Market Liquidity and Funding Liquidity

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Motivation

- **Market liquidity**
  - ease of trading an asset
  - asset specific

- **Funding liquidity**
  - availability of funds
  - agent specific

- these liquidity concepts *are mutually reinforcing*
  - funding liquidity to dealers, hedge funds, investment banks etc.
    \[\Rightarrow\] enhances trading and market liquidity
  - market liquidity improves collateral value, i.e. lowers margins
    \[\Rightarrow\] eases funding restriction
Stylized Facts on Market Liquidity

1. **Sudden liquidity “dry-ups”**

2. **Commonality of liquidity**

3. **Correlated with volatility**

4. **Flight to quality**
   - Acharya-Pedersen (2005)

5. **Moves with the market**
   - E.g. Amihud (2002)
Leverage and Margins

- Financing a *long position* of $x_t^{j+} > 0$ shares at price $p_t^j = 100$:
  - Borrow 90 dollar per share;
  - Margin/haircut: $m_t^{j+} = 100 - 90 = 10$
  - Capital use: $10x_t^{j+}$

- Financing a *short position* of $x_t^{j-} > 0$ shares:
  - Borrow securities, and lend collateral of 110 dollar per share
  - Shortsell securities at price of 100
  - Margin/haircut: $m_t^{j-} = 110 - 100 = 10$
  - Capital use: $10x_t^{j-}$

  Margins must be financed with capital: $x^j = x_t^{j+} - x_t^{j-}$

$$\sum_j \left( x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-} \right) \leq W_t$$  \hspace{1cm} (1)

with perfect cross-margining

$$M_t \left( x_t^1, \ldots, x_t^J \right) \leq W_t$$  \hspace{1cm} (2)
Regulatory Capital Requirements

- Basel: banks
  - regulatory capital subject to constraint similar to (1)
  - alternatively, a bank can use its own model similar to (2)
- SEC Net Capital Rule: brokers
  - net capital = capital minus haircuts (compare to (1))
  - net capital must exceed a certain fraction of aggregate debt
- Regulation T: customers of brokers trading US equity
  - initial margin must be at least 50%
Basic Model Setup

- Time: $t = 1, 2, 3$ (later: infinite horizon)
- One asset with final asset payoff $v$ (later: assets $j = 1, \ldots, J$)
- Market illiquidity: $\Lambda_t = |E_t(v) - p_t|$
- Agents
  - Initial customers with supply $S(z, E_t[v] - p_t)$ at $t = 1, 2$
  - Complementary customers demand $D(z, E_2[v] - p_2)$ at $t = 2$
  - Risk-neutral dealers provide *immediacy* and face capital constraint

$$x \ m(\sigma, \Lambda) \leq \underbrace{B}_{\text{cash}} + \underbrace{x_0(E_1[v] - \Lambda)}_{\text{value of initial holding}}$$

The Situation

Proposition 1

(i) If $S(z, \Lambda)m(\sigma, \Lambda) + x_0\Lambda$ is decreasing in $\Lambda$, there exists a unique stable equilibrium for each level of dealer wealth $B$. The equilibrium market illiquidity $\Lambda^*(B)$ is continuously decreasing in dealer wealth $B$.

(ii) Otherwise, there are multiple equilibria for some wealth levels. There exists equilibrium selections $\Lambda^*(B)$ such that market illiquidity $\Lambda^*(B)$ is decreasing in dealer wealth $B$, but all equilibrium selections are discontinuous: there must be $B'$ such that illiquidity jumps discontinuously if wealth drops below $B'$. 

Example: Margin is *increasing* in market illiquidity, $m = 4 + \Lambda$
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\[ x(\Lambda; B=120) \]

\[ x(\Lambda; B=90) \]
Example: Margin is *increasing* in market illiquidity, $m = 4 + \Lambda$
Example: Liquidity Dry-ups/ Fragility

Illiquidity $\Lambda$ as a function of funding $B$

$\Lambda^*(B)$

multiplicity

Liquidity Spirals

- less trading
- funding problems
- higher margins
- losses on existing positions
- lower market liquidity

Brunnermeier and Pedersen (2006): Market Liquidity and Funding Liquidity
Liquidity Spirals

Proposition 2

If $\Lambda > 0$ in a stable equilibrium then

$$-\frac{\partial S}{\partial \Lambda} m - \frac{\partial m}{\partial \Lambda} S - x_0 > 0$$

and

$$d\Lambda dB = -\frac{1}{-\frac{\partial S}{\partial \Lambda} m - \frac{\partial m}{\partial \Lambda} S - x_0}$$

$$d\Lambda d\sigma = \frac{\frac{\partial m}{\partial \sigma} S}{-\frac{\partial S}{\partial \Lambda} m - \frac{\partial m}{\partial \Lambda} S - x_0}$$

Multiplier effects arise if

$$\frac{\partial m}{\partial \Lambda} S + x_0 > 0.$$
Example: Margin Spirals

Margin is *increasing* in market illiquidity $m = 4 + \Lambda$

Example: 1987 Crash

- Increased volatility caused banks to require more margin
- funding problems for marketmakers
  - failures at NYSE, Amex, OTC, trading firms, etc.
  - “thirteen [NYSE specialist] units had no buying power” because of their funding constraint (SEC (1988))

⇒ mutually reinforcing

Fed response:
- “calls were placed by high ranking officials of the FRBNY to senior management of the major NYC banks, indicating that ... they should encourage their Wall Street lending groups to use additional liquidity being supplied by the FRBNY to support the securities community”

Margin for S&P500 Futures

Margin requirement for CME members as a fraction of the S&P500 index level

Brunnermeier and Pedersen (2006)
Market Liquidity and Funding Liquidity
Overview of Talk

1. Time-series Properties of Liquidity
2. Cross-sectional Properties of Liquidity
   - Commonality
   - Flight to Quality
3. Endogenous Margin Setting Based on VaR
4. Related Literature

Dealer maximizes *expected profit per capital use*

- expected profit \( E_1[v^j] - p^j = \Lambda^j \)
- capital use \( m^j \)

Shadow cost of capital, funding liquidity, \( \phi = \max_j \frac{\Lambda^j}{m^j} \).

Dealers
- invest only in securities with highest ratio \( \frac{\Lambda^j}{m^j} \)
  (dealers determine price)
- do not invest in securities with lower ratio
  (customers determine price)

(If funding is abundant, \( \phi = 0 \) and \( \Lambda^j = 0 \) \( \forall j \).)
Proposition 3

If $B, E_1[v^1], \ldots, E_1[v^J]$ are random, the market liquidity of any two securities $j$ and $k$ comove.

$$\text{Cov} \left[ \Lambda^j, \Lambda^k \right] \geq 0.$$  

and market liquidity comoves with funding liquidity

$$\text{Cov} \left[ \Lambda^j, \phi \right] \geq 0$$

- **Intuition**: Funding liquidity is driving common factor.
Proposition 4

(i) (Quality = Liquidity) Assets with lower fundamental volatility have better market liquidity.

(ii) (Flight to Quality) The market liquidity differential between high and low fundamental volatility securities is bigger when dealer funding is tight:

\[ \sigma_j > \sigma_k \] implies under stated conditions that

\[
\left| \frac{\partial \Lambda^j}{\partial B} \right| \geq \left| \frac{\partial \Lambda^k}{\partial B} \right|.
\]

\[ \text{Cov}(\Lambda^j, \phi) \geq \text{Cov}(\Lambda^k, \phi). \]
Security 2 has larger fundamental volatility than security 1, \( \sigma^2 = 2 > 1 = \sigma^1 \)
Constant margins equal to vol.; \( S(z^j, \Lambda^j) = 20 - 2\Lambda^j \), so \( \bar{\Lambda} = 10 \).
Overview of Talk

1. Time-series Properties of Liquidity
2. Cross-sectional Properties of Liquidity
3. Endogenous Margin Setting Based on VaR
   - Stabilizing Margins - the Cushioning Effect
   - Destabilizing Margins
4. Related Literature
Value at Risk (VaR) specification of margin

\[
\text{Pr}( - (p_{t+1} - p_t) \geq m ) = \pi \\
\text{Pr}( (v_{t+1} - v_t) + (\Lambda_{t+1} - \Lambda_t) \geq m ) = \pi
\]

fundamental risk  market liquidity risk
Stabilizing Margins: the Cushioning Effect

- Fully informed financiers
- Complementary customers arrive in $t = 2$ with certainty
  - $\Lambda_2 = 0 \Rightarrow$ no liquidity risk $\Rightarrow$ loan value $l_1$ independent of $\Lambda_1$
  - $m_1 = p_1 - l_1 = v_1 - l_1 - \Lambda_1$

Proposition 5

$m_1$ is decreasing in $\Lambda_1$. 

Fundamental volatility is stochastic and has ARCH structure.
- large change in fundamental value $\Delta v_t \Rightarrow$ next periods volatility is high

Imperfectly informed financiers: observe only $\Delta p_t$
- due to fundamental shock
- due to large order by initial customers

Large customer shock $\Rightarrow$ large price shock
Financier thinks that it might be due to fundamental shock
VaR implies higher margins since
1. fundamental vol is estimated to be higher
2. price will not rebound after a fundamental shock
## Related Theoretical Literature

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<td>Asym. information: Gennotte-Leland (1990)</td>
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<td>Margin Spiral</td>
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<td>Flight to Quality</td>
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<td>Conditions for destabilizing margins</td>
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*Paper links literatures on: asset pricing, microstructure, limits of arb, corporate finance, macro, GE*
1. Sudden liquidity “dry-ups”
   - fragility
   - liquidity spirals
   - due to destabilizing margins (financiers imperfectly informed + ARCH)

2. Commonality of liquidity:
   - these funding problems affect many securities

3. Market liquidity correlated with volatility:
   - volatile securities requires more capital to finance

4. Flight to quality / flight to liquidity:
   - when capital is scarce, traders withdraw more from “capital intensive” high-margin securities

5. Market liquidity moves with the market
   - because funding conditions do