Risk and Liquidity in a System Context

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Pricing claims in a system context

Some assets (e.g. loans) are claims against other parties

Value of my claim against $A$ depends on value of $A$’s claims against $B, C$, etc.

But $B$ or $C$ may have claim against me.

Balance sheet strength, spreads, asset prices fluctuate together

Equity value of financial system as a whole is value of “fundamental assets”
“While many believe that irresponsible borrowing is creating a bubble in housing, this is not necessarily true. At the end of 2004, U.S. households owned $17.2 trillion in housing assets, an increase of 18.1% (or $2.6 trillion) from the third quarter of 2003. Over the same five quarters, mortgage debt (including home equity lines) rose $1.1 trillion to $7.5 trillion. The result: a $1.5 trillion increase in net housing equity over the past 15 months.”

Value of fundamental assets is tide that lifts all boats

Housing ⇒ mortgages ⇒ CDOs ⇒ claims against CDO holders . . .
Example of Housing

Property Price

\( v' \)

\( v \)

property stock held by young

Supply of property from old
Balance Sheet Approach

Financial system is a network of interlinked balance sheets

Everything is marked to market

Risk-neutrality in pricing

• no role for risk aversion, but spreads fluctuate due to fluctuations in fundamental asset price

• fluctuations in risk appetite arising from solvency constraints
Leverage over the Financial Cycle

From Adrian and Shin (06)

Asset and Liability growth of Bank Holding Companies

1975q1 1980q1 1985q1 1990q1 1995q1 2000q1 2005q1

- Asset Growth
- Liability Growth
Leverage over the Financial Cycle: Investment Banks

Asset and Liability growth of Investment Banks

1975q1  1980q1  1985q1  1990q1  1995q1  2000q1  2005q1
IB Asset Growth  IB Liability Growth
Framework

- $n$ entities in financial system
- risky endowments realized at date $T$ with means $\{w_i\}$
- single fundamental asset, price $v$
- zero coupon debt of $i$ with face value $\bar{x}_i$ payable at $T$
- risk-free interest rate is zero
Balance Sheets

$x_i$ is market value of $i$’s debt

$a_i$ is market value of $i$’s assets

$e_i$ is market value of $i$’s equity

\[ a_i = e_i + x_i \]

If $i$ holds proportion $\pi_{ji}$ of $j$’s debt,

\[ a_i = w_i + vy_i + \sum_j \pi_{ji}x_j \]
Lemma 1. There exist functions $\{f_i\}$ such that

$$x_i = f_i(a_i, \theta)$$

\[ (1) \]
where each $f_i$ is non-decreasing in $a_i$, and is bounded above by $\bar{x}_i$ and

$$\theta = (v, w, \bar{x})$$

**Lemma 2.** The market value of equity is non-decreasing in $a_i$. That is, the function $e_i$ defined as

$$e_i \equiv a_i - f_i(a_i, \theta)$$  \hspace{1cm} (2)

is non-decreasing in $a_i$. 

System

\[ x_1 = f_1(a_1(x), \theta) \]
\[ x_2 = f_2(a_2(x), \theta) \]
\[ \vdots \]
\[ x_n = f_n(a_n(x), \theta) \]

where \( x = (x_1, x_2, \cdots, x_n) \).

Solve for fixed point \( x \) in:

\[ x = F(x, \theta) \]
Unique solution

**Theorem 3.** There is a unique profile of debt prices \( x(\theta) \) that solves
\[
x = F(x, \theta).
\]

**Theorem 4.** \( x(\theta) \) is increasing in \( \theta \).

Result follows from

(i) Tarski’s fixed point theorem

(ii) fact that \( \{f_i\} \) are contraction mappings
**Argument for Uniqueness**

Suppose there are distinct solutions $x, x'$. 

By Tarski, $x \leq x'$ and $x_i < x'_i$ for some $i$

Equity value of the system under $x$ is strictly lower than under $x'$

Equity value of the system is value of fundamental assets

Contradiction.
Solvency Constraints

Value of all assets and liabilities determined by

\[ \theta = (v, w, \bar{x}) \]

Constraints on equity/debt ratio

\[ \frac{a_i - x_i}{x_i} \geq r^* \]

Spreads

\[ \frac{\bar{x}_i}{x_i} - 1 \]
Sale $s_i$ to restore solvency

\[
\frac{w_i + v(y_i - s_i) + b_i - (x_i^0 - v s_i)}{x_i^0 - v s_i} \geq r^* 
\]  

(3)

\[
s_i = \min \left\{ y_i, \max \left\{ 0, \frac{(1 + r^*) x_i^0 - w_i - vy_i - b_i}{r^* v} \right\} \right\} 
\]  

(4)
Asking the Right Questions

• What role for mark-to-market accounting?

• “Domino” channel versus price mediated channel

• Large players versus small players

• Correlations in downturns

• “Risk appetite”