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Stefano Eusepi

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Abstract

Central bank communication plays an important role in shaping market participants' expectations. This paper studies a simple nonlinear model of monetary policy in which agents have incomplete information about the economic environment. It shows that agents' learning and the dynamics of the economy are heavily affected by central bank transparency about its policy rule. A central bank that does not communicate its rule can induce "learning equilibria" in which the economy alternates between periods of deflation coupled with low output and periods of high economic activity with excessive inflation. More generally, initial beliefs that are arbitrarily close to the inflation target equilibrium can result in complex economic dynamics, resulting in welfare-reducing fluctuations. On the contrary, central bank communication of policy rules helps stabilize expectations around the inflation target equilibrium.

Key words: monetary policy, nonlinear dynamics, learning, liquidity traps

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Eusepi: Federal Reserve Bank of New York (e-mail: stefano.eusepi@ny.frb.org). The author thanks Roc Armenter, Jess Benhabib, and Todd Keister for useful comments and discussions.

1 Introduction

Recent monetary research has emphasized the role that imperfect knowledge and learning have in policy design. A growing number of papers study the performance of simple monetary policy rules under the assumption that private agents and the central bank are learning about the 'true' model of the economy -see Evans and Honkapohja (2008) for a survey of the literature. In this environment, expectations can become unanchored as agents' learning process can drift away from the equilibrium predicted under rational expectations. Policy rules are evaluated according to their impact on expectations' dynamics, in particular their ability to induce stability under learning.

This paper analyzes the global properties of a simple nonlinear monetary model with learning and, in particular, it explores how the dynamics of expectations are affected by central banks' communication about their policy rules. Uncertainty about monetary policy concerns the behavior of the short term nominal interest rate. The central bank interest rate rule contains information about its long run objectives, stabilization trade-offs and expectations about the state of the economy. As in Eusepi and Preston (2007), a transparent central bank gives full information about its policy rule, enhancing the predictability of the nominal interest rate and achieving expectations stabilization. Conversely, an opaque central bank, by not disclosing sufficient details about its rule, impairs the market participants' ability to forecast future policy. If market participants are not informed about the monetary policy rule, even policy rules that are optimal under rational expectations can generate instability under learning.

Eusepi and Preston's (2007) results are based on linear approximations around a deterministic steady state. However, *global* analysis can uncover important dynamics that are ignored in a linear approximation. In fact, the main contribution of this paper is to study learning dynamics away from the steady state(s) equilibria and how they are affected by central bank communication.

The model considered describes a cashless economy with monopolistic competition, nominal rigidities and a Taylor-type monetary policy rule, consistent with the zero-bound on the nominal interest rate. As shown by Benhabib et. al. (2001b), this class of policy rules imply two steady state equilibria, one where inflation is consistent with the target set by the central bank and the other where inflation is below target: a 'liquidity trap equilibrium'. The existence of multiple steady states suggests that any analysis based on a log-linear approximation around the inflation target steady state might lead to misleading conclusions about the stability properties of the policy rule. For example, Eusepi (2007), Evans and Honkapohja (2005) and Evans et al. (2007) show that accounting for the liquidity trap equilibrium has important consequences for policy design under learning.

In this paper, agents use learning rules that are consistent with the nonlinear environment; Evans and Honkapohja's (1995) nonlinear framework is extended to a multivariate model. Despite the learning algorithm differs from the (mostly used) ones consistent with linear models, *local* stability results are consistent with the previous literature.

First, in a calibrated version of the model it is shown that the inflation target steady state is locally stable under learning if the policy rule is communicated to the public, but it can become unstable if market participants ignore the rule and have to learn.[1] In this latter case convergence is obtained only with a policy rule that responds aggressively to output.

The key intuition for instability is that under an opaque regime market participants fail to anticipate *systematic* changes in the future path of the nominal interest rate. As monetary policy becomes less effective in managing expectations, the monetary authority reacts too much and too late, causing swings in expectations and macroeconomic instability.

Second, the liquidity trap equilibrium is shown to be locally unstable[2] under learning, independently of central bank communication: instability occurs because 'passive' monetary policy fuels the well-known 'cumulative process' of diverging inflation expectations and aggregate demand.

Independently of central bank communication, *global* analysis reveals a richer set of results. First, global analysis shows the existence of a 'corridor of stability'. Small shocks to expectations induce temporary fluctuations and (in some cases) convergence back to the

[1] Eusepi and Preston (2007) analyze the stability conditions in a different model environment, both in terms of agents's decisions rules and learning algorithms.

[2] Similar results can be found in Bullard and Mitra (2002), Evans and Honkapohja (2003) and Eusepi (2007).

equilibrium where inflation is at its target. In contrast, sufficiently large shocks drive the economy on a deflationary spiral, preventing convergence back to the equilibrium. Thus, no steady state equilibrium is globally stable.

Second, even *within* the corridor of stability the economy sufficiently large shocks drive the economy into prolonged periods of deflation and slow economic growth. These wide and persistent swings in expectations would not be detected from local analysis of the inflation target steady state. It is in fact shown that the liquidity trap steady state has a strong influence on learning dynamics, even in cases where initial expectations are close to the inflation target steady state.

Third, for some parameter values that induce local instability of the inflation target steady state, the economy is shown to converge to an 'learning equilibrium cycle' where output, inflation and the nominal interest rate fluctuate around the steady state. The size of the fluctuations depends on the policy response to output. For sufficiently low output responses, the economy converges to a 'liquidity-trap cycle', where the economy alternates persistent phases of deflation and low output with phases of rapid expansion and inflation above target.

Central bank communication of the policy rule is shown to have a significant effect both on the local and global properties of the economy. Compared to opacity, it enlarges significantly the corridor of stability in the economy and, for empirically plausible calibrations, prevents the existence of welfare-reducing cycles. Finally, as communication makes monetary policy more effective, central bank intervention reduces the size of temporary fluctuations around the inflation target equilibrium, thus preventing the economy from sliding into extended periods of deflation.

More generally, the results of the paper show that the perils of global indeterminacy as discussed in Benhabib et. al. (2001, 2003) are not confined to models with perfect foresight. One possible objection to global indeterminacy results (under perfect foresight) is that multiple equilibria can arise only if agents in the economy hold expectations that are far away from the steady state equilibrium and therefore might not be robust to the introduction of learning. In contrast, this paper shows that learning dynamics can exhibit complicated paths even when initial expectations are arbitrarily close to the steady state.

The paper is organized in two sections. The second section describes the model and the learning algorithm and the third section shows the numerical results. The technical appendix describes the model solution.

2 The Model

2.1 A simple monetary economy

The economy is populated by a continuum of identical consumer-producers and by a monetary authority. For simplicity, I consider a cashless economy. The economic environment is deterministic.

Consumer-producer. Each yeoman farmer j maximizes an intertemporal utility of the form

$$\sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{(C_s(j))^{1-\sigma}}{1-\sigma} - \frac{(H_s(j))^{1+\chi}}{1+\chi} - \frac{\psi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \Pi^* \right)^2 \right]$$

where C_t denotes the consumption aggregator of a continuum of differentiated goods $C_{j,t}$,

$$C_t = \left[\int_0^1 (C_{j,t})^{\frac{\theta}{1-\theta}} dj \right]^{\frac{\theta}{1-\theta}}, \ \theta > 1$$

and H_t denotes the amount of hours worked. Each agent produces a differentiated good in a monopolistically competitive market. The good is sold at the price $P_t(j)$: changing prices has a quadratic utility cost which depends on the parameter ψ. Financial markets are incomplete, and the only non-monetary asset that is possible to trade is a one period riskless bond. The agent's flow budget constraint is

$$B_t(j) \leq R_{t-1} B_{t-1}(j) + P_t(j) Y_t(j) - P_t C_t(j) + T_t$$

where B_t denotes the riskless bond, R_t denotes the gross interest paid on the bond T_t denotes a transfer from the government. The total demand for the differentiated good, $Y_t(j)$, needs to satisfy the constraint

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t$$

where Y_t denotes aggregate demand and P_t is the price index defined as

$$P_t = \left[\int_0^1 (P_{j,t})^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}}.$$

The production function for each differentiated good is

$$Y_t(j) = H_t^{\alpha}(j)$$

where α denotes the returns to scale to labor. The agents' problem is then to choose a sequence for $C_t(j)$, $B_t(j)$, $P_t(j)$ and $H_t(j)$ to maximize the intertemporal utility and satisfy the flow budget constraint, aggregate demand, production function and the transversality condition

$$\lim_{s \to \infty} \prod_{k=1}^{s} \frac{1}{R_{t+k}} B_{t+s+1} = 0,$$

taking as given R_t, T_t, Y_t and P_t and given an initial zero wealth, i.e. $B_0(j) = 0$ for each j.

Predicting monetary policy. In order to emphasize the role of central bank communication, it is assumed the consumption and pricing decisions are taken one period in advance, before observing the current nominal interest rate. This has no implications under rational expectations (here perfect foresight). However, it alters the learning problem that agents face because households have to *forecast* the nominal interest rate. According to permanent income theory, optimal consumption decisions depend on the expected path of the real interest rate in the indefinite future[3]. Agents have to forecast future monetary policy independently of decisions delays. In the nonlinear framework considered in this paper optimal infinite horizon decisions rule are extremely hard to analyze. In the interest of simplicity I adopt the 'Euler approach' to learning, where consumption decision rules are derived directly from the Euler equation and ignoring the intertemporal budget constraint[4]. As a consequence, consumption decision rules involve only a one-period forecasting horizon. Here the assumed delay in the consumption decision implies that consumption in the current period depends on the *expected* interest rate in the current period. For further discussion about alternative

[3]Eusepi and Preston (2007) consider the log-linear optimal consumption decision rule, given agents' beliefs. This is also known as the 'anticipated utility' approach, see Sargent (1999) for example.

[4]The intertemporal budget constraint is verified ex-post in the simulations.

approaches to decision rules under learning see for example Evans and Honkapohja (2008) and Preston (2005).

Model solution. The problem first order conditions yield the Euler equation

$$C_t\left(j\right)^{-\sigma} = \hat{E}_{t-1}\left[\frac{\beta R_t C_{t+1}\left(j\right)^{-\sigma}}{\pi_{t+1}}\right] \tag{1}$$

where again, agents choose consumption *before* observing the current nominal interest rate. In the symmetric equilibrium we have $C_t(j) = C_t$, $Y_t(j) = Y_t$ and $H_t(j) = H_t$. Also, goods' market clearing imposes $C_t = Y_t$ and a zero net supply of bonds implies $\int B_t(j)dj = B_t(j) = 0$ in every period. Each producer faces the same real marginal cost

$$s_t = \frac{Y_t^{\frac{\chi+1-\alpha}{\alpha}+\sigma}}{\alpha} \tag{2}$$

where the labor supply decision is taken using all the information available in the period.[5] Finally, the first order conditions for the price decision and the equilibrium condition $P_t\left(j\right) = P_t$ give[6]

$$\Pi_t = \frac{\Pi^*}{2} + \frac{1}{2}\sqrt{\left(\Pi^*\right)^2 + 4\hat{E}_{t-1}\left[\beta\Pi_{t+1}(\Pi_{t+1} - \Pi^*) + \theta\psi^{-1}C_t^{1-\sigma}\left(s_t - \mu^{-1}\right)\right]}, \tag{3}$$

where, the price is set in advance and depends on the expected marginal cost (as a function of aggregate demand) and on the expected inflation one period ahead. Pricing decisions in this simple model do not involve forecasting directly the evolution of the nominal interest rate. Following the Euler approach, prices depend on one-period-ahead forecast of future inflation and current demand conditions.

Policy rule. The central bank sets the nominal interest rate according to

$$R_t = 1 + \left(R^* - 1\right)\hat{E}_{t-1}\left[\left(\frac{\Pi_t}{\Pi^*}\right)^{\frac{\phi_\pi}{R^*-1}}\left(\frac{Y_t}{\bar{Y}}\right)^{\frac{\phi_y}{R^*-1}}\right], \tag{4}$$

which is a standard Taylor-type rule, responding to expected inflation and output. Log-linearizing (4) around the inflation target steady state gives

$$\hat{R}_t = \phi_\pi \hat{E}_{t-1}\hat{\pi}_t + \phi_y \hat{E}_{t-1}\hat{y}_t$$

[5] With price and consumption decisions set in advance this assumption allowes for market clearing.

[6] Here I assume for simplicity that firms observe the current price level when setting prices, while they set prices before observing aggregate demand. See also the appendix for more details on (3).

where throughout the paper it is assumed that $\phi_\pi > 1$, so that the Taylor principle is satisfied. This rule has three differences with the most commonly used Taylor-type rules. First, the interest rate is set in response to *expected* output and inflation. This reflects the plausible assumption that the central bank does not have full information about the current state of the economy - see McCallum (1999) on this point. Second, there is no notion of output-gap: the main focus is on simple and implementable rules. Third, the nonlinear policy rule is consistent with a zero bound on the nominal interest rate.[7]

2.2 Learning

It is assumed that agents do not form decisions under rational expectations from the outset. The remainder of the paper studies the evolution of the economy under learning dynamics. Learning is modeled following Evans and Honkapohja (1995, 2001). Using (1), (2), (3) after imposing the market equilibrium conditions, the model reduced form solution takes the form

$$Z_t = H\left(\hat{E}_{t-1}G\left(Z_t, Z_{t+1}\right)\right), \tag{5}$$

where $Z_t = \begin{pmatrix} Y_t & \pi_t & R_t \end{pmatrix}'$ and where the functions $H(\cdot)$ and $G(\cdot)$ are defined in the appendix. Both private sector and central bank expectations are defined as

$$\theta_{t-1} = \hat{E}_{t-1}G\left(Z_t, Z_{t+1}\right), \tag{6}$$

where, as common in the learning literature, in order to avoid simultaneity issue, current economic decisions are taken by using last period's estimates. Agents attempt to learn about the (perfect foresight) steady state(s) of the system, $\bar{\theta}$, coinciding with the fixed point(s)

$$\bar{\theta} = G\left(H\left(\bar{\theta}\right), H\left(\bar{\theta}\right)\right). \tag{7}$$

where agents' beliefs are self-confirming. Here only deterministic equilibria are considered, but the stability results can be extended to noisy equilibria -see Evans and Honkapohja (1995).

[7]Fiscal policy is does not play a role in the paper. It is summarized by the expression

$$T_t = M_t - M_{t-1}.$$

Notice that given the nonlinear relationship among variables, economic agents do not take expectations on each variable separately.[8] Rather, they form expectations about the function $G(\cdot)$. which is a *nonlinear combination* of the variables in Z_t. For example, the element of $G(\cdot)$ corresponding to inflation dynamics is

$$G^{\pi}(Z, Z) = \beta\Pi(\Pi - \Pi^{*}) + \theta\psi^{-1}Y^{1-\sigma}\left(\frac{Y^{\frac{\chi+1-\alpha}{\alpha}+\sigma}}{\alpha} - \mu^{-1}\right).$$

Agents observe past values of the vector $G(Z_{t-h-1}, Z_{t-h})$, for $h = 0, ..., -\infty$ and use the following estimator

$$\theta_t = \gamma\sum_{h=0}^{\infty}(1-\gamma)^h G(Z_{t-1-h}, Z_{t-h})$$

which is a distributed lag with exponentially declining weights. The weights depend on the fixed gain parameter γ. Higher values of γ imply heavier discount of past data. The fixed gain reflects agents' belief that the steady state might be changing over time. This belief is further justified by the existence of multiple steady states in the model. Notice that, whereas in a stochastic environment constant gain learning does not converge to a point limit, in a deterministic environment convergence can occur. The updating of the estimator can be written in recursive form as

$$\theta_t = \theta_{t-1} + \gamma\left[G(Z_{t-1}, Z_t) - \theta_{t-1}\right]. \tag{8}$$

Combining (5) and (6), and inserting in (8) gives

$$\theta_t = \theta_{t-1} + \gamma\left[G(H(\theta_{t-2}), H(\theta_{t-1})) - \theta_{t-1}\right] \tag{9}$$

which describes the law of motion of agents' beliefs. Output, inflation and the nominal interest rate are then determined according to

$$Z_t = H(\theta_{t-1}).$$

The fixed point(s) of the system (9), defined in (7), include the steady state equilibria under perfect foresight. However, the global analysis of the system can uncover other fixed points induced by agents' learning behavior. Bullard (1994), for example, shows the existence of "learning equilibria" in a simple overlapping-generations model.

[8] There is no assumption of point expectations.

2.3 Central bank transparency and output determination

As in Eusepi and Preston (2007), central bank communication is modelled as market participant's information about the policy rule. If private agents understand the policy rule, output is determined according to

$$\begin{aligned} Y_t &= H^y\left[\hat{E}_{t-1}G_C^y\left(Y_t, Y_{t+1}, \Pi_t, \Pi_{t+1}\right)\right] \\ &= \left\{\beta\hat{E}_{t-1}\left[\frac{Y_{t+1}^{-\sigma} + \left(R^* - 1\right)\left(\frac{\Pi_t}{\Pi^*}\right)^{\frac{\phi_\pi}{R^*-1}}\left(\frac{Y_t}{\bar{Y}}\right)^{\frac{\phi_y}{R^*-1}}Y_{t+1}^{-\sigma}}{\Pi_{t+1}}\right]\right\}^{-\frac{1}{\sigma}}. \end{aligned}$$

That is agents make explicit use of their knowledge of the policy rule to forecast the current real interest rate. As a result, they only need to form expectations about current and future output and inflation. In the case the policy rule is not communicated, agents ignore the relationship between the nominal interest rate and expected output and inflation. Output is then determined according to

$$\begin{aligned} Y_t &= H^y\left[\hat{E}_{t-1}G_{NC}^y\left(R_t, Y_{t+1}, \Pi_{t+1}\right)\right] \\ &= \left\{\beta\hat{E}_{t-1}\left[\frac{R_tY_{t+1}^{-\sigma}}{\Pi_{t+1}}\right]\right\}^{-\frac{1}{\sigma}} \end{aligned}$$

where the nominal interest rate appears among the variables that are used for estimation and forecasting. The underlying assumption is that under an opaque central bank market participants do not have clear information about the *form* of the policy rule, which reflects for example the central bank's objectives and forecasts.

3 Learning dynamics

3.1 Calibration

The analysis of the model's local and global dynamics is conducted with numerical simulations. The simple model is calibrated at a quarterly frequency. The benchmark calibration is summarized in Table 1. The agents' discount rate is chosen to be $\beta = 0.99$ which implies a steady state real interest rate of 4% in annualized terms. The parameters χ and α, regulating the labor supply and the production function are set to $\chi = 0$ and $\alpha = 1$, implying a infinitely elastic labor supply and a constant returns to scale production function. The parameter ψ which measures the degree of nominal rigidities is calibrated as follows. The model with quadratic cost of pricing implies the same log-linear inflation equation as the Calvo model, more used in quantitative analysis, that is

$$\hat{\pi}_t = \beta \hat{E}_{t-1} \hat{\pi}_{t+1} + \xi \hat{E}_{t-1} \hat{s}_t$$

where

$$\begin{aligned} \xi &= (1 - p_n)(1 - \beta p_n) / p_n \\ &= \frac{\theta}{\psi \mu} \bar{Y} \end{aligned} \tag{10}$$

and p_n is the probability that a firm is not allowed to change the price. The quantitative literature offers different estimates of p_n. In the benchmark calibration p_n is set equal to 0.78, somewhat higher than the more common estimates, but consistent with the absence of real rigidities.[9] The parameter ψ is then chosen so that (10) holds. The elasticity of demand θ is set to 9, implying a markup of roughly 11%.

There is considerable uncertainty about the parameter σ^{-1}, the intertemporal elasticity of substitution of consumption, which in the macro literature ranges between 1 and 1/3: for the benchmark calibration it is set to $\sigma = 1.5$.

INSERT TABLE 1 ABOUT HERE

Regarding the policy rule, I assume an inflation target $\Pi^* = 1.0061$ (2.5% in annualized terms), which implies a steady state annualized nominal interest rate of 6.5% while the inflation coefficient, ϕ_π, is set to 1.5. The constant gain in the learning algorithm is set to $\gamma = 0.5$.

[9] In the presence of real rigidities, the Phillips curve is

$$\hat{\pi}_t = \beta \hat{E}_{t-1} \hat{\pi}_{t+1} + \tilde{\xi} \omega \hat{y}_t$$

where

$$\tilde{\xi} = \xi \frac{1}{1 + \omega \theta},$$

and ω depends on the amount of real rigidities. This implies a flatter Phillips curve for a given p_n- see Woodford (2003) .

The choice is somewhat arbitrary. In a stochastic environment with structural change the choice of γ involves a trade-off between tracking the change and reducing the volatility of the estimates. Absent structural change, the constant gain is generally treated as a free parameter and it is therefore chosen to maximize the fit of the model. Examples in the literature are Orphanides and Williams (2007), Milani (2007), Eusepi and Preston (2008), Carceles-Proveda and Giannitsaru (2007). Quantitative stochastic learning models with constant gain adopt much lower values (i.e. $\gamma = 0.05$). In the deterministic environment considered here the constant gain is chosen to imply realistic economic fluctuations at a quarterly frequency. The implications of different choices for γ for local and global dynamics are discussed in the next sections.

Finally, the output response in the policy rule, ϕ_y, is allowed to take different values in alternative experiments. As discussed below, the choice of ϕ_y has important implications for learning dynamics.

3.2 Local stability analysis

The model displays two steady state equilibria. At the 'inflation target' (IT in the sequel) equilibrium inflation is consistent with central bank's preferences and output can be shown to be

$$\bar{Y} = \left(\frac{\alpha}{\mu}\right)^{\frac{\chi+1-\alpha}{\alpha}+\sigma},$$

which is independent of monetary policy. There also exists a 'liquidity trap' steady state (LT in the sequel) where inflation is equal to $\Pi^{LT} < \Pi^*$ and Π^{LT} depends on the specific calibration of the model and, in particular, the monetary policy rule. The more active the rule is at the IT steady state (i.e. the higher ϕ_π), the lower the value of Π^{LT}. In the benchmark calibration[10] the LT steady state implies deflation (around -1.3% in annualized terms). Because of nominal rigidities, at the LT steady state output is low (around 0.1% lower than $\bar{Y}$) and depends on the details of the policy rule. As it is well-known in the literature[11], under rational expectations the IT equilibrium is locally determinate with $\phi_\pi > 1$, while the

[10] The value of Π^{LT} is not significantly affected by the different values of ϕ_y.
[11] See for example Benhabib et al. (2001a) and Bullard and Mitra (2002).

LT equilibrium is locally indeterminate. In the model version presented here, these results hold for all parameter values. However local determinacy does not imply local stability under learning. This section summarizes the local stability conditions under learning and how they relate to the model's parameters. Given that the main focus is on global dynamics, the analysis in the paper is numerical and based on the benchmark calibration. Alternative calibration exercises are shown in Table 2. Linearizing the system (9) around both steady states[12] yields the following results:

1. independently of the communication regime, the LT steady state is unstable under learning;

2. holding fixed the other parameters, a policy rule that responds tenuously to output leads to instability of the IT steady state. Even with an 'active' policy rule (i.e. $\phi_\pi > 1$) stability requires a *sufficiently low* ratio ϕ_π/ϕ_y: the response to inflation cannot be too aggressive (relative to output);

3. the threshold ratio is *higher* in a regime of communication;

4. the higher the constant gain, γ, the lower the ratio ϕ_π/ϕ_y required to obtain stability.

The first three points reformulate existing results in the literature. Evans and Honkapohja (2005), Eusepi (2007) and McCallum (2000) show the instability of the LT equilibrium under alternative learning schemes. The basic intuition is that passive monetary policy fails to move real interest rates in response to changes in inflation expectations, leading to locally explosive beliefs dynamics. Consider a increase in inflation expectations. Given that at the LT steady state the nominal interest rate increases *less* proportionally than inflation expectations, the real interest rate decreases, inducing an increase in aggregate demand. As output rises above steady state, so does inflation which further fuels inflation expectations. Hence inflation expectations are driven to a divergent path.

Eusepi and Preston (2007) show that in a regime of no communication, the IT equilibrium can be unstable under learning, depending on the policy response to the output gap. The main

[12] For details see the appendix.

intuition for this instability result is as follows. If the central bank does not communicate its policy rule, agents fail to anticipate *systematic* changes[13] in the nominal interest rate. As a result, monetary policy becomes less effective: policy changes affect aggregate demand and inflation with a delay. Consider a sudden increase in inflation expectations. Because of agents' failure to anticipate a higher future nominal interest rates output increases, further stimulating the initial increase in inflation expectations and driving the economy towards a divergent path. Responding to output is highly inefficient under rational expectations but proves to be beneficial under learning. In fact, a change in output affects *future* inflation expectations (via marginal cost): a sufficiently strong response to output can prevent large changes inflation expectations and maintain the stability of the IT equilibrium.

Finally, the last point is novel to this paper. In most of the literature, stability under learning is analyzed in terms of E-stability conditions, corresponding to the case were where $\gamma \rightarrow 0$ - see the appendix. For γ arbitrarily small, under the regime of central bank communication the inflation target equilibrium is stable for every parameter values. This is not true for the case of a positive gain. The finding suggests that in models with constant gain learning, a positive response to output is needed in order to maintain macroeconomic stability. This is particularly relevant because most of the empirical work on learning assumes constant gain algorithms.[14] The table below describes the threshold values for ϕ_y for alternative calibrations and under different assumptions about communication and the fixed gain.

TABLE 2 ABOUT HERE

As γ increases, stability requires a stronger response to output. Notice that, for a meaningful comparison with the popular Taylor rule the output coefficient should be expressed in annualized terms, $\phi_y^A = 4\phi_y$. Consider for example a regime of no-communication. In the benchmark calibration with $\gamma = 0.05$ any coefficient on output $\phi_y < 0.16$ leads to local instability of the IT equilibrium. In annualized terms this implies $\phi_y^A = 0.64$, larger than 0.5, the coefficient on the Taylor rule. The required coefficient is higher if the policy rule is

[13] By systematic it is intended those changes that are implied by the rule and therefore predictable.

[14] In stochastic models, constant gain learning implies that agents' beliefs do not converge to a point estimate but to an invariant distribution centered around the rational expectations beliefs. Convergence to the invariant distribution is related to the E-Stability conditions.- see Evans and Honkaphja (2001) for details.

more aggressive to inflation or if prices are relatively flexible. In both cases a given change in expectations has a larger effect on current output, inflation and the nominal interest rate.

3.3 Global dynamics

This section contains the main results of the paper. It discusses the global properties of the model under the benchmark calibration and, in particular, the effects of central bank communication on learning dynamics.

3.3.1 Regime of opacity (no-communication)

Stable IT steady state. In this first experiment the policy response to output is set to $\phi_y = 0.285$, consistent with stability of the IT equilibrium.[15] Figure 1 displays the phase diagram describing output and inflation dynamics. In all figures, output is expressed in percentage deviations from its IT steady state equilibrium, while inflation and the nominal interest rate are expressed in annualized percentage terms. The LT steady state is a saddle[16] while the IT steady state is a sink. The phase diagram suggests two important observations.

First, the stable manifold of the LT equilibrium (dotted line) delineates the basin of attraction of the IT steady state. Global analysis uncovers the existence[17] of a 'corridor of stability'. Small changes in expectations result in convergence back to the steady state. Large shocks driving the system outside the basin of attraction induce a divergent path involving declining output, inflation and the nominal interest rate. In the latter case, the outcome is deflation and a zero nominal interest rate, while agents' beliefs set on an explosive path. In this scenario, the model's predictions become less informative as agents' decisions eventually violate admissibility constraints such as positive consumption. It is plausible to assume that *before* the feasibility constraint are met either a change in policy or a change in the agents' learning process would occur. Benhabib et al. (2002), Evans and Honkapohja (2003) and Evans et. al. (2007), for example, show that a shift to a monetary growth rule and the

[15] This value is not inconsistent with what reported in the table -the table reports stability values with a two-digit approximation.

[16] The diagram is two dimensional. Notice that the other eigenvalues are all inside the unit circle. Only one eigenvalue is outside.

[17] The terms goes back to Leijonhufvud ([1973] 1981), discussing the stabilizing effects of market forces in response to demand shocks.

coordination between the monetary and fiscal policy can push the economy out of deflation. Also, agents' learning rules need not be time invariant. Marcet and Nicolini (2003) propose a learning rule which depends on the state of the economy. In fact, when considering large shocks the assumption of a time-invariant policy rule and a time-invariant learning algorithm becomes less realistic. For this reason, the behavior of the economy outside the stability corridor is outside the scope of this paper. In the sequel we focus on the case of smaller shocks.

FIGURE 1 ABOUT HERE

The second observation concerns the LT steady state. As it is clear from the above, despite local instability the LT equilibrium has an important role in determining the global dynamics of the system. Within the corridor of stability, shifts in expectations can lead to prolonged episodes of low output growth and deflation. For example, consider a shock that drives the economy sufficiently close to the stable manifold (point A). Here output is below its steady state while inflation is above target. From this initial condition, inflation and output start declining until they get close to the liquidity trap equilibrium (point B). Given that the LT steady state is unstable, the economy slowly begin reverting back to the steady state (point C). Notice that convergence is oscillatory and can require a long transition to the steady state. The dynamics of the economic system is influenced by the 'saddle' connection between the LT steady state and the IT steady state. In other words, the LT unstable manifold connects the LT steady state and the IT steady state. Trajectories originating close to the liquidity trap equilibrium (but inside the corridor of stability) eventually converge back to the IT steady state. Benhabib et al. (2001b) discuss the existence of the saddle connection under perfect foresight[18].

In order to get an intuition for this result, consider the following example. As pessimistic expectations induce a drop in output and inflation the policy rule prescribes a decrease in the nominal interest rate. In absence of central bank communication the change in policy has initially only a limited effect on aggregate demand and therefore does not stop the economy

[18]Benhabib et al. (2001b) describe in depth the mathematical results behind the global dynamics of this model; the Kopell and Howard theorem about the existence of a saddle connection and the Hobf bifurcation.

from getting closer to the LT equilibrium. As the economy gets close to the liquidity trap, the policy rule becomes *passive* (from its active stance near the IT steady state) preventing fast convergence back to the inflation target. After a prolonged period of deflation and slow output growth, the lower interest rate finally stimulates spending and the economy start reverting back towards the IT equilibrium. However, the nominal interest is now too low: the economy overshoots the target as inflation accelerates and output grows above steady state. Eventually convergence obtains.

FIGURE 2 ABOUT HERE

Unstable IT steady state. For sufficiently low values of ϕ_y the IT equilibrium becomes a source. As the ϕ_y decreases and the steady state becomes unstable an attracting (stable) cycle emerges around the IT steady state. This 'learning equilibrium' is absent in the perfect foresight model: it is a product of agents' learning dynamics[19]. From initial conditions that can be arbitrarily close to the inflation target the economy converges to an equilibrium cycle where output, inflation and the nominal interest rate display endogenous fluctuations. Figure 2 shows the phase-diagram for $\phi_y = 0.279$. The saddle connection with the IT steady state breaks, as the unstable manifold folds onto itself, generating a closed curve around the steady state. Any shock within the corridor of stability drives the economy to the equilibrium cycle. The lower panel shows the time evolution of the variables at the learning equilibrium where economic expansions above equilibrium are followed by low output and inflation below target. Again, absence of central bank communication generates policy-induced fluctuations as monetary policy responds too much and too late to changes in expected inflation and output. The size of economic fluctuations depends on the specific values of ϕ_y: the further away from the bifurcation point, the larger the equilibrium cycle. Global analysis of the system reveals interesting dynamics: as ϕ_y becomes smaller the stable and unstable manifolds become extremely close. As shown in Figure 3 the cycle now includes the whole basin of attraction of the inflation target equilibrium[20].

[19] More precisely, the model under perfect foresight does not undergoes an bifurcation. The IT steady state is locally determinate for any value of ϕ_y. In the learning model, as the steady state loses its stability the economy undergoes a supercritical Hobf bifurcation that generate the equilibrium cycle.

[20] In the limit as the stable and unstable manifold 'merge' the period of the cycle converges to infinity. In

FIGURE 3 ABOUT HERE

This limiting case is obtained for $\phi_y = 0.2697$. Any shock that moves output and inflation inside the basin of attraction drives the system to this 'liquidity trap cycle' where the economy repeatedly goes though a protracted phase of deflation and negative growth, followed by a phase of over-expansion with inflation above target[21]. Numerical experiments show that in the benchmark calibration the cycle exist for values of ϕ_y between 0.2697 and 2.84. This might appear a rather narrow range of parameter values, but even in the case of convergence to the IT steady state, extremely persistent oscillations around the steady state can be found for higher values of ϕ_y.

3.3.2 The benefits of central bank communication

Central bank communication has a strong influence on the local properties of the economic system. As shown above, the IT steady state is locally stable for a wider set of parameters. One additional implication is that under communication, *no* equilibrium cycles[22] exist for parameter values that have been considered in the previous section.

However, central bank communication is also shown to be very important when global dynamics are concerned. Figure 4 shows the basin of attraction for the benchmark economy with $\phi_y = 0.285$ and under a regime of communication. Again, the basin of attraction is delimited by the dotted line originating from the LT steady state. One important aspect of central bank communication is that the economy's corridor of stability becomes substantially wider. This in an important aspect of policy stabilization that would be omitted if the analysis is restricted to a linearized economy. In order to get a sense of what this means consider the following example. Comparing Figure 1 and Figure 4 it is immediate to see that under communication convergence to the IT equilibrium occurs even after a shock that decreases output by -1.5% and increases inflation to above 5%. The same shock would set the economy

other words, the fraction of time that the economy spends in deflation increases as ϕ_y decreases, until it gets to 1.

[21] For even smaller values all trajectories are explosive. Output and inflation keep falling and the nominal interest rate converges to zero.

[22] In fact, as mentioned above, equilibrium cycles emerge as the IT steady state becomes unstable under learning.

on a deflationary spiral in a regime of no-communication. The intuition is again that under a regime of communication private agents are able to anticipate policy changes dictated by the policy rule, making monetary policy more effective in the face of adverse shocks.

FIGURE 4 ABOUT HERE

A second important benefit of communication involves the economy's responses to shocks *within* the stability corridor. First, as mentioned at the beginning of this section, for the range of parameters considered in section 3.3.1, the IT equilibrium is stable under learning and therefore no learning equilibria exist.

Second, Figure 5 shows the response of the economy to a shock to expectations that lowers output 1% below steady state. For this experiment the response on output is chosen at a high level, $\phi_y = 0.35$ (implying a value of 1.4 in the Taylor rule!) in order to show that lack of communication can be costly even in the case of aggressive policies. In a regime of no-communication, the economy slides into a prolonged period of deflation, low economic growth (with output below steady state) and the nominal interest rate below steady state values. A similar trajectory as described in Figure 1.

FIGURE 5 ABOUT HERE

Conversely, under communication output and inflation drop but inflation stays positive. The initial decrease in the nominal interest rate is sufficient to stimulate output and lead to convergence back to the inflation target in a (relatively) short period of time. More generally, the transition to the steady state is shorter and smoother than in absence of communication. Again, these complex dynamics can be captured only by fully analyzing the global dynamics of the economy.

3.4 Sensitivity analysis

The results outlined in the previous sections are robust in changes to the models' parameters. Different values for σ, ψ, ϕ_π and γ affect the quantitative results but do no alter the qualitative behavior of the system. Numerical experimentation with these parameters has confirmed that the qualitative results of the paper are robust to plausible alternative parametrization[23]. All experiments below are conducted under the regime of no-communication. Local stability analysis shows that changes in the constant gain affect the threshold values of the ratio ϕ_π/ϕ_y that delivers stability under learning. From the global perspective, this affects the values of ϕ_y for which learning equilibria exist (i.e. values of ϕ_y for which the steady state becomes unstable). As shown in Table 2, for $\gamma \in [0.05, 0.5]$ these values are empirically plausible, involving a response to output comparable with the coefficient on the Taylor rule. Another effect of a change in γ is a slower convergence behavior of the learning process. In other words, changing γ affects the time scale of fluctuations but leaves the equilibrium cycles unchanged.

Changes in the intertemporal elasticity of substitution of consumption σ^{-1} can be shown to change the magnitude of the output's cycles. Higher values of σ^{-1}, implying a higher sensitivity of aggregate demand to expected changes in the nominal interest rate, increases the threshold value of ϕ_y to about 1 and widens output fluctuations. For example, setting $\phi_y = 0.92$, at the equilibrium cycle output fluctuations reach deviations of 2% from steady state values, but leave inflation fluctuations unchanged with respect to benchmark. Opposite effects are obtained[24] for $\sigma = 3$.

Changes in the degrees of nominal rigidities affect the threshold values of ϕ_y, as shown in Table 2, but do not have any significant impact on the magnitudes of the equilibrium cycles. Finally, changes in ϕ_π have important quantitative effects. The more aggressive the policy rule the wider the fluctuations in output and inflation. This because a higher ϕ_π implies higher deflation at the LT steady state. For example, setting $\phi_\pi = 2.5$ (which might reflect the post-Volker era) leads to oscillations in output of 2% (from steady state), while inflation fluctuates between 4% and less than -2%. This shows how more aggressive policies could ultimately lead to higher fluctuations in absence of central bank transparency.

[23] The experiments involve, $\sigma \in [0.5, 3]$, $p_n \in [0.5, 0.85]$, $\phi_\pi \in [1.5, 3]$ and $\gamma \in [0.05, 0.6]$. Only one paramer at a time was allowed to differ from benchmark in the experiment.

[24] In this case the threshold value becomes $\phi_y = 0.57$ and output fluctuations become as low as 0.2% deviations from steady state.

3.5 Discussion

Global indeterminacy and learning. Benhabib et. al. (2002, 2003) analyze a similar monetary model under perfect foresight and show that focusing on local analysis can be misleading. In their example, the model displays local determinacy but global indeterminacy. Equilibrium paths arbitrarily close to the IT equilibrium can converge to a liquidity trap or a cycle around the inflation target steady state. Their conclusion: simple Taylor-type rules could lead to multiple equilibria. The first general concern with this result is whether such complicated dynamics under perfect foresight can be robust to some form of learning behavior. The second concern, stressed by Woodford (2003), involves the interpretation of the result. Benhabib et al. (2001b, 2003) find that arbitrarily small deviations from the IT steady state would lead the economy to another equilibrium. But in fact this can only happen if *expectations of inflation in future periods are far away from the inflation target.* Conversely, for inflation expectations close to target, we should expect convergence back to equilibrium (as in the case of adaptive learning).

This paper addresses both issues. First, it is shown that complicated dynamics can actually occur as a *result* of learning dynamics. Second, despite local determinacy of the IT steady state under perfect foresight, there exist dynamic paths that converge to learning equilibria from *initial inflation expectations that can be arbitrarily close to the steady state.* More generally, the paper shows that even in the case of a locally stable IT equilibrium, under a no-communication regime small shock can induce persistent deviations in output and inflation.

Time invariant policies and the liquidity trap. A second criticism to this modeling approach is that the adaptive nature of the expectations formation mechanism prevent agents from reacting to announcement effects. For example, Eggertsson and Woodford (2004) and Svensson (2003) show how commitment to an inflationary policy, contingent to the state of the economy, can lead the economy outside a liquidity trap. But, these policies are proved to be stabilizing precisely because they are *fully understood* by market participants. Lack of communication or lack of full credibility, as modeled in this paper, prevents agents from anticipating future policy moves, thus restricting the policy options available to the central bank. Under a regime of no-communication announcement do not play any role. In contrast, the paper shows that a transparent central bank can successfully stabilize shocks (at least within the corridor of stability) without deviations from their policy rule.

4 Conclusions

The paper shows that central bank communication can have important effects on learning dynamics with consequences for economic stabilization. It is shown that in a regime of no communication central banks can generate policy-induced fluctuations, where the economy fall in prolonged periods of low economic activity and deflation, followed by overheating and high inflation. Conversely, central bank communication limits the effects of adverse shocks by keeping expectations anchored around the inflation target equilibrium.

5 Appendix

5.1 Model solution

The yeoman optimization problem is

$$\max_{C_t(j),H_t(j),B_t(j),P(j)} \hat{E}_{t-1}\sum_{s=t}^{\infty}\beta^{s-t}\left[\frac{(C_s(j))^{1-\sigma}}{1-\sigma}-\frac{(H_s(j))^{1+\chi}}{1+\chi}-\frac{\psi}{2}\left(\frac{P_s(j)}{P_{s-1}(j)}-\Pi^*\right)^2\right]+$$
$$+\hat{E}_{t-1}\sum_{s=t}^{\infty}\beta^{s-t}\lambda_s^1\left[-B_s(j)+R_{s-1}B_{s-1}(j)+P_s(j)\,H_t^{\alpha}(j)-P_sC_s(j)+T_s\right]+$$
$$+\hat{E}_{t-1}\sum_{s=t}^{\infty}\beta^{s-t}\lambda_s^2\left[\left(\frac{P_s(j)}{P_s}\right)^{-\theta}Y_s-H_s^{\alpha}(j)\right],$$

where λ_t^1 and λ_t^2 denote the Lagrange multipliers. Combining the first order conditions for $C_t(j)$ and $B_s(j)$ in the symmetric equilibrium we obtain the Euler equation

$$C_t^{-\sigma}=\hat{E}_{t-1}\left[\frac{\beta R_t C_{t+1}^{-\sigma}}{\Pi_{t+1}}\right].$$

The first order condition with respect to $P_t(j)$ gives in the symmetric equilibrium (where $P_t(j)=P_t$)

$$\Pi_t(\Pi_t-\Pi^*)=\hat{E}_{t-1}\left[\beta\Pi_{t+1}(\Pi_{t+1}-\Pi^*)+\theta\psi^{-1}C_t^{1-\sigma}\left(\frac{C_t^{\frac{\chi+1-\alpha}{\alpha}+\sigma}}{\alpha}-\mu^{-1}\right)\right],$$

where I use the assumption that firms observe P_t when choosing $P_t(j)$. Solving the quadratic equation in Π_t equation (3) in the text obtains[25]. Finally, solving for the labor supply decision gives

$$s_t = \frac{C_t^{\frac{\chi+1-\alpha}{\alpha}+\sigma}}{\alpha},$$

the real marginal cost, where I use the assumption that labor supply is chosen using all information available in the current period.

5.2 Learning

The nonlinear dynamical system can be written equation by equation. Agents take past averages of the following three functions. For the output equation, in the case of no-communication we have

$$\begin{aligned} \theta_t^y &= \theta_{t-1}^y + \gamma \left[G_{NC}^y (Z_{t-1}, Z_t) - \theta_{t-1}^y \right] \\ &= \theta_{t-1}^y + \gamma \left[\frac{\beta R_{t-1} Y_t^{-\sigma}}{\Pi_t} - \theta_{t-1}^y \right], \end{aligned}$$

while in the case where agents know the rule we obtain

$$\begin{aligned} \theta_t^y &= \theta_{t-1}^y + \gamma \left[G_C^y (Z_{t-1}, Z_t) - \theta_{t-1}^y \right] \\ &= \theta_{t-1}^y + \gamma \left[\frac{1 + (R^* - 1) \left[\left(\frac{\Pi_{t-1}}{\Pi^*} \right)^{\frac{\phi_\pi}{R^*-1}} \left(\frac{Y_{t-1}}{\bar{Y}} \right)^{\frac{\phi_y}{R^*-1}} \right] Y_t^{-\sigma}}{\Pi_t} - \theta_{t-1}^y \right]. \end{aligned}$$

Notice that the two updating rules have different consequences. This depend on the crucial assumption that agents have to forecast future policy, i.e. the consumption decision is taken *before* observing the current interest rate. For the inflation equation we get

$$\begin{aligned} \theta_t^\pi &= \theta_{t-1}^\pi + \gamma \left[G^\pi (Z_{t-1}, Z_t) - \theta_{t-1}^\pi \right] \\ &= \theta_{t-1}^\pi + \gamma \left[\beta \Pi_t (\Pi_t - \Pi^*) + \theta \psi^{-1} Y_{t-1}^{1-\sigma} \left(\frac{Y_{t-1}^{\frac{\chi+1-\alpha}{\alpha}+\sigma}}{\alpha} - \mu^{-1} \right) - \theta_{t-1}^\pi \right], \end{aligned}$$

[25] I select the root with a positive sign. The only one with an economic meaning.

and finally for the policy rule

$$\begin{aligned}\theta_t^r &= \theta_{t-1}^r + \gamma\left[G^r\left(Z_{t-1}, Z_t\right) - \theta_{t-1}^r\right] \\ &= \theta_{t-1}^r + \gamma\left[\left(\frac{\Pi_t}{\Pi^*}\right)^{\frac{\phi_\pi}{R^*-1}}\left(\frac{Y_t}{\bar{Y}}\right)^{\frac{\phi_y}{R^*-1}} - \theta_{t-1}^r\right].\end{aligned}$$

It is then possible to re-write the system in terms of agents' estimators only by using

$$Z_t = H\left(\theta_{t-1}\right).$$

In particular

$$Y_t = H^y\left(\theta_{t-1}^y\right) = \left(\theta_{t-1}^y\right)^{-\frac{1}{\sigma}},$$

$$\Pi_t = H^\pi\left(\theta_{t-1}^\pi\right) = \frac{\Pi^*}{2} + \frac{1}{2}\sqrt{\left(\Pi^*\right)^2 + 4\theta_{t-1}^\pi}$$

and

$$R_t = H^r\left(\theta_{t-1}^r\right) = 1 + \left(R^* - 1\right)\theta_{t-1}^r.$$

This gives the following system

$$\theta_t^y = \theta_{t-1}^y + \gamma\left[\frac{\beta\left(1 + \left(R^* - 1\right)\theta_{t-2}^r\right)\theta_{t-1}^y}{\frac{\Pi^*}{2} + \frac{1}{2}\sqrt{\left(\Pi^*\right)^2 + 4\theta_{t-1}^\pi}} - \theta_{t-1}^y\right], \tag{11}$$

or

$$\theta_t^y = \theta_{t-1}^y + \gamma\left[\beta\frac{\theta_{t-1}^y + \left(R^* - 1\right)\left(\frac{H^\pi\left(\theta_{t-2}^\pi\right)}{\Pi^*}\right)^{\frac{\phi_\pi}{R^*-1}}\left(\frac{\left(\theta_{t-2}^y\right)^{-\frac{1}{\sigma}}}{\bar{Y}}\right)^{\frac{\phi_y}{R^*-1}}\theta_{t-1}^y}{\frac{\Pi^*}{2} + \frac{1}{2}\sqrt{\left(\Pi^*\right)^2 + 4\theta_{t-1}^\pi}} - \theta_{t-1}^y\right] \tag{12}$$

under communication. Finally

$$\theta_t^\pi = \theta_{t-1}^\pi + \gamma\left[\begin{array}{c}\beta H^\pi\left(\theta_{t-1}^\pi\right)\left(H^\pi\left(\theta_{t-1}^\pi\right) - \Pi^*\right) \\ +\theta\psi^{-1}\left(\theta_{t-2}^y\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\left(\theta_{t-2}^y\right)^{-\frac{\chi+1-\alpha}{\sigma\alpha}-1}}{\alpha} - \mu^{-1}\right) - \theta_{t-1}^\pi\end{array}\right] \tag{13}$$

and

$$\theta_t^r = \theta_{t-1}^r + \gamma\left[\left(\frac{H^\pi\left(\theta_{t-1}^\pi\right)}{\Pi^*}\right)^{\frac{\phi_\pi}{R^*-1}}\left(\frac{\left(\theta_{t-1}^y\right)^{-\frac{1}{\sigma}}}{\bar{Y}}\right)^{\frac{\phi_y}{R^*-1}} - \theta_{t-1}^r\right], \tag{14}$$

which gives a five dimensional nonlinear dynamical system. Notice that in the regime of communication the equation (14) is independent of the rest of the system. (In other words, the system reduces to a four dimensions.)

5.3 Linearized model

Linearizing the system (9) in the case of no communication yields

$$Z_t^{nc} = AZ_{t-1}^{nc}$$

where $Z_t^{nc} = \begin{bmatrix} \hat{\theta}_t^y & \hat{\theta}_t^\pi & \hat{\theta}_t^R & \hat{\theta}_{t-1}^y & \hat{\theta}_{t-1}^R \end{bmatrix}'$ and $\hat{\theta}$ denotes the variable in deviations from its steady state value. In a regime of communication, the linearized system becomes

$$Z_t^c = BZ_{t-1}^c$$

where $Z_t^{nc} = \begin{bmatrix} \hat{\theta}_t^y & \hat{\theta}_t^\pi & \hat{\theta}_{t-1}^y & \hat{\theta}_{t-1}^\pi \end{bmatrix}'$. Local stability obtains if and only if all eigenvalues of the matrix A (B) are inside the unit circle. The matrices A and B are calculated numerically[26]. The stability condition is related to E-stability, as discussed in Evans and Honkapohja (2001). E-stability obtains if and only if all eigenvalues of

$$\frac{\partial}{\partial\theta}\left[G(\theta) - \theta\right]$$

have real parts less than one. The two stability conditions deliver the same result in the case the gain $\gamma \to 0$ - see Evans and Honkapohja (2001).

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LIST OF TABLES AND FIGURES

Table I

θ	β	σ	Π^*	ϕ_π	α	p_n	χ	γ
9	0.99	1.5	1.006	1.5	1	0.78	0	0.5

Table II

Benchmark	$\boldsymbol{\gamma} = \mathbf{0.005}$	$\boldsymbol{\gamma} = \mathbf{0.05}$	$\boldsymbol{\gamma} = \mathbf{0.1}$	$\boldsymbol{\gamma} = \mathbf{0.5}$
Communication	0	0	0.02	0.1
No Communication	0.14	0.16	0.17	0.29
$\mathbf{p}_n$**= 0.6**	$\boldsymbol{\gamma} = \mathbf{0.005}$	$\boldsymbol{\gamma} = \mathbf{0.05}$	$\boldsymbol{\gamma} = \mathbf{0.1}$	$\boldsymbol{\gamma} = \mathbf{0.5}$
Communication	0	0.04	0.09	0.52
No Communication	0.67	0.72	0.79	1.2
$\boldsymbol{\phi}_\pi$**= 2.5**	$\boldsymbol{\gamma} = \mathbf{0.005}$	$\boldsymbol{\gamma} = \mathbf{0.05}$	$\boldsymbol{\gamma} = \mathbf{0.1}$	$\boldsymbol{\gamma} = \mathbf{0.5}$
Communication	0	0.01	0.04	0.27
No Communication	0.25	0.28	0.31	0.57

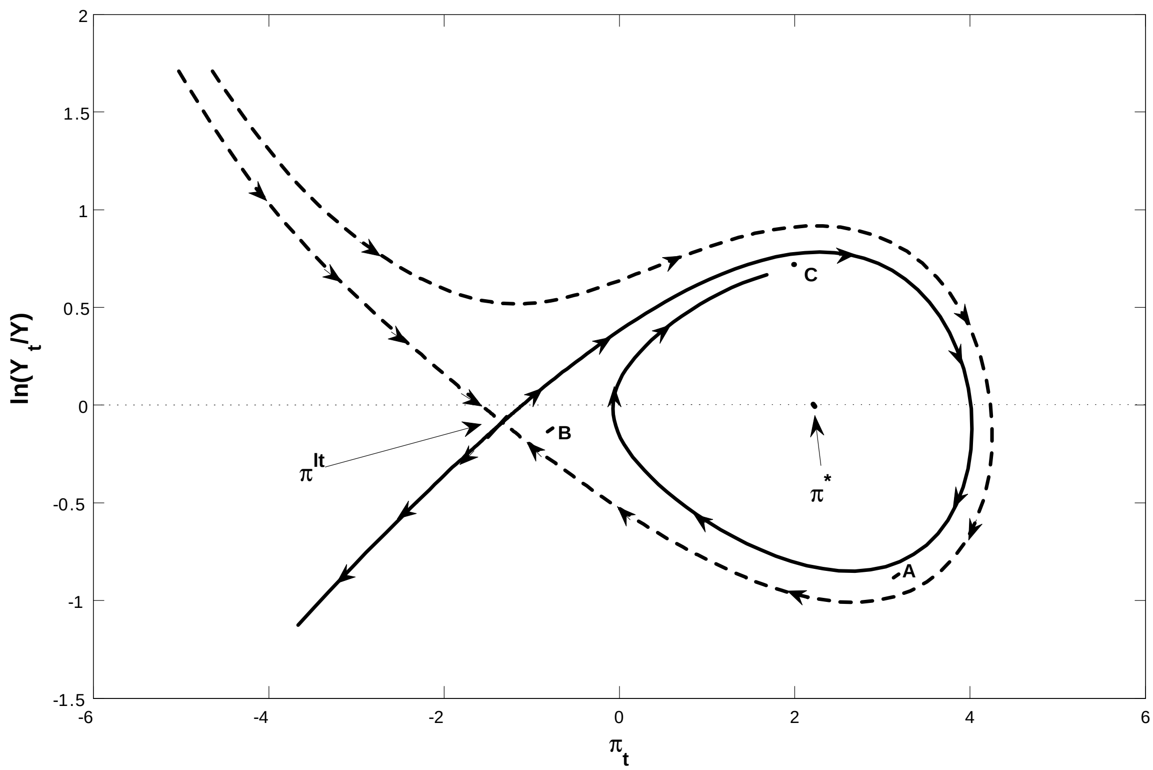

Figure 1: Phase diagram for in the economy in the regime of communication.

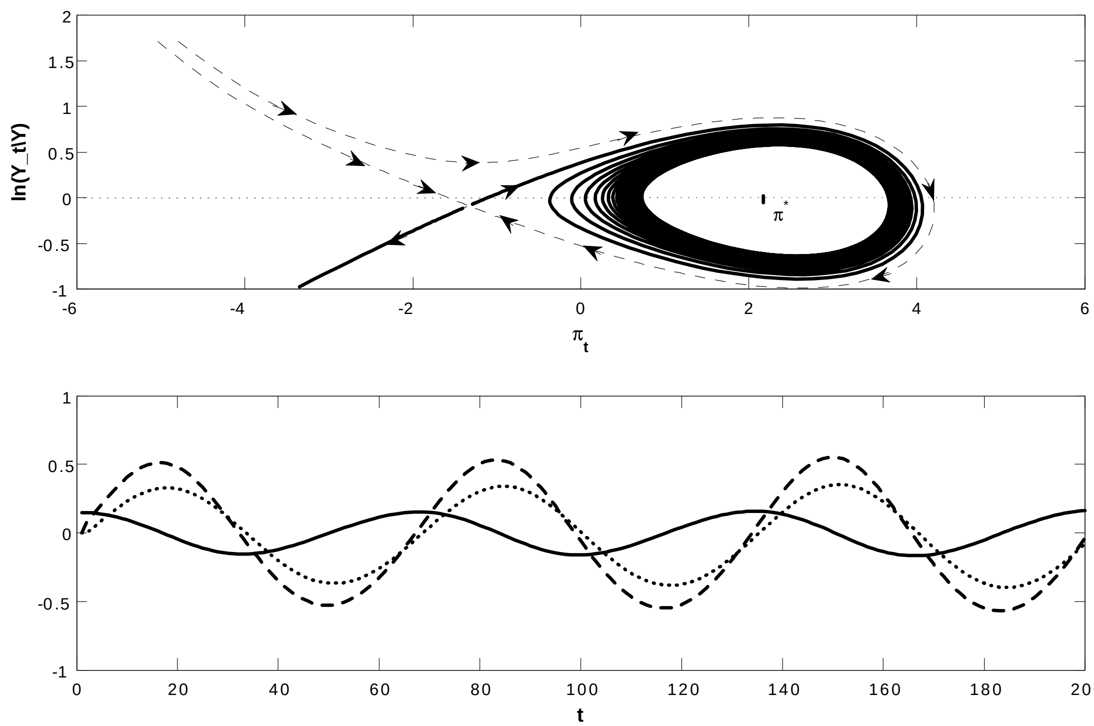

Figure 2: The upper panel shows the phase-diagram with the learning cycle around the IT steady state. The lower panel show the evolution of output (solid line), inflation (dotted line) and the nominal interest rate (dashed line). In the lower panel varibles are expressed in percentage deviations from their IT steady state values.

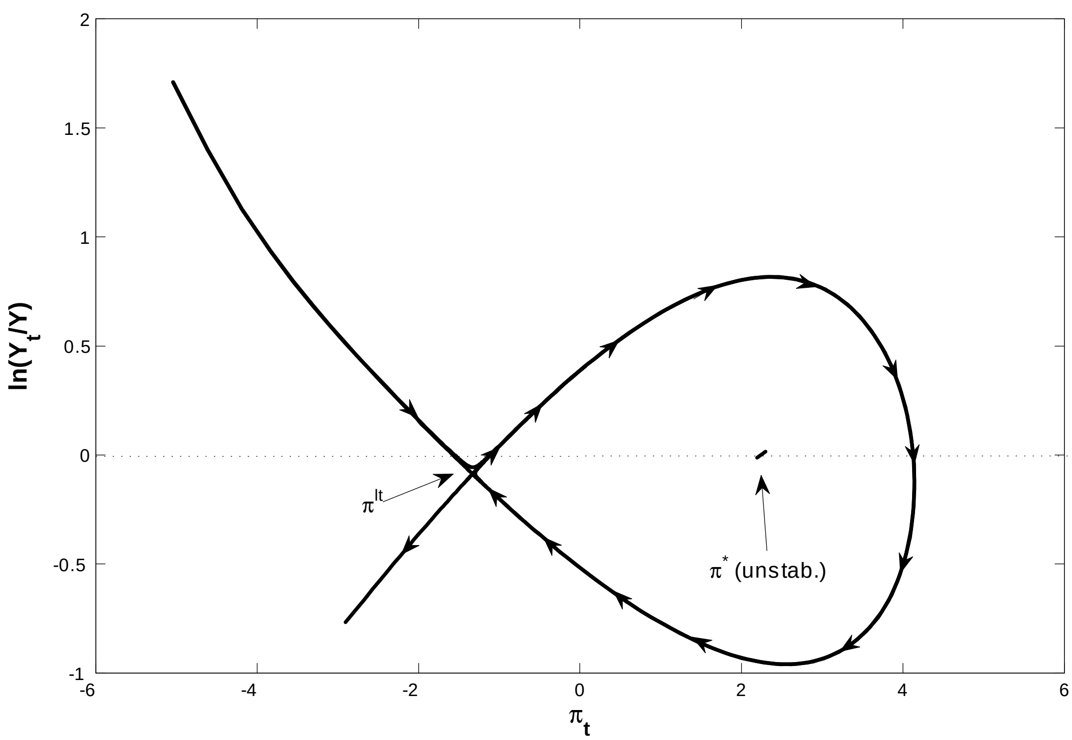

Figure 3: The liquidity trap cycle in the no communication regime.

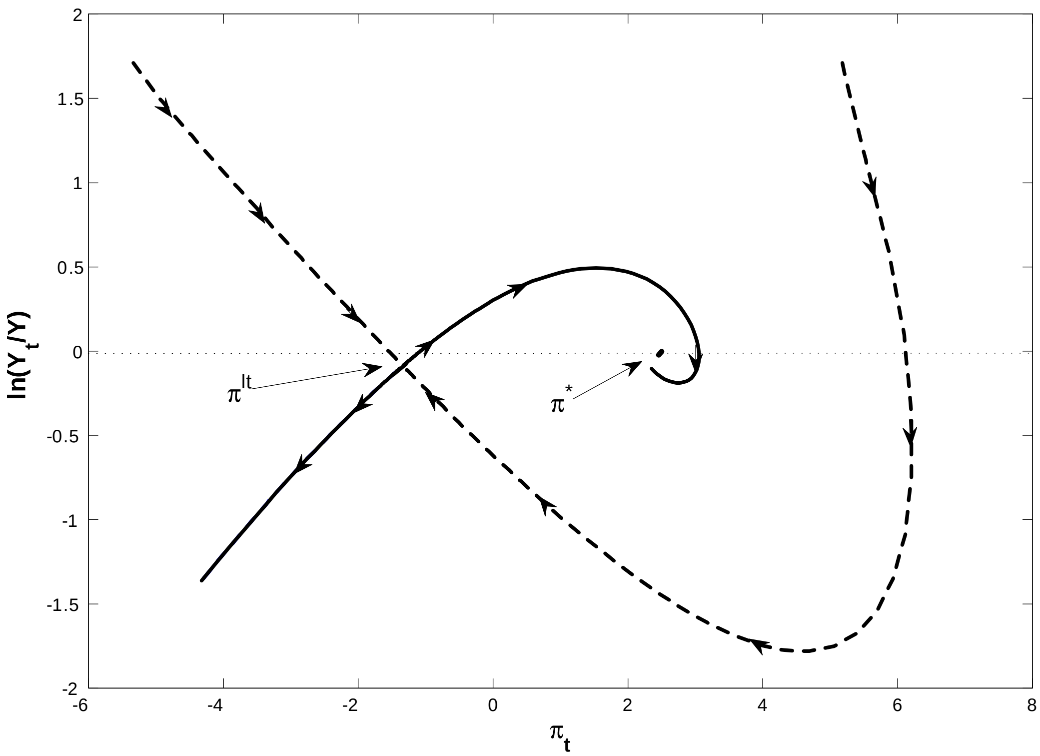

Figure 4: Phase diagram in a regime of communication.

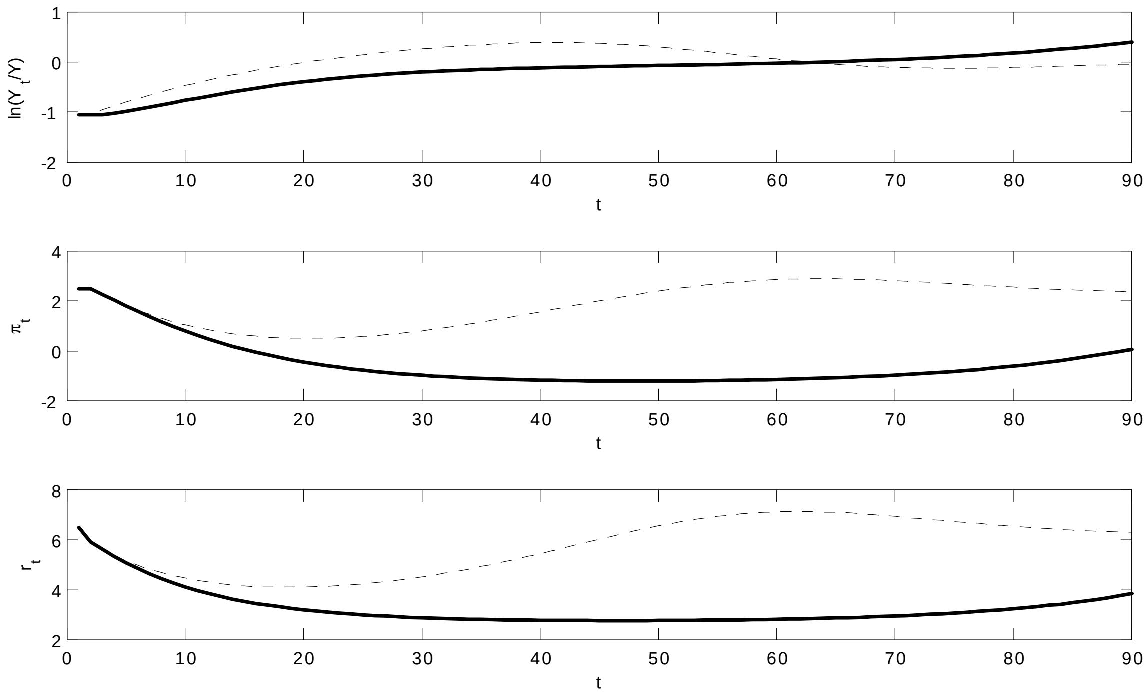

Figure 5: Negative expectations drive the economy to liquidity trap in a regime of no communication (solid line). Communication leads the economy back to the inflation target (dashed line).