Up Close It Feels Dangerous: “Anxiety” in the Face of Risk

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Abstract

We model an “anxious” agent as one who is more risk averse with respect to imminent risks than distant risks. Such horizon-dependent risk aversion preferences describe well-documented features of (i) individual behavior, (ii) equilibrium asset prices, and (iii) endogenously arising institutions. In particular, based on a utility function that captures individual subjects’ behavior in the lab, we predict a downward-sloping term structure of risk premia, and show that costly delegated portfolio management is a strategy to cope with dynamic inconsistency with respect to intra-temporal risk-return trade-offs.

Key words: anxiety, risk aversion, dynamic inconsistency

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1 Introduction

There is ample evidence that people behave in more risk averse ways with respect to risks that are close in time compared to risks that are distant. We term such behavior horizon-dependent risk aversion (HDRA), or more informally ‘anxiety’.\(^1\) Despite abundant experimental evidence that people exhibit HDRA preferences, economists have not yet developed a way of formally thinking about such preferences and the implications for economics and finance. This paper takes first steps toward such a framework by modeling an agent whose risk aversion explicitly depends on the temporal distance to the resolution and payoff of a lottery.

Figure 1 illustrates HDRA with a simple example. In both the top and the bottom comparison the agent has to choose between a risky alternative on the left and a safe alternative on the right. In the top comparison the risk is distant. As a result, the agent has low risk aversion with respect to the gamble. If her risk aversion is low enough, she may choose the risky over the safe alternative. In the bottom comparison the risk is imminent. As a result, the agent has high risk aversion and may choose the safe over the risky alternative. The agent’s preference implies different choices depending on the temporal distance of the risk. In particular, she may pull back from risks she previously intended to take, even absent new information and even if beliefs have not changed for any other reason.

As an intuitive example, consider a parachute jump. An agent may sign up for a jump several days or weeks in advance, thinking the thrill of the jump will be well

\(^1\)The New Oxford American Dictionary defines anxiety as a healthy “feeling of worry, nervousness, or unease, typically about an imminent event or something with an uncertain outcome” (emphasis added). This paper does not discuss anxiety disorder, which is a psychopathological condition.
worth the risk of an accident. However, when looking out the plane’s door at the moment of truth, the agent is likely to reconsider and may decide not to jump. Such behavior of parachutists, as well as similar examples, e.g. stage fright of performers, have been studied extensively in the psychology literature (Section 2 provides a discussion).

The parachuting example suggests that HDRA has its proximate cause in an emotional reaction to the proximity of risk. We discuss evidence supporting this interpretation in Section 3. In our analysis, however, we follow a traditional “revealed preference” approach: we use a utility function that captures the observed behavior without making a formal claim as to the reasons for such preferences.

The behavior our HDRA utility function captures is different from the behavior captured by related but conceptually orthogonal non-standard preferences, such as time-varying risk aversion, a preference for the timing of resolution of uncertainty, or preferences with non-exponential discounting, which include the quasi-hyperbolic discounting case (see Section 4). Specifically, quasi-hyperbolic discounting represents dynamic inconsistency for inter-temporal consumption-savings tradeoffs and gives rise to a demand for illiquid assets and other commitment devices to prevent overconsumption and facilitate saving (Laibson, 1997). Uncertainty or risk is not a central element in such models. In contrast, HDRA represents dynamic inconsistency for intra-temporal risk-return tradeoffs and therefore has implications in many domains of decision-making under uncertainty. For example, we show that HDRA can address key features of equilibrium asset prices, and particular variation in the cross-sectional pricing of risk, for which non-standard time preferences have no implications (Luttmer and Mariotti, 2003).

Modeling preferences with HDRA presents several challenges, in particular if one wants to maintain dynamic consistency for inter-temporal tradeoffs. We show that in a time-separable framework with general discounting and general utility indexes, the only way to have HDRA without dynamic inconsistency for inter-temporal consumption tradeoffs is to restrict analysis to a two-period setting. In a setting with more than two periods, HDRA necessarily leads to dynamic inconsistency in consumption even when the increased flexibility of non-exponential discounting is taken into account. This insight complements that of Strotz (1955): To achieve dynamic consistency, not only does discounting have to be exponential; we show that it is
also necessary that the utility indexes be identical.\footnote{As an alternative to the two-period framework used in the present paper, Andries, Eisenbach, and Schmalz (2014) drop time separability and generalized Epstein and Zin (1989) preferences to disentangle (consistent) time preferences from (inconsistent) risk preferences, and derive asset pricing implications in a fully dynamic model.}

After discussing these modeling challenges, we apply our model to a stylized stock market. We show that investors with HDRA require more compensation for short-run risks than for long-run risks. A key feature of equilibrium asset prices emerges as a direct consequence: the term structure of risk premia is downward-sloping – a fact first documented by van Binsbergen, Brandt, and Koijen (2012) that has aroused much attention in the recent asset pricing literature. This prediction is noteworthy as standard asset pricing models predict a flat or upward-sloping term structure of risk premia.

Given the potential of dynamically inconsistent risk-taking, we can distinguish between ‘naive’ and ‘sophisticated’ HDRA agents. We show that only an agent who is sophisticated about her dynamic inconsistency is willing to pay for commitment devices to take risk. In particular, lacking the resolve to personally manage an equity portfolio, she is willing to pay a fee to delegate her investment decisions. Thus, sophisticated agents with HDRA preferences generate a demand for delegated portfolio management, even if these services are costly and known to underperform passive benchmarks that are available at low cost (Gruber, 1996). Moreover, the HDRA model predicts that demand for investment advice is particularly strong for agents who would otherwise not invest in risky assets at all, as documented by Foerster et al. (2014).

In comparison to the existing literature, which has provided models that are able to generate the evidence in each of the three domains individually (individual behavior, asset pricing, and demand for delegated portfolio management), our model jointly explains all three.

The paper proceeds as follows. Section 2 presents experimental evidence for our main assumption: risk aversion decreasing with temporal distance. Section 3 discusses potential origins of HDRA preferences. Section 4 contrasts HDRA with related non-standard preferences. Section 5 discusses challenges in modeling HDRA and derives the model used in this paper. We apply the model in Section 6 to analyze the implications of HDRA on asset prices. In Section 7, we investigate how
sophisticated HDRA agents respond to the potential of dynamically inconsistent behavior with respect to intra-temporal risk-return tradeoffs. Section 8 concludes.

2 Experimental Evidence

This section reviews evidence that temporal distance affects risk-taking behavior. Horizon-dependent risk aversion is very well documented experimentally. Subjects tend to be more risk averse when a risk is temporally close than when it is distant, both in across-subject and within-subject studies.

Jones and Johnson (1973) have subjects participate in a simulated medical trial for a new drug; each subject has to decide on a dose of the drug to be administered. The subjects are told that the probability of experiencing unpleasant side-effects increases with the dose – but so does monetary compensation. More risk averse subjects should then choose lower doses than less risk averse subjects. The study finds that subjects choose higher doses when they are to be administered the next day than when they are to be administered immediately. Interestingly, the difference disappears if the decision can be revisited the next day (no commitment), suggesting that subjects may anticipate their preference reversals.

Welch (1999) documents preference reversals caused by stage fright. He finds that 67% of subjects who agree to tell a joke in front of a class the following week in exchange for $1 “chicken out” when the moment of truth arrives. In contrast, none of those who decline initially change their mind.

Noussair and Wu (2006) as well as Coble and Lusk (2010) use the protocol of Holt and Laury (2002), a widely used method in experimental economics, to elicit risk aversion. Subjects are presented with a list of choices between two binary lotteries. The first lottery always has two intermediate prizes, e.g. ($10.00, $8.00), while the second lottery always has a high and a low prize, e.g. ($19.25, $0.50). Going down the list, only the respective probabilities of the two prizes change, varying from (0.1, 0.9) to (0.9, 0.1). As probability mass shifts from the second prize to the first prize of both lotteries, the second lottery becomes increasingly attractive compared to the first lottery. Subjects are asked to pick one of two lotteries for each of the probability distributions. The probability distribution at which a subject switches from the “safe” lottery to the “risky” lottery is a proxy for the subject’s risk aversion. Noussair
and Wu (2006) use this protocol for a within-subject design with real payoffs, having each subject make choices for resolution and payout to occur immediately and also for risks and payouts that occur three months later. The study finds that four times more subjects are more risk averse for the present than for the future than the other way around. Coble and Lusk (2010) use the protocol for an across-subject design and find the same pattern with average risk aversion decreasing in the temporal distance of the risk.

In a different type of experiment, Baucells and Heukamp (2010) let subjects choose between two binary lotteries, a “safer” and a “riskier” one. Different treatments vary the delay until the lotteries are resolved and paid out. The study finds that more subjects choose the riskier lottery as the delay increases. Sagristano, Trope, and Liberman (2002) also have subjects choose between two lotteries and find the same effect of temporal horizon.

Finally, some studies elicit risk aversion by asking subjects for their certainty equivalents for different lotteries; a lower certainty equivalent corresponds to higher risk aversion. In Onculer (2000), subjects state their certainty equivalent for a lottery to be resolved and paid immediately, as well as for the same lottery to be resolved and paid in the future. The study finds that subjects state significantly lower certainty equivalents for the immediate lottery than for the future lottery. Abdellaoui, Diecidue, and Onculer (2011) conduct a similar study with real payoffs and find equivalent results.

3 Potential Origins

While our model follows the tradition of revealed preferences, we feel that a brief discussion of potential origins of HDRA is in place. In particular, we find it intuitively plausible that HDRA arises due to the effect of emotions on decision making and the fact that emotional responses are stronger for more salient cues.

Loewenstein, Weber, Hsee, and Welch (2001) point out that cognitive evaluations of risk do not depend on temporal distance; in contrast, emotional reactions to risk such as fear and anxiety increase as the risk draws closer (see also Loewenstein, 1987, 1996; Monat and Lazarus, 1991; Paterson and Neufeld, 1987). The authors point out that when such departures between thoughts and emotions occur, feelings
often exert a dominating influence on behavior. As a result, agents tend to behave in more risk averse ways with respect to risks at shorter horizons, even when cognitive evaluations of the risk remain constant.

Indeed, research in psychology documents a robust link between temporal proximity of risk and ‘anxiety’ as an emotional response. Some studies even document both horizon-dependent risk aversion preferences and an ‘anxiety-prone’ emotional response jointly. For example, the study by Jones and Johnson (1973), previously discussed in section 2, also measures higher stress levels for subjects deciding over immediate doses than for subjects deciding over delayed doses. Monat (1976) and Breznitz (2011) inform subjects that they will receive an electric shock (presumably of an uncertain strength given a subject-specific scale). The temporal distance varies across different treatment groups. Heart rate, and in the latter study also galvanic skin response and self-reported anxiety are all higher when the shock is closer in time. Fenz and Epstein (1967), Fenz and Jones (1972) and Roth, Breivik, Jørgensen, and Hofmann (1996) investigate the emotional response of parachutists approaching the time of a jump. Novice parachutists exhibit a similar dynamic of physiological measures and self-reports of anxiety as in the above experiments, while expert parachutists have a somewhat attenuated response to the proximity of the jump, suggesting an adaptive nature of ‘anxiety.’ Lo and Repin (2002) and Lo, Repin, and Steenbarger (2005) find similar psychophysiological responses to risk taking among securities traders.

While intuitively plausible, we do not claim that emotions are indeed the driver of the observed behavior. One reason is that in some theories of emotions, cognition drives emotions rather than the other way around (Gross and Barrett, 2011). As a result, cognition and emotions may not be as cleanly separated as suggested above.

Trope and Liberman (2003) offer the by psychologists widely accepted “construal theory” to explain choice behavior that differs by horizon. The theory proposes that the mental representation of events depends on the temporal distance to the event. Indeed, neurological evidence indicates that “separate neural systems value immediate and delayed monetary rewards” (McClure, Laibson, Loewenstein, and Cohen, 2004). As a consequence of different representations that come in different levels of abstraction, people make different decisions. The objective of this paper is to provide an economic model of such behavior that is useful to study the implications
of these notions from the psychology literature.

In sum, while the evidence on horizon dependence in the emotional response to risk as well as theories used by psychologists to explain horizon-dependent decision making are consistent with and plausibly linked to the horizon-dependent risk choices we discuss, we make no claim as to HDRA’s emotional or psychological origins. We take the standpoint of traditional economics: we observe choice and infer preferences, which we subsequently take as given when modeling behavior in different contexts. In the following sections, we examine a preference that reflects the experimental choice behavior without relying on any specific underlying driver of such behavior.

4 Distinction from Related Theories

In this section we distinguish ‘anxiety’ from existing theories that are related but conceptually orthogonal.

4.1 Preference for the Timing of Resolution of Uncertainty

\[ \frac{1}{2} 3 \succ \frac{1}{2} 0 \]

Figure 2: Preference for later resolution of uncertainty

The seminal paper by Kreps and Porteus (1978) is the first to consider a preference ranking between lotteries that differ in the timing of resolution of a given risk while the timing of the payoff is held constant. Figure 2 illustrates such a preference. In contrast, HDRA manifests itself in comparisons of lotteries that are resolved and paid out at the same time and ranks them differently depending on temporal distance. Further, Kreps and Porteus (1978) explicitly rule out dynamically inconsistent behavior. In contrast, we allow for dynamic inconsistency.
4.2 Time-Changing Risk Aversion

A large literature in asset pricing assumes that agents’ effective risk aversion changes over time, for example as a result of habit formation (Constantinides, 1990; Campbell and Cochrane, 1999). Figure 3 illustrates the choices of an agent who is more risk averse in one period than in another. In contrast, HDRA preferences are not time-varying. An anxious agent’s effective risk aversion changes as a function of temporal distance to risk, not as a function of calendar time. Further, models of time-changing risk aversion are typically dynamically consistent.

4.3 Dynamically Inconsistent Time Preferences

Agents with dynamic-inconsistency problems have been studied at least since Strotz (1955). Work in the tradition of Phelps and Pollak (1968) and Laibson (1997) focuses on inconsistent time preferences, e.g. modeled as quasi-hyperbolic discounting. The agent resolves inter-temporal consumption tradeoffs differently depending on the time horizon: if the time horizon is short, the agent is more impatient than if the time horizon is long. We study an orthogonal dimension by assuming that the agent’s risk preferences are dynamically inconsistent. The agent resolves intra-temporal risk tradeoffs differently depending on the time horizon: if the horizon is short, the agent is more risk averse than if the horizon is long. Thus we emphasize that agents can be dynamically inconsistent independently in the dimensions of inter-temporal consumption and intra-temporal risk.

4.4 Other Theories

HDRA belongs with a set of theories that emphasize the impact of salience on decision making – temporal distance is but one dimension of salience. For example,
in Bordalo, Gennaioli, and Shleifer (2012, 2013), the context makes certain aspects
of lotteries more or less salient. This approach can account for several empirically
relevant phenomena that are different from those accounted for by HDRA. Epstein
and Kopylov (2007) have a model of ‘cold feet’ in which agents become more pes-
simistic as risks approach, i.e. their subjective beliefs change. In contrast, HDRA is
motivated by experimental evidence in which the objective probabilities are known
to the subjects. Therefore, we keep beliefs fixed, but allow risk preferences to vary
with the horizon. Epstein (2008) provides an axiomatization for a two-period model
similar to ours. In contrast to our work, he uses the term ‘anxiety’ when an agent
is more risk-averse for distant risks, evoking a notion of anticipatory feelings. Such
anticipatory feelings are also an important aspect in Caplin and Leahy (2001) who
expand the prize space to mental states and explain a set of economic phenom-
ena different from the ones addressed in this paper. In contrast, we leave emotions
outside the model, and focus on the observable behavior. Finally, since our agent
may exhibit a preference for commitment, HDRA is also related to the model of

5 Model

Capturing the experimental evidence on horizon-dependent risk aversion discussed
in Section 2 raises several questions. On the one hand we need to decide on how to
model the observed preferences. As we show below, building on the time-separable
utility framework widely used in economics presents important challenges. On the
other hand, due to the potential for dynamic inconsistency, we need to decide how
to solve the model; we can assume the agent to be naive or sophisticated about the
dynamic inconsistency.

Suppose we want to build on the standard time-separable model of expected
utility. Denoting an uncertain inter-temporal consumption stream from period $t$
onwards by $\tilde{C}_t = (\tilde{c}_t, \tilde{c}_{t+1}, \tilde{c}_{t+2}, \ldots, \tilde{c}_T)$, we can generalize the standard model by
using the following utility function:

$$U_t(\tilde{C}_t) = \mathbb{E}\left[\delta_0 u_0(\tilde{c}_t) + \delta_1 u_1(\tilde{c}_{t+1}) + \cdots + \delta_{T-t} u_{T-t}(\tilde{c}_T)\right].$$

(1)
This utility function has both general discount factors $\delta_h$ and general von Neumann-Morgenstern utility indexes $u_h$ for every horizon $h$ relative to the current period $t$. In the standard model, discounting is geometric, $\delta_h = (\delta)^h$ for all $h$, and the utility index do not depend on the horizon, $u_h = u$ for all $h$.

Consider two lotteries $\tilde{x}$ and $\tilde{y}$ such that $\tilde{x} = \tilde{y} + \mu + \tilde{\varepsilon}$ with $\mu$ a constant and $\tilde{\varepsilon}$ a mean-zero lottery independent of $\tilde{y}$; such lotteries represent a typical risk-reward tradeoff where $\tilde{x}$ is “high risk, high reward” and $\tilde{y}$ is “low risk, low reward.” To capture the evidence of an HDRA agent in period $t$ who prefers the risky lottery $\tilde{x}$ if it is delayed, e.g. to period $t + 1$, but prefers the safe lottery $\tilde{y}$ if it is immediate, the general framework (1) has to satisfy:

For $h = 1$:
$$E[\delta_1 u_1(\tilde{x})] > E[\delta_1 u_1(\tilde{y})]$$

For $h = 0$:
$$E[\delta_0 u_0(\tilde{x})] < E[\delta_0 u_0(\tilde{y})].$$

Given the assumptions on $\tilde{x}$ and $\tilde{y}$, these conditions can be satisfied only if $u_0$ is more risk averse than $u_1$. It is important to note that the discount factors $\delta_0$ and $\delta_1$ cancel out of the two conditions above. This illustrates the conceptual difference between intra-temporal risk tradeoffs and inter-temporal consumption tradeoffs; the discount factors of a time-separable model affect the latter but not the former. The experimental evidence can therefore not be addressed by relaxing the standard assumption of geometric discounting.

The experimental evidence cited in Section 2 mainly contrasts imminent risks with delayed risks rather than risks with differing delays. We can therefore simplify the general model in (1) using only two utility indexes $v$ and $u$ by setting $u_0 = v$ for immediate risks and $u_h = u$ for delayed risks at all horizons $h \geq 1$ and assuming that $v$ is more risk averse than $u$ by the Arrow-Pratt measure of absolute risk aversion:

$$U_t(\tilde{C}_t) = E[\delta_0 v(\tilde{c}_t) + \delta_1 u(\tilde{c}_{t+1}) + \cdots + \delta_{T-t} u(\tilde{c}_T)]$$

with $$-\frac{v''(c)}{v'(c)} \geq -\frac{u''(c)}{u'(c)}$$ for all $c$. (3)

Given that the phenomenon of HDRA is conceptually orthogonal to phenomena of horizon-dependent impatience such as Laibson (1997), it would be desirable to keep the model free of any impatience elements so that we can cleanly identify the
implications of HDRA. However, the time-separable approach taken in (1) and (2) has the problem of confounding the dynamically inconsistent risk preferences with dynamically inconsistent time preferences. Consider the following two deterministic consumption streams:

\[ \mathcal{C}_t = (c, c_L, c, c, \ldots) \quad \text{and} \quad \mathcal{C}'_t = (c, c, c_H, c, \ldots) \quad \text{with} \quad c_L < c_H. \]

The two consumption streams only differ in periods \( t + 1 \) and \( t + 2 \) and choosing between the two involves the inter-temporal tradeoff whether to receive the smaller \( c_L \) earlier or the larger \( c_H \) later. Since the consumption streams are deterministic, we want the HDRA agent to evaluate them the same in period \( t \) and in period \( t + 1 \). This imposes a restriction on the utility function (2):

\[ U_t(C_t) = U_t(C'_t) \iff U_{t+1}(C_{t+1}) = U_{t+1}(C'_{t+1}). \]

First, note that \( U_t(C_t) = U_t(C'_t) \) implies:

\[ \delta_1 u(c_L) + \delta_2 u(c) = \delta_1 u(c) + \delta_2 u(c_H). \]  

(4)

Second, note that \( U_{t+1}(C_{t+1}) = U_{t+1}(C'_{t+1}) \) implies:

\[ \delta_0 v(c_L) + \delta_1 u(c) = \delta_0 v(c) + \delta_1 u(c_H). \]  

(5)

Combining equations (4) and (5) we get:

\[ \frac{v(c_L) - v(c)}{u(c_L) - u(c)} = \frac{(\delta_1)^2}{\delta_0 \delta_2}. \]

We want this to hold for arbitrary \( c_L \) and \( c \), which implies:

\[ \frac{v'(c)}{u'(c)} = \frac{(\delta_1)^2}{\delta_0 \delta_2} \quad \text{for all } c. \]  

(6)

For any general horizon-dependent discounting, \( (\delta_1)^2 / (\delta_0 \delta_2) \) is always a constant so to satisfy (6) the utility indexes \( v \) and \( u \) can only differ by an affine transformation. This, however, rules out that \( v \) and \( u \) have different levels of risk aversion.
as required to represent HDRA behavior. Notice that an implication is that flexible time-discount factors cannot be used to render a utility function with horizon-dependent risk aversion dynamically consistent. In other words, to attain dynamic consistency in a time-separable model, not only do time-discount factors need to be exponential (as Strotz (1955) points out). We point out that utility indexes also have to be identical. As a consequence, the assumption of identical utility indexes ubiquitous in the literature is a special case with considerable loss of generality.

The key problem revealed in the above discussion is the link between inter-temporal substitution and risk aversion inherent in the time-separable model of (1) or (2). There are two solutions to this problem. The first is to depart from the time-separable model to a model that separates time and risk preferences – in the spirit of Epstein and Zin (1989) – yet allows for risk preferences to depend on the horizon and be dynamically inconsistent. The agent’s optimization problem then involves a game against her future selves and involves significant analytical complexity. We follow this approach in Andries et al. (2014).

In this paper, we use the second solution to the problem which is much simpler: We restrict analysis to a two-period model with \( t = 0, 1 \). As the example with the lotteries \( \tilde{x} \) and \( \tilde{y} \) above illustrates, a two-period setting is sufficient to represent the behavior revealed by the experimental evidence on HDRA, as well as the asset pricing evidence on the term structure of risk premia that has been accumulated thus far. In contrast to a setting with more than three periods, however, there is no scope for dynamically inconsistent time preferences. Dynamic inconsistency in inter-temporal tradeoffs requires two periods \( t = 1, 2 \) for the tradeoff and at least one prior period \( t = 0 \) where the agent resolves the tradeoff differently than in \( t = 1 \). The restriction to a two-period setting allows us to cleanly identify implications of HDRA without having to worry about confounding influences of conceptually orthogonal theories, and especially quasi-hyperbolic discounting. We therefore use

\[4\text{Note that recursive utility formulations such as } Epstein\text{ and Zin (1989) are also limited to the special case of dynamic consistency, by construction.}\]
the following setup in this paper:

\[ U_0(\tilde{c}_0, \tilde{c}_1) = \mathbb{E}[v(\tilde{c}_0) + \delta u(\tilde{c}_1)] \quad \text{and} \quad U_1(\tilde{c}_1) = \mathbb{E}[v(\tilde{c}_1)] \]

with \(-\frac{v''(c)}{v'(c)} \geq -\frac{u''(c)}{u'(c)}\) for all \(c\).

Finally, the potential for dynamic inconsistency raises the question of how to solve the model. The agent can be naive and not realize that in the future she will not want to follow through with plans made in the present. In that case, the agent simply maximizes the utility function \(U_t\) in every period \(t\), choosing the optimal values for the future from the perspective of period \(t\) and (wrongly) assuming that she will not reoptimize and choose different values in the future. Alternatively, the agent can be sophisticated in the tradition of Strotz (1955) and optimize subject to the constraint that she will also optimize in the future. We will consider the implications of both naive and sophisticated behavior in the following analysis.\(^5\)

6 Asset Pricing Applications

In this section, we examine the behavior of an anxiety-prone agent in a stylized financial market. A key feature of asset prices that has recently aroused much attention is a downward-sloping term structure of risk premia in the equity market (van Binsbergen et al., 2012, 2013). The authors empirically price a claim on the dividends of the S&P 500 index in the near future in contrast to the price of the S&P 500 itself which is a claim on all its future dividends.\(^6\) The return from holding the claim to only the short-term dividends is much higher than the return to holding the claim to all future dividends, as displayed in Table 1 adapted from Table 1 in van Binsbergen et al. (2012). Not only is the return on the short-term claim higher – 14.8% vs. 6.9% annualized – but also the Sharpe ratios show that the price of equity risk is almost twice as high for the short-term claim. These results reflect that the premium for risks in the near future is significantly higher than the pre-

\(^5\)The dynamic inconsistency also raises issues for welfare analysis. Since there is no generally accepted welfare criterion for dynamically inconsistent agents, we focus on purely positive analysis in this paper.

\(^6\)The authors price the dividend strips synthetically using the prices of options on the S&P 500 and a no-arbitrage condition (put-call-parity).
Table 1: Effect of horizon on returns

<table>
<thead>
<tr>
<th></th>
<th>ST claim</th>
<th>LT claim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean return</td>
<td>1.16%</td>
<td>0.56%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>7.80%</td>
<td>4.69%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.1124</td>
<td>0.0586</td>
</tr>
</tbody>
</table>

Note: The table is adapted from Table 1 in van Binsbergen et al. (2012) and shows properties of monthly returns on a short-term dividend strip and on the S&P 500 itself.

um for risks in the distant future. We now show how HDRA can account for this phenomenon.

We consider a standard asset pricing setup in discrete time with two periods $t = 0, 1$ and two assets. Asset 0 pays a random dividend $\tilde{x}_0$ at the end of period 0 while asset 1 pays a random dividend $\tilde{x}_1$ at the end of period 1. Each asset is in net supply of 1 and the dividends $\tilde{x}_t$ are i.i.d. At the beginning of period 0, the agent has to form a portfolio $(\phi_0, \phi_1)$ of the two assets as well as borrowing/lending a quantity $\xi_t$, for $t = 0, 1$. Given initial wealth $w$, the agent solves the following problem:

$$
\max_{\{\phi_0, \phi_1, \xi_0, \xi_1\}} \mathbb{E}[v(\tilde{c}_0) + \delta u(\tilde{c}_1)]
$$

s.t. $\tilde{c}_t = \tilde{x}_t \phi_t + \xi_t$ for $t = 0, 1$

$$
p_0 \phi_0 + \xi_0 + p_1 \phi_1 + \frac{\xi_1}{1 + r} \leq w.
$$

The first-order conditions for an interior solution yield:

$$
\mathbb{E}[v'(\tilde{c}_0) (\tilde{x}_0 - p_0)] = 0
$$

and

$$
\mathbb{E}[\delta u'(\tilde{c}_1) (\tilde{x}_1 - (1 + r) p_1)] = 0.
$$

For a mass of identical HDRA agents we have $\tilde{c}_0 = \tilde{x}_0$ and $\tilde{c}_1 = \tilde{x}_1$ which gives us the following result on risk premia. (All proofs are relegated to the appendix.)

**Proposition 1.** If $v$ is more risk averse than $u$, the risk premium on the short-term
claim is higher than the risk premium on the long-term claim:

\[ E[\tilde{x}_0] - p_0 > E[\tilde{x}_1] - (1 + r) p_1. \]

This result shows that the HDRA model can directly account for the downward-sloping term structure of risk premia. Leading asset pricing models predict the opposite.\(^7\)

To understand the link between the above proposition and the empirical evidence, it is important to recall the distinction we pointed out in section 4.1 between a preference for early resolution of uncertainty and HDRA. In the real world, immediate risk resolves for both short-term assets and long-term assets. In HDRA, however, not the horizon of resolution alone, but the horizon of the payouts affects utility. Only long-term assets, by definition, have risks that are associated with payouts in the distant future. Therefore, a preference for the early resolution of uncertainty cannot explain a downward-sloping term structure of risk premia. The HDRA model also suggests an explanation for the value premium: as the duration of value stocks is shorter than that of growth stocks (Dechow et al., 2004) and HDRA investors dislike short-term risk, a value premium ensues. Notably, this prediction arises directly from the utility specification, and foregoes assumptions about the correlation structure of consumption growth and stochastic discount factor (Lettau and Wachter, 2007).

### 7 Commitment Devices and Institutional Responses

An agent who plans for tomorrow according to preference \( u \), but realizes that the future self will disagree with these plans (because she will have preference \( v \)), may try to find ways to commit to a future plan of action. While Schelling (1984) and others have discussed the ethical aspects that such a possibility brings about, the present discussion is only concerned with the fact that – and the question how – the agent can restrict her future self’s behavior, simply by virtue of having a first-mover advantage.

\(^7\)van Binsbergen et al. (2012) show that the term structure of risk premia is constant or upward-sloping in models such as Campbell and Cochrane (1999) and Bansal and Yaron (2004).
advantage over her future selves.\footnote{Similarly, we make no claim as to the normative implications (Gollier, 2012) but merely provide a positive model.}

Hiring an agent to carry out future risk-taking decisions according to the current self’s preferences is one way to prevent future actions from conflicting with the current self’s plans. In an investment setting, it may be the case that the anxious self is too risk averse to invest in equity, although the agent realizes that doing so has long-run benefits compared to saving in a less risky alternative that yields lower average returns. In this situation it makes sense for the agent to delegate investment decisions to a portfolio manager. In fact, Vanguard explicitly lists behavioral coaching as one of the benefits of its investment advisory services, stating that “some of the most significant opportunities [for the advisor] to add value occur [...] when clients are tempted to abandon their well-thought-out investment plan” (Kinniry, Jaconetti, DiJoseph, and Zilbering, 2014). They posit that behavioral coaching generates an “advisor’s alpha” of 150 basis points, based on a Vanguard study comparing the average returns of IRA account holders who make unadvised changes to their portfolio allocation to the average returns of corresponding target-date retirement funds (Weber, 2013). These considerations suggest that the value of delegated portfolio management, in contrast to arrived notions, may not lie in picking underpriced stocks. Rather, delegated portfolio management may help individuals prevent what Campbell (2006) identifies as the number one investment mistake households make: non-participation in risky asset markets. In other words, the appropriate benchmark to evaluate the performance of delegated portfolio and investment advice for HDRA agents is not a passive risky asset return, but the risk-free rate.

The following model formalizes this intuition in a setting with two periods, $t = 0, 1$. Going backwards, at the beginning of period 1, the HDRA agent has to form a portfolio $(\phi_1, \xi_1)$ consisting of a risky asset and a risk-free asset. The price of the risky asset is $p$ and it pays off a random $\tilde{x}$ at the end of period 1. In period 0, the agent decides whether to delegate the investment decision to a manager. The manager charges a fee $f > 0$, and invests at time $t = 1$ as instructed at $t = 0$. The agent’s degree of sophistication plays a key role in the delegation decision.
At $t = 0$, a naive agent plans for $t = 1$ to invest in stocks an amount

$$\phi_{1, \text{plan}}^{\text{self}} = \arg \max_{\phi} E\left[u\left(w + (\bar{x} - p) \phi\right)\right].$$

If instead the agent were to delegate the investment decision, she would advise the manager to buy

$$\phi_{1, \text{delegate}} = \arg \max_{\phi} E\left[u\left(w + (\bar{x} - p) \phi - f\right)\right].$$

Note that the agent evaluates the risk to occur at time $t = 1$ according to $u$, whether investment is delegated or not. When considering delegation at $t = 0$, the naive agent thus compares

$$E\left[u\left(w + (\bar{x} - p) \phi_{1, \text{delegate}} - f\right)\right] \text{ vs. } E\left[u\left(w + (\bar{x} - p) \phi_{1, \text{plan}}^{\text{self}}\right)\right].$$

The next proposition immediately follows.

**Proposition 2.** Given a management fee $f > 0$, a naive HDRA agent never delegates the portfolio decision.

The naive agent’s comparison (7) is flawed, however. Once period $t = 1$ arrives, the risk is imminent and is evaluated according to the more risk averse $v$. Contrary to her plans at $t = 0$, the naive agent, if left to her own devices at $t = 1$, will only invest

$$\phi_{1, \text{actual}}^{\text{self}} = \arg \max_{\phi} E\left[v\left(w + (\bar{x} - p) \phi\right)\right].$$

Since $v$ is more risk averse than $u$, we know that $\phi_{1, \text{actual}}^{\text{self}} < \phi_{1, \text{plan}}^{\text{self}}$ (see Wang and Werner, 1994).

A sophisticated agent takes the future self’s optimization problem as given and therefore optimizes subject to constraint (8). She thus compares

$$E\left[u\left(w + (\bar{x} - p) \phi_{1, \text{delegate}}\right) - f\right] \text{ vs. } E\left[u\left(w + (\bar{x} - p) \phi_{1, \text{actual}}^{\text{self}}\right)\right].$$

The left hand sides of the comparisons in (7) and (9) are the same; a naive and
a sophisticated agent both correctly anticipate that a money manager will implement \( \phi_1 = \phi_1^{\text{delegate}} \). However, the right hand sides of the comparisons differ since \( \phi_1^{\text{self, actual}} < \phi_1^{\text{self, plan}} \). These comparisons lead to the following proposition.

**Proposition 3.** Given HDRA preferences \((v, u)\), a sophisticated agent delegates the portfolio decision if the management fee \( f \) is sufficiently small. Correspondingly, given a management fee \( f \), a sophisticated agent with HDRA preferences \((v, u)\) delegates the portfolio decision if \( v \) is sufficiently more risk averse than \( u \).

Consistent with the hypothesis that households choose financial advice in order to achieve a riskier portfolio allocation, Foerster et al. (2014) show that the foremost effect of financial advisors is to increase individuals’ risky asset market participation (by up to 67%), as well as the extent of such risky asset market participation (by up to 39%). In addition, individuals who choose financial advice are less likely to close their accounts after negative return episodes. Finally, a comparison between these authors’ OLS and IV estimates provides explicit support for the notion of demand for a commitment device to take risk: individuals who would otherwise not participate in the stock market are more likely to choose financial advice, a decision which subsequently increases their risky asset market participation.\(^9\)

An important assumption in deriving proposition 3 is that the agent cannot undo the delegation decision of period 0 once period 1 arrives. The commitment device that an anxiety-prone agent uses for risk-taking must be illiquid to some degree, similar to commitment devices a present-biased agent uses for saving, e.g. the “golden eggs” of Laibson (1997). Such illiquidity can be explicit, as in the case of Vanguard’s “Personal Advisor Services” which go beyond just the behavioral coaching alluded to above. The service directly prevents clients from trading on their own (Vanguard

\(^9\)To address the same puzzle, Gennaioli, Shleifer, and Vishny (2012) assume that agents delegate to “money doctors” because it reduces the perceived risk. Our model of anxiety predicts similar behavior based on a non-standard preference rather than a belief distortion. Of course, effort costs of managing one’s portfolio may also lead to delegation of investment management. However, effort costs cannot justify hiring a manager who underperforms the index on average, as buying index funds is virtually costless and free of effort. While buying the index is free of effort, but it is not free of short-term risk and associated “anxiety.” Self 0 may thus correctly anticipate that the anxious self 1 will underperform the market even more than a random portfolio manager by failing to invest in equity at all. Self 0 will therefore be willing to pay an investment manager, even if she expects her to underperform the market.
Advisers, Inc., 2014). However, we also observe more subtle institutional features that provide illiquidity in arrangements where risk-taking is delegated.

Fees are one obvious feature that discourages agents from undoing delegation arrangements once they are set up. HDRA thus provides one explanation for why redemption fees continue to feature prominently in the mutual fund industry, while management fees are increasingly being competed away (Khorana, Servaes, and Tufano, 2009). Different fees that also help agents commit to risk taking are, e.g., fees brokerage house charge for changing the ratio of equities and bonds in one’s managed investment portfolio and fees that are imposed if the total exposure to a certain asset class falls below a threshold.\(^{10}\) Another way to provide the desired illiquidity is to introduce delays. Especially high-risk forms of delegation such as hedge funds commonly impose initial lock-in periods and subsequent mandatory delays for withdrawals. Putting a temporal distance between the investor’s decision to pull out and the valuation and payout of the investment prevents HDRA investors from “chickening out,” as the delayed risky outcome is treated with lower risk aversion by the investor.

8 Conclusion

We model agents with horizon-dependent risk aversion who are more risk averse when risks are closer in time. The HDRA preference formulation we propose describes behavior of experimental subjects that cannot be described with existing modeling approaches. To give an example of potential applications, we link HDRA preferences to established asset pricing puzzles. Sophistication about the resulting dynamic risk inconsistency and the associated costs triggers institutional responses such as delegated portfolio management. In comparison to existing models that describe the above phenomena, our model provides a unified explanation for all of them.

While the present model is static, we derive implications for general equilibrium asset pricing with horizon-dependent risk aversion in a multi-period model in Andries et al. (2014). HDRA preferences also have implications for information acquisition

\(^{10}\)The cost of having to provide liquidity without delay cannot necessarily explain such restrictions since the fund could easily charge the investor directly for such liquidation costs.
and belief formation which we study in a separate paper (Eisenbach and Schmalz, 2012). In that paper we show why it may be beneficial to a sophisticated anxiety-prone agent to hold overconfident beliefs as a commitment device to take risk, and how such self-delusion can lead to excessive risk taking.

The model’s new testable predictions include that sophisticated individuals exhibit a demand for commitment to take risk. Testing this prediction experimentally seems a promising road for future research. The model can be extended easily to generate testable implications also in other domains such as corporate finance, individual investor behavior, and household finance.
References


Appendix

Proof of Proposition 1. Since \( v \) is more risk averse than \( u \) we have

\[
- \frac{v''(x)}{v'(x)} > - \frac{u''(x)}{u'(x)}
\]

\[
\Rightarrow - \frac{d}{dx} \log v'(x) > - \frac{d}{dx} \log u'(x).
\]

Integrating both sides over some interval \([a, b]\) yields

\[
\frac{v'(b)}{v'(a)} < \frac{u'(b)}{u'(a)}
\]

and the reverse inequality for \( b < a \). For general \( a, b \) we therefore have

\[
\left( \frac{u'(a)}{u'(b)} - \frac{v'(a)}{v'(b)} \right) (a - b) > 0.
\]

Taking expectations for random \( \tilde{a} \) we get

\[
\frac{\mathbb{E}\left[u' (\tilde{a}) (\tilde{a} - b) \right]}{u'(b)} > \frac{\mathbb{E}\left[v' (\tilde{a}) (\tilde{a} - b) \right]}{v'(b)}.
\]

(10)

Substituting in the price \( p_0 \) for \( b \) and the dividend \( \tilde{x} \) for \( \tilde{a} \) the RHS is zero by the first order conditions and we get

\[
\mathbb{E}\left[u' (\tilde{x}) (\tilde{x} - p_0) \right] > 0
\]

Using the first order conditions, this implies that \( p_0 < (1 + r) p_1 \) and since \( \mathbb{E}[\tilde{x}_0] = \mathbb{E}[\tilde{x}_1] \) we have

\[
\mathbb{E}[\tilde{x}_0] - p_0 > \mathbb{E}[\tilde{x}_1] - (1 + r) p_1
\]

as desired. \( \square \)
Proof of Proposition 2. The fund management fee $f$ effectively reduces wealth and the agent is always worse off with lower wealth:

$$
\frac{d}{df} \max_{\phi} \mathbb{E}[u(w + (\bar{x} - p) \phi - f)] < 0 \quad (11)
$$

Given the definitions of $\phi_1^{\text{delegate}}$ and $\phi_1^{\text{self, plan}}$, the behavior of a naive agent follows immediately from (7) which is equivalent to the following inequality:

$$
\max_{\phi} \mathbb{E}[u(w + (\bar{x} - p) \phi - f)] < \max_{\phi} \mathbb{E}[u(w + (\bar{x} - p) \phi)].
$$

Thus, a naive agent will never delegate. \qed

Proof of Proposition 3. Turning to a sophisticated agent, given the definition for $\phi_1^{\text{self, actual}}$ we have:

$$
\max_{\phi} \mathbb{E}[u(w + (\bar{x} - p) \phi)] > \mathbb{E}[u(w + (\bar{x} - p) \phi_1^{\text{self, actual}})]. \quad (12)
$$

From condition (11) follows that there exists an $\bar{f} > 0$ such that:

$$
\max_{\phi} \mathbb{E}[u(w + (\bar{x} - p) \phi - \bar{f})] = \mathbb{E}[u(w + (\bar{x} - p) \phi_1^{\text{self, actual}})]. \quad (13)
$$

For any $f \in [0, \bar{f})$ we therefore have

$$
\mathbb{E}[u(w + (\bar{x} - p) \phi_1^{\text{delegate}} - f)] > \mathbb{E}[u(w + (\bar{x} - p) \phi_1^{\text{self, actual}})]
$$

so the sophisticated agent will choose delegation. Given the results of Wang and Werner (1994), the inequality (12) is stronger the greater the difference in risk aversion between $v$ and $u$. Therefore the critical value $\bar{f}$ defined in (13) is smaller the greater the difference in risk aversion between $v$ and $u$. \qed