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Abstract

We explore the cross-sectional pricing of volatility risk by decomposing equity market volatility into short- and long-run components. Our finding that prices of risk are negative and significant for both volatility components implies that investors pay for insurance against increases in volatility, even if those increases have little persistence. The short-run component captures market skewness risk, which we interpret as a measure of the tightness of financial constraints. The long-run component relates closely to business cycle risk. Furthermore, a three-factor pricing model with the market return and the two volatility components compares favorably to benchmark models.

Key words: asset pricing, stochastic volatility, cross section of returns

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When market volatility is stochastic, intertemporal models predict that asset risk premia are not only determined by covariation of returns with the market return, but also covariation with the state variables that govern market volatility. To study this prediction, we model the log-volatility of the market portfolio as the sum of a short- and a long-run volatility component. This approach parsimoniously captures shocks to systematic risk at different horizons.

Market volatility is a significant cross sectional asset pricing factor as shown by Ang et al. (2006).[1] Their two-factor model with the market return and market volatility does reduce pricing errors compared to the capital asset pricing model (CAPM), though not by as much as the Fama-French model. In contrast, our benchmark asset pricing model with the market return and the two volatility components as cross sectional pricing factors achieves lower pricing errors than the Fama and French (1993) model for size and book-to-market sorted portfolios. Our finding that the short- and long-run volatility components have negative, highly significant prices of risk is robust across sets of portfolios, sub-periods, and volatility model specifications.

Consistent with previous research, we also find that the average compensation for volatility risk is positive. This is because the prices of risk of both volatility components are negative, and average sensitivities to the volatility components are also negative. Our two-factor decomposition shows that the average risk premium for short-run volatility is 0.17% monthly versus 0.23% monthly for long-run volatility.

Across individual portfolios, we see a wide dispersion in sensitivity to the volatility components, which generates cross sectional variation in the risk premia attributed to these factors. For example, short-run volatility risk premia across the growth-value dimension range from -0.22% to 0.41%, while long-run risk premia range from 0.16% to 0.30% along the growth-value dimension. Because the dispersion of average returns across the book-to-market portfolios is the most important source of failure for the CAPM, the short-run component is an important cross sectional pricing factor.

To interpret the economics of short- and long-run volatility as pricing factors, we relate these two factors to a measure of the tightness of financial constraints and to the business cycle. We use the skewness of market returns as an indicator of the tightness of financial constraints, since return skewness arises endogenously in pricing theories with financial constraints (Hong and Stein (2003), Yuan (2005)). Intuitively, shocks to market skewness are particularly costly when financial constraints of investors are binding. Industrial production growth is our proxy for the business cycle; we use this measure because market volatility moves with the business cycle (Schwert (1989a) and (1989b)).

In our empirical analysis, we find that the risk premium of the short-run component correlates highly with the risk premium of market skewness, while the risk premium of the long-run component correlates highly with the risk premium of industrial production growth. Furthermore, market skewness is a significant pricing factor in the cross section of size and book-to-market sorted portfolios; however, including the short-run volatility component as additional factor makes skewness insignificant. The significance of industrial production innovations is eliminated by the inclusion of the long-run component as a pricing factor.

Our pricing results deepen our understanding of the relationship between risk and return. An extensive literature shows that the time series relation between market risk and return is ambiguous (see, in particular, French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), and Wu (2001)). Our cross sectional pricing approach allows us to distinguish between the asset pricing effects of shocks to volatility and the static risk-return tradeoff. Volatility risk premia compensate investors for the risk that volatility might increase in the future. Our finding that prices of risk are negative and significant for both volatility components implies that investors are willing to pay for insurance against increases in volatility risk, even if those increases have little persistence.

The remainder of the paper is organized as follows. In Section I, we present the volatility components model and its intertemporal asset pricing implications. Section II describes the main cross sectional asset pricing results and relates the volatility components to our measures of financial constraints and the business cycle. In Sections III and IV, we present robustness results. Section V concludes.

I. The short- and long-run components of market volatility

The literature on the time series of market risk shows that aggregate volatility is subject to shocks at different frequencies (Engle and Lee (1999)). Intertemporal asset pricing models predict that the set of state variables that determines systematic risk also determines expected returns of individual assets or portfolios of assets (Merton (1973)). Our analysis combines these insights from the volatility and the asset pricing literature. In this section, we present a model of market return volatility that parsimoniously captures short- and long-run volatility factors. We also present the asset pricing restrictions that will be tested in later sections.

A. Specification of volatility dynamics

Starting with Engle and Lee (1999), many studies find that two-component volatility models outperform one-component specifications in explaining equity market volatility.[2] In addition, two component volatility models perform well in the option pricing literature (see Xu and Taylor (1994), Bates (2000), and Christoffersen, Jacobs, and Wang (2006)). Nelson (1991) shows that conditionally log-normal models of volatility perform better than square-root or affine volatility specifications. In modeling market risk, we incorporate these features and specify the dynamics of the market return in excess of the risk-free rate R_t^M and its conditional volatility $\sqrt{v_t}$ as:

Market return: $$R_{t+1}^M = \mu_t^M + \sqrt{v_t}\varepsilon_{t+1} \tag{1a}$$

Market volatility: $$ln\sqrt{v_t} = s_t + l_t \tag{1b}$$

Short-run component: $$s_{t+1} = \theta_4 s_t + \theta_5 \varepsilon_{t+1} + \theta_6 \left(\left|\varepsilon_{t+1}\right| - \sqrt{2/\pi}\right) \tag{1c}$$

Long-run component: $$l_{t+1} = \theta_7 + \theta_8 l_t + \theta_9 \varepsilon_{t+1} + \theta_{10} \left(\left|\varepsilon_{t+1}\right| - \sqrt{2/\pi}\right). \tag{1d}$$

In equation (1a), ε_t is a normal i.i.d. error term with zero expectation and unit variance, and μ_t^M is the one-period expected excess return. The log-volatility in equation (1b) is the sum of two components s_t and l_t. Each component is an AR(1) processes with its own rate of mean reversion. Without loss of generality, let l_t be the slowly mean-reverting, long-run component and s_t be the quickly mean-reverting, short-run component ($\theta_4 < \theta_8$). We normalize the unconditional mean of s_t to be zero.

The terms $|\varepsilon_{t+1}| - \sqrt{2/\pi}$ in equations (1c) and (1d) are the shocks to the volatility components. Their expected values are equal to zero, given the normality of ε_t. For these error terms, equal sized positive or negative innovations result in the same volatility change, although the magnitude can be different for the short- and long-run components (θ_6 and θ_{10}). We also allow for an asymmetric effect of returns on volatility by including the market innovation in equations (1c) and (1d) with corresponding coefficients θ_5 and θ_9.

The market model defined by equations (1a) – (1d) converges to a continuous-time, two factor stochastic volatility process (see Nelson (1990)). Chernov et al. (2003) estimate a variety of specifications including the continuous time limit of our equations (1a) – (1d). They find that a linear specification with jumps fits the market return data as well as a log-linear specification with two components. An advantage of our specification is that it can be estimated in discrete time via maximum likelihood.

B. Equilibrium asset pricing restrictions

Expected returns are endogenous. The key insight of intertemporal equilibrium models such as Merton's (1973) intertemporal capital asset pricing model (ICAPM) is that state variables of the return generating process are state variables of the pricing kernel. In our setting with a two-component volatility process, the equilibrium pricing kernel thus depends on both the short- and long-run volatility components as well as the excess market return (R_t^M). We denote returns on asset i in excess of the risk-free rate by R_t^i. The equilibrium expected excess return for asset i is:

$$E_t\left(R_{t+1}^i\right)=\gamma_t Cov_t\left(R_{t+1}^i,R_{t+1}^M\right)+F_s Cov_t\left(R_{t+1}^i,s_{t+1}\right)+F_l Cov_t\left(R_{t+1}^i,l_{t+1}\right), \tag{2}$$

where γ_t is the coefficient of relative risk aversion, and F_s and F_l are (negatively) proportional to changes in the marginal utility of wealth due to changes in the state variables s_t and l_t.

Equation (2) shows that expected returns depend on three risk premia. The first risk premium arises from the covariance of the asset return with the market return, multiplied by relative risk aversion γ_t. This is the risk-return tradeoff in a static CAPM model. The second and third risk premia depend on the covariance of the asset return with the innovations in the short- and long-run factors. These are scaled by the impact of changes in the volatility factors on marginal utility of wealth (the terms F_s and F_l).[3]

We allow for time varying relative risk aversion in equation (2) to accommodate non-separable preferences. Equation (2) can be derived in Duffie and Epstein's (1992) economy with stochastic differential utility.[4]

In this intertemporal pricing framework, the evolution of market volatility is exogenous. Thus, market volatility in our setup can be linked to the variability of fundamental economic variables (for example, Cox, Ingersoll, and Ross (1985) and Tauchen (2005) address the equilibrium pricing of this type of risk). Market volatility can also reflect uncertainty generated endogenously by financial constraints as suggested by Cuoco (1997) as well as Detemple and Serrat (2003).

C. Estimation of the volatility components

In the case of the market portfolio, equation (2) implies that the conditional expected return depends on its conditional variance (the static risk-return tradeoff) and the volatility components (the terms associated with intertemporal hedging). To specify a market return model that captures the dependence of expected returns on the state variables of the economy, our benchmark definition of μ_t^M is:[5]

$$\mu_t^M = \theta_1 + \theta_2 s_t + \theta_3 l_t. \tag{3}$$

We interpret equation (3) as a first-order approximation to the functional relationship of the expected market return μ_t^M with the volatility components s_t and l_t. This specification does not allow the separate identification of the static risk-return trade-off and the dynamic hedging component of volatility risk, but the cross sectional approach does allow such identification.

We estimate our volatility model using daily market excess returns. We use daily data in order to improve the estimation precision (see Merton (1980)), and we time aggregate to a monthly frequency for our cross sectional analysis.[6] The value-weighted (cum-dividend) *Center for Research in Security Prices (CRSP)* portfolio return is our measure of the market return, and the three-month Treasury rate is our proxy for the risk-free rate. We estimate the volatility model from 1962/7/3 to 2005/12/31.

Summary statistics for the daily market excess return are given in the first row of Table I, and estimation results for the volatility model are shown in the remaining rows of the table.

[Table I]

In the expected return equation, we find that short-run volatility has a significant, negative coefficient (θ_2), while the long-run volatility component has a positive coefficient (θ_3) significant at the 10% level. The expected market return thus depends positively on long-run volatility (the risk-return trade off), but negatively on short-run volatility. This finding might explain why previous papers often have difficulty detecting a time-series relationship between risk and expected returns.[7]

We identify the short- and long-run components by their relative degrees of autocorrelation: the short-run volatility component has an autoregressive coefficient (θ_4) of 0.333, and the long-run component has an autoregressive coefficient (θ_8) of 0.989. While the long-run component is highly persistent, it is not permanent; we reject the hypothesis that θ_8=1 at the 1% level. Because the short- and long-run components determine log-volatility additively, we are not able to identify the means of the two components separately, and we estimate only the mean of the long-run component (θ_7).

We find that negative returns increase short- and long-run volatility more than positive returns. In our volatility model, this asymmetric impact of market return innovations on market volatility captures the time-varying skewness of market returns. The asymmetric effect for the short-run component (θ_5) is more than twice as large in magnitude as this effect for the long-run component (θ_9). Thus, we would expect short-run volatility to be closely linked to market skewness, since a negative return shock disproportionately increases short-run volatility, which further raises the likelihood of another a large move (either up or down).[8]

In Figure 1, we graph three estimates of market risk: conditional volatility from our model, implied volatility from the Chicago Board Options Exchange Volatility Index (VIX), and realized volatility from daily returns. It is apparent that all three measures are quite similar. As would be expected in the presence of a volatility risk premium, implied volatility appears to be a biased estimate of conditional and realized volatility (see Fleming (1999), Rosenberg (2000), and Bollerslev and Zhou (2005)).

[Figure 1] [Figure 2] [Figure 3]

Our long-run volatility component is highly correlated with the low frequency component of market volatility estimated using other techniques. To illustrate this point, we plot the long-run component together with the trend component of Hodrick and Prescott (1997) filtered daily squared returns in Figure 2. We see that these estimates of long-run volatility track each other closely. In Figure 3, we graph the time series of the short-run volatility component, which is clearly much less persistent than the long-run component.

For our cross sectional pricing tests, we aggregate daily innovations of the volatility components to a monthly frequency by subtracting the short- and long-run component from the value expected 21 days earlier. We then sum these innovations over the days in each month, and denote the monthly innovations of the short- and long-run components as *sres* and *lres*. Summary statistics of the innovations and the other pricing factors are shown in Table II.

[Table II]

II. The cross section of returns in size and book-to-market sorted portfolios

Fama and French (1992 and 1993) show that expected returns for size and book-to-market sorted portfolios are particularly difficult to fit with the single-factor CAPM. To address this failure of the CAPM, Fama and French (1993) develop the value and size asset pricing factors (high-minus-low *HML* and small-minus-big *SMB*). In this section, we focus on pricing the size and book-to-market sorted portfolios. We use the Fama-French three-factor model as a benchmark for the following asset pricing tests.[9]

A. The cross section of factor loadings, prices of risk, and risk premia 1963 to 2005

We estimate the unconditional beta representation of equation (2), which states that a portfolio's expected return is equal to the sum of its factor loadings times the prices of risk. In the first stage, we obtain loadings for each portfolio from time series regressions (Table III). In the second stage, we estimate prices of risk using monthly cross sectional regressions (Table IV). We then calculate risk premia for each portfolio using the factor loadings times the prices of risk (Table V). The standard errors reported in the tables are adjusted to incorporate estimation error in the volatility factors and in the factor loadings. A description of the estimation methodology for the factor loadings, prices of risk, and standard errors is given in the Appendix.

As shown in Table III, factor loadings on short- and long-run volatility exhibit significant variability across both the size and book-to-market dimensions. We see that growth stocks have positive loadings on short-run volatility, while value stocks tend to have negative loadings. Growth stocks thus provide insurance against short-run volatility shocks – after controlling for the market return and long-run volatility.

We can interpret the positive loadings of growth stocks on short-run volatility risk using insights from Pastor and Veronesi (2003). Growth stocks are typically young firms with a high degree of uncertainty about future profit growth. Pastor and Veronesi show that investor learning about firms' growth opportunities implies a positive dependence of returns with respect to volatility (after controlling for other pricing factors, particularly the market return). The cross sectional heterogeneity of volatility exposures along the value-growth dimension could also reflect differences in the duration of cash flows which might be linked to the differences in duration of the volatility shocks (Lettau and Wachter (2007)). In addition, diversity in the option value of growth opportunities across firms due to heterogeneous adjustment costs should create variability in volatility loadings along the value-growth dimension (see Gomes, Kogan, and Zhang (2003) and Zhang (2005)).

[Table III]

In Table IV, we analyze the pricing of volatility risk in the cross section of the 25 size and book-to-market sorted portfolios. Ang et al. (2006) show that market variance has a negative price of risk for the 1986 to 2000 sample period. We also identify a negative price for market variance risk in the 1963 to 2005 sample period (column iv) using innovations in estimated market variance from our two factor model.[10]

Following the prediction of the ICAPM that the state variables of market volatility should be priced factors, we go on to explore the pricing of each volatility component. In column (v) of Table IV, we see that both volatility components are significant pricing factors at the 1% level, and both components have negative prices of risk. These negative prices of volatility risk mean that assets with high returns in states of the world with high volatility are expensive (i.e. they have low expected returns). The price of risk of the short-run component of -0.21% monthly implies that an asset with a short-run beta of unity requires a 21 basis point lower return than an asset with zero exposure to the short-run component.

[Table IV]

In terms of pricing performance, our three factor volatility model compares favorably with four alternative benchmarks. We report the sum of squared pricing errors as well as the root-mean-squared pricing error (RMSPE) to evaluate the pricing performance of the different models (details are given in the Appendix). For example, our three factor model has a root-mean-squared pricing error of 0.13 (column v), while the Fama-French three factor model has a root-mean-squared pricing error of 0.14 (column ii). Besides the Fama-French model, we present three other benchmark asset pricing models: the CAPM (column i), a model with the market return and the momentum factor of Carhart (1997) (column iii), and a model analogous to Ang et al. (2006) with the market and innovations to the market variance as risk factors (column iv). None of these models performs as well as a model with the market return and the two volatility components as shown by their RMSPE's which range from 0.24 to 0.31.

In columns (vi) and (vii), we report prices of risk when the short- and long-run volatility components enter as separate factors. Each of the components is significant at the 1% level, and the prices of risk are similar to the model in which both components enter simultaneously. It is also noteworthy that the two factor model with the market and the short-run component (column vi) outperforms the other two factor models in Table IV (columns iii and iv).

Figure 4 provides a graphical comparison of different asset pricing specifications. In this figure, we display a scatter plot of average excess returns for the 25 size and book-to-market sorted portfolios against predicted returns from four different models. The models we include are: the CAPM, the Fama-French model, the Ang et al. (2006) model, and our volatility components model. We see that the model with the market return and variance as pricing factors (lower-left panel) only slightly improves upon the CAPM (upper-left panel). The fit of the market return and market variance model is not as good as the Fama-French model (upper-right panel). Notably, our three-factor model with the market and two volatility components (lower-right panel) produces a fit that is comparable to the Fama-French model. Considering that the Fama-French factors are constructed to address the mispricing in the size and book-to-market sorted portfolios, these plots provide an illustration of the accurate pricing properties of the volatility components model.

[Figure 4]

When we augment our benchmark three factor model with the *HML* and *SMB* factors, all five factors are significant, and the volatility components stay significant at the 1% level (Table IV, column viii). In this five-factor pricing model, pricing errors are slightly smaller than in either the volatility components three factor model or the Fama-French three factor model. So, the volatility components and the Fama-French factors capture some orthogonal sources of priced risk. Yet, Figure 4 suggests that many of the portfolios are priced similarly by the Fama-French factors and the volatility components.

To better understand this apparent tension, we take a closer look at the pricing errors for each portfolio. We find that the volatility components model more accurately prices small and large growth stocks (with a contribution of 0.08% and 0.02% to the difference in sum of squared pricing errors), while the Fama-French model prices a variety of portfolios marginally better (with contributions of 0.01% or 0.02% to the difference in sum of squared pricing errors for a variety of portfolios). These differences drive the result that all of the factors together are statistically significant so that the combination of the volatility components and the Fama-French factors improves pricing performance.

Why do the volatility components explain the average returns of growth stocks? Growth stocks have high market betas but low average returns. Controlling for the market return, we find that growth stocks also have positive exposures to short-run volatility risk, while the exposure of value stocks is negative. Combined with the negative price of volatility risk, our three-factor model predicts average returns of growth stocks that are lower than the CAPM benchmark.

For a portfolio that is long growth stocks and short value stocks, the magnitude of this effect is -0.63% monthly. We compute this portfolio return using the average growth portfolio risk premium minus the average value portfolio risk premium attributable to short-run volatility risk. These average returns are reported in the middle panel of Table V (-0.22% - 0.41% = -0.63%). Thus, in our pricing model, the low average returns of growth stocks are explained by the insurance they provide against short-run volatility risk.

[Table V]

Furthermore, we find that the long-run component explains some of the value spread (0.30% - 0.16% = 0.14% monthly, from the bottom panel of Table V), but much less than the short-run component. There is a large spread in short-run volatility risk premium between small and large stocks (-0.12% - 0.31% = - 0.43%, middle panel of Table V), but this spread in risk premia is more than offset by the long-run volatility premia (0.53% + 0.07% = 0.60%, bottom panel of Table V). The average short-run risk premium over all portfolios – a measure of the market risk premium associated with short-run volatility – is 0.17% monthly, and the average risk premium for the long-run component is somewhat larger (0.23%).

B. Interpreting the short- and long-run volatility components

In this section, we present cross sectional pricing results to facilitate interpretation of the short- and long-run volatility components. For this purpose, we investigate the relationship of the volatility components with two drivers of market volatility: the tightness of financial constraints and business cycle risk.

We use market skewness as an indicator of the tightness of financial constraints. Intuitively, an increase in market skewness makes financial constraints more binding. Indeed, Hong and Stein (2003) and Yuan (2005) show that financial constraints endogenously generate skewed asset returns, and that this skewness is priced in equilibrium. In addition, market skewness might be related to portfolio hedging (see Gennotte and Leland (1990)), or the price impact of the trades of large portfolio managers (see Gabaix et al. (2006)).

Previous work that analyzes the pricing of market skewness includes Rubinstein (1973), Kraus and Litzenberger (1976), and Harvey and Siddique (2000). In these papers, skewness risk is priced because investors dislike skewness or more generally higher order moments of returns. While these papers provide preference based motivations for the pricing of market skewness, the pricing of skewness risk in our setup follows directly from the intertemporal hedging of volatility risk. The time variation in volatility, in turn, drives time variation in skewness. Economically, we view the price of skewness risk as a proxy for the tightness of financial constraints, and we interpret disutility of skewness as a reduced form representation.

Bansal and Viswanathan (1993), Bansal, Hsieh, and Viswanathan (1993), and Dittmar (2002) investigate the premium due to portfolio covariance with market volatility (co-skewness) and the premium due to portfolio covariance with market skewness. The latter premium is called co-kurtosis by Fang and Lai (1997) and Christie-David and Chaudhry (2001). These papers motivate the pricing of higher moments of the market return distribution by taking higher order Taylor approximations to the equilibrium pricing kernel. Bansal and Viswanathan (1993), Bansal, Hsieh, and Viswanathan (1993), and Dittmar (2002) measure market volatility and market skewness using squared and cubed market returns at the same frequency (weekly or monthly) as their cross sectional regressions.

We choose a somewhat different definition of market skewness that has several advantages. In particular, we estimate market skewness from daily market return data, using the sample skewness within each month. Our use of higher frequency (daily) data allows us to obtain more precise estimates in our lower frequency (monthly) cross sectional regressions. This is analogous to our market volatility estimation approach, which is based on daily returns rather than squared monthly returns. We also adopt a normalized estimate of skewness (third moment divided by cubed standard deviation) rather than just cubed returns. This skewness definition effectively orthogonalizes the skewness factor to the volatility factor and makes it easier to distinguish each effect. The market skewness innovations in our cross sectional regressions are residuals from a monthly autoregressive model with one lag.

We choose a macroeconomic measure of business conditions, because market volatility moves with the business cycle (Schwert (1989a) and (1989b)). The existing literature analyzing the cross sectional pricing of business cycle risk includes Chen, Ross, and Roll (1986), Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Vassalou (2003).

Since gross domestic product is only available at a quarterly frequency and our analysis is monthly, we use industrial production growth as our proxy for the state of the business cycle. Industrial production data is from the Federal Reserve Board of Governors *G.17* release. For our cross sectional regressions, we compute industrial production growth innovations as an autoregressive model with three lags. Summary statistics for the skewness and industrial production factors are provided in Table II.

In Table VI, we assess the performance of market skewness and industrial production innovations as pricing factors. For both of these variables, we then add short-run and long-run volatility separately to see if the volatility factors encompass the information in skewness or business cycle risk.

[Table VI]

We find that market skewness and industrial production growth are significant asset pricing factors in addition to the market return (Table VI, columns ii and v). Adding short-run volatility as pricing factor makes skewness insignificant (column iii). Adding long-run volatility makes industrial production insignificant (column vii), but does not reduce the significance of market skewness (column iv). The short-run volatility component thus appears to capture shocks to market skewness, while the long-run component captures business cycle risk. An additional advantage of our factors over sample skewness and the industrial production risk factor is that our factors produce smaller pricing errors. Our short-run volatility factor also reduces pricing errors considerably more than the market skewness risk factor.

In columns (viii) and (ix) of Table VI, we see that market skewness and industrial production are jointly significant at the 1% level when added to a three factor model. However, if we also include the short- and long-run components in a five factor model, both skewness and industrial production become insignificant, while the short and long-run components are significant at the 1% level.

In Table VII, we extend the comparison of the volatility components on the one hand, and market skewness and industrial production on the other hand. We now assess factor correlations based on prices of risk, factor loadings, and risk premia. Across these measures, we see that short-run volatility is most highly correlated with market skewness, and long-run volatility is most highly correlated with industrial production growth.

In particular, the price of short-run volatility risk is negatively correlated with the price of skewness risk (Table VII, top panel). Thus, times when insurance against skewness risk is more expensive correspond to times of more negative prices of short-run volatility risk. Furthermore, portfolios that have higher loadings on market skewness tend to have lower loadings on short-run volatility (Table VII, middle panel). As a result of the negative time series correlation of prices of risk and the negative cross sectional correlation of factor loadings, market risk premia of short-run volatility and skewness are strongly positively correlated (Table VII, bottom panel).

[Table VII]

The negative correlation between the price of short-run volatility risk and the price of skewness risk is driven by the strong short-run leverage effect in the stochastic volatility model (see Table I). Negative shocks to the market return increase short-run volatility more than positive shocks to the market return, giving rise to the negative skewness in the market return distribution. Thus, the short-run volatility component embeds a market skewness effect and is naturally interpreted as a skewness factor.

III. Additional portfolio sorts and time periods

In this section, we present pricing results for alternative sets of portfolios and alternative sample periods. This analysis ensures that the significance of the volatility components is not specific to the particular set of portfolios or sample period used in the pricing tests.

A. *Volatility sorted portfolios*

An important alternative set of test assets consists of portfolios sorted on exposures to the volatility components. This sorting procedure also provides another estimate of the risk premia associated with the volatility components. We use 25 portfolios sorted first on exposure to short-run volatility and then long-run volatility for the cross sectional tests presented in the next section. We sort first on the short-run and then the long-run exposures, because we find that a double sort yields months with missing observations for some portfolios.

For each stock in the monthly *CRSP* data, we estimate factor loadings with respect to short-run and long-run volatility innovations. To estimate loadings for month *t*, we include all stocks that have at least 60 non-missing returns between January 1958 and month *t-1* (we re-estimate the volatility components model starting in 1957 for this purpose). Loadings are estimated using an expanding window of data. For each month in the 1963/7 to 2005/12 sample period, we form quintile portfolios using the loadings estimated with returns up to the previous month. All our portfolios are value weighted. Summary statistics for the volatility exposure sorted portfolios are reported in Table VIII.

[Table VIII]

By construction, the sensitivities of the short-run volatility portfolios with respect to the short-run volatility factor are increasing along the short-run quintiles, and the sensitivities of the long-run volatility portfolios are increasing along the long-run quintiles. Exposures to the volatility components are negative for each of the portfolios.

Table VIII also shows that portfolios with larger negative exposure to either short- or long-run volatility risk have higher expected returns. This is consistent with our finding that the volatility components have a negative price of risk. Since volatility has a negative price of risk, higher negative exposure to volatility risk should lead to higher expected returns.

The monthly excess return for the quintile with the most negative exposure to short-run volatility is 0.98%, while it is 0.78% for the quintile with the least negative exposure. For the long-run volatility component, the quintile with the highest negative exposure has monthly excess returns that average 1.11%, versus 0.68% for the quintile with the lowest exposure.

The difference in the average return between the first and the fifth short-run quintile also provides an estimate of the short-run volatility risk premium, which is 0.20% monthly. This estimate is only slightly higher than the average short-run risk premium of 0.17% obtained from the Fama-MacBeth regressions in the size and book-to-market sorted portfolios (middle panel of Table V). The long-run volatility risk premium implied by the sorts is 0.33% monthly (1.11% - 90.68%), which is somewhat higher than the long-run risk premium in the size and book-to-market sorted portfolios of 0.23% (bottom panel of Table V).

B. Cross sectional regressions using the volatility sorted portfolios and other portfolios

In column (i) of Table IX, we report the results from Fama-MacBeth regressions for the 25 volatility sorted portfolios. In column (ii), we pool the 25 volatility sorted portfolios with the 25 book-to-market sorted portfolios. In a misspecified pricing model, we would expect the prices of risk of the volatility components to change substantially across different sets of test assets. Thus, pooling the volatility-sorted and size and book-to-market sorted portfolios provides a specification check.

In columns (iii) – (vi), we report cross sectional pricing results for sub-periods (1963/7 to 2005/12 excluding 1987/10, 1986/03 to 2005/12, 1963/07 to 1986/02, and 1988/01 to 2005/12). Additionally, we estimate prices of risk in a pooled set of earnings-to-price, cash-flow-to-price, dividend yield and momentum sorted portfolios (column vii). In columns (viii) and (ix), we report pricing results for out-of-sample tests. Here, the volatility components and factor loadings are estimated out-of-sample over 20-year rolling windows starting in 1926, so the first cross sectional test starts in 1946 (column viii). We also show the out-of-sample results starting in 1963 (column ix).

[Table IX]

In all ten tests, both short- and long-run volatility are highly significant pricing factors with negative prices of risk. We find that magnitudes of the prices of risk for the volatility components are fairly similar across time and across portfolio sorts, which is a useful further specification test.

To determine whether our results are robust to model misspecification, we recalculate the significance levels for our three factor volatility components model (Table IV, column v) with Shanken and Zhou (2006) standard errors. As shown in column (x), the volatility components remain highly significant. This provides additional support for our benchmark specification.

IV. Robustness analysis of the volatility model

To further examine the robustness of our cross sectional pricing results, we test the pricing of volatility factors estimated using a number of alternative volatility model specifications. In Panel A of Table X, we report estimation results for three alternative specifications: the exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model of Nelson (1991), the generalized autoregressive conditional heteroskedasticity components (GARCH-components) model of Engle and Lee (1999), and the GARCH-GJR model of Glosten, Jagannathan, and Runkle (1993). In each alternative specification, we include the market variance in the expected return equation.

[Table X]

We use the Bayesian information criterion to compare models, since the models in Table X are non-nested. Our EGARCH components model achieves the lowest information criterion, indicating that it is preferable to the other three specifications. In Panel A, we see that all four specifications pass the Ljung-Box *Q*-test on the squared standardized residuals. The cross sectional regressions in Panel B show that all models have negative and highly significant prices of volatility risk.

More importantly, our volatility components model produces the lowest cross sectional pricing errors when compared to the other specifications (Table X, Panel B). The lowest root-mean-squared pricing error among the alternative models is 0.25 versus 0.13 for the benchmark model.

In these cross sectional tests, we use the of log market variance innovations as a pricing factor for the EGARCH and GARCH-GJR models. This makes the pricing results more comparable to our benchmark model in which *sres* and *lres* are also log-volatility innovations. And, in fact, we see that the single factor GARCH models (Panel B, columns ii and iv) have prices of risk very close to the estimated price of risk for the benchmark model short-run component.

One might also expect relatively similar pricing performance when comparing the Engle and Lee (1999) GARCH volatility components with our benchmark EGARCH volatility components, since both are two-factor volatility models. However, we cannot take logs of the GARCH components for our cross sectional tests, because the short-run component is sometimes negative. This results in the differently scaled prices of risk that we observe when we compare results in (i) and (iii) of Panel B. It is also not surprising that to see disparities in the prices of short-run volatility risk, because the persistence of the GARCH short-run component is much higher than that of the EGARCH component (autoregressive coefficients of 0.86 and 0.33, respectively).

In addition, we find taking logs of volatility factors seems to generally improve cross sectional pricing performance. For example, the root-mean-squared pricing errors using EGARCH log-market variance as a pricing factor is 0.25 (Table X, Panel B, column ii) compared to 0.29 for EGARCH market variance (not shown). This is likely to be the reason that the GARCH components model's pricing performance is the weakest among these alternative market risk models.

In Panel A of Table XI, we present time series and cross sectional results of our volatility components model using seven alternative specifications of the predictable component of the market return (see equation 2). We report our benchmark specification in column (i). In columns (ii)-(iv), we vary the volatility components included in the return equation. In columns (v)-(vii), we add autoregressive and moving average terms to the market return equation.

[Table XI]

Since some of the alternative models contain more explanatory variables than the benchmark model, it is not surprising that they achieve higher likelihoods. However, using the Bayesian information criterion to balance explanatory power versus parsimony, we see that the models are very similar. In addition, all models pass the Ljung-Box *Q*-test.

In terms of cross sectional pricing, Panel B of Table XI shows that the benchmark model is superior to the alternatives with the lowest sum of squared pricing errors. Alternative specifications perform reasonably well with sum of squared pricing errors ranging from 0.45 to

0.88. We also see that including market variance (column i versus iv) with the short- and long-run components results in only a small change in pricing accuracy. In contrast, using market variance alone in the expected return equation results in a noticeable deterioration in the sum of squared pricing errors (column i versus ii).

V. Conclusions

Intertemporal models predict that financial asset risk premia are not only due to covariation of returns with the market return, but also covariation with the state variables that govern market volatility. We model the log-volatility of the market portfolio as the sum of a short- and a long-run volatility component to form pricing factors that reflect shocks to systematic volatility at different horizons.

Our empirical results demonstrate that shocks to systematic volatility are more important determinants of equity returns than has been previously shown. We find that prices of risk are negative and significant for both volatility components indicating that investors require compensation in order to hold assets that depreciate when volatility rises, even if the volatility shocks have little persistence. Our analysis links the short-run volatility component to market skewness risk, which we interpret as a measure of the tightness of financial constraints. The long-run volatility component relates closely to business cycle risk. Our three factor pricing model with the two volatility components and the market return as pricing factors compares favorably to benchmark models in explaining the cross section of equity return.

Appendix

Shanken (1992) and Jagannathan and Wang (1998) derive consistent standard error estimators for the two-step Fama and MacBeth (1973) estimation technique. Both papers show that standard errors of the risk-premia (λ) must be adjusted to account for the estimation error of the factor loadings (β). Our approach adds another layer of estimation error, specifically error in the estimation of the volatility factors (F), which are used in the loadings regressions. Using results from Murphy and Topel (1985) and Jagannathan and Wang (1998), we derive the appropriate standard error adjustments for our three-step procedure.

In the first step, we use maximum likelihood to estimate the parameters of the volatility components model (θ). The daily log-likelihood function is

$$f_{1t^d}\left(\theta; s_{t^d}, l_{t^d} \mid R^M_{t^d}\right) = -\ln(2\pi) - \left(s_{t^d-1} + l_{t^d-1}\right) - \frac{\left(R^M_{t^d} - \theta_1 - \theta_2 s_{t^d-1} - \theta_3 l_{t^d-1}\right)^2}{2v_{t^d-1}}, \tag{A1}$$

where $t^d=1,...,T^d$ is the daily time index, T^d is the total number of daily observations, and the daily market excess return is denoted $R^M_{t^d}$. The estimate of the covariance matrix for the parameters θ is denoted by V_1. It is computed as the outer product of gradients using numerical derivatives where the *i-th* element of the gradient is $\left[f_{1t}\left(\theta_i+\varepsilon\right) - f_{1t}\left(\theta_i\right)\right]/\varepsilon$, so that

$$V_1 = \left[\sum_{t^d=1}^{T^d} \frac{\partial f_{1t^d}}{\partial \theta'} \frac{\partial f_{1t^d}}{\partial \theta}\right]^{-1}. \tag{A2}$$

Next, the factor loadings β^i ($K \times 1$) for each portfolio are estimated by regressing portfolio returns on pricing factors:

$$\beta^i = \left(F_c^{'} F_c\right)^{-1} F_c^{'} R_c^i, \quad \text{for } i = 1, ..., N \tag{A3}$$

where F_c=F-1_T. $\overline{F}$ is the $T\times K$ matrix of demeaned pricing factors, and $R_c^i = R^i - 1_T \overline{R}^i$ is the $T\times 1$

vector of demeaned excess portfolio returns for portfolio *i*.

We then stack the factor loadings portfolio by portfolio to obtain a $NK\times 1$ vector denoted by β. To estimate the $N\times N$ covariance matrix of residuals (Φ), we use the sample covariance matrix of the $1\times N$ vector of time series residuals (u_t). Therefore, the log-likelihood function conditional on the set of pricing factors $F_t(\theta)$ for month *t* is given by:

$$f_{2t}\left(\beta \mid R_t, F_t(\theta)\right) = -N\ln(2\pi) - 0.5\ln\left|\Phi\right| - 0.5u_t\Phi^{-1}u'_t. \tag{A4}$$

The covariance matrix of residuals (Φ) is constant, and (A4) defines the likelihood function to a system of seemingly unrelated regressions (SUR). Therefore, (A3) is the solution to maximizing (A4) with respect to the parameter vector θ (see Greene (2003)).

To estimate standard errors for the factor loadings (V_2), we evaluate the gradient of the likelihood function numerically:

$$V_2 = \left[\sum_{t=1}^{T} \frac{\partial f_{2t}}{\partial \beta'} \frac{\partial f_{2t}}{\partial \beta}\right]^{-1}. \tag{A5}$$

This estimate of the variance covariance matrix (V_2 which is $NK\times NK$) does not take the sampling error of the generated volatility factors into account.

We correct for estimation error in the loadings regression using the two-step procedure of Murphy and Topel (1985). Murphy and Topel show that the original parameter covariance matrix (V_2) must be corrected for estimation error to arrive at a consistent estimate (V_2^*). The formula is given as follows:

$$V_2^* = V_2 + V_2C_2\left(V_1 \otimes I_N\right)C_2'V_2 - V_2\left(\Gamma_2\left(V_1 \otimes I_N\right)_1 C_2' + C_2\left(V_1 \otimes I_N\right)\Gamma_2'\right)V_2, \tag{A6}$$

where $C_2 = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial f_{2t}}{\partial \beta'}\frac{\partial f_{2t}}{\partial \theta}$, $\Gamma_2 = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial f_{2t}}{\partial \beta'}\left(1_N' \otimes \frac{\partial f_{1t}}{\partial \theta}\right)$, and $\partial f_{1t}/\partial\theta$ denotes the average of the daily gradients $\partial f_{1t^d} / \partial \theta$ in month t from the likelihood function (A1). The derivative $\partial f_{2t} / \partial \beta$ is the gradient of the likelihood function (A4). The gradient of likelihood (A4) with respect to the parameters of the volatility model θ is $\partial f_{2t} / \partial \theta$. We compute $\partial f_{2t} / \partial \theta$ numerically as $\left[f_{2t}\left(\beta \mid R_t, F_t\left(\theta_i + \varepsilon\right)\right) - f_{2t}\left(\beta \mid R_t, F_t\left(\theta_i\right)\right)\right] / \varepsilon$ for each θ_i. 1_N denotes the $N \times 1$ vector of ones.

When pricing factors are not estimated, there are no first stage parameters. Therefore, the derivative of the likelihood function in the loadings regression with respect to these parameters is zero ($\partial f_{2t} / \partial \theta' = 0$, $\Gamma_2 = 0$, and $C_2 = 0$). Then, V_2^* equals the original covariance matrix estimate V_2. Otherwise, the second stage standard errors depend in the first stage standard errors (V_1) and covariances of scores from the first and second stages.

Following Fama and MacBeth (1973), our third stage estimates of factor risk premia are from monthly cross sectional regressions:

$$\lambda_t = \left(vecinv\left(\beta\right) vecinv\left(\beta\right)' \right)^{-1} vecinv\left(\beta\right) R_t, \text{ for } t = 1, \ldots, T \tag{A7}$$

where $vecinv(\beta)$ denotes the $K \times N$ matrix of stacked β^i's such that $vec(vecinv\,(\beta))=\beta$, and R_t is the $N \times 1$ cross section of portfolio returns at time t. The estimated prices of risk are contained in $\overline{\lambda}$ (a $K \times 1$ vector), which is the time series average of the λ_t's.

We then use Jagannathan and Wang (1998) to obtain standard errors for the risk premia (V_3^*) that are adjusted for error in the estimated factor loadings. We also correct for volatility factor estimation error by using the Murphy and Topel (1985) adjusted standard errors (V_2^*), rather than the OLS standard errors (V_2) from the loadings regressions. To further adjust for possible heteroskedasticity and autocorrelation, we estimate covariance matrices in the Jagannathan and Wang (1998) formula using Newey and West (1987):

$$V_3^* = T^{-1}\left(V_3 + W_3 - G_3\right) \tag{A8}$$

$$V_3 = \frac{1}{T}\sum_{t=1}^{T}\left(\lambda_t - \bar{\lambda}\right)\left(\lambda_t - \bar{\lambda}\right)' + \sum_{j=1}^{q}\frac{q-j}{q}\frac{1}{T}\sum_{t=j+1}^{T}\left(\left(\lambda_t - \bar{\lambda}\right)\left(\lambda_{t-j} - \bar{\lambda}\right)' + \left(\lambda_{t-j} - \bar{\lambda}\right)\left(\lambda_t - \bar{\lambda}\right)'\right)$$

$$W_3 = \left(\beta'\beta\right)^{-1}\beta'\left(I_N \otimes \bar{\lambda}\right)' V_2^*\left(I_N \otimes \bar{\lambda}\right)\beta\left(\beta'\beta\right)^{-1}$$

$$\begin{aligned} G_3 = &\sum_{t=1}^{T}\left(\left(\lambda_t - \bar{\lambda}\right)g_t'\beta\left(\beta'\beta\right)^{-1} + \left(\beta'\beta\right)^{-1}\beta' g_t\left(\lambda_t - \bar{\lambda}\right)'\right) \\ &+ \sum_{j=1}^{q}\frac{q-j}{q}\sum_{t=j+1}^{T}\left(\left(\lambda_{t-j} - \bar{\lambda}\right)g_t'\beta\left(\beta'\beta\right)^{-1} + \left(\beta'\beta\right)^{-1}\beta' g_{t-j}\left(\lambda_t - \bar{\lambda}\right)'\right) \end{aligned}$$

$$g_t = \left(I_N \otimes \left(\left(F - \bar{F}\right)'\left(F - \bar{F}\right)\right)^{-1}\lambda\right)' vec\left(\left(F_t - \bar{F}\right)' u_t\right),$$

where u_t is the $1 \times N$ row vector of errors from the time series regression. We use q = int(4(T/100)$^{2/9}$) for the Newey and West (1987) autocorrelation adjustment.

Without the Newey-West adjustment in G_3 and V_3 and with $V_2^* = V_2$, (A8) reduces to the expression of Jagannathan and Wang (1998, Theorem 1). This can be seen by replacing (A7) into (A8), and substituting analytically for V_2. The term V_3 then corresponds to the original covariance matrix of pricing errors as proposed by Fama and MacBeth (1973). The terms G_3 and W_3 adjust for the estimation errors of the previous stages.

We calculate the average pricing error for each portfolio, the sum of squared portfolio pricing errors, and the root-mean-squared pricing errors respectively using:

$$\bar{\alpha}^i = \bar{R}^i - \beta^i\bar{\lambda}\,,\ \sum_{i=1}^{N}\left(\bar{\alpha}^i\right)^2, \text{ and } \sqrt{\sum_{i=1}^{N}\left(\bar{\alpha}^i\right)^2 / N}\,. \tag{A9}$$

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ENDNOTES

[1] Papers that focus on the cross sectional pricing implications of time-varying volatility but do not study the pricing implications of intertemporal hedging include Engle, Bollerslev, and Wooldridge (1988), Harvey (1989), and Schwert and Seguin (1990). Ozoguz (2004) uses the ex-ante uncertainty of a regime switching model of the market premium as an asset pricing factor.

[2] Engle and Rosenberg (2000), Alizadeh, Brandt, and Diebold (2002), Bollerslev and Zhou (2002), Chacko and Viceira (2003), and Chernov et al. (2003) find that two-component volatility specifications outperform one-factor models for market return volatility.

[3] Merton (1973) and Cox, Ingersoll, Ross (1985) derive general results that allow us to write expression (2). Abel (1988) and Gennotte and Marsh (1992) — in discrete and continuous time, respectively — solve for equilibrium expected returns when cash-flow volatility follows a one-component, square-root process and investors have constant relative risk aversion preferences. With Kreps and Porteus (1978) preferences that separate the coefficient of relative risk-aversion from the intertemporal elasticity of substitution, Tauchen (2005) derives an approximate solution in a discrete-time setting with two volatility components. Bansal and Yaron (2004) quantify aggregate risk premia in a setting with one volatility component and Kreps-Porteus preferences.

[4] Our equation (2) can be derived in Duffie and Epstein's (1992) stochastic differential utility setting by replacing their equation (17) into (36), imposing market clearing, and then solving for the expected returns of individual assets. Duffie and Epstein's $-wJ_w/J_{ww}$ then corresponds to our γ_t, their vector $-wJxw/Jw$ corresponds to our $[F_s, F_l]$, and the vector of state variables x corresponds to the short- and long-run volatility components $[s, l]$ in our setting.

[5] It is unlikely that one would be able to identify hedging demands based on the signs and significance of estimated coefficients in the mean equation, because these coefficients might be affected by the particular functional form chosen for the volatility equation.

[6] Nelson (1992) shows that estimation error due to volatility model misspecification decreases as observation frequency increases. Andersen et al. (2003) demonstrate that volatilities can be precisely estimated without a parametric model using high frequency data; however, intraday data is not available over our full sample period. Campbell, Lo, and MacKinley (1997) point out potential biases in volatility estimation arising from the use of high-frequency data such as the effects of non-synchronous trading and bid ask bounce. While these effects can be important for individual stocks, the impact for a broadly diversified portfolio should be small.

[7] Since Merton's (1980) investigation of the time series of the risk-return tradeoff, the literature has found a negative, insignificant, or positive risk-return tradeoff depending on the specification. See, in particular, French, Schwert, and Stambaugh (1987), Turner, Startz, and Nelson (1989), Baillie and DeGennaro (1990), Campbell and Hentschel (1992), Glosten, Jagannathan, and Runkle (1993), Scruggs (1998), Harvey (2001), Brandt and Kang (2004), Ghysels, Santa-Clara, and Valkanov (2005), and Guo and Whitelaw (2006).

[8] The asymmetric relationship between return innovations and volatility is documented in one-factor contexts by Campbell and Hentschel (1992), Glosten, Jagannathan, and Runkle (1993), Zakoian (1994), Andersen, Benzoni, and Lund (2002), and Eraker, Johannes, and Polson (2003), and Bollerslev and Zhou (2005), among others. Engle and Lee (1999) model the asymmetric relation between returns and the short-run component of volatility, but not the long-run component. Chernov et al. (2003) also find that both the short-run and long-run

components of stock market volatility exhibit a leverage effect. Wu (2001) and Tauchen (2005) model economic mechanisms that give rise to the leverage effect.

[9] In our analysis, returns to the size and book-to-market sorted portfolios and the value, size and momentum factors (HML, SMB, and UMD) are from the website of Kenneth French at http://www.dartmouth.edu/~kfrench/. We use the *research* size and value factors for our analysis. These factors have better cross sectional pricing properties than the benchmark factors.

[10] There are a number of differences between our specification in column (iv) and the asset pricing model proposed by Ang et al. (2006). These authors use the VIX implied volatility index, which is only available since 1986, while we use estimated volatility since 1963. Their cross sectional regressions are based on a volatility factor mimicking portfolio, while we use market variance innovations directly as a factor. Lastly, they construct volatility innovations using monthly changes. We use innovations of an AR(2) process, since we detect significant autocorrelation in the differenced variance series.

Table I

Time-Series Estimation of the Volatility Components Daily 1962/7/3 to 2005/12/31

This table reports the summary statistics of the daily market excess return and the maximum likelihood estimates of the volatility components model. The market excess return is measured as the cum-dividend return of the value weighted *CRSP* portfolio in excess of the three-month Treasury bill rate. The standardized error term ε is assumed to be distributed normally with mean zero and variance one. The variance of the market excess return v is defined as $v=exp(2(s+l))$, where l denotes the long-run volatility component and s the short-run volatility component.

Summary statistics of market excess return (10951 days)

Mean	Median	Std. Dev.	Skewness	Kurtosis
0.030	0.054	0.883	-0.748	21.188

Market excess returns: $R^M_{t+1} = \theta_1 + \theta_2 s_t + \theta_3 l_t + \sqrt{v_t}\,\varepsilon_{t+1}$

	θ_1	θ_2	θ_3
coef.	0.045	-1.568	0.038
std. err.	0.014	0.140	0.022
p-value	0.001	0.000	0.086

Short-run component: $s_{t+1} = \theta_4 s_t + \theta_5 \varepsilon_{t+1} + \theta_6\left(|\varepsilon_{t+1}| - \sqrt{2/\pi}\right)$

	θ_4	θ_5	θ_6
coef.	0.333	-0.069	-0.002
std. err.	0.036	0.005	0.004
p-value	0.000	0.000	0.655

Long-run component: $q_{t+1} = \theta_7 + \theta_8 l_t + \theta_9 \varepsilon_{t+1} + \theta_{10}\left(|\varepsilon_{t+1}| - \sqrt{2/\pi}\right)$

	θ_7	θ_8	θ_9	θ_{10}
coef.	-0.002	0.989	-0.032	0.061
std. err.	0.001	0.001	0.002	0.003
p-value	0.003	0.000	0.000	0.000
p-value of θ_8=1:		0.000		

	10 lags	20 lags
Ljung-Box Q-statistic of ε^2	10.58	15.47
p-value	2.21	0.75

Table II
Summary Statistics of Pricing Factors Monthly 1963/7 to 2005/12

The daily innovations of the short-run component (*s*) and the long-run component (*l*) from the volatility components model (Table I) are aggregated to a monthly frequency (*sres, lres*) as described in Section I.C. The market variance (*v*) is aggregated to a monthly frequency, and variance innovations (*vres*) are estimated as residuals of a monthly autoregressive process with two lags. The market skewness factor is calculated from daily market excess returns for each month, and its residuals are obtained from a monthly autoregression. The industrial production factor is computed as residuals of an autoregressive process of monthly industrial production growth with three lags.

Pricing factor	Mean	Std. Dev.	Skewness	Kurtosis
Short-run volatility (*sres*)	0.00	0.47	0.04	3.04
Long-run volatility (*lres*)	0.05	3.77	0.57	3.80
Market variance (*vres*)	0.05	9.78	6.22	82.30
Excess market return	0.47	4.42	-0.50	5.06
Value factor (*HML*)	0.44	2.92	0.02	5.50
Size factor (*SMB)*	0.25	3.24	0.52	8.43
Momentum factor (*UMD)*	0.85	4.03	-0.65	8.43
Market skewness	0.00	0.55	-0.33	3.94
Industrial production	0.00	0.01	0.35	3.50

Table III
Factor Loadings of the 25 Size and Book-to-Market Sorted Portfolios Monthly 1963/7 to 2005/12

This table reports factor loadings from regressions of each size and book-to-market portfolio return on the market return, short-run volatility innovations (*sres*), and long-run volatility innovations (*lres*). Standard errors are adjusted for heteroskedasticity and estimation error of the volatility components (see Appendix). Significance at the 1%-level is denoted by ***, at the 5%-level by **, and at the 10%-level by *.

Multivariate loadings on the market factor

	Small	Size 2	Size 3	Size 4	Large
Growth	1.63 ***	1.51 ***	1.38 ***	1.33 ***	1.04 ***
Book-to-Market 2	1.20 ***	1.07 ***	1.01 ***	1.01 ***	0.86 ***
Book-to-Market 3	0.96 ***	0.87 ***	0.80 ***	0.81 ***	0.77 ***
Book-to-Market 4	0.89 ***	0.80 ***	0.68 ***	0.62 ***	0.54 ***
Value	0.89 ***	0.78 ***	0.76 ***	0.70 ***	0.53 ***

p -value that all 25 loadings are equal = 0.00%

Multivariate loadings on short-run volatility innovations (*sres*)

	Small	Size 2	Size 3	Size 4	Large
Growth	2.74 *	1.17	0.46	0.72	0.08
Book-to-Market 2	0.72	-0.44	-0.63	-0.55	-1.03 *
Book-to-Market 3	-0.35	-0.83	-1.29	-1.54 **	-0.95
Book-to-Market 4	-0.10	-1.02	-1.90 **	-2.79 ***	-2.63 ***
Value	-0.20	-1.99 *	-1.83 *	-2.78 ***	-2.96 ***

p -value that all 25 loadings are equal = 4.27%

Multivariate loadings on long-run volatility innovations (*lres*)

	Small	Size 2	Size 3	Size 4	Large
Growth	-0.26 ***	-0.14 ***	-0.08 **	0.00	0.07 ***
Book-to-Market 2	-0.27 ***	-0.17 ***	-0.09 ***	-0.03	0.04
Book-to-Market 3	-0.22 ***	-0.22 ***	-0.13 ***	-0.05	0.03
Book-to-Market 4	-0.25 ***	-0.19 ***	-0.11 ***	-0.05	0.03
Value	-0.31 ***	-0.21 ***	-0.16 ***	-0.05	-0.01

p -value that all 25 loadings are equal = 0.00%

Table IV

Pricing the Cross-Section of 25 Size and Book-to-Market Sorted Portfolios Monthly 1963/7 to 2005/12

This table reports summary statistics of the cross-sectional Fama-MacBeth (1973) regressions for the size and book-to-market sorted portfolios of Fama and French (1993). In the first stage, portfolio returns are regressed on the pricing factors to obtain factor loadings (reported in Table III). In the second stage, for each month, portfolio returns are regressed on the loadings, giving an estimate of the price of risk for each factor. The *t*-statistics are adjusted for autocorrelation and heteroskedasticity, estimation error in factor loadings, and estimation error of the volatility components (see Appendix). Innovations to the short-run component (*sres*), the long-run component (*lres*), and the variance of excess returns (*vres*) are from the volatility-components model reported in Table I. RMSPE denotes the root mean square pricing error.

		Benchmarks				Volatility Components				
		(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
Excess market return	coef.	0.65 ***	0.43 **	0.54 **	0.56 ***	0.34 *	0.43 *	0.45 **	0.42 **	0.35 *
	t-stat	2.92	2.18	2.39	2.75	1.64	1.85	2.24	2.04	1.65
Short-run volatility (*sres*)	coef.					-0.21 ***	-0.23 ***		-0.18 ***	-0.27 ***
	t-stat					-4.18	-5.10		-3.38	-6.60
Long-run volatility (*lres*)	coef.					-2.02 ***		-1.92 ***	-3.58 ***	-2.19 ***
	t-stat					-3.10		-3.14	-4.21	-3.18
Market variance (*vres*)	coef.				-3.23 **					
	t-stat				-1.98					
Value factor (*HML*)	coef.		0.49 ***						0.44 ***	
	t-stat		3.17						2.83	
Size factor (*SMB*)	coef.		0.24						0.26 *	
	t-stat		1.57						1.64	
Momentum factor (*UMD*)	coef.			-3.16 **						1.87 ***
	t-stat			-2.41						3.45
Sum of squared pricing errors		2.38	0.49	1.45	2.16	0.44	0.64	1.64	0.38	0.40
RMSPE		0.31	0.14	0.24	0.29	0.13	0.16	0.26	0.12	0.13

Table V

Factor Risk Premia of the 25 Size and Book-to-Market Sorted Portfolios Monthly 1963/7 to 2005/12

This table reports the risk premia of each size and book-to-market portfolio return on the market return, short-run volatility innovations (*sres*), and long-run volatility innovations (*lres*). The risk premia are calculated by multiplying factor loadings (from Table III) times prices of risk (from Table IV, column v).

	Market risk premium					
	Small	Size 2	Size 3	Size 4	Large	Average
Growth	0.56	0.52	0.48	0.46	0.36	0.48
Book-to-Market 2	0.41	0.37	0.35	0.35	0.30	0.36
Book-to-Market 3	0.33	0.30	0.27	0.28	0.27	0.29
Book-to-Market 4	0.31	0.28	0.23	0.21	0.18	0.24
Value	0.31	0.27	0.26	0.24	0.18	0.25
Average	0.38	0.35	0.32	0.31	0.26	0.32
	Short-run volatility risk premium					
	Small	Size 2	Size 3	Size 4	Large	Average
Growth	-0.57	-0.25	-0.10	-0.15	-0.02	-0.22
Book-to-Market 2	-0.15	0.09	0.13	0.12	0.22	0.08
Book-to-Market 3	0.07	0.17	0.27	0.32	0.20	0.21
Book-to-Market 4	0.02	0.21	0.40	0.59	0.55	0.35
Value	0.04	0.42	0.38	0.58	0.62	0.41
Average	-0.12	0.13	0.22	0.29	0.31	0.17
	Long-run volatility risk premium					
	Small	Size 2	Size 3	Size 4	Large	Average
Growth	0.53	0.27	0.17	-0.01	-0.15	0.16
Book-to-Market 2	0.55	0.35	0.19	0.07	-0.08	0.21
Book-to-Market 3	0.44	0.45	0.25	0.11	-0.05	0.24
Book-to-Market 4	0.50	0.38	0.23	0.10	-0.06	0.23
Value	0.62	0.43	0.31	0.10	0.01	0.30
Average	0.53	0.38	0.23	0.07	-0.07	0.23

Table VI

Pricing the Cross-Section of 25 Size and Book-to-Market sorted Portfolios Monthly 1963/7 to 2005/12

This table reports summary statistics of the cross-sectional Fama-MacBeth (1973) regressions for the size and book-to-market sorted portfolios of Fama and French (1993). In the first stage, portfolio returns are regressed on the pricing factors to obtain factor loadings. In the second stage, for each month, portfolio returns are regressed on the loadings, giving an estimate of the price of risk for each factor. Market skewness is calculated from daily market excess returns for each month and its residuals are obtained from a monthly autoregression. Industrial production denotes the time-series innovations of an AR(3) process of industrial production growth. Short- and long-run volatility are the innovations to the volatility components (*sres* and *lres*) from the volatility model of Table I. The *t*-statistics are adjusted for autocorrelation and heteroskedasticity, estimation error in factor loadings, and estimation error of the volatility components (see Appendix). RMSPE denotes the root mean square pricing error.

			Skewness			Industrial Production			Skewness and IP	
		(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(xi)
Excess market return	coef.	0.34*	0.67***	0.39*	0.39**	0.47**	0.36*	0.45**	0.40**	0.36*
	t-stat	1.64	3.05	1.85	1.94	2.36	1.71	2.25	2.01	1.73
Short-run volatility (*sres*)	coef.	-0.21***		-0.25***			-0.20***			-0.16***
	t-stat	-4.18		-4.74			-4.25			-3.97
Long-run volatility (*lres*)	coef.	-2.02***			-2.39***			-2.55***		-2.39***
	t-stat	-3.10			-3.92			-3.56		-3.88
Market Skewness	coef.		0.48**	-0.15	0.76***				0.82***	0.37
	t-stat		2.09	-0.54	2.53				3.49	1.61
Industrial production	coef.					0.01***	0.00	0.00	0.01***	0.00
	t-stat					3.15	1.28	-1.04	6.17	-0.06
Sum of squared pricing errors		0.44	1.90	0.60	0.47	1.77	0.50	1.62	0.58	0.38
RMSPE		0.13	0.28	0.15	0.14	0.27	0.14	0.25	0.15	0.12

Table VII
Correlations of Prices of Risk, Factor Loadings, and Risk Premia

This table reports the time series correlations of prices of risk, the cross sectional correlations of factor loadings, and the pooled time series and cross sectional correlations of risk premia for the pricing models of columns (ii), (iii) and (v) of Table IV and columns (ii) and (v) of Table VI.

Time series correlations of prices of risk

	Short-run volatility (*sres*)	Long-run volatility (*lres*)
Market return skewness	-78%	49%
Industrial production	7%	-98%
Value factor (*HML*)	-59%	-20%
Size factor (*SMB*)	26%	-88%
Momentum factor (*UMD*)	89%	-9%

Cross sectional correlations of factor loadings

	Short-run volatility (*sres*)	Long-run volatility (*lres*)
Market return skewness	-84%	61%
Industrial production	22%	-94%
Value factor (*HML*)	-83%	-13%
Size factor (*SMB*)	59%	-93%
Momentum factor (*UMD*)	85%	-15%

Cross sectional / time series correlations of risk premia

	Short-run volatility (*sres*)	Long-run volatility (*lres*)
Market return skewness	54%	22%
Industrial production	-2%	95%
Value factor (*HML*)	52%	10%
Size factor (*SMB*)	-1%	85%
Momentum factor (*UMD*)	79%	-4%

Table VIII
Portfolios Sorted on Volatility Component Sensitivities Monthly 1963/7 to 2005/12

Portfolios are formed by first sorting the universe of *CRSP* stocks into short-run and long-run volatility exposure quintiles. Exposures are calculated as univariate factor loadings using an expanding window from 1958/01, up to the month before portfolio formation. Each stock is required to have at least 60 months of returns before being included into the portfolio. Portfolio returns are value weighted. Innovations to the short-run component (*sres*) and the long-run component (*lres*) are from the volatility-components model reported in Table I.

		Short-run volatility quintiles				
		1	2	3	4	5
Average excess return		0.98	0.93	0.93	0.85	0.78
Loading on short-run volatility (*sres*)	coef.	-13.60	-10.88	-9.45	-7.94	-5.61
	t-stat	-26.85	-31.84	-32.82	-32.77	-24.95

		Long-run volatility quintiles				
		1	2	3	4	5
Average excess return		1.11	1.02	0.87	0.80	0.68
Loading on long-run volatility (*lres*)	coef.	-0.87	-0.73	-0.65	-0.56	-0.47
	t-stat	-10.78	-11.78	-12.37	-12.65	-12.52

Table IX

Pricing the Cross-Section of other Test Assets and Sample Periods

This table reports summary statistics of the cross-sectional Fama-MacBeth (1973) regressions using the set of portfolios and sample periods reported below. The t-statistics are adjusted for heteroskedasticity and autocorrelation, for the estimation error in factor loadings, and for the estimation error in the volatility components (see Appendix). RMSPE denotes the root mean squared pricing error.

		(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Excess market return	coef.	0.48 **	0.42 **	0.39 *	0.58 **	0.21	0.60 **	0.44 **	0.94 ***	0.88 ***	0.34 *
	t -stat	2.22	1.95	1.85	1.94	0.69	2.05	2.12	5.68	4.34	1.64
Short-run volatility (*sres*)	coef.	-0.12 **	-0.18 ***	-0.22 ***	-0.22 **	-0.19 ***	-0.22 ***	-0.17 ***	-0.42 ***	-0.20 ***	-0.21 ***
	t -stat	-2.07	-3.85	-4.28	-2.37	-3.03	-2.62	-4.33	-2.64	-4.48	-4.29
Long-run volatility (*lres*)	coef.	-2.39 **	-1.83 ***	-2.08 ***	-2.70 ***	-2.22 ***	-2.92 ***	-1.25 *	-2.14 ***	-2.52 ***	-2.02 ***
	t -stat	-2.23	-3.23	-3.18	-2.46	-2.53	-2.56	-1.86	-3.94	-2.86	-3.05
Sum of squared pricing errors		0.30	1.02	0.52	1.13	0.48	1.09	0.84	0.74	0.92	0.44
RMSPE		0.11	0.14	0.14	0.21	0.14	0.21	0.15	0.17	0.19	0.13

Column (i): 25 volatility exposure sorted portfolios 1963/07 to 2005/12

Column (ii): 25 volatility exposure plus 25 book-to-market sorted portfolios 1963/07 to 2005/12

Column (iii): 25 size and book-to-market sorted portfolios 1963/07 to 2005/12, excluding 1987/10

Column (iv): 25 size and book-to-market sorted portfolios 1986/03 to 2005/12

Column (v): 25 size and book-to-market sorted portfolios 1963/07 to 1986/02

Column (vi): 25 size and book-to-market sorted portfolios 1988/01 to 2005/12

Column (vii): 36 portfolios sorted on earnings/price (10), cash-flow/price (10), dividend yield (10), and size and momentum (6) 1963/07 to 2005/12

Column (viii): 25 size and book-to-market sorted portfolios, 20-year rolling out-of-sample factors and loadings, 1946/07 to 2005/12

Column (ix): 25 size and book-to-market sorted portfolios, 20-year rolling out-of-sample factors and loadings, 1963/07 to 2005/12

Column (x): 25 size and book-to-market sorted portfolios, Shanken and Zhou (2006) standard errors, 1963/07 to 2005/12

Table X
Comparison to Alternative Market Risk Models

Panel A: The estimation of alternative volatility model specifications uses the data described in Table I. The Bayesian Information Criterion of Schwarz (1978) allows comparison of goodness of fit across models. The benchmark model is the EGARCH components specification described in Section I.A.

Panel B: These cross-sectional pricing results use the volatility components estimated using the models in Panel A. The market variance pricing factor is in logs for the EGARCH and GARCH-GJR models. RMSPE denotes the root mean square pricing error.

Panel A: Time-series estimation, daily 1962/7/3 to 2005/12/31

(i) Benchmark specification

$R_{t+1} = 0.04^{***} - 1.57^{***} s_t + 0.04^{*} l_t + \sqrt{v_t}\, \varepsilon_{t+1}$

$s_{t+1} = 0.33^{***} s_t - 0.07^{***} \varepsilon_{t+1} - 0.002\, (|\varepsilon_{t+1}| - \sqrt{2/\pi})$

$l_{t+1} = 0.00^{***} + 0.99^{***} l_t - 0.03^{***} \varepsilon_{t+1} + 0.06^{***} (|\varepsilon_{t+1}| - \sqrt{2/\pi})$

Log-likelihood	-12,051
Ljung-Box of ε^2 (10 lags)	10.58
Bayesian Information Criterion	2.21

(ii) EGARCH, Nelson (1991)

$R_{t+1} = 0.02^{**} + 0.03^{*} v_t + \sqrt{v_t}\, \varepsilon_{t+1}$

$\ln(v_{t+1}) = -.12^{***} + 0.98^{***} \ln(v_t) - 0.07^{***} \varepsilon_{t+1} + 0.15^{***} |\varepsilon_{t+1}|$

Log-likelihood	-12,271
Ljung-Box of ε^2 (10 lags)	6.94
Bayesian Information Criterion	2.25

(iii) GARCH-components, Engle and Lee (1999)

$R_{t+1} = 0.03^{***} + 0.05^{***} v_t + \sqrt{v_t}\, \varepsilon_{t+1}$

$v_{t+1} = q_t + 0.88^{***} (v_t - q_t) + (v_t\, \varepsilon^2_{t+1} - q_t)\, [\, I_{\varepsilon<0}\ 0.11^{***} - 0.01]$

$q_{t+1} = 1.63 + 0.99^{***} q_t + 0.03^{***} v_t\, (\varepsilon^2_{t+1} - 1)$

Log-likelihood	-12,334
Ljung-Box of ε^2 (10 lags)	5.20
Bayesian Information Criterion	2.26

(iv) GARCH-GJR, Glosten, Jagannathan, Runkle (1993)

$R_{t+1} = 0.03^{***} + 0.01 v_t + \sqrt{v_t}\, \varepsilon_{t+1}$

$v_{t+1} = 0.01^{***} + 0.91^{***} v_t + v_t\, \varepsilon^2_{t+1}\, [\, I_{\varepsilon<0}\ 0.09^{***} + 0.03^{***}]$

Log-likelihood	-12,315
Ljung-Box of ε^2 (10 lags)	4.67
Bayesian Information Criterion	2.25

Panel B: Cross-sectional pricing, monthly 1963/7 to 2005/12, size and book-to-market sorted portfolios

	(i) Benchmark	(ii) EGARCH	(iii) GARCH-components	(iv) GARCH-GJR
Excess market return	0.34 *	0.43 **	0.48 **	0.43 **
Short-run volatility	-0.21 ***		-6.18 ***	
Long-run volatility	-2.02 ***		-3.30 ***	
Market variance		-0.19 ***		-0.22 ***
Sum of squared pricing errors	0.44	1.61	1.74	1.58
RMSPE	0.13	0.25	0.26	0.25

Table XI
Specification Analysis of the Volatility Components Model

Panel A: The time series estimation of the daily volatility components model 1962/7/3 to 2005/12/31 uses the data described in Table I. The models incorporate different expected return specifications; the baseline model is in column (i). The Bayesian Information Criterion of Schwarz (1978) allows comparison of goodness of fit across models. The estimated time series model is:

$$R^M_{t+1} = \theta_1 + \theta_2 s_t + \theta_3 l_t + \theta_v v_t + \theta_R R^M_t + \theta_\varepsilon \sqrt{v_{t-1}}\varepsilon_t + \sqrt{v_t}\varepsilon_{t+1}$$

$$s_{t+1} = \theta_4 s_t + \theta_5 \varepsilon_{t+1} + \theta_6 \left(|\varepsilon_{t+1}| - \sqrt{2/\pi}\right) \qquad l_{t+1} = \theta_7 + \theta_8 l_t + \theta_9 \varepsilon_{t+1} + \theta_{10}\left(|\varepsilon_{t+1}| - \sqrt{2/\pi}\right)$$

Panel B: Cross sectional asset pricing results using the 25 size and book-to-market sorted portfolios 1963/7 to 2005/12 monthly as in Table IV. RMSPE is the root mean squared pricing error.

Panel A: Time series estimation	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
θ_1	0.04 ***	0.05 ***	0.01	0.02	-0.04	-0.05	-0.04
θ_2	-1.57 ***		-1.62 ***	-1.59 ***	-0.13 **	-0.26 ***	-0.22 ***
θ_3	0.04 *			0.02	-0.04	-0.05	-0.05
θ_v		-0.02	0.04 **	0.02	0.07 ***	0.09 ***	0.08 ***
θ_R					0.18 ***		0.05
$\theta\varepsilon$						0.16 ***	0.12 **
θ_4	0.33 ***	0.86 ***	0.33 ***	0.33 ***	0.88 ***	0.88 ***	0.88 ***
θ_5	-0.07 ***	-0.06 ***	-0.07 ***	-0.07 ***	-0.06 ***	-0.06 ***	-0.06 ***
θ_6	0.00	0.03	0.00	0.00	0.03	0.03	0.03
θ_7	0.00 ***	0.00 ***	0.00 **	0.00 ***	0.00 ***	0.00 ***	0.00 ***
θ_8	0.99 ***	1.00 ***	0.99 ***	0.99 ***	1.00 ***	1.00 ***	1.00 ***
θ_9	-0.03 ***	-0.01 ***	-0.03 ***	-0.03 ***	-0.01 ***	-0.01 ***	-0.01 ***
θ_{10}	0.06 ***	0.04 ***	0.06 ***	0.06 ***	0.04 ***	0.04 ***	0.04 ***
Log-likelihood	-12050.8	-12177.7	-12050.6	-12050.4	-11501.9	-12020.4	-12019.0
Ljung-Box of ε^2 (10 lags)	10.58	8.97	9.87	10.15	8.35	8.89	8.78
Bayesian Information Criterion	2.21	2.23	2.21	2.21	2.21	2.21	2.21
Panel B: Cross sectional pricing	**(i)**	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**
Excess market return	0.34 *	0.65 ***	0.35 *	0.35 *	0.61 ***	0.62 ***	0.62 ***
Short-run volatility (*sres*)	-0.21 ***	-0.64 ***	-0.20 ***	-0.21 ***	-1.16 ***	-0.93 ***	-1.00 ***
Long-run volatility (*lres*)	-2.02 ***	-4.39 ***	-2.01 ***	-2.03 ***	-4.13 ***	-4.52 ***	-4.41 ***
Sum of squared pricing errors	0.44	0.84	0.45	0.45	0.63	0.68	0.67
RMSPE	0.13	0.18	0.13	0.13	0.16	0.17	0.16

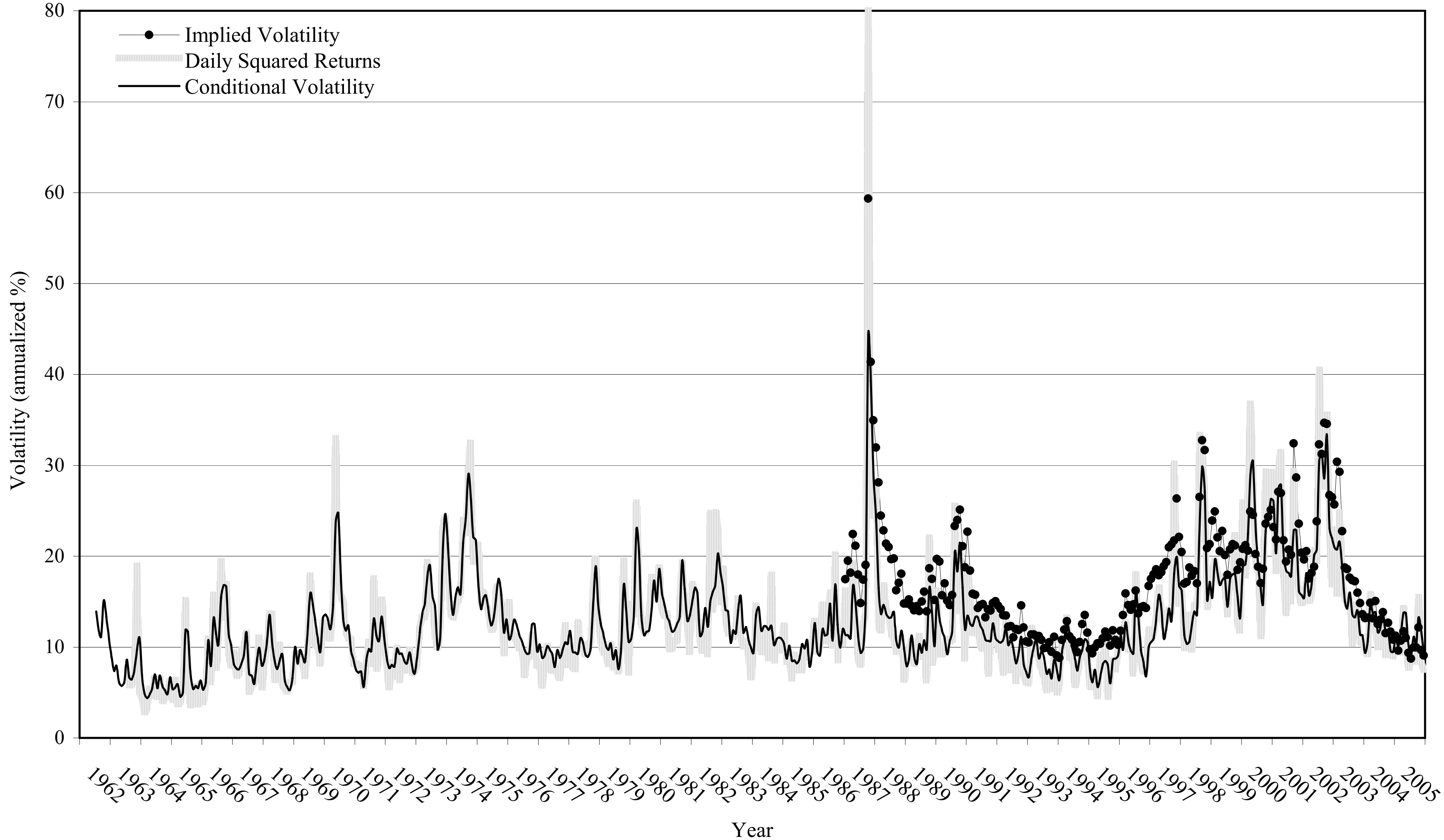

Figure 1. Monthly market volatility (annualized). This figure plots three measures of the annualized standard deviation of the market return at a monthly frequency for 1962/7 to 2005/12. The first measure is the implied volatility of the S&P100 stock index from the VIX (starting in 1986/01, dotted line). The second measure is derived from daily squared returns (grey thick line). The third measure is the conditional volatility from the volatility components model shown in Table I (black solid line).

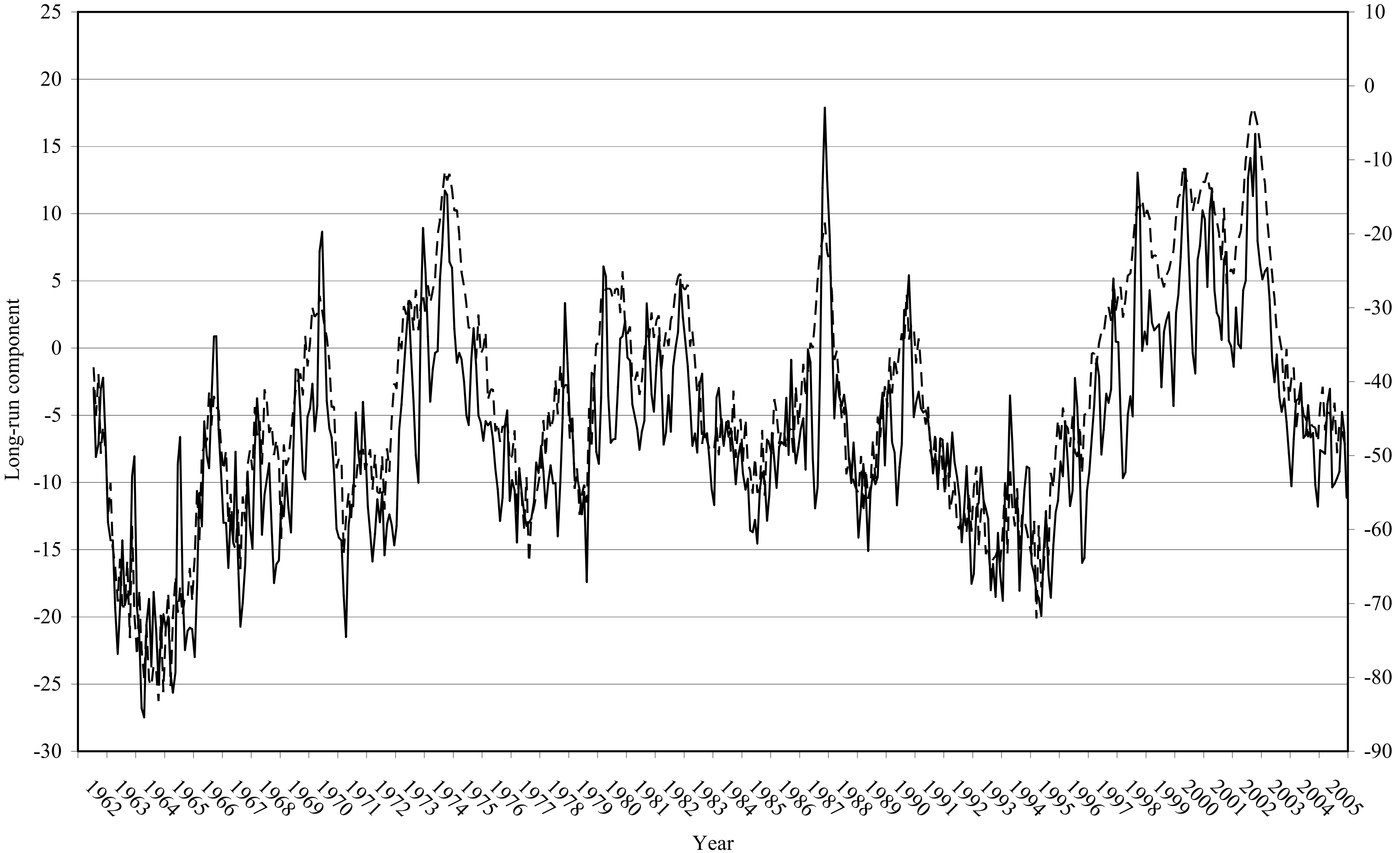

Figure 2. The long-run volatility component. This figure plots the estimated long-run volatility component (l) at a monthly frequency from 1962/7 to 2005/12 (solid line). The conditional variance of the excess market return is defined as $v=exp\,(2\,(s+l\,))$, where s is the short-run component of volatility (Figure 3). The estimate of l is from the stochastic volatility model that is reported in Table I. The Hodrick and Prescott (1997) filtered long-run component is obtained by applying the Hodrick-Prescott filter to the log of daily squared returns (dashed line).

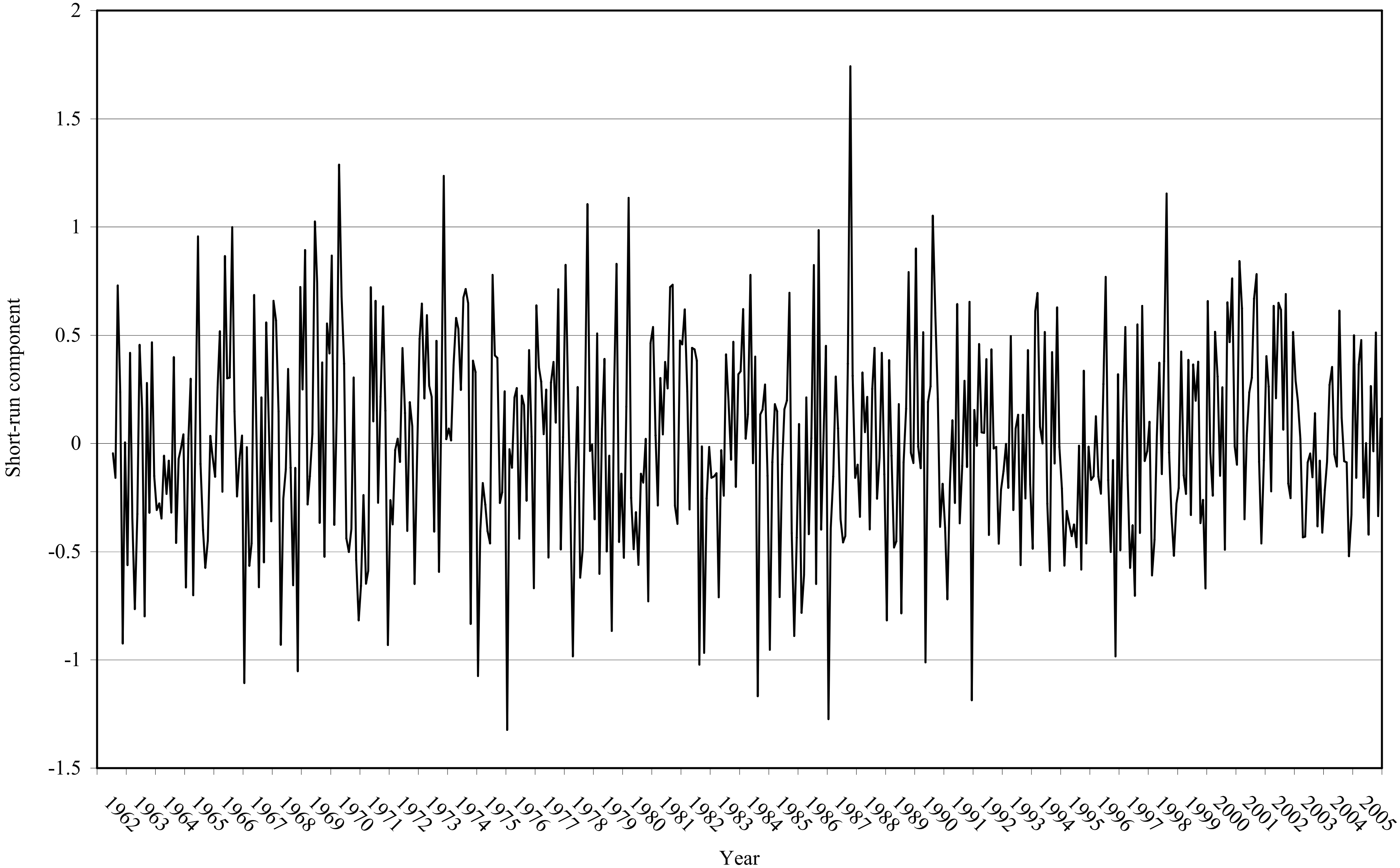

Figure 3. The short-run volatility component. This figure plots the estimated short-run volatility component s at a monthly frequency from 1962/7 to 2005/12. The conditional variance of the excess market return is defined as $v=exp\,(2\,(s+l\,))$, where l is the long-run component of volatility (Figure 2). The estimate of s is from the stochastic volatility model that is reported in Table I.

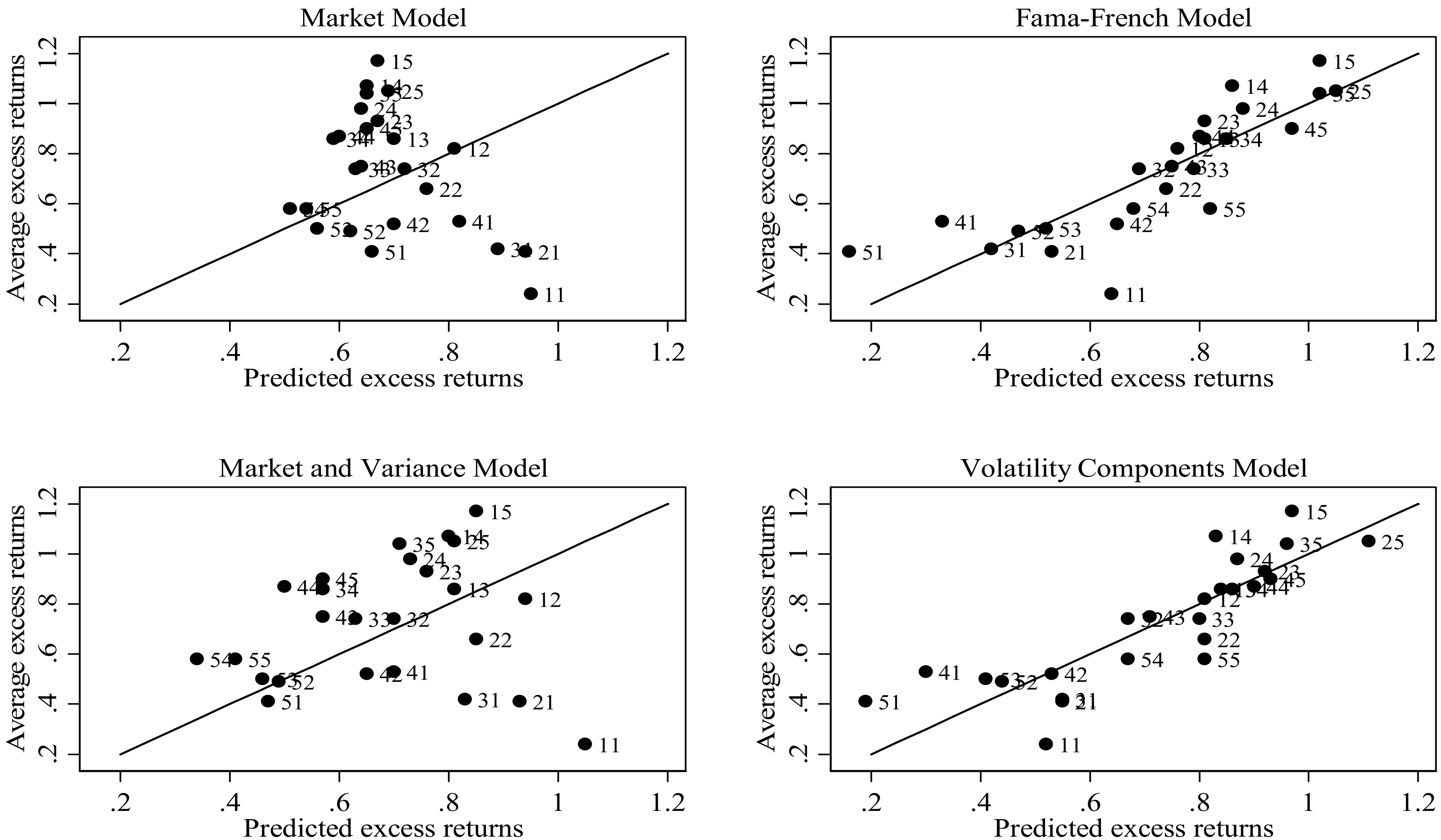

Figure 4. Actual versus predicted returns. This figure shows the average excess returns for the size and book-to-market sorted portfolios against the predicted returns from the models reported in columns (i), (ii), (iv) and (v) of Table IV. Portfolio *ij* corresponds to the *i-th* book-to-market quintile and the *j-th* size quintile.