A Note on Measuring Illiquidity Jumps*

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1 Background on Jump Estimation

A recent literature in financial econometrics is concerned with measuring the jump component in intraday asset returns.[1] Under very general conditions, no arbitrage implies that asset returns admit a decomposition of the form

$$dX_t = \underbrace{b_t\, dt + \sigma_t\, dW_t}_{\text{continuous variation}} + \underbrace{dJ_t}_{\text{jump component}}, \qquad t \in [0, T] \tag{1}$$

where X_t is the asset's log price, and W_t is a Brownian motion.

In the jump estimation literature, observed asset returns are treated as discrete samples of this continuous-time process. For simplicity, we assume that we have n observation times, and that these observation times are equidistant and labeled $i\Delta_n$ for $i = 0, 1, \ldots, T/\Delta_n$. In this notation, our discrete log-returns are $\{\Delta_i^n X\}_{i=1}^{T/\Delta_n}$, where $\Delta_i^n X \equiv X_{i\Delta_n} - X_{(i-1)\Delta_n}$ is the i^{th} observed return.

Notice that $\Delta_i^n X$ is the confluence of the three terms in (1): a deterministic drift b_t, a measure of continuous random variation $\sigma_t dW_t$, and a measure of large, discontinuous variation dJ_t. One of the goals of the jump estimation literature is to statistically measure when a given move in observed prices, $\Delta_i^n X$, is due mostly to the continuous component, or the jump component.

2 Measuring Jumps in Illiquidity

We apply the logic of "separating continuous moves from jump moves" to measuring illiquidity jumps, which is one way of statistically quantifying the notion of sudden liquidity evaporations. Let L_t denote an illiquidity index that has been demeaned, so that $E[\Delta_i^n L] = 0$, enabling us

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[1] See Aït-Sahalia and Jacod (2014) for an introduction and survey.

to abstract from modeling a drift term.[2] Thus, we think of illiquidity as dynamically evolving according to a process with a continuous component and a jump component,

$$dL_t = \sigma_t \, dW_t + dJ_t. \tag{2}$$

If the illiquidity index does not jump, then this simplifies to $dL_t = \sigma_t dW_t$, which heuristically says that dL_t is distributed $N(0, \sigma_t^2 dt)$ by definition of a Brownian motion. Hence, we can interpret any observed moves that are statistically unlikely to have been generated by a $N(0, \sigma_t^2 dt)$ random variable as evidence for the presence of a jump increment dJ_t.

The σ_t term is sometimes referred to as continuous local variation: continuous because it determines the variance of a Brownian increment, and local because it is time-varying and hence describes Gaussian variation that is local to time t. Since σ_t is unobserved, it must be estimated. In our blog, we use a trailing 12-month window of daily observations on the illiquidity index, $\Delta_i^n L$, and compute the bipower variation

$$BV_{t-\Delta_w,t} = \frac{\pi}{2} \sum_i |\Delta_i^n L| \, |\Delta_{i-1}^n L|, \tag{3}$$

where Δ_w is the 12-month window. Barndorff-Nielsen and Shephard (2004) show that this measure asymptotically converges to $\int_{t-\Delta_w}^t \sigma_s^2 ds$ as the sampling interval $\Delta_n \to 0$, leading naturally to an estimate $\hat{\sigma}_t^2 = \frac{1}{\Delta_w} BV_{t-\Delta_w,t} \approx \frac{1}{\Delta_w} \int_{t-\Delta_w}^t \sigma_s^2 ds \to \sigma_t^2$ as $\Delta_w \to 0$. Thus, this asymptotic approximation requires both the sampling interval Δ_n as well as the local estimation window Δ_w to shrink to zero as the sample size increases. Note that as opposed to standard realized variance or even a GARCH estimate, the bipower variation measure asymptotically excludes the jump increments in the computation of the local variation. In other words, we wish to avoid a situation wherein our estimate of local variance σ_t^2 includes dJ_t increments, as those would wrongly inflate our estimate of σ_t^2. Also of note is that the asymptotic theory stipulates that the sampling interval Δ_n and local volatility window Δ_w should tend to zero, and does not say that the length of time T needs to tend to infinity. Hence these liquidity jump measurements should improve with higher frequency data, which is intuitively appealing in the context of measuring flash events.

With this estimate in hand, we consider moves in L_t to be a jump when the standardized increment $Z_i \equiv \frac{\Delta_i^n L}{\hat{\sigma}_{i\Delta_n} \sqrt{\Delta_n}} > 2$, which means that the illiquidity index has moved more than two standard deviations relative to local variation.[3] By counting all such jumps in a given window of time, one can measure the illiquidity index's jump intensity.

3 Motivation, Discussion, and Caveats

We took this approach in order to quantitatively and objectively capture sudden deteriorations in relative liquidity conditions. Some of these deteriorations are well-known and readily observable –

[2] Examples of illiquidity indexes are discussed below.
[3] This Z statistic is the basis for the formal jump test in Lee and Mykland (2008).

the equity market flash crash in May 2010 and the October 15, 2014 flash rally in Treasuries come to mind. Others are more subtle and have at times been termed liquidity illusions or liquidity mirages: while bid-ask spreads and other metrics are historically favorable for equities and Treasuries, a large trade or other liquidity event may trigger sudden withdrawals of liquidity providers, which should statistically appear as illiquidity jumps. Thus, our measure of illiquidity jump intensities is an attempt to measure the relative stability of liquidity conditions.

We also note that our focus on liquidity metrics is a novel application of jump estimation techniques, which have historically been applied to prices. The absence of arbitrage and trading frictions mathematically implies that returns follow a semimartingale and hence permit a decomposition similar to (1). This theory makes no promises with regard to quantities, which form the basis of our liquidity metrics. Nevertheless, there are many situations in which stochastic processes provide a useful description for observed time series, and hence we adopt this reduced-form approach to our illiquidity index. The assumption is that increments to the liquidity index are mostly Gaussian (conditional on a time-varying and stochastic local volatility), but on occasion experience large jumps, as in a flash crash. Thus using the theory of jump estimation in this context seems like a reasonable application.

Finally, our focus on measuring jumps relative to *local* variation σ_t stems from the vastly changing nature of liquidity over our sample period. Since the 1990s, financial markets have experienced everything from electronification to changes in tick size to competitive entry. Hence, what was considered liquid in 2000 may not be liquid by 2015 standards, which suggests that it is difficult to compare $\Delta_i^n L$ for an i that is early in the sample to an i that is late in the sample. The focus on local variation therefore mitigates this effect, because it separates changes in liquidity that are normal relative to current trading conditions from changes in liquidity that are more severe than recently experienced.

4 A Remark on the Illiquidity Index Construction

The jump measurement procedure is applied to an illiquidity index L_t. Candidates for L_t that we have considered are the first principal component of a basket of illiquidity indicators, including bid-ask spreads, Amihud (2002) price impact, Roll (1984) effective bid-ask spread, inverse depth (when available), volume, and intraday price dispersion – a list that is by no means exhaustive, but is guided by our desire to detect illiquidity events with our jump estimation procedure.

A simple rule of thumb is that the index should be constructed in a way such that the jump estimator detects extreme events like the equity market flash crash on May 6, 2010 (while keeping false positives to a minimum). If a liquidity variable is so noisy that it masks such key events (for example by inflating local volatility $\hat{\sigma}_t$), then it is excluded from the index. Next, it is desirable to use high-frequency intraday liquidity metrics. For equities and Treasuries, we have used information at the 1-minute frequency. This is because (a) flash crashes are often short-lived, and so measures like end-of-day bid-ask spreads could completely miss the intraday illiquidity event, and (b) the jump test's asymptotic theory is based on so-called *fill-in* asymptotics, so that the sampling interval $\Delta_n \to 0$. However, for reasons of comparability to our daily corporate bond metrics, we aggregated this intraday information to the daily frequency and conducted the jump test on daily data. In ongoing work, therefore, we are refining these procedures to higher frequencies, a wider collection of liquidity metrics, and larger cross-sections of securities.

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