Structural Estimates of the U.S. Sacrifice Ratio

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Abstract -- This paper investigates the statistical properties of the U.S. sacrifice ratio -- the cumulative output loss arising from a permanent reduction in inflation. We derive estimates of the sacrifice ratio from three structural VAR models and then conduct Monte Carlo simulations to analyze their sampling distribution. While the point estimates of the sacrifice ratio confirm the results reported in earlier studies, we find that the estimates are very imprecise and that the degree of imprecision increases with the complexity of the model used. That is, increases in the number of structural shocks widen our confidence intervals. We conclude that the estimates provide a very unreliable guide for assessing the output cost of a disinflation policy.

KEY WORDS: Disinflation; Identification; Vector Autoregression; Structural shocks.

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I. Introduction

The successful conduct of monetary policy requires policymakers both to specify a set of objectives for the performance of the economy and to understand the effects of policies designed to attain these goals. Stabilizing prices, one of the dual goals of U.S. monetary policy, is no different. It is generally agreed that permanently low levels of inflation create long-run benefits for society, increasing the level and possibly the trend growth rate of real output.[1] There is also a strong belief that engineering inflation reductions involve short-term costs associated with a corresponding loss in output. Policymakers' decisions on the timing and extent of inflation reduction depend on a balancing of the costs and benefits of moving to a new, lower level of inflation, which in the end requires estimates of the size of each.

The purpose of our work is to investigate the size of one aspect of this: the output cost of disinflationary policy, usually referred to as the **sacrifice ratio**. The sacrifice ratio is the cumulative loss in output, measured as a percent of one-year's GDP, associated with a one percentage point permanent reduction in inflation.[2]

While the sacrifice ratio is a key consideration for policymakers, estimating its size is a difficult exercise, as it requires the identification of changes in the stance of monetary policy and an evaluation of their impact on the path of output and inflation. Neither one of these determinations is straightforward, as it is difficult to gauge the timing and extent of shifts in policy as well as to separate the movements in output and inflation into those that were caused by policy

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[1]See Barro (1996) and the collected articles in Feldstein (1999).

[2]As Filardo (1998) notes, the sacrifice ratio likely understates the true cost of disinflation because it neglects any personal costs borne by unemployed workers and their families.

and those that were not.   Finally, even if an estimate of the sacrifice ratio can be constructed, it is critical for policymakers to know something about the precision of the estimate. It is one thing to say that our best guess is that reducing inflation one percentage point entails a loss of 3 percentage points of GDP. But it is entirely another to go on and say that the output loss could be anywhere from 0 to 5 or 10 percentage points of GDP.

This paper focuses on the U.S. sacrifice ratio and investigates a number of issues associated with its construction and behavior. We examine quarterly data over the period 1959-97 and use structural vector autoregressions (SVARs) to identify shifts in monetary policy and analyze their impact on output and inflation. Within this framework, we consider models of increasing complexity, beginning with Cecchetti's (1994) two-variable system, then considering Shapiro and Watson's (1988) three-variable system, and finally examining Gali's (1994) four-variable system. In each of these models, we derive estimates of the sacrifice ratio under a different set of identifying restrictions for the structural shocks. To assess the reliability of the sacrifice ratio estimates, we undertake a series of simulation exercises -- Monte Carlo experiments -- that allow us to construct confidence intervals.

Our analysis generates estimates of the sacrifice ratio that vary substantially across the models, ranging from just over 1 to nearly 10. That is, we estimate that for inflation to fall one percentage point, somewhere between 1 and 10 percent of one year's GDP must be sacrificed. While the point estimates are broadly consistent with the results of previous studies, the analysis also suggests that the sacrifice ratio is very imprecisely estimated. For example, a 90 percent confidence interval covers zero for the point estimate in each model. Further, the sacrifice ratio estimates display greater imprecision as the models allow for additional structural shocks. While the simplest two-variable system indicates that the true value of the sacrifice ratio lies in the interval between -0.4 and +3.2, the four-variable system suggests that the true value is somewhere between -49 and +68. We do not take these latter estimates too literally, as the values seem extremely implausible given our reading of the recent history. Nevertheless, they serve as a caution to policymakers in that the point estimates would seem to provide a very unreliable guide for gauging the output cost of a disinflation policy.

II. The Sacrifice Ratio

It is generally believed that attempts on the part of a monetary authority to lower the inflation rate will lead to a period of increased unemployment and reduced output. The reason why disinflationary episodes have this affect on real economic activity is because inflation displays a great deal of persistence or inertia. That is, price inflation (measured by indices such as the consumer price index) tends to move slowly over time, exhibiting very different behavior from things like stock or commodity prices. Thus, the adjustment process during a disinflation requires the monetary authority to slow aggregate demand growth, creating a period of temporary slack in the economy that will lower the inflation rate only eventually.

A number of explanations have been offered for inflation's slow adjustment and the absence of costless disinflations. Fuhrer (1995) provides an overview of this discussion and focuses on three possibilities. First, inflation persistence may arise from the overlap and non-synchronization of wage and price contracts in the economy. Wages and prices adjust at different times, and to each other, and so once one starts to rise, the other does too. Stopping the process takes time. Second, people's inflation expectations may adjust slowly over time, being based on a sort of adaptive mechanism. Because decisions about wages and prices depend on expectations of future changes, slow adaptation is self-fulfilling, creating inertia. And third, if people do not believe that the monetary authority is truly committed to reducing inflation, then inflation will not fall as rapidly. That is, the credibility of the policymaker is important in determining the dynamics of inflation, with less credibility leading to more persistence.

The view that reductions in inflation are accompanied by a period of decreased output (relative to trend) has generated considerable debate among economists on how to lessen the costs of disinflation. Some discussions have focused on the speed of disinflation and whether the monetary authority should adopt a gradualist approach or subject the economy to a "cold turkey" remedy.[3] Other discussions have focused on identifying the sources of inflation persistence and analyzing their implications for the pursuit of cost-reducing strategies.

While these theoretical discussions raise a number of important issues, the design and implementation of disinflation policies clearly require agreement about the quantitative impact of monetary policy on output and inflation. Measurement of the sacrifice ratio is a prerequisite to any study attempting to evaluate its key determinants or the impact of alternative policies on the cost of disinflation.

With this practical issue in mind, a number of authors have estimated sacrifice ratios for the U.S. using a variety of techniques. Okun (1978) examines a family of Phillips curve models and derives estimates that range from 6 to 18 percent of a year's GNP, with a mean of 10 percent. Gordon and King (1982) use traditional and vector autoregression (VAR) models to obtain

[3]See Okun (1978), Gordon and King (1982), Taylor (1983), Sargent (1983), Schelde-Andersen (1992) and Ball (1994).

estimates of the sacrifice ratio that range from 0 to 8. Mankiw (1991) examines the 1982-85 Volcker disinflation and uses Okun's law to arrive at a "back-of-the-envelope" estimate of 2.8. More recently, Ball (1994) examines movements in trend output and trend inflation over various disinflation episodes and obtains estimates that vary from 1.8 to 3.3.

While the estimates calculated by Ball (1994) and Mankiw (1991) are roughly the same order of magnitude, suggesting that a consensus may exist about the size of the sacrifice ratio, there are several remaining issues. First, prior studies do not, in our view, adequately control for the impact of nonmonetary factors on the behavior of output and inflation. Consider the plots of the quarterly growth rate of real GDP and the consumer price index (CPI) displayed in **Figure 1**. While some of the movements in output and inflation over the post-World War II period are surely attributable to monetary policy actions, it is unreasonable on either theoretical grounds or from visual inspection to believe they all are. Thus, computing a meaningful estimate of the sacrifice ratio requires more than simply calculating a measure of the association between output and inflation during arbitrarily selected episodes. Rather, it depends critically on isolating which movements are the result of monetary influences.

Secondly, previous studies such as Gordon and King (1982) fail to account for the policy process. Some of the actions undertaken by a monetary authority are intended to accommodate or offset shocks to the economy. However, the analysis of Gordon and King does not allow the movements in a policy variable to be separated into those associated with a shift in policy and those reflecting a systematic (or endogenous) response to the state of the economy. This type of decomposition, which is necessary to assess the effects of monetary policy on the economy, requires the specification and estimation of a structural economic model.

Another important issue concerns the periods selected for the empirical analysis. Studies such as Ball (1994) focus solely on specific disinflationary episodes -- periods when contractionary monetary policy are thought to have resulted in the reduction in both inflation and output. However, it is not obvious that estimates of the sacrifice ratio should exclude a priori episodes in which inflation and output are both increasing. Such an approach would only be justified if there were an accepted asymmetry in the impact of monetary policy on output and prices.[4] In the absence of such evidence, economic expansions would contain episodes in which output and inflation increased as a result of expansionary monetary policy. Such episodes would then be as informative about the sacrifice ratio as a disinflation.[5]

Filardo (1998) is an exception, as he presents evidence that the sacrifice ratio for the U.S. varies across three regimes corresponding to periods of weak, moderate and strong output growth. Using a measure of the sacrifice ratio similar to that used in this study, Filardo derives sacrifice ratio estimates of 5.0 in the weak growth regime and 2.1 in the strong growth regime.[6]

While the findings of Filardo offer new and interesting insights into the output cost of disinflation, his study, along with previous work, is silent on the accuracy of the estimates. Specifically, there has been no serious attempt to characterize the statistical precision of sacrifice

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[4]This discussion abstracts from other considerations such as Ball's identification of disinflation episodes or the construction of his measures of trend output and trend inflation.

[5]Cecchetti (1994) refers to the increase in output from a higher inflation rate as the "benefit ratio". If monetary policy has symmetric effects on output an inflation, then the benefit ratio and sacrifice ratio would simply be mirror images of each other.

[6]Filardo's results associate the moderate growth regime with a Phillips curve whose slope is essentially zero. Filardo does not report an estimate for the sacrifice ratio for this regime because the within-regime value would approach infinity.

ratio measures. Estimation of economic relationships and magnitudes inherently involves some uncertainty and it is extremely important to quantify their reliability. For example, policymakers may be reluctant to undertake certain policy actions unless they can attach a high degree of confidence to the predicted outcomes.

Characterizing the precision of sacrifice ratio estimates is a primary goal of our analysis. We do this by studying the properties of three structural vector autoregression (SVAR) models. We now turn to a discussion of the SVAR estimation methodology and a description of the models that we use to construct sacrifice ratio estimates.

III. Structural Vector Autoregressions and Monetary Policy Shocks

The structural vector autoregression (SVAR) approach that we adopt here remains a popular technique for analyzing the effects of monetary policy on output and prices. A SVAR can be viewed as a dynamic simultaneous equations model with identifying restrictions based on economic theory. In particular, the SVAR relates the observed movements in a variable to a set of structural shocks -- innovations that are fundamental in the sense that they have an economic interpretation. In the formulation of their identification assumptions, the models we employ appeal to economic theories which then allow us to interpret one of the structural innovations as a monetary policy shock. For this reason, we find the SVAR methodology attractive in evaluating monetary policy's impact on output and inflation, and giving us a measure of the sacrifice ratio.

The SVAR approach decomposes monetary policy into a systematic and a random component. The systematic component can be thought of as a reaction function and describes the historical response of the monetary authority to movements in a set of key economic variables.

The random component signifies actions on the part of the monetary authority that can not be explained by the reaction function and is labeled as "monetary policy shocks".[7] The monetary policy shocks are a principal focus of the remaining analysis and ultimately allow us to answer questions concerning the quantitative impact of monetary policy on output and prices.[8]

Our approach for calculating an estimate of the sacrifice ratio can be illustrated within a relatively simple system that only includes output and inflation. Following Cecchetti (1994), we consider the following structural VAR model:

(1)

$$(1 - L)y_t = \Delta y_t = \sum_{i=1}^{n} b_{11}^{i}\Delta y_{t-i} + b_{12}^{0}\Delta\pi_t + \sum_{i=1}^{n} b_{12}^{i}\Delta\pi_{t-i} + \epsilon_t^{y}$$

$$(1 - L)\pi_t = \Delta\pi_t = b_{21}^{0}\Delta y_t + \sum_{i=1}^{n} b_{21}^{i}\Delta y_{t-i} + \sum_{i=1}^{n} b_{22}^{i}\Delta\pi_{t-i} + \epsilon_t^{\pi}$$

where y_t is the log of output at time t, π_t is the inflation rate between time t-1 and t, and $\epsilon_t = [\epsilon_t^{y}, \epsilon_t^{\pi}]'$ is a vector innovation process that contains the shocks to aggregate supply (ϵ_t^{y}) and aggregate demand (ϵ_t^{π}).[9] It is assumed that ϵ_t has zero mean and is serially uncorrelated with covariance matrix $E[\epsilon_t\epsilon_t'] = \Omega$ for all t.

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[7]This point underscores the importance of differentiating between changes in monetary policy that reflect a systematic (or endogenous) response to the state of the economy and a shift in the stance of policy. Specifically, our evaluation of the sacrifice ratio corresponds to a 'pure' monetary tightening and does not arise as a consequence of (or as a response to) other shocks.

[8]The SVAR approach is not without its limitations. The estimated effects of shocks can vary substantially as a result of slight changes in identifying restrictions. Thus, it may be quite important to consider a set of models when drawing inferences based upon the SVAR approach.

[9]The model includes the change (or first difference) of inflation which allows shocks to have a permanent effect on the level of inflation. The differencing of the inflation series is also required to account for its nonstationary behavior over the sample period. We later discuss the results of unit root tests for inflation and output.

Our primary interest is in the impact of the structural shocks on output and inflation over time. To evaluate these magnitudes, we can look at the vector-moving-average (VMA) representation of (1) which provides the impulse responses of the system to the structural shocks. This is written as:

$$\textbf{(2)} \quad \begin{aligned} (1 - L)y_t &= A_{11}(L)\epsilon^y_{t-i} + A_{12}(L)\epsilon^\pi_{t-i} = \sum_{i=0}^{\infty} a^i_{11}\epsilon^y_{t-i} + \sum_{i=0}^{\infty} a^i_{12}\epsilon^\pi_{t-i} \\ (1 - L)\pi_t &= A_{21}(L)\epsilon^y_{t-i} + A_{22}(L)\epsilon^\pi_{t-i} = \sum_{i=0}^{\infty} a^i_{21}\epsilon^y_{t-i} + \sum_{i=0}^{\infty} a^i_{22}\epsilon^\pi_{t-i} \end{aligned}$$

where $A_{ij}(L)$ is a polynomial in the lag operator L. If we initially use aggregate demand shocks to identify shifts in monetary policy, then (2) provides a particularly convenient representation to assess the dynamic impact of a monetary policy shock on output and inflation. An estimate of the sacrifice ratio can then be computed based on the structural impulse response functions from (2).

For inflation, the sum of the coefficients in $A_{22}(L)$ measures the effect of a monetary policy shock on its level. In the case of output, however, the sacrifice ratio requires us to consider the *cumulative* effect on its level resulting from the incidence a monetary policy shock. This quantity can be expressed as a function of the coefficients in $A_{21}(L)$. Taken together, the relative impact of monetary policy on output and inflation, and hence the sacrifice ratio, over the time horizon τ is just the ratio of these effects and can be calculated as:

$$\textbf{(3)} \quad S_{\epsilon^\pi}(\tau) = \frac{\sum_{j=0}^{\tau}\left(\dfrac{\partial y_{t+j}}{\partial \epsilon^\pi_t}\right)}{\left(\dfrac{\partial \pi_{t+\tau}}{\partial \epsilon^\pi_t}\right)} = \frac{(\sum_{i=0}^{0} a^i_{12}) + (\sum_{i=0}^{1} a^i_{12}) + \ldots + (\sum_{i=0}^{\tau} a^i_{12})}{(\sum_{i=0}^{\tau} a^i_{22})} = \frac{(\sum_{i=0}^{\tau}\sum_{j=0}^{i} a^i_{12})}{(\sum_{i=0}^{\tau} a^i_{22})} .$$

For a disinflationary monetary strategy undertaken at time t, the numerator measures the cumulative output loss through the first τ periods, while the denominator is the difference in the level of inflation τ periods later.[10] **Figure 2** illustrates the relationship between the sacrifice ratio and the structural impulse response functions for output and inflation.

Because the structural shocks are not observable, we obtain estimates of the structural impulse responses by using the reduced form VAR representation of (1) in conjunction with identifying restrictions.[11] One set of identifying restrictions is based on the assumption that the structural shocks are uncorrelated and have unit variance. This results in $\Omega = I$, where I is the identity matrix. Following Blanchard and Quah (1989), our additional identifying restriction for the model is that aggregate demand shocks have no permanent effect on the level of output. This latter restriction is equivalent to the condition that $A_{12}(1) = \sum_{i=0}^{\infty} a_{12}^{i} = 0$.

The previous analysis assumes that the economy is driven by only two sets of shocks and associates shifts in monetary policy with aggregate demand shocks. While the two-variable system is useful for illustrating the SVAR methodology and may provide a good approximation for analyzing the relative importance of nominal shocks and real shocks, this framework could yield misleading estimates of the sacrifice ratio. Specifically, the current restrictions fail to identify separate components of the aggregate demand shock. Thus, the estimated monetary policy shock would not only encompass policy shifts, but also other shocks related to government spending or

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[10]The numerator of the sacrifice ratio in equation (3) is calculated as the sum of the changes in output. A more accurate measure would take into account the timing of the output losses by incorporating a real interest rate and accumulating the sum of discounted changes in output.

[11]The Technical Appendix provides further details on the identification of SVAR models.

shifts in consumption or investment functions.

To provide a more detailed analysis, we also derive estimates of the sacrifice ratio from models developed by Shapiro and Watson (1988) and Gali (1992). These models allow us to decompose the aggregate demand shock into individual components and therefore can be used to judge the sensitivity of the results to alternative measures of the monetary policy shock.

Following Shapiro and Watson (1988), we consider a three-variable system given by:

$$\begin{bmatrix} \Delta y_t \\ \Delta \pi_t \\ (i_t - \pi_t) \end{bmatrix} = A(L) \begin{bmatrix} \epsilon_t^y \\ \epsilon_t^{LM} \\ \epsilon_t^{IS} \end{bmatrix} \tag{4}$$

where i_t is a short-term nominal interest rate and $(i_t - \pi_t)$ is an <u>ex-post</u> real interest rate, and A(L) is a 3 x 3 matrix of polynomial lags. Based on this model, we are able to identify three structural shocks, where ϵ_t^y continues to denote an aggregate supply shock and where the aggregate demand shock is now decomposed into an LM shock and an IS shock denoted, respectively, by ϵ_t^{LM} and ϵ_t^{IS}.

We identify the structural shocks using both short-run and long-run restrictions. The Blanchard-Quah restriction allows us to identify the aggregate supply disturbance. In addition, we discriminate between IS and LM shocks by assuming that monetary policy has no contemporaneous effect on output which implies that $a_{12}^0 = 0$. For the calculation of the sacrifice ratio estimate, we identify monetary policy shifts with the LM shocks.[12]

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[12]While we refer to this as the "Shapiro-Watson model", our model actually differs from theirs in two small ways. First, Shapiro and Watson decompose aggregate supply innovations into a technology shock and a labor supply shock. Second, they also include oil prices as an

The Gali (1992) model, which allows for the identification of a fourth structural disturbance, can be written as:

$$\textbf{(5)} \qquad \begin{bmatrix} \Delta y_t \\ \Delta i_t \\ (i_t - \pi_t) \\ (\Delta m_t - \pi_t) \end{bmatrix} = A(L) \begin{bmatrix} \epsilon_t^{y} \\ \epsilon_t^{MS} \\ \epsilon_t^{MD} \\ \epsilon_t^{IS} \end{bmatrix}$$

where m_t is the log of the money supply and $(\Delta m_t - \pi_t)$ is the growth of real money balances. The structural demand shocks ϵ_t^{MS}, ϵ_t^{MD} and ϵ_t^{IS} denote a money supply shock, a money demand shock and an IS shock, respectively.[13]

The identification of the structural shocks are again based on short-run and long-run restrictions. We retain the Blanchard-Quah restriction and assume that the three aggregate demand disturbances have no permanent effect on the level of output. Further, we follow Gali and adopt two additional assumptions. The first is that neither money demand nor money supply affect output contemporaneously ($a_{12}^0 = a_{13}^0 = 0$). The second assumption is that contemporaneous prices do not enter the money supply rule which implies that $b_{23}^0 + b_{24}^0 = 0$. Our estimate of the sacrifice ratio uses money supply shocks to identify changes in the stance of monetary policy.

While equation (3) can be used to derive an estimate of the sacrifice ratio, there is some uncertainty about the true value. This consideration would normally suggest that we compute and

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exogenous regressor in their SVAR system.

[13]Although the inflation rate does not appear as an individual variable in the Gali (1992) model, we can recover its impulse response function from the estimated impulse response functions for Δi_t and $(i_t - \pi_t)$

report a standard error to determine the imprecision attached to the estimate. In the case of the sacrifice ratio, however, the calculation of a standard error is difficult, as $S_{\epsilon^{\pi}}(\tau)$ is a function of the parameters in the structural VMA (the a_{12}^{i}'s and a_{22}^{i}'s). Moreover, while the application of the *Delta method* might appear to offer a solution to this problem, this technique would be computationally demanding because of complexities associated with the nature of the impulse response functions in equation (2).[14]

As an alternative approach for gauging the reliability of the sacrifice ratio estimates, we employ *Monte Carlo* methods. In using these techniques, our goal is to approximate the exact sampling distribution of the estimator of the sacrifice ratio in equation (3). The approximation is derived using a bootstrap method and involves a large number of replications. The procedure can be briefly described as follows.

Let θ denote the vector of parameters that constitute the reduced form VAR model and let $\hat{\theta}$ and $\{\hat{\mu}_1,\ \hat{\mu}_2,\ \ldots\ \hat{\mu}_T\}$ denote, respectively, the estimated parameter vector and residuals of the reduced form VAR using the observed set of data.[15] The estimated parameter vector $\hat{\theta}$ along with the estimated residuals $\{\hat{\mu}_1,\ \hat{\mu}_2,\ \ldots\ \hat{\mu}_T\}$ can be used to generate a full sample of artificial

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[14]The Delta method would require an estimate of the variance-covariance matrix of the parameters in (2) as well as the calculation of the first-derivatives of the function [equation (3)] with respect to the estimated parameter vector. However, obtaining estimates of the variance-covariance matrix of the impulse response functions is an extremely difficult task. While there are methods that allow for such a calculation in the case of the two-variable system, the three- and four-variable systems are too complex for these methods to be feasible. Therefore, we adopt a considerably easier and more tractable approach for assessing the precision of the sacrifice ratio estimates. In addition, consistency of the estimates produced by the Delta method relies on the assumption of normality which is a condition that is likely violated in the sample.

[15]The estimated residuals $\hat{\mu}_t$ denote the (n x 1) vector of innovations to the reduced form VAR at time t.

data.[16] A reduced from VAR can then be estimated for the simulated data and the estimated parameter vector $\hat{\theta}_1^*$ in conjunction with the identification restrictions can be used to produce an estimate $S_1(\hat{\theta}_1^*)$ of the sacrifice ratio $S_{\epsilon^\pi}(\tau)$. The process can be repeated and a second full sample of artificial data can be generated and used to produce a second estimate $S_2(\hat{\theta}_2^*)$. In this manner, a series of N simulations can be undertaken and used to construct N estimates of the sacrifice ratio denoted by $S_1(\hat{\theta}_1^*)$, $S_2(\hat{\theta}_2^*), \ldots,$ $S_N(\hat{\theta}_N^*)$. We can then construct confidence intervals for $S_{\epsilon^\pi}(\tau)$ based on the range that includes the specified percent of the values for $S_i(\hat{\theta}_i)$.

Monte Carlo methods can also provide insights into the presence of bias in the point estimates. Specifically, we can use information from the simulations to report bias-corrected point estimates of the sacrifice ratio using the following formula:

(6) $$\tilde{S}_{\epsilon^\pi}(\tau) = \hat{S}_{\epsilon^\pi}(\tau) - [\bar{S}_{\epsilon^\pi}(\tau) - \hat{S}_{\epsilon^\pi}(\tau)]$$

where $\tilde{S}_{\epsilon^\pi}(\tau)$ is the bias-adjusted point estimate of the sacrifice ratio, $\hat{S}_{\epsilon^\pi}(\tau)$ is the point estimate from estimation of the SVAR model, and $\bar{S}_{\epsilon^\pi}(\tau)$ is the mean of the Monte Carlo draws. The term in brackets measures the estimated bias.

The analysis now turns to a presentation of the results and a discussion of the point

[16]The individual observations for an artificial sample are generated by randomly drawing a single realization from $\{\hat{\mu}_1,\ \hat{\mu}_2,\ \ldots\ \hat{\mu}_T\}$ and adding the realization to the estimated equations in the reduced form VAR based on the historical data. The full sample of artificial data is then constructed by repeating this process T times, where it is assumed that the sampling of the estimated innovations takes place with replacement. Because of the presence of lagged terms in the VAR system, the presample values of the variables are set equal to those actually observed in the data.

estimates and their associated confidence intervals.[17]

IV. Empirical Results

We construct estimates of the sacrifice ratio from the three structural VAR models using quarterly data over the sample period 1959:Q1-1997:Q4. Output is measured by real GDP and inflation by the percentage change in the consumer price index (CPI). The short-term interest rate represents the yield on three-month Treasury bills and the monetary aggregate is measured by M1. The Data Appendix describes the data in further detail.

Estimation of the sacrifice ratio also requires the selection of horizons for the long-run restriction on aggregate demand shocks and to calculate the dynamic response of output and inflation to monetary policy shocks. Following Cecchetti (1994), we assume that aggregate demand shocks completely die out after twenty years and compute estimates of the sacrifice ratio based on the response of output and inflation occurring five years after a shift in monetary policy. That is, we truncate the structural VMA representations at eighty quarters and set τ equal to twenty quarters.

It is also worth noting that preliminary analysis of the data provided support for the model specifications. In particular, we examined the stationarity properties of the various series in order to determine the appropriate differencing of the data. The results from the application of Dickey-Fuller (1979) unit root tests provided evidence that the *ex post* real interest rate and the growth

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[17] We restrict our attention to nonrecursive structural VAR models in this study. This consideration is principally based on the observation that small recursive systems can produce anomalous responses on the part of variables to a monetary policy shock. The usual practice for resolving this outcome is to include additional variables. However, this approach would limit our ability to study systems characterized by a small number of structural shocks and to analyze the sensitivity of the results to various decompositions of the aggregate demand shock.

rate of real money balances are stationary variables, but that both output and inflation contain a unit root. These latter findings not only offer support for some of our key identifying restrictions, but are also consistent with the concept of a sacrifice ratio.[18]

Table 1 presents the point estimates of the sacrifice ratio and **Figure 3** displays the estimated responses of output and inflation to a one-unit monetary policy shock across the three models. The point estimates should be interpreted as the cumulative output loss associated with a permanent one-percentage point decline in the rate of inflation measured on an annual basis.[19]

The Cecchetti and Shapiro-Watson models yield almost identical estimates for the sacrifice ratio of 1.38 and 1.28, respectively. Interestingly, these estimates are also very similar to the values calculated by Ball (1994) and Schelde-Andersen (1992) for the Volcker disinflation.[20] In contrast, the Gali model generates a markedly higher estimate of 9.87 which is much closer to the mean value obtained by Okun (1978).

An examination of the impulse response functions in **Figure 3** reveals a pattern that is qualitatively similar across models and accords with the predicted effects of a monetary tightening. The typical path of output after a monetary policy shock is a decline followed by an

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[18]The presence of a unit root in the output process allows the long-run restriction on the effects of aggregate demand shocks to be well-defined and meaningful. As previously discussed, the presence of a unit root in the inflation process allows for permanent shifts in its level.

[19]The lag length of the reduced form VAR was set equal to eight for the Cecchetti model and equal to four for the Shapiro-Watson and Gali models. The estimates of the sacrifice ratio are based on a different transformation of the data for output and prices. Output growth is measured at a quarterly rate in percentage terms, while inflation is measured at an annual rate in percentage terms.

[20]Ball (1994) and Schelde-Andersen (1992) obtain estimates of 1.8 and 1.4, respectively.

eventual return to its initial level.[21] Inflation also decreases in response to the monetary policy shock. Unlike the case for output, inflation does not return to its initial level in the long-run and instead displays a permanent decline on the order of 0.5 - 1.0 percent.

There are some differences in the magnitude of the response of inflation and output across the models which appear to explain the higher sacrifice ratio estimate from the Gali model. Relative to the Cecchetti and Shapiro-Watson models, the Gali model suggests a deeper and more protracted output decline along with a smaller long-run decrease in inflation. Both of these effects lead to a higher estimate for the sacrifice ratio.

Although the point estimates in **Table 1** are consistent with those reported in previous studies, we also need to consider and address the issue of their precision. **Table 2** presents the results from simulating the distribution of the estimated sacrifice ratio using the bootstrap procedure outlined in the previous section. The simulated distributions are based on 10000 replications and the corresponding density functions are depicted in **Figures 4-6**.[22]

There are two striking results that emerge from **Table 2** and **Figures 4-6**. The first is that expanding the two-variable system to identify separate components of the aggregate demand shock leads to highly variable estimates of the sacrifice ratio. In particular, the range of the confidence intervals for the Shapiro-Watson and Gali models indicates that the results are extremely sensitive to the measure of monetary policy shocks. Second, a 90% confidence interval for the estimated sacrifice ratio includes zero for all three models. That is, we cannot with any

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[21]The nature of the identification restrictions in the Shapiro-Watson and Gali models precludes a contemporaneous response of output to the monetary policy shock.

[22]For purposes of presentation, the frequency density functions only include the observations falling in the 2.5%-97.5% fractile range.

reasonable degree of certainty rule out the possibility that $S_{\epsilon^{\pi}}(\tau)=0$. Taken together, these findings would suggest that we have little understanding about the quantitative impact of monetary policy on output and inflation.

A closer examination of the results from the Monte Carlo simulations offers further insights into the properties of the structural estimates of the sacrifice ratio. For the Cecchetti model, the simulated distribution of the sacrifice ratio estimates is nonnormal, although it is reasonably symmetric. In addition, the mean of the bootstrapped estimate of the sacrifice ratio is almost identical to the original estimate providing little evidence of bias. While the 90% confidence interval covers zero, an 80% confidence interval yields a positive range for the sacrifice ratio of 0.09 to 2.75.

For the Shapiro-Watson and Gali models, the simulated distributions of the sacrifice ratio estimates reveal a markedly different picture. The distributions are nonnormal, asymmetric and extremely long tailed. Further, there is now increased evidence of bias in the point estimates, particularly in the case of the Gali model.[23] More dramatic, however, are the 90% confidence intervals which yield ranges for the sacrifice ratio that are highly implausible and speak directly to the unreliable nature of the point estimates.

A complete exploration into the source(s) of the imprecision of the sacrifice ratio estimates is beyond the scope of this paper. However, it is possible to gain some insight into this issue. It is worth noting from the Technical Appendix that the sacrifice ratio is a function of the structural impulse responses which themselves depend on estimates of two quantities. The first is

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[23]The bias-corrected point estimates of the sacrifice ratio for the Shapiro-Watson and Gali models are 3.885 and 15.691, respectively.

the matrix A_0 which measures the contemporaneous effects of structural shocks on the variables of the system. The second is the lag polynomial matrix C(L) which provides the impulse response functions of the reduced from VAR.

Pagan and Robertson (1998) have recently shown that SVAR models can be cast in a generalized method of moments (GMM) framework and that the restrictions used to identify the structural shocks generate instruments for the estimation of A_0. In addition, they document substantial randomness in SVAR estimates of a liquidity effect and provide evidence that the poor quality of instruments used in estimation is an important source of the imprecision.

The results of Pagan and Robertson (1998) suggest that instrument quality may be a relevant consideration for our results. Because the formula in equation (3) involves sums and ratios of structural impulse response functions, the use of weak or invalid instruments would lead to imprecision in the estimate of A_0 that would likely be exacerbated in the calculation of the sacrifice ratio. This would be in addition to any randomness associated with estimates of the impulse response functions of the reduced form VAR which are already known to be characterized by considerable variability.[24]

V. Conclusion

This study examines the output cost of disinflation for the U.S. Across various models, the estimates of the sacrifice ratio imply that a permanent one percentage point reduction in inflation

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[24]We do not attempt to provide a detailed investigation into the relative contributions of A_0 and C(L) to the variability of the sacrifice ratio estimates. This is beyond the scope of the paper and would be an extremely difficult task owing to the sacrifice ratio being a complicated function of the structural impulse response functions which are themselves a product of the random elements A_0 and C(L).

entails a loss of 1.3-10 percent of a year's real GDP. The confidence intervals around the point estimates, however, indicate that none of the point estimates differs from zero at conventional levels of statistical significance. Further, the high degree of imprecision associated with the estimates suggests that our knowledge about the actual impact of monetary policy on the behavior of the economy is quite limited.

Because the models examined in Pagan and Robertson (1998) and in this study share similar identification schemes, future research may need to be particularly conscious of the issue of instrument quality. For example, the evidence in **Figures 5-6** is entirely consistent with the idea that the identifying restrictions used in estimation may be tenuous and generate weak or invalid instruments. This interpretation would be particularly important for the results from the Shapiro-Watson model. Specifically, the assumption that money does not have a contemporaneous effect on output is a commonly adopted restriction used to identify monetary policy shocks in a variety of structural VAR models.[25] If this restriction is problematic, then it would appear that current empirical methodologies may be extremely limited in their ability to identify robust structural relationships.[26] Thus, while a better understanding of the true costs of disinflation would be of particular interest and importance to policymakers, we are skeptical that current data and econometric techniques can provide a meaningful set of estimates.

[25]This includes both nonrecursive and recursive structural VAR models. It is also important to note that the imprecision of the sacrifice ratio estimates in the Shapiro-Watson model is not due to a reduction in the degrees of freedom. Because of different lag lengths, the degrees of freedom in the Cecchetti and Shapiro-Watson models are approximately equal.

[26]Alternative identification schemes are unlikely to change this outcome. Unlike standard situations which allow for the application of instrumental variables procedures, the instrument set available for the estimation of SVAR models is very restricted. This lack of valid instruments may impose severe restrictions on the nature of the structural shocks that can be identified.

Table 1

Estimated Sacrifice Ratio for the United States: 1959:Q1 - 1997:Q4

Cumulative 5-Year Output Loss as a Percentage of Real GDP	
Model	$S_{\epsilon^{\pi}}(\tau)$
Cecchetti Two-Variable System	1.376
Shapiro-Watson Three-Variable System	1.277
Gali Four-Variable System	9.871

Table 2

Simulated Distribution of $S_{\epsilon^{\pi}}(\tau)$: 1959:Q1 - 1997:Q4

Monte Carlo Experiment Based on 10,000 Replications				
Model	Mean	Median	90% Confidence Interval	Other Statistics
Cecchetti Two-Variable System	1.395	1.380	(-0.400 , 3.240)	Sk=0.12 Ku=2.95
Shapiro-Watson Three-Variable System	-1.331	-1.800	(-36.294 , 36.869)	Sk=-6.28 Ku=3150
Gali Four-Variable System	4.051	6.010	(-48.983 , 67.621)	Sk=-24.2 Ku=1831

The terms Sk and Ku denote the skewness and kurtosis of the simulated density functions of the sacrifice ratio for each model.

TECHNICAL APPENDIX

The unrestricted Vector Autoregression (VAR) representation (excluding deterministic variables) is given by:

(A.1) $$X_t - D_1 X_{t-1} - D_2 X_{t-2} - \ldots - D_k X_{t-k} = D(L)X_t = \mu_t, \; E[\mu_t \mu_t^{\prime}] = \Sigma \;\forall t$$

where X_t is an (n x 1) vector of endogenous variables, D(L) is a *k*th-order lag polynomial matrix, E denotes the unconditional expectations operator, and μ_t is the (n x 1) vector of innovations to the system whose variance-covariance matrix is given by Σ .

Equation (A.1) can be estimated and inverted to yield its unrestricted vector moving-average (VMA) representation:

(A.2) $$X_t = \mu_t + C_1 \mu_{t-1} + C_2 \mu_{t-2} + \ldots = C(L)\mu_t$$

The structural VAR (SVAR) model provides a representation where the endogenous variables are expressed in terms of underlying economic shocks. In particular, the analogues of equations (A.1) and (A.2) for the structural system can be written as:

(A.3) $$B_0 X_t = B_1 X_{t-1} + B_2 X_{t-2} + \ldots + B_k X_{t-k} + \epsilon_t, \; E[\epsilon_t \epsilon_t^{\prime}] = \Omega \;\forall t$$

(A.4) $$X_t = A_0 \epsilon_t + A_1 \epsilon_{t-1} + \ldots = A(L)\epsilon_t$$

Equations (A.2) and (A.4) imply that:

(A.5) $$E[(A_0 \epsilon)(A_0 \epsilon)^{\prime}] = A_0 \Omega A_0^{\prime} = \Sigma = E[(\mu)(\mu^{\prime})]$$

and

$$A(L) = C(L)A_0. \tag{A.6}$$

The ability to link the unrestricted VAR to a structural VAR model hinges crucially on the estimation of the matrix A_0. Because A_0 has (n x n) unique elements, complete identification requires a total of n^2 restrictions. This is a necessary but not sufficient condition for identification, as sufficiency requires the matrix A_0 to be invertible.

In addition to assuming that Ω is the identity matrix, there are three other sets of identifying restrictions that are employed in the estimation of A_0. Two of these sets focus on the effects of the structural shocks on particular variables and involve short-run restrictions [A_0 restrictions] or long-run restrictions [A(1) restrictions]. The other set involves restrictions on the coefficients of contemporaneous variables in specific structural equations [B_0 restrictions].

DATA APPENDIX

This Appendix discusses the data used in the estimation. All data are quarterly. The estimates are conducted over the sample period 1959:Q1-1997:Q4.

Output:

The output series *GDPH* is measured as real gross domestic product in chain-weighted 1992 dollars.

Prices:

The price data are a quarterly average of the monthly consumer price series *PCU* for all urban consumers.

Interest Rates:

The interest rate data are a quarterly average of the monthly series *FTBS3* which represent the yield on three-month Treasury bills.

Money Stock:

The data on the money stock are for M1 and are a quarterly average of the series *FM1*. The measures of M1 prior to 1959 are taken from the *Federal Reserve Bulletin*.

References

Ball, Laurence M. 1994. "What determines the sacrifice ratio?" In *Monetary Policy*, ed. N. Gregory Mankiw. Chicago: University of Chicago Press.

Barro, Robert J. 1996. "Inflation and Growth." Federal Reserve Bank of St. Louis. *Economic Review* (May/June).

Blanchard, Olivier J. and Danny Quah. 1989. "The dynamic effects of aggregate demand and supply disturbances." *American Economic Review* 79 (September): 655-73.

Cecchetti, Stephen G. 1994. "Comment." In *Monetary Policy*, ed. N. Gregory Mankiw. Chicago: University of Chicago Press.

Dickey, David A. and Wayne A. Fuller. 1981. "Likelihood ratio statistics for autoregressive time series with a unit root." *Econometrica* 49 (November): 1057-72.

Feldstein, Martin S. 1999. ed. *The Costs and Benefits of Achieving Price Stability*. Chicago: University of Chicago Press, forthcoming.

Filardo, Andrew J. 1998. "New evidence on the output cost of fighting inflation." Federal Reserve Bank of Kansas City. *Economic Review* (Third Quarter).

Fuhrer, Jeffrey C. 1995. "The persistence of inflation and the cost of disinflation." Federal Reserve Bank of Boston. *New England Economic Review* (January/February).

Gali, Jordi. 1992. "How well does the IS-LM model fit postwar U.S. data?" *Quarterly Journal of Economics* 107 (May): 709-38.

Gordon, Robert J., and Stephen R. King. 1982. "The output cost of disinflation in traditional and vector autoregressive models." *Bookings Papers on Economic Activity* 1:205-42.

Mankiw, N. Gregory. 1991. *Macroeconomics*. New York: Worth Publishers.

Okun, Arthur M. 1978. "Efficient disinflationary policies." *American Economic Review* 68 (May): 348-52.

Pagan Adrian R. and John C. Robertson. 1998. "Structural models of the liquidity effect." *Review of Economics and Statistics* 80 (May): 202-17.

Sargent, Thomas J. 1983. "Stopping moderate inflations: The methods of Poincare and Thatcher." In *Inflation, debt, and indexation,* eds. Rudiger Dornbusch and Mario H. Simonsen. Cambridge: M.I.T. Press.

Schelde-Andersen, Palle. 1992. “OECD country experiences with disinflation.” In *Inflation, disinflation, and monetary policy*, ed. Adrian Blundell-Wignell. Reserve Bank of Australia: Ambassador Press.

Shapiro, Matthew D. and Mark W. Watson. 1988. “Sources of business cycle fluctuations.” In *NBER Macroeconomics Annual*, eds. Olivier J. Blanchard and Stanley Fischer. Cambridge: M.I.T. Press.

Taylor, John B. 1983. “Union wage settlements during a disinflation.” *American Economic Review* 73 (December): 981-93.

Figure 1A: Quarterly Growth Rate of Output

Real GDP: 1959:Q1 - 1997:Q4

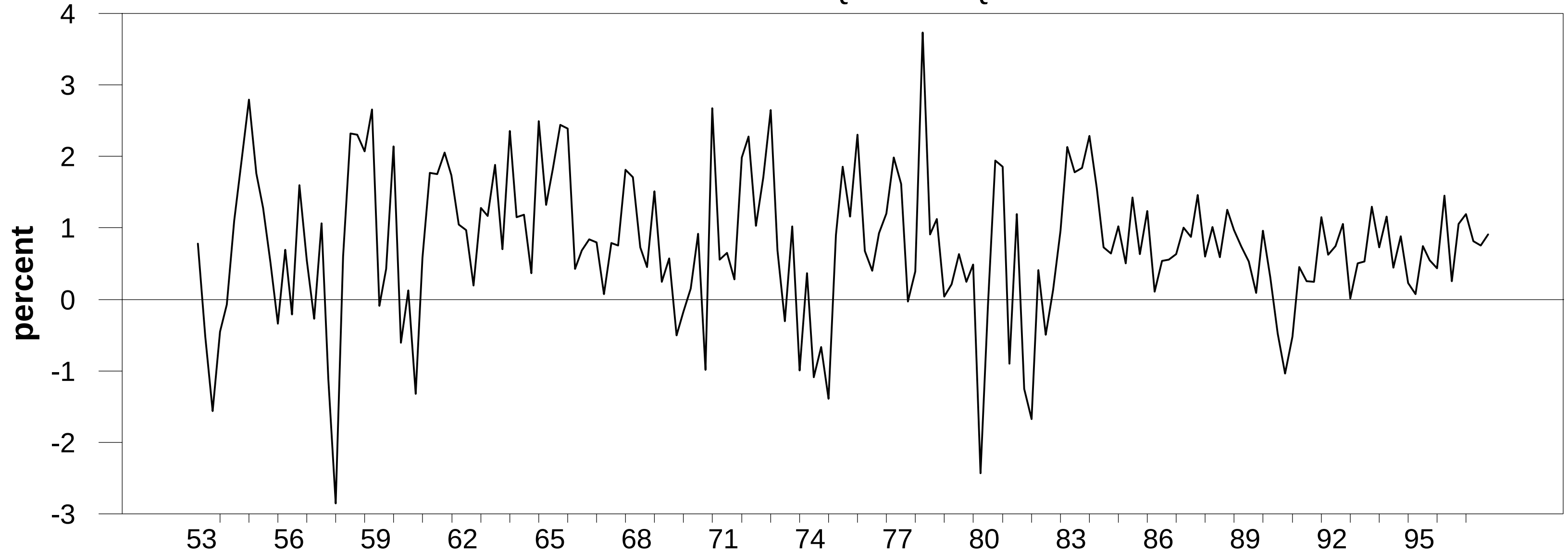

Figure 1B: Quarterly Growth Rate of Prices

CPI Inflation: 1959:Q1 - 1997:Q4

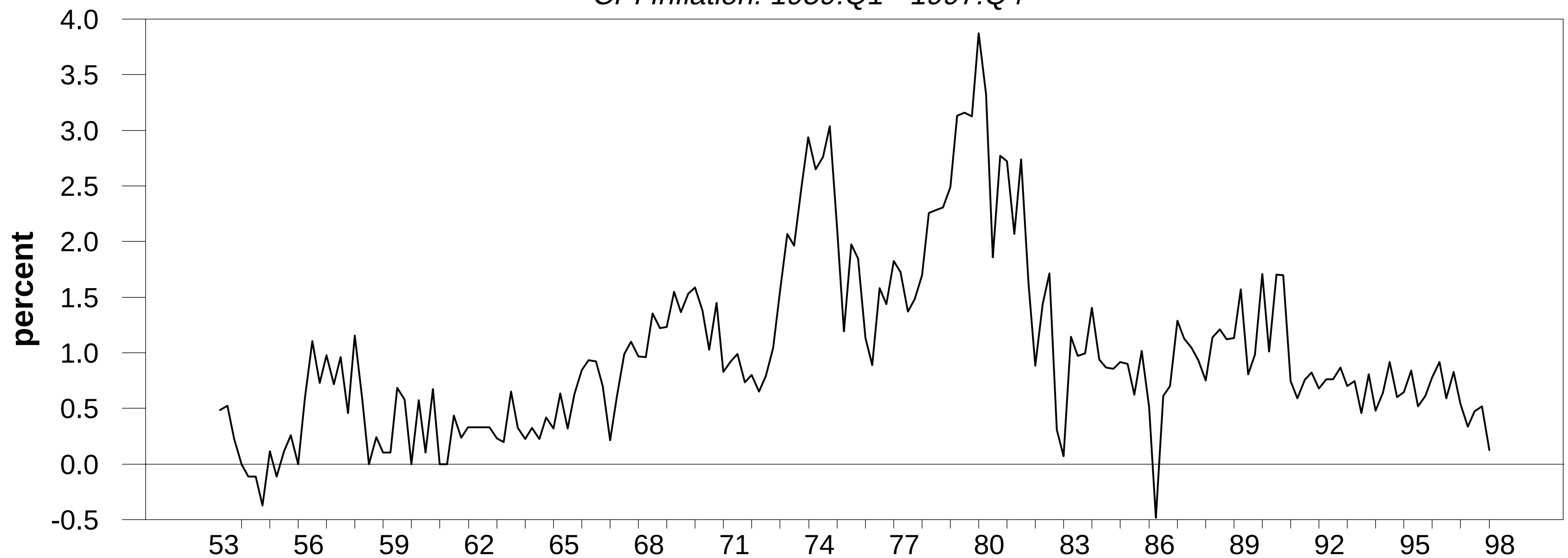

Figure 2

Calculation of the Sacrifice Ratio: Hypothetical Responses of Output and Inflation to a Contractionary Monetary Policy Shock

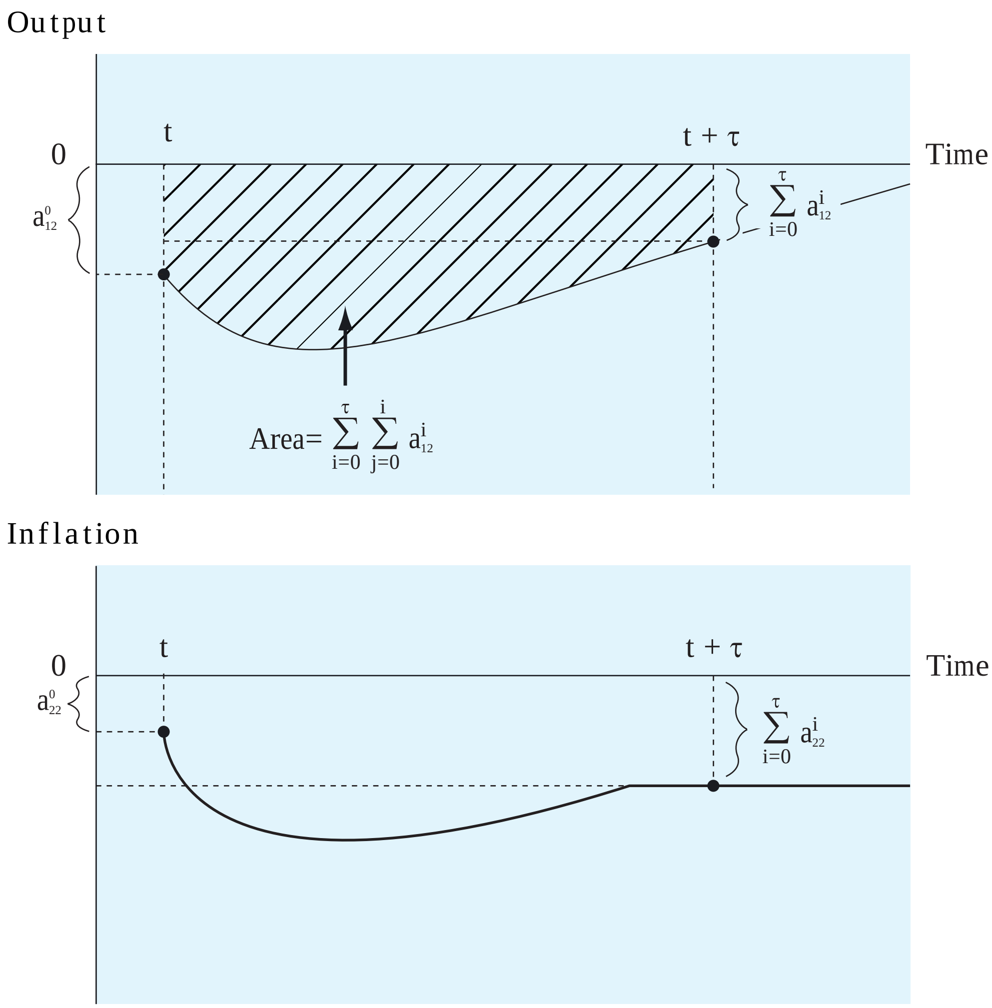

Figure 3A: Dynamic Response to a Monetary Policy Shock

Real GDP - Cecchetti

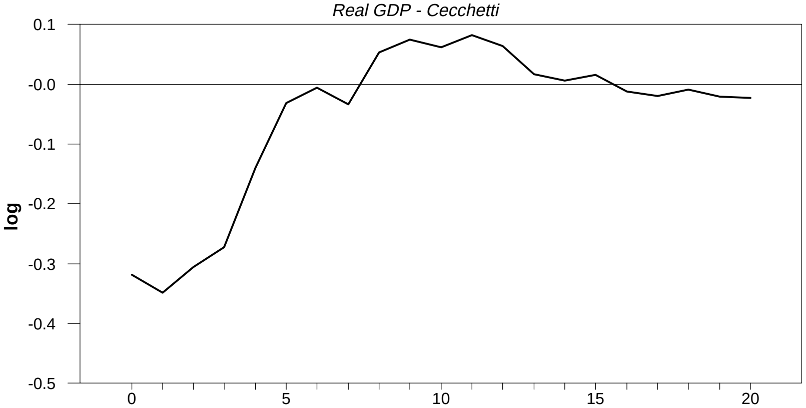

Figure 3B: Dynamic Response to a Monetary Policy Shock

Inflation - Cecchetti

Figure 3C: Dynamic Response to a Monetary Policy Shock

Real GDP - Shapiro-Watson

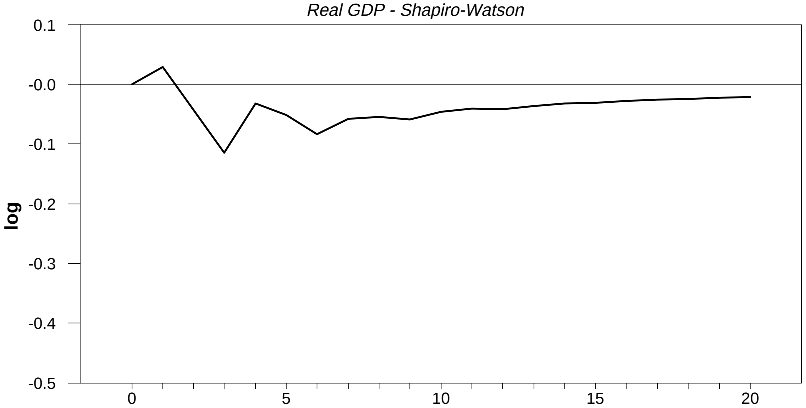

Figure 3D: Dynamic Response to a Monetary Policy Shock

Inflation - Shapiro-Watson

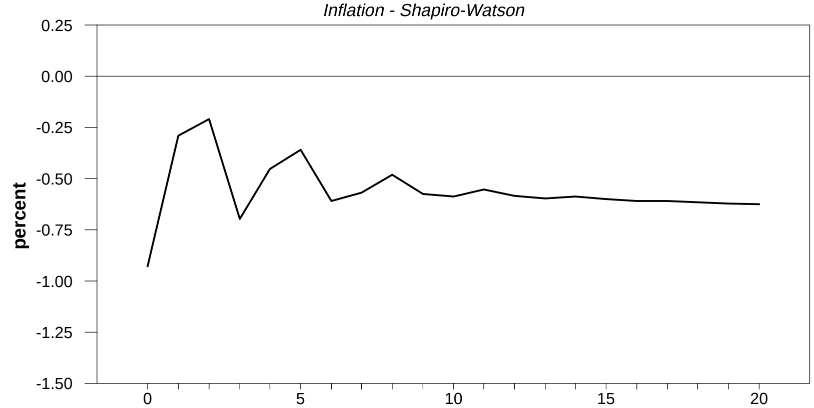

Figure 3E: Dynamic Response to a Monetary Policy Shock

Real GDP - Gali

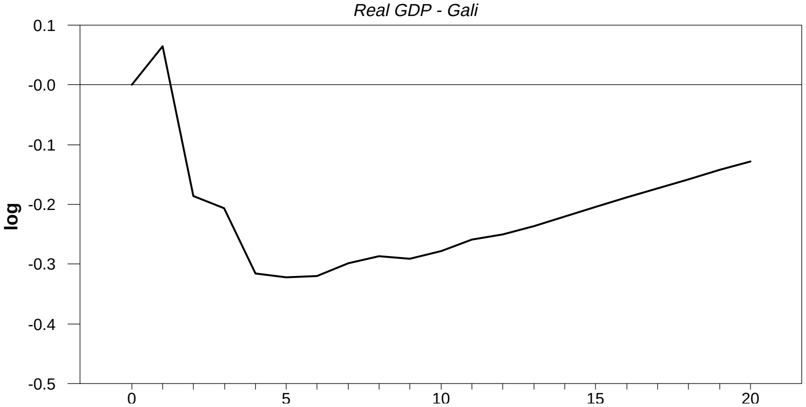

Figure 3F: Dynamic Response to a Monetary Policy Shock

Inflation - Gali

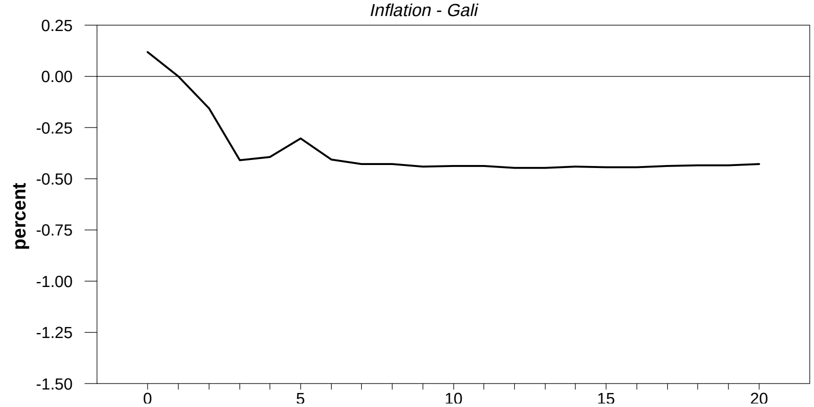

Figure 4: Frequency Distribution for the Cecchetti Model
2.5%-97.5% Fractile Range

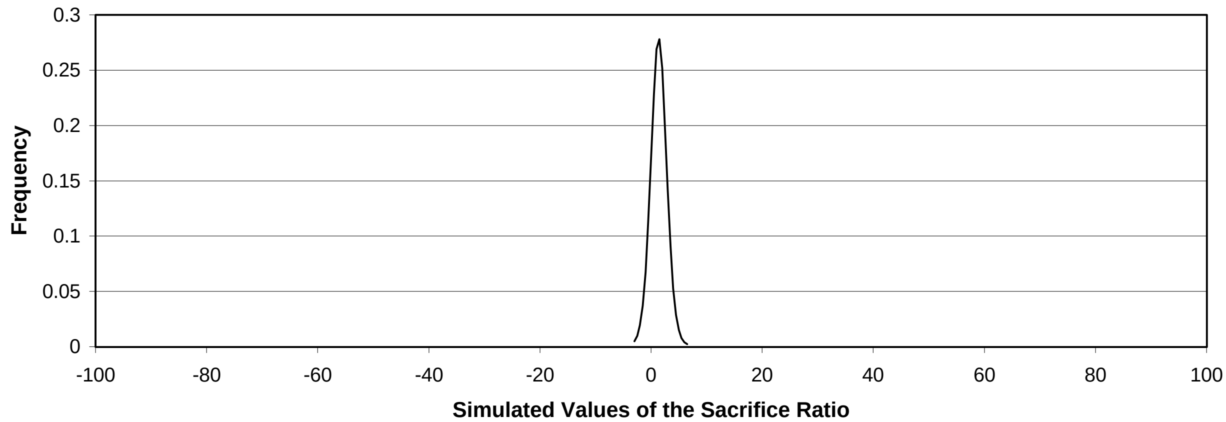

Figure 5: Frequency Distribution for the Shapiro-Watson Model
2.5%-97.5% Fractile Range

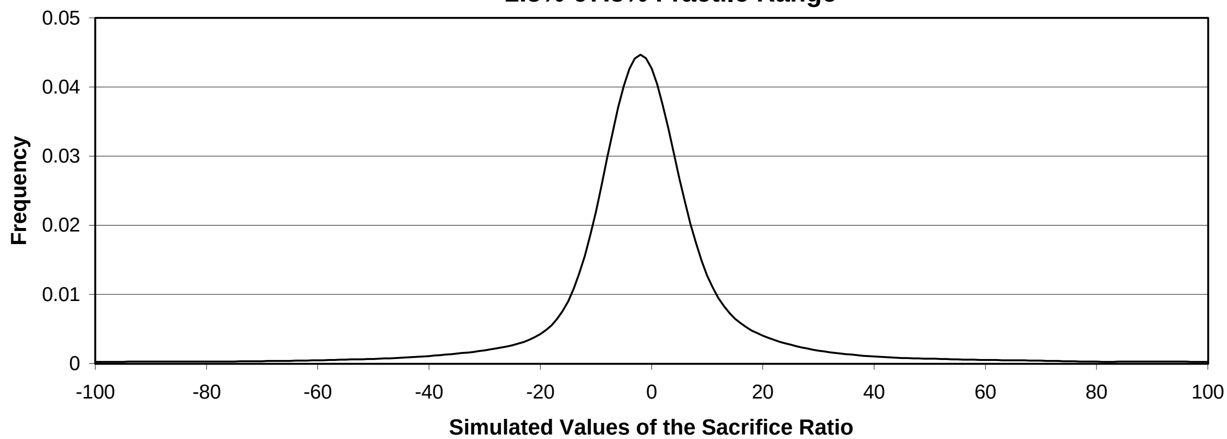

Figure 6: Frequency Distribution for the Gali Model
2.5%-97.5% Fractile Range