Risk and Liquidity in a System Context

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Pricing claims in a system context

Some assets (e.g. loans) are claims against other parties

Value of my claim against A depends on value of A's claims against $B, C,$ etc.

But B or C may have claim against me.

Balance sheet strength, spreads, asset prices fluctuate together

Equity value of financial system as a whole is value of "fundamental assets"

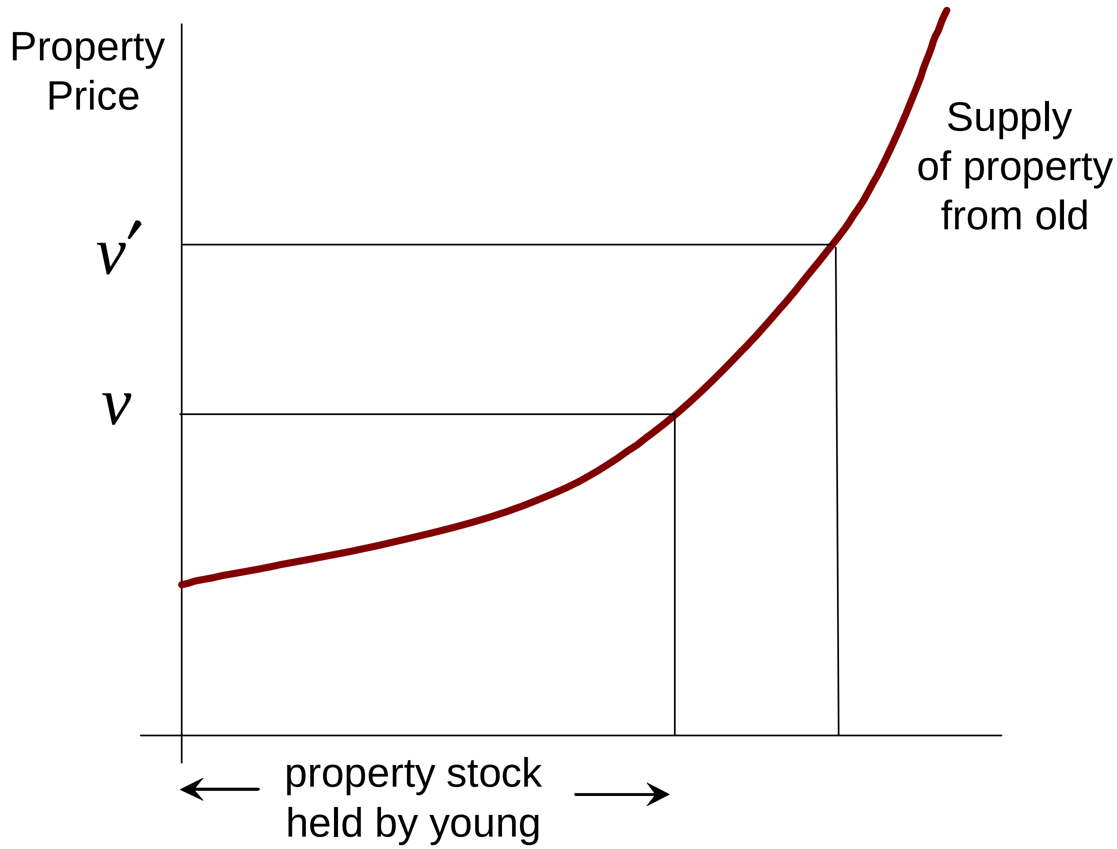
Marking to market

"While many believe that irresponsible borrowing is creating a bubble in housing, this is not necessarily true. At the end of 2004, U.S. households owned \$17.2 trillion in housing assets, an increase of 18.1% (or \$2.6 trillion) from the third quarter of 2003. Over the same five quarters, mortgage debt (including home equity lines) rose \$1.1 trillion to \$7.5 trillion. The result: a \$1.5 trillion increase in net housing equity over the past 15 months."

Value of fundamental assets is tide that lifts all boats

Housing $\Rightarrow$ mortgages $\Rightarrow$ CDOs $\Rightarrow$ claims against CDO holders . . .

Example of Housing



Balance Sheet Approach

Financial system is a network of interlinked balance sheets

Everything is marked to market

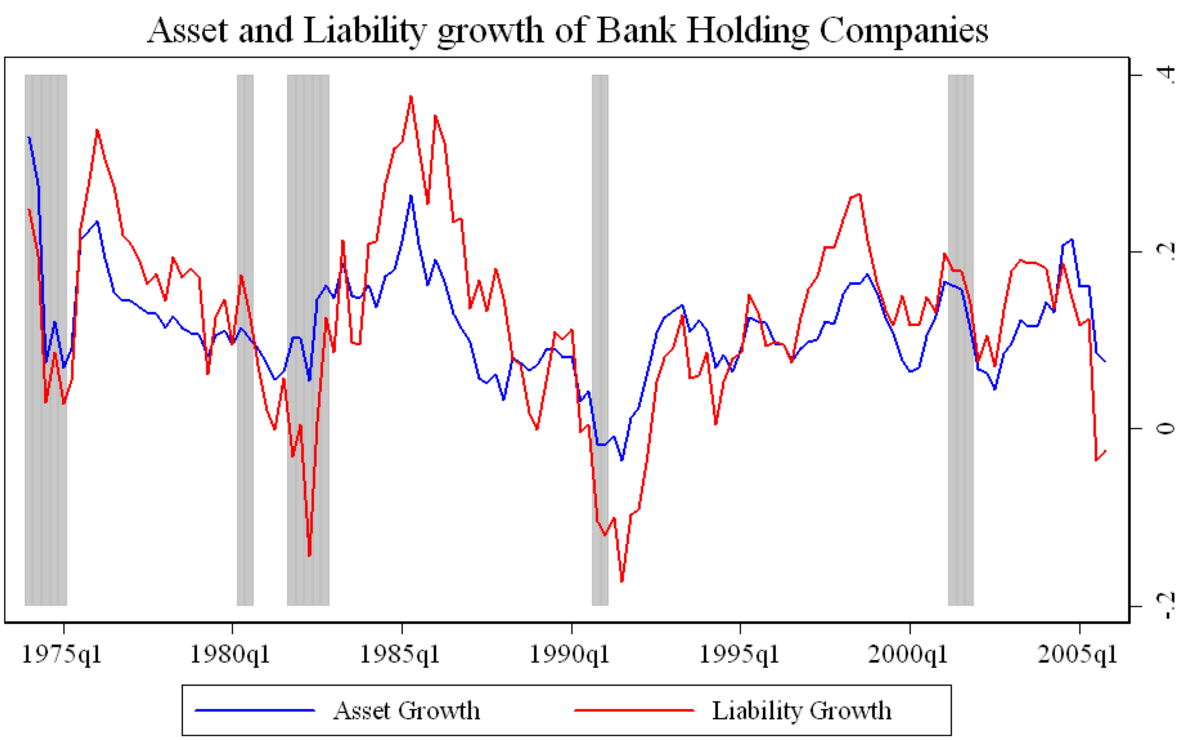
Risk-neutrality in pricing

- no role for risk aversion, but spreads fluctuate due to fluctuations in fundamental asset price

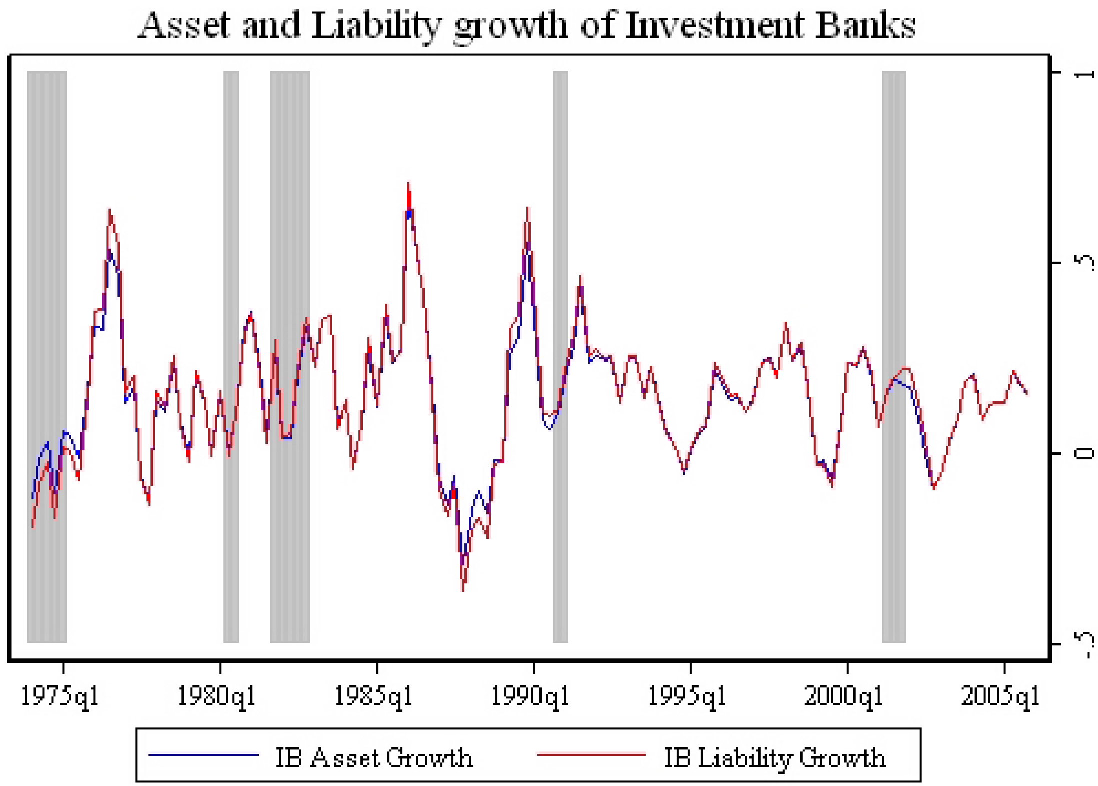
- fluctuations in *risk appetite* arising from solvency constraints

Leverage over the Financial Cycle

From Adrian and Shin (06)



Leverage over the Financial Cycle: Investment Banks



Framework

- n entities in financial system
- risky endowments realized at date T with means $\{w_i\}$
- single fundamental asset, price v
- zero coupon debt of i with face value $\bar{x}_i$ payable at T
- risk-free interest rate is zero

Balance Sheets

x_i is market value of i's debt

a_i is market value of i's assets

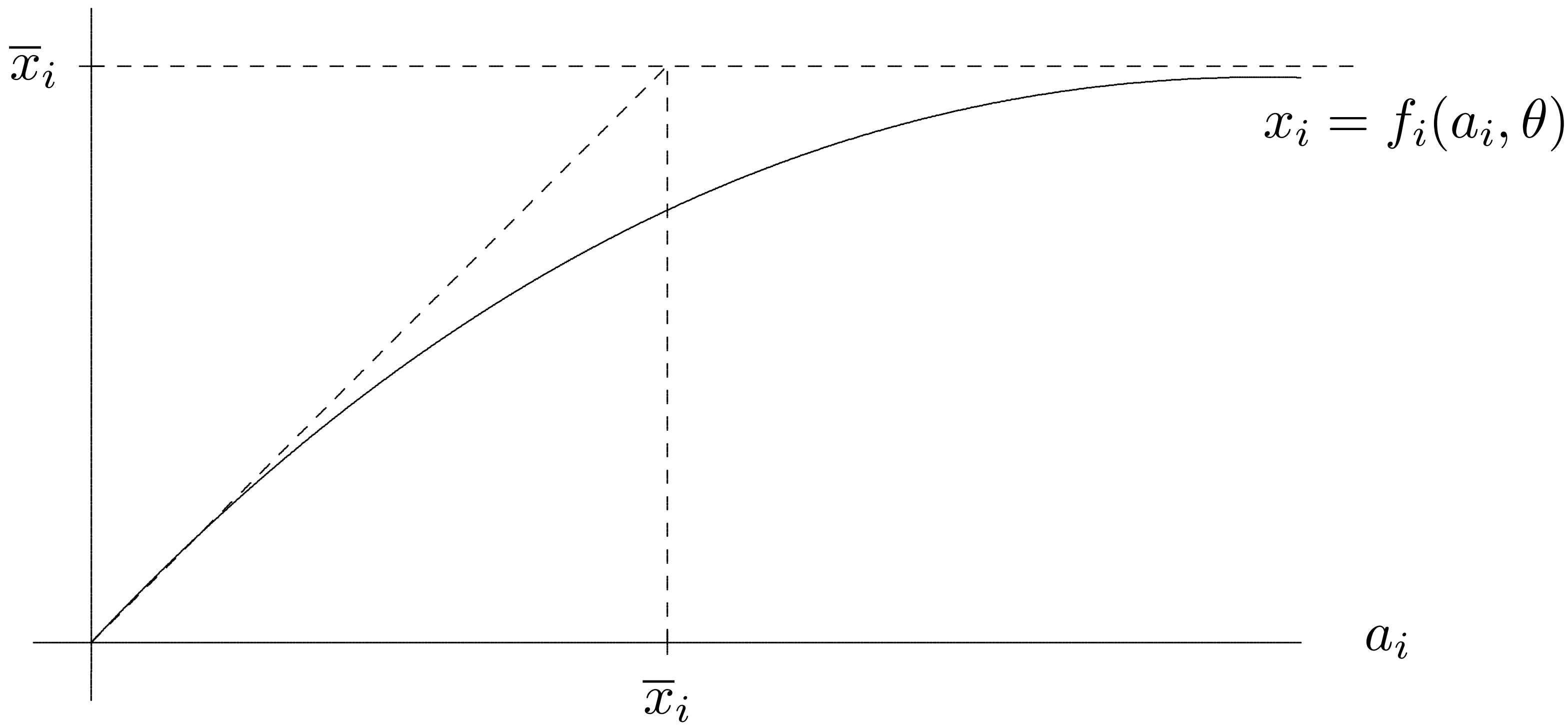
e_i is market value of i's equity

$$a_i = e_i + x_i$$

If i holds proportion π_{ji} of j's debt,

$$a_i = w_i + vy_i + \sum_j \pi_{ji} x_j$$

Merton (1974)



Lemma 1. *There exist functions* $\{f_i\}$ *such that*

$$x_i = f_i\left(a_i, \theta\right) \tag{1}$$

where each f_i is non-decreasing in a_i, and is bounded above by $\bar{x}_i$ and

$$\theta = (v, w, \bar{x})$$

Lemma 2. *The market value of equity is non-decreasing in a_i. That is, the function e_i defined as*

$$e_i \equiv a_i - f_i(a_i, \theta) \tag{2}$$

is non-decreasing in a_i.

System

$$\begin{aligned} x_1 &= f_1\left(a_1\left(x\right), \theta\right) \\ x_2 &= f_2\left(a_2\left(x\right), \theta\right) \\ &\vdots \\ x_n &= f_n\left(a_n\left(x\right), \theta\right) \end{aligned}$$

where $x = (x_1, x_2, \cdots, x_n)$.

Solve for fixed point x in:

$$x = F\left(x, \theta\right)$$

Unique solution

Theorem 3. *There is a unique profile of debt prices $x(\theta)$ that solves $x = F(x, \theta)$.*

Theorem 4. *$x(\theta)$ is increasing in θ.*

Result follows from

(i) Tarski's fixed point theorem

(ii) fact that $\{f_i\}$ are contraction mappings

Argument for Uniqueness

Suppose there are distinct solutions x, x'.

By Tarski, $x \leq x'$ and $x_i < x'_i$ for some i

Equity value of the system under x is strictly lower than under x'

Equity value of the system is value of fundamental assets

Contradiction.

Solvency Constraints

Value of all assets and liabilities determined by

$$\theta = (v, w, \bar{x})$$

Constraints on equity/debt ratio

$$\frac{a_i - x_i}{x_i} \geq r^*$$

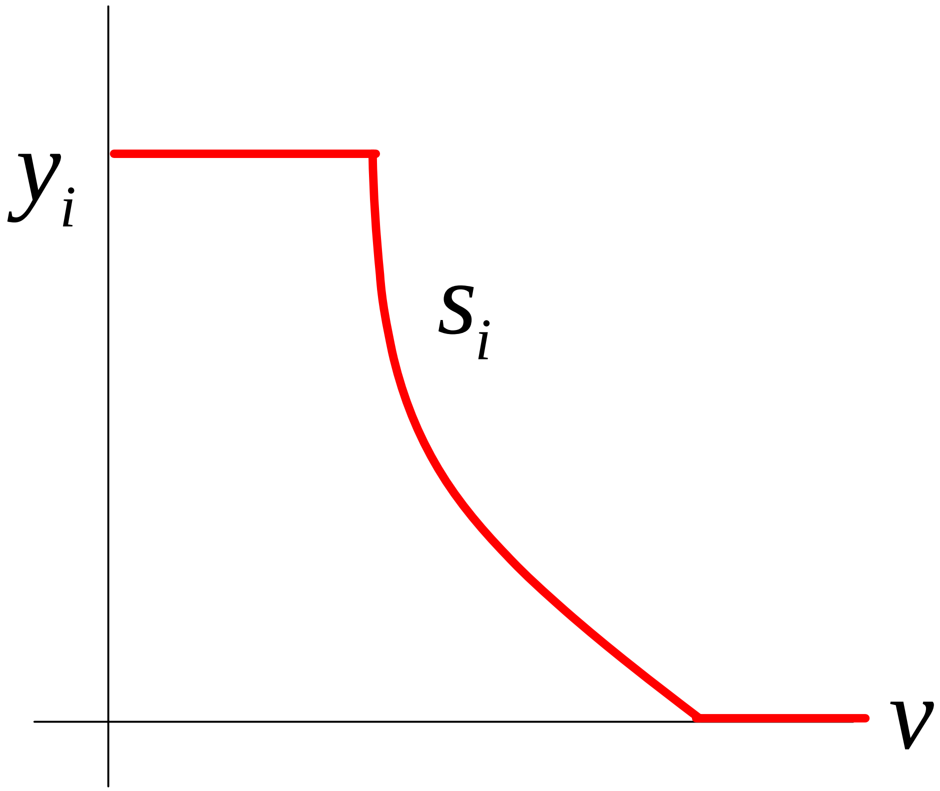
Spreads

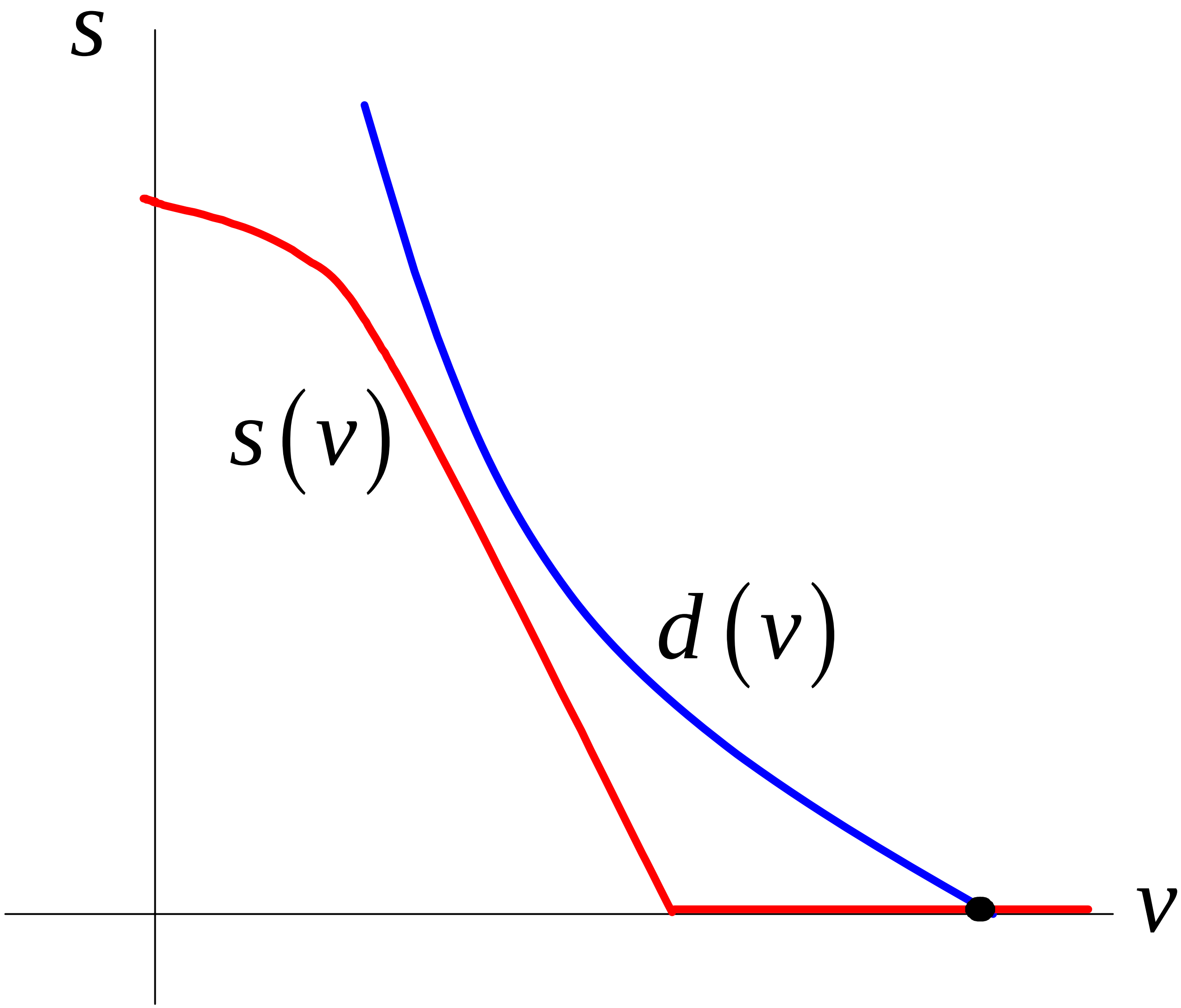
$$\frac{\bar{x}_i}{x_i} - 1$$

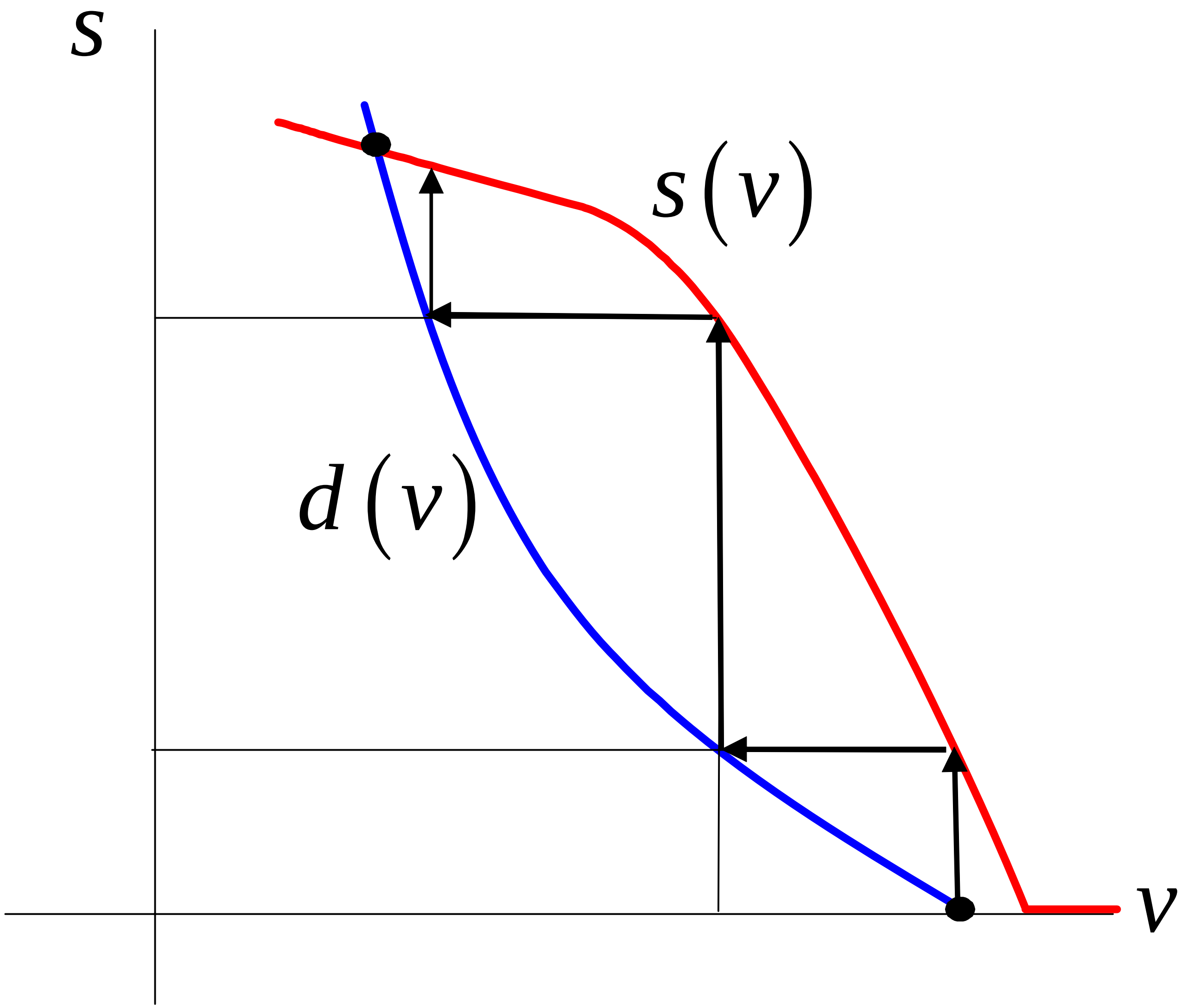
Sale s_i to restore solvency

$$\frac{w_i + v\left(y_i - s_i\right) + b_i - \left(x_i^0 - vs_i\right)}{x_i^0 - vs_i} \geq r^* \tag{3}$$

$$s_i = \min\left\{y_i, \max\left\{0, \frac{\left(1 + r^*\right) x_i^0 - w_i - vy_i - b_i}{r^* v}\right\}\right\} \tag{4}$$


y_i
s_i
v


s
s(v)
d(v)
v



s
$s(v)$
$d(v)$
v

Asking the Right Questions

- What role for mark-to-market accounting?
- "Domino" channel versus price mediated channel
- Large players versus small players
- Correlations in downturns
- "Risk appetite"