in accord with the view that reserves are non-substitutable for banks between days of the same reserve period, monetary policy can drive predictable wedges in overnight rates across any two days.

The second prediction is fairly consistent with U.S. data, which display a clear correlation of means and volatilities of overnight rates for each reserve period (see Tables A1-A2, and the extended discussion in Furfine, 2000) and for other calendar days. In particular, high and volatile rates tend to cluster on settlement and end-quarter days. However, non-U.S. markets provide little support for the same prediction: our estimates show almost no sign correlation between mean and variance coefficients. For instance, while settlement days display pervasively high interest volatility, they display above-average mean rates only in the United States and Germany. Along the workweek, the only highest-volatility day that is

also the highest-mean day is Thursday in the United Kingdom; in no market is the lowest-volatility day also the lowest-mean day. There is some correlation in sign between mean and variance coefficients at end-months, end-quarters, and end-years: in 12 instances where both mean and volatility coefficients were statistically significant (out of 24 possible instances), 11 agreed in sign, showing both higher mean and volatility. On the other hand, we found mean and variance coefficients for days before and after holidays to be either insignificant or to display no correlation in sign. Altogether, this evidence suggests that introducing penalties on daily liquidity overdrafts or other frictions on banks' ability to shift reserve holdings across days contributes (at best) marginally to explaining overnight interest rate behavior in the world's largest money markets.

## 6 Conclusions

A simple model of an interbank market in which a central bank intervenes — with limited commitment — to stabilize interest rates around a target, can replicate the main qualitative properties of money market rates' volatility in the world's largest interbank markets. These properties include a tendency for interest rate volatility to behave cyclically in countries with reserve averaging, a tendency for volatility to rise in periods when a central bank is less likely to accommodate shocks to banks' liquidity, and — most interesting — a non-linear response of volatility to movements of market rates within official interest rate corridors. To replicate these properties, our model relies only on parameterized differences in central banks' commitment to interest rate smoothing at high frequency, pointing to this aspect of monetary policy execution as a key factor shaping the behavior of money markets and short-term interest rates around the world.

Our agenda for further research is topped by two items. First, allowing for heterogeneity in banks' behavior and exposure to shocks should be useful to capture features of the interbank trading environment that our representative-agent model with instantaneous trading is not designed to emulate. Chief among these features are a tendency for small banks to

be lenders and large banks to be borrowers in the market, and the sharp changes in interest volatility observed in intra-day data (including a systematic rise in volatility at day's end).

A second goal should be to incorporate the high-frequency, institution-driven perspective of our and of related studies, into a framework more suitable for asset pricing. At this stage, our contribution should be seen as complementary to that of studies such as Piazzesi (2001) and Farnsworth and Bass (2002), whose focus on asset pricing accompanies a much more stylized description of banks' trading environment than that provided by our study. A first step for future research bridging these two perspectives would be to parameterize the nonlinearities in interest rate behavior that we document and explain here, use them to augment otherwise-standard pricing models, and see if this improves the empirical performance of the model against short-term bond price data.

## Appendix A. Data, estimation, and technical results

**Data.** We collected daily transaction-weighted rates on unsecured overnight interbank loans and official interest rate corridors for the G-7 countries and the Euro Zone. The main source of data was Datastream, integrated by series of official rates (and, for Canada, of market rates) obtained from the relevant central banks. We also assembled information on central banks' procedures, reserve requirements, starting and ending dates of reserve periods, target interest rates, and market and target exchange rates. We collected data beginning as early as January 1, 1985, until April 4, 2002, but estimated each country's model only over the most recent period without major institutional change. Accordingly, the U.S. sample begins in January 1991, when reserve requirements for non-transaction deposits were eliminated. The Japanese sample ends in March 1999, when the "zero interest policy" was implemented. The German and Italian samples end at end-1998, when the Euro Zone sample begins. The French sample ends in June 1994, when changes in data reporting and in Banque de France procedures eliminated virtually all volatility in overnight rates. The Italian sample begins in December 1990, when reserve averaging was introduced. The Canadian sample begins in July 1994, when reserve requirements were lowered to zero.

We defined ceilings and floors of interest rate corridors in accord with definitions provided in BIS (1997) and individual central banks' publications. We used rates on standing facilities as ceiling rates for the Euro Zone, Germany, France, the United Kingdom, Italy, and Canada, and as floor rates for the Euro Zone and Canada. We used, as ceiling rates, penalty rates on end-period reserve overdrafts for the United States and pre-1995 Japan, and the discount rate — which has been used as a cap for market rates since March 1995 — for Japan in more recent years; and, for floor rates, we used the compensatory rate on end-period excess reserves for Italy, the official reverse-repo rate for France, and the discount rate for Germany. For countries lacking facilities aimed at keeping rates above zero — the United States, Japan, and the United Kingdom — we set the floor rate at zero.<sup>13</sup>

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<sup>13</sup>The German and Japanese discount windows operated very differently. Since 1995, Japan's discount

**Estimation.** We assumed the error terms  $\nu_t$  to be distributed as Student- $t$  variables, and estimated the model by maximum likelihood, using numerical optimization.<sup>14</sup>

We estimated our model independently for each country. To a large extent, this was the only possible choice, since most of our samples did not overlap in time.<sup>15</sup> Another problem, however, was that our non-linear, highly-parameterized model was computationally very intensive to estimate, requiring more than a day per regression, for some of our samples, on a standard desktop computer. Convergence and identification of global maximum likelihood certainly would have been a problem in simultaneous estimation.

For each sample, we followed a general-to-specific search to obtain our final specification. We started by estimating a model encompassing all variables in (2) and (3). We then dropped the least significant variables (in groups of three or four at a time) and re-estimated the model, iteratively, until we obtained specifications with all parameters significant at least at the 0.10 percent level, with the following exceptions to this rule. First, we retained the full set of reserve-period coefficients  $\delta_{m_t}$  in the mean equation if the coefficients were jointly significant. We did the same for the coefficients  $\xi_{m_t}$  in the variance equation and for the five week-day coefficients in both the mean and variance equations. We also retained all rate has been set at a penalty, thus effectively capping market rates. The German window, instead, was a rationed, below-market facility, providing a floor to market rates, as banks could repay outstanding loans before lending in the market at below-discount rates. Until January 2003, the U.S. discount rate was also set below market. However (see Peristiani, 1998), borrowing banks incurred additional non-pecuniary costs, including special Fed scrutiny, limits on further loans, and the cost of revealing financial weakness by being supported by the Fed. These costs suggest that U.S. market rates could be capped at a level equal to the discount rate *plus* non-pecuniary costs of discount borrowing. However, we preferred to use the penalty rate on reserve deficiencies as a ceiling for U.S. rates, since this is a readily available measure, while non-pecuniary costs of discount borrowing are difficult to estimate and are both bank-specific and time-varying.

<sup>14</sup>As in Andersen and Lund (1997), we smoothed the function  $|\nu_t|$  at the origin by using, for  $|\nu_t| < \frac{\pi}{2K}$ , the twice-differentiable approximation  $|\nu_t| = \frac{\frac{\pi}{2} - \cos(K\nu_t)}{K}$ , with  $K$  set at 20.

<sup>15</sup>Notably, the Euro Zone sample was wholly disjoint from those of Germany, France, and Italy. However, because we used for each country the longest, most recent sample of institutionally homogeneous data, problems of non-overlapping samples were pervasive.

the EGARCH parameters in our final specification, even when, for instance, the asymmetric response parameter  $\theta$  was not statistically significant.

To select the autoregressive EGARCH structure, we analyzed sample-by-sample the correlogram of squared standardized residuals. The result was an EGARCH(1,1) model for all countries except the United Kingdom, which required an EGARCH(2,2). Standard tests revealed insignificant coefficients for higher lags and no residual conditional heteroskedasticity.

We generally avoided testing for structural breaks in the coefficients, since the large number of observations would have led us to accept almost any break. Again, we made one exception to this rule. For France, even casual visual inspection of the data revealed a dramatic decline in volatility beginning in May 1992. At this time, the Banque de France had modified its operating procedures by intervening much more aggressively to reduce interest volatility and by clearly indicating to market participants its plan to provide funds at a stable repo rate. Accordingly, we introduced a constant shift dummy in the level of the variance as of May 1992 and allowed the EGARCH parameters to break at this time.

**Technical results.** We briefly review the results not discussed in the text. These are reported in Tables A1-A5 which, along with Table 1, completely document our estimation.

We found fixed effects on mean interest rates (the coefficients  $\delta_{m_t}$  and  $\delta_{c_t}$  in (2); see Table A2-A3) to be erratic in sign and significance across countries. The only fairly robust cross-country pattern in mean rates was the tendency for rates to rise at end-quarters (Table A3), a feature likely to reflect, in part, window-dressing effects previously recognized in U.S. data.<sup>16</sup> Week-day and holiday-related patterns in volatility (the coefficients  $\xi_{c_t}$  in Table A4) also proved rather non-robust across countries, except for greater end-quarter and end-year volatility in most samples, and greater volatility in the first day of each reserve period.

Changes in official rates had generally predictable effects on market rates (Table A5). Target, floor, and ceiling rate changes, in particular, were positively correlated with changes in market rates, with coefficients all ranging between zero and one. In countries for which

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<sup>16</sup>See, for instance, Allen and Saunders (1992).

we had access to interest rate targets, we found most end-period gaps of market rates from target rates to disappear in the first day of the following period.

Estimated features of the error terms were qualitatively similar to those documented by Ball and Torous (1999) for one-month rates. In particular, we found pervasive GARCH effects, with evidence of persistent volatility: the coefficients  $\lambda$  and  $\alpha$  in Table A4 are everywhere strongly significant. We also found the error terms to display clear fat tails, with degrees of freedom of the  $t$ -distribution estimated mostly between 2 and 4. Finally, we found some evidence of asymmetric effects of positive and negative shocks to rates (the coefficient  $\theta$  is significant in five of the eight regressions).

### Appendix B. Proofs of Results 2 and 3

**Proof of Result 2:** Since, by Result 1,  $r_T(\tilde{c}_T)$  is continuous and decreasing in  $\tilde{c}_T$ , then  $E_{T^{a.m.}}[r_T(\tilde{c}_T)] = \int r_T(c_{T-1} + m_T + \nu)dG(\nu)$  is continuously decreasing in  $(c_{T-1} + m_T)$  over the whole range  $[0, R]$ , and therefore over the sub-range  $[r^F, r^C]$  as well. Then, by the intermediate value theorem, there is a unique  $m^*$  that solves  $E_{T^{a.m.}}[r_T] = r^*$ , and this is continuous and strictly decreasing in  $c_{T-1}$  over  $c_{T-1} = [-\infty, \infty]$ . Since the truncation  $m_T = \min\{\max\{m^*, -m\}, m\}$  is a weakly monotonic transformation, Result 2 follows.  $\square$

**Remark.** For reference below, note that truncating  $m^*(c_{T-1})$  to  $[-m, m]$  divides the support  $c_{T-1} = [-\infty, \infty]$  into three intervals: **i)** a lower interval  $c_{T-1} = [-\infty, \underline{c}^T]$ , where  $m_T(c_{T-1}) = m$ , and  $E_{T^{a.m.}}[r_T]$  is strictly decreasing in  $c_{T-1}$ : for these values of  $c_{T-1}$ , intervention is already as high as possible, so that the rate expected for day  $T$  at intervention time rises as  $c_{T-1}$  falls from  $\underline{c}^T$  to  $-\infty$ ; **ii)** an upper interval  $c_{T-1} = [\bar{c}^T, \infty]$ , where  $m_T(c_{T-1}) = -m$ , and  $E_{T^{a.m.}}[r_T]$  is also strictly decreasing in  $c_{T-1}$ : intervention is already as low as possible, so that the rate expected for day  $T$  falls as  $c_{T-1}$  rises from  $\bar{c}^T$  to  $\infty$ ; and **iii)** an intermediate interval  $c_{T-1} = [\underline{c}^T, \bar{c}^T]$ , where  $m_T(c_{T-1}) = m^*(c_{T-1})$  is decreasing in  $c_{T-1}$ , and  $E_{T^{a.m.}}[r_T] = r^*$ : intervention equals the level needed keep  $E_{T^{a.m.}}[r_T]$  fixed at  $r^*$ . (The time superscripts in  $\underline{c}^T$  and  $\bar{c}^T$  denote the dependence of  $\underline{c}^T$  and  $\bar{c}^T$  on

time, but not on the state variables:  $\underline{c}^T$  and  $\bar{c}^T$  are day-specific functions of parameters only.) Therefore,  $E_{T^{a.m.}}[r_T \mid m_T(c_{T-1})]$  is everywhere non-increasing in  $c_{T-1}$ , and strictly decreasing over non-degenerate intervals. The same remark applies by replacing  $T$  everywhere with  $t$ .

**Proof of Result 3:** The proof is by recursion: We assume Result 3 to hold at  $t + 1$ , and show that it also holds at  $t$ . The recursion is completed by observing that, by Result 1, Result 3 holds at  $T$ .

To establish uniqueness we only need to observe that the quadratic equation (9) has a unique positive, increasing solution for  $r_t$  as a function of  $E_t[r_{t+1}]$ , and that  $E_t[r_{t+1}]$  is unique by the uniqueness property of conditional expectations.

To establish the monotonicity of  $r_t(\tilde{c}_t)$ , then, we only need to establish that  $E_t[r_{t+1}]$  is strictly decreasing in  $\tilde{c}_{t+1}$ . But this follows from  $E_t[r_{t+1}] = E_t[E_{t+1^{a.m.}}[r_{t+1}]] = \int_{-\infty}^{\infty} E_{t+1^{a.m.}}[r_{t+1}(c_t + \varepsilon + m_t + \nu_t)]dF(\varepsilon)$ , since  $E_{t+1^{a.m.}}[r_{t+1}(c_t + \varepsilon + m_t + \nu_t)]$  is non-increasing in  $c_t + \varepsilon + m_t + \nu_t$ , it is strictly decreasing on a positive-probability range of  $c_t + \varepsilon + m_t + \nu_t$  (see the Remark at the end of the previous proof), and  $F(\varepsilon)$  is strictly increasing over  $[-\infty, \infty]$ .  $\square$

### Appendix C. Outline of the solution method

To solve our model numerically, we begin by defining grids for the state variables  $c_t$  and  $\tilde{c}_t$ ,  $t = 1, \dots, T$ , and a discrete approximation for the normal distributions of  $\nu_t$  and  $\varepsilon_t$ . We then start with the analytical solution for  $r_T(\tilde{c}_T)$  in (6) to solve for  $m_T(c_{T-1})$ . To this end, for each node on the grid for  $c_{T-1}$ , we obtain  $m_T(c_{T-1})$  by computing the expectation  $E_{T^{a.m.}}[r_T(\tilde{c}_T)]$  in (7) over the possible realizations of  $\nu_T$ , given the known values of  $r_T(\tilde{c}_T)$ . Whenever simulated realizations of  $\nu_t$  (and, later, of  $\varepsilon_t$ ), added to  $c_{T-1}$ , yield a value of  $\tilde{c}_T$  not on the grid's nodes (which happens with probability one), the appropriate value of  $r_T(\tilde{c}_T)$  is linearly interpolated from the adjacent nodes.

Stepping back to day  $T - 1$ , we solve for  $r_{T-1}(\tilde{c}_{T-1})$  for each node on the grid for  $\tilde{c}_{T-1}$ , by computing the expectation  $E_{T-1}[r_T(\tilde{c}_T)]$  in (9) over the possible realizations of  $\varepsilon_{T-1}$  and

$\nu_T$ , given the known values of  $r_T(\tilde{c}_T)$ . To calculate  $c_T = \tilde{c}_{T-1} + \varepsilon_{T-1} + m_T + \nu_T$  we also include, for each realization of  $c_{T-1} = \tilde{c}_{T-1} + \varepsilon_{T-1}$ , the value of  $m_T(c_{T-1})$  obtained above.

We then step back to the morning of  $T-1$  to solve for  $m_{T-1}(c_{T-2})$ , then to the afternoon of  $T-2$  to solve for  $r_{T-2}(\tilde{c}_{T-2})$ , and so on, for  $t = T-3, \dots, 2, 1$ . This yields a complete family of solutions for the  $T$  vectors  $r_1(\tilde{c}_1), \dots, r_T(\tilde{c}_T)$ .

This solution can be used to compute volatility statistics by Montecarlo. In the examples discussed in the text, we simulated 50,000 reserve periods by drawing 50,000 sequences  $\{\varepsilon_t\}$  and  $\{\nu_t\}$ , each including  $T$  i.i.d., normally-distributed values of  $\varepsilon_t$  and  $\nu_t$ , and simulating the resulting sequences of  $\{r_t(\tilde{c}_t)\}$ . Importantly, since the solution method does not suffer from curse of dimensionality (the solution's time rises linearly with both  $T$  and the precision of the grids), the grids for  $c_t$  and  $\tilde{c}_t$  can be chosen — essentially — as fine as desired. (We chose grids for  $c_t$  and  $\tilde{c}_t$  mapping into grids for  $r_t$  with steps no larger than 1 basis point.)

Figure 1  
Corridor and Overnight Market Rates

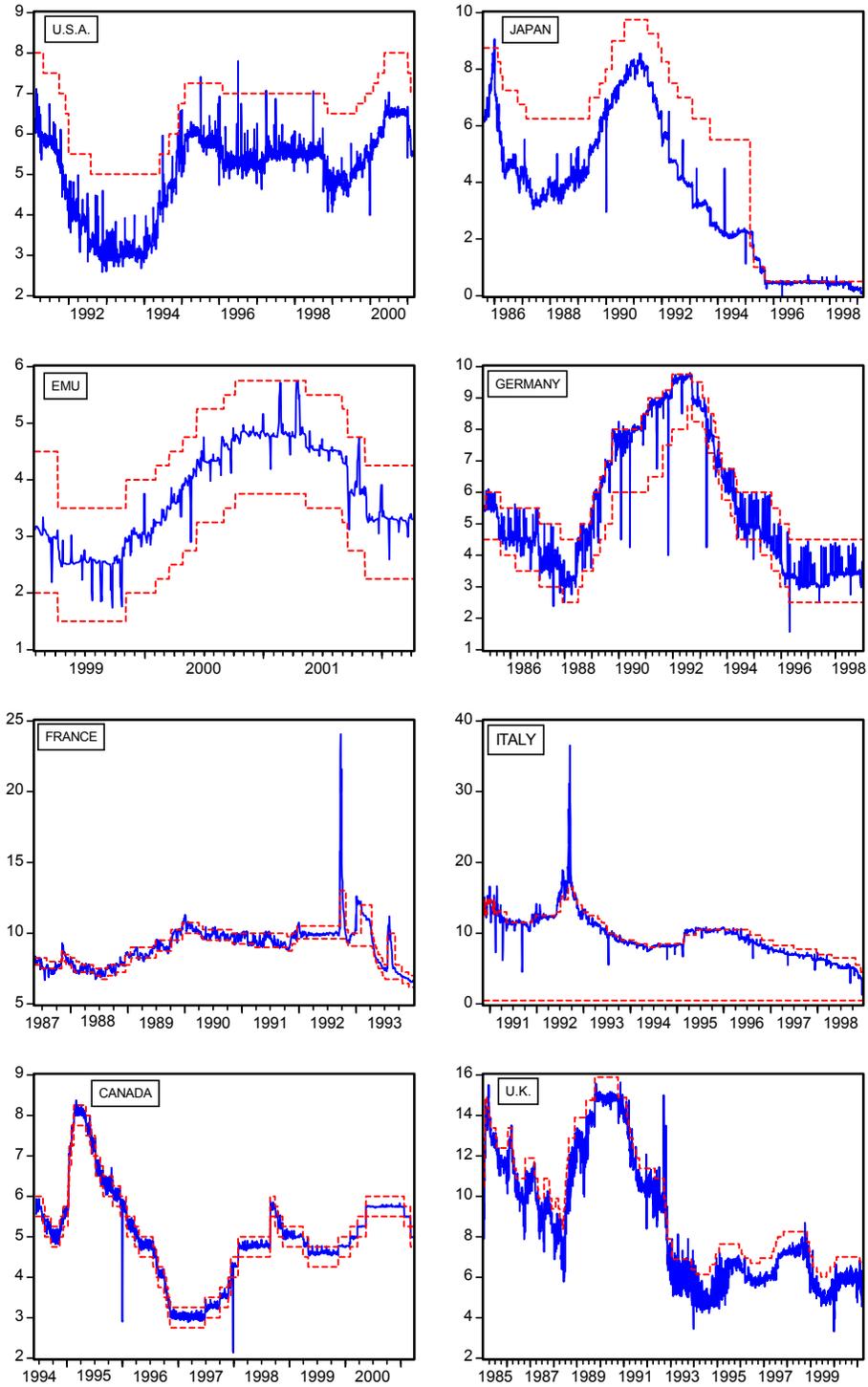


Figure 2  
 Estimated standard deviation of overnight rates  
 (ratio to standard deviation in the last day and 95 percent confidence band)

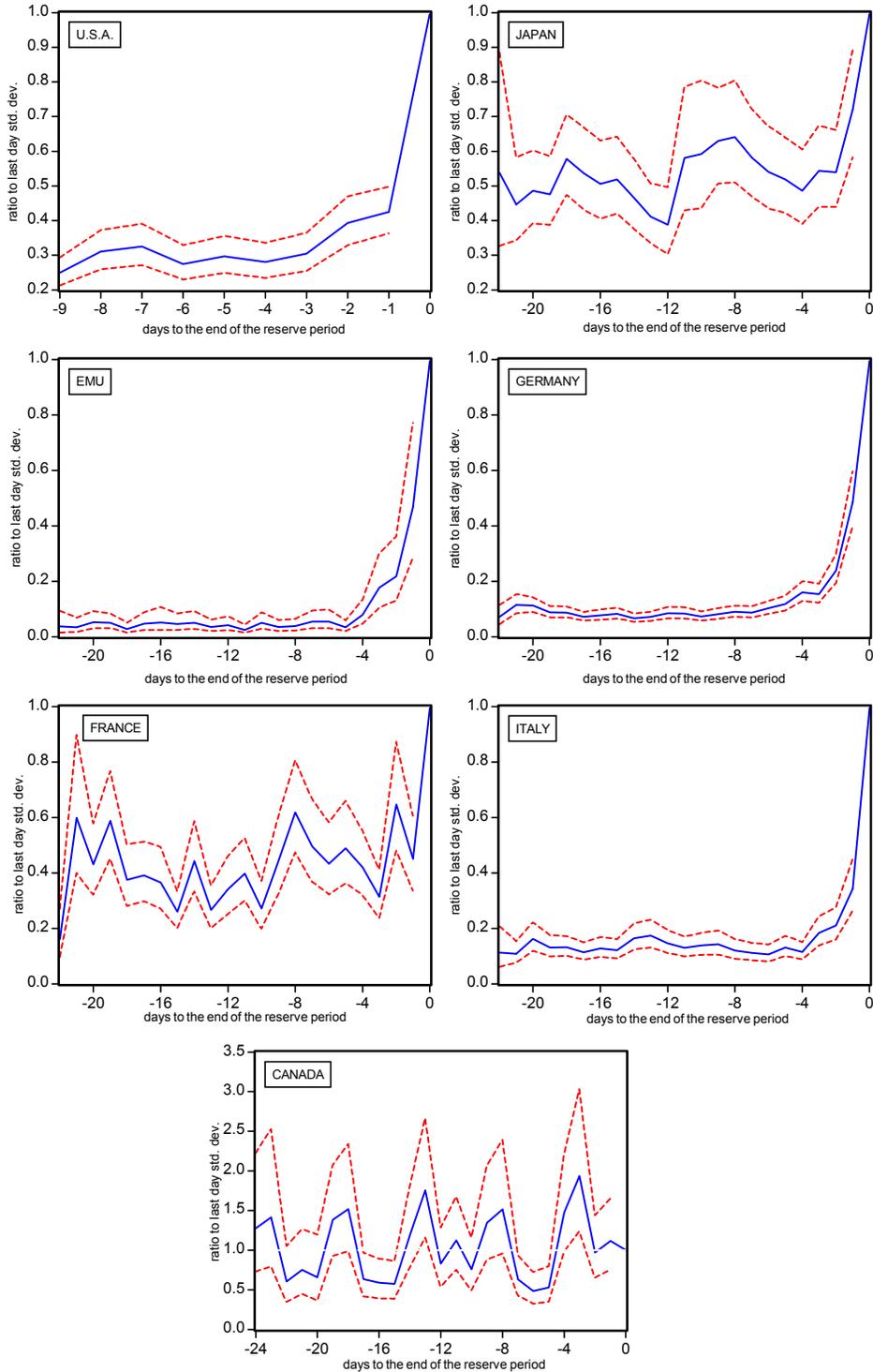


Figure 3

Overnight interest rate curves for different values of  $k$

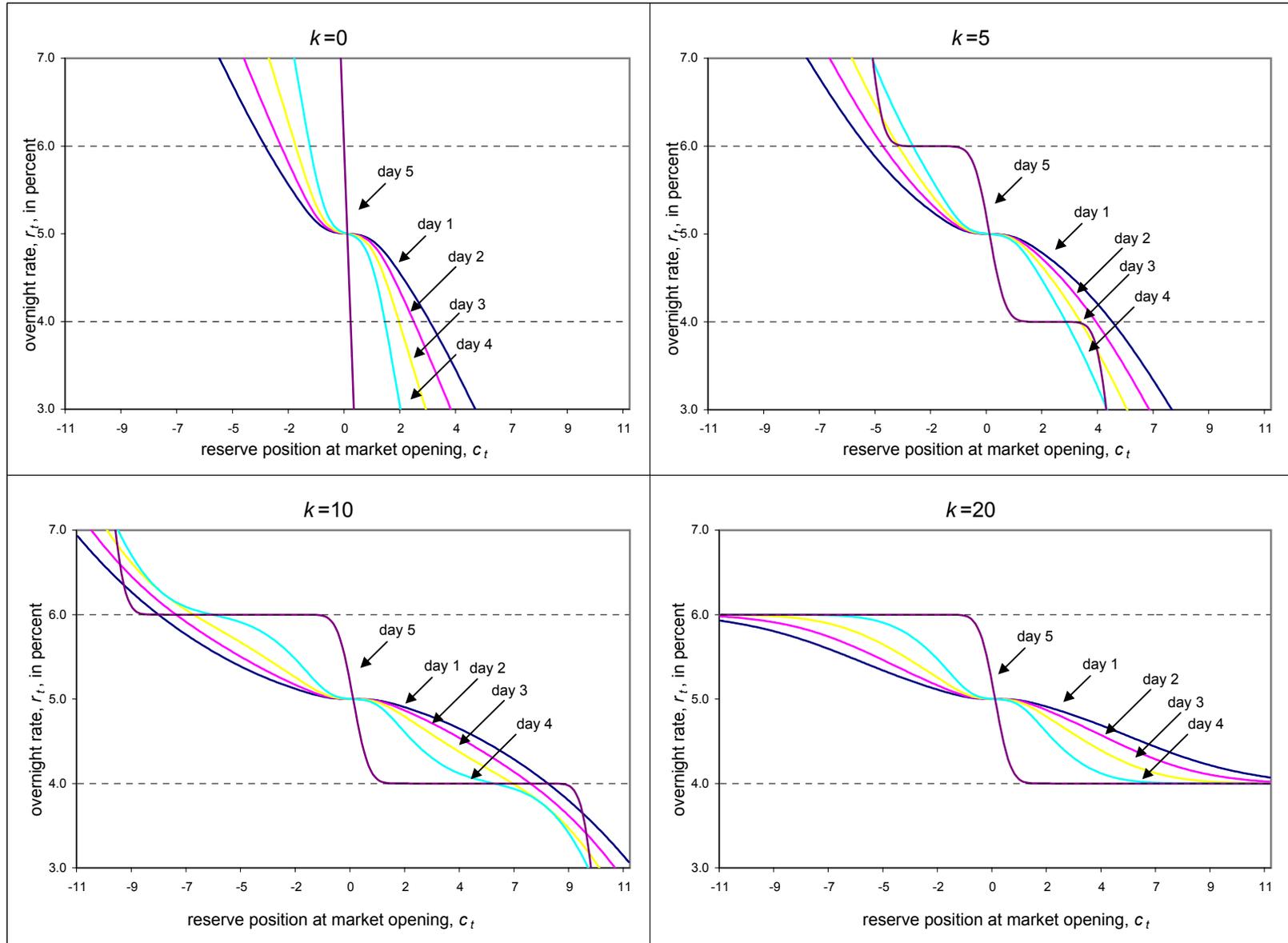


Figure 4

Conditional volatility of overnight rates over the reserve period

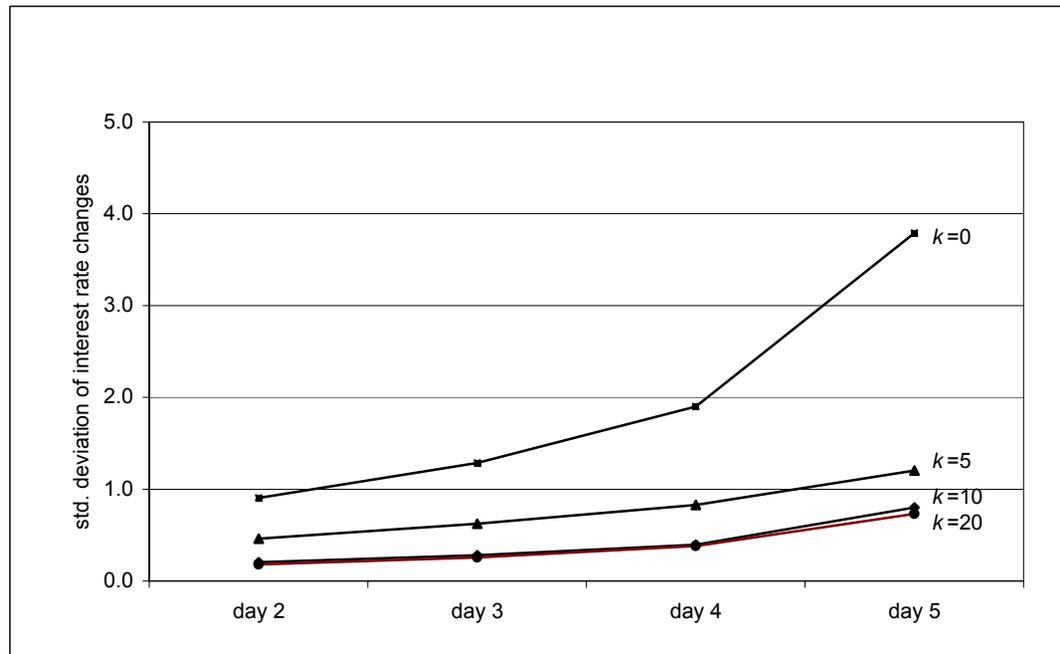
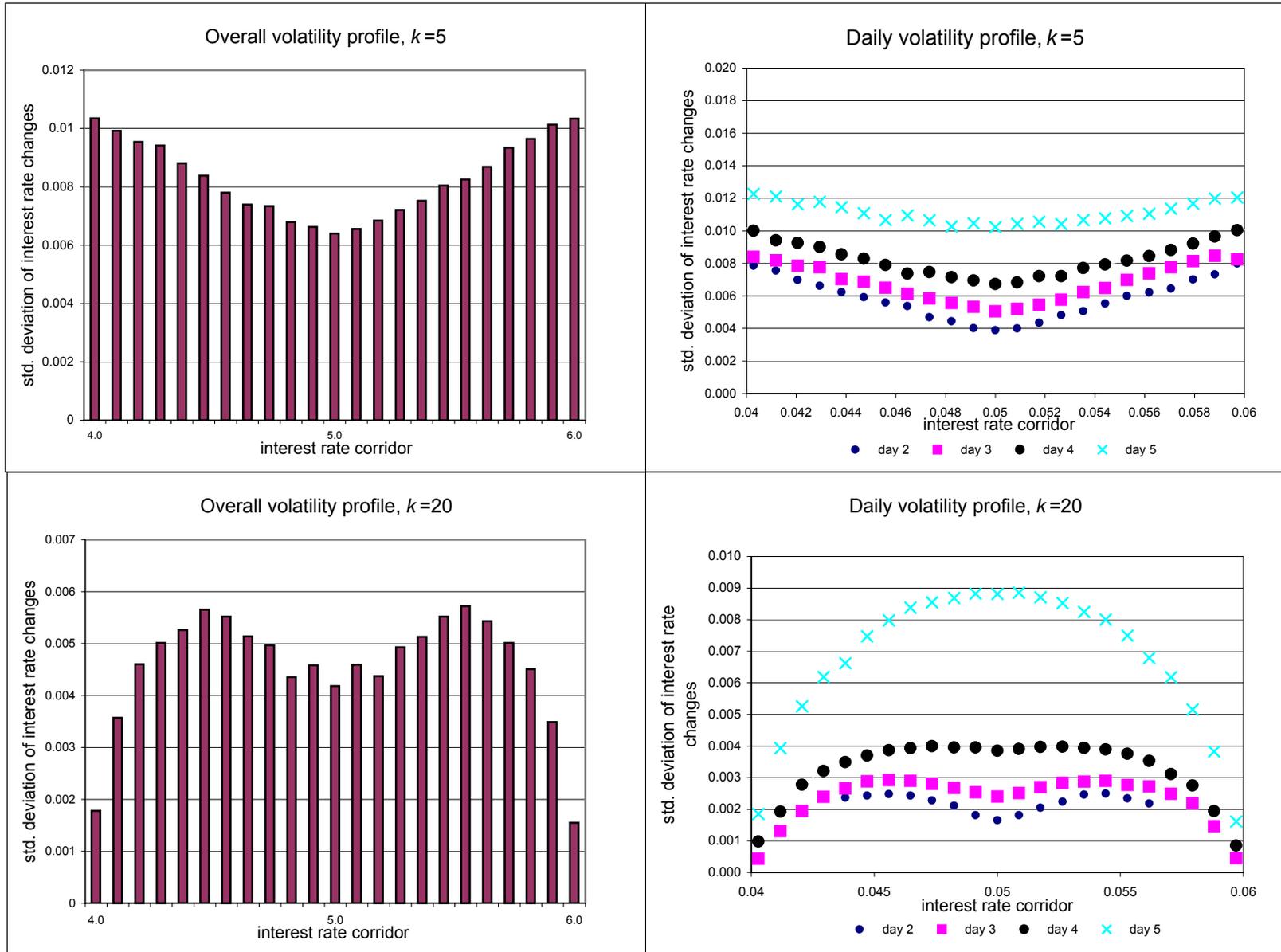


Figure 5

Conditional volatility of overnight rates over the corridor



**Table 1**

**Interest and Exchange Rate Corridor Effects on Volatility**  
(standard errors in parentheses; \*\* and \* indicate significance at 5% and 10% level)

	United States	Japan	Euro Zone	Germany	France	Italy	Canada	United Kingdom
Position of exchange rate in its corridor (ERM divergence indicator), $x_t$				0.032 ** (0.008)	0.021 ** (0.005)	0.050 ** (0.015)		0.008 (0.006)
Position of overnight rate in its corridor, $h_t$	-4.581 ** (1.669)	-1.223 * (0.662)	-7.117 ** (3.307)	-0.534 ** (0.248)	0.095 ** (0.038)	2.049 ** (0.596)	0.402 ** (0.055)	3.529 ** (0.913)

**Table A1**

**Reserve Period Effects on Volatility,  $\xi_{mt}$**

(ratio to standard deviation of last day of reserve period, standard errors in parentheses;

\*\* and \* indicate significance at 5% and 10% level of difference from 1)

	United States	Japan	Euro Zone	Germany	France	Italy	Canada
Days from end-period:							
0	1	1	1	1	1	1	1
1	0.425 ** (0.033)	0.720 ** (0.077)	0.468 ** (0.122)	0.486 ** (0.050)	0.451 ** (0.068)	0.343 ** (0.046)	1.116 (0.225)
2	0.393 ** (0.035)	0.539 ** (0.055)	0.218 ** (0.058)	0.238 ** (0.026)	0.647 ** (0.098)	0.210 ** (0.029)	0.969 (0.197)
3	0.305 ** (0.028)	0.544 ** (0.059)	0.177 ** (0.049)	0.153 ** (0.017)	0.313 ** (0.044)	0.183 ** (0.026)	1.937 ** (0.449)
4	0.280 ** (0.025)	0.486 ** (0.054)	0.079 ** (0.022)	0.160 ** (0.018)	0.420 ** (0.058)	0.115 ** (0.016)	1.476 (0.308)
5	0.297 ** (0.027)	0.519 ** (0.054)	0.034 ** (0.009)	0.118 ** (0.013)	0.489 ** (0.074)	0.131 ** (0.018)	0.528 ** (0.112)
6	0.275 ** (0.025)	0.540 ** (0.059)	0.054 ** (0.017)	0.103 ** (0.011)	0.433 ** (0.065)	0.106 ** (0.015)	0.482 ** (0.101)
7	0.325 ** (0.030)	0.582 ** (0.063)	0.054 ** (0.016)	0.086 ** (0.010)	0.495 ** (0.075)	0.111 ** (0.016)	0.630 ** (0.128)
8	0.310 ** (0.028)	0.641 ** (0.073)	0.038 ** (0.010)	0.089 ** (0.010)	0.618 ** (0.083)	0.120 ** (0.018)	1.513 (0.359)
9	0.249 ** (0.020)	0.629 ** (0.069)	0.035 ** (0.010)	0.081 ** (0.010)	0.443 ** (0.070)	0.142 ** (0.022)	1.347 (0.297)
10		0.591 ** (0.092)	0.050 ** (0.015)	0.072 ** (0.008)	0.271 ** (0.043)	0.138 ** (0.020)	0.757 (0.166)
11		0.581 ** (0.089)	0.025 ** (0.007)	0.083 ** (0.010)	0.397 ** (0.056)	0.129 ** (0.018)	1.124 (0.231)
12		0.388 ** (0.048)	0.042 ** (0.013)	0.084 ** (0.010)	0.341 ** (0.052)	0.146 ** (0.021)	0.829 (0.188)
13		0.411 ** (0.043)	0.035 ** (0.010)	0.071 ** (0.008)	0.267 ** (0.039)	0.173 ** (0.025)	1.757 ** (0.377)
14		0.465 ** (0.051)	0.050 ** (0.016)	0.067 ** (0.008)	0.442 ** (0.063)	0.164 ** (0.023)	1.193 (0.257)
15		0.519 ** (0.056)	0.045 ** (0.015)	0.082 ** (0.010)	0.259 ** (0.033)	0.121 ** (0.017)	0.575 ** (0.118)
16		0.505 ** (0.056)	0.051 ** (0.021)	0.077 ** (0.009)	0.365 ** (0.056)	0.127 ** (0.018)	0.588 ** (0.125)
17		0.537 ** (0.059)	0.046 ** (0.016)	0.071 ** (0.008)	0.390 ** (0.054)	0.113 ** (0.016)	0.635 ** (0.137)
18		0.578 ** (0.058)	0.027 ** (0.009)	0.086 ** (0.010)	0.375 ** (0.056)	0.132 ** (0.018)	1.519 (0.338)
19		0.476 ** (0.049)	0.050 ** (0.013)	0.087 ** (0.010)	0.588 ** (0.079)	0.131 ** (0.019)	1.385 (0.288)
20		0.486 ** (0.053)	0.052 ** (0.016)	0.112 ** (0.013)	0.431 ** (0.064)	0.162 ** (0.025)	0.657 * (0.208)
21		0.447 ** (0.060)	0.033 ** (0.013)	0.114 ** (0.017)	0.598 ** (0.125)	0.108 ** (0.019)	0.750 (0.204)
22		0.538 ** (0.140)	0.037 ** (0.019)	0.071 ** (0.018)	0.161 ** (0.044)	0.112 ** (0.037)	0.604 * (0.177)
23							1.413 (0.434)
24							1.274 (0.373)

**Table A2**

**Reserve Period Effects on Mean,  $\delta_{mt}$**

(mean difference from first day of the reserve period; standard errors in parentheses;  
**\*\*** and **\*** indicate significance at 5% and 10% level)

	United States	Japan	Euro Zone	Germany	France	Italy	Canada
Days from end-period:							
0	-1.255 (2.080)	-3.132 ** (1.356)	-8.326 * (4.419)	8.522 * (4.622)	-7.082 (7.126)	-8.623 (6.323)	-6.839 (5.008)
1	-10.617 ** (1.483)	-3.342 ** (1.290)	-10.556 ** (2.690)	-5.519 ** (2.161)	-6.394 (6.184)	-6.172 (3.884)	-7.479 (4.949)
2	-2.863 ** (1.323)	-2.856 ** (1.259)	-4.085 ** (1.990)	-9.744 ** (1.531)	-7.227 (6.048)	-7.685 ** (3.470)	-6.412 (4.881)
3	-12.102 ** (1.161)	-3.127 ** (1.231)	-3.410 * (1.726)	-10.450 ** (1.362)	-6.961 (5.773)	-7.692 ** (3.294)	-9.546 * (4.814)
4	-7.076 ** (1.044)	-3.371 ** (1.207)	-2.699 * (1.522)	-10.218 ** (1.292)	-8.892 (5.710)	-8.930 ** (3.177)	2.521 (4.563)
5	-9.298 ** (0.941)	-3.139 ** (1.194)	-2.703 * (1.505)	-9.508 ** (1.197)	-8.410 (5.592)	-8.068 ** (3.097)	-9.636 ** (4.439)
6	-7.004 ** (0.827)	-3.184 ** (1.174)	-2.614 * (1.499)	-9.161 ** (1.157)	-3.624 (5.409)	-8.075 ** (3.012)	-9.168 ** (4.425)
7	-1.186 * (0.694)	-2.957 ** (1.157)	-2.240 (1.493)	-8.744 ** (1.121)	1.033 (5.301)	-7.094 ** (2.961)	-9.022 ** (4.411)
8	-8.171 ** (0.443)	-2.231 * (1.130)	-2.157 (1.459)	-8.244 ** (1.096)	2.285 (5.115)	-8.266 ** (2.908)	-11.034 ** (4.388)
9	0	-1.827 * (1.102)	-2.187 (1.448)	-7.537 ** (1.061)	1.897 (4.710)	-10.549 ** (2.850)	2.002 (4.220)
10		-1.335 (1.071)	-2.178 (1.441)	-6.770 ** (1.038)	3.082 (4.528)	-8.328 ** (2.764)	-8.670 ** (4.113)
11		0.218 (0.942)	-1.976 (1.432)	-6.230 ** (1.019)	5.307 (4.428)	-7.476 ** (2.639)	-5.468 (4.066)
12		1.330 * (0.715)	-1.940 (1.416)	-5.951 ** (0.993)	5.986 (4.269)	-6.782 ** (2.560)	-3.670 (3.918)
13		1.623 ** (0.667)	-1.444 (1.408)	-5.269 ** (0.962)	7.933 * (4.153)	-5.274 ** (2.451)	-5.428 (3.811)
14		1.709 ** (0.649)	-0.839 (1.406)	-5.047 ** (0.940)	7.117 * (4.066)	-7.502 ** (2.278)	3.366 (3.417)
15		1.665 ** (0.623)	-0.036 (1.372)	-4.713 ** (0.924)	3.639 (3.850)	-8.006 ** (2.049)	-9.065 ** (3.254)
16		1.330 ** (0.594)	0.666 (1.314)	-4.566 ** (0.895)	-1.624 (3.775)	-7.543 ** (1.950)	-10.029 ** (3.215)
17		0.638 (0.550)	1.395 (0.876)	-4.344 ** (0.867)	-4.646 (3.651)	-7.416 ** (1.827)	-10.694 ** (3.169)
18		0.741 (0.506)	1.389 * (0.658)	-3.796 ** (0.840)	-1.983 (3.496)	-6.292 ** (1.721)	-13.736 ** (3.128)
19		0.281 (0.433)	1.201 * (0.633)	-2.922 ** (0.806)	0.223 (3.347)	-4.833 ** (1.607)	-1.831 (2.920)
20		0.320 (0.375)	0.916 (0.610)	-1.484 * (0.753)	0.041 (2.908)	-3.897 ** (1.456)	-10.804 ** (2.733)
21		-0.122 (0.289)	0.543 (0.358)	-0.176 (0.609)	-0.806 (2.542)	-3.388 ** (1.060)	-11.174 ** (2.568)
22		0	0	0	0	0	-11.176 ** (2.292)
23							-12.764 ** (2.084)
24							0

**Table A3**

**Calendar Effects on Mean,  $\delta_{ct}$ , and Previous Period's Effects on First Day's Mean**  
(standard errors in parentheses; \*\* and \* indicate significance at 5% and 10% level)

	United States	Japan	Euro Zone	Germany	France	Italy	Canada	United Kingdom
Monday		0		0		0		0
Tuesday <sup>1</sup>		0.049 (0.111)		0.376 ** (0.116)		-0.440 (0.331)		-0.910 (1.172)
Wednesday <sup>1</sup>		0.225 * (0.131)		-0.346 ** (0.144)		-1.386 ** (0.401)		-1.197 (1.071)
Thursday <sup>1</sup>		0.219 * (0.126)		-1.323 ** (0.149)		-1.162 ** (0.402)		0.946 (0.894)
Friday <sup>1</sup>		-0.501 ** (0.105)		-1.237 ** (0.133)		-0.383 (0.337)		-3.187 ** (0.634)
Day before end of months 1,2,4,5,7,8,10,11	0.023 ** (0.007)	0.010 ** (0.003)						
End of months 1,2,4,5,7,8,10,11	0.119 ** (0.008)	0.027 ** (0.005)	0.051 ** (0.007)				0.014 ** (0.003)	0.061 ** (0.013)
Day after end of months 1,2,4,5,7,8,10,11	-0.027 ** (0.009)		-0.035 ** (0.006)					-0.029 * (0.016)
Day before end of quarter								
End of quarter	0.280 ** (0.039)	0.067 ** (0.019)	0.069 ** (0.016)	0.145 ** (0.073)		0.030 ** (0.015)	0.019 ** (0.006)	0.153 ** (0.024)
Day after end of quarter	-0.176 ** (0.042)	-0.060 ** (0.016)	-0.076 ** (0.015)	0.039 * (0.023)				-0.102 ** (0.023)
Day before end of year								
End of year	-0.572 ** (0.067)		0.279 ** (0.056)	0.354 ** (0.149)	0.125 ** (0.005)	0.225 ** (0.077)		0.120 ** (0.053)
Day after end of year	0.830 ** (0.095)		-0.325 ** (0.053)	0.158 ** (0.034)	-0.188 ** (0.007)	-0.246 ** (0.105)		
Day before 1-day holiday								
Day after 1-day holiday	0.063 ** (0.027)					-0.061 ** (0.019)		
Day before 3-day holiday	-0.032 ** (0.010)	-0.009 ** (0.004)						-0.198 ** (0.037)
Day after 3-day holiday	0.244 ** (0.013)	0.006 ** (0.004)		0.030 ** (0.013)				0.167 ** (0.031)
Day before 4-day holiday								-0.120 ** (0.048)
Day after 4-day holiday				0.029 ** (0.009)	-0.113 ** (0.023)			0.125 ** (0.055)
Effect on first day of the period of difference from target rate in previous day, <b>M</b>	-0.796 ** (0.019)		-1.013 ** (0.022)				-0.583 ** (0.174)	
Effects on first day of the period of change in previous period's:								
last day, <sup>2</sup> $\phi_1$	-0.054 ** (0.022)	-0.847 ** (0.059)		-0.958 ** (0.018)	-0.347 ** (0.088)	-0.927 ** (0.030)		
day before last, $\phi_2$		-0.708 ** (0.082)		-0.980 ** (0.041)		-0.791 ** (0.073)		
second day before last, $\phi_3$		-0.560 ** (0.088)		-0.629 ** (0.053)		-0.588 ** (0.113)		
third day before last, $\phi_4$				-0.786 ** (0.092)		-0.716 ** (0.109)		
fourth day before last, $\phi_5$				-0.611 ** (0.079)				

<sup>1</sup> Mean difference from Monday (basis points).

<sup>2</sup> For France, coefficient estimated from pre-May 1992 data.

**Table A4**

**Calendar Effects on Variance,  $\xi_{ct}$ , and EGARCH Parameters**  
(standard errors in parentheses; \*\* and \* indicate significance at 5% and 10% level)

	United States	Japan	Euro Zone	Germany	France	Italy	Canada	United Kingdom
Monday		1	1	1	1		1	1
Tuesday <sup>2</sup>		0.869 ** (0.046)	1.239 ** (0.131)	0.935 (0.049)	1.066 (0.079)		1.053 (0.118)	0.792 ** (0.032)
Wednesday <sup>2</sup>		0.888 ** (0.049)	1.527 ** (0.168)	1.165 ** (0.062)	0.969 (0.070)		1.195 (0.139)	0.905 ** (0.039)
Thursday <sup>2</sup>		0.886 ** (0.046)	1.273 ** (0.139)	1.037 (0.058)	0.908 (0.066)		1.305 ** (0.147)	1.024 (0.045)
Friday <sup>2</sup>		0.942 (0.050)	1.251 ** (0.130)	1.160 ** (0.063)	1.113 (0.078)		1.316 ** (0.157)	0.992 (0.043)
End of months 1,2,4,5,7,8,10,11, or the previous and following days		0.391 * (0.219)	2.063 ** (0.430)				0.494 ** (0.172)	
End of quarter, or the previous and following days	2.138 ** (0.202)	2.928 ** (0.265)	3.154 ** (0.538)	0.438 ** (0.222)	1.015 ** (0.248)		1.032 ** (0.269)	
End of year, or the previous and following days	2.009 ** (0.396)	1.609 ** (0.244)	2.390 ** (0.726)		0.829 * (0.470)	1.982 ** (0.436)		
Day before 1-day holiday	1.252 ** (0.361)							
Day after 1-day holiday	1.259 ** (0.351)							
Day before 3-day holiday	0.420 * (0.262)			0.971 ** (0.344)				0.889 ** (0.297)
Day after 3-day holiday	0.812 ** (0.236)			1.212 ** (0.376)				0.862 ** (0.250)
Day before 4-day holiday				0.980 * (0.537)				0.668 ** (0.326)
Day after 4-day holiday								0.909 ** (0.389)
Post-May 1992 period					-8.067 ** (0.278)			
First day of the maintenance period, $\psi_1$		0.809 ** (0.230)	3.643 ** (0.485)	2.584 ** (0.252)	1.208 ** (0.326)	1.218 ** (0.294)		
EGARCH parameters:								
$\lambda$	0.429 ** (0.047)	0.966 ** (0.005)	0.778 ** (0.033)	0.895 ** (0.011)	0.980 ** (0.001)	0.937 ** (0.010)	0.971 ** (0.007)	1.329 ** (0.093)
$\alpha$	0.957 ** (0.119)	0.474 ** (0.035)	1.642 ** (0.440)	0.673 ** (0.101)	0.169 ** (0.017)	0.706 ** (0.078)	0.205 ** (0.030)	0.604 ** (0.041)
$\theta$	0.456 ** (0.075)	0.032 (0.024)	0.121 (0.105)	-0.030 (0.032)	0.171 ** (0.018)	-0.102 ** (0.042)	0.181 ** (0.029)	-0.103 ** (0.039)
$\lambda(2)$								-0.335 ** (0.092)
$\alpha(2)$								-0.456 ** (0.044)
$\theta(2)$								0.070 * (0.035)
Degrees of freedom of $t$ -distribution	2.623 ** (0.188)	2.848 ** (0.155)	2.332 * (0.221)	2.279 ** (0.094)	2.130 ** (0.028)	2.641 ** (0.171)	3.798 ** (0.325)	4.263 ** (0.270)

<sup>2</sup> Ratio to standard deviation of Monday, with \*\* and \* indicating significance at 5% and 10% level of difference from 1.

**Table A5****Effects of Changes in Official Rates**

(standard errors in parentheses; \*\* and \* indicate significance at 5% and 10% level)

	United States	Japan	Euro Zone	Germany	France	Italy	Canada	United Kingdom
Effects on mean, ( :								
day $t$ change when target is changed by 1 on the same day	0.438 ** (0.044)	0.871 ** (0.227)	0.306 ** (0.067)	0.062 ** (0.019)			0.842 ** (0.028)	0.552 ** (0.067)
day $t$ change when ceiling is changed by 1 on the same day				0.215 ** (0.037)	0.048 ** (0.007)	0.252 ** (0.087)	0.106 ** (0.022)	
day $t$ change when floor is changed by 1 on the same day		0.729 ** (0.066)				0.167 * (0.096)		
Effects on variance, T :								
$t$ is the day of a target change		2.501 ** (1.122)	1.334 ** (0.659)				0.767 ** (0.179)	0.841 ** (0.182)
$t$ is the day of a ceiling change	1.213 ** (0.401)			1.373 ** (0.351)	1.814 ** (0.179)	0.733 ** (0.243)		
$t$ is the day of a floor change		2.672 ** (0.493)						

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