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# Federal Reserve Tools for Managing Rates and Reserves

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## **Federal Reserve Tools for Managing Rates and Reserves**

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### **Abstract**

The Federal Reserve announced in January 2019 that it would maintain an ample supply of reserves amid its balance sheet reduction. We model the impact of reserves on banks' liquidity and balance sheet costs. In competitive general equilibrium, the optimal supply of reserves equates bank deposit rates to the interest rate paid on excess reserves (IOER), consistent with ample reserves. Raising the Fed's overnight reverse repo rate up to IOER would increase liquidity, expediently reduce the overabundance of reserves, and stabilize the volatility of overnight market rates. Empirical analysis supports our model and can explain recent puzzles in money market rates.

Key words: banks, balance sheet costs, liquidity, Federal Reserve, reserves, overnight reverse repurchases

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# 1 Introduction

The Federal Reserve’s balance sheet grew dramatically in the past decade to over \$4 trillion in size, with reserves increasing from roughly \$10 billion in 2008 to a peak of \$2.8 trillion in 2014. In January 2019, the Federal Reserve announced it would maintain an ample supply of reserves at the conclusion of its ongoing balance sheet reduction.<sup>6</sup> However, there is a debate between advocates for maintaining very abundant reserves, as has been the case post-crisis, versus reverting to very scarce reserves, as pre-crisis.

We analyze the optimal supply of reserves in a general equilibrium model of banks. Our model delivers new insights and important policy guidance. In particular, we show that the optimal supply of reserves is achieved when bank deposit rates equal the interest rate on excess reserves (IOER). The corresponding supply of reserves equates banks’ marginal liquidity and balance sheet costs. We also show that it is optimal to set the overnight reverse repurchase (RRP) rate equal to the IOER rate. Such a setting endogenously brings reserves to their optimal level and absorbs bank liquidity shocks to stabilize the volatility of overnight market rates.

Our results derive from three main ingredients. First, reserves are banks’ most immediate source of liquidity, and the cost of borrowing reserves on the interbank market is higher when reserves are scarcer. Second, moral hazard necessitates capital requirement regulation. Third, capital requirements create balance sheet costs for banks, because equity is costly relative to deposits. Deposits provide households with liquidity whereas equity does not.

We conduct an empirical analysis of the dependence of wholesale deposit rates on reserves and short-term Treasury securities, which our model suggests have opposite effects on bank balance sheet costs. We also test the significance of an additional impact on bank balance sheet costs, the expansion by the FDIC of its assessment base in April 2011. The results are supportive of our model and provide new insight on the role that short-term Treasury securities, as well as reserves, play in the determination of bank borrowing rates. In particular, our analysis can explain the greater than anticipated rise in short-term rates relative

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<sup>6</sup>See the Federal Open Market Committee (FOMC) announcement at <https://www.federalreserve.gov/newsevents/pressreleases/monetary20190130c.htm>. Reserves are widely expected to decline to roughly \$1 trillion. For example, see the New York Fed’s survey of primary dealers’ expectations at: <https://www.newyorkfed.org/medialibrary/media/markets/survey/2018/dec-2018-spd-results.pdf>.

to IOER over the past year that has puzzled many market observers.<sup>7</sup>

Our main result, that the Fed should set the quantity of reserves such that the equilibrium bank deposit rate equals IOER, follows from the trade-off between the benefit of reserves to provide liquidity to banks with the cost of reserves that follows from their requirement to be held only by banks. Our model also highlights that it would be unwise to try to go back to the pre-crisis regime, with a bank deposit rate exceeding the implicit interest rate on reserves of zero. Overall, our results fall within the broad guidance of the FOMC's most recent March 2019 normalization principles and plans.<sup>8</sup> Importantly, we provide the first guide as to how an optimal moderate supply of reserves may be determined.

By focusing on the economic frictions that affect short-term rates broadly, our model also sheds new light on the important role that the Fed's overnight RRP plays in U.S. money markets. Overnight RRP are available to some non-bank as well as bank counterparties at a rate set at a spread below IOER. We show that, in equilibrium, overnight RRP with non-banks increase welfare by absorbing bank liquidity shocks. This reduces balance sheet costs and increases bank liquidity by enabling a higher optimal supply of reserves. The overnight RRP rate can be increased to equal IOER to ensure that the overnight RRP provides the maximum welfare value. Increasing the overnight RRP rate to IOER can also efficiently reduce the current overabundance of reserves to their optimal level more rapidly than relying on the Fed's current strategy of gradual asset run-offs alone. The overnight RRP also stabilizes the volatility of interest rates better than IOER does alone, something that is clearly observed empirically. Overall, we find that the overnight RRP has many benefits and should not be phased out, as the FOMC has stated it intends to do.<sup>9</sup>

The optimal quantities of reserves and the overnight RRP, which determine the size of the Fed's balance sheet, is a topic of growing policy and market attention. Several authors have recently analyzed this topic with contrasting views. Greenwood, Hansen, and Stein (2015, 2016) argue for a large Fed balance sheet to supply large amounts of overnight RRP for financial stability reasons. They reason that overnight RRP can act similarly to their documented findings of Treasury bills crowding out banks' production of money-like assets, complementing the work of Nagel (2016).<sup>10</sup> Gagnon and Sack (2014) advocate for setting

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<sup>7</sup>In response to a reporter's question in 2018 about why this rise is occurring, Fed Chairman Powell replied "Really, no one [knows why]...I think there's a lot of probability on the idea of just high [Treasury] bill supply leads to...higher money market rates generally... We don't know that that's the only effect and, you know, we're just going to have to be watching and learning." Source: <https://www.federalreserve.gov/mediacenter/files/FOMCpresconf20180613.pdf>.

<sup>8</sup>Source: <https://www.federalreserve.gov/newsevents/pressreleases/monetary20190320c.htm>.

<sup>9</sup>Source: <https://www.federalreserve.gov/monetarypolicy/policy-normalization.htm>.

<sup>10</sup>Additional advocates for a large Fed balance sheet include Cochrane (2014), who argues for sizable overnight RRP to crowd out private short-term debt creation and large reserves to satiate monetary liquidity

the overnight RRP rate at IOER and maintaining an abundance of reserves as optimal. We show that the overnight RRP rate set to IOER will prevent an abundant quantity of reserves in equilibrium, and that a moderate equilibrium quantity of reserves is optimal. Sims (2016) argues for a small Fed balance sheet based on the maturity mismatch between the Fed’s assets and reserves liabilities, which creates risks to the Fed’s net worth, political support, and policy independence. Williamson (2019) shows that excessive reserves cause bank balance sheet costs because of binding capital requirements. He argues for reducing the overabundance of reserves either directly or else indirectly through the overnight RRP to improve welfare.<sup>11</sup>

In contrast to these recent studies, which generally advocate either for a very “large” Fed balance sheet, as is currently the case, or a very “small” Fed balance sheet, as before 2008, we find that the Fed balance sheet size should be neither too small nor too large. Rather, the optimal size is determined by a moderate quantity of reserves and overnight RRP. Increasing the quantity of reserves beyond a certain point provides diminishing liquidity benefits that are eventually outweighed by the costs of bank balance sheet expansion. Decreasing the quantity of reserves too much creates increasing bank illiquidity costs that eventually outweigh the benefits of the reduction in bank balance sheet costs.

The paper proceeds with an institutional background on the Fed’s balance sheet policies, followed by the analysis of optimal reserves and then the overnight RRP. Proofs and extensions are located in the appendices.

## 2 Institutional background

Historically, the Federal Reserve supplied a scarce amount of reserves in the banking system to maintain positive interest rates. Depository institutions (DIs) in the U.S. hold ‘required reserves’ for reserve requirements and ‘excess reserves’ for precautionary reasons to meet

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needs. Duffie and Krishnamurthy (2016) argue for sizable overnight RRP to produce a more direct and effective transmission of interest rate policy to money markets and financial markets. Bernanke (2016) supports extensive overnight RRP and large reserves for better establishing the Fed as lender-of-last-resort. Additional arguments for large reserves are for banks’ provision of payment services (Curdia and Woodford, 2011), broad liquidity services (Goodfriend, 2002), and the effective transmission of monetary policy (Reis, 2016). Gertler and Karadi (2013) argue for a large amount of Fed assets to help circumvent bank leverage constraints.

<sup>11</sup>Stein (2012) and Kashyap and Stein (2012) advocate for small reserves to maintain their scarcity value and limit bank creation of money-like deposits for financial stability reasons. Bindseil (2016) argues that central banks should have small balance sheets in order to focus on their core mandate.

liquidity shocks. DIs in need borrow reserves from each other at the federal funds rate.<sup>12</sup> This rate represented the marginal cost for banks' most immediate liquidity shocks and influenced other bank funding rates and money market rates through arbitrage and substitution. The Fed targeted the fed funds rate by adjusting the supply of reserves using open market operations (OMOs) as its primary policy. In an OMO, the open market trading desk at the Federal Reserve Bank of New York would buy or sell government securities with 'primary dealer' counterparties either on a temporary basis (using repurchase agreements, i.e., RPs) or on a permanent basis (using outright transactions).<sup>13</sup> For example, purchasing Treasuries would increase the supply of reserves and decrease the fed funds rate.<sup>14</sup>

This scarcity method for managing short-term rates was no longer available once the level of reserves dramatically increased starting in late 2008. Reserves grew as a by-product of the Fed's crisis liquidity operations and subsequent large-scale asset purchases (LSAPs, i.e. quantitative easing) aimed at lowering longer-term rates.<sup>15</sup> With a supply of reserves far beyond banks' demand for reserve requirements and precautionary liquidity, banks' have a zero-interest rate marginal value for reserves that do not pay interest.

**IOER** To manage rates with abundant reserves, the Fed prepared a variety of new policy tools.<sup>16</sup> The Fed began to pay IOER to DIs in October 2008, following Congressional authorization included in the Emergency Economic Stabilization Act of 2008.<sup>17,18</sup> The Fed distinguishes between IOER as its primary policy rate and the fed funds rate as its policy target. In December 2008, the FOMC lowered IOER to 25 basis points and set the fed funds target rate to a range of 0 to 25 basis points, its effective zero bound.<sup>19</sup> Since December 2015, the FOMC has raised IOER and the target range for the fed funds rate several times.

**Overnight RRP** The overnight RRP is economically analogous to collateralized loans made to the Fed by its expanded set of 164 counterparties, which include DIs and non-DIs

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<sup>12</sup>A few other type of institutions, such as government-sponsored enterprises (GSEs), also participate in the federal funds market.

<sup>13</sup>The list of primary dealers is at <http://www.newyorkfed.org/markets/primarydealers.html>. Assets eligible for OMOs are Treasuries, agency debt, and agency mortgage-backed securities.

<sup>14</sup>See Ennis and Keister (2008) and Keister et al. (2008) for a more detailed introduction to traditional Federal Reserve monetary policy and OMOs.

<sup>15</sup>Gagnon et al. (2011) provide an analysis of the first LSAP. Details on the Fed's asset purchase program are at: <https://www.federalreserve.gov/faqs/what-were-the-federal-reserves-large-scale-asset-purchases.htm>.

<sup>16</sup>The list of Fed policy tools is at: <https://www.federalreserve.gov/monetarypolicy/policytools.htm>.

<sup>17</sup>Source: <http://www.newyorkfed.org/newsevents/speeches/2009/dud090729.html>.

<sup>18</sup>In principle, the Federal Reserve could set a different rate for the interest on required reserves and the interest on excess reserves. In practice, the two have been the same since 2009.

<sup>19</sup>Source: <http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20081216.pdf>.

such as money market mutual funds (MMFs), GSEs, and securities dealers.<sup>20</sup> The overnight RRP does not change the size of the Fed’s balance sheet but modifies the composition of its liabilities by reducing reserves. Small-value testing of the overnight RRP began in 2009 with fixed-quantity auctions before switching in 2013 to a fixed-rate facility.<sup>21</sup> The overnight RRP with a fixed-rate set at the bottom of the fed funds target range was formally adopted by the FOMC in March 2015.<sup>22</sup>

**Normalization** In September 2017, the Fed announced its normalization plan to gradually reduce its balance sheet by stopping the reinvestment of capped amounts of proceeds from its maturing Treasuries and agency mortgage-backed securities.<sup>23</sup> This strategy allows for a gradual run-off of the Fed’s assets and hence gradual decrease in the supply of reserves. The end-point for this run-off is scheduled for September 2019, and the quantity of reserves will continue to gradually decline for a further amount of time offset by gradual increases in currency and other non-reserve Fed liabilities.<sup>24</sup>

## 3 Model

In this section, we develop the model with interest on reserves to determine the optimal supply of reserves in section 4. In section 6, we add the overnight RRP to the model to analyze its optimal policy use.

### 3.1 Set up

The economy lasts for three dates 0, 1, 2, and is populated by competitive risk-neutral households who maximize expected utility, banks and firms that maximize expected profits, and a central bank and government. At date 0, households have an endowment of  $g$  goods. Banks, firms, and the central bank do not receive an endowment.

At date 0, firms borrow from banks to buy goods as inputs into their production technology. The government buys goods for an exogenous amount of fiscal consumption financed

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<sup>20</sup>Pre-crisis, RRP’s with primary dealers were used on a very infrequent basis by the Fed as a part of standard OMOs. Details on RRP’s and the list of RRP counterparties are at the following two links, respectively: <http://data.newyorkfed.org/aboutthefed/fedpoint/fed04.html> [https://www.newyorkfed.org/markets/rrp\\_counterparties](https://www.newyorkfed.org/markets/rrp_counterparties).

<sup>21</sup>Source: <http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20130731.pdf>.

<sup>22</sup>Source: <https://www.federalreserve.gov/monetarypolicy/policy-normalization.htm>.

<sup>23</sup>Details are at: <https://www.federalreserve.gov/monetarypolicy/fomcminutes20170920.htm>.

<sup>24</sup>Details are at: <https://www.federalreserve.gov/newsevents/pressreleases/monetary20190320c.htm>.

by issuing government bonds. Households use proceeds from selling goods to acquire government bonds, bank deposits, and preferred bank equity, which we refer to as *equity*. The central bank buys government bonds with newly created reserves, which act as the nominal unit of account in the economy.<sup>25</sup> Reserves can only be held by banks and earn a rate of interest set by the central bank.

To create a motive for interbank trading we assume that bank deposit markets are segmented and banks are subject to a liquidity shock at date 1. Households can only deposit at the banks in their sector. For simplicity, we also assume that households only hold equity in banks in their sector and firms only borrow from banks in their sector. These latter two assumptions are not required for the results.

The liquidity shock takes the form of a deposit withdrawal from households in one of the sectors to buy government bonds. The shocked-sector banks may meet the withdrawal with their reserve holdings. If they do not have enough reserves, they can borrow in the interbank market. Households in the other sector sell bonds and deposit the proceeds with their banks.

At date 2, firms sell the goods they produced. Households receive the proceeds from their deposit, equity, and government bond holdings, as well as the (common equity) residual profits of their sector's banks and firms. Households also pay a lump-sum tax out of their nominal revenues and buy goods from firms to consume.

**Ingredients** Our results derive from three standard ingredients from the banking literature, which are detailed in the next subsection. These frictions are added in a reduced form manner to focus the analysis on the logical implications of these ingredients for central bank policy. First, interbank market frictions lead to bank liquidity costs when there is a scarcity of reserves. Second, bank moral hazard arises from inefficient risky projects that banks can take to shift risk onto depositors. The government, as regulator, imposes capital requirements in the form of a minimum amount of equity. Third, equity is relatively costly because it does not provide households the liquidity value of deposits and bonds.

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<sup>25</sup>We consider only the central bank and households as holders of bonds, as banks and firms would not hold bonds in equilibrium.

## 3.2 Optimizations

**Households** Following the recent literature establishing the liquidity value of money-like assets, households have a liquidity factor on deposits and bonds, denoted by  $\theta > 1$ .<sup>26</sup> We assume that the size of the liquidity shock is proportional to the bank assets held by households in the shocked sector. Those assets are deposits, denoted by  $D$ , and (preferred) equity, denoted by  $E$ . The amount of early withdrawals can thus be written as  $D^W \equiv \lambda(D + E)$ , where  $\lambda$  is the size of the liquidity shock.

Each household receives the return on the equity it holds ( $R^E E$ ) as well as the residual profits from the representative bank ( $\Pi^{Bj}$ ) and firm ( $\Pi^{Fj}$ ) in its sector, and pays a lump sum tax  $\Upsilon$ . In addition, the households' nominal income depends on whether it is in the shocked sector,  $j = s$ , or nonshocked sector,  $j = n$ , which is realized at date 1. We use  $\mathbf{1}$  as the indicator function to show the additional returns received by the representative shocked and nonshocked households, respectively, where  $\mathbb{E}[\mathbf{1}_{j=s}] = \mathbb{E}[\mathbf{1}_{j=n}] = \frac{1}{2}$ .

The household in the shocked sector decreases its deposits by  $D^W$  and increases its holdings of government bonds by a corresponding amount, denoted  $B_1^s$ . The household in the nonshocked sector sells  $B_1^n$  government bonds and deposits a corresponding amount with the bank in its sector, denoted  $D_1$ . In addition, this household has to hold the additional preferred equity,  $E_1$ , that the bank in its sector issues. Formally, we can write the household's expected nominal income as:

$$\begin{aligned} \Pi^{Hj} &= R^E E + \Pi^{Bj} + \Pi^{Fj} - \Upsilon \\ &+ \mathbf{1}_{j=s} \theta [R^D (D - D^W) + R^B (B^H + B_1^s)] \\ &+ \mathbf{1}_{j=n} \{R^{E1} E_1 + \theta [R^D D + R^{D1} D_1 + R^B (B^H - B_1^n)]\}. \end{aligned} \quad (1)$$

Note that equity doesn't benefit from the the liquidity factor  $\theta > 1$ .

A household's utility is the real value of its nominal income at date 2,  $u^j \equiv \frac{\Pi^{Hj}}{P_2}$ . The

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<sup>26</sup>The liquidity value of money-like short-term bank debt is the centerpiece of Greenwood, Hanson and Stein (2015, 2016), based on a monetary benefit service for households in Stein (2012) and a transactions value from lower information sensitivity in Dang et al. (2009, 2015, 2017), developed by Gorton and Pennacchi (1990), and motivated by the liquidity value of deposits originating in Diamond and Dybvig (1983) and Bryant (1980). More broadly, the safety and liquidity premium of Treasuries is demonstrated by Krishnamurthy and Vissing-Jorgensen (2011, 2012), Caballero and Krishnamurthy (2009), and Krishnamurthy (2002).

household optimization is:

$$\left. \begin{aligned} \max_{\mathcal{Q}^{Hj}} \mathbb{E}[u^j] \\ \text{s.t. } D + E + B^H \leq G \\ P_1^B B_1^s \leq R^W D^W \\ D_1 + E_1 \leq P_1^B B_1^n \\ B_1^n \leq B^H. \end{aligned} \right\} \quad (2)$$

where  $\mathcal{Q}^{Hj} \equiv \{D, E, B^H, D_1, E_1, B_1^j\}_j$ .  $G$  denotes the household's nominal quantity of goods at date 0. Without loss of generality, we normalize the price level at date 0 to equal one, so  $G = g$ .<sup>27</sup> The first three inequalities in optimization (2) are the budget constraints at date 0 for each household and at date 1 for the shocked and nonshocked households, respectively. The term structure of deposit returns implies that the return on early withdrawals at date 1 at the shocked bank is  $R^W = \frac{R^D}{R^{D1}}$ . The last inequality is a feasibility constraint.

**Firms** Firms demand loans to buy the quantity  $L$  of date-0 goods, which they use to produce goods sold at date 2. The price,  $P_2$ , at which these goods are sold is endogenously determined. The firm chooses  $L$  to maximize profits:

$$\max_L \Pi^{Fj} = P_2 \int_0^L r(\hat{L}) d\hat{L} - R^L L. \quad (3)$$

We assume that the marginal real return on production by firms is greater than one and follows standard Inada conditions with  $r(L) > 0$ ,  $r'(L) < 0$ ,  $r(0) = \infty$ , and  $r(\infty) = 1$ .

**Banks** We start this section by deriving the banks' profits. Banks earn interest income on their assets: loans, reserves, and potentially interbank loans. They pay interest on their liabilities: deposits, equity, and potentially interbank borrowing.

At date 0, banks supply loans,  $L$ , and hold reserves,  $M$ . The amount of reserves held by a bank at the end of date 1 depends on whether or not the bank is in the shocked sector. The bank in the shocked sector has to pay  $R^W D^W$  to withdrawing depositors and can borrow reserves in the interbank market. We denote its interbank borrowing by  $I^s$ . The bank in the nonshocked sector accrues reserves corresponding to additional deposits,  $D_1$ , and equity,  $E_1$ , and can lend in the interbank market. Its interbank market loans are denoted by  $I^n$ . In addition, banks in both sectors receive interest from the central bank on the reserves they

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<sup>27</sup>Nominal quantities and returns are denoted with capital letters, while real quantities and returns are denoted with lowercase letters.

hold at the end of date 0. Formally, we have

$$M_1^s = R^M M + I^s - R^W D^W \quad (4)$$

$$M_1^n = R^M M - I^n + D_1 + E_1 \quad (5)$$

for the shocked and nonshocked banks, respectively.

With this, we can write the date 2 profit for a bank in sector  $j \in \{n, s\}$  as:

$$\begin{aligned} \Pi^{Bj} = & R^L L + R^M M_1^j \quad (6) \\ & - \mathbf{1}_{j=s} [R^E E + R^D (D - D^W) + R^I I^s] \\ & - \mathbf{1}_{j=n} [R^E E + R^D D + R^{E1} E_1 + R^{D1} D_1 - R^I I^n + \int_0^{I^n} Y(\hat{I}^n) d\hat{I}^n]. \end{aligned}$$

On the RHS of equation (6), the first line represents the interest income on reserves and loans. The second line represents the shocked bank's outstanding liabilities at date 2. The third line represents the nonshocked bank's outstanding liabilities in the first four terms, the interbank loan asset in the fifth term, and the interbank cost in the last term, to which we now turn.

**Liquidity costs** In the last term of equation (6),  $Y(I^n)$  represents the marginal cost of monitoring interbank loans. We assume that  $Y(\cdot)$  is a convex function with  $Y(0) = 0$  and  $Y'(\cdot) > 0$  to capture the fact that as a bank's interbank lending exposure grows, more monitoring is necessary. Hence, the cost of providing liquidity to the interbank market increases with the amount of liquidity provided.

In practice, banks assign maximum allowable counterparty exposure limits that imply an increasing marginal shadow cost of interbank lending that is ultimately prohibitive. The interbank cost can also be interpreted as representing other costs in the interbank market, such as search costs, that increase when reserves are more scarce in equilibrium.<sup>28</sup>

Below we show that, in equilibrium, the shocked bank bears the entire interbank cost through the interbank rate it pays because that bank has an inelastic demand for interbank borrowing to meet its early withdrawals.<sup>29</sup>

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<sup>28</sup>The large literature on interbank monitoring costs originates with Rochet and Tirole (1996) and is detailed and broadly developed by Freixas and Rochet (2008) and Rochet (2008). Interbank market costs are also extensively studied in the more recent literature on the search and matching frictions in the bilateral, OTC fed funds market. See, e.g., Afonso and Lagos (2015), Armenter and Lester (2017), Ashcraft and Duffie (2007), Atkeson et al. (2015), and Bech and Monnet (2016). Limits to interbank borrowing capacity are studied as arising from liquidity and credit constraints by Acharya and Skeie (2011) and Ashcraft et al. (2011), and from moral hazard in monitoring by Acharya et al. (2012).

<sup>29</sup>Hence, a more general model of various interbank market costs paid directly by both the shocked and nonshocked banks would not change the paper's results.

**Capital requirements and balance sheet costs** Capital requirements arise because of bank moral hazard that takes the form of risk-shifting onto depositors. A bank can take an unobservable, negative expected NPV project that has a positive or negative return with equal probability. Specifically, each bank can take the project at date 0 for a marginal return, if there is a positive realization, of  $R^\alpha(A)$  on the bank's date 0 assets  $A$ , where

$$A = L + M.$$

The nonshocked bank can also take the project at date 1 for a marginal return, if there is a positive realization, of  $R^\alpha(A + A_1)$  on the bank's new date 1 assets  $A_1$ .  $A_1$  is equal to the amount of the bank's new liabilities at date 1:

$$A_1 = D_1 + E_1.$$

We assume that  $R^\alpha(\cdot)$  is a convex function with  $R^\alpha(0) = 0$  and  $R^{\alpha'}(\cdot) > 0$ , which implies that the bank's ability for risk-taking is increasing in the size of the bank's balance sheet at dates 0 and 1.

In practice, as banks grow larger, they can undertake greater amounts of hidden risk-taking, such as through derivatives and off-balance sheet exposures that do not require more verifiable initial investments as with loans. For example, banks are the predominant participant in the multi-trillion dollar market for interest rate swaps. Determining whether derivatives are used for hedging or speculating depends on sophisticated quantitative models of detailed bank-specific information on its assets and liabilities.

The bank chooses whether to take the risky project at dates 0 and 1 to maximize profit subject to the expected return required by equity. If there is a negative realization on the project, the bank and equityholders bear a complete loss. The depositors bear a partial loss and lose their liquidity value on deposits, such that  $\theta = 1$ . Hence, the risky project is a form of risk-shifting that is socially inefficient.

The government as regulator imposes a capital requirement that is sufficient to incentive banks not to take the risky project. The full analysis analyzed in appendix B shows that the capital requirement is

$$\begin{aligned} \underline{E}(A) &\equiv \frac{R^\alpha(A)A}{R^E} \\ \underline{E}_1(A, A_1) &\equiv \frac{R^\alpha(A+A_1)A_1}{R^{E1}}. \end{aligned} \tag{7}$$

The capital requirement acts as a "balance sheet cost" because it increases the cost of

growing a bank’s balance sheet.<sup>30</sup> Frictions related to bank balance sheet size are motivated in part by the analysis of market observers. For example, interbank broker Wrightson ICAP (2008) voiced concerns that large reserves could “clog up bank balance sheets.” Under Basel III, U.S. banks are subject to a leverage ratio that is higher, and assessed on a broader base, than was the case pre-crisis. Thus, regulatory-based bank balance sheet costs are likely to be relevant.<sup>31</sup> An additional source of balance sheets costs is the FDIC deposit insurance assessment fees that are applied to all non-equity bank liabilities and increase with banks’ balance sheet size. The balance sheet cost in our model can be interpreted as also capturing the effect of the FDIC assessment.<sup>32</sup> Furthermore, banks tended to reduce the size of their balance sheets as the quantity of reserves increased, which Martin et al. (2016) demonstrate may be explainable by the presence of balance sheet costs and a large level of reserves partially crowding out bank lending.<sup>33</sup>

**Bank optimization** We can now write the bank’s optimization, which is given by

$$\left. \begin{aligned} \max_{\mathcal{Q}^{Bj}} \mathbb{E}[\Pi^{Bj}] \\ \text{s.t. } L + M \leq D + E \\ M_1^j \geq 0 \quad \text{for } j \in \{n, s\} \\ E \geq \underline{E} \\ E_1 \geq \underline{E}_1, \end{aligned} \right\} \quad (8)$$

where  $\mathcal{Q}^{Bj} \equiv \{L, M, D, E, I^j, D_1, E_1\}_j$ . The first two inequalities are the bank’s budget constraints for dates 0 and 1, respectively.<sup>34</sup> The last two inequalities are the bank’s capital

<sup>30</sup>The impact of balance sheet costs based on the leverage and capital constraints for financial intermediaries has been studied for asset pricing, financial crises, and monetary policy in the recent literature, most notably by Adrian and Shin (2008, 2009, 2014), Adrian et al. (2014), He and Krishnamurthy (2012, 2013), He et al. (2010), and He et al. (2017).

<sup>31</sup>Portfolio manager of the Fed’s balance sheet Simon Potter states that “An increase in bank reserves that increases bank assets makes regulatory leverage ratios more binding, raising the shadow marginal cost of bank balance sheets...as the level of reserves declines during normalization, marginal balance sheet costs should fall.” Source: <https://www.newyorkfed.org/newsevents/speeches/2014/pot141007>.

<sup>32</sup>Armenter and Lester (2017) show in a calibrated model that bank balance sheet costs are increasing in reserves and deposits and are driven by direct expenses, such as FDIC fees, and indirect expenses, such as capital requirements and leverage ratios.

<sup>33</sup>Additionally, NY Fed President William Dudley states that “to the extent that the banks worry about their overall leverage ratios, it is possible that a large increase in excess reserves could conceivably diminish the willingness of banks to lend.” Source: <http://www.newyorkfed.org/newsevents/speeches/2009/dud090729.html>.

<sup>34</sup>For simplicity, we abstract from required reserves for banks, as they do not play a meaningful role and would not alter our results. Bennett and Peristiani (2002) show that reserve requirements have been largely avoided in the U.S. since the 1980s by sweep accounts. The small amounts of remaining reserve requirements are largely met by vault cash that banks hold for retail purposes and so pose little cost for banks.

requirements to issue a minimum amount of equity,  $\underline{E}$  and  $\underline{E}_1$  at dates 0 and 1, respectively.

**Central bank and government budget constraints** The budget constraint for the central bank at date 0 is

$$B^{CB} \leq M, \quad (9)$$

which shows that the quantity of bonds that the central bank buys as assets is limited by the quantity of reserves the central bank supplies at date 0.<sup>35</sup> In section 3.3 below, an equilibrium is defined for any choice of the supply of reserves,  $M$ . The central bank's optimization problem for choosing the optimal supply of reserves is presented in section 4.2 on welfare and optimal policy. We consider the return on reserves ( $R^M$ ) as exogenous since it does not affect welfare in our model and allows us to focus on the optimal supply of reserves for any given rate on reserves.<sup>36</sup>

At date 2, the government pays the return  $R^B B$  on bonds and receives the lump sum tax  $\Upsilon$ . The central bank receives the return  $R^B B^{CB}$  on its bond holdings and pays banks the return  $R^{M^2} M$  on reserves, where  $R^{M^2}$  denotes  $(R^M)^2$ . The date 2 consolidated budget constraint for the government and the central bank is

$$R^B B + R^{M^2} M \leq \Upsilon + R^B B^{CB}.$$

Since  $M$  and hence  $B^{CB}$  are chosen by the central bank,  $B$  and  $R^{M^2}$  are taken as exogenous, and  $R^B$  is an endogenous equilibrium variable, the lump sum tax  $\Upsilon$  required<sup>37</sup> to meet net government liabilities at date 2 is

$$\Upsilon \geq R^B (B - B^{CB}) + R^{M^2} M. \quad (10)$$

**Goods market at dates 0 and 2** At date 0, firms buy  $L$  goods, the government buys  $B$  goods, and households sell their endowment of goods  $g$  at a price normalized to one. For simplicity, we do not consider households storing goods, which they would not do in equilibrium except for instances of extreme balance sheet costs.

At date 2, firms sell their production of goods,  $\int_0^L r(\hat{L}) d\hat{L}$ . The household in sector  $j$  buys consumption goods  $c^j$  with its net nominal revenue, which is equal to the household's

<sup>35</sup> Abstracting from currency as a liability of the central bank does not effect our analysis.

<sup>36</sup> We use the term 'rate' interchangeably with 'return' to refer to gross returns rather than the net rate of return, except where 'net' rate is specified.

<sup>37</sup> Since the government tax is determined based on the central bank's choice of reserves, the determination of  $P_2$  in equilibrium represents a simple form of the fiscal theory of the price level with monetary dominance and passive fiscal policy. Note also that the quantities  $B$ ,  $B^{CB}$ ,  $M$ , and  $\Upsilon$  are normalized to refer to per-sector quantities.

income given by equation (1) with the household liquidity factor set to one. Thus,  $c^j P_2 = \Pi^{Hj}(\theta = 1)$ . The marginal real cost for interbank lending across sectors by the nonshocked bank is  $y(I^n) = \frac{Y(I^n)}{P_2}$ , which is considered to be a cost in terms of real resources. These resources are acquired by the nonshocked bank buying a total of  $\int_0^{I^n} y(\hat{I}^n) d\hat{I}^n$  goods at date 2.

### 3.3 Equilibrium

**Assumptions** To focus on a size of liquidity shocks that is consistent with common money market flows, we assume that  $\lambda$  that is less than  $\frac{R^M}{R^W}$ ,  $\frac{D}{D+E}$ , and  $\frac{B-M}{G-(B-M)}$ . This ensures that, at date 1, the amount of interbank borrowing weakly decreases in the quantity of reserves, shocked household withdrawals of deposits are feasible, and nonshocked household sales of bonds are feasible, respectively.

**Definition 1** *An equilibrium in the economy consists of the two-period returns for date 0 assets  $(R^L, R^D, R^E, R^B) > 0$ , the one-period returns for date 1 assets  $(R^I, R^{D1}, R^{E1}) > 0$ , the price of bonds at date 1  $P_1^B > 0$ , and the price of goods at date 2  $P_2 > 0$ ; such that, at the optimizing quantities  $Q^{Hj}$  for the household in each sector  $j \in \{n, s\}$  given by (2),  $L$  for each firm given by (3), and  $Q^{Bj}$  for the bank in each sector  $j \in \{n, s\}$  given by (8); and at the central bank and government exogenous quantities  $M$  and  $B$ , and endogenous quantities  $B^{CB}$  and  $\Upsilon$  given by binding budget constraints (9) and (10), respectively; markets clear for:*

- (a) deposits ( $D$ ), equity ( $E$ ) and loans ( $L$ ) within each sector  $j \in \{n, s\}$  at date 0;
- (b) deposits ( $D_1$ ) and equity ( $E_1$ ) within sector  $j = n$  at date 1;
- (c) reserves at date 0 ( $M$ ) and date 1,  $M_1^n + M_1^s = 2R^M M$ ;
- (d) bonds at date 0,  $B^{CB} + B^H = B$ ;
- (e) bonds at date 1,  $B_1 \equiv B_1^n = B_1^s$ ;
- (f) interbank loans at date 1,  $I \equiv I^n = I^s$ ;
- (g) goods at date 0,  $L + B = g$ ; and
- (h) goods at date 2,  $\sum_{j \in \{n, s\}} c^j + \int_0^{I^n} y(\hat{I}^n) d\hat{I}^n = 2 \int_0^L r(\hat{L}) d\hat{L}$ .

**Proposition 1** *There exists a unique equilibrium.*

We proceed by analyzing the equilibrium effects of reserves on bank liquidity and balance sheet costs in order to then determine the optimal supply of reserves.

## 4 Reserves

Banks face both benefits and costs of increasing their reserves holdings. On the one hand, holding more reserves creates a buffer against demand shocks, which protects the bank from having to fund withdrawals with costly interbank loans. On the other hand, banks face capital requirements and must hold more equity when they hold more reserves. Equity is costly because it is not liquid from the perspective of households.

### 4.1 Bank liquidity and balance sheet costs

We start by defining the net cost for banks to hold reserves:

**Definition 2** *The net cost of reserves,  $C(M)$ , is defined as the IOER-deposit rate spread:*

$$C(M) \equiv R^{M^2} - R^D. \quad (11)$$

This represents the compensation banks must receive to be willing to hold reserves in equilibrium. We will show that the net cost for banks to hold reserves is equal to the bank's balance sheet cost minus the bank's liquidity cost.

**Balance sheet costs** The balance sheet cost is the (expected) marginal capital requirement cost for a marginal increase in balance sheet size at date 0. A marginal increase in the bank's balance sheet size at date 0 leads to expected additional amounts of required equity of  $\frac{dE(A)}{dA}$  at date 0 and  $\frac{dE_1(A)}{dA}$  at date 1.

We define the *balance sheet cost* as

$$K(A) \equiv (R^E - R^D) \frac{dE(A)}{dA} + \frac{1}{2}(R^{E1} - R^{D1}) \frac{dE_1(A)}{dA}, \quad (12)$$

where the spreads  $(R^E - R^D) = (\theta - 1)R^D$  and  $(R^{E1} - R^{D1}) = (\theta - 1)R^{D1}$  represent the premiums on equity returns relative to deposit returns at dates 0 and 1, respectively. These spreads are required by households to hold equity, which does not provide households the liquidity value ( $\theta > 1$ ) on deposits.

**Liquidity costs** Consider the bank that faces a withdrawal shock at date 1. If its holding of reserves,  $R^M M$ , is greater than the amount withdrawn,  $R^W D^W$ , then the bank is liquid and does not need to access the interbank market. If instead,  $R^W D^W \geq R^M M$ , then the bank is illiquid and needs to borrow the difference,

$$I = (R^W D^W - R^M M)^+, \quad (13)$$

in the interbank market.

Since the shocked bank's demand for funds in the interbank market is completely inelastic when  $I > 0$ , it bears the cost of an interbank transaction,  $Y(I)$ , in equilibrium. This cost takes the form of a spread between the interbank rate and the interest rate on reserves,  $R^I - R^M = Y(I > 0) > 0$ .

The net marginal liquidity value of reserves represents the value that a bank saves in expected liquidity costs from interbank borrowing. With a one-half probability, a bank has the liquidity shock and pays the marginal interbank cost  $Y(I)$  on its interbank borrowing.

Since  $D^W = \lambda A$ , and bank assets increase linearly with reserves in equilibrium,  $\frac{dA(M)}{dM} = 1$ , the quantity of the shocked bank's interbank borrowing given in equation (13) decreases with a marginal increase in reserves:  $\frac{dI(M)}{dM} = -(R^M - \lambda R^W)\mathbf{1}_{I>0} < 0$ . Equivalently, the bank's interbank borrowing is reduced by  $\frac{d(-I)}{dM} = (R^M - \lambda R^W)\mathbf{1}_{I>0}$ . This amount reflects that a marginal increase in reserves at date 0 provides the return  $R^M$  available to pay out for withdrawals. Funding the increase in date 0 reserves with an increase in the bank's date 0 liabilities increases the shocked bank's date 1 withdrawals at the margin by an amount  $\frac{dD^W(A)}{dA} = \lambda$  that is paid the return  $R^W$ .

Thus, a marginal increase in reserves reduces a bank's expected marginal interbank borrowing cost by  $\frac{1}{2}Y(I)\frac{d(-I)}{dM}$ , which we define as the *bank liquidity cost*. Since

$$\frac{1}{2}Y(I)\frac{d(-I)}{dM} = \frac{1}{2}(R^M - \lambda R^W)Y(I), \quad (14)$$

the bank liquidity cost is equal to the bank's net marginal liquidity value of reserves.

**Proposition 2** *The net cost of reserves is equal to the bank's balance sheet cost minus the bank's liquidity cost:*

$$C(M) = K(A) - \frac{1}{2}Y(I)\frac{d(-I)}{dM}. \quad (15)$$

The net cost of reserves represents the net marginal effect of an increase in reserves on bank profits. Since an increase in reserves is funded by an increase in deposits, the competitively determined IOER-deposit rate spread naturally captures this marginal effect on profits.<sup>38</sup>

Since the Fed is the monopoly supplier of reserves, the Fed's choice of reserves determines the equilibrium  $C(M)$  for each bank. An increase in the Fed's supply of reserves in the

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<sup>38</sup>An increase in reserves increases one-for-one bank assets and hence bank liabilities, i.e., deposits plus equity. A partial increase in bank liabilities could occur instead if an increase in reserves only leads to a partial increase in bank assets. Such is the case when there is a partial decrease in bank loans caused by a crowding-out effect from reserves. This occurs at the zero lower bound on real deposit rates, when households would store some of their goods at date 0. For simplicity, we do not consider this situation in the paper, as it does not meaningfully alter our results.

banking system increases the equilibrium size of each bank's balance sheets and hence balance sheet costs.

More plentiful reserves also lower bank liquidity costs. More reserves imply a decrease in the amount of interbank borrowing required to meet liquidity shocks. This results in a lower marginal interbank market cost,  $Y(I)$ , and lower interbank rate spread to IOER,  $R^I - R^M = Y(I)$ . The interbank rate spread is positive if there is a positive amount of interbank borrowing in equilibrium:  $I > 0$ . This occurs if there is at least a partial scarcity of reserves,  $M < \bar{M}$ , where

$$\bar{M} \equiv \frac{\lambda R^W (G - B)}{R^M - \lambda R^W}$$

is the threshold amount of reserves in the banking system below which the interbank market is active.

With an overabundance of reserves,  $M \geq \bar{M}$ , bank liquidity shocks can be met without interbank borrowing. This demonstrates how the large increase in reserves beginning in late 2008 led to the actual interbank component of the fed funds market effectively disappearing.<sup>39</sup> With no bank liquidity costs, the deposit rate relative to IOER is determined solely by bank balance sheet costs:  $R^D = R^{M^2} - K(A)$ .

Moreover, a decrease in the government's supply on bonds,  $B$ , holding reserves constant, has the opposite effect of an increase in the central bank's supply of reserves,  $M$ , holding  $B$  constant, as stated in the following proposition.

**Proposition 3** *In the region of overabundant reserves,  $M \geq \bar{M}$ , the IOER-deposit rate spread,  $R^{M^2} - R^D = K(A)$ , increases equivalently from a decrease in bonds ( $B$ ) as from an increase in reserves ( $M$ ).*

The opposite effects of bonds and reserves can be understood in two ways. First, in equilibrium, bank assets at date 0 are  $A = G - (B - M) = L + M$ , which shows that banks assets  $A$ , and hence balance sheet costs  $K(A)$ , increase equally from a decrease in  $B$  as from an increase in  $M$ . When  $B$  decreases, the government buys fewer goods at date 0, so loans  $L$  increase one-for-one. Thus, a decrease in  $B$  increases  $A$ . Second, and equivalently, household assets at date 0 are  $G = D + E + B^H$ , where  $B^H = B - B^{CB} = B - M$ . Thus, household assets are  $G = D + E + (B - M)$ . A decrease in  $B$ , holding  $M$  constant, means that households

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<sup>39</sup>The remaining activity in the fed funds market has consisted primarily of GSE lending to banks. GSEs lend in the fed funds market because they hold reserves but are not eligible for IOER (Bech and Klee, 2011). Banks borrow from GSEs in a similar manner and at similar rates as from depositors. Klee et al. (2016) show that since late 2008, the fed funds rate has provided a weak anchor and transmission mechanism to other short-term funding rates. Gagnon and Sack (2014) highlight that the fed funds rate does not perform a policy role when there is an abundance of reserves. Hence, IOER has effectively replaced the fed funds rate as the marginal rate for bank liquidity.

increase their bank holdings ( $D + E$ ) one-for-one with their decrease in household bonds,  $B^H$ , which decrease one-for-one with a decrease in  $B$ .

## 4.2 Economic welfare and optimal policy

In this section, we show that the central bank's optimal supply of reserves is determined by equating banks' liquidity and balance sheet costs. This is achieved when the bank deposit rate is equal to IOER.

The trade-off banks face in deciding whether or not to expand their balance sheets is also the main driver of overall welfare in our model. More reserves increase bank liquidity and reduce interbank borrowing costs. However, the increase in banks' balance sheets raises equity requirements, which decreases households' liquid assets. Both of these margins affect not only bank profits and equilibrium returns, but also welfare.

The optimal supply of reserves is the quantity that maximizes welfare, which is household expected utility and is equal to the total surplus in the economy. To maximize welfare, the central bank chooses the optimal supply of reserves,  $M^*$ , that maximizes expected household utility  $\mathbb{E}[u^j]$ . Since increasing reserves increases welfare by the amount of the bank liquidity cost and decreases welfare by the amount of the bank balance sheet cost, the optimal quantity of reserves equates these two marginal bank costs. This result is stated in the following proposition.

**Proposition 4** *At the central bank's optimal supply of reserves ( $M^*$ ) for maximizing welfare,  $\mathbb{E}[u^j]$ , bank liquidity costs equal bank balance sheet costs:*

$$\frac{1}{2}Y(I)\frac{d(-I)}{dM} = K(A). \quad (16)$$

The central bank's optimization problem gives a first order condition that is equivalent to setting the bank's net cost of reserves equal to zero,  $C(M) = 0$ , which also directly leads to equation (16). These results lead to the following proposition.

**Proposition 5** *The optimal supply of reserves is a moderate quantity  $M^* \in (0, \bar{M})$ . At this optimal quantity ( $M^*$ ), the date 0 net cost of reserve holdings equals zero:*

$$C(M^*) = R^{M^2} - R^D = K(A) - \frac{1}{2}Y(I)\frac{d(-I)}{dM} = 0. \quad (17)$$

This proposition establishes three important implications. First, a positive amount of reserves is always desirable to provide banks with liquid assets to mitigate the cost of interbank trading. Second, in contrast, is that some amount of bank illiquidity and interbank lending is

optimal. If the amount of reserves is so large that banks have no liquidity costs, then there is a benefit to decreasing the amount of reserves to mitigate the equity cost from banks' balance sheets. Thus,  $M^* \in (0, \bar{M})$  provides an optimal interior solution.

Third, and perhaps most novel, is that the net cost of reserve holdings equals zero. A particularly interesting consequence of this result is that the spread between IOER and the deposit rate,  $R^{M^2} - R^D$ , is zero under the optimal choice of reserves. This provides a sharp characterization of the optimal supply of reserves in terms of economic variables that are easily observable.

The intuition for this result again lies in weighing the two key frictions in our model. On the one hand, banks must take into consideration the cost of capital caused by equity requirements. If this were the only effect present in our model, then we would obtain the result that the deposit rate would be below IOER regardless of the amount of reserves in the banking system. Once a bank borrows enough deposits to fund the bank's loans to firms, any further amount of deposits lead to a bank accumulating reserves. Banks would demand additional deposits only if the deposit rate is low enough below IOER to cover the marginal capital cost from balance sheet expansion.

Our second friction implies that having more reserves reduces banks' cost of having to borrow in the interbank market. This in turn gives banks an incentive to compete for deposits, consequently bidding up their rate. When the supply of reserves is below the optimum, reserves provide banks a greater net liquidity value than balance sheet cost, which is reflected by a negative net cost of reserves:  $C(M < M^*) < 0$ . Banks competition for reserves, and hence deposits, leads to the deposit rate above IOER:  $R^D > R^M$ . This is consistent with the deposit rate at a positive spread above the return on reserves, equal to one, before IOER was introduced in 2008, and when there was a scarcity of reserves:  $R^D > R^{M^2} = 1$ . This also shows that a partial scarcity of reserves,  $M < M^*$ , is required to maintain positive net deposit rates when IOER is at the zero lower bound.

Since the cost of capital increases with reserves, and the cost of interbank trading decreases with reserves, the supply of reserves should be chosen to equalize these costs at the margin. In our model, this corresponds to the equilibrium relation  $R^D = R^{M^2}$ , which serves as an optimum policy rule for the supply of reserves. At this point, there is a partial scarcity of reserves that creates a positive spread of the interbank rate above IOER,  $R^I - R^M = Y(I)$ .

In the context of current Federal Reserve policy, our model gives a sharp prediction regarding the optimal supply of reserves and also how to achieve it. In particular, reserves should be decreased until bank deposit rates increase to the level of IOER. This situation characterizes the equilibrium in which marginal costs arising from bank illiquidity are equated

with marginal capital costs. From the perspective of a policymaker, our model shows that observing interest rate spreads can serve as a benchmark for measuring the welfare effects of current policy.

### 4.3 Reducing overabundant reserves

We can interpret the Fed lowering the current overabundance of reserves within the context of the model by considering at the beginning of date 0 a *starting* quantity of reserves and central bank bond holdings notated by  $M'$  and  $B^{CB'} = M'$ , respectively. We define the optimal quantity of bonds held by the Fed as  $B^{CB*}$ , which implies that  $B^{CB*} = M^*$ . Within date 0, the optimal choice of reserves,  $M = M^*$ , can be implemented by the Fed selling a quantity  $\Delta = B^{CB'} - B^{CB*}$  of its bonds. Households buy these bonds by withdrawing  $\Delta$  of deposits. For this deposit withdrawal, the households' banks pay  $\Delta$  reserves to the Fed. Since banks hold reserves at accounts with the Fed, the banks' payment to the Fed reduces reserves by  $\Delta$  in the banking system, and these reserves are extinguished. The Fed's outstanding reserves liabilities are decreased by  $\Delta$ . Hence, as  $B^{CB'}$  is reduced to  $B^{CB*}$ , reserves in the banking system are reduced from  $M'$  to  $M^*$ .

An equivalent interpretation is for the Fed to reduce reserves from the starting point of  $M'$  with the gradual rolling off of bonds from the Fed's balance sheet as bonds mature without the reinvestment of the proceeds. Assume that instead of the Fed selling bonds,  $\Delta = B^{CB'} - B^{CB*}$  is the amount of bonds the Fed holds that mature and roll off the Fed's balance sheet within date 0. The government issues and sells a new amount of bonds equal to  $\Delta$  to keep the aggregate supply of bonds constant at  $B$  within date 0. Households buy the new bonds with deposits by their banks paying  $\Delta$  reserves to the government. The government (i.e. the Treasury Department) also holds a reserves account at the Fed. The government pays the  $\Delta$  reserves received to the Fed for the Fed's  $\Delta$  maturing bonds. Hence,  $B^{CB'}$  is reduced to  $B^{CB*}$  and  $M'$  decreases to  $M^*$ , which is an equivalent outcome to the Fed selling bonds directly.

This interpretation method can also be applied to illustrate how the supply of reserves increases from Fed bond purchases, such as during the Fed's LSAP program. From a starting quantity of reserves, the Fed purchases bonds at date 0 from households by creating and paying reserves to the households' banks for credit to the households' deposit accounts. In particular, from a starting quantity of zero reserves and Fed bond holdings, the initial introduction of reserves into the banking system can be interpreted as follows. At the start of date 0, banks can lend to firms, which buy goods from households, who in turn deposit

at the banks. These transactions can occur with inside money created by banks and do not require banks to hold reserves. Likewise, households can start by buying all of the bonds sold by the government, with the government using the proceeds to buy goods from households. The Fed then buys bonds from households by paying reserves to the households' banks. Alternatively, the Fed can buy bonds directly from the government by paying reserves to the government's (i.e. Treasury Department's) reserve account at the Fed. The government pays for some of the goods it buys from households by paying its reserves to the households' banks for credit to the household's deposit accounts.

## 5 Empirical analysis

The empirical implications of our model are novel and testable. Specifically, the model draws a tight link between the quantities of reserves and bonds and the IOER-deposit rate spread.

Since the introduction of IOER in October 2008, the amount of reserves has been sufficiently large to eliminate the need for interbank borrowing, which corresponds to  $M \geq \bar{M}$  in our model. Without bank liquidity costs, the deposit rate decreases below IOER with an increase in reserves driven by the increase in bank balance sheet costs,  $R^D = R^{M^2} - K(A)$ , following from proposition 3. An increase in  $B$  has the opposite effect of increasing the deposit rate relative to IOER.

Consequently, our empirical analysis estimates the relationship between the IOER-deposit rate spread and the quantities of reserves and bonds for 2009-18.<sup>40</sup> In addition, we estimate the effect of the April 2011 FDIC policy action to increase the base for its assessments to include all (non-equity) liabilities of bank holding companies. We test the hypothesis that the increase in the FDIC assessment represents an increase in bank balance sheet costs as reflected by an increase in the spread between IOER and the market rates at which banks borrowed, as our model would suggest.

### 5.1 Variable definitions and Data

The deposit rate in our model,  $R^D$ , is the object of our empirical analysis. We use two prominent wholesale bank borrowing rates for the empirical implementations of  $R^D$ . We

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<sup>40</sup>Note that the level of reserves and Treasuries have been determined independently from the IOER-deposit rate spread. Reserves in the early years of this decade were dominated by the LSAP policies and have been on an "auto-pilot" reduction path since 2017. Treasury securities outstanding are a result of fiscal deficits. Choices of reserves and Treasuries may have been slightly affected by IOER but not by the IOER-deposit rate spread.

focus on wholesale rates, as they best correspond to the deposit rates in our model that competitive banks must pay when competing for household funding against households’ alternative option of holding government bonds. The two rates are the overnight AA financial commercial paper (CP) rate and the overnight Eurodollar rate. The CP rate measures the rate on overnight promissory notes issued by credit-worthy financial corporations, with the main issuers consisting of U.S. commercial bank subsidiaries of bank holding companies and foreign banks.<sup>41</sup> The overnight Eurodollar rate is the common measure of banks’ cost of wholesale deposits.<sup>42</sup>

The variable  $M$  is measured by bank reserves issued by Federal Reserve Banks. To create an empirical counterpart for the variable  $B$ , government bonds, we must first ask what features bonds have in the model. In the model, we interpret households holding bonds through the MMF as an intermediary, as shown in appendix C. MMF holdings of Treasuries are restricted by the SEC’s rule 2A-7, promulgated under the Investment Company Act of 1940, which limits the duration of securities that they may purchase and hold. Specifically, rule 2A-7 states that a MMF may not “[a]cquire any instrument with a remaining maturity of greater than 397 calendar days.” As a result, our empirical implementation of the variable  $B$  in the model is a proxy for the universe of Treasury issues, including Bills, Notes, and Bonds and issues of Federal Financing Banks, with remaining time to maturity of less than 397 days. Specifically, we use the readily available data series of these Treasury securities with less than a year to maturity, which we refer to hereafter as ‘Treasuries.’ We also consider an estimation specification using the difference between the quantities of Treasuries and reserves as the independent variable, which accounts for the possibility of an empirical relationship between the two variables that is outside of the scope of our model.

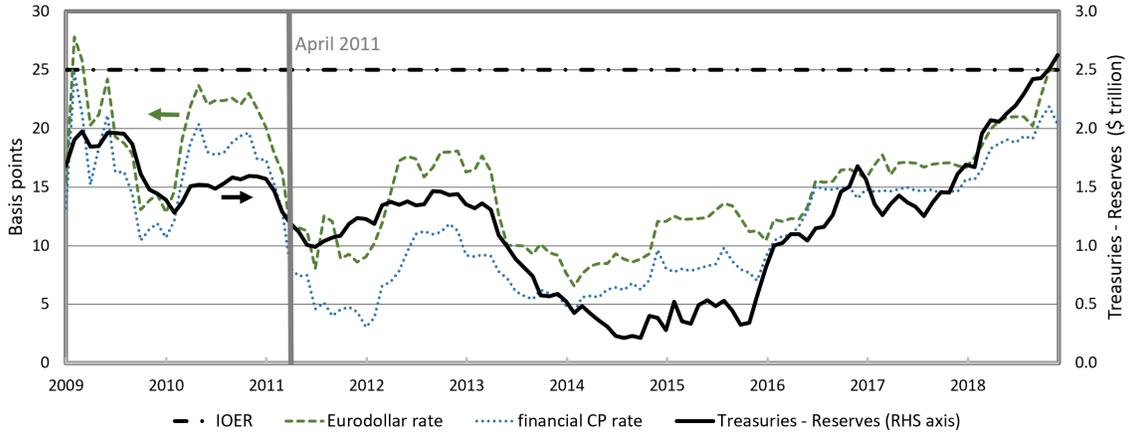
We obtain the monthly averages of IOER, the CP rate, and the Reserve Balances with Federal Reserve Banks from FRED at the Federal Reserve Bank of St. Louis. We obtain overnight Eurodollar rates from the ICAP Capital Markets Eurodollar Rates series via Bloomberg using the daily midmarket price series, and we calculate a simple monthly average for the Eurodollar rate from the daily rates. For simplicity, all references to rates throughout this empirical section refer to net interest rates. We obtain the Treasuries outstanding with one year or less to maturity from Quandl, and we measure average current month Treasuries as the average of the reported month-end amounts for the current and prior month. In order to study bank borrowing rates relative to IOER, as in the model, we normalize our interest

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<sup>41</sup>See Kacperczyk and Schnabl (2010).

<sup>42</sup>Specifically, it measures the rate wholesale depositors receive on U.S. dollar denominated overnight deposits at foreign banks or at overseas branches of U.S. banks.

Figure 1: Bank borrowing rates, Treasuries, and Reserves



IOER, Eurodollar, and financial CP rates are measured on the LHS y-axis. Treasuries minus Reserves are measured on the RHS y-axis. Treasuries refer to those outstanding with less than or equal to one year to maturity. Rates, reserves and treasuries are monthly averages. Rates are normalized to hold IOER at a constant rate of 25 basis points. The vertical line at April 2011 indicates when the expansion of the FDIC's assessment base began.

rate variables relative to a normalization of IOER adjusted to a constant rate of 25 basis points. Hence, the normalization takes effect starting with the increase of actual IOER above 25 basis points beginning in late 2015. Specifically, our normalized rate ( $r_t$ ) relates to the actual rate ( $R_t$ ) and the actual interest on reserves ( $IOER_t$ ), where the subscript  $t$  indicates the month  $t$ , as follows:

$$r_t = R_t - (IOER_t - 25 \text{ basis points}).$$

Figure 1 shows how bank borrowing rates have fluctuated since 2009, and how the difference between Treasuries (of less than one-year maturity) and reserves have fluctuated in the same time period. The LHS axis measures the (normalized) Eurodollar, CP, and IOER rates. The RHS axis measures the difference between Treasuries and reserves. The vertical line at April 2011 marks the date of the FDIC's expansion of its assessment base to include all liabilities of bank holding companies, which include overnight wholesale deposits.

## 5.2 Empirical specification

Our model specifies a relationship between wholesale deposit rates and the outstanding amount of Treasuries and reserves. Figure 1 is suggestive that such a relationship is present using the empirical variables we employ. A basic regression in the level of either bank borrowing rate on the level of Treasuries and reserves yields significant coefficients of the right signs. However, since we are dealing with time-series variables, we specify an estimation

utilizing difference variables to avoid spurious correlation.

A basic regression model in levels posits that  $r_t = a + bM_t + cB_t + u_t$ . We allow for the possibility of autocorrelation, so that  $u_t = \rho u_{t-1} + \varepsilon_t$ , where  $\varepsilon_t$  follows a normal distribution with mean equal to zero and variance  $\sigma^2$ . The coefficient  $\rho$  can vary between  $-1$  and  $1$ , allowing for the possibility of a unit root, i.e., it is possible that  $|\rho| = 1$ . That allows us to take differences, and we insert a lagged level of the interest rate on the right side of the equation. Doing so is a standard approach in accommodating a unit root. If the interest rate follows a stationary autoregressive process, the coefficient estimates will be unbiased because the lag of the interest rate is included, and its coefficient estimate should be between  $-1$  and  $1$ . If, in contrast, it is nonstationary, the coefficient on the lagged level should be zero. Together, these yield the following estimation specification:

$$\Delta r_t = \alpha_{INT} + \alpha_{RES}\Delta M_t + \alpha_{BOND}\Delta B_t + \alpha_{LAG}r_{t-1} + \varepsilon_t. \quad (18)$$

We also wish to test whether the expansion of the FDIC's assessment base in April 2011 represented an increase in balance sheet costs for banks, which would tend to depress the wholesale deposit rate. The expansion of the base could also increase balance sheet costs only temporarily, if it were the case that banks were to adjust their deposit pricing over time, or if institutions exempt from the assessment, such as foreign banking organizations, were to expand their borrowing in response. We wish to test whether such an expansion of the assessment base had only temporary or more permanent effects on the wholesale deposit rate.

We introduce an indicator variable,  $\mathbf{1}_t^{FDIC}$ , to represent the periods after which the FDIC changed its assessment formula to assess all liabilities, including Eurodollar deposits and CP issuances, of bank holding companies, in April 2011:

$$\mathbf{1}_t^{FDIC} = \begin{cases} 0: t < \text{April 2011} \\ 1: t \geq \text{April 2011} \end{cases}$$

Let  $\Delta \mathbf{1}_t^{FDIC} = \mathbf{1}_t^{FDIC} - \mathbf{1}_{t-1}^{FDIC}$  represent the change in the level of the indicator functions at month  $t$ .

As in McAndrews et al. (2016), we can capture permanent effects of the FDIC policy changes by including the change in the levels of the indicator function in equation (18) above, as shown here in equation (19):

$$\Delta r_t = \alpha_{INT} + \alpha_{RES}\Delta M_t + \alpha_{BOND}\Delta B_t + \alpha_{LAG}r_{t-1} + \alpha_{FDIC}\Delta \mathbf{1}_t^{FDIC} + \varepsilon \quad (19)$$

This would be appropriate if the whole regime following the implementation of the FDIC's new assessment formula led to lower borrowing rates being offered by banks, for example.

If, alternatively, there is only a temporary effect of the policies, it would be appropriate to include lagged changes of the indicator variables, as in the following equation (20), which allows the possibility that the effect of the FDIC change would be reduced with time:

$$\Delta r_t = \alpha_{INT} + \alpha_{RES} \Delta M_t + \alpha_{BOND} \Delta B_t + \alpha_{LAG} r_{t-1} + \alpha_{FDIC} \Delta \mathbf{1}_t^{FDIC} + \beta_{FDIC} \Delta \mathbf{1}_{t-1}^{FDIC} + \varepsilon. \quad (20)$$

### 5.3 Results

Our empirical strategy is to estimate equations (19) and (20). We estimate two specifications, one using the CP rate, and the other using the Eurodollar rate. All variables are expressed as monthly averages. The results are presented in Table 1.

The results are consistent with our model. Columns (1) and (2) of the table report estimation results for equations (19) and (20), respectively, using the CP rate as the dependent variable. In both specifications, an increase in reserves by \$1 trillion decreases the CP rate relative to IOER, and hence increases the spread between IOER and the CP rate, by about 8.6 basis points. Similarly, in both specifications, an increase in Treasuries by the same amount decreases the spread between the IOER rate and the CP rate by about 5.3 basis points. Columns (3) and (4) report estimation results for the same two equations using the Eurodollar rate as the dependent variable. The corresponding estimates for these regressions are a 9.5 basis points increase and a 4.9 basis points decrease in the IOER-Eurodollar spreads, respectively. In both specifications for both interest rate spreads, we find that the expansion of the FDIC’s assessment base expands the spread by about 4 basis points throughout the period—no reversal of the effect is observed, as seen in the second and fourth columns of Table 1. Levels of adjusted R-squared suggest a good fit for such estimations of differences (rather than in levels).

For columns (1)-(4), the final two rows of the table report the F-statistic and p-value from a two-sided Wald test of the null hypothesis that  $\alpha_{RES} = -\alpha_{BOND}$ . In both specifications for both interest rates, we cannot reject the null hypothesis that  $\alpha_{RES} = -\alpha_{BOND}$  even at the 20% level. This suggests that we also consider a specification using  $B - M$  as the independent variable, which our model finds is sufficient for determining interest rate spreads. In addition, by treating  $B - M$  as our independent variable, we account for the possibility that the two variables are empirically related outside of the considerations of our model.

This leads us to estimate equation (21) shown in the last two columns of Table 1:

$$\Delta r_t = \alpha_{INT} + \alpha_{BOND-RES} \Delta (B_t - M_t) + \alpha_{LAG} r_{t-1} + \alpha_{FDIC} \Delta \mathbf{1}_t^{FDIC} + \beta_{FDIC} \Delta \mathbf{1}_{t-1}^{FDIC} + \varepsilon. \quad (21)$$

While equation (21) discards some information that is contained in the separate variances

Table 1: Bank borrowing rates, Treasuries, and reserves

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	$\Delta r_{cp,t}$	$\Delta r_{cp,t}$	$\Delta r_{ed,t}$	$\Delta r_{ed,t}$	$\Delta r_{cp,t}$	$\Delta r_{ed,t}$
$\Delta M_t$	-8.597*** (2.028)	-8.571*** (2.037)	-9.523*** (2.195)	-9.535*** (2.207)		
$\Delta B_t$	5.297** (2.441)	5.261** (2.453)	4.920* (2.652)	4.935* (2.667)		
$\Delta(B_t - M_t)$					7.192*** (1.486)	7.625*** (1.613)
$\Delta 1_{FDIC,t}$	-4.092** (1.643)	-4.099** (1.650)	-3.831** (1.786)	-3.828** (1.794)	-4.244** (1.643)	-4.028** (1.791)
$\Delta 1_{FDIC,t-1}$		-0.519 (1.642)		0.220 (1.787)	-0.503 (1.641)	0.237 (1.791)
$r_{CP,t-1}$	-0.0697** (0.0295)	-0.0703** (0.0297)			-0.0698** (0.0297)	
$r_{ED,t-1}$			-0.0799** (0.0344)	-0.0796** (0.0346)		-0.0803** (0.0347)
Constant	0.896** (0.379)	0.907** (0.382)	1.322** (0.548)	1.315** (0.553)	0.862** (0.379)	1.271** (0.553)
Observations	119	119	119	119	119	119
Adjusted R <sup>2</sup>	0.222	0.216	0.214	0.207	0.216	0.203
F-test	0.981	0.979	1.616	1.599		
Prob > F	0.324	0.324	0.206	0.209		

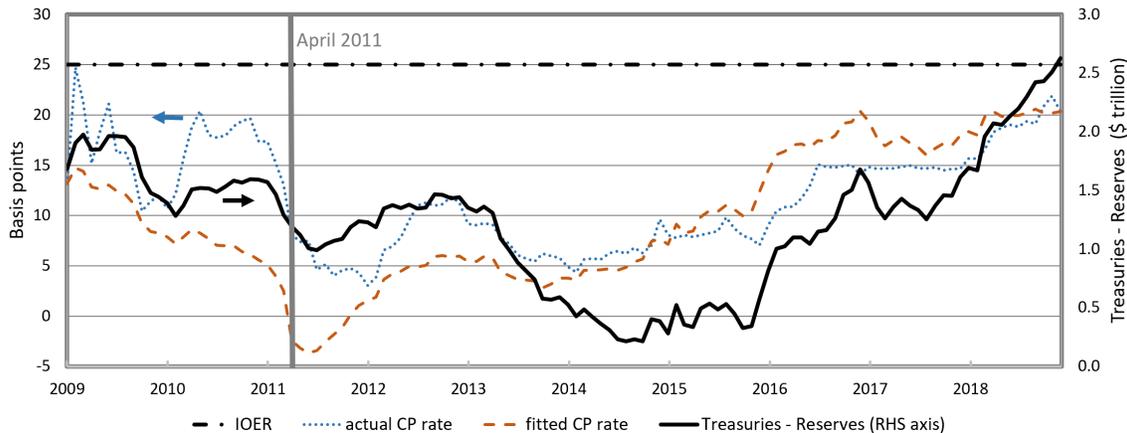
Standard errors in parentheses

\*\*\* p&lt;0.01, \*\* p&lt;0.05, \* p&lt;0.1

of  $B$  and  $M$ , the variable  $\Delta(B_t - M_t)$  is consistent with our model. Furthermore, we find that using the combined variable  $\Delta(B_t - M_t)$  to estimate equation (20) leads to only a slightly lower adjusted R-squared than using the two variables separately, for both interest rates. This suggests that equation (21) retains most of the relevant information for the correlation with  $r_t$ , relative to equation (20).

Figure 2 displays the levels of the (normalized) actual ( $r_t$ ) and fitted ( $\hat{r}_t$ ) values of the CP rate and IOER. The fitted values  $\hat{r}_t$  are the levels derived from the fitted values of  $\Delta \hat{r}_t$  estimated in equation (21). Visual inspection suggests a close fit between the fitted and actual values of the CP rate, especially in the more recent years, when overnight rates relative to IOER have increased well beyond what financial markets had anticipated and

Figure 2: Actual and fitted financial CP rate



IOER, Eurodollar, and financial CP rates are measured on the LHS y-axis. Treasuries minus Reserves are measured on the RHS y-axis. Treasuries refer to those outstanding with less than or equal to one year to maturity. Rates, reserves and treasuries are monthly averages. Rates are normalized to hold IOER at a constant rate of 25 basis points. The vertical line at April 2011 indicates when the expansion of the FDIC's assessment base began.

have puzzled many market observers.<sup>43</sup> Our theory and empirical results provide a sound basis to explain such recent rate moves.<sup>44</sup>

## 6 Overnight RRP

Our model can also be used to study the impact of the overnight RRP (ONRRP) and the parameters of that facility that maximize welfare. In this section, we introduce the ONRRP and consider its impact on rates and reserves.

We show that the optimal ONRRP rate is equal to IOER.<sup>45</sup> At that rate, the supply of reserves is endogenously set at its optimal level. In addition, the ONRRP allows banks to insure themselves against the risk of shocks at a lower cost. A further benefit of the ONRRP is that it stabilizes deposit and bond rates. In appendix D, we show that other supplementary policy tools the Fed has tested are not as effective as the overnight RRP for managing rates and reserves and should not be used.

<sup>43</sup>See, for example, Richard Leong, “Market Rate Rise May Thwart Fed’s Balance Sheet Plan,” Reuters, June 27, 2018, at <https://www.reuters.com/article/us-usa-fed-ioer-analysis/market-rate-rise-may-thwart-feds-balance-sheet-plan-idUSKBN1JN2HO>.

<sup>44</sup>Our consideration of using all Treasury securities of less than or equal to one year, as justified by MMF holdings, results in a stronger empirical explanation for the rise in in short-term rates relative to IOER than does other recent research, such as by Smith (2019), that considers only Treasury bills.

<sup>45</sup>This is consistent with a proposal by Gagon and Sack (2014).

## 6.1 Equilibrium effects

Expanding the model, the central bank can offer households the option to invest in a one-period ONRRP at date  $t \in \{0, 1\}$ .<sup>46</sup> The central bank offers the ONRRP across sectors, and the return for investing  $Q_t$  in the ONRRP is  $R^{Q_t} \leq R^M$ . The ONRRP can either have a fixed rate, in which case the quantity is determined by the market, or a fixed aggregate quantity, in which case the rate is market determined. Total central bank liabilities, which are reserves and the ONRRP, determine the size of central bank assets,

$$B^{CB} = M + Q_0, \quad (22)$$

where quantities are normalized to a per-sector basis. To simplify the analysis, we apply two conditions that hold under the optimal central bank policy:  $2Q_1 \leq R^W \lambda A + 2(M - M^*)^+$  and  $2R^M M > R^W \lambda A$ .<sup>47</sup> Formal details of the model and equilibrium with the addition of the ONRRP are presented in the proof of the following lemma. This lemma shows that the ONRRP acts as a floor for the bank deposit rate by competing for household investments at dates 0 and 1.

**Lemma 1** *The overnight RRP sets a floor on deposit rates at dates 0 and 1:  $R^D \geq R^{Q_0} R^{Q_1}$  and  $R^{D_1} \geq R^{Q_1}$ , which are binding for  $Q_0 Q_1 > 0$  and  $Q_1 > 0$ , respectively.*

We proceed by analyzing the case of a fixed-rate ONRRP with the same rate at dates 0 and 1,  $R^{Q_0} = R^{Q_1}$ .<sup>48</sup>

## 6.2 The ONRRP and the optimal supply of reserves

Setting the ONRRP rate equal to IOER endogenously yields the optimal quantity of reserves at date 0. This is an alternative to the central bank directly reducing the supply of reserves through a lower quantity of bonds it holds at date 0.

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<sup>46</sup>Banks would be indifferent or prefer not to invest in the ONRRP because the equilibrium ONRRP return is less than or equal to IOER. We show in appendix C that the results of households investing directly in the ONRRP are identical in the more institutionally realistic setting of allowing households to hold shares in MMFs, which in turn invest in the ONRRP and bonds.

<sup>47</sup>The first condition states that the aggregate quantity of the ONRRP is weakly less than the size of the liquidity shock plus any overabundance of aggregate reserves, which precludes the ONRRP from triggering date 1 withdrawals in excess of those caused by the liquidity shock. The second condition states that the supply of aggregate reserves entering date 1 is greater than the size of the liquidity shock, which allows the potential for the ONRRP to absorb the full liquidity shock.

<sup>48</sup>It is possible to show an equivalence between a fixed-rate ONRRP and a fixed-quantity ONRRP if the central bank chooses the quantity supplied appropriately.

**Proposition 6** *Given an initial amount of reserves  $M' > M^*$ , the optimal supply of reserves can be implemented with a fixed-rate ONRRP with its rate set equal to IOER,  $R^{Q0} = R^{Q1} = R^M$ .*

In an economy without the ONRRP and with  $M' > M^*$ , the deposit rate at the beginning of date 0 would be below IOER:  $R^{D'} < R^{M^2}$ . In contrast, in an economy with the same initial stock  $M'$  and with an ONRRP at a rate  $R^{Q0} R^{Q1} > R^{D'}$ , the supply of reserves would endogenously decrease as households prefer to invest at the ONRRP than at banks. The decrease in bank deposits from households reduces the banks' balance sheet cost, allowing them to increase the rate they offer on deposits. Households will invest at the ONRRP until the deposit rate rises to  $R^D = R^{Q0} R^{Q1}$ . If the ONRRP rate is set equal to IOER, the bank deposit rate will increase to IOER as well,  $R^D = R^{M^2}$ . The quantity demanded at the ONRRP at that rate is  $Q_0 = M' - M^*$ , which brings the date 0 amount of reserves to the optimum level  $M = M^*$ .

Hence, our model suggests that the Fed could quickly reach the optimal level of reserves by setting the ONRRP rate to be equal to IOER. In contrast to the current normalization strategy, which relies on letting the Fed's assets mature, setting the ONRRP rate equal to IOER would allow market participants to endogenously determine the supply of reserves. Note that this strategy does not affect the asset side of the Fed's balance sheet, since the quantity of bonds at date 0 remains  $B^{CB'} = M'$ .

In our model, any reduction in the bond holdings of the Fed, down to a lower bound  $B^{CB^*} = M^*$ , would result in a reduction in the takeup quantity at the ONRRP, rather than an increase in the bank deposit rate. Proposition 6 above implies that any positive takeup at the ONRRP is consistent with an optimal allocation when the ONRRP rate is set equal to IOER.

### 6.3 Reducing the cost of liquidity shocks

The ONRRP has an additional benefit in our model. It reduces the cost for banks of holding a larger buffer of reserves for insuring themselves against the risk of shocks. The value of reserves as a buffer against liquidity shocks is maximized at date 0 when reserves are equally distributed among banks. However, without the ONRRP, liquidity shocks lead to an imbalance of reserves beyond what is used for interbank lending at the nonshocked bank, leading to new date 1 assets  $A_1$  and the costly equity requirement  $E_1$ .

The ONRRP can eliminate this date 1 equity cost by absorbing the liquidity shock if its rate is set high enough. As the ONRRP rate is increased, the unshocked household invests

a greater amount in the ONRRP rather than in additional bank deposits and equity at date 1. Setting the ONRRP rate equal to IOER is sufficient for the ONRRP take-up to equal the full amount of the liquidity shock.<sup>49</sup> At the optimal supply of reserves, the ONRRP has a quantity at date 0 of  $Q_0 = 0$ , and has an aggregate quantity at date 1 equal to the full amount of the liquidity shock,  $2Q_1 = R^W \lambda A$ , which corresponds to a per sector quantity of  $Q_1 = \frac{1}{2} R^W \lambda A$ .<sup>50</sup> As a result, the nonshocked bank's new date 1 assets ( $A_1$ ) and equity ( $E_1$ ) are zero. This illustrates how a fixed-rate ONRRP can absorb liquidity shocks when they occur.

For example, the ONRRP has seen large increases in take-up at quarter-ends, as some banks reduce the size of their balance sheet, leaving nonbank investors with opportunities to invest their cash (see Egelhof et al., 2017). The ONRRP also saw a large increase in take-up following the SEC's 2016 MMF reform.

Since the cost of holding reserves is lower when the ONRRP is available, there is a higher optimal amount of reserves  $M^{**}$  supplied by the central bank to lower bank liquidity costs in line with the lower bank balance sheet costs.

**Proposition 7** *The optimal central bank policy is implemented by the overnight RRP rate equal to IOER,  $R^{Q0} = R^{Q1} = R^M$ , and an optimal supply of reserves that is greater with the overnight RRP than without it,  $M^{**} \in (M^*, \bar{M})$ . Bank liquidity costs equal balance sheet costs,  $\frac{1}{2}Y(I)\frac{d(-I)}{dM} = K(A)$ , and the net cost of reserve holdings equals zero:  $C(M^{**}) = R^{M^2} - R^D = 0$ .*

The optimal supply of reserves is still relatively moderate, as it maintains a partially active interbank market with  $M^{**} < \bar{M}$ . In contrast to the current literature, which typically advocates either for a very large or a very small size of the Fed's balance sheet, our model instead suggests a moderate size. The optimal quantity of reserves and use of the ONRRP imply that the optimal size for the central bank balance sheet is at minimum equal to  $B^{CB^{**}} = M^{**}$ . At this size, the composition of the central bank's liabilities fluctuate between a quantity  $M^{**}$  of reserves at date 0 when there is no liquidity shock, and a quantity  $M^{**} - Q_1$  of reserves and  $Q_1 = \frac{1}{2} R^W \lambda A$  of the ONRRP at date 1 when there is a liquidity shock. Following from section 6.2, a larger size of the central bank balance sheet can only be optimal with a greater size of the ONRRP at both dates 1 and 2.

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<sup>49</sup>The necessary condition for the ONRRP rate to absorb the full liquidity shock is given in the proof of the following proposition 7.

<sup>50</sup>A higher initial supply of reserves is reduced following the subsection above with an additional quantity of the ONRRP at both dates 0 and 1.

## 6.4 Stabilizing money market rates

A fixed-rate ONRRP can also reduce the volatility of money market rates that otherwise arises if there are volatile bank liquidity shocks. The model can be extended to allow for a shock of random size. In this case, the shock  $\lambda$  becomes a random variable, denoted  $\tilde{\lambda}$ , that takes a realization,  $\lambda^i$ , where  $i \in \{h, l\}$  corresponds to a relatively high or low shock state, with  $\lambda^h > \lambda^l$  and  $\mathbb{E}[\tilde{\lambda}] \in (\lambda^l, \lambda^h)$ . At date 0, the central bank chooses its policy for the ONRRP, along with reserves, before the realization of  $\tilde{\lambda}$  at date 1.

The next proposition shows that the ONRRP can stabilize the bank deposit rate and the one-period return on government bonds, where the one-period holding return on bonds between dates 0 and 1 is  $P_1^B$  and between dates 1 and 2 is  $R^{B1} \equiv \frac{R^B}{P_1^B}$ .

**Proposition 8** *The overnight RRP with rate equal to IOER implements a constant one-period rate at dates 0 and 1 for deposits and bonds equal to IOER:  $R^W = P_1^B = R^M$  and  $R^{D1} = R^{B1} = R^M$  for both shock size states  $i \in \{h, l\}$ . In contrast, with no overnight RRP, the date 1 deposit and bond rate is volatile with a higher rate in the low shock state than in the high shock state.*

The ONRRP with a rate  $R^{Q0} = R^{Q1} = R^M$  has an equilibrium takeup of the full amount of the liquidity shock  $R^W \lambda^i A$ , regardless of the shock size state  $i \in \{h, l\}$ . As a consequence, the equilibrium rates on deposits and bonds acquired at date 1 equal  $R^{Q1} = R^M$ .<sup>51</sup> In contrast, without the ONRRP, date 1 deposit and bond rates decrease in the size of the liquidity shock. A larger liquidity shock absorbed by the nonshocked bank requires larger date 1 equity ( $E_1$ ) and hence lower date 1 deposit rates. Hence,  $R^{D1}(\lambda^h) < R^{D1}(\lambda^l)$ , and by substitution between bonds and deposits,  $R^{B1}(\lambda^h) < R^{B1}(\lambda^l)$ .

In addition, with random shock sizes, the optimal welfare with  $M^{**}$  reserves is maintained with the ONRRP rate set at IOER, since the ONRRP eliminates required date 1 equity in both the high and low shock states.

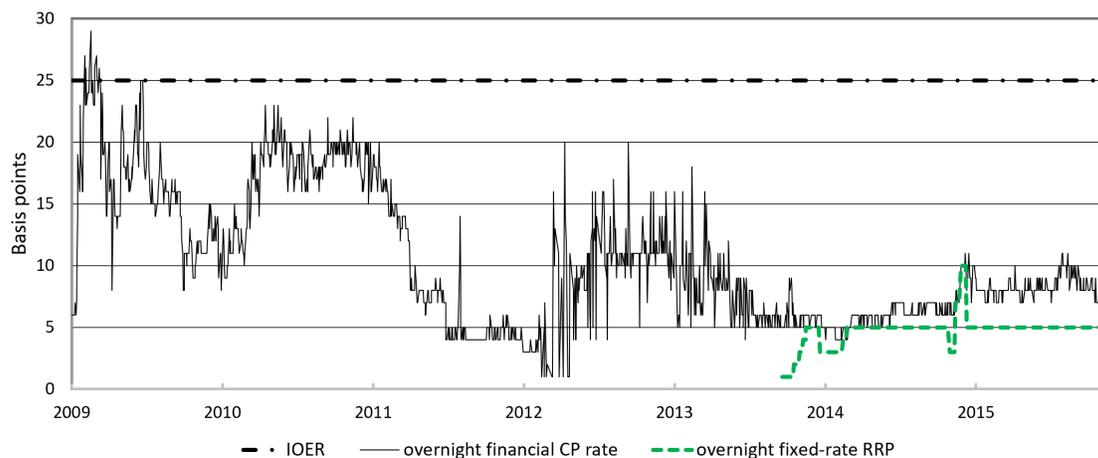
Moreover, the following corollary shows that with a random shock size, the ONRRP can only stabilize rates using a fixed-rate rather than fixed-quantity implementation.

**Corollary 1** *The overnight RRP with a fixed-quantity supply cannot implement a constant date 1 deposit and bond rate.*

For any fixed-quantity,  $Q_1$ , the date 1 deposit and bond rates equal the equilibrium ONRRP rate and are greater in the low state than high state:  $R^{D1}(\lambda^l) > R^{D1}(\lambda^h)$ . The Fed originally

<sup>51</sup>To implement constant rates, the ONRRP rate set at IOER is sufficient; the necessary condition for the ONRRP rate to be high enough is given in the proof of the previous proposition 8.

Figure 3: Overnight financial CP rate and overnight fixed-rate RRP



The overnight financial CP rates on the last day of each month are excluded because of window-dressing effects.

tested the ONRRP using a fixed-quantity supply before testing and then adopting the fixed-rate implementation. Our model suggests that a fixed-rate ONRRP has important benefits compared to a fixed-quantity facility.

Figure 3 illustrates that the fixed-rate ONRRP greatly reduced volatility and acted as a strong floor for overnight bank funding rates, as exemplified by the overnight AA financial CP rate, which is seen as the lowest of the bank borrowing rates in Figure 1 and is the most volatile of these rates when measured at higher frequencies.<sup>52</sup>

Figure 3 plots the daily overnight financial CP rate starting in 2009 and the daily ONRRP starting in September 2013, when the ONRRP was switched from a fixed-quantity to a fixed-rate implementation. The two series are plotted through December 2015, when IOER was first increased as part of the FOMC’s series of interest rate hikes and the ONRRP rate spread below IOER was increased to 25 basis points. The figure shows that the fixed rate on the ONRRP was set at five basis points for most of the time period, although at brief times it was set at rates ranging from one to ten basis points for testing purposes.

We observe that the volatility of the overnight CP rate is strikingly reduced during the period when the overnight fixed-rate ONRRP is in place. We also observe that the overnight CP rate rarely falls below the ONRRP rate. Whereas, prior to the ONRRP facility, the overnight CP rate repeatedly falls to one basis point, an effective zero lower bound, from

<sup>52</sup>Figure 2 data sources are, for IOER, <https://www.federalreserve.gov/datadownload/Choose.aspx?rel=PRates>; for the overnight AA financial commercial paper (CP) rate, <https://www.federalreserve.gov/releases/cp/>; and for the fixed-rate ONRRP, <https://apps.newyorkfed.org/markets/autorates/tomo-search-page>. The overnight financial CP rates on the late day of each month are excluded because of window-dressing effects.

February through April 2012.

While the Fed's exercises with the ONRRP led to confidence of it acting as a floor on funding rates, the Fed's current normalization plan is to eventually phase out its use. We argue for the continued use of the ONRRP. Beyond acting as a floor on overnight rates, the ONRRP stabilizes the volatility of overnight rates, reduces bank balance sheet costs, and enables a higher optimal quantity of reserves.

Additional advantages of the ONRRP likely exist that are outside the formal benefits we consider in this model. Lower volatility and uncertainty of overnight funding rates and quantities can support a more elastic demand for short-term assets more broadly, which may further help transmit the stabilization effect of the ONRRP to other money market assets. For example, Klee et al. (2016) shows empirically that the fixed-rate ONRRP has strongly reduced the volatility and increased the co-movement of overnight funding rates. These results complement the Duffie and Krishnamurthy (2016) findings that the ONRRP provides a better transmission of monetary policy than IOER alone, which is the basis for their advocacy for the continued use of the ONRRP.

## 7 Concluding remarks

The optimal quantity of reserves and use of the overnight RRP have been publicly debated primarily between advocates for either a large or small Fed balance sheet size. We analyze the optimal Fed balance sheet based on the impact of reserves and the overnight RRP on the banking system.

Our model provides a sharp result that the optimal supply of reserves is determined by equating their impact at the margin on bank liquidity costs and balance sheet costs. We derive an optimal policy rule, which states that reserves should be reduced until the bank deposit rate rises to equal IOER. Our empirical analysis supports our model and provides a strong explanation for the determination of the IOER-deposit rate spread based on the quantity of reserves and short-term Treasuries outstanding. We also demonstrate that the Fed should establish the overnight RRP as a permanent policy tool rather than end its use as currently planned. With the overnight RRP rate set at IOER, it increases the optimal quantity of reserves by absorbing bank liquidity shocks, which reduces bank balance sheet costs. It also expediently reduces reserves to their optimum and stabilizes overnight interest rates.

The optimal quantity of reserves and use of the overnight RRP implies that a moderate size of the Fed balance sheet is optimal. A likely reason that a moderate size has not yet

been given more attention is that such a size has not been previously used by central banks in practice or studied in the academic literature. Historically, central banks operated using a *corridor* or *channel* system without paying interest on reserves. Maintaining positive (net) interest rates required central banks to create extreme scarcity of reserves. The advent of a *floor* system with relatively large central bank balance sheets did not occur until 2006, starting with the Reserve Bank of New Zealand. Positive rates can be supported when there is a large supply of reserves by paying interest on reserves. The Fed and other central banks only expanded their balance sheets with large quantities of reserves starting in 2008 as a by-product of large liquidity operations during the financial crisis and large asset purchases in response to weak economies.

The academic study of central bank balance sheet policy was traditionally based on a corridor system following Poole (1968). The central bank inelastic supply of reserves intersects with banks' downward-sloping aggregate demand for reserves in the region of a small quantity of reserves and a very active interbank market. Goodfriend (2002) originates the study of a floor system, which corresponds to banks' elastic demand at the interest rate on reserves in the region of a large quantity of reserves and an inactive interbank market.

Our solution of a moderate quantity of reserves is unique in the literature. This quantity corresponds to the small region where banks' demand for reserves is downward-sloping, just before it kinks and is inelastic. This region is characterized by a partially active interbank market and has not been previously considered in the literature or in central bank practise. This result introduces a novel paradigm for future research on the analysis and optimality of central bank policy more broadly. Our findings also provide a new system for the practice of central bank policy, with predictions and interpretations of the relevant short-term interest rates and relative spreads. Such research will continue to shape the balance sheet policy decisions faced by the Federal Reserve, as well as soon to be faced by other central banks with massively expanded balance sheets including the ECB, Bank of England, and Bank of Japan.

## Appendix A: Proofs

**Proof of Proposition 1.** Necessary first order conditions and sufficient second order conditions hold for  $L$  in the firm optimization (3);  $Q^{Bj}$  in the bank optimization (8); and  $Q^{Hj}$  in the household optimization (2). With market clearing, and equal date 0 asset holdings across sectors by ex-ante symmetry, all constraints bind for the agents with the exception of  $B_1^n \leq B^H$ ,  $M_1^n \geq 0$ , and potentially  $M_1^s \geq 0$ . Binding household date 1 budget constraints give  $D_1 + E_1 = R^W D^W$ . Market clearing for the interbank market and reserves at date 1 imply  $A_1 = R^W \lambda A$ ; hence, since  $D^W \equiv \lambda A$ , we have  $A_1 = D_1 + E_1$ .

In the household optimization (2), first order conditions with respect to  $D$ ,  $E$ ,  $D_1$ ,  $E_1$  and  $B_1^n$ , with binding constraints and market clearing, give  $B_1 = R^W D^W$ ,  $P_1^B = R^W$ , and the returns  $R^B = R^D$ ,  $R^{D1} = \frac{R^B}{P_1^B}$ ,  $R^E = \theta R^D$  and  $R^{E1} = \theta R^{D1}$ . Market clearing and binding bank and household constraints give  $A = G - B + M$ .

In the bank optimization (8), first order conditions with respect to  $L$ ,  $M$ ,  $I^j$ ,  $D$ ,  $E$ ,  $D_1$  and  $E_1$ , with binding constraints and market clearing, along with the household optimization results above, give  $I = (R^W \lambda A - R^M M)^+$  and the following returns:

$$R^{D1} = R^M - \left(\frac{\theta-1}{\theta}\right) R^{E1} \frac{\partial E_1(A_1)}{\partial A_1} \quad (23)$$

$$R^I = R^M + Y(I) \quad (24)$$

$$R^L = R^{M^2} + \frac{1}{2} R^M Y(I) \quad (25)$$

$$R^L = \left(1 - \frac{1}{2}\lambda\right) R^D + \frac{1}{2}\lambda R^W R^I + \left(\frac{\theta-1}{\theta}\right) \left[ R^E \frac{\partial E(A)}{\partial A} + \frac{1}{2} R^{E1} \frac{\partial E_1(A)}{\partial A} \right]. \quad (26)$$

Substituting for  $R^L$  with its expression in equation (25) into the first order condition with respect to  $L$  for the firm optimization (3) gives  $P_2 = \frac{R^{M^2}}{r(L) - \frac{1}{2} R^M y(I)}$ .

**Proof of Proposition 2.** In equation (26), substituting for  $R^I$  with its expression in equation (24), substituting for  $R^D$  in the term  $-\frac{1}{2}\lambda R^D$  with  $R^W R^{D1}$ , then substituting for  $R^{D1}$  with its expression in equation (23), and simplifying, gives

$$R^L = R^D + \frac{1}{2}\lambda R^W Y(I) + \left(\frac{\theta-1}{\theta}\right) \left\{ R^E \frac{\partial E(A)}{\partial A} + \frac{1}{2} R^{E1} \left[ \frac{\partial E_1(A)}{\partial A} + \lambda R^W \frac{\partial E_1(A_1)}{\partial A_1} \right] \right\}. \quad (27)$$

In the balance sheet cost definition, equation (12), substituting for  $R^E$  and  $R^{E1}$  with  $\theta R^D$  and  $\theta R^{D1}$ , respectively, from the household's first order conditions and simplifying gives

$$K(A) = \frac{\theta-1}{\theta} \left[ R^E \frac{dE(A)}{dA} + \frac{1}{2} R^{E1} \frac{dE_1(A)}{dA} \right]. \quad (28)$$

Since  $\frac{dE_1(A)}{dA} = \frac{\partial E_1(A)}{\partial A} + \frac{\partial E_1(A_1)}{\partial A_1} \frac{\partial A_1(A)}{\partial A}$ , and  $\frac{\partial A_1(A)}{\partial A} = \lambda R$ , the third term on the RHS of equation (27) equals and can be substituted with  $K(A)$ , which shows that the expected

return on loans is equal to the (expected) cost of funding a loan:

$$R^L = R^D + K(A) + \frac{1}{2}\lambda R^W Y(I), \quad (29)$$

The term  $\frac{1}{2}\lambda R^W Y(I)$  is the expected marginal cost of interbank borrowing to cover depositor liquidity shock withdrawals .

Substituting for  $R^L$  in equation (29) with the expression for  $R^L$  in equation (25), and solving for  $R^{M^2}$ , gives the bank's implicit funding cost for holding reserves:

$$R^{M^2} = R^D + K(A) - \frac{1}{2}(R^M - \lambda R^W)Y(I). \quad (30)$$

This states that in equilibrium, the exogenous return on reserves must equal the deposit rate cost for funding reserves, plus the balance sheet cost, minus the net marginal liquidity value of reserves  $\frac{1}{2}(R^M - \lambda R^W)Y(I)$ . This term is nonnegative following from the assumption that  $\lambda < \frac{R^M}{R^W}$ .

Substituting for  $\frac{1}{2}(R^M - \lambda R^W)Y(I)$  with  $\frac{1}{2}Y(I)\frac{d(-I)}{dM}$  (based on equation (14)) into equation (30), then using the derived expression for  $R^{M^2}$  to substitute for  $R^{M^2}$  in equation (11), and simplifying, gives the proposition's result of equation (15).

**Proof of Proposition 3.** The proposition follows directly from the explanation following the statement of the proposition.

**Proof of Proposition 4.** The central bank's optimization to maximize welfare is  $\max_M \mathbb{E}[u^j]$ .<sup>53</sup>

At equilibrium returns and quantities, the objective function is equal to

$$\mathbb{E}[u^j] = \frac{1}{P_2} \left[ P_2 \int_0^L r(\hat{L}) d\hat{L} + (\theta - 1)R^D G - (R^E - R^D)E - \frac{1}{2}(R^{E1} - R^{D1})E_1 - \frac{1}{2} \int_0^I Y(\hat{I}) d\hat{I} \right]. \quad (31)$$

The first term is the total production of goods in a sector. The second and third terms represent the household liquidity value  $(\theta - 1)$  received on the amount of household endowment  $G$  that is invested at date 0 in liquid deposits and bonds but not equity. To see this,

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<sup>53</sup>To focus on central bank policy for balance sheet size, we do not examine optimal price inflation,  $P_2$ , arising from interest rate policy,  $R^M$ . The price level is determined as a simple application of the fiscal theory of the price level as in Stein (2012), Cochrane (2005, 2014), Sims (2013), and Freixas et al. (2009). Examining price inflation would show neo-Fisherian inflationary impacts (Cochrane 2014) from higher nominal interest rates,  $R^M$ , since the equilibrium price level  $P_2 = \frac{R^{M^2}}{r(L) - \frac{1}{2}R^M y(I)}$  represents a modified version of the Fisher equation with inflation equal to the ratio of the nominal policy rate to the net real marginal return of production in the economy. Instead, we consider the direct welfare effects of the central bank optimizing over reserves,  $M$ , where interest rate policy,  $R^M$ , holds second-order price and rate effects fixed. Equivalent would be a log-linearization approach using a first-order approximation to a Taylor series expansion of the price and rates. Also equivalent would be to assume that rates of interest are additive rather than compounded over the two periods in the model, which reflects the practice for interest on reserves paid by the Fed at biweekly periods rather than daily.

substitute for  $G$  with  $D + B^H + E$  from the household budget constraint in the second term. Substitute for  $(R^E - R^D)$  with  $(\theta - 1)R^D$ , the equity premium, in the third term. The second and third terms of equation (31) simplify to equal the net liquidity value on deposits and bonds,  $(\theta - 1)(D + B^H)R^D$ , which also reflects that the return on bonds must equal the return on deposits in equilibrium,  $R^B = R^D$ , to ensure that households are indifferent between holding bonds and deposits. The fourth term of equation (31) is the expected cost of the equity premium on additional equity at date 1.<sup>54</sup> Finally, the last term of equation (31) is the expected total interbank market cost.

This last term,  $-\frac{1}{2}\int_0^I Y(\hat{I})d\hat{I}$ , increases with a marginal increase in reserves by the amount

$$\frac{d}{dM} \left[ -\frac{1}{2}\int_0^I Y(\hat{I})d\hat{I} \right] = -\frac{1}{2}Y(I)\frac{dI}{dM} = \frac{1}{2}Y(I)\frac{d(-I)}{dM} > 0,$$

which is equal to the bank liquidity cost.

A marginal increase in reserves increases the quantities of equity in the third and fourth terms of equation (31) by  $\frac{dE(A)}{dM} = \frac{dE(A)}{dA}$  and  $\frac{dE_1(A)}{dM} = \frac{dE_1(A)}{dA}$ , respectively. This reflects that since  $A = L + M$ , bank assets directly increase with reserves in equilibrium:  $\frac{dA(M)}{dM} = 1$ . Thus, a marginal increase in reserves decreases welfare by an amount  $(R^E - R^D)\frac{dE(A)}{dA} + \frac{1}{2}(R^{E1} - R^{D1})\frac{dE_1(A)}{dA}$ , which is equal to the bank balance sheet cost,  $K(A)$ .

Hence, the first order condition of the central bank's optimization,

$$\frac{d\mathbb{E}[u^j(M)]}{dM} = \frac{1}{2}Y(I)\frac{d(-I)}{dM} - K(A) = 0,$$

gives the proposition's result in equation (16).

Finally, for  $I > 0$ , we have  $R^W \lambda A < R^M M$ , which substituting for  $A$  with  $G - B + M$  and solving for  $M$  gives  $M < \bar{M}$  and implies that  $I = 0$  for  $M \geq \bar{M}$ .

**Proof of Proposition 5.** We first establish, with the following lemma, that  $\frac{dC(M)}{dM} > 0$ , which provides a sufficient second order condition to show that there is an interior optimum for the central bank's optimization of  $M$ .

**Lemma 2** *A bank's net cost of reserves increases with the supply of reserves in the banking system:  $\frac{dC(M)}{dM} > 0$ .*

**Proof of Lemma 2.** Since  $\frac{dA(M)}{dM} = 1$ , we have based on equation (28)

$$\frac{dK(A)}{dM} = \frac{dK(A)}{dA} = \left(\frac{\theta-1}{\theta}\right)\frac{d}{dA}\left[R^E\frac{dE(A)}{dA} + \frac{1}{2}R^{E1}\frac{dE_1(A)}{dA}\right],$$

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<sup>54</sup>The new deposit or bond liquid assets that a household acquires at date 1 replace the household's bonds that are sold or deposits that are withdrawn at date 1, respectively, and so do not receive additional liquidity benefits. However, the amount of date 1 bond sales that go toward new date 1 equity decreases the household's liquidity benefit.

which is positive since  $R^\alpha(\cdot)$  is convex, and we have

$$\begin{aligned}\frac{dI(M)}{dM} \Big|_{R^M M \geq R^W \lambda A} &= 0 \\ \frac{dI(M)}{dM} \Big|_{R^M M < R^W \lambda A} &= -(R^M - \lambda R^W).\end{aligned}$$

To evaluate  $\frac{dC(M)}{dM} = \frac{d}{dM}[K(A) - \frac{1}{2}(R^M - \lambda R^W)Y(I)]$  for  $R^M M \geq R^W \lambda A$ ,

$$\begin{aligned}\frac{dY(I)}{dM} &= \frac{dY(I)}{dI} \frac{dI(M)}{dM} = 0 \\ \frac{dC(M)}{dM} &= \frac{dK(A)}{dA} > 0.\end{aligned}$$

For  $R^M M < R^W \lambda A$ ,

$$\begin{aligned}\frac{dY(I)}{dM} &= \frac{dY(I)}{dI} \frac{dI(M)}{dM} = -(R^M - \lambda R^W) \frac{dY(I)}{dI} \\ \frac{dC(M)}{dM} &= \frac{dK(A)}{dA} + \frac{1}{2}(R^M - \lambda R^W)^2 \frac{dY(I)}{dI},\end{aligned}$$

which implies that  $\frac{dC(M)}{dM} > 0$  since  $Y'(I) > 0$ . This establishes the lemma.

**Proof of Proposition 5 (continued).** Since  $\frac{d(-I)}{dM} = \frac{d(-I)}{dA} = (R^M - \lambda R^W)$  for  $M < \bar{M}$ , and  $Y(I) = 0$  for  $M \geq \bar{M}$  following from  $I(M \geq \bar{M}) = 0$ , we can write

$$\frac{1}{2}Y(I) \frac{d(-I)}{dA} = \frac{1}{2}(R^M - \lambda R^W)Y(I).$$

Substituting for  $\frac{1}{2}(R^M - \lambda R^W)Y(I)$  with  $\frac{1}{2}Y(I) \frac{d(-I)}{dA}$  into equation (17) gives  $C(M^*) = K(A) - \frac{1}{2}Y(I) \frac{d(-I)}{dA}$ , from which  $C(M^*) = 0$  follows directly from proposition 4.

To establish that  $M^* < \bar{M}$ , note that since  $Y(I) = 0$  for  $M \geq \bar{M}$ , we have  $C(M \geq \bar{M}) = K(A) > 0$ . Since  $\frac{dC(M)}{dM} > 0$ , we have that  $M^* < \bar{M}$  is required for  $C(M^*) = 0$ . To establish that  $M^* > 0$ , note that for  $R^M = 1$ ,  $R^D \geq 1$ , and an arbitrarily small amount of reserves  $\check{M} > 0$ , as were all the case pre-IOER, we have that  $C(\check{M}) = 1 - R^D \leq 0$ . Since  $\frac{dC(M)}{dM} > 0$ , as reserves decrease from  $\check{M}$  to zero,  $C(M)$  decreases and hence  $C(0) < 0$ . Thus,  $M^* > 0$  is required for  $C(M^*) = 0$ .

As discussed in footnote 53 regarding the central bank optimization, an alternative approach, which is equivalent to holding second-order price and rate effects fixed, is to instead use a log-linearization with a first-order approximation to a Taylor series expansion of price and rate effects, which results in a central bank objective function

$$\mathbb{E}[w^j] = \int_0^L r(\hat{L})d\hat{L} + (\theta - 1)W - \left(\frac{\theta-1}{\theta}\right) (R^E E + \frac{1}{2}R^{E1} E_1) - \frac{1}{2} \int_0^I y(\hat{I})d\hat{I} \quad (32)$$

to replace the expression in equation (31). A second alternative approach, which is also equivalent, is to assume that one-period rates of interest are additive rather than compounded

over the two periods in the model, which also results in the central bank objective function equation (32). The first order condition for reserves of this objective function equation (32) gives the same result,  $\frac{1}{2}Y(I)\frac{d(-I)}{dA} = K(A)$ , for the optimal level of reserves,  $M^*$ , which leads to the same result that  $C(M^*) = 0$ .

**Proof of Lemma 1.** The ONRRP is incorporated into the model by redefining the household dates 0 and 1 budget constraints as  $D + E + B^H + Q_0 \leq G$ ,  $P_1^B B_1^s + Q_1^s \leq R^W D^W + R^{Q0} Q_0$ , and  $D_1 + E_1 + Q_1^n \leq P_1^B B_1^n + R^{Q0} Q_0$ , respectively; adding  $\mathbf{1}_{[j=s]}\theta R^{Q1} Q_1^s$  and  $\mathbf{1}_{[j=n]}\theta R^{Q1} Q_1^n$  to the household income  $\Pi^{Hj}$  in equation (1); and adding  $Q_0$ ,  $Q_1^s$  and  $Q_1^n$  to  $Q^{Hj}$ . The budget constraints are redefined for the central bank as  $B^{CB} = M + Q_0$  for date 0 and for the consolidated government and central bank as  $\Upsilon = R^B(B - B^{CB}) + R^{M^2} M + R^{Q1} Q_1$  for date 2. The central bank's additional choice variables are  $Q_0$  and  $Q_1$ . Added to the definition of an equilibrium for a fixed-quantity ONRRP are the returns  $R^{Q0} > 0$  and  $R^{Q1} > 0$ ; and market clearing conditions at date 0, for  $Q_0$  household demand and central bank supply, and at date 1,  $\frac{1}{2}(Q_1^s + Q_1^n) = Q_1$ .

Market clearing, binding budget constraints, and equal initial asset holdings across sectors give  $A = W - B + M + Q_0$  and  $A_1 = [R^W \lambda A - 2(Q_1 - R^{Q0} Q_0)]^+$ . First order conditions give  $R^W \geq R^{Q0}$  and  $R^{D1} \geq R^{Q1}$ , which bind for  $Q_0 > 0$  and  $Q_1 > 0$ , respectively, and hence imply that  $R^D \geq R^{Q0} R^{Q1}$ , which binds for  $Q_0 Q_1 > 0$ .

The equilibrium definition is revised for a fixed-rate ONRRP at date  $t = 0$ , date  $t = 1$ , or both dates  $t \in \{0, 1\}$ , by excluding  $Q_t$  and including  $R^{Qt}$  as choice variables for the central bank. The equilibrium outcome with a fixed-rate ONRRP at dates 0 and/or 1 is equivalent to the equilibrium outcome with a fixed-quantity ONRRP since there exists a one-to-one mapping between the equilibrium  $R^{Qt}$  and  $Q_t$  for all cases.

**Proof of Proposition 6.** For starting reserves  $M' > M^*$  without the ONRRP, with  $B^{CB'} = M'$ , we have  $R^{D'} < R^{M^2}$  since  $C(M') > 0$ . With the ONRRP at rate  $R^{Q0} = R^{Q1} = R^M$ , first order conditions from the household's optimization require  $R^D = R^{M^2}$  and  $R^{D1} = R^M$ , which following from lemma 1 requires  $Q_0 = M' - M^*$ . Hence,  $M = B^{CB'} - Q_0 = M^*$ .

**Proof of Proposition 7.** The optimal central bank policy is the joint quantities of reserves and the ONRRP,  $\mathcal{Q}^{CB} \equiv \{M, Q_0, Q_1\}$ , that maximize household expected utility,  $\mathbb{E}[u^j]$ . The maximization of  $\mathbb{E}[u^j]$  over  $\mathcal{Q}^{CB}$  subject to the central bank budget constraints given in the proof of lemma 1 give the first order conditions  $Q_1 = \frac{1}{2}R^W \lambda A$  and  $M = M^{**}$ . The optimal ONRRP quantity  $Q_1 = \frac{1}{2}R^W \lambda A$  results in  $A_1 = 0$  following from  $A_1 = [R^W \lambda A - 2(Q_1 - R^{Q0} Q_0)]^+$  derived in the proof of lemma 1,  $E_1(A_1 = 0) = 0$  from equation (7), and hence

welfare equal to

$$\mathbb{E}[u^j] = \frac{1}{P_2} \left[ P_2 \int_0^L r(\hat{L}) d\hat{L} + (\theta - 1)(R^D D + R^B B^H) - \frac{1}{2} \int_0^I Y(\hat{I}) d\hat{I} \right]. \quad (33)$$

Equation (33) is modified from welfare without the ONRRP by setting  $E_1 = 0$  in equation (31). Thus,  $M^{**}$  is equivalently given by the maximization of equation (33) with respect to reserves.

In order to implement the optimal quantities  $Q_0 = 0$  at date 0 and  $Q_1 = \frac{1}{2}R^W \lambda A$  at date 1 with optimal reserves  $M^{**}$ , the necessary ONRRP rate is  $R^{Q0} = R^{Q1} \in [\bar{R}^{D1}, R^M]$ , where  $\bar{R}^{D1} \equiv R^M - \left(\frac{\theta-1}{\theta}\right)R^{E1} \frac{\partial E_1(A_1)}{\partial A_1} \Big|_{A_1=0}$ , following from the bank's and household's first order conditions. (Note that with an overabundance of initial reserves  $M' > M^{**}$ , the ONRRP rate of  $R^{Q0} = R^{Q1} = R^M$  implements the optimal ONRRP quantities  $Q_0 = M' - M^{**}$  and  $Q_1 = \frac{1}{2}R^W \lambda A + R^{Q0}Q_0$ ).

The results of  $C(M^{**}) = K(A) - \frac{1}{2}Y(I) \frac{d(-I)}{dA}$ ,  $\frac{1}{2}Y(I) \frac{d(-I)}{dA} = K(A)$ , and hence  $C(M^{**}) \equiv R^{M^2} - R^D = 0$  follow from the proofs of these results in propositions 2, 4 and 5 with  $E_1 = A_1 = 0$ , respectively. In particular, equating the solutions for  $R^L$  from the first order conditions with respect to  $A$  and  $M$  in the bank optimization; substituting for  $R^D$  in the term  $-\frac{1}{2}\lambda R^D$  with  $R^W R^{D1}$ ; substituting for  $R^{D1}$  with any choice of  $R^{Q1} \in [\bar{R}^{D1}, R^M]$ ; substituting for  $K(A_1 = 0)$  with  $\frac{1}{2}Y(I) \frac{d(-I)}{dA}$  following from the first order condition with respect to  $M$  for the optimization of  $\mathbb{E}[u^j]$ ; and simplifying, gives  $C(M^{**}) \equiv R^{M^2} - R^D = 0$ .

Following from the proof of lemma 2 with  $E_1 = A_1 = 0$ , we have  $\frac{dC(M)}{dM} > 0$ , which establishes a sufficient second order condition for an interior optimum for  $M$ . The derivative of  $\mathbb{E}[u^j]$  with respect to  $M$  with  $Q_1 = \frac{1}{2}R^W \lambda A$  is greater than the corresponding derivative of  $\mathbb{E}[u^j]$  with  $Q_1 = 0$  because of the omission of the  $-\frac{1}{2}R^{E1}E_1$  term in equation (31), which gives  $M^{**} > M^*$ . The derivative of  $\mathbb{E}[u^j]$  with  $Q_1 = \frac{1}{2}R^W \lambda A$  is negative for  $M \geq \bar{M}$ , which gives  $M^{**} < \bar{M}$ .

**Proof of Proposition 8.** First, note that  $R^{B1} \equiv \frac{R^B}{P^B} = R^{D1}$  is shown in the proof for proposition 1. Next, replace  $\lambda$  by  $\psi\lambda^h + (1 - \psi)\lambda^l$  for a generic probability  $\psi \in (0, 1)$  in the agent optimizations and market clearing conditions of the model. For all  $\lambda^i \in (0, 1)$  for  $i \in \{h, l\}$  such that  $\lambda^h > \lambda^l$ , and for any choice of  $R^{Q1} \in [\bar{R}^{D1}, R^M]$ , where  $\bar{R}^{D1} \equiv R^M - \left(\frac{\theta-1}{\theta}\right)R^{E1} \frac{\partial E_1(A_1)}{\partial A_1} \Big|_{A_1=0}$ , we have  $R^{D1} = R^{B1} = R^{Q1}$ , with  $Q_1 = \frac{1}{2}R^W \lambda^i A$ , for  $i \in \{h, l\}$ . Whereas, with no ONRRP ( $Q_1 = 0$ ), we have  $R^{D1}(\lambda^i) = R^{B1} = R^M - \left(\frac{\theta-1}{\theta}\right)R^{E1} \frac{\partial E_1(A_1)}{\partial A_1} \Big|_{A_1=R^W \lambda^i A}$ , which is decreasing in  $\lambda^i$  since  $R^\alpha(\cdot)$  is convex, and hence  $R^{D1}(\lambda^h) = R^{B1}(\lambda^h) < R^{D1}(\lambda^l) = R^{B1}(\lambda^l)$ .

**Proof of Corollary 1.** Following from the proof of proposition 8, for any  $\lambda^i \in (0, 1)$  for

$i \in \{h, l\}$  such that  $\lambda^h > \lambda^l$ , and for any  $Q_1$  that is not conditional on the state  $i \in \{h, l\}$ , we have  $R^{D1}(\lambda^h) = R^{B1}(\lambda^h) < R^{D1}(\lambda^l) = R^{B1}(\lambda^l)$ .

## Appendix B: Bank capital requirements

At dates 0 and 1, a bank can unobservably take a risky project with a negative expected NPV and a realized marginal return of either  $R^\alpha(\cdot) > 0$  or  $\beta(\cdot) < 0$  with equal probability.  $|\beta(\cdot)|$  is sufficiently large that depositors bear a partial loss and lose their liquidity value on deposits such that  $\theta = 1$ , which implies that the risky project is a form of bank risk-shifting that is socially inefficient. The government has the option of providing a bail-out to depositors by using a lump sum tax on all households to pay for the loss on deposits at a bank that has a negative realization of the risky project.

The bank chooses whether to take the risky project at dates 0 and 1 in order to maximize profit subject to the expected return required by equityholders. Specifically, if the bank takes the risky profit, it pays equity a sufficient return to equity when there is a positive realization to compensate for the loss to equity when there is a negative realization. Since equity has a zero return if there is a negative realization, equity receives an additional return of  $R^E E$  ( $R^{E1} E_1$ ) if there is a positive realization for a project taken at date 0 (date 1). Hence, the profit for bank  $j \in \{n, s\}$  conditional on having a positive realized return from risk-shifting on its new date 0 assets  $A$  is

$$\Pi^{Bj,RS_0} = \Pi^{Bj} + R^\alpha(A)A - R^E E, \quad (34)$$

or on its new date 1 assets  $A_1$  is

$$\Pi^{Bj,RS_1} = \Pi^{Bj} + R^\alpha(A + A_1)A_1 - R^{E1} E_1. \quad (35)$$

The first term of equations (34) and (35) is the bank's profit  $\Pi^{Bj}$  from equation (6) if the bank does not take the risk-shifting project at either dates 0 or 1. The second terms of equations (34) and (35) are the returns  $R^\alpha(A)A$  and  $R^\alpha(A + A_1)A_1$  from a positive realization of risk-shifting at dates 0 and 1, respectively. The third terms of equations (34) and (35) subtract the additional return paid to equity  $R^E E$  and  $R^E E_1$  for positive realizations.

Since risk-shifting is socially inefficient, the government as regulator imposes an equity capital requirement that incentives banks not to take the risky project. The constraint for a bank not to take the risk-shifting project at date  $t \in \{0, 1\}$  is

$$\mathbb{E}_t[\Pi^{Bj,RS_t}] \leq \mathbb{E}_t[\Pi^{Bj}] \quad \text{for } t \in \{0, 1\}. \quad (36)$$

Note that we could add an additional constraint for the bank not to take the risk-shifting project at both dates 0 and 1, but that would be redundant as fulfillment of the no risk-shifting constraints for dates 0 and 1 individually guarantees fulfillment of such an additional constraint.

Substituting for  $\Pi^{Bj,RS_0}$ ,  $\Pi^{Bj,RS_1}$  and  $\Pi^{Bj}$  from their expressions in equations (34), (35), and (6), respectively, equation (36) with a binding inequality gives the minimum capital requirements to prevent risk-shifting as

$$\begin{aligned}\underline{E}(A) &\equiv \frac{R^\alpha(A)A}{R^E} \\ \underline{E}_1(A, A_1) &\equiv \frac{R^\alpha(A+A_1)A_1}{R^{E1}},\end{aligned}$$

which is the result given in equation (7). Setting the capital requirement at the minimum to prevent risk-shifting at each date 0 and 1 minimizes welfare costs. Equity issued at date 0 decreases date 0 deposits and is more costly in expectation for welfare as well as for bank profits than equity issued at date 1, where the measure of welfare is given by equation (31). Date 1 equity is only necessary for the nonshocked bank with date 1 inflows. Thus, reducing expected date 1 required equity with additional date 0 equity is inefficient.

The capital requirement is a source of economic inefficiency, since equity does not provide the liquidity value of deposits, but it acts as a constrained-efficient mechanism to solve the time-inconsistency problems for the bank and government. If the project was verifiable, complete contracts for deposits would provide costless commitment for the bank not to take the project without requiring costly equity.

If the government could ex-ante commit against bailouts to depositors, banks would issue equity equal to the capital requirement as a market discipline commitment device to avoid otherwise facing higher deposit rates from the loss of liquidity value. Without commitment, the government imposes the capital requirement in place of market-discipline based equity. Alternatively, the government could ensure the depositor liquidity benefit by providing deposit insurance. However, this is less efficient than capital requirements because it would not provide the bank a commitment device against taking the risky project. This is true even if the bank were charged the expected government cost of the deposit insurance, unless such charges could be contingent on verifiability of the bank's actual risk-taking.

## Appendix C: Money market funds

The MMF is incorporated into the model including the overnight RRP (ONRRP), which is presented in section 6 and detailed in the proof for lemma 1. There are competitive MMFs that operate across both sectors represented by a single price-taking MMF. At date 0, the MMF buys  $B^{MF}$  bonds, invests  $Q_0^{MF}$  in the date 0 ONRRP, and issues  $S$  shares to households. At date 1, the MMF invests  $Q_1^{MF}$  in the date 1 ONRRP, redeems  $S^W$  shares and issues  $S_1$  new shares. MMF bond and share quantities are normalized to a per-sector basis.

Instead of transacting in bonds, households acquire and redeem MMF shares. (Allowing households to transact both in bonds and MMF shares is equivalent).

Specifically, households buy  $S$  shares instead of buying  $B^H$  bonds and investing  $Q_0$  in the ONRRP at date 0. The shocked household uses its early withdrawal of  $D^W$  to buy  $S_1$  additional shares instead of buying  $B_1^s$  bonds and investing  $Q_1^s$  in the ONRRP at date 1. The nonshocked household redeems  $S^W$  shares instead of selling  $B_1^n$  bonds. Returns at date 2 are  $R^S$  for shares issued at date 0 and  $R^{S1}$  for shares issued at date 1. The return on shares issued at date 0 and redeemed early at date 1 is  $R^{SW}$ . Households receive the liquidity benefit factor  $\theta$  on MMF shares, as with deposits and bonds. In particular, the date 0 MMF shares provide a promised redemption return  $R^{SW}$ , which parallels the return  $R^W$  on early deposit withdrawals.

The MMF profit is

$$\Pi^{MF} = R^B B^{MF} + R^{Q1} Q_1^{MF} - R^S (S - S^W) - R^{S1} S_1. \quad (37)$$

At date 2, the MMF receives the  $R^B$  return on its  $B^{MF}$  bonds bought at date 0. The MMF pays the return  $R^S$  on the  $(S - S^W)$  outstanding shares issued at date 0 and the return  $R^{S1}$  on shares issued at date 1.

The MMF maximizes its expected profit,  $\Pi^{MF}$ , as follows:

$$\left. \begin{aligned} \max_{Q^{MF}} \mathbb{E}[\Pi^{MF}] \\ \text{s.t. } B^{MF} + Q_0^{MF} &\leq S \\ R^{SW} S^W + Q_1^{MF} &\leq S_1 + R^{Q0} Q_0^{MF}, \end{aligned} \right\} \quad (38)$$

where  $Q^{MF} \equiv \{B^{MF}, S, S^W, S_1, Q_0^{MF}, Q_1^{MF}\}$ . The two inequalities are the MMF budget constraints for dates 0 and 1, respectively. The first inequality states that the amount  $B^{MF}$  the MMF pays to buy bonds is limited to the amount  $S$  received from issuing shares at date 0. The second inequality states that the amount  $R^{SW} S^W$  paid for early redemptions is limited to the amount  $S_1$  received from issuing shares at date 1.<sup>55</sup>

The model is updated with the substitution of  $S$  for  $(B^H + Q_0)$ ,  $B^{MF}$  for  $B^H$ ,  $Q_0^{MF}$  for  $Q_0$ ,  $Q_1^{MF}$  for  $\frac{1}{2}(Q_1^s + Q_1^n)$ ,  $S^W$  for  $B_1^n$ ,  $\frac{S_1}{R^{SW}}$  for  $B_1^s$ , and  $R^{SW}$  for  $P_1^B$ . The household income

<sup>55</sup>Note that MMFs are not required to issue equity to create money-like assets with liquidity benefits, as MMFs invest in government bonds and the ONRRP without the ability for risk-shifting. Stein (2012) makes this essential point by referring to the “more benign forms of money creation, for example, money market fund accounts backed exclusively by Treasury bills.”

$\Pi^{Hj}$  in equation (1) and optimization in equation in (2) are updated, respectively, as

$$\begin{aligned}
\Pi^{Hj} &= R^E E + \Pi^{Bj} + \Pi^{Fj} - \Upsilon \\
&+ \mathbf{1}_{[j=s]} \theta [R^D (D - D^W) + R^S B^S + R^{S1} S_1] \\
&+ \mathbf{1}_{[j=n]} \{ \theta [R^D D + R^{D1} D_1 + R^S (B^S - S^W)] + R^{E1} E_1 \}.
\end{aligned}$$

$$\left. \begin{aligned}
&\max_{\mathcal{Q}^{Hj}} \mathbb{E}[u^j] \\
&\text{s.t. } D + E + S \leq G \\
&\quad S_1 \leq R^W D^W \\
&\quad D_1 + E_1 \leq R^{SW} S^W \\
&\quad S^W \leq S.
\end{aligned} \right\} \quad (39)$$

The definition of an equilibrium is updated as follows. The returns  $(R^S, R^{SW}, R^{S1}) > 0$  are added. The MMF optimizing quantities  $\mathcal{Q}^{MF}$  given by (38) are added to the agents' optimizing quantities. Market clearing is added for MMF shares at date 0,  $S$ . Market clearing for bonds at date 1 is replaced by market clearing for MMF shares at date 1,  $S_1^s = S_1$ .

**Proposition 9** *The returns on MMF shares are equal to the corresponding returns on deposits and bonds for equivalent one- or two-period holding periods:  $R^{SW} = R^W = P_1^B$ ,  $R^{S1} = R^{D1} = \frac{R^B}{P_1^B}$ , and  $R^S = R^D = R^B$ . The MMF quantities are equal to the corresponding quantities without the MMF:  $S = B^H + Q_0$ ,  $B^{MF} = B^H$ ,  $Q_0^{MF} = Q_0$ ,  $Q_1^{MF} = \frac{1}{2}(Q_1^s + Q_1^n)$ ,  $S^W = B_1^n$ , and  $S_1 = P_1^B B_1^s$ . The MMF makes zero profits, and the results of the paper are unchanged.*

**Proof.** Necessary first order conditions and sufficient second order conditions hold for  $\mathcal{Q}^{MF}$  in the MMF optimization (38) and the revised household optimization (39). With market clearing and symmetric date 0 asset holdings; the results for corresponding equilibrium returns and quantities follow directly from the binding budget constraints and first order conditions with respect to  $S$ ,  $D$ ,  $S_1$  and  $D_1$  in the revised household optimization (39); the binding budget constraints and first order conditions with respect to  $\mathcal{Q}^{MF}$  in the MMF optimization (38); the term structure on deposit returns  $R^D = R^W R^{D1}$ ; and the equilibrium results  $R^B = R^D$ ,  $R^{D1} = \frac{R^B}{P_1^B}$ , and  $P_1^B = R^W$  following the proof of proposition 1. Substituting for the corresponding equilibrium returns and quantities into equation (37), the MMF profit is zero, and all of the results and proofs of the paper hold directly and unchanged. ■

The MMF is not a required component within the model since it does not affect the results of the paper. However, a value-added role for the MMFs can easily be incorporated

into the model to make the MMF a necessary feature for achieving the optimality results of the paper. The household liquidity benefit for money-like assets can be more narrowly defined as to apply only to household assets that have a contracted one-period return. This definition implies that the liquidity benefit would apply to deposits and shares but not to bonds, which makes the role of the MMF critical. The sale at date 1 of bonds bought at date 0 occurs in the bond market. Hence, the one-period return for date 0 bonds is based on the bond market transaction rather than a contracted one-period return as is provided by the MMF along with banks.

## Appendix D: Other policy tools

Before creation of the ON RRP, the Fed first developed in 2008 longer-term RRP and the term deposit facility (TDF) for offer to banks to supplement IOER for managing rates and reserves.<sup>56</sup> The Fed has kept these other tools as an option to use as needed.<sup>57</sup> We argue against their use by extending our model to show that the term RRP is not as effective as the ON RRP, and the TDF is inefficient for reducing the overabundance of reserves.

**Background** Term RRP are similar to the ON RRP but have a longer-term maturity than overnight. The TDF allows DIs to make term deposits of reserves at a rate above IOER.<sup>58</sup> Each of these tools have been developed and tested with both fixed-quantity auctions at market determined rates and at fixed-rates with market determined quantities. Small-value testing of the term RRP from 2009 until 2015 and the TDF since 2010 have been conducted for a range of maturities up to 28 days.

**Analysis** The term RRP and TDF are offered as two-period assets at date 0. We assume that the TDF is only available to banks, as in practice. The term RRP has a return of  $R^{TM}$  and quantity of  $Q^{TM}$ , while the TDF has a return of  $R^{TD}$  and quantity of  $Q^{TD}$ . The central

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<sup>56</sup>NY Fed President Dudley referred to the RRP and TDF as the “suspenders” supporting the IOER “belt” for the Fed to “retain control of monetary policy” in a 2009 speech. Source: <http://www.newyorkfed.org/newsevents/speeches/2009/dud090729.html>.

<sup>57</sup>In previous normalization principles and plans, the FOMC states: “During normalization, the Federal Reserve intends to use an overnight reverse repurchase agreement facility and other supplementary tools as needed to help control the federal funds rate.” Source: <https://www.federalreserve.gov/monetarypolicy/policy-normalization-discussions-communications-history.htm>.

<sup>58</sup>Details on the TDF are at: <https://www.federalreserve.gov/monetarypolicy/tdf.htm>. The TDF was approved April 2010, following the approval of amendments to Regulation D (Reserve Requirements of Depository Institutions), allowing Federal Reserve Banks to offer term deposits to institutions eligible to earn interest on reserves. Source: <http://www.federalreserve.gov/newsevents/press/monetary/20100430a.htm>.

bank's budget constraint is  $B^{CB} = M + Q_0 + Q^{TM} + Q^{TD}$  at date 0. Full details of the model extension are provided in the proof for proposition 10 below.

The term RRP is similar to the ONRRP, but since it is a term transaction, the date 0 and date 1 rates and quantities must be equal. While the term RRP with a rate set to IOER endogenizes the supply of reserves, it cannot stabilize rates in the way the ONRRP does. And, as a consequence, it cannot implement optimal welfare with the supply of reserves  $M^{**}$ , which requires a greater ONRRP quantity at date 1 to absorb liquidity shocks than at date 0.

The TDF is not as effective as the term RRP because it does not reduce the size of banks' balance sheets. RRPs are held by non-banks (households in our model), so every dollar invested in the term RRP reduces the balance sheet of the banking sector by a dollar. In contrast, term-deposits must be held by banks. The TDF simply takes the place of reserves as an illiquid asset on banks' balance sheets that cannot be used to pay out for depositors' date 1 withdrawals. Thus, the TDF achieves lower welfare than the term RRP because it does not reduce balance sheet costs. The TDF also does not absorb liquidity shocks as does the ONRRP to support a higher optimal supply of reserves or stabilize rates.

**Proposition 10** *The term RRP and TDF reduce welfare relative to the optimal use of the ONRRP.*

**Proof.** The term RRP and TDF are incorporated into the model with the ONRRP, formalized in the proof of lemma 1, by updating the following. The household date 0 budget constraint is  $D + E + B^H + Q_0 + Q^{TM} \leq G$ , and  $\theta R^{TM} Q^{TM}$  is added to the household income  $\Pi^{Hj}$ . The bank's budget constraint at date 0 is  $A = L + M + Q^{TD} \leq D + E$ , and  $R^{TD} Q^{TD}$  is added to the bank profit  $\Pi^{Bj}$ . The budget constraint for the central bank is  $B^{CB} = M + Q_0 + Q^{TM} + Q^{TD}$  at date 0 and for the consolidated government and central bank at date 2 is  $\Upsilon = R^B(B - B^{CB}) + R^{M^2} M + R^{Q^1} Q_1 + R^{TM} Q^{TM} + R^{TD} Q^{TD}$ . Added to the definition of an equilibrium are the returns  $R^{TM} > 0$  and  $R^{TD} > 0$  and market clearing for  $Q^{TM}$  and  $Q^{TD}$  at date 0.

The optimal central bank policy is the choice of  $\mathcal{Q}^{CB} \equiv \{M, Q_0, Q_1, Q^{TM}, Q^{TD}\}$  to maximize  $\mathbb{E}[u^j]$ , which following the proof of proposition 7 gives the optimal condition  $K(A) = \frac{1}{2} Y(I) \frac{d(-I)}{dA}$  with a result of  $M = M^{**}$ ,  $Q_1 = \frac{1}{2} R^W \lambda A$  and  $Q_0 = Q^{TM} = Q^{TD} = 0$ . Results are equivalent if the term RRP and/or TDF are offered at fixed rates rather than fixed quantities since there exists a one-to-one mapping between the equilibrium  $R^{TM}$ ,  $R^{TD}$ ,  $Q^{TM}$  and  $Q^{TD}$  for all cases. ■

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