NONLINEAR RISK[1]

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Abstract

This paper proposes a flexible framework for analyzing the joint time series properties of the level and volatility of expected excess stock returns. An unobservable dynamic factor is constructed as a nonlinear proxy for the market risk premia with its first moment and conditional volatility driven by a latent Markov variable. The model allows for the possibility that the risk-return relationship may not be constant across the Markov states or over time. We find a distinct business cycle pattern in the conditional expectation and variance of the monthly value-weighted excess return. Typically, the conditional mean decreases a couple of months before or at the peak of expansions, and increases before the end of recessions. On the other hand, the conditional volatility rises considerably during economic recessions. With respect to the contemporaneous risk-return dynamics, we find an overall significantly negative relationship. However, their correlation is not stable, but instead varies according to the stage of the business cycle. In particular, around the beginning of recessions, volatility increases substantially reflecting great uncertainty associated with these periods, while expected returns decrease anticipating a decline in earnings. Thus, around economic peaks there is a negative relationship between conditional expectation and variance. However, towards the end of a recession expected returns are at its highest value as an anticipation of the economic recovery, and volatility is still very high in anticipation of the end of the contraction. That is, the risk-return relation is positive around business cycle troughs. This time-varying behavior also holds for non-contemporaneous correlations of these two conditional moments.

KEY WORDS: Expected Excess Return, Risk premia, Conditional Variance, Dynamic Factor, Markov Process.

JEL Classification Code: C32 , E32, E44, G12

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1. INTRODUCTION

In the last twenty years great progress has been made in modeling the relation between risk and expected return. Most of this research has focused on the single-period risk-return tradeoff among different securities. There is general agreement that riskier securities are rewarded by larger expected returns, within a given time period. However, there are less obvious conclusions about the joint dynamics of risk and return over time. On a market-wide level, there is no consensus in most related empirical work concerning the temporal behavior of both stock market returns and their volatility, although there is substantial evidence of nonlinearity in their dynamics.[1] In particular, recent findings show that a distinct pattern is revealed in expected stock returns and their conditional variances when they are grouped according to the state of the business cycle.[2] This implies that stocks may bear more risk at some times than others, but it is not indisputable whether investors require larger risk premium on average during times when stocks are more risky.

Theory also does not yield unambiguous insights about the relationship between risk and excess return. Backus and Gregory (1993), for example, find that theoretical models are consistent with virtually any sort of relationship between excess return and its conditional variance proxying for risk, depending on model preferences and the probability structure across states. Further, using equilibrium asset pricing models, one would expect the relationship between excess return and variables proxying for corporate cash flows and investors' discount rates to be nonlinear.

Related empirical research has focused on modeling the dynamics of time-varying conditional second moments of stock returns as proxies for risk premia. From a theoretical point

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[11] Fama and Schwert (1977), Campbell (1987), Nelson (1991) or Glosten, Jagannathan, and Runkle (1993), among others, find a negative relation between conditional expected stock return and variance. On the other hand, Chan, Karolyi, and Stulz (1992) find no statistically significant relationship between expected return and conditional variance in the U.S. stock market. Others, such as French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1992) find a positive relation between expected returns and conditional second moments.

[2] For example, Whitelaw (1994), Fama and French (1989) and particularly Perez-Quiros and Timmermann (1996) find evidence of a significant state dependence in the conditional distribution of stock returns, with financial variables proxying for risk forecasting business cycle phases.

of view the predictability of the level and volatility of returns should be connected.[3] Thus, rather than modeling them separately, considerable effort has gone into modeling their joint dynamic behavior. New models such as ARCH, GARCH and stochastic volatility (SV) have been developed to capture the persistence in the volatility of returns. The main empirical framework of the joint determination of the conditional mean and variance of returns is the ARCH-M, in which time varying conditional second moments account for changes in risk premia. The underlying assumption of these models is that risk premium on assets can be represented as linear increasing functions of their conditional covariance with the market.[4]

In this paper we are particularly interested in constructing an empirical framework that does not impose an a priori structure between the conditional mean and volatility of stock returns. We estimate an unobservable dynamic factor as a nonlinear proxy for the market risk premia with first and second conditional moments driven by a latent two-state Markov variable. That is, we consider the possibility that market return and its volatility are not necessarily related together directly but are a function of a third variable - the Markov process, which represents the state of financial market conditions.

In addition to offering a flexible description of the joint time series properties of the level and volatility of expected stock returns, our approach captures potential asymmetric responses by investors to changes in risk, depending on their perception of the state of business conditions. The two Markov states can be interpreted as bull and bear markets.[5] These values could be associated with an increasing relation between mean and variance for the market returns. However, they could be associated with low mean and high variance and high mean and low variance as well. In our framework, expected stock returns can be higher or lower during periods when the market is more volatile. It could be the case, for example, that in those times investors desiring to hedge against risk might move back and forth from stock to bonds, driving changes in

[3] That is, predictability of the level implies predictability of volatility. However, if the level of returns is difficult to predict, it does not imply that the volatility should be.

[4] Further, the ARCH-M restricts the conditional mean of excess returns to be positive despite the evidence from regressions that in certain periods the excess return is predicted to be negative (e.g. Whitelaw 1994, Perez-Quiros and Timmerman 1996, Pesaran and Timmerman 1995, among others).

[5] In stock market jargon, bear market are periods of persistent decrease in stock prices. Thus, bear markets are also associated with periods when the excess return is negative.

expected stock returns and the direction of the risk-return relation according to the stage of the economy.

The proposed framework allows the use of multivariate information with a parsimonious variance-covariance structure to produce the sort of predictions obtained from regression models. In contrast, most ARCH, GARCH and SV models use only the information contained in returns. The multivariate information is introduced by constructing a stock market index, subject to switches between bull and bear markets, from a range of financial variables, as in Chauvet and Potter (1997). We also examine the risk-return relationship for stocks from different firm sizes, which captures potential asymmetric behavior of returns across financial states, depending on different market capitalization. Ultimately, forecasts of excess returns can be obtained from forecasts of the mean and volatility of the stock market index, which allows analysis of their behavior across the Markov states. We study the dynamics of their contemporaneous correlation as well as correlations at leads and lags.

In terms of results, we find a significant asymmetric behavior of conditional excess returns according to firm size. In particular, excess returns on stocks of small firms, as proxied by the CRSP equal-weighted index, are more reactive to changes in the state of financial markets than large firms. In addition, a business cycle pattern is present in the conditional expectation and variance of the value-weighted excess return. Typically, the conditional mean decreases a couple of months before or at the peak of expansions, and increases before the end of recessions. On the other hand, the conditional volatility rises considerably during economic recessions.

With respect to the risk-return relation, during bear markets expected excess returns are low while the conditional volatility is high. In bull markets, the conditional mean increases while the volatility decreases. However, the contemporaneous correlation is not stable, but varies according to the state of the business cycle. In particular, around the beginning of recessions, volatility increases substantially reflecting great uncertainty associated with these periods, while expected returns decrease anticipating a decline in earnings. Thus, around economic peaks there is a negative relationship between conditional expectation and variance. However, towards the end of a recession expected returns are at its highest value as an anticipation of the economic recovery, and volatility is still very high in anticipation of the near end of the contraction. That is, the risk-return relation is positive around business cycle troughs. This time-varying behavior also holds for non-contemporaneous correlations of these two conditional moments.

The paper is organized as follows. The second section describes the model and interprets nonlinear risk premium within the Markov switching dynamic factor framework. The third and fourth sections discuss estimation and analytical description of the updating and conditional variances of the excess returns. In the fifth section, the empirical results are presented and compared to extant literature. The sixth section concludes and suggests directions for future research.

2. MODEL DESCRIPTION

We propose modeling expected excess returns on stocks, Y_{kt}, as a function of a common unobserved dynamic factor, F_t, and individual idiosyncratic noises, ε_{kt}. The factor captures market-wide comovements underlying these stocks, and it is a parsimonious proxy for the market risk premium:

$$Y_{kt} = \lambda_{jk} F_t + \varepsilon_{kt}, \qquad k = 1,...,4;\ j = 0,1 \tag{1}$$

$$\varepsilon_{kt} \sim \text{ i.i.d.} N(0, \Sigma).$$

In a first specification, Y_{kt} is a 4x1 vector of monthly excess stock return (defined as the difference between continuously compounded stock returns and a T-bill rate) on the valued-weighted index, the equal-weighted index, IBM stock, and GM stock. In a second specification, Y_{kt} includes other financial variables such as price-earnings ratio, dividend yield, the 3-month T-bill rate, in addition to the excess return on the valued-weighted index. The factor loadings λ_{jk}, measure the sensitivity of the k^{th} series to the market risk premia, F_t in Markov state j. The factor loading for the value-weighted excess return is set equal to one in both states to provide a scale for the unobservable variable F_t.[6]

In order to examine potential changes in conditional excess return and in its volatility across different states of the financial markets, we allow the first and second moments of the

[6] Generally, researchers set the factor variance to one or give it a scale in the same units as one of their regression coefficients. For the case in which only the mean switches, normalization can be achieved by setting the factor variance to one. For the models we are interested here, with a switching factor variance, normalization is attained by setting one of the factor loadings to unity.

factor to switch regimes according to a Markov variable, S_t, representing the state of financial conditions:

(2) $$F_t = \alpha_1 + \alpha_0 S_t + \phi F_{t-1} + h_{S_t} \qquad S_t = 0,1$$

$$F_t \sim (m_{S_t}, q_{S_t}) \qquad h_{S_t} \sim \text{i.i.d. } N(0,\ s^2_{h_{S_t}}),$$

that is, financial markets can be either in an expansion period (bull market), S_t=0; or in a contraction state (bear market), S_t=1, with the switching ruled by the transition probabilities of the first-order two-state Markov process, p_{ij}=Prob[S_t=j|S_{t-1}=i], $\sum_{j=0}^{1} p_{ij} = 1$, i, j = 0,1. The dynamic factor is, therefore, a representation of nonlinear market risk across Markov states. Cyclical variation in the nonlinear risk is generated from shocks common to each of the Y_{kt} observable variables, η_t, and all idiosyncratic movements arise from the term ε_{kt}. That is, we assume that η_t and ε_{kt} are mutually independent at all leads and lags, for all k = 1,…,4, for each model specification.

The dynamic factor is the common element among the financial variables and is produced as a nonlinear combination of the observable variables Y_{kt}. This factor has a time-varying conditional mean and variance and, therefore, should play a role in determining the time series behavior of market risk premia. This framework does not impose a priori relation between the level and volatility of excess returns. Thus, conditional volatility could either be higher or lower in the bear market than in the bull market.

Different specifications are estimated in which the factor loadings λ_{jk} may or may not switch across states. Modeling the factor loadings as state dependent allows the model to capture potential asymmetric behavior of returns across financial market states, depending on the size of the firm. We use the excess returns on IBM and GM stocks to represent large firms, and the excess returns on the equal-weighted index to proxy for the dynamic behavior of small firms. The excess return on the value-weighted index represents market premium.

3. ESTIMATION AND ANALYSIS OF CONDITIONAL MOMENTS

The parameters of the model are estimated using a nonlinear discrete version of the Kalman filter combined with Hamilton's (1989) nonlinear filter in one algorithm, as suggested by Kim (1994). The model is cast in state-space form, where equations (1) and (2) are, respectively, the measurement and transition equations The goal of the nonlinear filter is to form forecasts of the factor and the associated mean squared error matrices, based not only on information available up to time t-1, $I_{t-1} \equiv [Y'_{t-1}, Y'_{t-2}, \ldots, Y'_1]'$, but also on the Markov state S_t taking on the value j, and on S_{t-1} taking on the value i. That is:

(3) $$F_{t|t-1}^{(i,j)} = E(F_t \mid I_{t-1}, S_t = j, S_{t-1} = i)$$

(4) $$\theta_{t|t-1}^{(i,j)} = E[(F_t - F_{t|t-1})(F_t - F_{t|t-1})' \mid I_{t-1}, S_t = j, S_{t-1} = i)],$$

where $F_{t|t-1} = E(F_t \mid I_{t-1})$. The nonlinear Kalman filter is:

(5) $$F_{t|t-1}^{(i,j)} = \alpha_j + \phi F_{t-1|t-1}^{i}$$
(6) $$\theta_{t|t-1}^{(i,j)} = \phi^2 \theta_{t-1|t-1}^{i} + \sigma_{s_t}^2$$
(prediction equations)

(7) $$F_{t|t}^{(i,j)} = F_{t|t-1}^{(i,j)} + K_t^{(i,j)} N_{t|t-1}^{(i,j)}$$
(8) $$\theta_{t|t}^{(i,j)} = (I - K_t^{(i,j)} \lambda_j) \theta_{t|t-1}^{(i,j)}$$
(updating equations)

where $\alpha_j = \alpha_0 + \alpha_1^j$, $K_t^{(i,j)} = \theta_{t|t-1}^{(i,j)} \lambda_j' [Q_t^{(i,j)}]^{-1}$ is the Kalman Gain, $N_{t|t-1}^{(i,j)} = Y_t - \lambda_j F_{t|t-1}^{(i,j)}$ is the conditional forecast error of Y_t, and $Q_t^{(i,j)} = \lambda_j \theta_{t|t-1}^{(i,j)} \lambda_j' + \Sigma$ is its conditional variance. Hamilton's nonlinear filter is:

(9) $$\text{Prob}(S_{t-1} = i, S_t = j \mid I_{t-1}) = p_{ij} \sum_{h=0}^{1} \text{Prob}(S_{t-2} = h, S_{t-1} = i \mid I_{t-1})$$

From this joint conditional probabilities, the density of Y_t conditional on S_{t-1}, S_t, and I_{t-1} is:

(10) $$f(Y_t \mid S_{t-1} = i, S_t = j, I_{t-1}) = \{(2\pi)^{-k/2} |Q_t^{(i,j)}|^{-1/2} \exp(-\frac{1}{2} N_{t|t-1}^{(i,j)'} Q_t^{(i,j)-1} N_{t|t-1}^{(i,j)})\}$$

The joint probability density of states and observations is then calculated by multiplying each element of (9) by the corresponding element of (10):

(11) $$f(Y_t, S_{t-1}=i, S_t=j \mid I_{t-1}) = f(Y_t \mid S_{t-1}=i, S_t=j, I_{t-1})\, \text{Prob}(S_{t-1}=i, S_t=j \mid I_{t-1})$$

The probability density of Y_t given I_{t-1} is:

(12) $$f(Y_t \mid I_{t-1}) = \sum_{j=0}^{1} \sum_{i=0}^{1} f(Y_t, S_{t-1} = i, S_t = j \mid I_{t-1})$$

The joint probability density of states is calculated by dividing each element of (11) by the corresponding element of (12):

(13) $$\text{Prob}(S_{t-1} = i, S_t = j \mid I_t) = f(Y_t, S_{t-1} = i, S_t = j \mid I_{t-1}) / f(Y_t \mid I_{t-1})$$

Finally, summing over the states in (13), we obtain the filtered probabilities of bull or bear markets:

(14) $$\text{Prob}(S_t = j \mid I_t) = \sum_{i=0}^{1} \text{Prob}(S_{t-1} = i, S_t = j \mid I_t)$$

The link between the two filters arises as an approximation introduced through $F^{j}_{t|t}$ and $q^{j}_{t|t}$, which truncates the forecasts at each iteration. The approximation is required to make the filter computationally tractable, since at each date t the nonlinear filter computes 4 forecasts, and at each iteration the number of possible cases is multiplied by the number of states. The approximation consists of a weighted average of the updating procedures by the probabilities of the Markov state:

(15) $$F^{j}_{t|t} = \frac{\sum_{i=1}^{M} \text{Prob}[S_{t-1} = i, S_t = j|I_t] F^{(i,j)}_{t|t}}{\text{Prob}[S_t = j|I_t]},$$
$$q^{j}_{t|t} = \frac{\sum_{i=1}^{M} \text{Prob}[S_{t-1} = i, S_t = j|I_t] \{q^{(i,j)}_{t|t} + (F^{j}_{t|t} - F^{(i,j)}_{t|t})(F^{j}_{t|t} - F^{(i,j)}_{t|t})'\}}{\text{Prob}[S_t = j|I_t]}.$$

The nonlinear filter allows recursive calculation of the predicted equations using only observations on $\{Y_{kt}, k=1,\ldots,4\}$ given values for the parameters in ϕ, λ_j, α_j, p_{ij}, Σ and $s^2_{h_{S_t}}$, and initial inferences for the factor, $F^{j}_{t|t}$, the mean squared error, $q^{j}_{t|t}$, and the joint probability of the Markov-switching states. The outputs are their one-step updated values. This permits estimation of the unobserved state vector as well as the probabilities associated with the latent Markov state. A by-product of this algorithm is the conditional likelihood of the observable variable, which can be evaluated at each t. The log likelihood function is:

(16) $\text{Log } f(Y_T, Y_{T-1}, \ldots|I_0)) =$

$$\sum_{t=1}^{T} \log \sum_{j=0}^{1} \sum_{i=0}^{1} \{(2p^{-k/2}|Q^{(i,j)}_t|^{-1/2} \exp(-\frac{1}{2} N^{(i,j)'}_{t|t-1} Q^{(i,j)-1}_t N^{(i,j)}_{t|t-1})\} \text{Prob}(S_{t-1}=i, S_t=j \mid I_{t-1}).$$

The filter evaluates this likelihood function at each t, which can be maximized with respect to the model parameters using a nonlinear optimization algorithm. Thus, the factor is constructed as a nonlinear combination of the observable variables weighted by the probabilities of the Markov state, using information available through time t:

(17) $$F_{t|t} = E(F_t|I_t) = \sum_{j=0}^{1} \text{Prob}(S_t=j \mid I_t)\ F^{j}_{t|t}$$

The conditional moments of the excess returns are obtained from forecasts of the mean and volatility of the dynamic factor. From equations (1), (2) and from the nonlinear algorithm, the conditional expectation of excess returns are:

$$(18) \qquad E(Y_t|I_{t-1}) = \sum_j \lambda_j\{\alpha_j + \phi\, E(F_{t-1}|I_{t-1})\}\ \mathrm{Prob}(S_t = j|I_{t-1})$$

Notice that the value-weighted market excess return, $E(er_t^{vw}|I_t) = E(F_t|I_t)$ given that its corresponding factor loading is set to one, $\lambda_{vw}=1$. The conditional variances of excess returns are obtained from the Kalman iterations:

$$(19) \qquad \mathrm{Var}(Y_t|I_{t-1}) = \sum_j \sum_i \lambda_j\{q_{t|t-1}^{(i,j)}\ \mathrm{Prob}(S_t = j, S_{t-1} = i|I_{t-1})\}\lambda_j' + \Sigma,$$

which corresponds to the conditional variance of the forecast error of Y_t.

As seen in equations (18) and (19), the model does not impose a priori relation between the level and volatility of excess returns. In fact, expected excess return and its conditional volatility may not be related together directly but may be a nonlinear function of the state of financial market conditions, as represented by the Markov process. Thus, expected excess returns and conditional volatility could be positively or negatively associated or, they could exhibit no relationship at all.

4. EMPIRICAL RESULTS

Data and Models

Three specifications of the nonlinear dynamic factor model are estimated for monthly data from 1954.02 to 1997.12, in an application to the post-war U.S. financial market. In Models 1 and 2 Y_{kt} is composed of the excess return on the CRSP value-weighted index (VW), on the CRSP equal-weighted index (EW), on the IBM stock, and on the GM stock. The excess return is defined as the difference between continuously compounded stock returns and the 3-month T-bill rate in annual terms. In Model 3, Y_{kt} includes the 3-month T-bill rate (TB3), the S&P 500 price-earnings ratio (P/E) and dividend yield (Dyield), in addition to the value-weighted excess return. These data are from the 1997 release of the DRI Basic Economic Database. For state dependent factor loadings, as in Model 2, equation (1) is rewritten as:

$$
\begin{array}{ccccccc}
 & Y_{kt} & & l_k^{st} & & & \varepsilon_{kt} \\
(1') & \begin{vmatrix} er_{vwt} \\ er_{ewt} \\ er_{ibmt} \\ er_{gmt} \end{vmatrix} & = & \begin{vmatrix} 1 \\ l_{vew}^{st} \\ l_{vibm}^{st} \\ l_{vgm}^{st} \end{vmatrix} & F_t & + & \begin{vmatrix} e_{vwt} \\ e_{ewt} \\ e_{ibmt} \\ e_{gmt} \end{vmatrix}.
\end{array}
$$

For state dependent factor loadings and using financial variables other than excess returns, as in Model 3, equation (1) is substituted by:

$$
\begin{array}{ccccccc}
 & Y_{kt} & & l_k^{st} & & & \varepsilon_{kt} \\
(1'') & \begin{vmatrix} er_t^{vw} \\ \Delta \ln dyield_t \\ \Delta TB3_t \\ \Delta (P/E)_t \end{vmatrix} & = & \begin{vmatrix} 1 \\ l_{dyield}^{st} \\ l_{tb3}^{st} \\ l_{p/e}^{st} \end{vmatrix} & F_t & + & \begin{vmatrix} e_{wt} \\ e_{dyieldt} \\ e_{tb3t} \\ e_{p/et} \end{vmatrix}
\end{array}
$$

4.1. The Dynamic Financial Factors

In Models 1 and 2, excess returns are conditioned on the Markov process and on a latent factor that captures comovements on past values of different measures of excess returns. In Model 1, the mean and the volatility of the dynamic factor switch regimes, while in Model 2 we also allow the factor loadings to vary across the Markov states. Thus, we can examine the risk-return relationship for stocks from different firm sizes, and compare the results for potential asymmetric behavior of returns across financial states, depending on different market capitalization. In Model 3, excess return on the value-weighted index is conditioned on a switching latent factor constructed from comovements underlying past values of other financial variables, as described above. This framework, as in Chauvet and Potter (1997), allows the use of multivariate information with a parsimonious variance-covariance structure to produce the sort of predictions obtained from regression models — but in a nonlinear setting.

The maximum likelihood estimates are shown in Table 1. In all models, the two Markov states are statistically significant: state 0 exhibits negative mean, high volatility and a shorter average duration, which is associated with the short lasted and nervous bear markets. State 1 has a positive mean, low volatility and a longer average duration, capturing the features of bull markets. These results are similar to those found in Chauvet and Potter (1997) and Chauvet (1998b).

The likelihood ratio test for the null of one state model against the alternative of a Markov switching model has an unknown sampling distribution, since several of the classical assumptions of asymptotic distribution theory do not hold. Thus, we test for the number of states using the approach proposed by Garcia (1998), which is based on Hansen (1993). This likelihood ratio test provides strong evidence for the two-state model.[7]

With respect to the model assumptions, Brock, Dechert, and Scheinkman's (1996) BDS test for nonlinear models fails to reject the hypothesis of i.i.d. disturbances.[8] In addition, the one-step ahead forecast errors are not predictable by lags of the observable variables and their pairwise covariances are approximately zero.

Figure 1 plots the dynamic factor obtained from Model 2 against the value-weighted and equal-weighted excess stock returns.[9] The dynamic factor is highly correlated with these observed excess returns, particularly at turning points, representing bear and bull markets. Figure 2 shows the actual and conditional excess return on the value-weighted from Model 3. A remarkable feature of this model in comparison with linear regression models of excess returns is that here the expected excess returns mimic closely the volatility of the realized excess returns.

In all models, we set the factor loading of the value-weighted index to one (λ_{vw}=1).[10] Thus, we can compare the sensitivity of the other components to the factor in the same units as the value-weighted excess returns. Model 1 captures the empirical observation that small firms, as represented by the equal-weighted excess returns, are more reactive to the market (λ_{ew}=1.14), while stock returns on large firms such as IBM (λ_{ibm}=0.96) or GM (λ_{GM}=0.97) are less correlated with the market risk.

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[7] Although Garcia's critical values are designed for a univariate AR(1) regime switching model and the test is parameter dependent, the value of the likelihood ratios obtained here are about 3 times larger than the highest value in Garcia's table for the 1% significance level.

[8] For a vector $\varepsilon_t^m = \varepsilon_t, \varepsilon_{t+1},..., \varepsilon_{t+m-1}$, we use m=2, 3 and we set the distance d between any two vectors, ε_t^m and ε_s^m equal to the standard deviation of ε_t. The test estimates the probability that these vectors are within the distance d.

[9] The dynamic factors obtained from each of the models are qualitatively similar.

[10] The normalization affects only the scale of the factor. None of the time series properties of the dynamic factor or the correlation with its components is affected by the choice of the parameter scale.

Allowing the factor loadings to switch regimes, as in Model 2, we can capture the asymmetric behavior of returns depending on the size of the firm across financial market states. Table 2 summarizes these findings. While in bull markets excess stock returns of large and small firms exhibit a similar behavior (λ values around one), in bear markets firm size makes a difference. That is, during periods of low market excess return, small firms are the most reactive to market risk (λ_{ew}=1.41), while large firms are much less sensitive to the market (λ_{IBM}=0.85, λ_{GM}=0.98). That is, stock returns of large firms decrease less than small firm returns during bear markets.

To verify these results further, we fit an AR(0) univariate Markov switching model to each of the four components of the factor in Models 1 and 2, allowing both the mean and the volatility of the variables to switch regime. The estimated filtered probabilities of bear markets are plotted against NBER-dated recessions in Figure 3. The results confirm the nonlinearities underlying the factor model. The probabilities of bear markets from excess returns on small firms, as proxied by the equal-weighted index, are the most volatile and strongly react to most of the economic recessions in the sample data. On the other hand, the probabilities of bear markets for IBM and GM excess stock returns are less volatile and correspond less closely to the NBER dated economic recessions.

Table 3 reports dating of the U.S. stock market cycle phases. The framework adopted in this paper provides probabilities that can be used as filtering rules for dating turning points. We use information from the frequency distribution of the smoothing probabilities from Model 2 to define turning points: a peak (trough) occurs if the smoothing probabilities of bear markets are greater (smaller) than their mean plus one-half their standard deviation. The results for our sample data confirm the empirical observation that there have been more bear markets (10) than recessions (7), as measured by the NBER. With the exception of the 1960-61 recession, all others in the sample data were associated with a bear market. Generally, bear markets begin a couple of months before a recession and end in the middle of it, anticipating economic recovery. These findings are illustrated in Figure 4, which shows the smoothed probabilities of bear markets and the NBER recessions.

4.2. Conditional Moments of the Financial Factors

In order to empirically investigate the relationship between conditional expected excess return and its volatility, we derive these moments as described in Equations 18 and 19 of Section 3. Given the richer framework provided by state dependent factor loadings, we will analyze the findings from Models 2 and 3.[11] The models yield first and second conditional moments for each of the four components of the dynamic factor. In Model 2, the dynamic risk-return relationship for the value-weighted (VW), equal-weighted (EW), IBM, and GM stocks are all very similar. Thus, we will focus mainly on the results for the value-weighted excess return factor.

Figures 5 and 6 plot the conditional expectation and variance of the value-weighted excess return factor and NBER-dated recessions obtained from Models 2 and 3, respectively. Typically, the conditional mean decreases a couple of months before or at the peak of expansions, and increases before the end of recessions. On the other hand, the conditional volatility increases during economic recessions.

The conditional volatilities are very similar for both models, although the conditional expectations are less so. The results suggest that when conditioned only to past values of excess returns and to the state of the economy as proxied by the Markov process (Model 2), expected excess returns exhibit a very distinct bull and bear markets pattern. In particular, it has approximately the same unconditional mean (0.06 a year) and median (0.10 a year) as the realized value-weighted excess returns, and most values of the conditional expectation are close to the realized unconditional median. In contrast, when conditioned on other financial variables in addition to the Markov process (Model 3), expected excess return is less concentrated around the median. It mimics more closely the realized value-weighted excess return, particularly the amplitude of its oscillations (Figures 2 and 6). In both models expected excess return conditional on financial variables also displays business cycle dynamics – it decreases during expansions until reaching a minimum in the middle of a recession, and increases in the second half of a recession, reaching a maximum at its trough.

The counter-cyclical behavior of the conditional variance is also found by Whitelaw (1994), Timmerman and Perez-Quiros (1996), Harrison and Zhang (1997), Schwert (1989) or Kandell and Stambaugh (1990). However, the results in the literature for the conditional

[11] The likelihood ratio between Models 1 and 2 rejects Model 1 at the 0.5% significance level.

expectation are mixed. For example, Harrison and Zhang report procyclical or countercyclical expected returns depending on the conditioning variables.[12] Fama and French (1989), Whitelaw, and Timmerman and Perez-Quiros find a pattern for expected excess returns similar to ours, particularly for the results from Model 3.

Contemporaneous Relationship

Figure 7 plots scatter diagrams for the conditional expectation and conditional variance obtained from Model 2, for bear and bull markets as dated in Table 3.[13] During bear markets, expected excess returns are low while the conditional volatility is high. In bull markets, the conditional mean increases while the volatility decreases.[14] These findings are summarized in Figure 8, which shows the covariance of the conditional mean and variance in the form of a scatter-plot. That is, when the level and volatility of expected return are conditioned only to a Markov state variable and no a priori association is imposed on them, we find a significant contemporaneous negative risk-return relationship at the monthly frequency.

A negative but weak relationship is also found by Glosten, Jagannathan, and Runkle (1993), using a GARCH-M model. As discussed in Backus and Gregory (1993), negative, nonmonotonic or positive relationship between the first and second conditional moments of stock returns can arise from equilibrium models. The empirical literature reports mixed findings depending on the way the moments are modeled and the conditional variables used. The analysis of conditional moments from Model 2 can add to the discussion in that it reflects expectations based only of past information on different measures of excess returns and on the state of the economy, as represented by the Markov process. If the history of excess return subsumes to some extent all publicly available information from financial and economic variables, Model 2 has the advantage of tapering the problem of obtaining different results depending on the conditional

[12] The role of conditioning and mis-specification in determining the direction of the relationship is discussed by Glosten, Jagannathan, and Runkle (1993), Harvey (1991), and Pagan and Hong (1991), among others, particularly when a symmetric relation between risk-return is imposed.

[13] These results hold if we use different procedures to date bear/bull markets as well, such as different threshold values for calling a turn, or using the smoothing probabilities from Model 3, or using the bear market dating suggested by Niemira and Klein (1994).

[14] As discussed in Chauvet (1998b), stock market phases are closely associated with economic fluctuations.

financial variables chosen.

Using financial variables in addition to the Markov state, as in Model 3, allows us to study the role of conditioning variables and to compare our results to existing literature. As in Chauvet and Potter (1997), we condition excess stock returns on other financial variables that proxy for the market risk premia, such as price-earnings, dividend yield, and interest rates. These variables have been extensively used in related empirical work to verify the risk-return relationship. Figure 9 plots scatter diagrams for the mean and volatility of the excess return on the value-weighted conditional on financial variables for the whole sample. Notice that although the overall contemporaneous relation is still negative, the relation between these two moments is weaker than as found in Model 2, in which stock returns are conditioned only to the state of the economy. In particular, during bear markets the conditional expectation decreases and the volatility increases for low values of the conditional expectation. In bull markets the reverse occurs, also for low values of the conditional expectation (Figure 10). In fact, a closer examination of these diagrams suggest that there is a nonlinear behavior of these moments, depending on whether conditional expectations are positive or negative. Dividing the sample into periods when the conditional expectation is positive or negative shows a remarkable result – the risk-return relation is weakly positive if we exclude periods of negative conditional excess returns, and significantly negative otherwise (Figure 10). This nonlinear behavior may be behind the diversity of empirical results found in the literature regarding the risk-return relation.

Table 4 summarizes these results, showing the contemporaneous correlation between conditional expectation and variance of excess returns across financial cycle phases. For Model 2, the relation is negative independently on the stage of the financial cycle. However, when excess returns is conditioned on other financial variables in addition to the Markov state, the correlation is -0.91 for times when the conditional expectation is negative and 0.44 for periods when it is positive.

This finding may arise from the dynamics of conditional expected return near the trough of business cycle recessions, when expected return is at its highest value as an anticipation of the end of the recession, and volatility is still very high. In fact, we find that the conditional volatility is also at its highest values near peaks and troughs of business cycles (Figures 5 and 6).

Even though some would claim that intuition may point to a positive risk-return relation — during times of high volatility, investors might move from stock to bonds, driving expected returns up, the direction of the relation seems to depend on the state of the economy. Our results do not contradict this intuition, but indicates that it holds for some periods and not for others. First, we find that conditional variance moves up and down during economic recessions, reflecting the great uncertainty of these periods. The net effect is an increase in volatility during those times. Second, immediately before and during economic recessions, expected excess return reach its minimum and its maximum values. In addition, similar to the conditional volatility, expected excess returns also display up and down movements during bad times. Since the decrease in the expected excess returns is substantial (reaching negative values) at the peak of economic expansions, a net negative contemporaneous relationship between risk-return dominates for the whole sample. That is, the relation is strongly negative in the first half of recessions, and positive in the second half. This suggests that the risk-return dynamic relationship can be better understood if studied within and as a function of the different stages of the economy.

This result can also be illustrated by examining whether the Sharpe ratio is stable over time. To investigate this, we examine the linkages between the Sharpe ratio and fluctuations in economic activity. The Sharpe ratio or the price of risk corresponds to the conditional mean divided by the square root of the variance:

$$SR = E(Y_t|I_{t-1})/\sqrt{Var(Y_t|I_{t-1})}.$$

Table 5 reports a series of regressions of the price of risk on measures of business cycles, such as a 0/1 dummy variable representing recessions as dated by the NBER, changes in industrial production, and changes in the business cycle indicator generated by Chauvet (1998a).[15] We find that the regression coefficients are statistically significant in all the regressions and the Sharpe ratio displays a strong countercyclical business pattern (negative for the NBER recession dummy and positive for the others). This is also illustrated in Figures 11 and 12: in bear markets the conditional mean is low and the volatility is high, implying that the Sharpe ratio is low, while in bull markets, with a high conditional mean and low volatility, the Sharpe ratio is much higher, for both Models 2 and 3.

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[15] This monthly coincident indicator is constructed from a Markov switching dynamic factor using economic variables that move contemporaneously with business cycles, such as: sales, personal income, industrial production, and employment.

This time-varying risk-return relationship over the business cycle is also found in excess returns on large firms, as represented by IBM and GM stocks, and in small firms, as proxied by the excess returns on the equal-weighted index. Figure 13 plots the Sharpe ratio for the IBM, GM, and equal-weighted excess returns, respectively. Again, the price of risk falls during bear markets and increases in bull markets for each of the four components of the dynamic factor.

Non-Contemporaneous Correlations

We find that the contemporaneous relationship between expected excess returns and the conditional variance is time-varying within economic recessions. We further examine their correlation at leads and lags. Tables 6 and 7 and Figure 14 show the cross-correlogram between conditional excess return and variance for 30-month leads and lags of the conditional variance. For Model 2, their cross-correlation is negative and significant up to 9 months for leads and lags and weakly positive for leads and lags from 20 to 30 months. The offset correlations are symmetric implying that cyclical variations in risk and return are negatively related but coincident. Also, conditional variance seems to slightly lead its expected excess return. For Model 3, the relation is weaker. It is significantly negative up to two lags of the conditional variance and statistically insignificant for higher lags. For leads of the conditional variance, the relation is negative and significant up to 9 months. This result is also seen in Figure 14, which plots the cross-correlation between conditional expectation and variance against 30 leads and lags of the conditional variance. For Model 3, there is a negative and significant correlation between these two moments for small leads and lags, but using Granger causality and spectral analysis we find that expected excess return slightly leads volatility. No strong conclusion can be drawn from this, since the relationship between these two moments may be driven by a third variable - the state of the economy, as examined here. However, it seems that when excess returns are expected to be low, an immediate increase in market volatility follows as investors seek to move their position to hedge against noise, reflecting learning about the data as the state of the economy changes. This result is in contrast with Whitelaw (1994), who finds a weak contemporaneous relation, but a strong offset correlation, in which volatility leads expected returns across business cycle phases. On the other hand, Harrrison and Zhang (1995) find small and negative contemporaneous and offset correlations at the monthly frequency. Using spectral analysis, these authors find that there is no significant lead or lag relationship between the conditional moments, but a time-varying contemporaneous relationship. The difference in the results may arise from the alternative conditioning variables underlying these studies and the fact that Whitelaw assumes a linear risk-return relation.

Based on the findings of the previous session, an interesting question is whether the non-contemporaneous relationship is also non-stable for subsamples of the data. Table 8 reports the cross-correlation for periods of negative or positive conditional expectation. In fact, when the conditional expectation is positive, we find that offset correlations are positive and significant for up to 2 leads and lags. However, restricting the sample for times when the conditional expectation is negative, the offset correlation is significant and negative up to 2 leads and lags (Figure 15).

5. CONCLUSIONS

This paper proposes an empirical framework that offers a flexible description of the joint time series properties of the level and volatility of expected stock returns. An unobservable dynamic factor is built as a nonlinear proxy for the market risk premia with first moment and conditional volatility driven by a latent Markov variable. That is, we consider the possibility that the market expected return and its conditional volatility are not necessarily related together directly but are a function of a third variable — the two-states Markov process, which can be interpreted as bull and bear markets.

We find a significant asymmetric behavior of conditional excess returns according to firm size. In particular, excess returns on small firm stocks are more reactive to changes in the state of financial markets than large firms. In addition, a business cycle pattern is present in the conditional expectation and variance of the value-weighted excess return factor. Typically, the conditional mean decreases a couple of months before or at the peak of expansions, and increases before the end of recessions. On the other hand, the conditional volatility rises considerably during economic recessions.

With respect to the risk-return relation, during bear markets expected excess returns decrease while the conditional volatility increases. In bull markets, the conditional mean increases while the volatility decreases. That is, when the level and volatility of expected return are conditioned only to a Markov state variable and no a priori association is imposed on them, we find an overall contemporaneous negative risk-return relationship at the monthly frequency. This negative relation is less significant if other conditional financial variables are included.

However, this contemporaneous correlation is not stable, but instead varies according to the state of the business cycle. Around peaks and during the first half of economic recessions as measured by the NBER, their relation is negative. However, during the second half of economic recessions, the trade-off between risk and return is positive. This result arises from the dynamics of conditional expected returns near business cycle peaks and trough. Around the beginning of recessions, volatility increases considerably reflecting great uncertainty associated with these periods, while expected returns decrease anticipating a decrease in earnings. Thus, there is a negative relationship between conditional expectation and variance. Towards the end of a recession, expected returns are at its highest value as an anticipation of the economic recovery, and volatility is still very high in anticipation of the end of the contraction. In fact, we find that the conditional volatility is at its highest values near peaks and troughs of business cycles. Thus, during times of high volatility, investors might move back and forth from stock to bonds, driving changes in expected returns and the direction of the relation depending on the stage of the economy. This time-varying behavior also holds for non-contemporaneous correlations. When the conditional expectation is positive, we find that offset correlations between conditional mean and variance are positive and significant for shorter leads and lags. However, restricting the sample for times when the conditional expectation is negative, the offset correlation is significant and negative. The results suggest that the contemporaneous and offset risk-return relationship change over time, as a result of the dynamics of conditional expected returns around business cycle peaks and troughs.

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Table 1: Maximum Likelihood Estimates
1954.2-1997.12

Model 1:
$Y_{kt} = \lambda_k F_t + \varepsilon_{kt}$
$F_t = \alpha_1 + \alpha_0 S_t + \phi F_{t-1} + h_{S_t}$ S_t=0,1
k = Excess Returns on VW, EW, GM, IBM

Model 2:
$Y_{kt} = l_k^{st} + F_t + \varepsilon_{kt}$
$F_t = \alpha_1 + \alpha_0 S_t + \phi F_{t-1} + h_{S_t}$ S_t=0,1
k = Excess Returns on VW, EW, GM, IBM

Model 3:
$Y_{kt} = l_k^{st} + F_t + \varepsilon_{kt}$
$F_t = \alpha_1 + \alpha_0 S_t + \phi F_{t-1} + h_{S_t}$ S_t=0,1
k = Excess Returns on VW, Changes in Dividend Yield, TB-3 month, P/E

Parameters	Model 1	Parameters	Model 2	Parameters	Model 3
α_1	0.117	α_1	0.131	α_1	0.038
	(0.027)		(0.024)		(0.015)
α_0	-0.351	α_0	-0.356	α_0	-0.113
	(0.172)		(0.137)		(0.079)
ϕ	0.018	ϕ	-0.010	ϕ	0.295
	(0.062)		(0.041)		(0.049)
$\sigma^2\varepsilon_{vw}$	0.013	$\sigma^2\varepsilon_{vw}$	0.013	$\sigma^2\varepsilon_{vw}$	0.135
	(0.006)		(0.006)		(0.009)
$\sigma^2\varepsilon_{ew}$	0.098	$\sigma^2\varepsilon_{ew}$	0.094	$\sigma^2\varepsilon_{dy}$	1.359
	(0.010)		(0.009)		(0.350)
$\sigma^2\varepsilon_{gm}$	0.363	$\sigma^2\varepsilon_{gm}$	0.363	$\sigma^2\varepsilon_{tb3}$	61.218
	(0.023)		(0.023)		(3.794)
$\sigma^2\varepsilon_{ibm}$	0.396	$\sigma^2\varepsilon_{ibm}$	0.394	$\sigma^2\varepsilon_{p/e}$	5.621
	(0.025)		(0.025)		(0.514)
λ_{vw}	1	λ^0_{vw}	1	λ^0_{vw}	1
	-		-		-
λ_{ew}	1.138	λ^0_{ew}	1.317	λ^0_{dy}	-9.134
	(0.041)		(0.067)		(0.737)
λ_{gm}	0.974	λ^0_{gm}	0.969	λ^0_{tb3}	-3.589
	(0.061)		(0.108)		(1.704)
λ_{ibm}	0.964	λ^0_{ibm}	0.887	$\lambda^0_{p/e}$	10.070
	(0.060)		(0.098)		(0.892)
p_{11}	0.957	λ^1_{vw}	1	λ^1_{vw}	1
	(0.023)		-		-
p_{00}	0.756	λ^1_{ew}	1.003	λ^1_{dy}	-11.204
	(0.153)		(0.055)		(1.001)
$\sigma^2_{\eta 1}$	0.145	λ^1_{gm}	0.978	λ^1_{tb3}	-2.683
	(0.019)		(0.097)		(1.922)
$\sigma^2_{\eta 0}$	0.608	λ^1_{ibm}	1.021	$\lambda^1_{p/e}$	10.859
	(0.174)		(0.089)		(1.036)
		p_{11}	0.960	p_{11}	0.948
			(0.018)		(0.023)
		p_{00}	0.814	p_{00}	0.703
			(0.093)		(0.133)
		$\sigma^2_{\eta 1}$	0.149	$\sigma^2_{\eta 1}$	0.066
			(0.016)		(0.012)
		$\sigma^2_{\eta 0}$	0.512	$\sigma^2_{\eta 0}$	0.306
			(0.100)		(0.090)
LogL(θ)	-1525.06	**LogL(θ)**	-1517.82	**LogL(θ)**	-4671.76

Asymptotic standard errors in parentheses correspond to the diagonal elements of the inverse hessian obtained through numerical calculation.

Table 2 - Firm Size Asymmetries Across States (Model 2)

Asymmetries	Bear Market	Bull Market
Market: λ_{vw}	1	1
Large Firms: λ_{IBM}	0.887	1.021
Large Firms: λ_{GM}	0.969	0.978
Small Firms: λ_{ew}	1.317	1.003
α_{st} / σ_{st}	-0.314	0.339

Table 3 - Dating of the U.S. Bear Markets Smoothed Probabilities, Model 2

NBER Recessions		Bear Markets	
Peak	Trough	Peak	Trough
1957:08	1958:04	1957:08	1957:12
1960:04	1961:02	-	-
-	-	1962:03	1962:10
-	-	1966:05	1966:09
1969:12	1970:11	1969:02	1970:09
1973:11	1975:03	1973:01	1975:02
-	-	1978:08	1978:11
1980:01	1980:07	1979:09	1980:04
1981:07	1982:11	1981:06	1982:02
-	-	1987:09	1987:11
1990:07	1991:03	1990:07	1990:10

The stock market is assumed to be in a Bear Market if the smoothed probabilities of bear markets, $P(S_t=0|I_T)$, is greater than their mean + ½ their standard deviation.

Table 4 - Contemporaneous Correlation Between Conditional Expectation and Conditional Variance of VW Excess Returns Across Business and Financial Cycles

Correlation: CE and CV during↓	Model 2	Model 3
Bear Market	-0.505	-0.443
Bull Market	-0.978	-0.152
Bear Market PN	-0.995	-0.766
Bull Market PN	-0.998	-0.098
CE<0	-0.985	-0.906
CE>0	-0.971	0.438
Full Sample	-0.995	-0.399

NBER refers to a 0/1 dummy variable taking the value 1 during recessions and 0 during expansions, as dated by the NBER. Bear and Bull markets refer to the smoothed probabilities of bear and bull markets, respectively, obtained from each model.

Table 5 - Individual Regressions of the Sharpe Ratio on Economic Variables

	Model 2			Model 3		
Independent Variable→	NBER	ΔlnIP	SFC	NBER	ΔlnIP	SFC
Coefficient	-1.214	0.294	0.020	-1.199	0.311	0.011
	(-12.115)	(6.873)	(8.416)	(-3.743)	(2.324)	(2.805)
Adj. R^2	0.217	0.086	0.130	0.021	0.009	0.015

t- statistic inside parentheses. NBER is a 0/1 dummy variable taking the value of one at NBER-dated recessions, ΔlnIP is the log first difference of Industrial Production, and SFC is a business cycle index built from a switching dynamic factor which is highly correlated with the log first difference of GDP (see Chauvet 1998a). We ran three simple regression of the Sharpe Ratio on a constant and each of the independent variables, for Models 2 and 3. The constant term is positive and significant in all equations.

Table 6 - Model 2
Correlogram: Conditional Expectation (CE) and Conditional Variance (CV) of the VW

i	CE of VW, CV of VW(-i)	CE of VW, CV of VW(+i)
0	-0.9950	-0.9950
1	-0.7631	-0.7336
2	-0.5475	-0.5195
3	-0.4402	-0.4213
4	-0.4191	-0.4018
5	-0.3931	-0.3708
6	-0.3136	-0.2912
7	-0.2319	-0.2206
8	-0.1886	-0.1799
9	-0.1678	-0.1636
10	-0.1437	-0.1350
11	-0.1127	-0.1061
12	-0.1296	-0.1292
13	-0.1332	-0.1294
14	-0.1364	-0.1322
15	-0.1049	-0.1126
16	-0.0967	-0.1022
17	-0.0922	-0.0894
18	-0.0720	-0.0621
19	-0.0497	-0.0405
20	-0.0139	-0.0142
21	0.0195	0.0186
22	0.0481	0.0465
23	0.0502	0.0391
24	0.0640	0.0586
25	0.0758	0.0727
26	0.0801	0.0802
27	0.0977	0.0977
28	0.0873	0.0863
29	0.0713	0.0668
30	0.0570	0.0498

Table 7 - Model 3
Correlogram: Conditional Expectation (CE) and Conditional Variance (CV) of the of the VW

i	CE of VW, CV of VW(-i)	CE of VW, CV of VW(+i)
0	-0.3993	-0.3993
1	-0.3330	-0.1192
2	-0.2066	-0.0164
3	-0.1358	-0.0011
4	-0.1500	0.0058
5	-0.2220	0.0083
6	-0.1948	0.0549
7	-0.1561	0.0261
8	-0.1550	0.0188
9	-0.1077	0.0126
10	-0.0524	0.0611
11	-0.0266	0.1109
12	-0.0549	0.0988
13	-0.0892	0.0514
14	-0.0330	0.0644
15	0.0139	0.0403
16	0.0235	0.0149
17	0.0021	0.0263
18	-0.0313	0.0414
19	0.0279	0.0424
20	0.0071	0.0091
21	-0.0479	0.0046
22	0.0148	0.0091
23	0.0472	-0.0194
24	0.0305	-0.0643
25	0.0338	-0.0902
26	0.0070	-0.0744
27	-0.0217	-0.0537
28	0.0035	-0.0188
29	-0.0238	-0.0079
30	-0.0589	0.0220

Table 8 - Model 3
Correlogram: Conditional Expectation (CE) and Conditional Variance (CV) of the Value-Weighted Excess Return

i	CE of VW, CV of VW(-i) CE<0	CE of VW, CV of VW(-i) CE>0	CE of VW, CV of VW(+i) CE<0	CE of VW, CV of VW(+i) CE>0
0	-0.9064	0.4383	-0.9064	0.4383
1	-0.3151	0.1756	-0.2111	0.1267
2	-0.1193	0.0820	-0.0702	0.0046
3	-0.0837	0.0347	-0.0542	-0.0109
4	-0.0975	-0.0572	-0.0614	0.1021
5	-0.1117	-0.0502	-0.0566	0.0304
6	0.0156	-0.0376	0.0063	0.0744
7	-0.0592	-0.0248	-0.0710	-0.0274
8	-0.0914	0.0002	-0.0764	-0.0333

9	-0.0837	0.0148	-0.0473	0.0173
10	-0.0610	-0.0156	-0.0421	0.0461

Figure 1 - Model 2 - Excess Return Factor (—), Value-Weighted Excess Returns (- - -), Equal-Weighted Excess Returns (⁻ ⁻ ⁻) Smoothed Using H-P Filter (λ=1) and NBER Dated Recessions (Shaded Area)

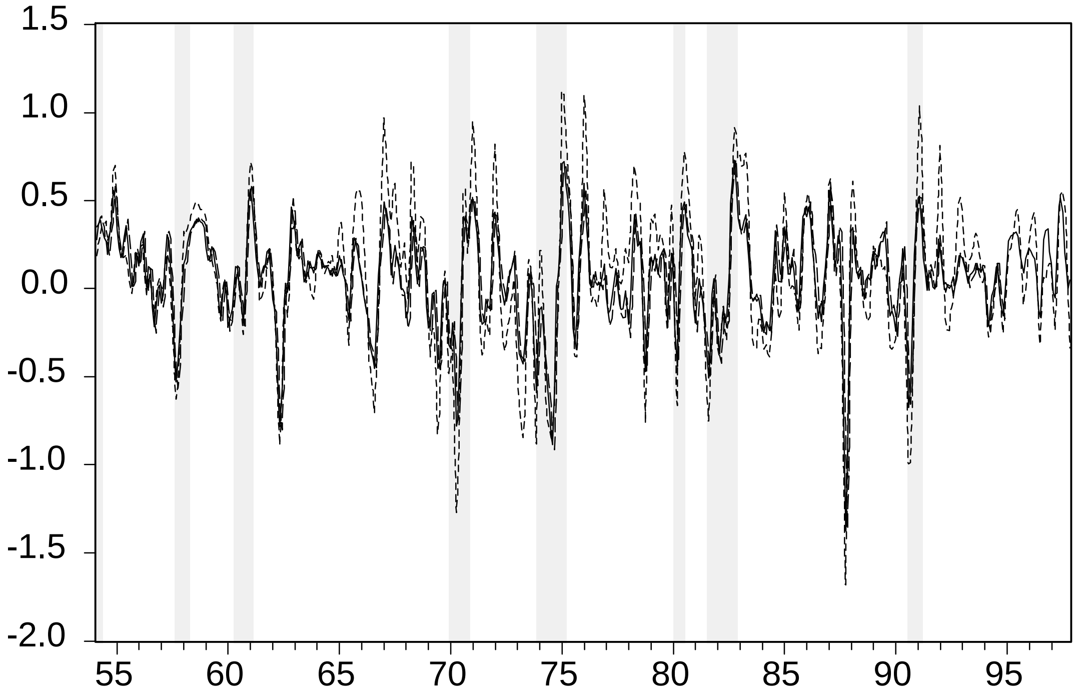


Figure 2 - Model 3 - Excess Return Factor (—), Conditional Expectation of the Value-Weighted Excess Return (- - -), and NBER Dated Recessions (Shaded Area)

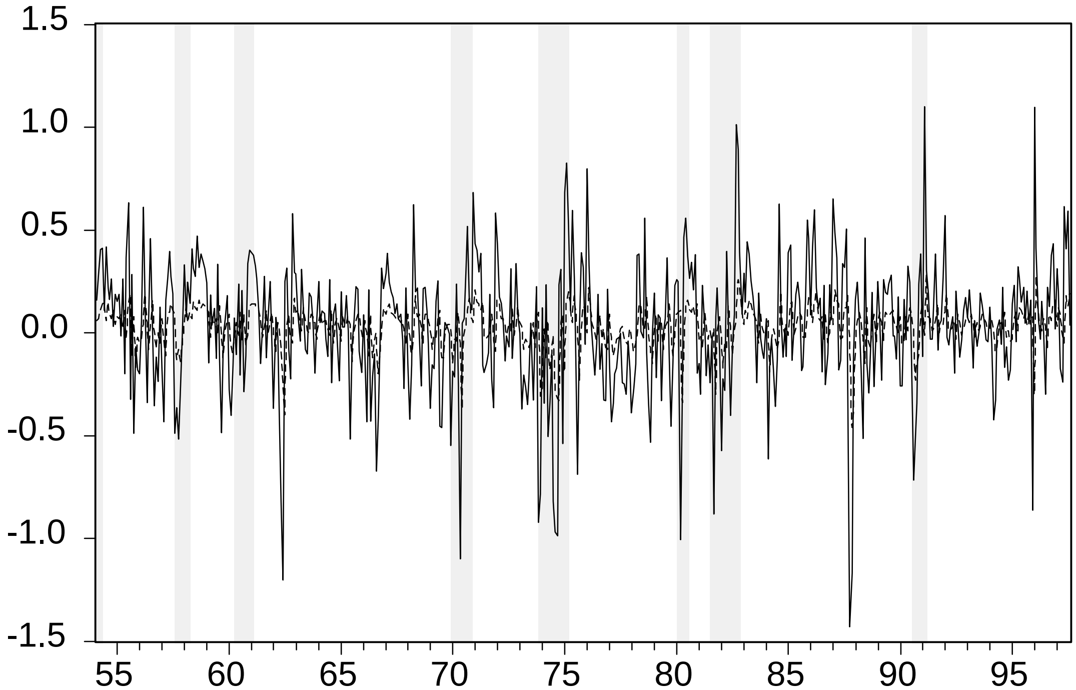

1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
55
60
65
70
75
80
85
90
95

Figure 3 - Smoothed Probabilities of Bear Market from Fitting a Univariate AR(0) Markov Switching Model to the Factor Components, and NBER Dated Recessions(Shaded Area)

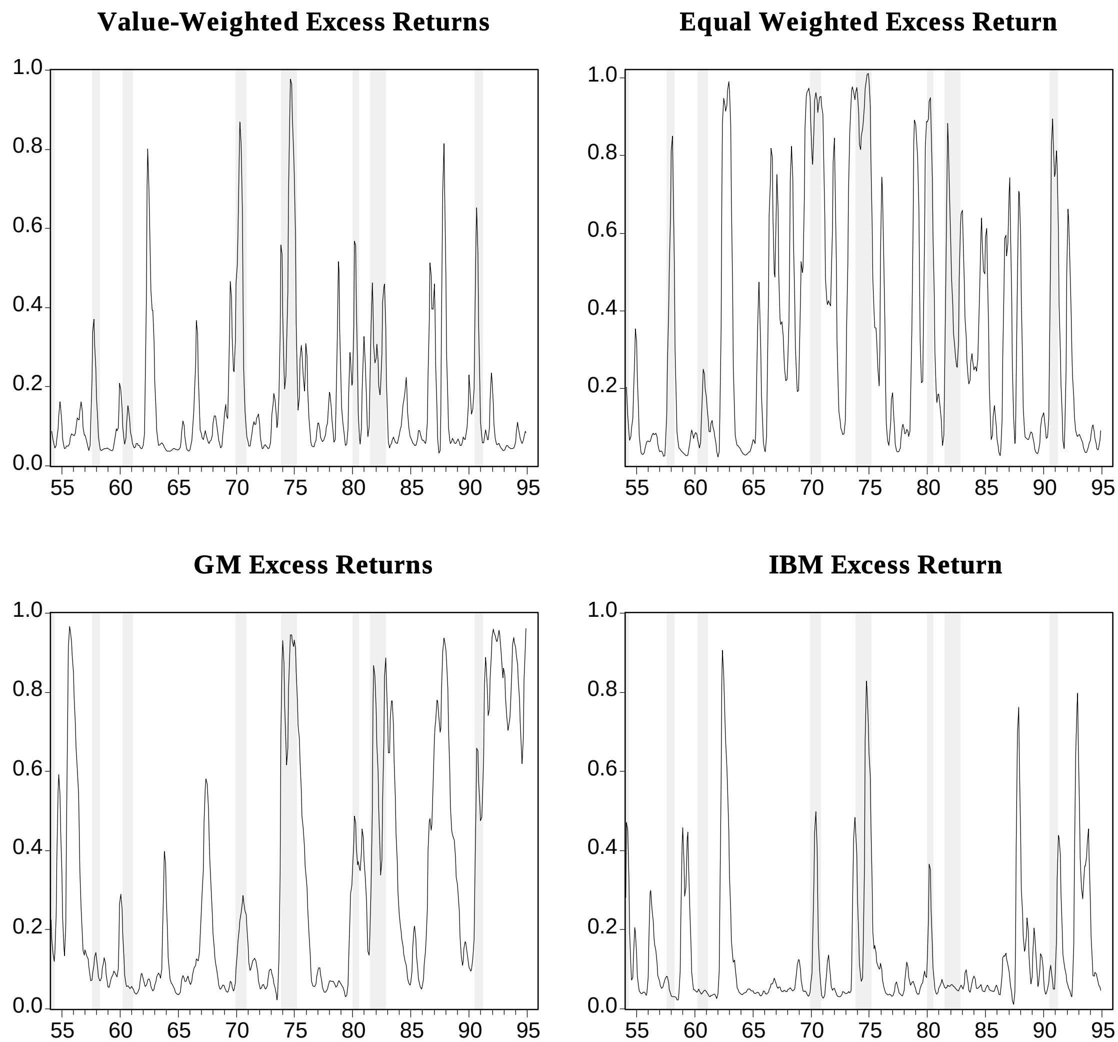


Figure 4 - Smoothed Probabilities of Bear Market from Model 3 and NBER Dated Recessions

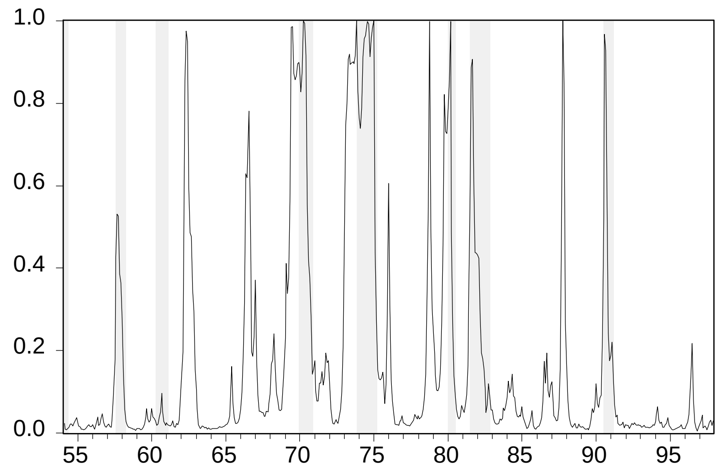


Figure 5 - Model 2 - Conditional Expectation (CE) and Conditional Variance (CV) of the VW Excess Return (—), and NBER Dated Recessions (Shaded Area)

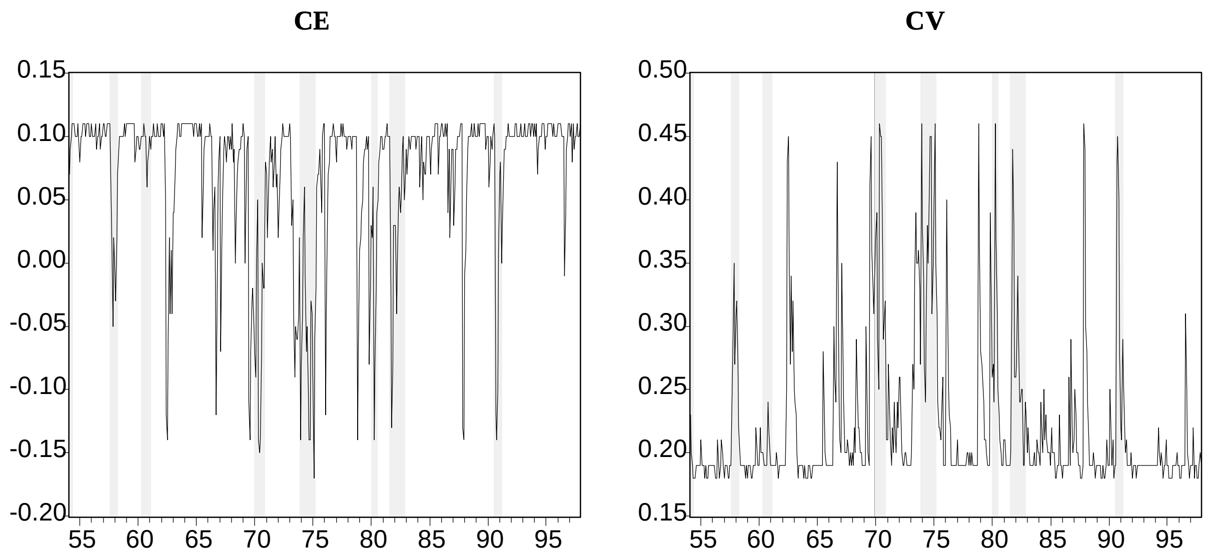


Figure 6 - Model 3 - Conditional Expectation (CE) (Estimated Series and Smoothed Series Using H-P Filter λ=10), Conditional Variance (CV) of the VW Excess Return, and NBER Dated Recessions(Shaded Area)

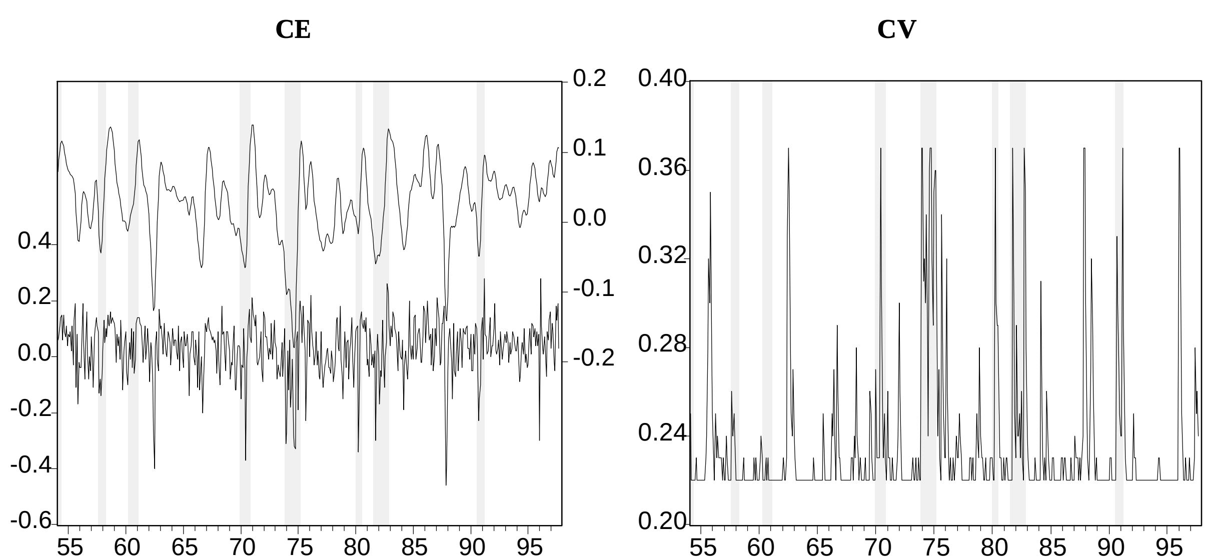


Figure 7 - Model 2: Scatter Diagrams - Conditional Expectation (CE) and Conditional Variance (CV) of the VW Excess Return - Bear and Bull Markets

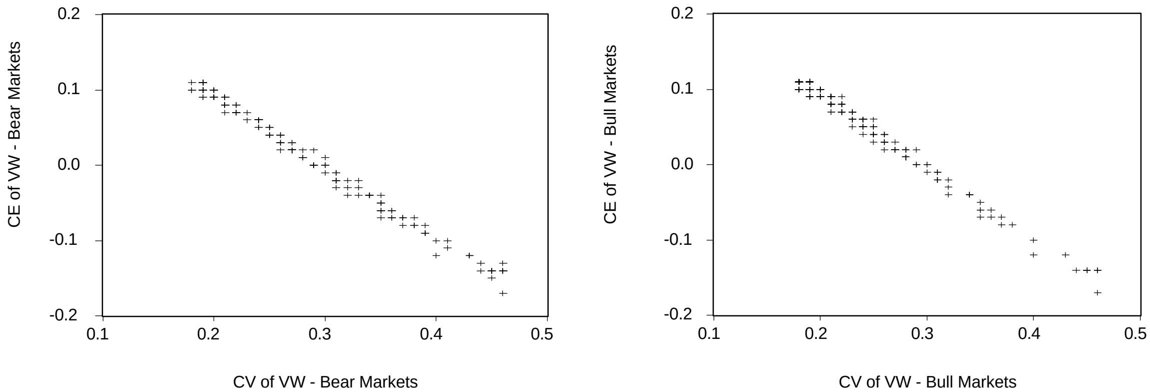


Figure 8 - Model 2: Scatter Diagram - Conditional Expectation (CE) and Conditional Variance (CV) of the VW Excess Return - Full Sample

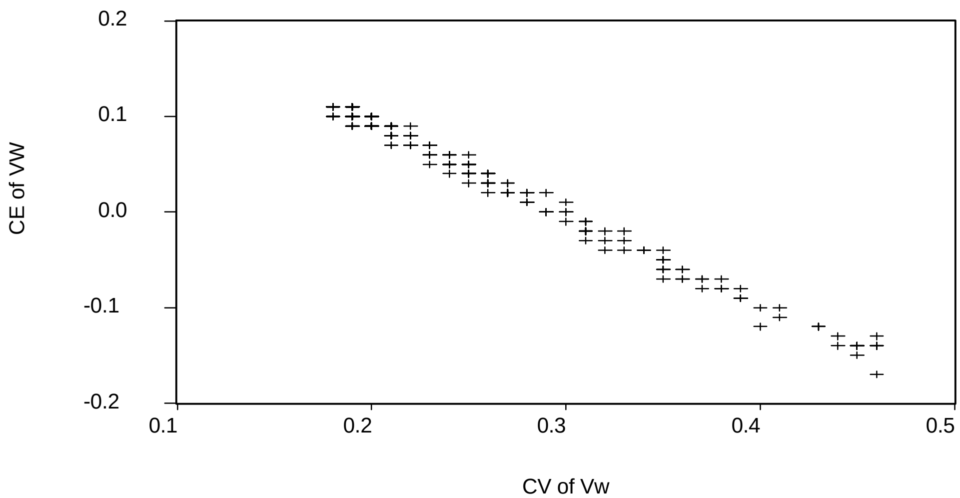


Figure 9 - Model 3: Scatter Diagram - Conditional Expectation (CE) and Conditional Variance (CV) of the VW Excess Return for the Full Sample

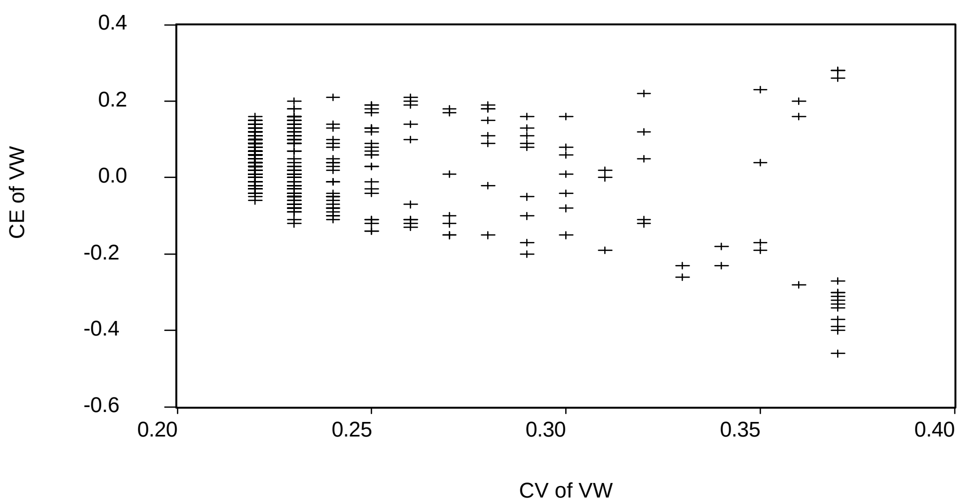


Figure 10 - Model 3: Scatter Diagrams - Conditional Expectation (CE) and Conditional Variance (CV) of the VW Excess Return - Bear and Bull Markets:

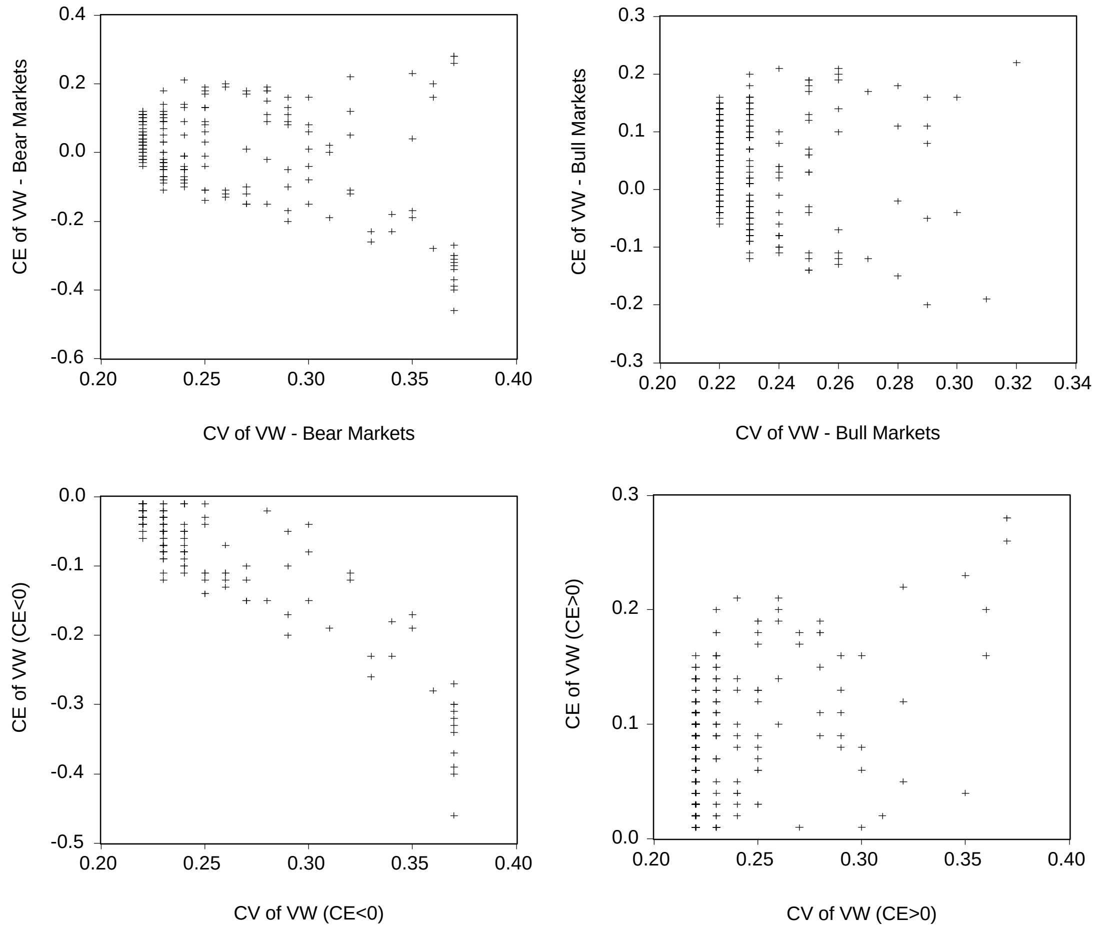


Figure 11 - Model 2: Sharpe Ratio of the VW Excess Return Factor (—), Model 2, and NBER Dated Recessions (Shaded Area)

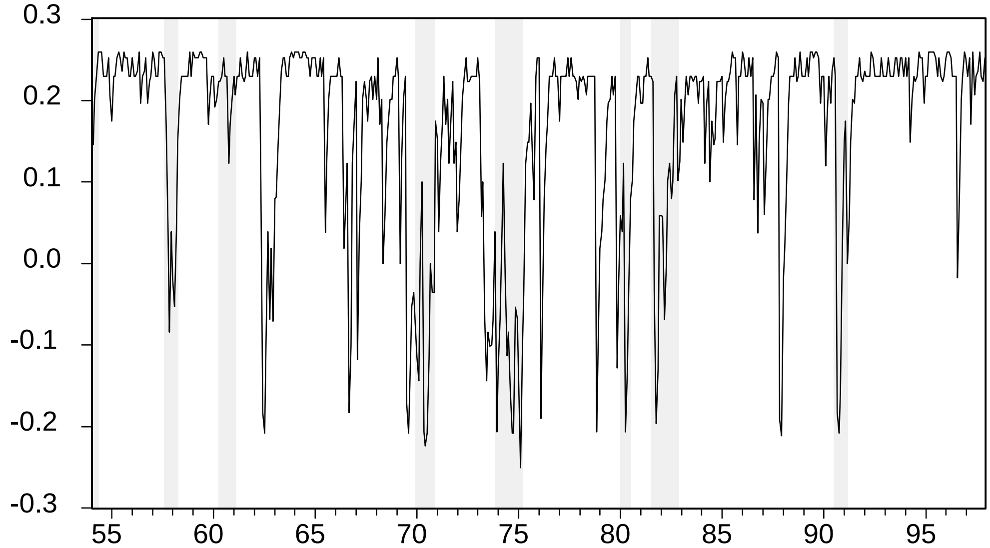


Figure 12 - Model 2 - Sharpe Ratio of the Components of the Factor (—), Model 2, and NBER Dated Recessions (Shaded Area)

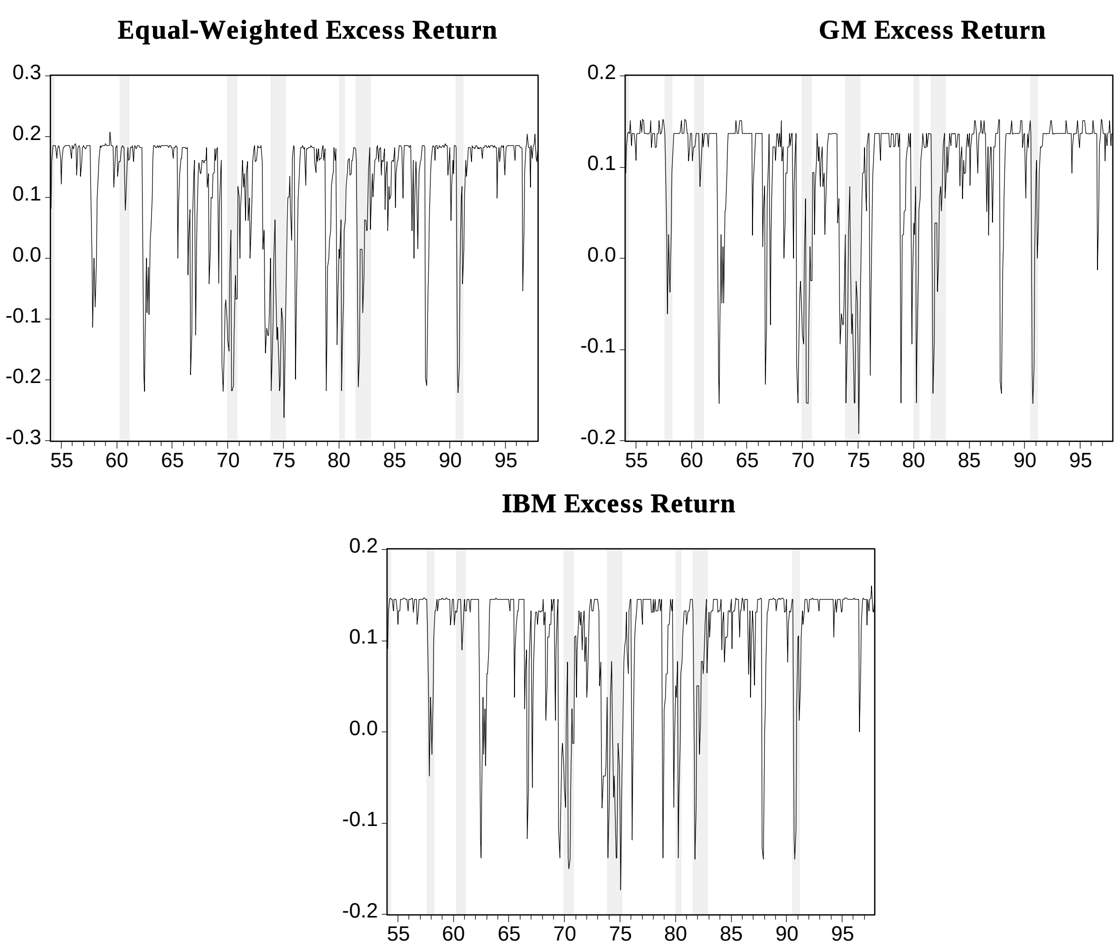


Figure 13 - Model 3: Sharpe Ratio of the VW Excess Return (Estimated Series and Smoothed Series Using H-P Filter λ=10), and NBER Dated Recessions (Shaded Area)

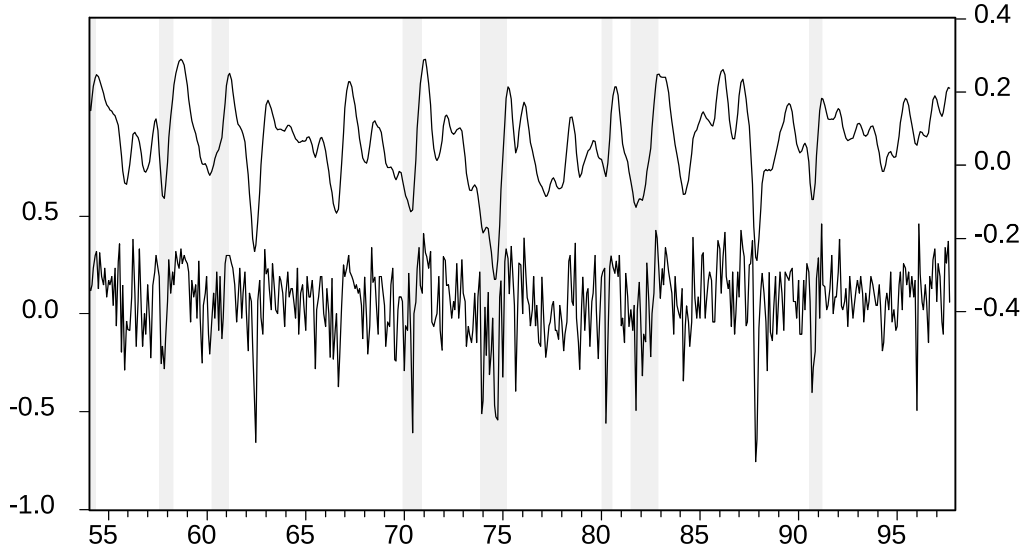


Figure 14 - Cross-Correlogram of Conditional Expectation (CE) and Conditional Variance (CV) of the VW Excess Return for 30 Lags (——) and Leads (- - -) of the Conditional Variance

Model 2 **Model 3**

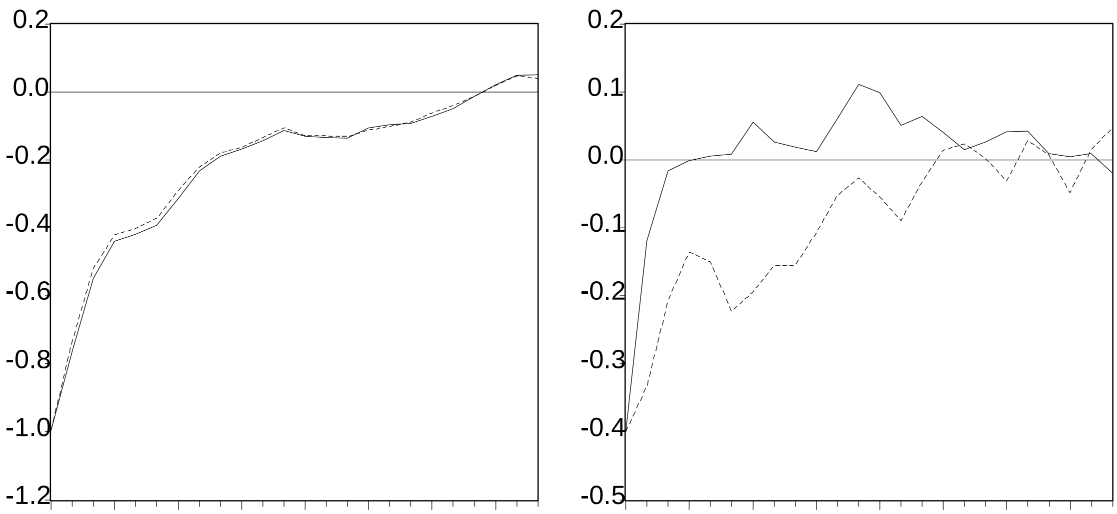
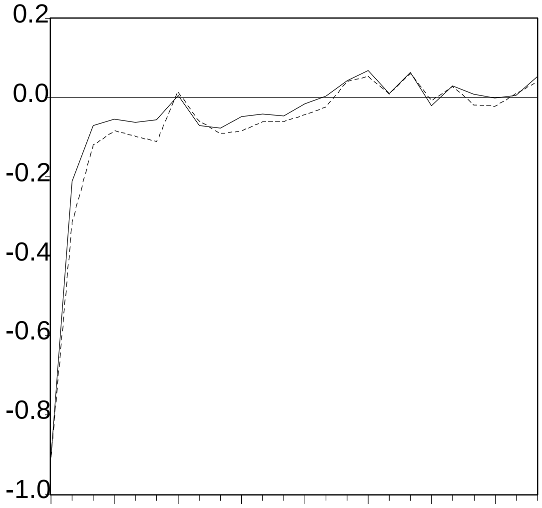
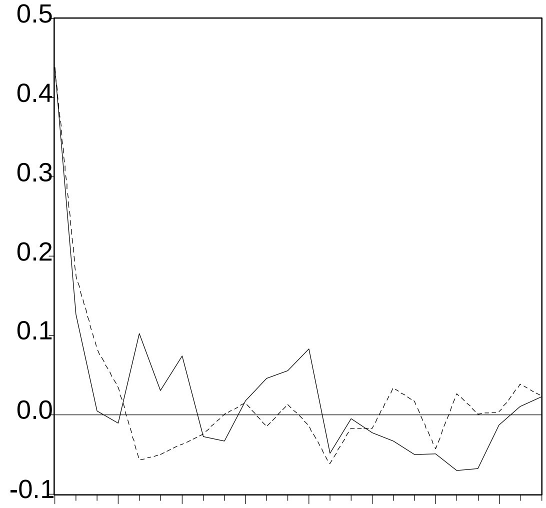


Figure 15 - Cross-Correlogram of Conditional Expectation (CE) and Conditional Variance (CV) of the VW Excess Return for 30 Lags (——) and Leads (- - -) of the Conditional Variance

CE < 0 **CE > 0**


0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0


0.5
0.4
0.3
0.2
0.1
0.0
-0.1