Federal Reserve Bank of New York
Staff Reports

Information Acquisition and Financial Intermediation

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Staff Report No. 571
September 2012

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JEL classification: G12, G19, E44, G00

Abstract

This paper considers the problem of information acquisition in an intermediated market, where the specialists have access to superior technology for acquiring information. These informational advantages of specialists relative to households lead to disagreement between the two groups, changing the shape of the intermediation-constrained region of the economy and increasing the frequency of periods when the intermediation constraint binds. Acquiring the additional information is, however, costly to the specialists, making them less likely to decrease their risky asset holdings when the intermediation constraint binds. I show that this behavior leads the equity capital constraint to bind more frequently, making asset prices in the economy more volatile. I find empirical evidence consistent with these predictions.

Key words: rational inattention, asset prices in intermediated economies

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1 Introduction

Standard theories of intermediated markets postulate that specialists have access to better information about risky assets than households and thus are able to invest in risky asset markets. However, this assumption does not take into account the costs associated with specialists' acquisition and processing of additional information. Papers like Hong et al. (2007) provide evidence that even sophisticated investors are inattentive to important economic news. This paper considers the information acquisition problem faced by the specialists in an intermediated market, and finds that the differences in beliefs between the specialists and the households lead to more frequent periods of intermediation shortage and more volatile asset prices.

My model builds on the model of financial intermediation of He and Krishnamurthy (2011). There are two types of agents in the economy: specialists and households. While the specialists have access to the risky asset market, households cannot invest directly in the risky asset. The two types of agents thus enter into an intermediation contract, with the specialists investing in the risky asset on behalf of the households. As in He and Krishnamurthy (2010), the intermediation relation is subject to an equity constraint, with the households unwilling to commit funds greater than given multiple of specialist wealth, ensuring that the specialists have "skin in the game".

The asset markets in the economy are modeled along the lines of Lucas (1978). There is a risky asset producing an exogenous but risky dividend stream. While the specialists can invest in the risky asset directly, the households cannot. Both types of agents in the economy can, however, lend and borrow through investing in (or, correspondingly, shorting) a riskless bond. The point of departure of this model from the previous literature is that the agents in this economy do not know the expected dividend growth rate. Instead, they use observations of dividends to learn about the true expected dividend growth rate.

In addition, the specialists also have access to a costly external signal, with the cost of observing the signal increasing in the additional information contained in the signal. While the signal cost is modeled as a monetary cost in this paper, the form of the cost function makes this monetary cost isomorphic to a utility cost for the specialists. Thus, the cost of observing the external signal can be interpreted as the effort exerted by the specialist in acquiring additional information about the assets held by the intermediary.

The differences in learning technologies lead to disagreement between the specialists and the households. While the differences in beliefs influence the total wealth of the intermediary (through the optimal allocation decisions of the household) and, thus, the behavior of asset prices in the economy, unlike the setting of Scheinkman and Xiong (2003), this disagreement does not lead to asset price bubbles. Intuitively, the risky assets in the economy are always held by the intermediary sector. Thus, even though there is disagreement between the specialists and the households, there is no resale motive in setting asset prices and bubbles do not occur. The risk premium, risky asset return volatility and the Sharpe ratio of the risky asset, however, all increase as the specialist becomes more optimistic relative to the household and as the household becomes more dominant in the economy.

The disagreement between specialists and households also impacts the shape of the intermediation-constrained region in the economy. In particular, while it is still true that the economy is intermediation-constrained when the household's wealth is large relative to that of the specialist, when the household is more optimistic than the specialist, the critical level of the relative wealth of the household decreases. Intuitively, when the household is more optimistic, it would like to invest a larger fraction of its wealth in the risky asset, allocating a larger fraction of its wealth to the intermediary. Notice that, since both the specialist and the household are myopic in this economy, the household

does not take into account the fact that the specialist has more precise information when making its portfolio allocation decision.

In the long-run information equilibrium of the economy, costly information acquisition makes the specialists less likely to decrease their risky asset holding when the intermediation constraint binds. Intuitively, as the cost of observing signals increases, the specialist is willing to participate more in the risky asset market to recuperate the costs associated with information acquisition. This in turns leads the risk premium, risky asset return volatility and the Sharpe ratio of the risky asset to increase less dramatically in the intermediation-constrained region, since the risky asset is distributed across a larger mass of agents.

This paper links the recent literature on financial intermediation in a macroeconomic setting with the literature on economic agents' limited capacity to process information, or rational inattention. The economic literature on rational inattention builds on the seminal papers by Sims (2003, 2006). The main premise of the rational inattention literature is that agents face a cost in processing the signals available to them in the public markets and thus optimally choose to observe only some of the information potentially available to them. The cost of information can either be a physical cost, with the agents limited in the rate of information transmission that they can process, or a monetary cost, increasing in the information transmission rate demanded by the agent. The current paper falls in the latter category, with the specialists in the economy paying a monetary cost out of their own wealth for observing more precise external signals. The information choice setting in this paper is closest to Detemple and Kihlstrom (1987) and Huang and Liu (2007). While both of these papers take the dynamics of asset prices in the economy as given and study the optimal portfolio choice problem of an investor faced with information acquisition costs, Huang and Liu (2007) solve the date zero optimal information choice

while Detemple and Kihlstrom (1987) allow the agent in the economy to dynamically update his information choice. Huang and Liu (2007) show that rational inattention may cause the representative investor to over– or underinvest. Furthermore, the optimal trading strategy is myopic with respect to future information choices.

Van Nieuwerburgh and Veldkamp (2010) and Kacperczyk et al. (2011) study the optimal portfolio and attention allocation between multiple assets. Van Nieuwerburgh and Veldkamp (2010) show that, given a fixed capacity to process information about expected asset returns, the investor that collects information before choosing the optimal portfolio allocation will systematically deviate from holding a diversified portfolio and may choose to invest instead in a diversified fund and a concentrated set of assets. In a similar setting, Kacperczyk et al. (2011) show that mutual fund managers optimally alter their information choice based on the state of the economy, leading to higher correlation of fund portfolio holdings with the aggregate information, higher dispersion in returns across funds and higher average fund performance in recessions than in expansions. Unlike the current paper, the fund managers of Kacperczyk et al. (2011) face a capacity constraint in information acquisition. Thus, the information friction in their paper can be interpreted as differences in skill between different fund managers. This paper differs from the above literature in that the model is dynamic, and asset prices are determined in equilibrium.

This paper is also related to the large literature in banking studying (dis)intermediation. While traditional models consider the problem in a static setting (see Diamond and Dybvig (1983); Allen and Gale (1994); Holmstrom and Tirole (1997); Diamond and Rajan (2005)), the more recent work (see e.g. He and Krishnamurthy (2011, 2010); Brunnermeier and Sannikov (2010); Haddad (2012)) studies the links between financial intermediation and asset prices in a dynamic setting. These papers, however, assume that the factors

underlying aggregate output (and, hence, prices) in the economy are perfectly observed.

The rest of the paper is organized as follows. Section 2 presents the economic environment faced by the agents in the economy. The theoretical equilibrium behavior of asset prices is examined in Section 3, while Section 4 provides a numerical illustration. I provide some motivation empirical evidence in Section 5. Section 6 concludes. Technical details are relegated to the appendix.

2 The Model

In this section, I describe the environment faced by the agents in the economy. Starting with the financial intermediation setting of He and Krishnamurthy (2010), I consider the case of imperfect information and the incentives to acquire more precise information.

2.1 Economic environment

In this paper, I consider a version of the Lucas (1978) endowment economy. There are two types of assets traded in asset markets: a risk-free bond in zero net supply, with (locally) risk less rate r_t and a risky asset in unitary supply. The risky asset is a claim to the dividends of the Lucas tree, with risky dividend growth given by:

$$\frac{dD_t}{D_t} = g_t dt + \sigma_d dZ_{dt}, \tag{2.1}$$

where D_0 is known by the agents in the economy, $\sigma_d > 0$ are constants, and dZ_t is the increment of the standard Brownian motion under the appropriate filtration. The expected dividend growth rate g_t is time-varying and evolves according to a mean-reverting

process:

$$dg_t = \kappa_g \left(\bar{g} - g_t\right) dt + \sigma_g dZ_{gt}, \tag{2.2}$$

where κ_g, $\bar{g}$ and σ_g constants, and dZ_{gt} is the increment of the standard Brownian motion, independent of dZ_{dt}. Notice that this specification corresponds to the continuous-time version of the long-run specification of aggregate consumption growth dynamic of Bansal and Yaron (2004). The mean-reverting expected dividend growth rate g_t corresponds to the long-run component of consumption growth in their specification; since the model in question is a general equilibrium model, aggregate dividends correspond to aggregate consumption. Denoting by P_t the price of the risky asset at time $t \geq 0$, the risky asset total return is given by:

$$dR_t = \frac{D_t dt + dP_t}{P_t}.$$

There are two types of agents in the economy, each of unit mass: households and specialists. As in He and Krishnamurthy (2010, 2011), I assume that the households cannot invest directly in the risky asset. This corresponds to the assumptions usually made in the literature on limited market participation (see e.g. Allen and Gale (1994); Basak and Cuoco (1998); Mankiw and Zeldes (1991); Vissing-Jorgensen (2002)) and is usually motivated by appealing to "informational" transaction costs that households face in order to invest directly in the risky asset market. While I do not investigate the optimal occupation choice, I make a step in that direction by allowing the specialists to have access to a better learning technology.

To circumvent the limited participation constraint, at each time $t > 0$, households and specialists are randomly matched to create a short-lived (lasting from time t to $t + dt$)

intermediary. The intermediary is subject to an equity constraint. In particular, denoting by w_t the time t wealth of the specialist and by H_t the time t contribution of the household to the intermediary, the equity constraint stipulates that:

$$H_t \leq m w_t. \tag{2.3}$$

That is, the household can only contribute up to a multiple m of specialist wealth to the intermediary. As in He and Krishnamurthy (2010), I assume that the specialist contributes all of his wealth to the intermediary, and that the intermediary profits are distributed between the specialist and the household in proportion to their relative wealth contributions.

Both the specialist and the household evaluate consumption paths using the log utility function. With this assumption, the expected lifetime utility of the specialist is given by:

$$\mathbb{E}\left[\int_0^{+\infty} e^{-\beta t} \log c_t dt\right],$$

where β is the time discount rate of agents in the economy and c_t is the specialist's consumption rate at time t, and the expected lifetime utility of the household is given by:

$$\mathbb{E}\left[\int_0^{+\infty} e^{-\beta t} \log c_{ht} dt\right],$$

where c_{ht} is the household's consumption rate at time t.

2.2 Learning

Unlike the previous literature, I assume that the agents in this economy do not know the true value of the expected dividend growth rate, g_t. Instead, agents can use observations

of the realized dividend growth process to learn about g_t. In addition, specialists can observe an external, unbiased signal of the expected dividend growth rate of the form:

$$de_t = g_t dt + \frac{\sigma_e}{\sqrt{n_t}} dZ_{et}, \tag{2.4}$$

where $\sigma_e > 0$ is a constants, n_t denotes the effort expanded by the specialist in acquiring information and dZ_{et} is the increment of the standard Brownian motion, independent of both dZ_{dt} and dZ_{gt}. I assume that the specialists face a capacity constraint in processing the external signal, with the total information transmission rate to the specialist from observations of both the realized dividend growth rate dD_t/D_t and the external signal bounded above by a constant. As in Sims (2003, 2006), this can be interpreted as a physical constraint in processing information. The actual act of observing the external signal is also costly to the specialist in a monetary sense, with the per-unit-of-wealth cost of the effort required to observe an external signal of a given precision proportional to the increase in the information transmission rate to the specialist. This second cost corresponds to the cost of information production (data collection and noise reduction). Both of these frictions prevent the specialist from "growing out" of the information constraint as his wealth increases. For a more detailed discussion of the different specifications of the information acquisition trade off, see Boyarchenko and Duarte (2012).

Given the strategic choice to acquire information, it is not immediate that standard filtering results apply. Detemple and Kihlstrom (1987), however, show that, even with strategic information choice, the specialist's beliefs are given by the Kalman-Bucy filter. In particular:

Lemma 2.1. *(Kalman-Bucy Filter)*

Given the time t information set $\mathcal{F}_t = \sigma^- \{D_s,\ e_s :\ s \leq t\}$, the specialist's inference at

time t of the expected dividend growth rate has a Gaussian distribution: $g_t|\mathcal{F}_t \sim N(\hat{g}_t, \gamma_t)$, with the inferred growth rate $\hat{g}_t$ evolving according to:

$$d\hat{g}_t = \kappa_g (\bar{g} - \hat{g}_t)\, dt + \frac{\gamma_t}{\sigma_d} d\tilde{Z}_{dt} + \frac{\gamma_t \sqrt{n_t}}{\sigma_e} d\tilde{Z}_{et}, \tag{2.5}$$

and the conditional variance of the belief as:

$$\frac{d\gamma_t}{dt} = -2\kappa_g \gamma_t + \sigma_g^2 - \gamma_t^2 \left(\frac{1}{\sigma_d^2} + \frac{n_t}{\sigma_e^2} \right). \tag{2.6}$$

Here, $d\tilde{Z}_{dt}$ and $d\tilde{Z}_{et}$ are independent increments of the standard Brownian motion under $\mathcal{F}_t$, given, respectively, by:

$$\begin{aligned} d\tilde{Z}_{dt} &= \sigma_d^{-1} \left(\frac{dD_t}{D_t} - \hat{g}_t dt \right) \\ d\tilde{Z}_{et} &= \frac{\sqrt{n_t}}{\sigma_e} (de_t - \hat{g}_t dt). \end{aligned}$$

Proof. See e.g. Theorem 10.2 of Liptser and Shiryaev (1977). □

Notice also that, under the specialist's information set $\mathcal{F}_t$, the signal vector evolves as:

$$\begin{aligned} \frac{dD}{D} &= \hat{g}_t dt + \sigma_d d\tilde{Z}_{dt} \\ de_t &= \hat{g}_t dt + \frac{\sigma_e}{\sqrt{n_t}} d\tilde{Z}_{et}. \end{aligned}$$

Recall that the household learns about the expected dividend growth rate g using observations of the realized dividend growth rate only. Denote by $\mathcal{F}_t^h = \sigma^- \{D_s : \ s \leq t\}$ the household's information set at time t. Similarly to the belief evolution of the specialist, the household's inference at time t of the expected dividend growth rate has a Gaussian distribution: $g_t|\mathcal{F}_t^h \sim N(\hat{g}_{ht}, \gamma_{ht})$, with the inferred dividend growth rate $\hat{g}_{ht}$ evolving

according to:

$$d\hat{g}_{ht} = \kappa_g \left(\bar{g} - \hat{g}_{ht}\right) + \frac{\gamma_{ht}}{\sigma_d} d\tilde{Z}^h_{dt},$$

and the conditional variance of the belief as:

$$\frac{d\gamma_{ht}}{dt} = -2\kappa_g \gamma_{ht} + \sigma_g^2 - \frac{\gamma_{ht}^2}{\sigma_d^2},$$

where $d\tilde{Z}^h_{dt}$ is the increment of the standard Brownian motion under $\mathcal{F}^h_t$, given by:

$$d\tilde{Z}^h_{dt} = \sigma_d^{-1} \left(\frac{dD_t}{D_t} - \hat{g}_{ht} dt \right).$$

Given the Gaussian structure of the observations-state system, the information transmission rate to the specialist is given by:

$$\begin{aligned} \frac{d\mathcal{I}}{dt}\left(g; D_t, e_t\right) &= \frac{1}{2}\mathbb{E}\left[\left(g_t - \hat{g}_t\right)^2 \left(\frac{1}{\sigma_d^2} + \frac{n_t}{\sigma_e^2} \right) \middle| \mathcal{F}_t \right] \\ &= \frac{\gamma_t}{2} \left(\frac{1}{\sigma_d^2} + \frac{n_t}{\sigma_e^2} \right). \end{aligned} \tag{2.7}$$

For a proof of the above result, see e.g. Turmuhambetova (2005). Compare this to the information transmission rate to the household:

$$\frac{d\mathcal{I}^h}{dt}\left(g; D_t\right) = \frac{\gamma_{ht}}{2\sigma_d^2}.$$

While the specialist and the household are both relatively uninformed, so that γ_{ht} and γ_t are of similar magnitude, the information transmission rate to the specialist is higher since the specialist observes an additional signal. When the specialist becomes relatively

more informed, so that γ_{ht} is much higher than γ_t, the information transmission rate to the household as higher. Intuitively, for the less informed agent, every new observation contains more new information than for the more informed agent.

I assume that the information transmission rate to the specialist is bounded above by a constant $\kappa > 0$:

$$\frac{d\mathcal{I}}{dt}(g; D_t, e_t) \leq \kappa \quad \forall t \geq 0,$$

and that the cost of observing an external signal of precision $\sqrt{n_t}/\sigma_e$ is proportional to the implied increase in the information transmission rate to the specialist:

$$w_t \theta \gamma_t \frac{n_t}{\sigma_e^2} dt.$$

The parameter θ, which measures the marginal cost of increasing signal precision, per unit of wealth$\times$variance, can be interpreted as the aversion of the specialist to exerting effort to observe additional signals of the long-term component of dividend growth rates. In the appendix, I show that the value function of the specialist is strictly increasing the precision of the external signal, so that he always chooses the signal precision to just satisfy the capacity constraint. Thus, the per-period cost of the external signal can be expressed as:

$$w_t \theta \left(2\kappa - \frac{\gamma_t}{\sigma_d^2}\right) dt.$$

Introduce $f_t = \hat{g}_t - \hat{g}_{ht}$ to be the disagreement between the specialist and the household about the expected dividend growth rate. Under the specialist's information set $\mathcal{F}_t$,

disagreement evolves as:

$$df_t = -\left(\kappa_g + \frac{\gamma_{ht}}{\sigma_d^2}\right) f_t dt + \left(\frac{\gamma_t - \gamma_{ht}}{\sigma_d}\right) d\tilde{Z}_{dt} + \frac{\gamma_t \sqrt{n_t}}{\sigma_e} d\tilde{Z}_{et}.$$

Thus, disagreement follows a mean-reverting process, with variation in the speed of mean-reversion determined by the information transmission rate to the household. Notice also that, when the household is more uncertain about its inference of the expected dividend growth rate, so that γ_{ht} is large, disagreement is negative correlated with the innovations to the dividend growth process.

Finally, we can parametrize the evolution of the risky asset return under the specialist's information set $\mathcal{F}_t$ as:

$$dR_t = \mu_{Rt} dt + \sigma_{Rd,t} d\tilde{Z}_{dt} + \sigma_{Re,t} d\tilde{Z}_{et},$$

and under the household's information set $\mathcal{F}_t^h$ as:

$$dR_t = \mu_{Rt}^h dt + \sigma_{Rd,t}^h d\tilde{Z}_{dt}^h.$$

Since the specialist and the household agree about the risky asset price in equilibrium, we can also represent the risky asset return under the specialist's information set as:

$$dR_t = \left(\mu_{Rt}^h + \frac{\sigma_{Rd,t}}{\sigma_d} f_t\right) dt + \sigma_{Rd,t}^h d\tilde{Z}_{dt}.$$

Equating coefficients, we see that, in equilibrium:

$$\sigma^h_{Rd,t} = \sigma_{Rd,t}$$

$$\sigma_{Re,t} = 0$$

$$\mu^h_{Rt} = \mu_{Rt} - \frac{\sigma_{Rd,t}}{\sigma_d} f_t.$$

Thus, even though the innovations in the external signal inform the specialist's beliefs, they do not directly enter into the risky asset returns process. Notice also that, when the household is more optimistic than the specialist about the expected dividend growth rate, so that $f_t < 0$, the household believes the expected risky asset return to be higher as well. In the following, I denote by $\pi_{Rt} = \mu_{Rt} - r_t$ the risk premium under the specialist's information set; under the household's information set, the risk premium is given by $\pi_{Rt} - \sigma_{Rd,t} f_t / \sigma_d$.

2.3 State variables

It is useful at this point to summarize the state variables in the economy and their evolutions under the specialist's information set. Since the specialist is the marginal investor in both the risky and the risk-free asset markets, asset prices satisfy his Euler equation and, hence, the specialist's information set is the relevant one in determining the time series behavior of returns. The full state vector in the economy is:

$$(\hat{g}_t, f_t, \gamma_t, \gamma_{ht}, y_t, w_t, w_{ht}).$$

Here, $y_t = w_{ht}/D_t$ is the relative wealth of the household. Since the households are constrained in their portfolio allocation decision, their relative wealth will be a driving

factor for asset prices and, hence, the wealth evolutions in the economy. An alternative specification would be to have the current dividend D_t as a state variable instead of y_t; however, this specification turns out to lead to more parsimonious asset pricing formulas. Parametrize the evolution of y_t under the specialist's information set as:

$$dy_t = \mu_{yt}dt + \sigma_{yd,t}\tilde{Z}_{dt}.$$

Notice that, since innovations to the external signal do not affect the evolution of the risky asset return, they will not affect the evolution of the relative wealth of the household. Notice also that, in an economy with households only, y_t corresponds to the inverse of the consumption-wealth ratio cay_t of Lettau and Ludvigson (2001), which has been shown to predict stock returns.

2.4 Specialist's problem

Since specialists are the only agents with access to the risky asset market, I assume that the specialists makes all of the investment decisions on the total capital of the intermediary and faces no portfolio restrictions in buying or short-selling either the risky asset or the risk-free bond. In particular, denote by α_{It} the fraction of intermediary capital invested in the risky asset at time t and by w_t the specialist wealth at time t. Notice that, since all of the specialist's wealth is invested in the intermediary, α_{It} is the effective exposure of the specialist to the risky asset. Then:

Proposition 2.2. *The specialist chooses his consumption rate, his information acquisition and the intermediary's exposure to the risky asset to solve:*

$$\max_{\{c_t,\alpha_{It},n_t\}} \mathbb{E}\left[\int_0^{+\infty} e^{-\beta t}\log c_t dt\right], \tag{2.8}$$

subject to the specialist's budget constraint:

$$dw_t = -c_t dt + w_t r_t dt + \alpha_{It} w_t \left(dR_t - r_t dt\right) - w_t \theta \left(2\kappa - \frac{\gamma_t}{\sigma_d^2}\right) dt.$$

The specialist's optimal consumption rule is:

$$c_t = \beta w_t, \tag{2.9}$$

the optimal risk exposure is:

$$\alpha_{It} = \frac{\pi_{Rt}}{\sigma_{Rd,t}^2}, \tag{2.10}$$

and the optimal external signal precision is:

$$n_t = \left(\frac{2\kappa}{\gamma_t} - \frac{1}{\sigma_d^2}\right) \sigma_e^2. \tag{2.11}$$

Thus, the specialist consumes a fixed proportion, β, of his wealth each period, and invests according to the standard myopic investment rule. The optimal signal precision increases with the precision of the specialist's beliefs. Intuitively, as the beliefs of the specialist become more precise, the same rate of information transmission is attained with a higher signal precision.

2.5 Household's problem

Consider now the household's problem. Denote by $\alpha_{ht} \in [0, 1]$ the fraction of household wealth allocated to the intermediary at time t. As in He and Krishnamurthy (2010), I assume that the household is precluded from shorting both the intermediary and the

risk-free bond. Then the following result holds.

Proposition 2.3. *Taking the specialist's wealth w_t and exposure choice α_{It} as given, the household solves:*

$$\max_{\{c_{ht},\alpha_{ht}\}} \mathbb{E}\left[\int_0^{+\infty} e^{-\beta t} \log c_{ht} dt\right], \tag{2.12}$$

subject to the household's budget constraint:

$$dw_{ht} = -c_{ht}dt + w_{ht}r_t dt + \alpha_{ht}\alpha_{It}w_{ht}\left(dR_t - r_t dt\right),$$

the intermediation constraint:

$$\alpha_{ht}w_{ht} \leq mw_t,$$

and the no shorting constraint: $\alpha_{ht} \in [0,1]$. The household's optimal consumption rule is:

$$c_{ht} = \beta w_{ht}, \tag{2.13}$$

and the optimal risk exposure in the unconstrained region is:

$$\alpha_{ht} = \frac{\pi_{Rt} - \sigma_{Rd,t}\sigma_d^{-1} f_t}{\alpha_{It}\sigma_{Rd,t}^2}. \tag{2.14}$$

Thus, in the unconstrained region, the household also acts as a standard myopic investor, consuming a constant proportion of its wealth each period.

2.6 Equilibrium

Definition 2.4. *An equilibrium in this economy is a set of price processes* $\{P_t\}$ *and* $\{r_t\}$*, and decisions* $\{c_t, c_{ht}, \alpha_{It}, \alpha_{ht}, n_t\}$ *such that:*

1. *Given the price processes, decisions solve the consumption-savings problems of the specialist (2.8) and the household (2.12).*

2. *Decisions satisfy the intermediation constraint.*

3. *The risky asset market clears:*

$$\alpha_{It}\left(w_t + \alpha_{ht} w_{ht}\right) = P_t. \tag{2.15}$$

4. *The goods market clears:*

$$c_t + c_{ht} = D_t. \tag{2.16}$$

Notice that, since the risk-free bonds are in equilibrium zero-net supply, the risky asset market clearing condition can be expressed as:

$$w_t\left(1 - \theta\left(\kappa - \frac{\gamma_t}{\sigma_d^2}\right)\right) + w_{ht} = P_t.$$

3 Asset prices

In this section, I characterize the asset prices in the economy. Notice that, since the households are (potentially) constrained in making their investment decisions by the intermediation constraint, the specialist is the marginal agent in both the risky and the

risk-free asset markets. In particular, the risk-free rate in the economy satisfies the Euler equation of the specialist but not necessarily that of the household.

3.1 Risky asset price

Begin by considering the risky asset price. Since the specialists and the households in this economy have log preferences, we can derive the risky asset price in closed form. Substituting the specialist's (2.9) and the household's (2.13) optimal consumption decisions into the goods market clearing condition (2.16), the price of the risky asset can be expressed as:

$$P_t = \frac{D_t}{\beta\sigma_d^2}\left[\sigma_d^2 - \theta\left(\kappa\sigma_d^2 - \gamma_t\right)\right] + \frac{\theta}{\sigma_d^2}\left(\kappa\sigma_d^2 - \gamma_t\right) w_{ht}.$$

Thus, the equilibrium price-dividend ratio is given by:

$$\frac{P_t}{D_t} = \frac{1}{\beta\sigma_d^2}\left[\sigma_d^2 - \theta\left(\kappa\sigma_d^2 - \gamma_t\right)\right] + \frac{\theta}{\sigma_d^2}\left(\kappa\sigma_d^2 - \gamma_t\right) y_t.$$

Recall that the economy is intermediation-constrained when the specialist has relatively low wealth, so that:

$$\alpha_{ht} = \frac{\pi_{Rt} - \sigma_{Rd,t}\sigma_d^{-1} f_t}{\alpha_{It}\sigma_{Rd,t}^2} > \frac{m w_t}{w_{ht}}.$$

Rewriting, we obtain:

$$y_t \geq \frac{m\pi_{Rt}}{\beta\left[(1+m)\,\pi_{Rt} - \sigma_{Rd,t}\sigma_d^{-1} f_t\right]}.$$

Thus, the y boundary of the intermediation-constrained region is increasing in the Sharpe

ratio and decreasing in the disagreement between the specialist and the household. Figure 1 plots the boundary between the unconstrained and the constrained regions as a function of the disagreement between the specialist and the household f (y axis) and the scaled wealth of the household y (x axis). As households become more wealthy as a share of total output in the economy, the constraint boundary decreases, making it more likely that the economy will be intermediation-constrained. Intuitively, as the relative wealth of households increases, the relative wealth of specialists must decrease, making the intermediation constraint tighter. Similarly, as the household becomes more optimistic relative to the specialist, so that f_t decreases, the constraint boundary decreases, making it more likely that the economy will be intermediation-constrained. Intuitively, as households become more optimistic about the expected dividend growth rate, the optimal share of their wealth allocated to the intermediary increases, making the intermediation constraint tighter.

Finally, notice that we can express the risky asset return as:

$$\begin{aligned} dR_t = & \frac{\hat{g}_t\left(\sigma_d^2 - \theta\left(\kappa\sigma_d^2 - \gamma_t\right)\right) + \theta(1-\beta y_t)\left[\sigma_g^2 - 2(\kappa_g + \kappa)\gamma_t\right]}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)} dt \\ & + \frac{\theta\beta y_t\left(\kappa\sigma_d^2 - \gamma_t\right)\left(r_t - \beta + \alpha_{ht}\alpha_{It}\pi_{Rt}\right)}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)} dt + \frac{\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\left(\beta\alpha_{ht}\alpha_{It}\sigma_{Rd,t}y_t - \sigma_d\right)}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)} d\tilde{Z}_{dt}. \end{aligned} \tag{3.1}$$

3.1.1 Return volatility

Substituting the household's optimal portfolio choice into the risky asset returns process (3.1), we obtain the following result.

Proposition 3.1. *The equilibrium risky asset return volatility satisfies:*

$$\sigma_{Rd,t} = \frac{\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\left(\beta\alpha_{ht}\alpha_{It}\sigma_{Rd,t}y_t - \sigma_d\right)}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)}.$$

Thus, in the intermediation-unconstrained region, the risky asset return volatility is given by:

$$\sigma_{Rd,t} = -\frac{\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\sigma_d}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)} + \frac{\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\beta y_t}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)}\left(\frac{\pi_{Rt}}{\sigma_{Rd,t}} - \frac{f_t}{\sigma_d}\right).$$

When the economy is in the intermediation-constrained region, the risky asset return volatility becomes:

$$\sigma_{Rd,t} = -\frac{\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\sigma_d}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)} + \frac{\theta\left(\kappa\sigma_d^2 - \gamma_t\right)(1-\beta y_t)}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)}\frac{\pi_{Rt}}{\sigma_{Rd,t}}.$$

Notice that, in the intermediation-constrained region, the risky asset return volatility does not depend on the disagreement between the specialist and the household while the disagreement does influence the volatility in the unconstrained region. Intuitively, in the intermediation-constrained region, the households are constrained in choosing their portfolio allocation, and are thus the inframarginal investors in the risky asset. Thus, the price of the risky asset in the constrained region reflects only the beliefs of the specialist. In the unconstrained region, both the specialist and the household are at their unconstrained optimum, so the risky asset price balances both of their beliefs. Notice also that, unlike the perfect information setting of He and Krishnamurthy (2011), the disagreement between the specialist and the household and the learning process of the specialist introduce stochastic volatility in the returns process even in the unconstrained region. The volatility of the returns process is increasing in the Sharpe ratio of the risky asset, both in the constrained and the unconstrained region of the economy. Notice also that, asymptotically, as $y_t \to +\infty$, which corresponds to the economy becoming increasingly more intermediation-constrained, $\sigma_{Rd,t}^2 \to -\pi_{Rt}$. Intuitively, as the household becomes infinitely large relative to the economy, asset prices converge to the shadow asset prices in

an economy where the household is the only agent in the economy, but cannot trade in the risky asset. At the other extreme, as $y_t \to 0$, so that the economy becomes increasingly less intermediation-constrained,

$$\lim_{y_t \to 0} \sigma_{Rd,t} = \frac{\theta \left(\kappa \sigma_d^2 - \gamma_t\right) \sigma_d}{\theta \left(\kappa \sigma_d^2 - \gamma_t\right) - \sigma_d^2}.$$

Thus, as the specialist becomes the dominant agent in the economy, the risky asset volatility depends only on the fluctuation in the beliefs of the specialist. In particular, in the long-run equilibrium of the information acquisition (with $\gamma_t = \frac{\sigma_g^2}{2(\kappa+\kappa_g)}$), the risky asset volatility will be constant.

3.1.2 Risk premium

We can also use the risky asset returns process (3.1) to obtain the equilibrium risk premium.

Proposition 3.2. *The equilibrium risk premium satisfies:*

$$\begin{aligned}\pi_{Rt} = -\beta &+ \frac{\theta \left[\sigma_g^2 - 2\left(\kappa_g + \kappa\right)\gamma_t\right]\left(1 - \beta y_t\right)}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)} + \frac{\left(\sigma_d - \beta y_t \alpha_{ht} \alpha_{It} \sigma_{Rd,t}\right)^2}{(1-\beta y_t)^2} \\ &+ \frac{\beta y_t \sigma_d^2 \left[r_t - \beta - \hat{g}_t + \alpha_{ht}\alpha_{It}\pi_{Rt}\right]}{\left(1-\beta y_t\right)\left(\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\right)}.\end{aligned}$$

Thus, in the intermediation-unconstrained region, the risk premium is given by:

$$\begin{aligned}\pi_{Rt} = -\beta &+ \frac{\theta \left[\sigma_g^2 - 2\left(\kappa_g + \kappa\right)\gamma_t\right]\left(1 - \beta y_t\right)}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)} + \frac{\beta y_t \sigma_d^2 \left[r_t - \beta - \hat{g}_t\right]}{\left(1-\beta y_t\right)\left(\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\right)} \\ &+ \frac{\beta y_t \sigma_d^2}{\left(1-\beta y_t\right)\left(\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\right)}\left(\frac{\pi_{Rt}}{\sigma_{Rd,t}} - \frac{f_t}{\sigma_d}\right)\frac{\pi_{Rt}}{\sigma_{Rd,t}} \\ &+ \left(\frac{\sigma_d}{1-\beta y_t} - \frac{\beta y_t}{1-\beta y_t}\left(\frac{\pi_{Rt}}{\sigma_{Rd,t}} - \frac{f_t}{\sigma_d}\right)\right)^2.\end{aligned}$$

When the economy is in the intermediation-constrained region, the risk premium becomes:

$$\pi_{Rt} = -\beta + \frac{\theta\left[\sigma_g^2 - 2\left(\kappa_g + \kappa\right)\gamma_t\right]\left(1-\beta y_t\right)}{\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)} + \frac{\beta y_t \sigma_d^2\left[r_t - \beta - \hat{g}_t\right]}{(1-\beta y_t)\left(\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\right)}$$
$$+ \frac{m\sigma_d^2}{\left(\sigma_d^2 - (1-\beta y_t)\theta\left(\kappa\sigma_d^2 - \gamma_t\right)\right)}\frac{\pi_{Rt}^2}{\sigma_{Rd,t}^2} + \left(\frac{\sigma_d}{1-\beta y_t} - m\frac{\pi_{Rt}}{\sigma_{Rd,t}}\right)^2.$$

Similarly to the risky asset return volatility, in the intermediation-constrained region, the risk premium does not depend on the disagreement between the specialist and the household while the disagreement does influence the risk premium in the unconstrained region. Asymptotically, as $y_t \to +\infty$, the risk premium becomes:

$$\lim_{y_t \to +\infty} \pi_{Rt} = -\frac{\beta}{1+m^2} - \frac{1}{1+m^2}\frac{\left[\sigma_g^2 - 2\left(\kappa_g + \kappa\right)\gamma_t\right]}{\left(\kappa\sigma_d^2 - \gamma_t\right)} - \left[r_t - \beta - \hat{g}_t\right].$$

In the long-run equilibrium of the information acquisition, this becomes

$$\pi_{Rt} \to -\frac{\beta}{1+m^2} - \left[r_t - \beta - \hat{g}_t\right].$$

At the other extreme, as $y_t \to 0$, the equilibrium risk premium converges to:

$$\lim_{y_t \to 0} \pi_{Rt} = -\beta + \frac{\theta\left[\sigma_g^2 - 2\left(\kappa_g + \kappa\right)\gamma_t\right]}{\sigma_d^2 - \theta\left(\kappa\sigma_d^2 - \gamma_t\right)} + \sigma_d^2.$$

As with the risky asset return volatility, as the specialist becomes the dominant agent in the economy, the equilibrium risk premium is determined by fluctuations in his beliefs. In the long-run equilibrium of the information acquisition, the risk premium becomes $\pi_{Rt} \to \sigma_d^2 - \beta$.

3.2 Risk-free rate

Since the specialist is the marginal investor in the risk-free market, the risk-free rate satisfies the specialist's Euler equation:

$$r_t dt = \beta dt + \mathbb{E}\left[\frac{dw_t}{w_t}\middle|\mathcal{F}_t\right] - var\left(\frac{dw_t}{w_t}\middle|\mathcal{F}_t\right).$$

Notice that we can represent:

$$\frac{dw_t}{w_t} = \frac{d\left(D_t - \beta w_{ht}\right)}{D_t - \beta w_{ht}} = \frac{dD_t/D_t - \beta dw_{ht}/D_t}{1 - \beta y_t}$$

This yields the following result.

Proposition 3.3. *The equilibrium risk-free rate is given by:*

$$r_t - \beta - \hat{g}_t = -\beta y_t \alpha_{ht} \alpha_{It} \pi_{Rt} - \frac{\left(\sigma_d - \beta y_t \alpha_{ht} \alpha_{It} \sigma_{Rd,t}\right)^2}{1 - \beta y_t}.$$

Thus, in the intermediation-unconstrained region, the risk-free rate is given by:

$$r_t - \beta - \hat{g}_t = -\beta y_t \left(\frac{\pi_{Rt}}{\sigma_{Rd,t}} - \frac{f_t}{\sigma_d}\right)\frac{\pi_{Rt}}{\sigma_{Rd,t}} - \frac{1}{1 - \beta y_t}\left(\sigma_d - \beta y_t \left(\frac{\pi_{Rt}}{\sigma_{Rd,t}} - \frac{f_t}{\sigma_d}\right)\right)^2.$$

When the economy is in the intermediation-constrained region, the risk-free rate becomes:

$$r_t - \beta - \hat{g}_t = -m(1 - \beta y_t)\frac{\pi_{Rt}^2}{\sigma_{Rd,t}^2} - \frac{1}{1 - \beta y_t}\left(\sigma_d - m\frac{\pi_{Rt}}{\sigma_{Rd,t}}\left(1 - \beta y_t\right)\right)^2.$$

Thus, the risk-free rate is increasing in the expected long-run dividend growth rate and decreasing in the Sharpe ratio in both the constrained and the unconstrained regions

of the economy. Asymptotically, as $y_t \to +\infty$, the risk-free rate becomes:

$$\lim_{y_t \to +\infty} r_t = -\infty.$$

Intuitively, as the household becomes the dominant agent in the economy, the demand for borrowing by the specialist decreases, while the supply of credit by the households increases, driving the equilibrium interest rate to $-\infty$. At the other extreme, as $y_t \to 0$, the equilibrium risk-free rate converges to:

$$\lim_{y_t \to 0} r_t = \beta + \hat{g}_t - \sigma_d^2.$$

Thus, as the specialist becomes the dominant agent in the economy, the risk-free rate is determined by his beliefs about the long-run expected dividend growth rate.

4 Numerical Illustration

In this section, I examine the behavior of the equilibrium asset prices and portfolio allocation choices for some calibrated parameters. For the parameters of the dividend growth process, the long-run mean of the dividend growth process, and the external signal, I adapt the calibration of Bansal and Yaron (2004) for my specification. The capacity of the specialist to process information, κ, is chosen to make the initial information choice 0:

$$\kappa = \frac{1}{2}\frac{\gamma_0}{\sigma_d^2},$$

and the prior variance of the specialist's (γ_0) and the household's (γ_{h0}) belief are chosen to be 1. I explore the impact of varying the marginal cost of observing better information, θ, on the equilibrium asset prices. As in He and Krishnamurthy (2010, 2011), I choose the

maximal leverage of the specialists, m, to be 4. As discussed in He and Krishnamurthy (2010), the choice of m corresponds to the equity capital constraints of banks and hedge funds, since these institutions are more likely to be the marginal buyers of risky assets during periods when equity capital is constrained. Finally, the time discount rate in the economy is chosen to be 1%. The parameters used for this numerical illustration are summarized in Table I.

4.1 Portfolio allocations

Begin by considering the optimal portfolio choices of the specialist and the household. Fig. 3 and 4 plot the percentage of intermediary capital allocated to the risky asset (α_I) and the percentage of household's wealth allocated to the intermediary (α_h), respectively. Notice first, that as the specialists become more optimistic relative to the households (so that f becomes more positive), α_I increases and α_h increases. Intuitively, as the specialists become more optimistic, they are willing to invest more in the risky asset while the opposite holds true for the households. In particular, when the specialists and the households agree on the expected dividend growth rate (so that $f = 0$) or when the specialists are pessimistic relative to the households (so that $f < 0$), the no shorting constraint begins to bind for the households, constraining their allocation to the intermediary to only be as large as their wealth. At the other extreme, when the household becomes extremely pessimistic relative to the intermediary, the household faces the constraint that it cannot short the intermediary and is forced to put all its wealth in the risk-free asset.

When the specialist is more optimistic than the household, as the relative importance of the household in the economy increases, the household's allocation into the intermediary increases, until the economy enters into the intermediation-constrained region. Intuitively, as the household becomes a larger fraction of the economy, asset prices are increasingly

more reflective of the household's belief, in addition to that of the specialist, making households more likely to invest in the risky asset. This effect also accounts for the opposite behavior of the specialist's portfolio choice: for large optimism on the part of the specialist, the fraction of intermediary capital allocated to the intermediary increases as the scaled wealth of the household increases while the household is constrained to invest 0, but decreases when the household is finally able to enter the market. When the beliefs of the specialist and the household are more in sync, the fraction of intermediary capital allocated to the risky asset increases for some range out the household's scaled wealth in the region where the household is not constrained, but decreases for larger values of y.

Notice that, while the marginal cost of information acquisition θ only has a small effect on the peak of intermediary investment in the risky asset, the cost does have a large effect on the degree of divestment when the intermediation constraint binds. In particular, for the case of low cost of information acquisition, the specialists divest more aggressively, driving up risk premia. This is similar to the hold-up problem encountered in the contracting literature[1]: when the cost of acquiring information about an asset is high, the specialist is willing to hold on to the asset longer to recuperate the costs associated with the investment in information.

Notice finally that either when the household is constrained to invest 0 is the intermediary (low values of y, high values of f) or when the household is constrained to invest 1 is the intermediary (low values of y, low values of f) or when the economy is in the intermediation-constrained region (high values of y, low values of f), the portfolio allocation choice does not depend on the disagreement between the specialist and the household.[2] Intuitively, because of one constraint or the other, the household is precluded

[1]See e.g. Klein et al. (1978); Williamson (1979, 1985); Grossman and Hart (1986); Hart and Moore (1990)

[2]See Fig. 2 for a graphical illustration of the four regions of the economy.

from participating effectively in the risky asset market and, thus, the difference between beliefs does not play a role. Disagreement does however determine for which relative sizes of the household sector the economy enters one of the constrained sectors.

4.2 Asset prices

Consider next risk premium in the economy (Fig. 5). Since the specialists are the agents that hold the risky assets in this economy, it is not surprising that the risk premium in the economy increases as the specialists become more optimistic. Further, as the relative wealth of the household increases, the risky asset is distributed across a smaller mass of specialists, increasing the required risk premium. This effect becomes more pronounced in the intermediation-constrained region, since the specialists cannot fully supplement their own funds with household contributions in clearing the risky asset market, driving the risk premium up. Notice also that, while the effect of changing the marginal cost of information (θ) was small for the optimal portfolio allocations, higher costs of processing information reduces the risk premium. Intuitively, since the information acquisition costs are paid by the specialist, increasing the cost of information is equivalent to increasing the long-run equilibrium time-discount rate of the specialist, lowering the risk premium that specialists are willing to pay for holding the risky asset.

The risky asset volatility (Fig. 6) mimics the behavior of the risk premium: as the specialists become more optimistic relative to the households or as the scaled household wealth becomes larger, the risky asset volatility increases. Intuitively, since disagreement is mean-reverting around zero, as the specialists become more optimistic, the probability of disagreement declining increases, increasing the volatility. As the household becomes more dominant in the economy, the risky asset is distributed among a smaller mass of specialists, so shocks to the dividend growth rate become amplified. Notice, however,

that the increases in the risk premium are not perfectly off-set by the increase in the risky asset volatility: the Sharpe ratio of the risky asset (Fig. 7) is also increasing in the disagreement between the specialist and the household and in the scaled wealth of the household.

Consider finally the risk-free rate in the economy (Fig. 8). As the specialist becomes more optimistic relative to the household, the risk-free rate decreases. Intuitively, as the specialist becomes more optimistic, the household is less willing to invest in the risky asset and, hence, more willing to invest in the risk-free asset, lowering the interest rate. The risk-free rate also decreases as the household becomes more dominant in the economy. Thus, as the scaled household wealth increases, the lending demand by specialists is distributed across a larger mass of households, lowering the risk-free rate. Notice that this is the risk-free asset market counterpart to the mechanism that increases the risk premium (and the Sharpe ratio) as specialists become less dominant in the economy. Since the asset markets must clear in equilibrium, the relative size of the natural owners of the two types of assets (households for the risk-free asset and specialists for the risky asset) impacts the level of the expected return in the corresponding markets.

Notice finally that, as with optimal portfolio choice, asset prices do not depend on the disagreement between the specialist and the household when either the household is constrained to invest 0 is the intermediary (low values of y, high values of f) or the household is constrained to invest 1 is the intermediary (low values of y, low values of f) or the economy is in the intermediation-constrained region (high values of y, low values of f). Since the households are constrained in their portfolio choice in those regions, they are infra-marginal investors and their beliefs do not impact asset prices.

5 Empirical Analysis

In this section, I conduct some simple exploratory empirical analysis to examine testable predictions of the model. First, during "normal" times, stock volatility should be directly impacted by the disagreement between the households and the specialists, whereas the individual beliefs of the households only enter indirectly through an impact on risk premia. Second, during periods when the specialists are liquidity constrained, household beliefs should have a direct impact on volatility. Finally, the beliefs of both specialists and households should command a risk premium during normal times, with the risk premium on household beliefs increasing during period when specialists are liquidity constrained. The empirical evidence is consistent with these predictions, although the degree of statistical significance varies across specifications.

5.1 Data

In the empirical analysis, I interpret the disagreement between specialists and households broadly and focus on variation in the pessimism about the overall future prospects of the economy. As a proxy for the beliefs of the households, I use the Michigan Survey of Consumer Expectations. Each month, 500 individuals are randomly selected from the contiguous United States (48 states plus the District of Columbia) to participate in the Surveys of Consumers. The questions asked cover three broad areas of consumer confidence: personal finances, business conditions, and future buying plans, with the respondents being asked to provide an assessment of both the current conditions and their future expectations. The index is then constructed as follows: the number of negative responses to each question is subtracted from the number of positive responses to the question. The three resulting numbers are then averaged, with the index ranging between

-100 and +100.

I use the "anxious" index produced by the Federal Reserve Bank of Philadelphia as a proxy for specialist beliefs. The anxious index measures the probability of a decline in real GDP, as reported in the Survey of Professional Forecasters. The survey asks panelists to estimate the probability that real GDP will decline in the quarter in which the survey is taken and in each of the following four quarters. The anxious index is the probability of a decline in real GDP in the quarter after a survey is taken. For example, in the survey taken in the first quarter of 2012, the anxious index is 13.4 percent, which means that forecasters believe there is a 13.4 percent chance that real GDP will decline in the second quarter of 2012.

To make the two measures of beliefs compatible, I scale them to have zero mean and unit variance. Further, since an increase in the Consumer Expectations Index implies an improvement in households' outlook, I reverse the sign of the Index before scaling. The measure of disagreement is then constructed as the difference between the scaled Consumer Expectations and the scaled Anxious Index.

Monthly observations of the two indexes are plotted in Fig. 9. As has been documented in the literature, both the Anxious Index and the Consumer Expectations Survey are leading indicators of the business cycle, with the two measures increasing during recessions. While both measures exhibit counter-cyclical behavior, they do no covary perfectly, as can be seen from the resulting measure of disagreement in Fig. 10. Disagreement between the specialists and the households increases during booms, and decreases during recessions.

5.2 Stock market volatility

According to the model, during normal times, an increase in the pessimism of households relative to the specialists should lead to an increase in stock market volatility as households start to exit the intermediation relation. To assess the significance of this association, I consider the following regression:

$$VC_t = a + bf_t + cVC_{t-1} + e_t.$$

Here, VC_t is the realized stock market volatility, computed using daily returns of the S& P 500 index within the given quarter. Adding the lagged observations of volatility removes most of the serial correlation in VC. I compute Newey-West standard errors with three lags, and verify that using six lags leads to identical conclusions.

Table II reports the OLS estimates of the coefficients of interest and their t-statistics. As the model predicts, an increase in the pessimism of households relative to the specialists leads to more volatility, with the coefficient $b > 0$ on the measure of disagreement. The coefficient is statistically significant in all three specifications, suggesting that stocks are indeed more volatile when specialists are more pessimistic than households.

The model also predicts that households beliefs *per se* should not impact volatility strongly during normal times (they do so only through an indirect impact on the risk premium) but, during periods when the specialists are liquidity-constrained, increased household pessimism should increase volatility. The last two regressions test this specification:

$$VC_t = a + bf_t + cVC_{t-1} + d\hat{g}_{ht} + e_t$$

$$VC_t = a + bf_t + cVC_{t-1} + d\hat{g}_{ht} + h\hat{g}_{ht}\mathbf{1}_{Recession} + e_t.$$

While the signs of the coefficient on household pessimism and on the interaction term between household pessimism and an NBER recession dummy are positive as predicted by the model, the coefficient on household beliefs is only marginally statistically significant and the interaction term coefficient is not statistically significant.

5.3 Equity risk premium

The model predicts that, during normal times, specialists' optimism and the relative pessimism of increase the risk premium paid on risky assets. To asset the significance of this association, I consider the following regression:

$$R_{t+1} = a + b\hat{g}_t + cf_t + e_t.$$

Here, R_{t+1} is the cumulative return on the CRSP value-weighted market portfolio over the quarter following the date of the survey. As before, I compute Newey-West standard errors with three lags.

Table III reports the coefficients of interest and their t statistics. Although the signs of the estimated coefficients on specialists' optimism and the disagreement between households and specialist coincide with the model-predicted ones, they are statistically insignificant. There might be no unconditional risk premium associated with these measures of specialists' beliefs and the disagreement between households and specialists. It is also possible, though, that 44 years of quarterly data is not enough to ensure sufficient power for this test.

The model also predicts, that conditional on the specialists being liquidity constrained, disagreement commands a smaller risk premium. The following regression tests that hypothesis by introducing an interaction term between disagreement and an NBER recession

dummy:

$$R_{t+1} = a + b\hat{g}_t + cf_t + df_t \mathbf{1}_{Recession} + e_t.$$

While, as expected, the coefficient on the interaction term between disagreement and the recession dummy is negative, it is also not statistically significant.

6 Conclusion

As information potentially available to (sophisticated) market participants increases, optimal information acquisition and the costs of acquiring and processing information have a larger effect on asset prices. This paper considers the problem of information acquisition in an intermediated market, with the specialist given access to superior technology of acquiring information. The disparity in the learning technologies of the specialist and the household introduces endogenous disagreement between the specialist and the household, changing the shape of the intermediation-constrained region of the economy, and increasing the frequency of periods of when the intermediation constraint binds. Relative optimism on the part of the specialist increases the risk premium, risky asset return volatility and the Sharpe ratio of the risky asset, but decreases the fraction of household wealth allocated to the intermediary sector. I find some empirical support for the model's key predictions.

In the long-run information equilibrium of the economy, costly information acquisition makes the specialists less likely to decrease their risky asset holding when the intermediation constraint binds. Intuitively, as the cost of observing signals increases, the specialist is willing to participate more in the risky asset market to recuperate the costs associated with information acquisition. This in turns leads the risk premium, risky asset return

volatility and the Sharpe ratio of the risky asset to increase less dramatically in the intermediation-constrained region, since the risky asset is distributed across a larger mass of agents.

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A Proofs

A.1 Proof of Proposition 2.2

Denote by J the value function of the specialist at time t:

$$J(w_t, \hat{g}_t, \gamma_t, f_t, \gamma_{ht}, y_t) = \max_{\{c_s, \alpha_{Is}, n_s\}} \mathbb{E}\left[\int_t^{+\infty} e^{-\beta s} \log c_s ds\right].$$

We will guess and verify that J has the form:

$$J(w_t, \hat{g}_t, \gamma_t, f_t, \gamma_{ht}, y_t) = \frac{1}{\beta} \log w_t + \Gamma(\hat{g}_t, \gamma_t, f_t, \gamma_{ht}, y_t),$$

where Γ is an unknown function to be determined. Then the HJB of the specialist is given by:

$$\begin{aligned}
\log w_t + \beta\Gamma = & \max_{c_t, \alpha_{It}, \sqrt{n_t}} \log c_t + \frac{1}{\beta w_t}\left(-c_t + w_t r_t + \alpha_{It} w_t \pi_{Rt} - w_t \frac{\theta}{2}\gamma_t \frac{n_t}{\sigma_e^2}\right) - \frac{1}{2\beta}\alpha_{It}^2 \sigma_{Rd,t}^2 \\
& + \Gamma_{\hat{g}} \kappa_g (\bar{g} - \hat{g}_t) + \frac{1}{2}\Gamma_{\hat{g}\hat{g}} \gamma_t^2 \left(\frac{1}{\sigma_d^2} + \frac{n_t}{\sigma_e^2}\right) - \Gamma_f \left(\kappa_g + \frac{\gamma_{ht}}{\sigma_d^2}\right) f_t \\
& + \frac{1}{2}\Gamma_{ff}\left(\left(\frac{\gamma_t - \gamma_{ht}}{\sigma_d}\right)^2 + \frac{\gamma_t^2 n_t}{\sigma_e^2}\right) - \Gamma_\gamma \left(2\kappa_g \gamma_t - \sigma_g^2 + \gamma_t^2 \left(\frac{1}{\sigma_d^2} + \frac{n_t}{\sigma_e^2}\right)\right) \\
& - \Gamma_{\gamma_h}\left(2\kappa_g \gamma_{ht} - \sigma_g^2 + \frac{\gamma_{ht}^2}{\sigma_d^2}\right) + \Gamma_y \mu_{yt} + \frac{1}{2}\Gamma_{yy}\sigma_{yd,t}^2 \\
& + \Gamma_{f\hat{g}}\left(\left(\frac{\gamma_t - \gamma_{ht}}{\sigma_d}\right)\frac{\gamma_t}{\sigma_d} + \frac{\gamma_t^2 n_t}{\sigma_e^2}\right) + \Gamma_{fy}\sigma_{yd,t}\left(\frac{\gamma_t - \gamma_{ht}}{\sigma_d}\right) + \Gamma_{\hat{g}y}\sigma_{yd,t}\frac{\gamma_t}{\sigma_d} \\
& + \phi_{1t}\frac{\gamma_t}{\sigma_e} n_t + \phi_{2t}\left(\kappa - \frac{\gamma_t}{2}\left(\frac{1}{\sigma_d^2} + \frac{n_t}{\sigma_e^2}\right)\right),
\end{aligned}$$

where $\phi_{1t} \geq 0$ is the time t Lagrange multiplier on the no forgetting constraint $n_t \geq 0$ and $\phi_{2t} \geq 0$ is the time t Lagrange multiplier on the capacity constraint. Taking the first order conditions, we obtain:

$$\begin{aligned}
[c_t]: &\quad c_t = \beta w_t \\
[\alpha_{It}]: &\quad \alpha_{It} = \frac{\pi_{Rt}}{\sigma_{Rd,t}^2} \\
[\sqrt{n_t}]: &\quad \frac{\theta}{\beta} = \left(\frac{1}{2}\Gamma_{\hat{g}\hat{g}} - \Gamma_\gamma + \frac{1}{2}\Gamma_{ff} + \Gamma_{f\hat{g}}\right)\gamma_t + \phi_{1t} - \frac{\phi_{2t}}{2}.
\end{aligned}$$

Since the first order condition for n_t does not depend on n_t, the specialist always chooses his attention allocation to be at the capacity constraint.

A.2 Proof of Proposition 2.3

Denote by J^h the value function of the household at time t:

$$J^h(w_{ht}, \hat{g}_t, \gamma_t, f_t, \gamma_{ht}, y_t) = \max_{\{c_{hs}, \alpha_{hs}\}} \mathbb{E}\left[\int_t^{+\infty} e^{-\beta_h s} \log c_{hs} ds\right].$$

Similarly to the specialist's problem, guess that J^h has the form:

$$J^h(w_{ht}, \hat{g}_t, \gamma_t, f_t, \gamma_{ht}, y_t) = \frac{1}{\beta_h} \log w_{ht} + \Gamma^h(\hat{g}_t, \gamma_t, \hat{g}_{ht}, \gamma_{ht}, y_t),$$

where Γ^h is an unknown function to be determined. Denote by $\lambda_t/(\beta_h w_{ht}) \geq 0$ the Lagrange multiplier on the time t intermediation constraint, $\eta_{1t} \geq 0$ the Lagrange multiplier on the time t no intermediary shorting constraint of the household and by η_{2t} the time t Lagrange multiplier on the no risk-free bond shorting constraint. Then the HJB of the household is given by:

$$\begin{aligned} w_{ht} + \beta_h \Gamma^h = & \max_{c_{ht}, \alpha_{ht}} \log c_{ht} + \frac{1}{dt} \mathbb{E}\left[d\Gamma^h \middle| \mathcal{F}_t^h\right] \\ & + \frac{1}{\beta_h w_{ht}} \left(-c_{ht} + w_{ht} r_t + \alpha_{ht} \alpha_{It} w_{ht} (\mu_{Rt}^h - r_t)\right) - \frac{1}{2\beta_h} \alpha_{ht}^2 \alpha_{It}^2 \left(\sigma_{Rd,t}^h\right)^2 \\ & + \frac{\lambda_t}{\beta_h w_{ht}} \left(m w_t - \alpha_{ht} w_{ht}\right) - \eta_{1t} \alpha_{ht} + \eta_{2t} \left(1 - \alpha_{ht}\right). \end{aligned}$$

Taking the first order conditions, we obtain:

$$\begin{aligned} [c_{ht}]: \quad & c_{ht} = \rho_h w_{ht} \\ [\alpha_{ht}]: \quad & \alpha_{ht} = \frac{\mu_{Rt}^h - r_t}{\alpha_{It} \left(\sigma_{Rd,t}^h\right)^2} - \frac{\lambda_t + \eta_{1t} + \eta_{2t}}{\alpha_{It}^2 \left(\sigma_{Rd,t}^h\right)^2}. \end{aligned}$$

In the unconstrained region, $\lambda_t = \eta_{1t} = \eta_{2t} = 0$, so that (2.14) obtains.

σ_d	0.15
κ_g	0.0210
$\bar{g}$	0.0015
σ_g	3.43×10^{-5}
σ_e	0.225
κ	22.22
θ	(0.5, 1)
m	4
β	1%

Table I: Parameters used for numerical illustrations. The parameters of the dividend growth process (σ_d), of the long-run mean of dividend growth (κ_g, $\bar{g}$, σ_g) and of the external signal (σ_e) are a version of the Bansal and Yaron (2004) calibration, modified to the setting of this paper. The capacity of the specialist to process information κ is chosen to make zero precision of the external signal in the initial period optimal. The maximal leverage of the specialists, m is taken from He and Krishnamurthy (2010).

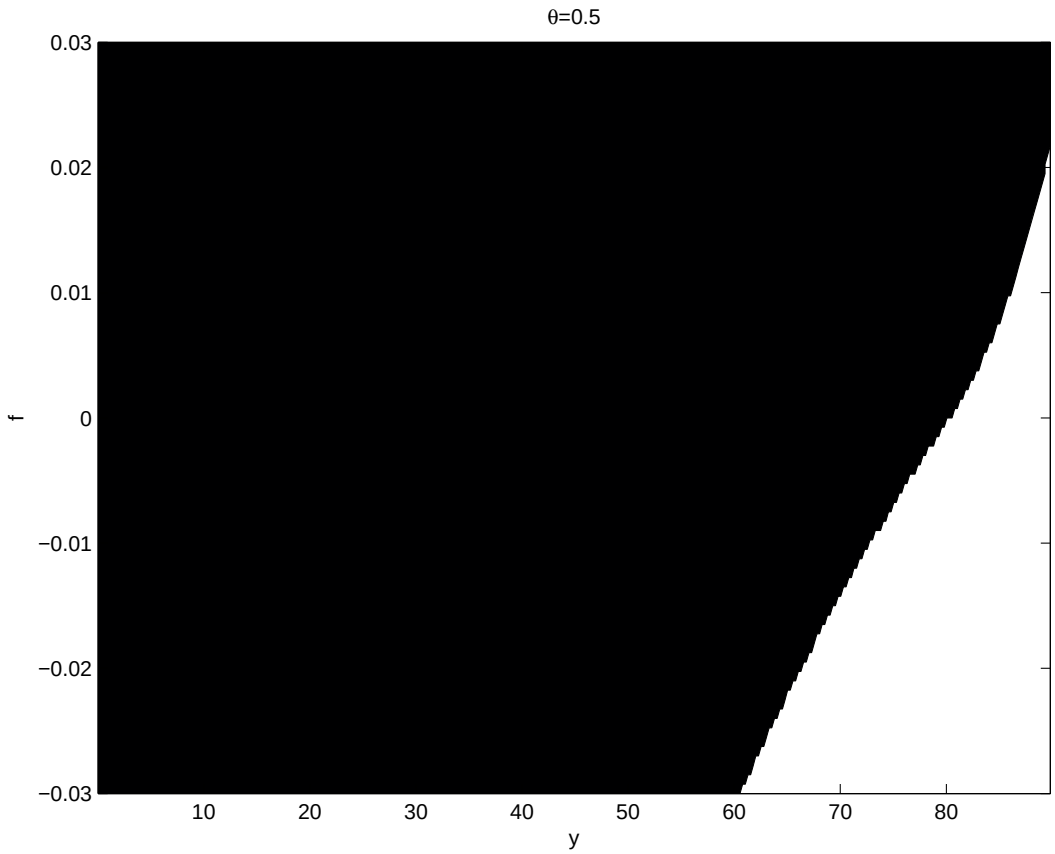

Figure 1: Intermediation-constrained and unconstrained regions of the economy as a function of the disagreement between specialists and households (f) and the relative wealth of the households (y). The unconstrained region is pictured in black and the constrained in white.

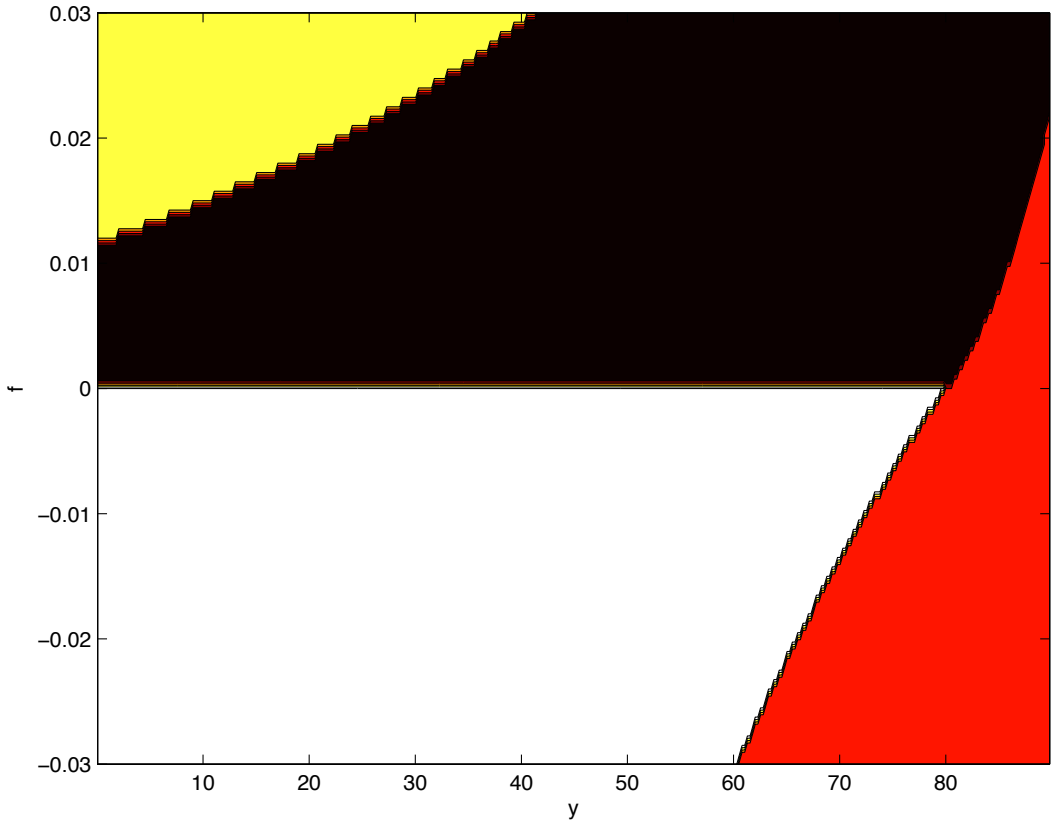

Figure 2: Intermediation-constrained (red), unconstrained (black), $\alpha_{ht} = 0$ (yellow) and $\alpha_{ht} = 1$ (white) regions of the economy as a function of the disagreement between specialists and households (f) and the relative wealth of the households (y).

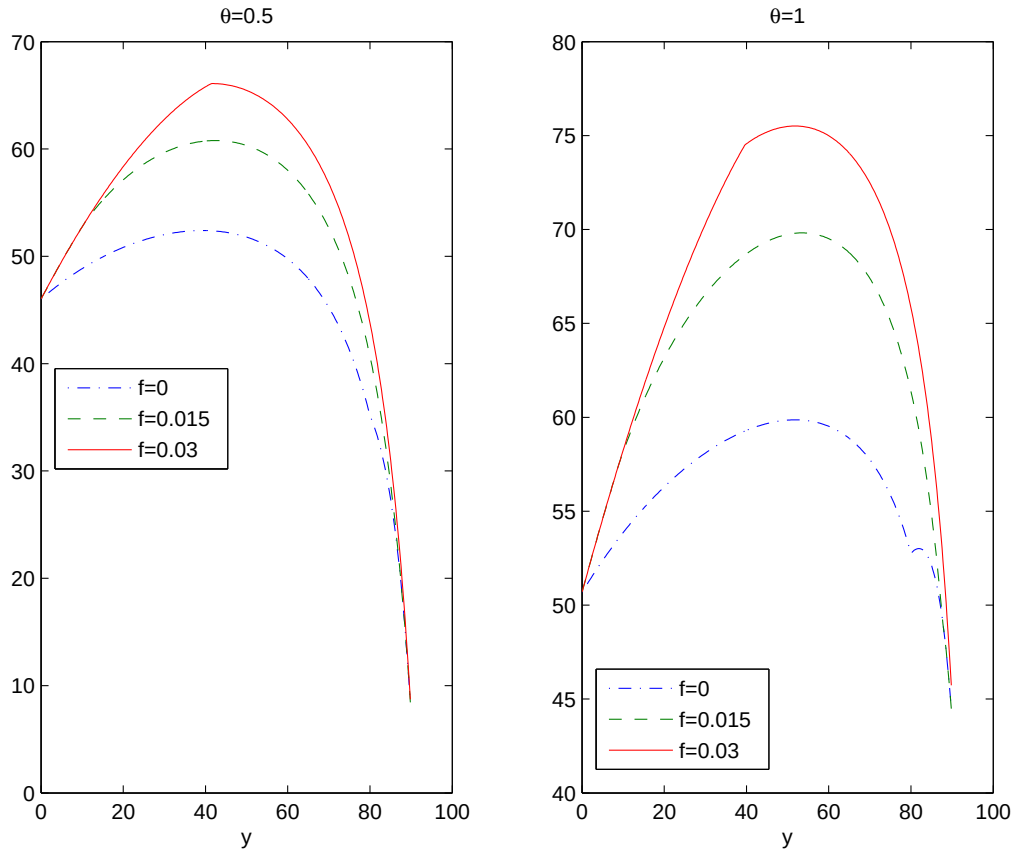

Figure 3: The percent of intermediary capital allocated to the risky asset (α_{It}) in the economy as a function of the disagreement between specialists and households (f) and the relative wealth of the households (y), for two different levels of the marginal cost of acquiring additional information (θ).

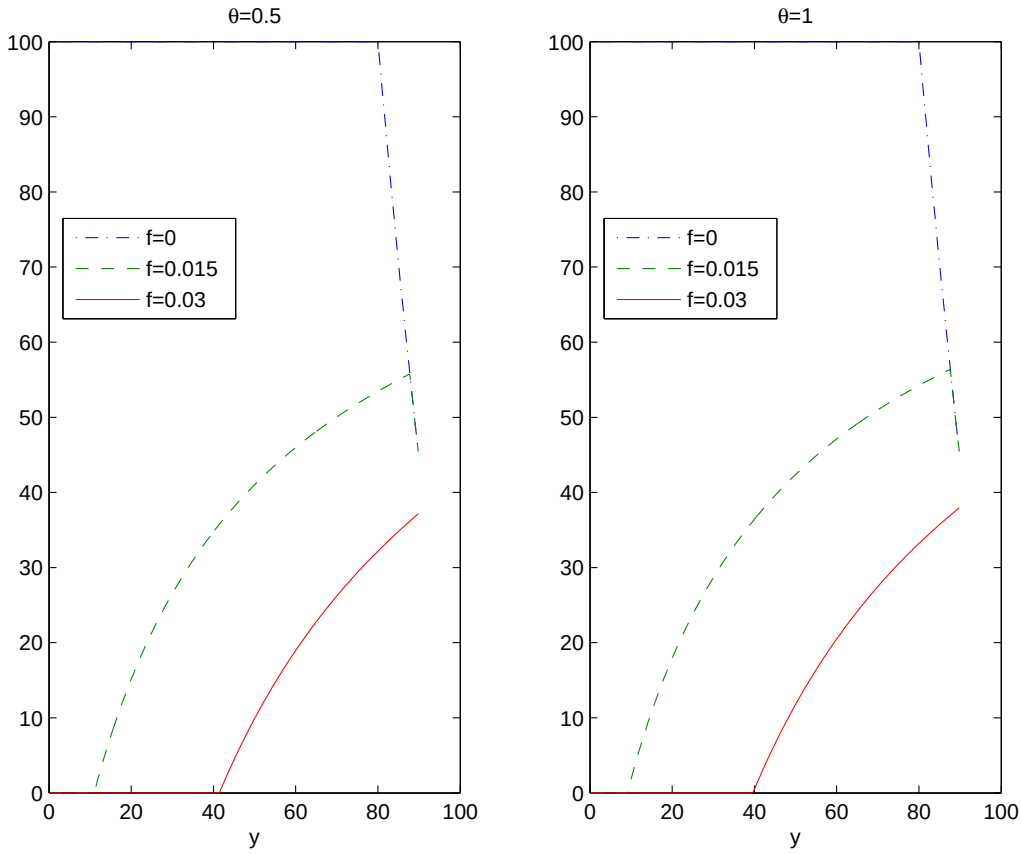

Figure 4: The percent of household wealth allocated to the intermediary (α_{ht}) in the economy as a function of the disagreement between specialists and households (f) and the relative wealth of the households (y), for two different levels of the marginal cost of acquiring additional information (θ).

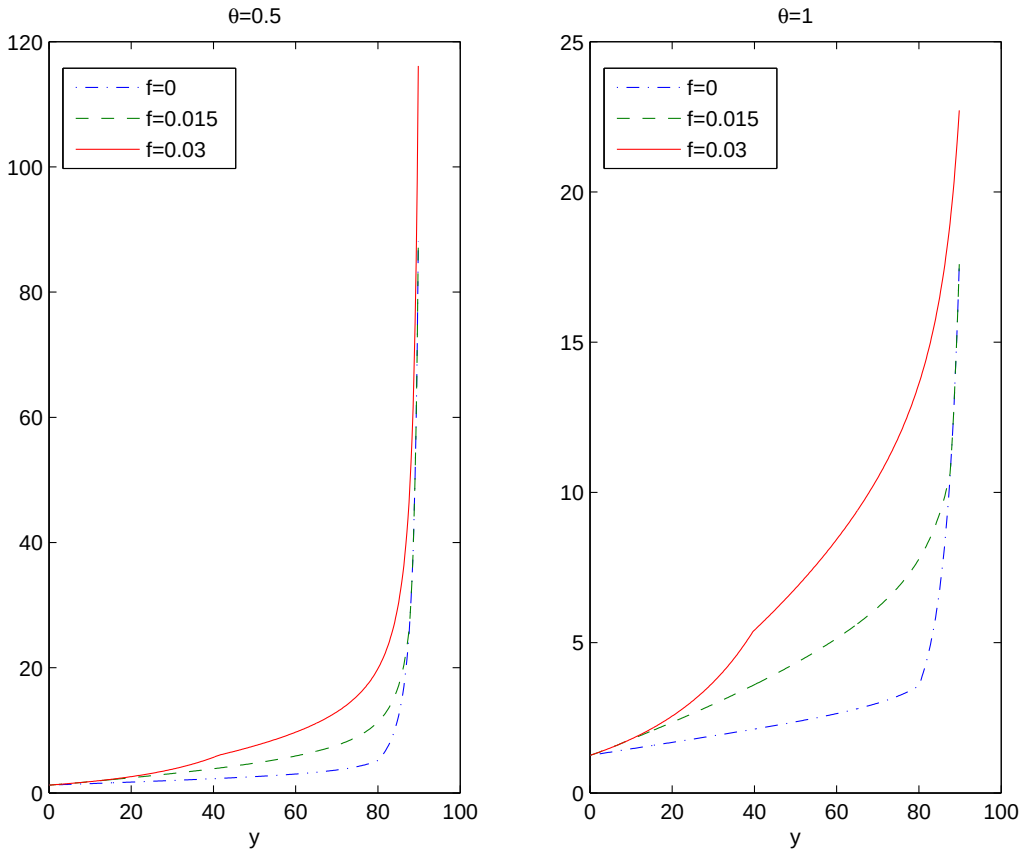

Figure 5: The risk premium (π_{Rt}) in the economy as a function of the disagreement between specialists and households (f) and the relative wealth of the households (y), for two different levels of the marginal cost of acquiring additional information (θ).The risk premium is reported in percentage terms.

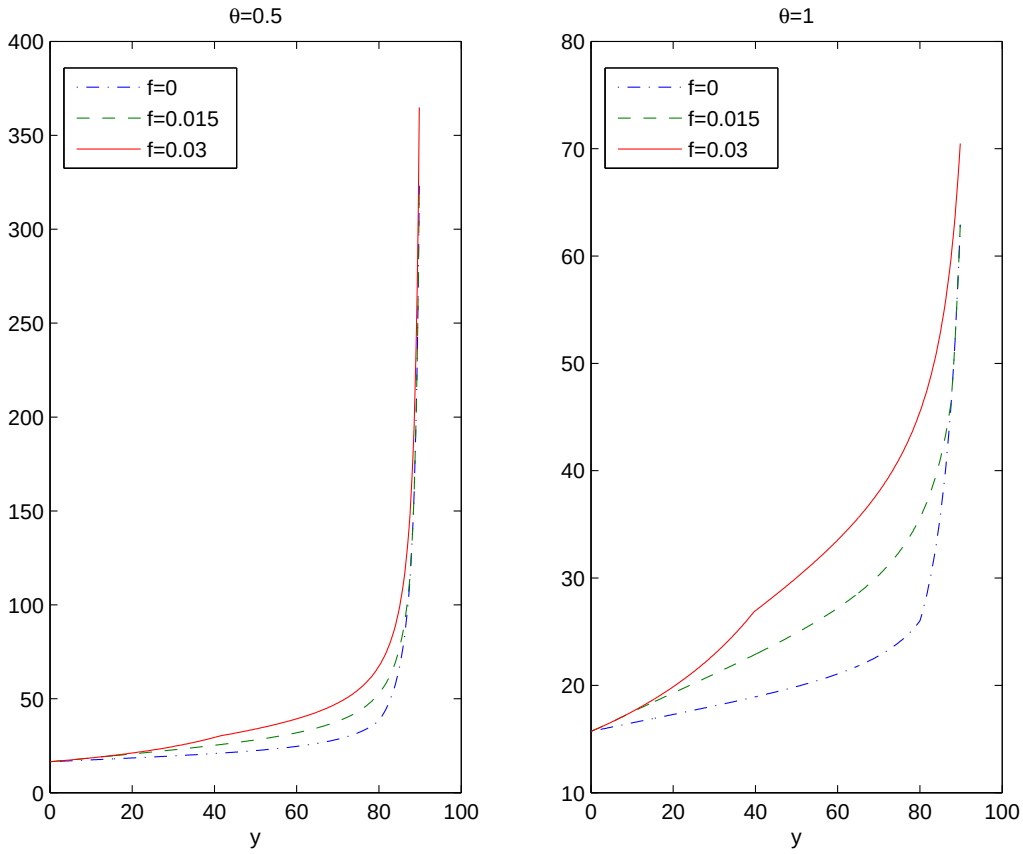

Figure 6: The risky asset volatility (σ_{Rt}) in the economy as a function of the disagreement between specialists and households (f) and the relative wealth of the households (y), for two different levels of the marginal cost of acquiring additional information (θ).The risky asset volatility is reported in percentage terms.

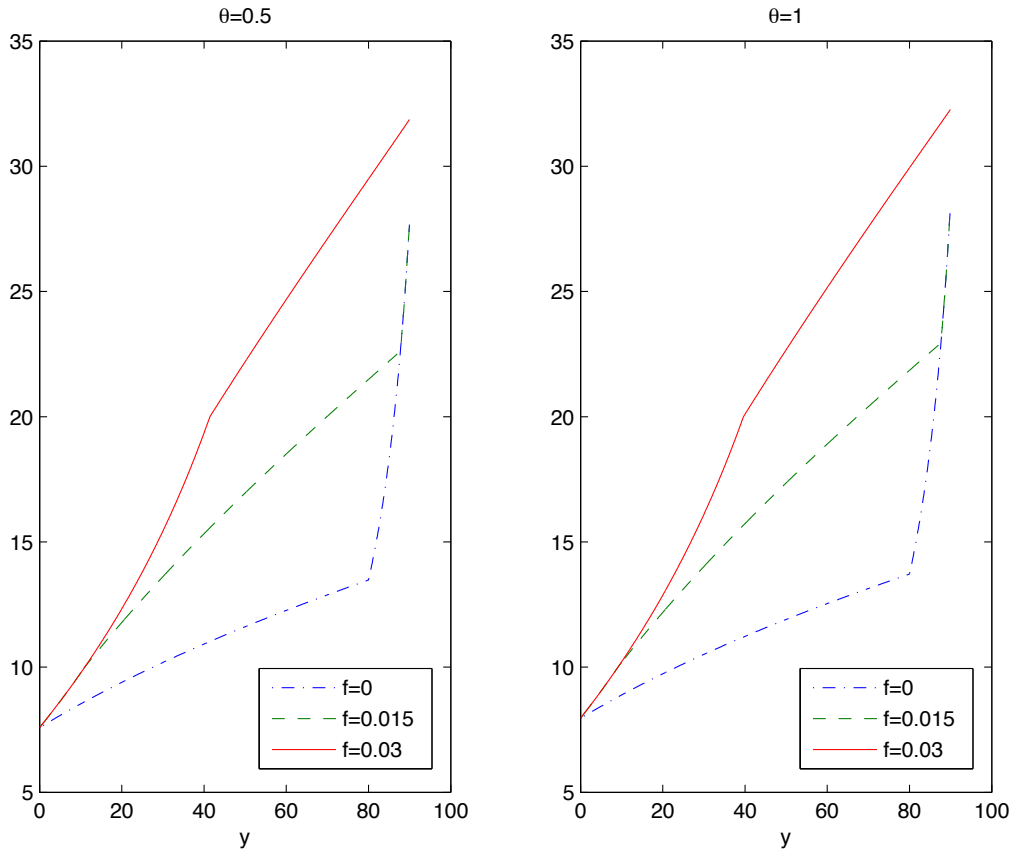

Figure 7: The Sharpe ratio (π_{Rt}/σ_{Rt}) in the economy as a function of the disagreement between specialists and households (f) and the relative wealth of the households (y), for two different levels of the marginal cost of acquiring additional information (θ).The Sharpe ratio is reported in percentage terms.

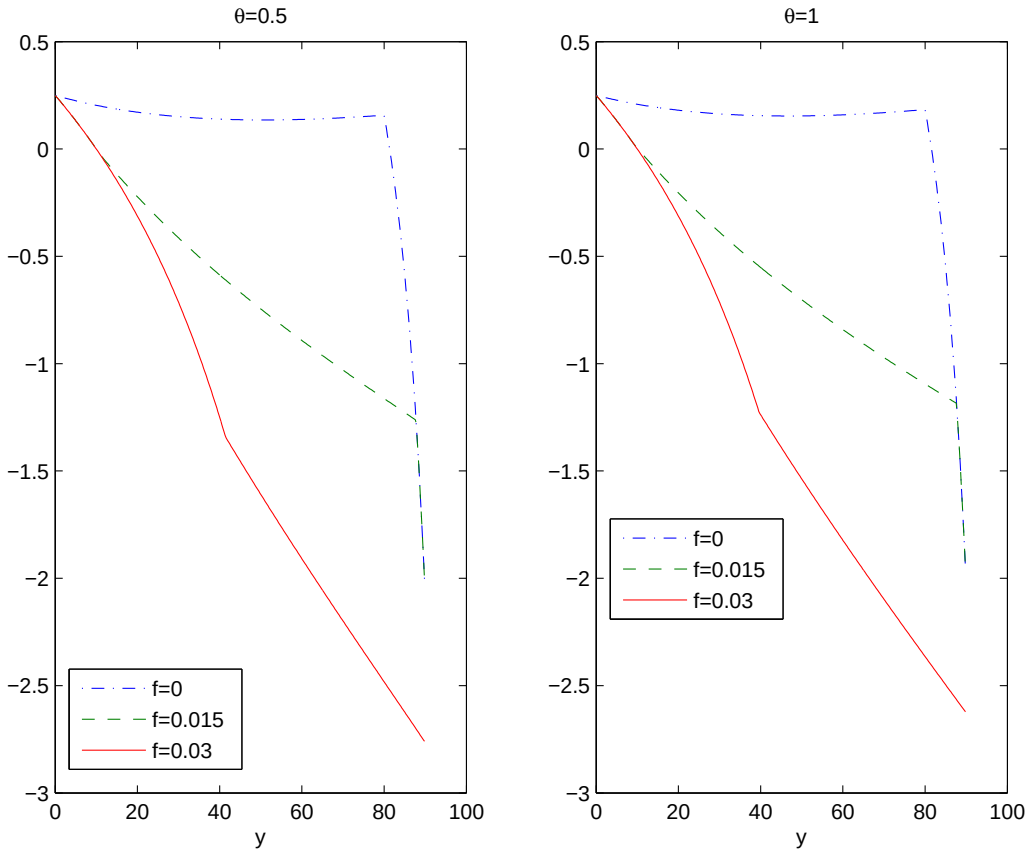

Figure 8: The risk-free rate (r_t) in the economy as a function of the disagreement between specialists and households (f) and the relative wealth of the households (y), for two different levels of the marginal cost of acquiring additional information (θ). The risk-free rate is reported in percentage terms.

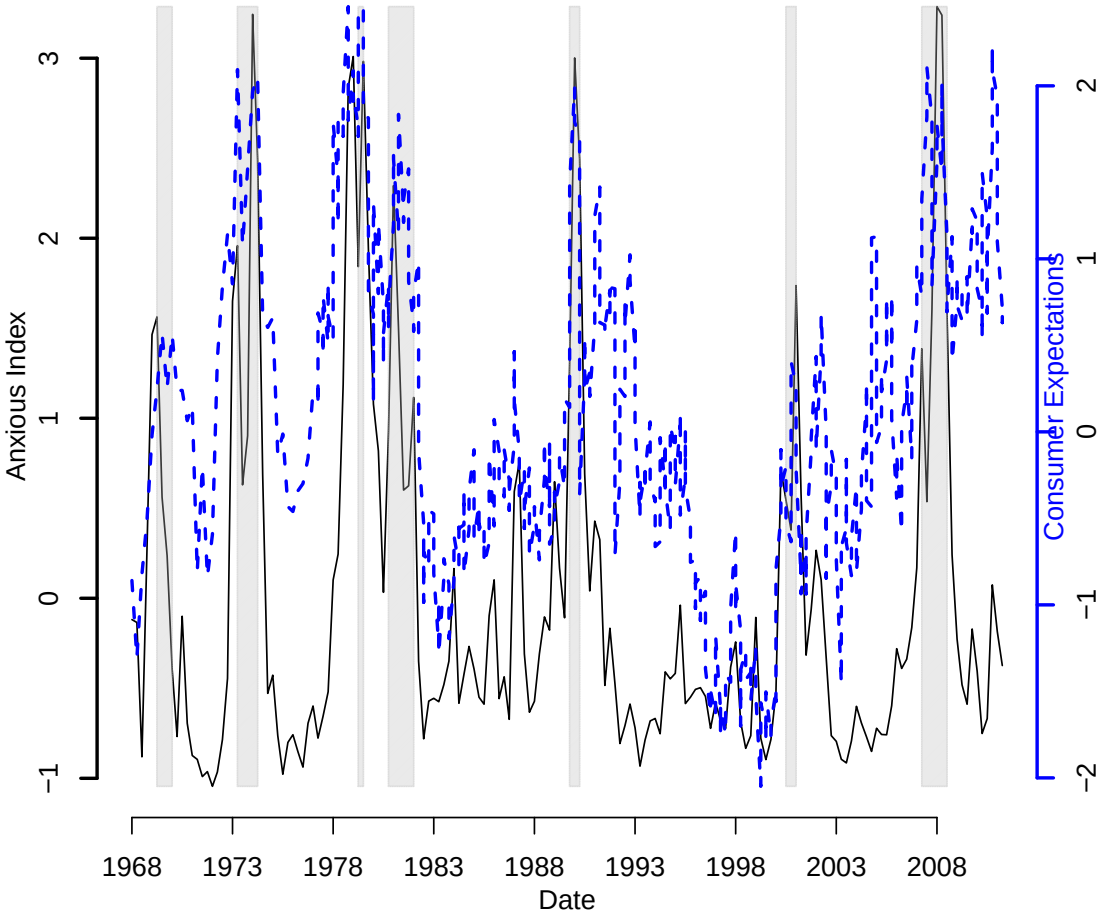

Figure 9: The time series evolution of the Philadelphia Fed Anxious Index (left scale, solid line) and the Michigan Consumer Expectations Survey (right scale, dashed line). Both time series have been scaled to have mean zero and unit variance. NBER recessions are highlighted in grey. The Michigan Consumer Expectations Survey has been transformed so that an increase in the level implies increased pessimism. Data source: Federal Reserve Bank of Philadelphia, Haver DLX.

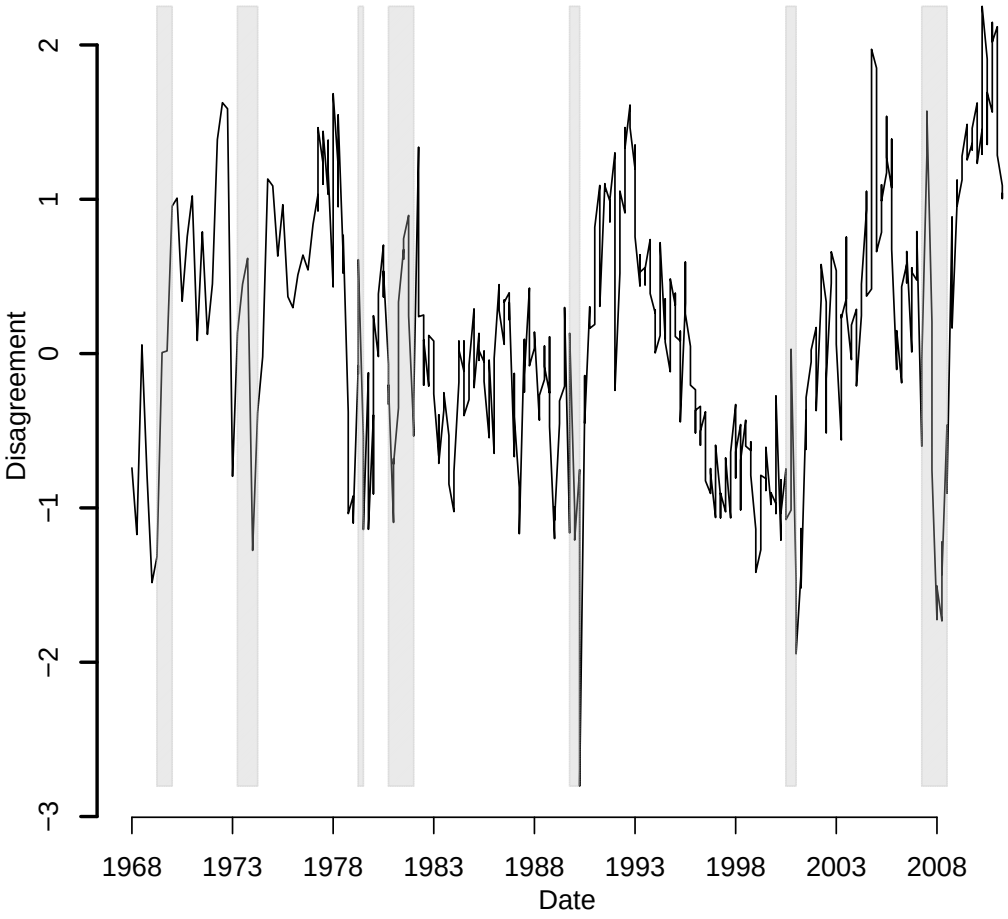

Figure 10: The time series evolution of the disagreement between specialists and households. Disagreement is measured as the difference between the Philadelphia Fed Anxious Index and the Michigan Consumer Expectations Survey, with the two components scaled to have mean zero and unit variance. NBER recessions are highlighted in grey. The Michigan Consumer Expectations Survey has been transformed so that an increase in the level implies increased pessimism. Data source: Federal Reserve Bank of Philadelphia, Haver DLX.

Table II: Disagreement and Return Volatility

This table addresses the questions "Does disagreement increase return volatility during normal times?" and "Do household beliefs impact volatility when economic conditions are worse?". The table reports the estimated slope coefficients and their t-statistics from the following regressions:

$$
\begin{aligned}
VC_t &= a + bf_t + cVC_{t-1} + e_t \\
VC_t &= a + bf_t + cVC_{t-1} + d\hat{g}_{ht} + e_t \\
VC_t &= a + bf_t + cVC_{t-1} + d\hat{g}_{ht} + h\hat{g}_{ht}\mathbf{1}_{Recession} + e_t.
\end{aligned}
$$

f_t is the measure of disagreement between specialists and households, computed as the difference between the Michigan Survey of Consumer Expectations Index and the Anxious Index of the Federal Reserve Bank of Philadelphia. VC_t stands for the realized volatility of the S& P 500 Index, computed from daily index returns within the quarter. $\hat{g}_{ht}$ denotes the household beliefs, as measured by he Michigan Survey of Consumer Expectations Index, and $\hat{g}_{ht}\mathbf{1}_{Recession}$ is the interaction term between household beliefs and an NBER recession dummy. The t-statistics are computed based on Newey-West standard errors with three lags. The sample period is Q4 1968 through Q1 2012.

Disagreement	HH Beliefs	HH Beliefs$\times\mathbf{1}_{Recession}$
0.04		
(1.92)		
0.06	.04	
(2.38)	(1.87)	
0.05	.03	.03
(2.31)	(1.40)	(.65)

Table III: Disagreement and Expected Returns

This table addresses the questions "Do specialist beliefs and disagreement between specialists and households command a risk premium during normal times?" and "Does the risk premium on disagreement decrease when economic conditions are worse?". The table reports the estimated slope coefficients and their t-statistics from the following regressions:

$$R_{t+1} = a + b\hat{g}_t + cf_t + e_t$$
$$R_{t+1} = a + b\hat{g}_t + cf_t + df_t\mathbf{1}_{Recession} + e_t.$$

R_{t+1} is the aggregate stock market return in excess of the one month T-bill rate over the quarter following quarter t. f_t is the measure of disagreement between specialists and households, computed as the difference between the Michigan Survey of Consumer Expectations Index and the Anxious Index of the Federal Reserve Bank of Philadelphia. $\hat{g}_t$ stands for specialists' beliefs, proxied for by the Anxious Index of the Federal Reserve Bank of Philadelphia. $f_t\mathbf{1}_{Recession}$ is the interaction term between disagreement and an NBER recession dummy. The t-statistics are computed based on Newey-West standard errors with three lags. The sample period is Q4 1968 through Q1 2012.

Anxious	Disagreement	Disagreement$\times\mathbf{1}_{Recession}$
-0.25	.85	
(-0.21)	(.86)	
-0.25	0.85	-0.008
(-0.24)	(0.82)	(-0.003)