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Bank Lending in Times of Large Bank Reserves

Antoine Martin
James McAndrews
David Skeie

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Abstract

The amount of reserves held by the U.S. banking system rose from under $50 billion in mid-2008 to over $1.5 trillion by mid-2011. Some economists argue that such a large quantity of bank reserves could lead to overly expansive bank lending as the economy recovers, regardless of the Federal Reserve's interest rate policy. In contrast, we show that the size of bank reserves has no effect on bank lending in a frictionless model of the current banking system, in which interest is paid on reserves and there are no binding reserve requirements. We also examine the potential for balance sheet cost frictions to distort banks' lending decisions. We find that large reserve balances do not lead to excessive bank credit and may instead be contractionary.

Key words: banking, lending, reserves, interest on reserves, Federal Reserve

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Martin, McAndrews, Skeie: Federal Reserve Bank of New York (e-mail: antoine.martin@ny.frb.org, jamie.mcandrews@ny.frb.org, david.skeie@ny.frb.org). The authors thank Todd Keister for very valuable conversations that contributed to this paper. They also thank Sha Lu and particularly Ali Palida for outstanding research assistance.

1 Introduction

The amount of reserves held by the U.S. banking system rose from under \$50 billion in mid-2008 to over \$1.5 trillion in mid-2011. These results are important because several economists and financial market participants claim that large levels of bank reserves will lead to overly expansive bank lending.[2] Despite such concerns, little formal analysis has been conducted to show such an effect under the current banking system. In contrast, other commentators on the economy claim that the large level of reserves held in the banking system is evidence of a lack of bank lending.

In this note, we present a basic model of the current U.S. banking system, in which interest is paid on bank reserves and there are no binding reserve requirements. We find that, absent any frictions, lending is unaffected by the amount of reserves in the banking system. The key determinant of bank lending is the difference between the return on loans and the opportunity cost of making a loan. We show that this difference does not depend on the quantity of reserves. Moreover, when we introduce frictions, in the form of a cost related to the size of a bank's balance sheet cost, increases in reserves may actually reduce bank lending and lead to a decrease in prices.

The current banking system in the United States and worldwide no longer resembles the traditional textbook model of fractional reserve banking. Historically, the quantity of reserves supplied by a central bank determines the amount of bank loans. Through the "money multiplier," banks expand loans to equal the amount of reserves divided by the reserve requirement. However, in many countries, reserve requirements have been reduced either to zero, or to such small levels that they are no longer binding.[3]

[2]In an interview, Philadelphia Fed President Charles Plosser expressed concern about what would occur "were all those excess reserves to start flowing out into the economy in the form of loans or purchases of other assets," and in a speech Dallas Fed President Richard Fisher also said that "the Fed must be 'wary' of excess reserves sparking an expansion of bank credit," Beckner (2009). Meltzer (2010) expresses similar concerns.

[3]Bennett and Peristiani (2002) show that reserve requirements have been largely avoided in the United States since the 1980s by sweep accounts, and that the remaining reserve requirements are largely met by vault cash that banks hold at branches and ATMs. As of mid-2008, required

Starting in the late 1980s, the Federal Reserve supplied the quantity of reserves needed to maintain its policy target—the federal funds rate—which is the interest rate at which banks lend reserves to each other in the interbank market. The Federal Reserve did not target the amount of reserves, the quantity of deposits or loans on banks' balance sheets, or broad measures of the money supply. In that regime, the federal funds rate represents a bank's alternative return on assets and hence is the required marginal return on bank lending. Banks expand their balance sheets so long as the marginal cost of funding is less than the marginal return on bank lending, abstracting from credit and liquidity risk. The federal funds rate sets the level of the required marginal return.

From 2007 through 2011, the Federal Reserve greatly expanded the scope of its tools to address the financial crisis and severe recession. Bank reserves increased rapidly after the Federal Reserve provided unprecedented unsterilized lending through several facilities after the bankruptcy of Lehman Brothers. Reserves continued to increase as the Federal Reserve purchased roughly $1.75 trillion in Treasury securities, agency mortgage-backed securities, and agency debt. Between September 2008 and mid-2011, bank reserves grew from $30 billion to over $1.5 trillion, as illustrated in Figure 1. To allow the Federal Reserve to continue targeting its policy rate even with large reserves outstanding, Congress accelerated previously granted authority for the Federal Reserve to pay interest on reserves in the Emergency Economic Stabilization Act of 2008. The Federal Reserve began paying interest on reserves on October 9, 2008. Paying interest on reserves allows the Federal Reserve to choose the required return on banks' reserves independently from the quantity of reserves in the banking system.[4]

[Figure 1. Large quantity of reserves in the banking system]

reserves were $71 billion, just 0.6 percent of total bank assets, and vault cash satisfied $43 billion of these requirements.

[4] For details and analysis, see Ennis and Keister (2008), Keister, Martin and McAndrewes (2008) and Keister and McAndrews (2009).

We introduce a new framework in which the role of fiat reserves that pay interest can be studied in a general equilibrium banking economy with a closed system of bank payments and central bank reserves. We include banking, corporate, and retail sectors, which transact in competitive markets for bonds, deposits, loans and goods. We first create a benchmark model that shows without frictions, bank lending quantities and interest rates are invariant to the level of reserves chosen by the central bank. Banks lend up to the point where the marginal return on loans equals the return on holding reserves, which is equal to the interest rate on reserves set by the central bank. This provides an indifference result for the quantity of reserves. In particular, while the size of banks' balance sheets expand with increases in reserves, all else equal, the lending decision for a bank is determined by the same marginal return condition as with the former method of monetary policy implementation. A loan is made at the margin if its return exceeds the marginal opportunity cost of reserves, whether that is the federal funds rate as with the prior regime, or the rate of interest on reserves as in the current regime. We also demonstrate that the quantity of reserves held in the banking system in the absence of binding reserve requirements or significant currency withdrawals is determined in the U.S. solely by the Federal Reserve. Aggregate bank reserves are independent of and provide no measure of the availability of bank credit or banks' willingness to lend.

We also study costs related to the size of a bank's balance sheet to examine whether the level of reserves affects bank lending under this friction. The concern that banks may face balance sheet costs has been raised by market observers.[5] Banks may have costs that are increasing in the size of their balance sheets because of agency costs or regulatory requirements for capital or leverage ratios. During the recent crisis, banks worked to reduce the size of their balance sheets and were slow to raise equity capital, suggesting an increase in balance sheet costs. The analysis shows that, with these increasing costs, large quantities of reserves may, surprisingly,

[5] For example, Wrightson ICAP (2008) expressed the concern that excess reserves could "clog up bank balance sheets," and see also Wrightson ICAP (2009); whereas, Ennis and Wolman (2012) study the distribution of reserves among banks and do not find evidence for such an effect.

have a contractionary effect on bank lending. Large balance sheet costs create a wedge between bank returns paid on deposits from returns received on assets. When returns paid on deposits cannot fall enough in the face of increasing balance sheet costs because of a lower bound, increases in reserves can partially crowd out lending.

The paper proceeds with the model presented in Section 2. Section 3 gives results for the benchmark case with no frictions and the cases with balance sheet costs. Section 4 concludes. Formal statements of each proposition and proofs are contained in the Appendix.

2 Model

We consider a competitive economy with household, firm, and banking sectors, a central bank, and a government. At date 0, the government issues bonds (B) that can be held by households (B^H), banks (B^B), or the central bank (B^{CB}):

$$B = B^H + B^B + B^{CB}. \tag{1}$$

Banks offer deposits (D) to households and provide loans to firms. Households have an endowment (E) that can be held in deposits (D), government bonds (B^H), or storage of goods (S),

$$E = D + B^H + S, \tag{2}$$

where goods have an implicit normalized price level of one at date 0. The central bank purchases bonds with an inelastic demand by issuing reserves (M),

$$B^{CB} = M, \tag{3}$$

and only banks can hold these reserves.

At date 1, firms produce output with a marginal real return $r(L)$ on a volume of loans (L). Firms sell their output to households at the date 1 price level of goods. This price is equal to inflation (Π), i.e. the relative price of goods between dates 0 and 1, because the date 0 price level is normalized to one. We define firms' marginal nominal return on the production and sale of their output as

$$R(L) \equiv \Pi r(L).$$

Note that we use uppercase letters to denote nominal amounts and lowercase letters to denote real amounts throughout the paper.

Firms pay a return (R^L) on loans, banks pay a return (R^D) on deposits, the government pays a return (R^B) on bonds, and the central bank pays a return (R^M) on reserves. The government, central bank, banks, firms, and households are price takers in all markets, which include the markets for bonds, deposits, loans, and goods. For simplicity, we abstract from credit risk, liquidity risk, and risk aversion.[6]

Next, we can write the optimization problems faced by firms, households, and banks. For simplicity, we model each of these sectors as a representative entity. A firm chooses loans, sells output for revenue ($\int_L R(\hat{L})d\hat{L}$), and repays loans at a return (R^L) in order to maximize profit. The firm's problem is

$$\max_L \int_L R(\hat{L})d\hat{L} - R^L L. \tag{4}$$

A household chooses how many deposits and bonds to hold, which after paying a lump sum tax (T) is used to purchase goods. Households keep any remaining endowment in storage, in order to maximize real consumption given as

$$\tfrac{1}{\Pi}(R^D D + R^B B^H - T) + S. \tag{5}$$

Substituting for deposits ($D = E - B^H - S$) from the household's budget constraint,

[6] During the financial crisis up through September 2008, there was less that \$100 billion in reserves in the banking system. At several points, banks appear to have had a demand for reserves for precautionary reasons that may have impacted interest rate spreads for liquidity reasons (see Ashcraft, McAndrews and Skeie, 2011). However, the focus in the current paper is for the time period starting in late 2009 and beyond, when reserves ranged in the several hundreds of billions of dollars. This level was determined by the Federal Reserve supply for the purchase of assets rather than by bank demand. The ample supply of reserves has easily satisfied any potential liquidity demand for reserves. For analysis of banking fragility in related nominal contracting frameworks, see Allen, Carletti and Gale (2011), Diamond and Rajan (2006), Martin (2006), and Skeie (2004, 2008); and for studies of central bank interest rate policy within these frameworks see see Diamond and Rajan (2009) and Freixas, Martin and Skeie (2011).

equation (2), the problem can be written as

$$\max_{B^H,S} \tfrac{1}{\Pi}[R^D(E - B^H - S) + R^B B^H - T] + S. \tag{6}$$

A bank receives deposits and must choose how many loans to finance (L), as well as how many reserves (M) and how many bonds (B^B) to hold, in order to maximize profits. The bank's problem is

$$\max_{L,M,B^B} R^L L + R^M M + R^B B^B - R^D D - \int_D C(\hat{D})d\hat{D}, \tag{7}$$

where $c(D)$ is the marginal real cost associated with the size, D, of the bank's balance sheet, and $C(D)$ is the marginal nominal balance sheet cost defined as

$$C(D) \equiv \Pi c(D).$$

The bank's balance sheet requires that

$$D = L + M + B^B, \tag{8}$$

so we can write

$$\max_{L,M,B^B} R^L L + R^M M + R^B B^B - R^D(L + M + B^B) - \int_{L+M+B^B} C(\hat{D})d\hat{D}. \tag{9}$$

The date 0 budget constraints for households, banks and the central bank, given by equations (2), (8), and (3), respectively, together imply household endowment is divided among loans, storage, and government bonds,

$$E = L + S + B.$$

For a given amount of government bonds, B, maximum lending occurs when there is no storage, which we denote by

$$\bar{L} \equiv E - B.$$

We take as exogenous the government's choice of the quantity of bonds,

$$B = \bar{B},$$

and the central bank's choice of the quantity of reserves and return on reserves,

$$
\begin{aligned}
M &= \bar{M}, \\
R^M &= \bar{R}^M,
\end{aligned}
$$

respectively. The central bank remits its net revenue $(R^B B^{CB} - R^M M)$ to the government, and the government sets the lump sum tax (T) to repay its debt:

$$T = R^B B - (R^B B^{CB} - R^M M). \tag{10}$$

We make the following assumptions on exogenous parameters and functions:

(A1): $r(L) > 1, r^{'}(L) < 0, r^{''}(L) > 0, r(0) = \infty, r(\infty) = 1$

(A2): $0 < \bar{M} < \bar{B} < E$

(A3) $c(D) \geq 0$, $c(0) = 0$, $c^{'}(D) \geq 0$, $c^{'}(0) > 0$ if $c(D) > 0$, $c(\bar{M}) < \infty$

Assumption (A1) states that the firm's technology is more productive than storage, along with standard Inada conditions. Assumption (A2) considers, for simplicity, monetary and fiscal policy parameters that are within the feasible limit of the economy. Assumption (A3) states that when balance sheet costs are positive, these costs are increasing in the size of the balance sheet.

Letting $\mathbb{R} = (\Pi, R^M, R^L, R^D, R^B)$ and $\mathbb{Q} = (S, M, L, D, B^{CB}, B^H, B^B)$, we define an equilibrium as prices $\mathbb{R} > 0$ and quantities $\mathbb{Q} > 0$ such that markets clear at $\mathbb{Q}$ given individual optimizations at $\mathbb{R}$.

In an interior solution, the first-order conditions for the firm, household, and bank would be:

$$
\begin{aligned}
L[R^L - R(L)] &= 0, && (11) \\
B^r(R^B - R^D) &= 0, && (12) \\
S(\Pi - R^D) &= 0, && (13) \\
L\left[R^L - R^D - C(D)\right] &= 0, && (14) \\
M\left[R^M - R^D - C(D)\right] &= 0, && (15) \\
B^B\left[R^B - R^D - C(D)\right] &= 0. && (16)
\end{aligned}
$$

Since households can invest in both government bonds and deposits, they must have the same return for any interior solution. In such cases, we write $R^D = R^B$. Since M and L are strictly positive, firms borrow loans to the point that their first order condition binds, $R(L) = R^L$. The marginal loan financed by banks has a return equal to the return paid on reserves, $R^L = R^M$.

3 Results

3.1 Benchmark case

We first consider the benchmark case with no balance sheet costs, $c(D) = 0$. The return on the marginal loan, $r(L)$, and hence the quantity of loans financed, L, is independent of the quantity of reserves, M. This provides our first basic result.

Proposition 1 *In the benchmark case with no balance sheet costs, there exists an equilibrium which is unique up to the allocation of bonds between households and firms. The quantity (L) and marginal return (R^L) of bank lending are independent of the quantity of reserves (M) issued by the central bank. Market returns (R^M, R^L, R^D, R^B) are equal to the return on reserves set by the central bank ($\bar{R}^M$) and are greater than inflation (Π).*

The marginal rate of return on loans is endogenous, depending on the real marginal return of the production function of the firm, the endogenous amount of loans, and the endogenous price at which the firm sells goods, giving an endogenous nominal marginal rate of return on loans the firm pays to the bank.

[Figure 2. Benchmark model with minimal reserves and no balance sheet costs]

Figures 2 and 3 illustrate the effects of reserves on the equilibrium in the benchmark case with no balance sheet costs. Panel A in each figure shows the available bonds, $\bar{B}$, for sale in the government bond market to the central bank, households and banks. The central bank determines the quantity of reserves by choosing the amount it supplies, M_0, to buy government bonds, B_0^H, represented by the perfectly elastic demand curve for bonds, $B_0^{CH,D}$. Households purchase the remainder of the supply of bonds not bought by the central bank. Panel B shows that in the deposit market, the remainder of the households' endowment, E, is held in deposits, D_0, implying a perfectly inelastic supply of deposits by households, D_0^S.

[Figure 3. Moderate level of reserves and no balance sheet costs]

Because households' quantity of bonds decrease with the central bank's holding of bonds, households' deposits increase with the level of reserves issued by the central bank. Banks have a perfectly elastic demand curve for deposits because any additional deposit gives the bank an additional reserve asset, which pays R^M. Panel C shows that in the loan market, firms' loan demand, L_0^D, determined by (11), is decreasing in the loan rate, R^L, and reflects that $r(L)$ is decreasing in L. Loan supply, L_0^S, is perfectly elastic at the return on reserves, $\bar{R}^M$, which is the banks' opportunity cost for holding loans.

The quantity of banks' loans remains unchanged since all equilibrium rates remain constant, highlighted by the result of $R(L) = R^L = R^M$. This is an especially robust relationship that holds throughout the paper, even when frictions are added in later sections. This expression can be rearranged to show how inflation is determined: $\Pi = \frac{R^M}{r(L)}$. The inflation rate is positive, $\Pi - 1 > 0$, if the central bank's policy rate, $R^M - 1$, is greater than the equilibrium real rate $r(L) - 1$. This is a restatement of the Fisher equation that nominal returns equal real returns multiplied by the gross rate of inflation: $R^M = r(L)\Pi$.

In Figure 2, with minimal reserves, equilibrium loans are equal to nearly the full quantity of deposits. In Figure 3, with a moderate level of reserves, loans no longer comprise the near entirety of banks' assets. Instead, the size of banks' balance sheets increase to fund both their loans to firms and the reserves issued by the central bank.

Figure 3 also demonstrates that the quantity of reserves held in the banking system is determined solely by the central bank's level of bond purchases. The level of bank reserves are independent of and unaffected by banks' supply of loans to firms. This states that whether bank reserves are high or low gives no indication of the amount of bank lending that is occurring. Equivalently, the amount of bank lending has no implication of the quantity of bank reserves held by banks.

3.2 Balance-sheet costs

Next, we consider the case of positive bank balance sheet costs. This is an important and natural friction to consider since market participants raised concern that banks' balance sheets may be too large (Wrightson ICAP, 2008 and 2009). Bank balance sheet costs may incorporate the costs of capital requirements and the shadow cost of potentially binding capital ratios.[7]

If $c(D) > 0$, then banks will reduce the size of their balance sheets by not holding bonds, $B^B = 0$. Households are at a corner solution, since they hold all the government bonds not held by the central bank. A positive balance sheet cost for banks $c(D) > 0$ does not necessarily affect the number of loans financed by banks, and $R^L = R^M$ still holds for moderate balance sheet costs and reserve quantities. Instead, banks reduce the return on deposits: $R^D = R^M - C(D) < R^M = R(L)$. The banks' return on the marginal loan, R^L, is not equal to banks' marginal funding costs, R^D, but rather is equal to the return on alternative assets, R^M, that banks can invest in: namely, reserves.

Proposition 2 *For moderate balance sheet costs, $c(D)$, and reserve levels ($\bar{M}$), the marginal return (R^L) of lending by banks equals the return on reserves ($\bar{R}^M$).*

[7] We do not explicitly model bank capital, which is implicitly incorporated in the bank balance sheet liabilities (D). As such, bank capital, which may need to be raised during times of distress to support continued or increased bank balance sheet size because of bank capital and leverage ratio requirements, may be an important part of bank balance sheet costs, $c(D)$. Carlson, Shan and Warusawitharana (2011) argue that higher capital ratios may support greater loan growth, particularly in times of distress, as they show evidence for during the recent financial crisis. The reluctance of many banks to raise capital during the crisis indicates that capital may be particularly costly to raise in times of distress.

These returns are greater than the return on deposits and bonds, which are equated ($R^D = R^B$), and which in turn remain above inflation (Π). The amount of bank sector lending (L) is independent of the amount of reserves ($\bar{M}$).

The households' supply of deposits and banks' demand for deposits is endogenous, and hence the size of the banks' balance sheet is endogenous. The government bond rate and deposit rate are both determined in equilibrium according to the households first order conditions. Because the households always hold bonds and deposits in equilibrium, the two rates must be equal to make the household indifferent in holding the two assets. When there are positive balance sheet costs, the deposit rate falls below the bank's return on its assets (the interest on reserves rate which equals the loan rate) in order for the bank to be willing to hold a marginal deposit and a marginal asset. Thus the government bond rate falls below the banks' other asset rates, and banks prefer then to hold reserves and loans but zero bonds.

[Figure 4. Moderate level of reserves and moderate balance sheet costs]

The invariance result of moderate balance sheet costs and reserves on bank lending is illustrated in Figure 4. The equilibrium returns on deposits and government bonds are equal and below the return on reserves: $R^D = R^B < \bar{R}^M$. The decrease in the return on deposits absorbs the balance sheet cost. Bank do not incur the balance sheet cost in their borrowing rates and do not pass the cost on through higher lending rates. Households receive the surplus from the banking sector at the margin. Households are willing to absorb the balance sheet costs as long as they receive a marginal real return by depositing at the bank, $\frac{R^D}{\Pi}$, that is greater than the return on storage of one. In contrast, banks are operating at a competitive zero profit condition and are not willing to absorb losses at the margin. Households' demand for bonds becomes downward sloping along the region corresponding to positive balance sheet costs as increasing deposits imply a decreasing deposit return offered by banks, which also decreases households' reservation rate for bonds. The bond return equates with the deposit return in equilibrium. However, as long as the return on deposits remains above inflation, the real return on deposits is greater than the return on storage, which is one. Households will continue to have an inelastic supply of deposits equal to their endowment that is not held in bonds. The quantity of bank lending is unchanged from the benchmark case of zero balance sheet costs.

Finally, for large enough reserves and balance sheet costs, the deposit return falls to such a low level that it cannot fully absorb the costs. This occurs when the deposit return, as given by $R^D = R(L) - C(D)$, falls to the lower bound given by the households' option to store goods instead of hold deposits. At this point, according to the households' first order condition (13), the real return on deposits is equal to that of storage, $\frac{R^D}{\Pi} = 1$. Together, these constraints imply that bank lending will be held down to a level such that the net marginal real rate of return on lending equals the marginal real bank balance sheet cost:

$$r(L) - 1 = c(D). \tag{17}$$

Lending in the economy can increase to the point that marginal real productivity of loans above the opportunity cost of storage equals the marginal real banking cost of intermediating loans. For large enough reserves M and balance sheet costs $c(D)$, where $D = L + M$, such that (17) holds, reserves partially crowd out bank lending.

Proposition 3 *For a large enough level of reserves (M) and balance sheet costs, $c(D)$, the return on deposits (R^D) and bonds (R^B) decrease to equal inflation (Π). Bank lending (L) and inflation (Π) are decreasing in the quantity of reserves (M).*

The volume of loans is always determined according to $R(L) = R$. Regardless of how high the balance sheet costs are, the bank is always indifferent between holding marginally more loans or reserves, and so the returns are equal. The bank chooses its optimal amount of reserves according to a demand curve for reserves. The central bank chooses a quantity of reserves to supply, which is a point on the bank's demand curve and hence satisfies the bank's optimal demand. We have endogenized the lower bound on deposit rates by including the household's option to store goods. This implies that in equilibrium the real rate of return on deposits, $\frac{R^D}{\Pi} - 1$, cannot fall below zero (or equivalently that a real return on deposits of $\frac{R^D}{\Pi}$ cannot fall below one. Without the availability of storage, lending would not be affected.

[Figure 5. Large level of reserves and large balance sheet costs]

Bank loans are equal to deposits held in excess of reserves, $L = D - M$. As shown in Figure 5, when bank balance sheet costs and reserves are large enough that R^D is at the lower bound, at the margin a unit of reserves increases balance sheet costs by $c(D)$. Constraint (17) requires a corresponding increase in the real return on loans and hence a reduction in lending. This decrease is held in storage by households. A marginal quantity of reserves is absorbed by a partial decrease in lending and a partial increase in the size of bank balance sheets.

The rate of return on deposits, $R^D - 1$, and on government bonds, $R^B - 1$, both decrease to equal the inflation rate, $\Pi - 1$, which gives a real return on deposits of $\frac{R^D}{\Pi} = 1$. The return on deposits is below the return on reserves by the balance sheet cost wedge $1 + c(D)$: $R^D = \frac{R^M}{1+c(D)} < R^M$. The rate of return on deposits cannot fall below the inflation rate because that would imply a real return on deposits below one: $\frac{R^D}{\Pi} < 1$. Households would prefer only to store goods.

In this crowding out case, reserves have a further deflationary effect by lowering inflation: $\Pi = \frac{R^M}{1+c(D)}$ and $\frac{\partial \Pi}{\partial M} < 0$. Inflation is lower but may still be positive, in which case deposit rates remain positive: $R^D - 1 = \Pi - 1 > 0$. However, a stronger result may also occur, in which inflation rates $\Pi - 1$ fall below zero, resulting in an actual deflation. Nominal deposit and bond rates would be negative, while real rates $\frac{R^D}{\Pi} - 1$ would remain equal to zero.

3.3 Discussion

We can discuss several potential extensions that lie outside the formal model, including the effect of macroeconomic shocks on bank lending; bank heterogeneity; and historical regimes for reserves.

To start, we examine how shifts in parameters can effect bank lending. First, we consider an increase in loan demand driven by a productivity shock. We compare the effect of an increase in the marginal return on firms' investment up to $\tilde{r}(\cdot) > r(\cdot)$ when there are minimal versus large reserves and balance sheet costs. With minimal reserves, an increase in productivity leads to a decrease in inflation since $R(\cdot) = \Pi r(\cdot) = R^M$. The marginal nominal return of bank lending is unchanged and there is no change in lending.

With large reserves and balance sheet costs, an increase in real productivity to $\tilde{r}(\cdot) > r(\cdot)$, for a given level of loans L, increases the left-hand side of equation (17). There is an increase in equilibrium loans to $\tilde{L}$, which moderates the equilibrium increase in productivity to $\tilde{r}(\tilde{L})$. There is an increase in deposits, $\tilde{D} - D = \tilde{L} - L$, and in bank balance sheet costs $c(\tilde{D})$, to the point that equation (17) holds: $\tilde{r}(\tilde{L}) - 1 = c(\tilde{D})$. The increase in loans are supported by a decrease in household storage of $S - \tilde{S} = \tilde{L} - L$. This shows that an increase in loan demand driven by a positive real productivity shock leads to an increase in bank lending. However, the increase in the equilibrium marginal return on loans to $\tilde{r}(\tilde{L})$, is complemented by a decrease in inflation to $\tilde{\Pi}$, because firms' nominal return on loans, $\Pi\tilde{r}(\tilde{L})$, is tied to the loan rate, R^L, and interest on reserves, R^M: $\Pi\tilde{r}(\tilde{L}) = R^L = R^M$. Again, we find overall that $\tilde{R}(\tilde{L}) = R^L$; the marginal nominal return on lending is unchanged.

Next, we consider an increase in loan demand that is driven by an increase in households' demand. We examine the effect of an increase in household endowment up to $\tilde{E} > E$, when there is a low or a moderate size of reserves and balance sheet costs. The increase in endowment leads to an increase in households' demand for deposits and an increase in inflation, which shifts out firms' demand for loans, lowering the firms' real return on investment. The increase in equilibrium deposits, loans, and inflation is given by $(\tilde{D} - D) = (\tilde{L} - L) = (\tilde{E} - E)$ and $(\tilde{\Pi} - \Pi) = (\frac{R^L}{r(\tilde{L})} - \frac{R^L}{r(L)})$.

The model allows for an instantaneous adjustment of deposits and loans regardless of the level of bank reserves. However, a lower velocity of money is required for a banking system with a higher level of reserves than one with a lower level of reserves. The banking sector lends the quantity of reserves it holds, M, to firms that buy goods from households, who deposit the reserves in the banking system. The reserves have to turnover $\frac{\tilde{L}-L}{M}$ times for an increase in deposits up to $\tilde{D}$ and in loans up to $\tilde{L}$. In practice, outside of the model, if there is heterogeneity among banks, it may take some time or cost for the adjustment process of banks that have sudden increased lending opportunities to attract deposits or interbank loans. A higher quantity of reserves requires a lower velocity of money and may lead to a slightly faster increase in lending in response to a sudden increase in loan demand. Hence, the level of reserves could affect the speed in which equilibrium levels of lending would adjust to shocks in the economy. For either driver of increased loan demand above, we see that faster adjustment cost speeds that may result from larger reserve levels produce more efficient outcomes.

These adjustment effects may also provide insight into the consideration of the extreme heterogeneity of the banking sector in the U.S. We model a representative bank that makes a representative type of loan to firms. In reality, banks in the U.S. vary tremendously in many features including bank size, sources of deposits, and focus of lending (for instance, see Janicki and Prescott, 2006). For example, banks provide commercial and industrial loans, real estate loans, and consumer loans. While aggregate reserves in the banking system are fixed by the Federal Reserve, the distribution of reserves among banks is not fixed and may depend on bank size, deposit sources, and lending focus. Outside of the model, we can consider that such variation among banks may lead to different speeds of adjustment to changes in bank borrowing and lending. For example, banks that have greater access to wholesale deposits can increase or decrease borrowing and hence lending faster than banks that rely more on retail deposits. However, we do not expect that variation among bank types or the speed of adjustments of bank borrowing and lending would lead to a significant change in our equilibrium results.

We can also use the model to compare the current regime of interest on reserves with past regimes. Historically, central banks used a reserve requirement ratio in order to create a demand for reserves that were not paid interest and to control the amount of bank loans through the money multiplier. For a reserve requirement ratio of ρ, the money multiplier is $\frac{1}{\rho}$. With a supply of reserves (M) as chosen by the central bank, and under a binding money multiplier constraint, the banking sector could hold a maximum amount of deposits equal to $D = \frac{M}{\rho}$ and provide a maximum amount of loans equal to $L = \left(\frac{1-\rho}{\rho}\right) M$. Over time, most central banks have either eliminated reserve requirements entirely or have allowed banks to largely avoid it, such as through sweep accounts in the U.S. Our model of bank lending, with interest on reserves and no meaningful reserve requirement, shows that the money multiplier is no longer relevant. Banks take deposits and lend to the point that the marginal return on loans $R(L)$ equals the return R^M paid by the central bank on reserves, the banks' alternative asset.

In past regimes that did not pay interest on reserves, reserve requirements were considered to impose a "tax" on banks. This tax is the return that banks had to forego by holding required reserves that paid no return, equal to $\rho D R(L)$. In comparison, under a policy of interest on reserves, banks no longer face the tax on required reserves. However, with a large quantity of reserves in the banking system, banks face the potential additional balance sheet costs from large levels of reserves, equal to $\int_D C(\hat{D}) d\hat{D} - \int_L C(\hat{L}) d\hat{L}$. Relative to the implicit tax on the modest level of required reserves that did not receive interest in past regimes, the balance sheet costs from reserves that are paid interest in the current regime would be smaller in times of low levels of reserves but would likely be much greater in times of large levels of reserves.

4 Conclusion

Perhaps because of its novelty, the large quantity of reserves in the banking system has generated a great amount of concern and debate. However, there is little analysis of how reserves impact bank lending when interest is paid on reserves. This paper presents a model of the current U.S. banking system that includes interest on reserves and no binding reserve requirements. The exercise is important because of expressed concerns that large reserves could lead to excessive lending by banks, despite little formal analysis of the issue.

We develop a complete yet parsimonious framework by fully specifying a general equilibrium economy with several competitive sectors and a closed system of reserves and payments within the banking system. We study households' supply of deposits, demand for bonds, and consumption goods; firms' demand for loans and supply of consumption goods; and banks' supply of loans and demand for deposits, bonds, and reserves. While we consider a representative competitive price-taking bank, it would be interesting in future research to eventually consider banks that are not fully price-taking, such as banks that may have some monopoly power on deposits and loans.

We show that without frictions, the amount of lending is independent of the amount of reserves in the banking system. We also demonstrate that the quantity of reserves is determined by the Federal Reserve and does not provide any measure of the willingness of banks to lend. We have kept our model simple and elementary in order to illustrate that the key determinant of bank lending is not fundamentally affected by the quantity of reserves. This point has been obscured by the traditional textbook model of the money multiplier, which, while simple, is not an elementary model. Rather, that model assumes that a particular constraint—namely, the money multiplier—is always binding.

Our conclusion is likely to hold in more sophisticated models. While we cannot exclude the possibility that a more complicated model would overturn this result, economists concerned that large reserves will generate excessive lending should articulate precisely which frictions in a banking model will necessarily lead to this result. In contrast to such concerns, we study a friction under which the quantity of reserves could crowd out bank lending and lead to a decrease in inflation. Banks may have increasing costs in the size of their balance sheets because of agency costs or regulatory requirements on capital or leverage. Under such a friction, the effect of large reserves is contractionary rather than expansionary.

Appendix: Proofs

Proof of Proposition 1. We will show that if $c(D) = 0$, then there exists an equilibrium $(\mathbb{Q}, \mathbb{R})$ where $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$, which is unique up to the allocation of bonds between households and firms.

In any equilibrium, we must have (1), (2), and (3), and the central bank choice of reserves and their return requires that $R^M = \bar{R}^M$ and $M = \bar{M}$. We first show that there does exist an equilibrium with $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$. Consider $R^M = R^L = R^D = R^B = \bar{R}^M$. We have banks indifferent between holding bonds, reserves, and loans, and households indifferent between holding deposits and bonds. Consider Π such that $r(\bar{L}) = R^L/\Pi$. By (A1) such a Π exists and $\Pi < R^L$. Now consider $D = \bar{L} + \bar{M}$, $B^B = 0$, $B^{CB} = \bar{M}$, $B^H = \bar{B} - \bar{M}$, $L = \bar{L}$ and $S = 0$. Clearly these quantities satisfy individual optimizations at the given prices, are non-negative given (A2), and clear the market. Thus, this is an equilibrium at $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$.

To show uniqueness we argue that $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$ must hold in any equilibrium, and that $L = \bar{L}$ and $S = 0$ in any equilibrium. This will imply that all equilibria are unique up to the allocation of bonds between households and firms since in equilibrium $M = \bar{M}$. Since $r(L) > 1$, we must have $R^L > \Pi$ in any equilibrium, otherwise firms first order conditions could never be satisfied. (A1) also requires that $L > 0$, which in turn implies that $R^M = R^L \geq R^B$ since $\bar{M} > 0$. Also, we must have $R^M = R^L = R^D$, for inequality would imply that banks would demand either zero or infinite quantities of deposits. Market clearing in the bond market then requires that $R^M = R^L = R^D = R^B$. Since we always must have $R^M = \bar{R}^M$, we have that $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$ in any equilibrium. Now $R^D = R^B > \Pi$ directly implies that $S = 0$, which in turn implies that $L = \bar{L}$ since households must expend their entire endowment. In sum, we have that any potential equilibrium must have $R^M = R^L = R^D = R^B = \bar{R}^M > \Pi$, $L = \bar{L}$ and $S = 0$. Thus, the equilibrium is unique up to the allocation of bonds between households and firms.

Proof of Proposition 2. We will show that if $c(D) > 0$ and $c(\bar{M} + \bar{L}) \leq r(\bar{L}) - 1$, then there exists a unique equilibrium $(\mathbb{Q}, \mathbb{R})$ where $L = \bar{L}$ and $R^M = R^L = \bar{R}^M > R^D = R^B \geq \Pi$.

Because of (A3), (11), and (14) we must have $R^L > R^D$. (A1) requires $L > 0$ and (A2) requires $M > 0$ in equilibrium, thus we must have $R^M = \bar{R}^M = R^L$. Once again, market clearing in the bond market then requires that $R^D = R^B \geq \Pi$. In sum, we have that $R^M = \bar{R}^M = R^L > R^D = R^B \geq \Pi$. Now, we show that there is an equilibrium with $L = \bar{L}$. We first find an R^D and Π such that $R^D = R^L - \Pi c(\bar{M} + \bar{L})$ and $\Pi = R^L/r(\bar{L})$; i.e., consumption of $\bar{L}$ must be optimal for both banks and firms. As (A3) guarantees that $\bar{L} > 0$, we have that $\Pi > 0$. Thus, $R^D = R^L - (R^L/r(\bar{L}))(c(\bar{M}+\bar{L})) < R^L$. Furthermore, $c(\bar{M}+\bar{L}) \leq r(\bar{L}) - 1$ implies that $R^D \geq R^L/r(\bar{L}) = \Pi$. Finally, setting $R^D = R^B$, we have an equilibrium where $\mathbb{Q} = (\bar{M}, \bar{L}, \bar{L} + \bar{M}, \bar{M}, B - \bar{M}, 0)$. To see that this is unique, consider a potential equilibrium loan quantity $L' \neq \bar{L}$. Clearly, $L' < \bar{L}$, but this implies that $S > 0$, since $R^M = R^L = \bar{R}^M > R^B$ implies that banks will not hold bonds and households need to expend all of their endowment. $S > 0$ implies that $R^D = R^B = \Pi$. However, if $c(\bar{M} + \bar{L}) \leq r(\bar{L}) - 1$, then $c(\bar{M} + L') < r(L') - 1$, which implies that $R^D > \Pi$ for L' to be optimal loan consumption for both banks and firms. Thus, L'cannot be an equilibrium and any potential equilibrium must have $L = \bar{L}$. Clearly, if $L = \bar{L}$ in equilibrium, then the only quantity vector that would clear the market is $\mathbb{Q}$. Thus, the equilibrium quantity vector is unique.

Proof of Proposition 3. We will show that if $c(\bar{M} + \bar{L}) > r(\bar{L}) - 1$, then there exists a unique equilibrium $(\mathbb{Q}, \mathbb{R})$, where $L < \bar{L}$, $R^M = R^L = \bar{R}^M > R^D = R^B = \Pi$, $\frac{\delta L}{\delta M} < 0$, and $\frac{\delta \Pi}{\delta M} < 0$.

Consider L such that $c(\bar{M} + L) = r(L) - 1$. Such an L exists and is greater than zero by (A3). Since $c(\bar{M}+\bar{L}) > r(\bar{L})-1$, $L < \bar{L}$. This L is optimally demanded by by both banks and firms when $R^D = \Pi$. Since $R^D = R^L - (R^L/r(L))(c(\bar{M} + L)) < R^L$ for $L > 0$, we must have $R^M = \bar{R}^M = R^L > R^D = R^B = \Pi$. Now consider $\mathbb{Q} = (\bar{M}, L, L + \bar{M}, \bar{M}, \bar{B} - \bar{M}, E - (\bar{B} - \bar{M}) - L)$. $\mathbb{Q}$ obviously clears the market at $R^M = \bar{R}^M = R^L > R^D = R^B = \Pi$. To see that $\mathbb{Q}$ is a unique quantity vector, it suffices to show that L is the only potential equilibrium loan quantity, for then market clearing would imply all other quantities would have to equate with $\mathbb{Q}$. Consider some $L^{'} \neq L$. $L^{'} > L$ would imply that $\Pi > R^D$ for $L^{'}$ to be optimal for both banks and firms, so $L^{'} > L$ cannot be an equilibrium. $L^{'} < L$ implies that $\Pi < R^D$ for $L^{'}$ to be optimal for both firms and banks. But this would imply that $S = 0$, and $L^{'}$ would not clear the market since $L^{'} < L < \bar{L}$. So $L^{'}$ cannot be an equilibrium loan quantity, and the only potential equilibrium loan quantity is L. Through implicit differentiation, we have $\frac{\delta L}{\delta M} = c^{'}(D)/(r^{'}(L) - c^{'}(D)) < 0$ by (A1). Similarly we have $\frac{\delta \Pi}{\delta M} = [-R^L(c^{'}(D))(1 + \frac{\delta L}{\delta M})]/(1 + c(D))^2$. Clearly $\left|\frac{\delta L}{\delta M}\right| < 1$, so we have that $\frac{\delta \Pi}{\delta M} < 0$.

References

[1] Allen, Franklin, Elena Carletti and Douglas Gale (2011). "Money, Financial Stability and Efficiency," working paper.

[2] Ashcraft, Adam, James McAndrews and David Skeie (2010). "Precautionary Reserves and the Interbank Market," *Journal of Money, Credit and Banking* 43(7).

[3] Beckner, Steven K. (2009). "Federal Reserve State of Play," imarketnews.com November 12, 2009. http://imarketnews.com/node/4617.

[4] Bennett, Paul and Stavros Peristiani (2002). "Are U.S. Reserve Requirements Still Binding?" Federal Reserve Bank of New York *Economic Policy Review* 8(1).

[5] Carlson, Mark, Hui Shan, and Missaka Warusawitharana (2011). "Capital Ratios and Bank Lending: A Matched Bank Approach," Finance and Economics Discussion Series No. 2011-34, Federal Reserve Board.

[6] Diamond, Douglas W. and Raghuram G. Rajan (2006). "Money in a Theory of Banking," *American Economic Review* 96.

[7] Diamond, Douglas W. and Raghuram G. Rajan (2009). "Illiquidity and Interest Rate Policy," NBER Working Paper No. 15197.

[8] Ennis, Huberto M. and Todd Keister (2008). "Understanding Monetary Policy Implementation," Federal Reserve Bank of Richmond *Economic Quarterly* 235-263.

[9] Ennis, Huberto M. and Alexander L. Wolman (2012). "Large Excess Reserves in the U.S.: A View from the Cross-Section of Banks," Federal Reserve Bank of Richmond Working Paper No. 12-05.

[10] Freixas, Xavier, Antoine Martin and David Skeie (2011). "Bank Liquidity, Interbank Markets and Monetary Policy," *Review of Financial Studies* 24(8).

[11] Janicki, Hubert P. and Edward Simpson Prescott (2006). "Changes in the Size Distribution of U.S. Banks: 1960–2005," Federal Reserve Bank of Richmond *Economic Quarterly* 92(4).

[12] Keister, Todd, Antoine Martin and James McAndrews (2008). "Divorcing Money from Monetary Policy," Federal Reserve Bank of New York *Economic Policy Review* 14(2).

[13] Keister, Todd and James McAndrews (2009). "Why Are Banks Holding So Many Excess Reserves?" *Current Issues in Economics and Finance*, Federal Reserve Bank of New York.

[14] Martin, Antoine (2006). "Liquidity Provision vs. Deposit Insurance: Preventing Bank Panics Without Moral Hazard," *Economic Theory* 28.

[15] Meltzer, Allan H. (2010). "The Fed's Anti-Inflation Exit Strategy Will Fail." *Wall Street Journal.* January 27, 2010. http://online.wsj.com/article/SB10001424052748704375604575023632319560448.html.

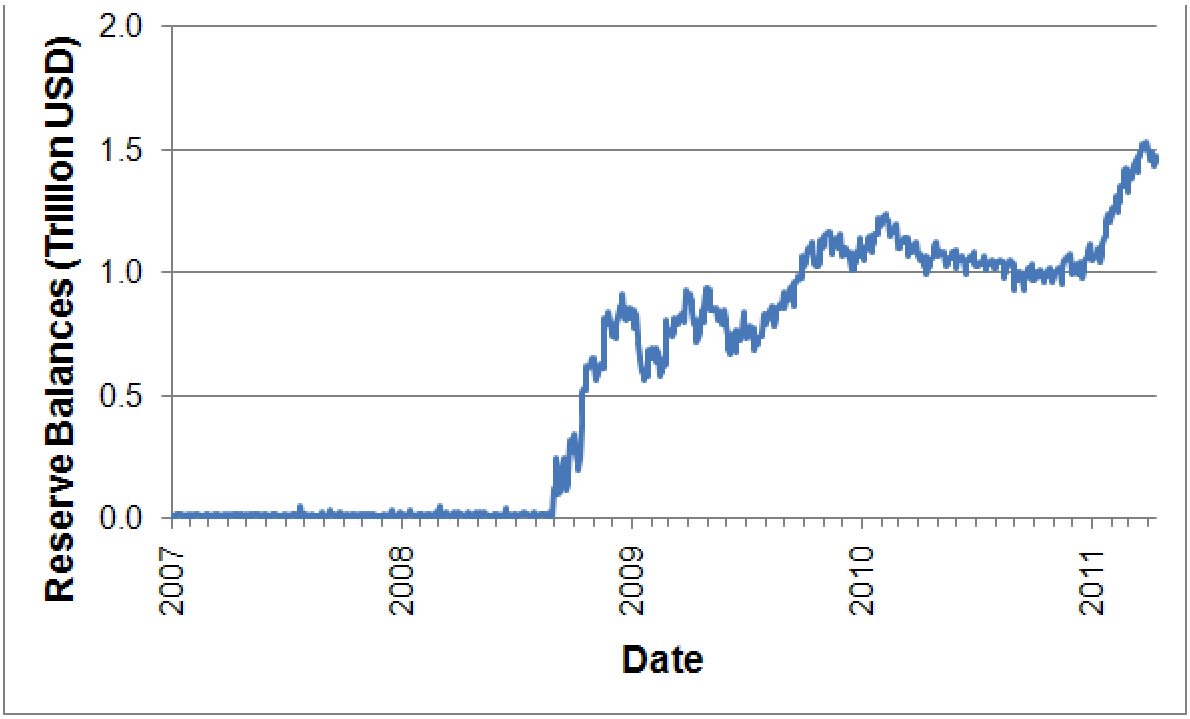
[16] Skeie, David (2004). "Money and Modern Bank Runs," dissertation, Princeton.

[17] Skeie, David (2008). "Banking with Nominal Deposits and Inside Money," *Journal of Financial Intermediation* 17(4).

[18] Wrightson ICAP (2008). *Money Market Observer,* October 6.

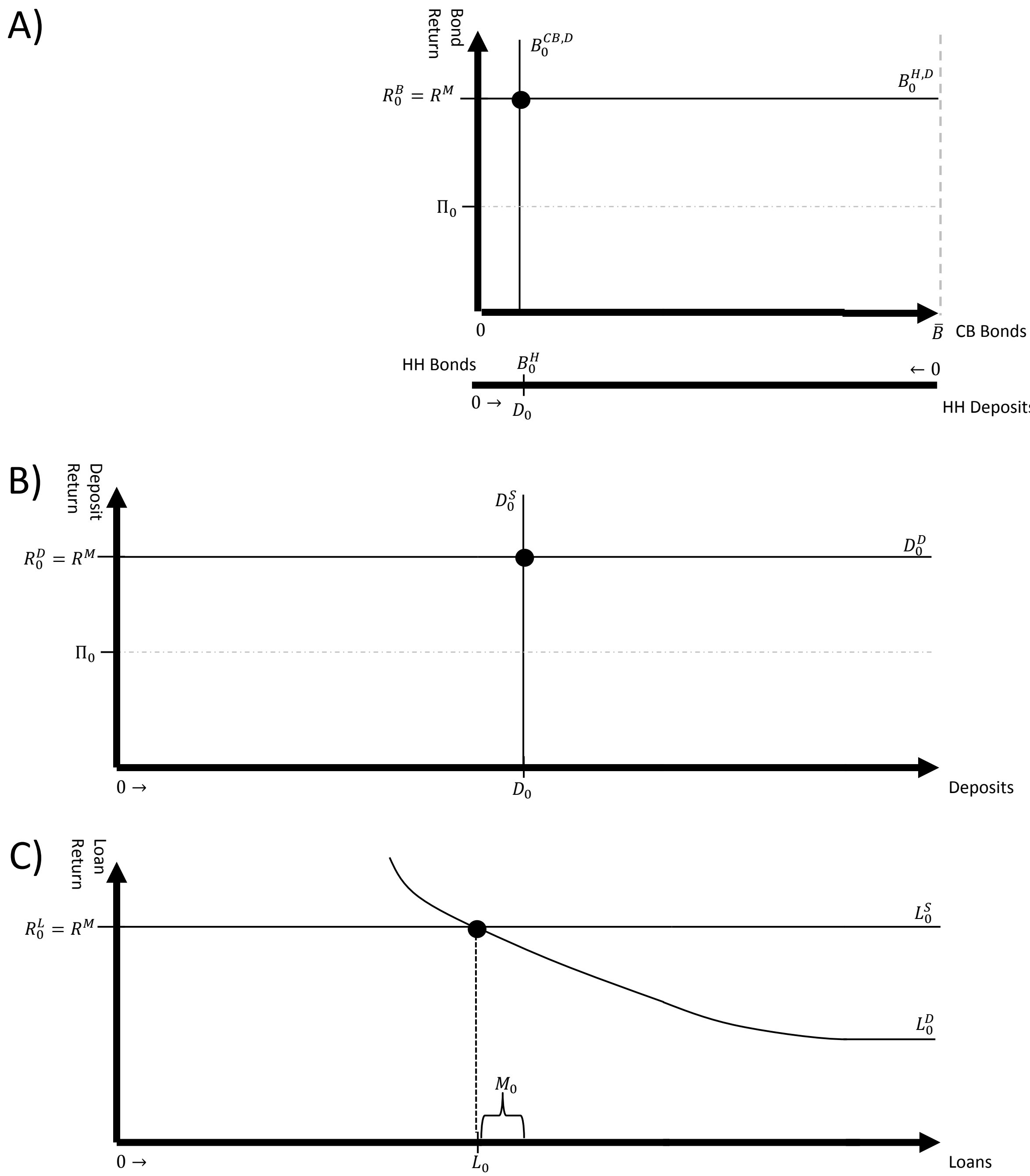
[19] Wrightson ICAP (2009). *Money Market Observer,* January 12.

Figure 1. Large quantity of reserves in the banking system



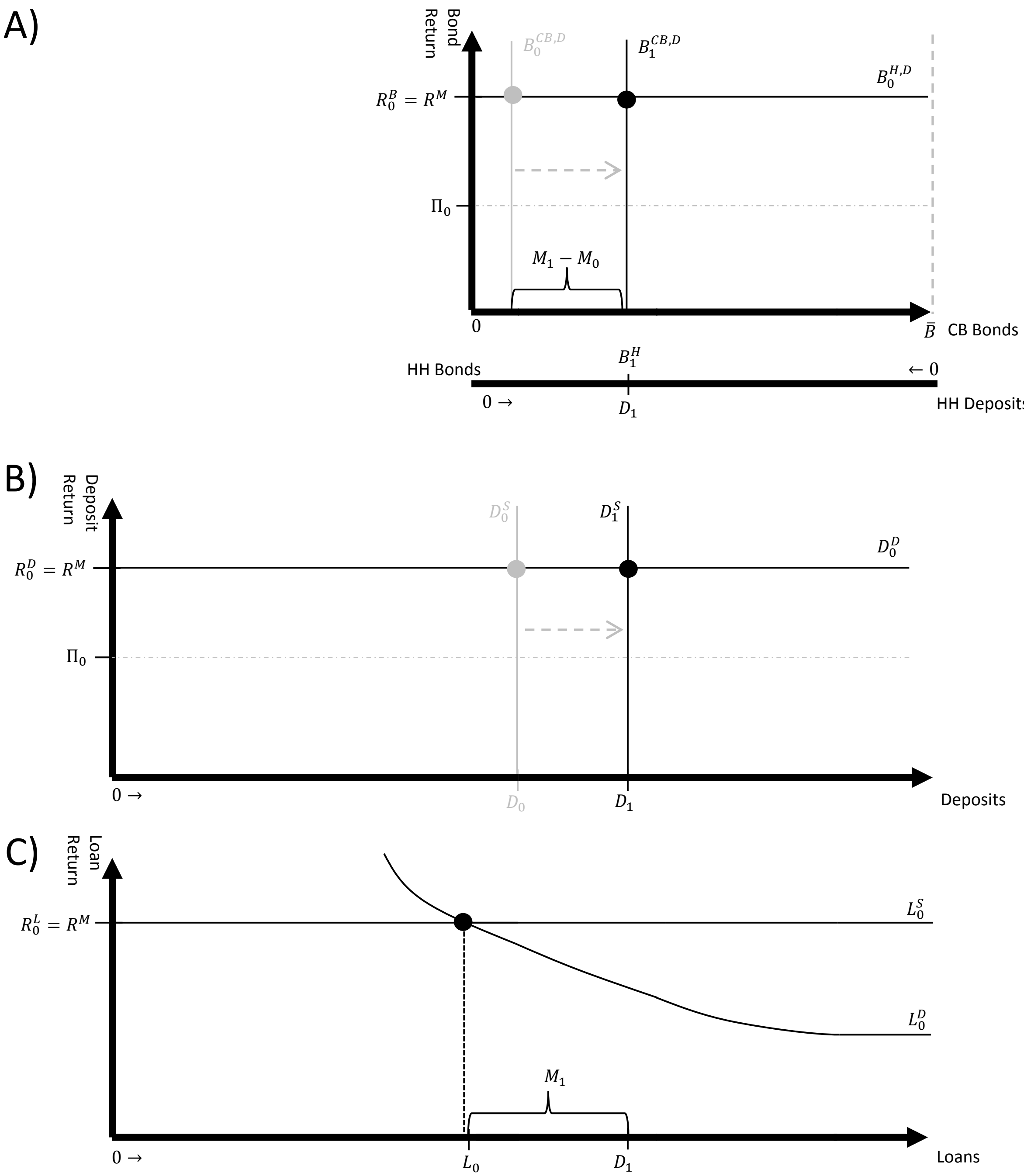
Source: Federal Reserve statistical release H.4.1: Factors Affecting Reserve Balances. Frequency: biweekly. Note: Reserve balances with Federal Reserve Banks are the difference between "total factors supplying reserve funds" and "total factors, other than reserve balances, absorbing reserve funds." This item includes balances at the Federal Reserve of all depository institutions that are used to satisfy reserve requirements and balances held in excess of balance requirements. It excludes reserves held in the form of cash in bank vaults, and excludes service-related deposits.

Figure 2. Benchmark model with minimal reserves and no balance sheet costs



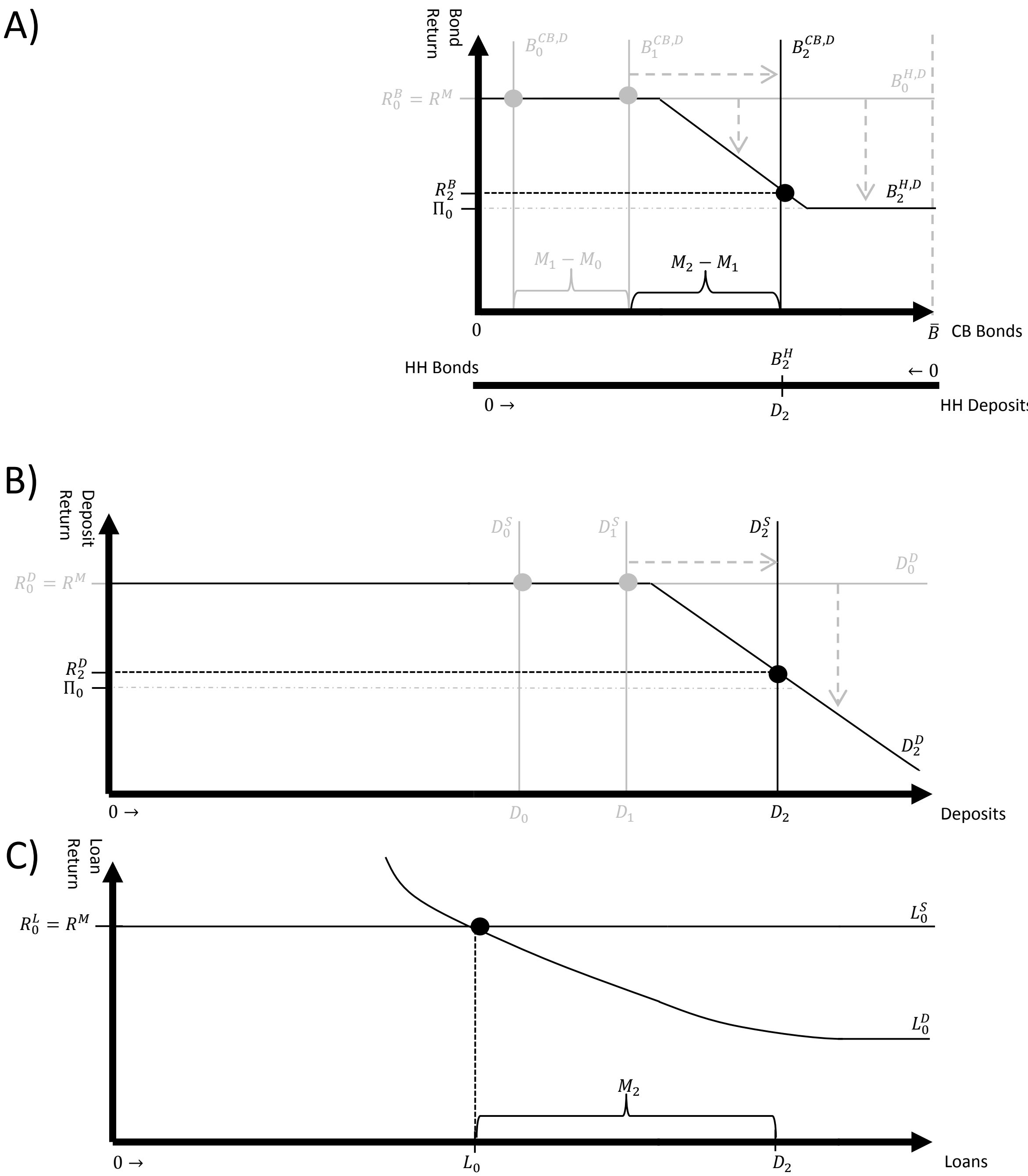
Notation: Demand and supply curves are denoted by a superscript '*S*' and '*D*', respectively.
A) Bond market: household bond holdings are increasing leftward and consequently central bank bond holdings are increasing rightward. The line underneath shows the breakdown of household assets with deposits increasing rightward and bond holdings increasing leftward. When there are minimal reserves, households hold almost the entire supply of bonds as shown by B_0^H. The remainder of their endowment is held as deposits D_0.
B) Deposit market: Equilibrium in the bond market results in a perfectly elastic supply curve for deposits at D_0, as shown by D_0^S. In equilibrium, deposits are therefore given by the quantity D_0.
C) Loan market: Equilibrium loans are equal to banks equilibrium deposit holdings minus reserves. A minimal level of reserves imply that loans comprise almost the entirety of bank assets. In equilibrium all returns are equated with the return on reserves R^M.
Note: Banks are indifferent between holding bonds and loans when they have equal returns and there are no balance sheet costs. We consider the case of no bank bond holdings in these figures for simplicity.

Figure 3. Moderate level of reserves and no balance sheet costs



A) Bond market: With a larger quantity of reserves and no balance sheet costs, households' bond holdings decrease by the amount of reserves to B_1^H. The increase in reserves is represented by an inelastic demand for bonds at the reserve level M_1. Households' bond holdings decrease to B_1^H, while their deposits increase by their decrease in bond holdings, which is equal to the increase in reserves.
B) Deposit market: Equilibrium in the deposit market once again is determined by households' inelastic supply of deposits D_1^S at D_1.
C) Loan market: Since deposit holdings increase by exactly the amount of reserves, equilibrium loans are left unchanged. Banks assets now consist of their original loan holdings plus reserves ($D_1 = L_1 + M_1$). Once again, equilibrium returns are left unchanged, with all being equated with the return on reserves.

Figure 4. Moderate level of reserves and moderate balance sheet costs

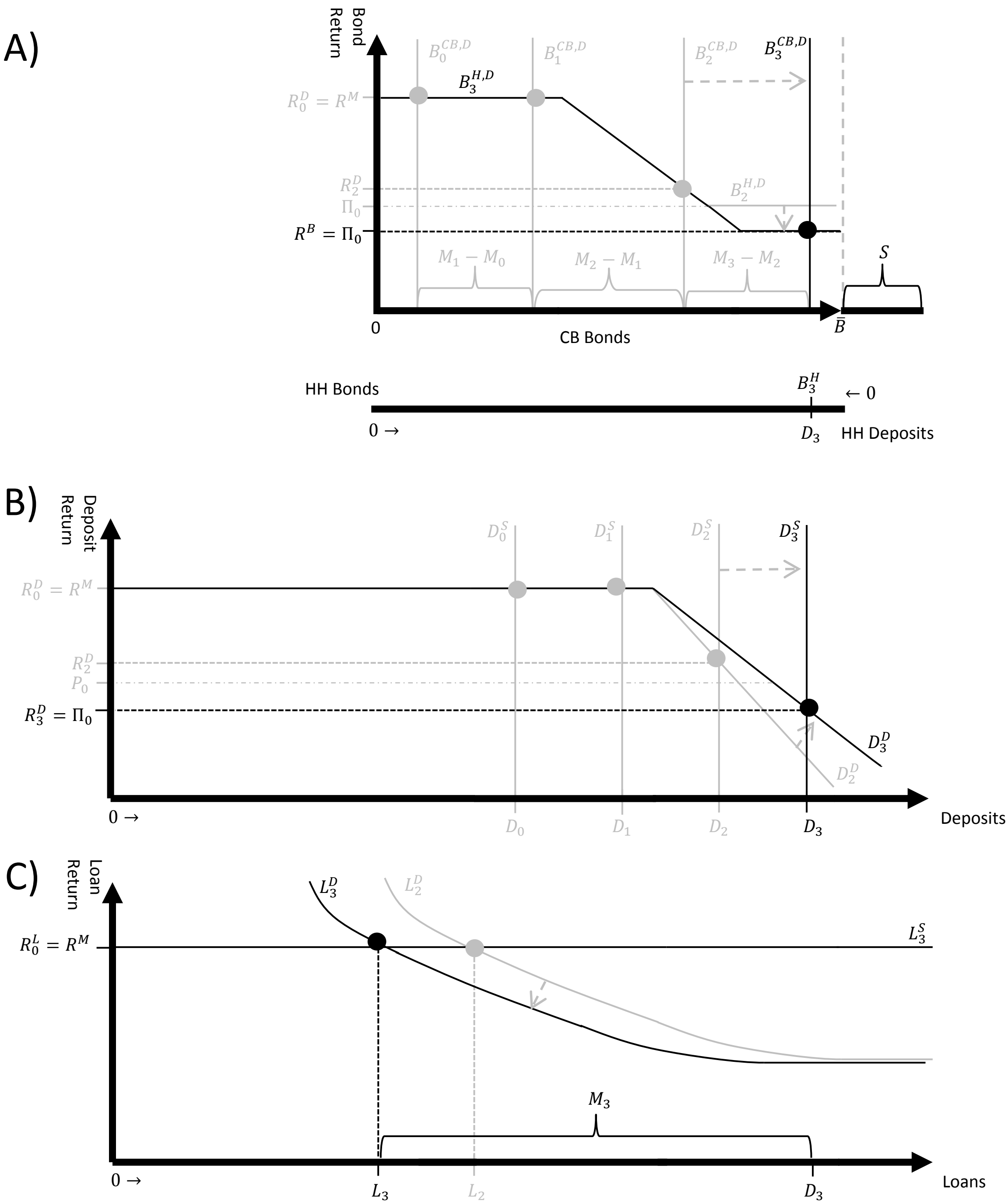


A) Bond market: With moderate reserves and balance sheet costs, the households bond demand curve, $B_2^{HH,D}$, becomes downward sloping for implied deposit quantities corresponding to regions where banks have positive balance sheet costs. This is to reflect that the equilibrium deposit return decreases as the deposit supply curve shifts outward in the deposit market. As a result, an increase in the reserve level to M_2 decreases the equilibrium bond return. Once again bond holdings fall to B_2^H, by the exact amount of the increase in reserves , $M_2 - M_1$.

B) Deposit market: The decrease in bond holdings is accompanied by an increase in the deposit supply to D_2^S. The deposit return decreases to compensate banks for their growing balance sheet costs along this region. Households willingly hold positive deposits as long as the deposit return is above that of the price level.

C) Loan market: Loans once again remain unchanged because deposits increase by the exact amount of the increase in reserves. Banks' balance sheets grow again but the increase is entirely because of the increase in reserves.

Figure 5. Large level of reserves and large balance sheet costs



A) Bond market: With excessive balance sheet costs, bond and deposit returns equate to gross inflation, and storage becomes positive taking up part of the household's assets. The bond demand curve shifts downward to reflect the decreasing price level.
B) Deposit market: The deposit demand curve shifts upward to with decreasing inflation until the deposit rate and inflation are also equated.
C) Loan market: Equilibrium loans decrease to L_3 as the decrease in inflation shifts loan demand leftward. Thus, deposits increase by strictly less than the increase in reserves.