# The New York Fed Staff Nowcast 2.0

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Nowcasts of GDP growth are designed to track the economy in real-time by incorporating information from several economic indicators. In April 2016, the New York Fed's Research Group launched the New York Fed Staff Nowcast, a dynamic factor model that updated estimates of current quarter GDP growth at a weekly frequency. The unprecedented fluctuations in many input series during the COVID-19 pandemic posed significant challenges to estimating the model, so the publication was suspended in September 2021. Recent time series econometric developments offer tools to better accommodate data volatility. In this report, we relaunch the New York Fed Staff Nowcast in a version that incorporates several novel features to address pandemic-related challenges. In Section 1, we describe the model and the data used. In Section 2, we describe how to interpret the output of the model by looking at the evolution of the New York Fed Staff Nowcast for 2023 Q1. In Section 3, we evaluate the model's performance and in Section 4 we analyze the performance as an early signal during recessionary and recovery periods. Finally, the appendix (A) provides details on the estimation approach.

# 1 The Model

The current version of the Staff Nowcast is a dynamic factor model that builds on the legacy Staff Nowcast (see Bok, Giannone, Caratelli, Sbordone, and Tambalotti (2018)), to which we defer for the general description of the use of dynamic factor models for forecasting.<sup>1</sup> The model assumes that a number of observed series ( $y_{1,t}...y_{n,t}$ ) are driven by a few latent common factors ( $f_{1,t}...f_{n_f,t}$ ) and idiosyncratic errors ( $e_{1,t}...e_{n,t}$ ) that are specific to each series. We write this model in vector form as

$$y_t = \mu + \iota g_t + \Lambda f_t + e_t, \tag{1}$$

where  $g_t$  captures a long-run growth trend, and  $f_t$  is a vector of five latent factors. Vector  $\iota$  and matrix  $\Lambda$  contain the factor loadings. The vector  $\mu$  holds time-invariant means for each variable, while  $e_t$  contains idiosyncratic errors. This specification includes several changes relative to the legacy model, which we discuss in turn in the following subsections.

 $<sup>^{1}</sup>$ For a complete treatment of dynamic factor models and their applications see Bai and Ng (2008) and Stock and Watson (2016).

Another change from the legacy model is our Bayesian estimation approach, which enables us to report probability intervals alongside each point estimate of real GDP growth. The model parameters are re-estimated every quarter (on the first Wednesday of the quarter), and the Staff Nowcast is updated each Friday to incorporate the effect of data released during the week.

## 1.1 Long-Run Trend

We introduce the latent long-term trend  $g_t$  in Equation 1 to account for changes in the average long-run growth rate of US real GDP. Antolin-Diaz, Drechsel, and Petrella (2017) document a gradual decline in output concentrated in the early 2000s. As in Antolin-Diaz, Drechsel, and Petrella (2022), we model the latent  $g_t$  as

$$g_t = g_{t-1} + \gamma_g v_{gt}$$
  $v_{gt} \sim N(0, 1)$  (2)

We set entries of the loading factor  $\iota$  in (1) corresponding to the output growth series to 1, while we let all other entries equal 0. We augment the static means with a time-varying trend, to account for the presence of periods of higher quarterly annualized growth rates of GDP and GDI in our sample. The estimated trend reflects the subtle decline in average long-run growth. As an illustration, Figure 1 plots the trend estimate with the 90% probability interval and year-over-year real GDP growth, highlighting this gradual decline.

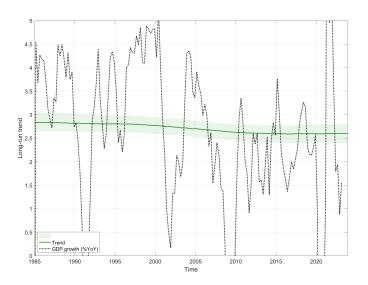


FIGURE 1. GDP Growth (%YoY) and Estimated Trend

### **1.2** Richer Dynamics in the Latent Factors

We address the challenges arising from pandemic-related data instabilities by introducing stochastic volatility and outlier adjustment to the latent variable dynamics. The transition equations of the model

are specified as follows:

$$f_t = \Phi_1 f_{t-1} + \dots + \Phi_4 f_{t-4} + \sigma_{ft} \odot s_{ft} \odot \varepsilon_{ft}$$
(3)

$$e_t = \phi e_{t-1} + \sigma_{et} \odot s_{et} \odot \varepsilon_{et}, \tag{4}$$

where  $\varepsilon_{ft}$  and  $\varepsilon_{et}$  are independently normally distributed errors, and are scaled by time-varying volatilities  $\sigma_{ft}$  and  $\sigma_{et}$  and discrete outliers  $s_{ft}$  and  $s_{et}$ . We denote the coefficients of the common factor lags as  $\Phi$ , and those of the idiosyncratic errors as  $\phi$ . Relative to the legacy model that restricts  $\Phi$  to a diagonal matrix, we allow the lags of one factor to impact the current value of another. This change allows our model to capture more general lead-lag relationships between variable subgroups. Antolin-Diaz et al. (2022) discuss the heterogeneous lead-lag dynamics present in the response of macroeconomic data to common shocks in more detail.

The time-varying volatilities evolve as

$$\ln \sigma_{ft}^2 = \ln \sigma_{f,t-1}^2 + \gamma_f \odot v_{ft} \qquad v_{ft} \stackrel{iid}{\sim} N(0_{n_f \times 1}, I_{n_f})$$

$$\ln \sigma_{et}^2 = \ln \sigma_{e,t-1}^2 + \gamma_e \odot v_{et} \qquad v_{et} \stackrel{iid}{\sim} N(0_{n,\times 1}, I_{n_s})$$
(5)

The elements  $s_{ft}$  and  $s_{et}$  of the outlier matrix are equal to 1 by default, and sparse discrete outliers distributed between 2 and 5 capture large one-time surprises in the data.

The new non-linear dynamics in the factor update equations reduce the model's sensitivity to large shocks. When the data reveal an increase in the number of large errors observed for a single series, the model attributes the higher variation to a growing  $\sigma_{et}$ . When larger errors occur more often across related series, the model upwardly revises the  $\sigma_{ft}$  corresponding to the common factor. Adjusting outliers this way allows the model to handle smaller surprises as normal without overreacting to the drastic deviations observed during the COVID-19 pandemic. The addition of these richer dynamics in the latent variables improves the forecasting performance of the model.

#### **1.3** Specification of the Factors and the Loading Structure

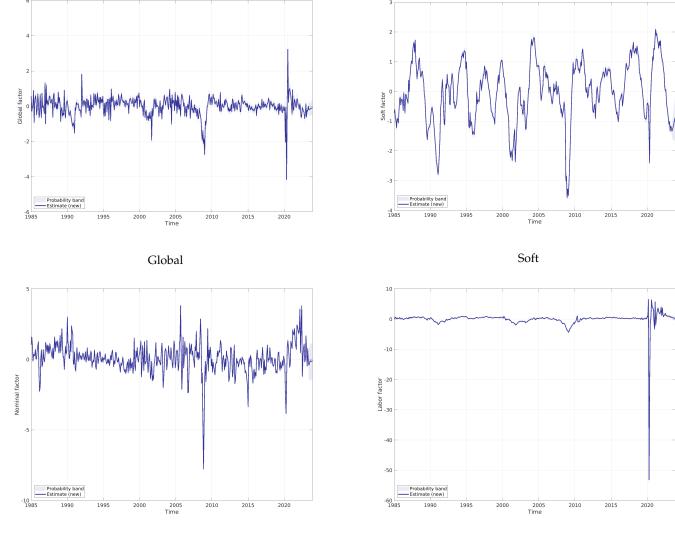
We updated the model's latent factor structure (see Table 1 for detail). As in the legacy model, every series loads on a single Global factor, while specific subgroups of series load on additional factors. Specifically, we include a "Soft" factor to capture local correlations in survey data and a "Labor" factor for series pertaining to labor market variables as in the legacy model. For the new specification, we replace the "Real" factor with a "Nominal" factor on which series that measure price levels or enter the model in nominal terms load. In addition, we include a fifth "COVID" factor that is only active from March to September of 2020 to capture correlated variation in several series impacted by the pandemic. For this factor, we restrict  $\sigma_{COVID,t}$  to 1 and let the outlier vector  $s_{COVID,t}$  scale the shocks. This adjustment prevents numerical issues that arise in early COVID quarters where we have a very limited period when

the factor is active.<sup>2</sup>

Figure 2 displays the evolution of four of the latent factors (as estimated on July 5, 2023) from the start of the sample in January 1985 through the end of 2023. Factor estimates are shown with their 68% probability intervals, which are very tight around the point estimates until the out-of-sample forecasts for May - December 2023. Figure 4 displays the COVID factor which is set to 0 before March and after September 2020. This factor assigns a large dip in economic activity to April and features a subsequent spike that peaks in June. The Labor factor captures a similar sharp drop in April but remains depressed longer. The factor rebounds in September and remains elevated until early 2023.

Figure 3 contains a subfigure for each latent factor displaying two objects: the factor volatility plotted with its 68% probability interval in blue (measured on the left axis), and the outliers that scale the shocks shaded in yellow (measured on the right axis). The plots of factor volatilities  $\sigma_{ft}$  spike in periods of irregular factor movement. The Global and Nominal  $\sigma$  plots show marked increases around periods of recession and expansion, with local peaks around 1990, 2008, and 2020. The stochastic volatility also allows the Labor factor to quickly adapt to a dramatic deterioration in the labor market in April 2020. The subplot of Labour in Figure 3 shows the model's attributions to  $\sigma$  and s during the pandemic that correlate with the factor drop seen in Figure 2. This latent factor behavior allows the new model to better handle the extreme data releases happening during the pandemic, which are discussed in further detail in Section 4.

<sup>&</sup>lt;sup>2</sup>Several approaches have been discussed in the recent literature to deal with the unprecedented data instabilities caused by the COVID-19 pandemic. The introduction of a COVID factor follows the idea that the COVID-19 pandemic represents a new shock, i.e., a new source of uncertainty not present in the economic data prior to 2020. This is related to Ng (2021) and, more specifically, to Maroz, Stock, and Watson (2021).



# FIGURE 2. Factors Over Time

Nominal

Labor

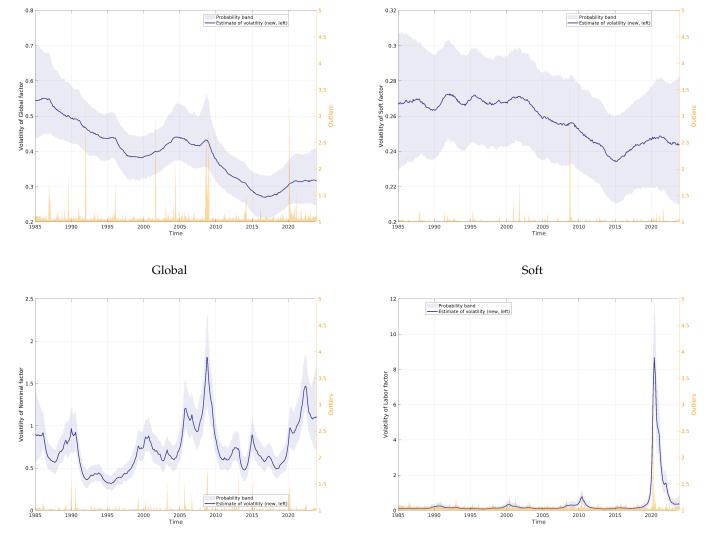
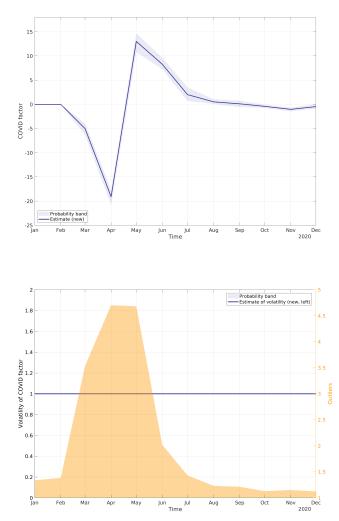


FIGURE 3. Factor Volatility Over Time

Nominal

Labor



# FIGURE 4. COVID Factor

# 1.4 Data

The series included in the vector  $y_t$  in equation 1 are released at either monthly or quarterly frequencies. Following Schorfheide and Song (2013), the monthly series enter our measurement equation directly  $(Y_{mt} = y_{mt})$ , and the quarterly series first undergo the following transformation:

$$Y_{it} = \frac{1}{9}(1 + 2L + 3L^2 + 2L^3 + L^4)y_{it}.$$
(6)

The series used in the model are listed in Table 1 and span a sample beginning in January 1985. For the estimation, we use all the data in the sample from 1985, unlike the legacy model which only used the most recent 15 years of data in the estimation. We decided to use the whole sample given that the richer specification of the new model allows it to accommodate and learn from older data. In particular, the time-varying trend growth and volatilities give the model enough flexibility to robustly handle data from periods characterized by higher growth or volatility. Furthermore, including more data spanning periods of recession and recovery allows for more precise estimation of underlying dynamics of the business cycle.

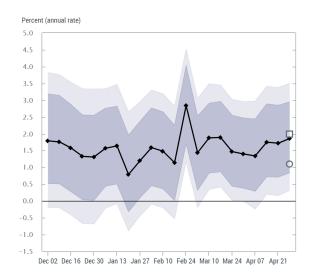
The series listed in Table 1 are the same as used in the legacy model except for two: we removed capacity utilization because of its collinearity with industrial production, and we removed the producer price index due to some data inconsistencies.

Data Series	Block	Units
	TGSNLC	
All employees: Total nonfarm		Level change (thousands)
JOLTS: Total job openings		Level change (thousands)
Civilian unemployment		Ppt. change
ADP nonfarm private payroll employment		Level change (thousands)
Nonfarm business sector: Unit labor cost		QoQ % change (annual)
ISM mfg.: PMI composite index		Index
ISM non-mfg.: NMI composite index		Index
ISM mfg.: Prices index		Index
ISM mfg.: Employment index		Index
Empire State Mfg. Survey: General business conditions		Index
Philly Fed Mfg. Business Outlook: Current activity		Index
Industrial production index		MoM % change
Manufacturers' new orders: Durable goods		MoM % change
Merchant wholesalers: Inventories: Total		MoM % change
Total business inventories		MoM % change
Manufacturers' shipments: Durable goods		MoM % change
Manufacturers' unfilled orders: All industries		MoM % change
Manufacturers' inventories: Durable goods		MoM % change
Retail sales and food services		MoM % change
Real personal consumption expenditures		MoM % change
New single-family houses sold		MoM % change
Housing starts		MoM % change
Value of construction put in place		MoM % change
Building Permits		Level change (thousands)
Exports: Goods and services		MoM % change
Imports: Goods and services		MoM % change
Import Price Index		MoM % change
Export price index		MoM % change
CPI-U: All items		MoM % change
CPI-U: All items less food and energy		MoM % change
PCE: Chain price index		MoM % change
PCE less food and energy: Chain price index		MoM % change
Real disposable personal income		MoM % change
Real gross domestic income		QoQ % change (annual)
Real gross domestic product		QoQ % change (annual)
Labor Manufacturing Hous	ing and constru	ction Income
0	national trade	Prices

TABLE 1. Data Series that enter the New York Fed Staff Nowcast 2.0

This is a list of all the data series that enter the New York Fed Staff Nowcast. The color-coded squares refer to the category to which each series belongs, as detailed in the legend. Filled squares in the "Block" column indicate the factors on which each data series loads in the model, with T, G, S, N, L, and C indicating the trend, global, soft, nominal, labor, and COVID-19 factors, respectively. "Units" indicates how the series enter the model.

# 2 How To Read the Model Output

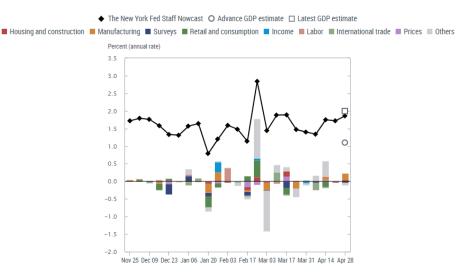


#### FIGURE 5. Nowcasting 2023:Q1

Figure 5 displays the Staff Nowcast for 2023 Q1. The third estimate of GDP has been released for Q1, allowing us to use it as an example to describe and explain how to interpret the model output. Generally, the publication of the Staff Nowcast for a given reference quarter commences one week after the publication of the second official GDP estimate for two quarters prior. For example, we started producing the Staff Nowcast for 2023 Q1 on Friday, December 9, 2022, following the second estimate of 2022 Q3 GDP. At this time, we also added a data point reporting the December 2 forecast for the first quarter. The black line represents the evolution of the Staff Nowcast, with the diamonds indicating the point estimate at each update, based on information available at that time. This figure also includes probability bands that measure the uncertainty of the estimate and are designed to contain the observed value for GDP growth with 50% and 68% probability. The circle and square at the right of the figure represent the advance and third official estimate of real GDP growth for comparison with the Staff Nowcast. The final point estimate for 2023 Q1 stood at 1.9%. The advance estimate from BEA was 1.1% which was revised upwards to a final estimate of 2.0%.

The contribution of each 'news' to the change in the Staff Nowcast is displayed in Figure 6. News is the difference between each data release throughout the week and the model predictions for that release. This 'news' is translated into an impact on the Staff Nowcast based on a 'weight' that represents the relevance of that variable. We aggregate the news by color-coded categories, as displayed in the key. The sum of the impacts of the week's news, represented by the colored bars of each category, is the change in the Staff Nowcast for that week. For example, in the update on Friday, January 20 2023, negative surprises in retail sales data and industrial production data drove a decrease in the point estimate.

#### FIGURE 6. Impact of Data Releases 2023:Q1



# 3 Model Performance

To evaluate the model's predictive power, we created a historical reconstruction of the Staff Nowcast that replicated the estimates we would have obtained in real-time.<sup>3</sup> For each quarter, starting with 2006:Q2, we began producing the Staff Nowcast one month before the start of the quarter and continued updating it until the day before the advance GDP estimate for that quarter was released. We re-estimated the model parameters on the first Wednesday of each quarter and updated the values each Friday with data vintages that preserved real-time data flow and revisions. The specification of the model is as discussed in Section 1 for the whole sample period.

## 3.1 Accuracy of Point Estimates

From the point estimates obtained in the historical reconstruction exercise, we computed the mean squared errors across quarters using three different forecast horizons. We used the point estimate from the model two months before the advance GDP release, one month before and one day before (final) to compare to the official GDP estimate. Table 2 displays a summary of the mean squared error (MSE) across quarters, calculated for those three forecast horizons, aggregated to three different time periods: the pre-pandemic (quarters 2006:Q2 to 2019:Q4), the post-pandemic (quarters 2020:Q4 to 2023:Q1), and total (all quarters excluding outlier quarters 2020:Q2 and 2020:Q2). Panel A calculates the MSE relative to the BEA advance estimate of GDP growth, while Panel B compares them relative to the third estimate. For the pre-pandemic period we also provide analogous mean squared errors for the legacy Staff Nowcast to compare relative performance. Finally, to focus on the performance of more recent quarters, Table 3

<sup>&</sup>lt;sup>3</sup>This reconstruction provides a representation of the predictions that would have been obtained in real time using this updated model framework. We acknowledge that the techniques used in this nowcasting model were developed more recently; therefore, we describe these forecasts as "historical reconstructions".

compares the final Staff Nowcast for quarters from 2020:Q1 through 2023:Q1 with the advance and the third BEA releases of GDP growth estimates.

Panel A: MSE Calculated Using Advance GDP Estimate								
Model		Pre-pandemic	Post-pandemic	Total				
Staff Nowcast 2.0		-	-					
	2 months out	2.2	8.0	3.8				
	1 month out	1.5	6.5	2.9				
	Final	1.3	7.7	2.7				
Legacy Staff Nowcast								
	2 months out	2.2	-	-				
	1 month out	1.4	-	-				
	Final	1.5	-	-				
Panel B: MSE Calculated Using Third GDP Estimate								
Panel B: N	ASE Calculated	Using Third GE	<b>PP Estimate</b>					
Panel B: Model	<b>MSE Calculated</b>	Using Third GE Pre-pandemic	<b>PP Estimate</b> Post-pandemic	Total				
	ASE Calculated	<u> </u>		Total				
Model	<b>MSE Calculated</b> 2 months out	<u> </u>		Total 4.8				
Model		Pre-pandemic	Post-pandemic					
Model	2 months out	Pre-pandemic 3.2	Post-pandemic 9.4	4.8				
Model	2 months out 1 month out	Pre-pandemic 3.2 2.4	Post-pandemic 9.4 7.6	4.8 3.8				
Model Staff Nowcast 2.0	2 months out 1 month out	Pre-pandemic 3.2 2.4	Post-pandemic 9.4 7.6	4.8 3.8				
Model Staff Nowcast 2.0	2 months out 1 month out Final	Pre-pandemic 3.2 2.4 2.1	Post-pandemic 9.4 7.6	4.8 3.8				

#### TABLE 2. Comparison of MSE

Panel A uses the first "advance" real GDP estimate from the Bureau of Economic Analysis for MSE calculations. Panel B uses the third estimate of real GDP. Pre-pandemic MSE is calculated using point estimates from quarters 2006:Q2 - 2019:Q4. Post-pandemic MSE calculations use point estimates from 2020:Q4 through 2023:Q1 (Panel A) or 2022:Q4 (Panel B). Total MSE is calculated excluding pandemic outlier quarters 2020:Q2 and 2020:Q3. Final Staff Nowcast MSE calculations use the point estimates created with data vintages from the day before each advanced GDP estimate is released. One month ahead MSE uses vintages from four weeks before the release, and two months ahead MSE uses vintages from eight weeks before the release.

Quarter	Final Staff Nowcast	Advance Estimate	Third Estimate
2020:Q1	0.6	-4.8	-5.0
2020:Q2	-36.4	-32.9	-31.4
2020:Q3	24.9	33.1	33.4
2020:Q4	3.4	4	4.3
2021:Q1	5.2	6.4	6.4
2021:Q2	2.8	6.5	6.7
2021:Q3	3.3	2.0	2.3
2021:Q4	3.2	6.9	6.9
2022:Q1	4.1	-1.4	-1.6
2022:Q2	1.9	-0.9	-0.6
2022:Q3	0.5	2.6	3.2
2022:Q4	1.0	2.9	2.6
2023:Q1	1.9	1.1	1.3

TABLE 3. Final Point Estimate and BEA GDP Estimates for Recent Sample Quarters

Final Staff Nowcasts are calculated using data vintages from the day before each advanced GDP estimate is released.

# 3.2 Decomposition of Staff Nowcast Movement

Each week the Staff Nowcast moves according to the impacts of new data releases, revisions to old data series, and revisions to the outlier matrices  $s_{ft}$  and  $s_{et}$ . Table 4 provides the share of Staff Nowcast movement (impact) attributed to each category of series over the period 2014-2022 and across some two-year subperiods that exclude the year 2020. The series are aggregated into categories according to Table 1. As Table 4 shows, the surveys have a larger contribution in the subperiods while the labor series have an outsized effect in the overall sample, which includes the year 2020 (see the last column). In the post-pandemic period, the labor series, the surveys and the parameter revisions have a larger impact than before. The Staff Nowcast also moves significantly more post-pandemic, reflecting higher uncertainty over the period.

Category	2014 - 2016	2017 - 2019	2021-2022	2014 - 2022
Income	3.4%	3.1%	1.0%	1.0%
International Trade	7.2%	6.3%	4.4%	2.6%
Housing & Construction	8.0%	8.3%	5.0%	3.4%
Labor	14.6%	13.7%	18.5%	41.5%
Manufacturing	11.5%	10.8%	8.6%	5.0%
Surveys	19.5%	21.3%	22.7%	16.2%
Retail & Consumption	10.1%	11.1%	10.4%	6.1%
Prices	8.0%	9.7%	5.0%	3.0%
Data Revisions	11.0%	9.4%	4.8%	4.0%
Parameter Revisions	6.8%	6.3%	19.0%	16.2%
Mean Quarterly Impact	6.6	4.8	26.5	25.5

TABLE 4. Share of Total Staff Nowcast Impact by Category

Parameter Revisions category also reflects updates of the outlier matrices  $s_{et}$  and  $s_{ft}$ .

# 4 Nowcasting Recessions and Recoveries

High-frequency forecasting tools such as the New York Fed Staff Nowcast provide useful early signals of economic contraction. In this section, we analyze the weekly Staff Nowcast movements during the COVID-19 Recession and the Great Recession using the historical reconstruction described in the previous section.

# 4.1 COVID-19 Recession

Although the COVID-19 recession was brief, defined by the NBER as lasting just two months (February-April 2020), its impact on the United States' economy was broad and deep. Given the sudden onset of the pandemic recession, we find it a valuable exercise to describe the weekly evolution of the reconstructed real-time GDP estimates for 2020:Q2 and 2020:Q3.

The first indication of a recession occurred when the 2020:Q2 Staff Nowcast incorporated the beginning-of-the-quarter parameter estimates on April 1st, as displayed in Figure 7. This estimation introduced the COVID factor which is active from March through September 2020 and is set to zero otherwise. The initial dip in the COVID factor was slight and only caused the Staff Nowcast to fall from 1.4% to -2.2% on April 3rd. Two weeks later, significant negative surprises in the Empire State and Philadelphia Fed Manufacturing surveys drove the Staff Nowcast down to -16.4%. The estimate remained relatively steady until April labor market figures were released in early May. The magnitude of the month-over-month drop in payroll employment figures from BLS and ADP far exceeded the model's forecasts, triggering a dramatic nosedive to -143.1%. The next update saw the Staff Nowcast rise to -105.4% when the Empire State Manufacturing survey came out above expectations, and the outlier matrices  $s_{et}$  and  $s_{ft}$  were revised. The last large movement for the quarter occurred at the beginning of June when the Staff Nowcast over the next few weeks to a final estimate of -36.6% <sup>4</sup>.

Despite large swings in the point estimates during 2020:Q2, the 50% and 68% probability intervals (not shown in the figure) remained relatively tight. While they spanned 2.0 and 2.9 points, respectively, at the beginning of the quarter, they grew to 4.5 and 6.5 points wide after the parameters were re-estimated in April, and reduced to 2.0 and 2.8 points by the end of the quarter.

Turning to 2020:Q3, Figure 7 illustrates the dramatic rebound in the Staff Nowcast. The estimate jumped from an initial -54.1% to +38.0% with the release of the May labor market data. Negative surprises from manufacturing surveys impacted the estimate by a few points in mid-June while the private payroll employment increase for June sent the Staff Nowcast to a high of +51.0% on July 2nd. The July parameter re-estimation tempered the estimate to +27.0% with further subsequent downward revisions as negative surprises from retail sales, manufacturing surveys, and industrial production were incorporated. Strengthening labor market data drove a seven percentage point increase at the beginning

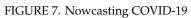
 $<sup>^{4}</sup>$ The BEA advance estimate of GDP growth for the quarter, released on July 30<sup>th</sup> 2020, was -32.9%. This was revised up to -31.4% in the third estimate.

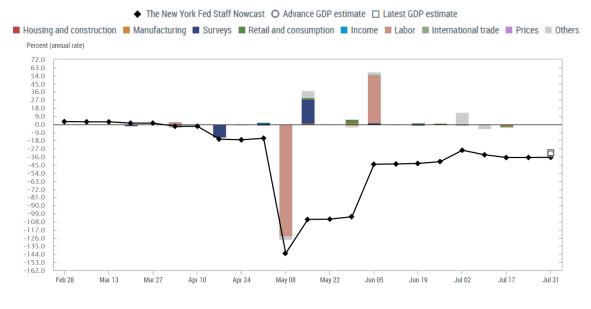
of August, but the gains were offset by outlier revisions, below-expectation retail sales, and increasing headline CPI the following week. August labor market data pushed the estimate back up to +25.3% at the beginning of September, which held steady until the end of the quarter. <sup>5</sup>

2020:Q3 had wider confidence intervals overall than Q2, reflecting the period's uncertainty. The 50% and 68% bands initially spanned 5.9 and 8.8 points, respectively, which jumped to 8.4 and 12.4 points after the July re-estimation. October re-estimation had a negligible effect on the probability interval width. The 50% and 68% bands dropped to a width of 1.8 and 2.7 points by the end of the quarter.

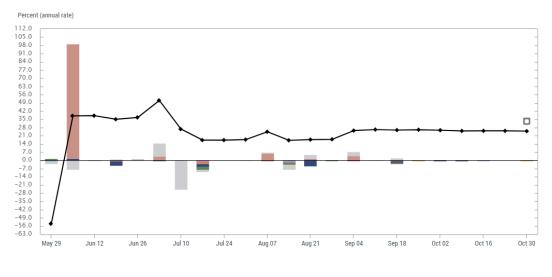
Overall, the model captures the dramatic sudden drop in economic activity in March-April 2020. The model's ability to effectively process large negative and positive data surprises over the pandemic period showcases its flexibility. The model appears to perform better over the COVID recessionary period. The final estimate for Q2 was 5.0% higher than the latest GDP estimate for the quarter. The larger span in probability bands in Q3 indicate the uncertainty over the recovery period and the final estimate was 8.5% lower than the latest GDP estimate for the quarter.

<sup>&</sup>lt;sup>5</sup>The BEA advance estimate of GDP growth for the quarter, released on October 29<sup>th</sup> 2020, was 33.1%. This was revised up marginally to 33.4% in the third estimate.











### 4.2 Nowcasting the Great Recession

Compared with the pandemic, the news received during the Great Recession and subsequent expansion did not move the Staff Nowcast as dramatically each week. Figure 8 details the progression over 2008:Q4 and 2009:Q3. These quarters were chosen as they mark the deepest recessionary period and the beginning of the recovery period.

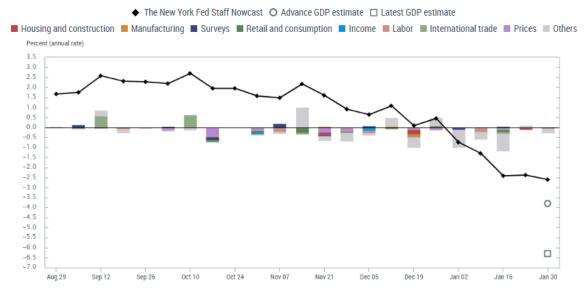
The Staff Nowcast for 2008:Q4, the deepest recessionary quarter according to the latest GDP estimate, began around 1.4% at the beginning of September. Positive news in the form of lower-than-expected import prices for both August and September bolstered the Staff Nowcast to a peak estimate of 2.7% on October 10th. The next week, the estimate fell to 1.9%, driven primarily by higher-than-expected CPI. For the remainder of the quarter, all data release categories nudged the estimate lower to a final value of -2.6%, with adjustments to the model's outlier classification contributing the most.<sup>6</sup>

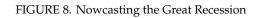
Turning to the post-recession quarter 2009:Q3, lower-than-expected CPI and positive surprise from the housing sector and manufacturing surveys pushed the estimate above 0 in mid-June. Higher international trade prices nudged the estimate down, but were offset the following week by positive June housing data. At the end of July, lower-than-expected estimates for Q2 GDP and June personal consumption data put the Staff Nowcast at just 0.3%. However, in the following four weeks the estimate jumped to 2.4% as it processed positive news from the labor market and manufacturing sectors, and low inflation. Data releases from nearly every sector came in above expectations on September 18th, bolstering the Staff Nowcast to 3%. Lukewarm September data from manufacturing, employment, and retail sales pulled the value down to a final estimate of 2.5%. <sup>7</sup>

Overall, the model appears to effectively process incoming news across both the recessionary and recovery periods. The 50% and 68% probability bands for 2008:Q4 (not shown in the figure) spanned 2.2 and 3.3 points at the beginning of the period and declined over the subsequent weeks. For the recovery period, the width of the bands were initially larger, spanning 3.3 and 4.6 points, and declined more notably to 1.8 and 2.6 points. The final Staff Nowcast was 3.7% higher than the latest GDP estimate for 2008:Q3 and 0.2% higher than the latest GDP estimate for 2009:Q3.

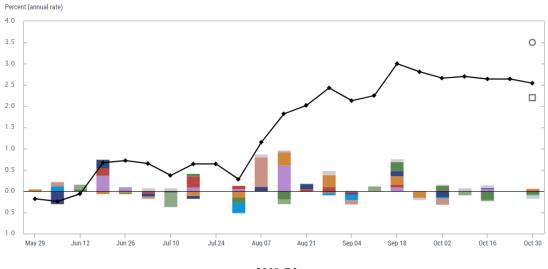
<sup>&</sup>lt;sup>6</sup>The BEA advance estimate of GDP growth for the quarter, released on January 30<sup>th</sup> 2009, was -3.8%. This was revised down to -6.3% in the third estimate.

<sup>&</sup>lt;sup>7</sup>The BEA advance estimate of GDP growth for the quarter, released on October 28<sup>th</sup> 2009, was 3.5%. This was revised down to 2.2% in the third estimate.











# A Details of Estimation Approach

We adopt a Bayesian approach to estimate the New York Fed Staff Nowcast. There are three main reasons to do this. First, a Bayesian approach is a convenient way of performing estimation of a dynamic factor model with the extended features we incorporated to deal with the challenges of the COVID-19 pandemic (such as stochastic volatility and outliers). Second, it allows us to construct probability intervals for the Staff Nowcast of GDP growth to report alongside point estimates, providing a more complete picture of the state of economic activity and the uncertainty around it. Third, it permits easy integration of different sources of uncertainty when computing point estimates and probabilities. For example, the probability bands we report reflect both estimation and filtering uncertainty.

In this section we discuss the details of the estimation algorithm. We rely on the Gibbs sampler—a particular Markov Chain Monte Carlo (MCMC) technique that samples from an approximation to the posterior distribution of parameters and latent factors by exploiting the existence of closed forms and efficient algorithms to simulate conditional distributions of a partition of the parameter space.

**Model equations.** Let  $Y_{mt}$  be an  $n_m$ -vector of monthly series and  $Y_{qt}$  an  $n_q$ -vector of quarterly series (imputed to the third month of each quarter). The model relates them to a time-varying trend  $g_t$ , unobservable factors  $f_t$  and errors  $e_t$  via the measurement equation:

$$Y_{t} = \begin{bmatrix} Y_{mt} \\ Y_{qt} \end{bmatrix} = \begin{bmatrix} I_{n_{m}} & 0_{n_{m} \times n_{q}} \\ 0_{n_{q} \times n_{m}} & \frac{1}{3}(1 + 2L + 3L^{2} + 2L^{3} + L^{4})I_{n_{q}} \end{bmatrix} y_{t},$$
$$y_{t} = \mu + \iota g_{t} + \Lambda f_{t} + e_{t}.$$

With  $n = n_m + n_q$ ,  $y_t$  is the *n*-vector of monthly-equivalent series for  $Y_t$ . If the *i*-th entry of  $Y_t$  is a monthly series we have  $Y_{it} = y_{it}$ , while if it is a quarterly series we have

$$Y_{it} = \frac{1}{3} \left\{ (\bar{y}_{it} + \bar{y}_{i,t-1} + \bar{y}_{i,t-2}) - (\bar{y}_{i,t-3} + \bar{y}_{i,t-4} + \bar{y}_{i,t-5}) \right\},\$$

where  $\bar{y}_{it}$  is such that  $\Delta \bar{y}_{it} = y_{it}$ , as in Mariano and Murasawa (2003). The long-run trend  $g_t$  is modeled after Antolin-Diaz et al. (2017) and Antolin-Diaz et al. (2022), with the entries of  $\iota$  corresponding to output and spending set to 1 and 0 otherwise.

The model for latent variables is

$$\begin{split} f_t &= \Phi_1 f_{t-1} + \dots + \Phi_{p_f} f_{t-p_f} + \sigma_{ft} \odot \varepsilon_{ft}, \\ e_t &= \phi_1 \odot e_{t-1} + \dots + \phi_{p_e} \odot e_{t-p_e} + \sigma_{et} \odot \varepsilon_{et}, \\ \varepsilon_{ft} &\stackrel{iid}{\sim} N(0_{n_f \times 1}, I_{n_f}), \quad \varepsilon_{et} \stackrel{iid}{\sim} N(0_{n_e \times 1}, I_{n_e}), \end{split}$$

with time-varying trend and volatilities given by

$$\begin{split} g_t &= g_{t-1} + \gamma_g v_{gt}, \\ \ln \sigma_{ft}^2 &= \ln \sigma_{f,t-1}^2 + \gamma_f \odot v_{ft}, \\ \ln \sigma_{et}^2 &= \ln \sigma_{e,t-1}^2 + \gamma_e \odot v_{et}, \\ v_{gt} &\stackrel{iid}{\sim} N(0,1), \quad v_{ft} \stackrel{iid}{\sim} N(0_{n_f \times 1}, I_{n_f}), \quad v_{et} \stackrel{iid}{\sim} N(0_{n_e \times 1}, I_{n_e}). \end{split}$$

**Parameters.** The parameter vector includes means, loadings, autoregressive coefficients, and standard deviations, i.e.,

$$\boldsymbol{\theta} = \left(\boldsymbol{\mu}, \boldsymbol{\gamma}_g, \boldsymbol{\Lambda}, \boldsymbol{\Phi}, \boldsymbol{\gamma}_f, \boldsymbol{\phi}, \boldsymbol{\gamma}_e\right).$$

As is well known, loadings  $\Lambda$  must be subject to certain restrictions for identification and we achieve this by specifying that certain factors (e.g., the soft, labor and nominal factors) only affect a subset of observables. The dimensions are as follows:

- $\mu$  is an *n*-vector;
- $\gamma_g$  is a scalar;
- $\Lambda$  is an  $n \times n_f$  matrix;
- $\Phi = (\Phi_1 \dots \Phi_{p_f})$  is  $n_f \times n_f p_f$  with the VAR coefficient matrices for  $f_t$  horizontally concatenated and each  $\Phi_\ell$  is  $n_f \times n_f$ ;
- $\gamma_f$  is an  $n_f$ -vector;
- $\phi = (\phi_1 \dots \phi_{p_e})$  is  $n_e \times p_e$  with the AR coefficient vectors for  $e_t$  horizontally concatenated and each  $\phi_t$  is  $n_e \times 1$ ; and
- $\gamma_e$  is an  $n_e$ -vector.

**Priors.** We use a combination of normal and inverse-gamma priors assuming independence among groups of parameters, i.e.,<sup>8</sup>

$$\begin{split} \mu &\sim N(m_{\mu}, P_{\mu}^{-1}), \\ \gamma_g &\sim 1/\sqrt{\Gamma(v_g/2, 2/(v_g s_g^2))}, \\ \text{vec}(\Lambda) &\sim N(m_{\Lambda}, P_{\Lambda}^{-1}), \quad (\text{subject to the identifying restrictions}) \\ \text{vec}(\Phi) &\sim N(m_{\Phi}, P_{\Phi}^{-1}), \\ \gamma_f &\sim 1/\sqrt{\Gamma_{n_f}(v_f/2, 2/(v_f s_f^2))}, \end{split}$$

<sup>&</sup>lt;sup>8</sup>Γ<sub>*K*</sub>(*α*, *β*) is a vector of *K* independent  $\Gamma(\alpha_k, \beta_k)$ -distributed random variables.

$$\begin{aligned} \mathrm{vec}(\phi) &\sim N(m_{\phi}, P_{\phi}^{-1}), \\ \gamma_e &\sim 1/\sqrt{\Gamma_n(v_e/2, 2/(v_e s_e^2))} \end{aligned}$$

The default choice for the prior (depending on the hyperparameters  $T_{\text{pre-mean}}$ ,  $T_{\text{pre-load}}$ ,  $T_{\text{pre-dyn}}$ ,  $T_{\text{pre-var}}$ ) is as follows:

- $m_{\mu} = 0_{n \times 1}$  and  $P_{\mu} = T_{\text{pre-mean}} I_n$ ;
- $s_{g}^{2}$  is calibrated to 1 and  $v_{g} = T_{\text{pre-var}}$ ;
- $m_{\Lambda} = 0_{nn_f \times 1}$  and  $P_{\Lambda} = T_{\text{pre-load}}(\Pi_{\Lambda} \otimes I_n)$  with preferred value  $\Pi_{\Lambda} = I_{n_f}$ ,
- $m_{\Phi} = 0_{n_t^2 p_f \times 1}$  and  $P_{\Phi} = T_{\text{pre-AR}}$  blk diag  $(\Pi_{\Phi}, 2^2 \Pi_{\Phi}, \dots, p_f^2 \Pi_{\Phi})$  with preferred value  $\Pi_{\Phi} = I_{n_f^2}$ ;<sup>10</sup>
- $s_f^2$  is  $n_f$ -vector calibrated with the static model ML estimates and  $v_f = T_{\text{pre-var}} \mathbf{1}_{n_f \times 1}$ ;
- $m_{\phi} = 0_{np_e \times 1}$  and  $P_{\Phi} = T_{\text{pre-AR}} \text{blk} \operatorname{diag} (I_n, 2^2 I_n, \dots, p_e^2 I_n);^{11}$
- $s_e^2$  is *n*-vector calibrated with the static model ML estimates and  $v_e = T_{\text{pre-var}} \mathbf{1}_{n_e \times 1}$ .

**Gibbs sampler.** We will assume  $p_f, p_e \le 4$ . Let  $n_X = 5(1 + n_f + n_a)$  and define the  $n_X$ -vector  $X_t$ ,

$$X_{t} = \begin{bmatrix} g_{t} \\ \vdots \\ g_{t-4} \\ f_{t} \\ \vdots \\ f_{t-4} \\ e_{qt} \\ \vdots \\ e_{q,t-4} \end{bmatrix}$$

where  $e_{qt}$  is the lower  $n_q$ -subvector of  $e_t$  (that is, the part of  $e_t$  with the measurement errors for the quarterly series). Let  $\theta$  be a value of the parameters. The goal is to obtain a new value of  $\theta$ . Let  $\theta_{\sim \vartheta}$ denote the parameter vector excluding  $\vartheta$ .

To implement the Gibbs sampler, we need to augment  $\theta$  by  $\{\sigma_{ft}, \sigma_{et}\}$ .

 $<sup>^9</sup>$ This structure implies that loadings are prior-independent between equations, but within-equation dependence is permitted. The choice  $\Pi_{\Lambda} = I_{n_f}$  imposes within-equation prior-independence.

<sup>&</sup>lt;sup>10</sup>This structure is a Minnesota-like prior: VAR coefficient matrices  $\Phi_1, \ldots, \Phi_p$  are assumed independent and with precisions proportional to the squared lag. <sup>11</sup>This structure is also a Minnesota-like prior.

# (1) DRAW THE LATENT VARIABLES. Consider the following state space representation:<sup>12</sup>

$$\begin{bmatrix} Y_{mt} - \phi_{m1} \odot Y_{m,t-1} - \dots - \phi_{mp_e} \odot Y_{m,t-p_e} \\ Y_{qt} \end{bmatrix} = D + HX_t + \epsilon_t,$$
  
$$\epsilon_t \stackrel{iid}{\sim} N(0_{n \times 1}, \Sigma_{\epsilon,t}),$$
  
$$X_t = FX_{t-1} + G\eta_t,$$
  
$$X_1 \sim N(\mu_1, \Sigma_1), \quad \eta_t \stackrel{iid}{\sim} N(0_{(1+n_f+n_q) \times 1}, \Sigma_{\eta,t}),$$

where  $\phi_{m\ell}$  is the upper  $n_m$ -subvector of  $\phi_\ell$ . Defining  $\mu_m, \mu_q, \iota_m, \iota_q, \Lambda_m, \Lambda_q, \phi_{m\ell}, \phi_{q\ell}, \sigma_{em,t}, \sigma_{eq,t}$  in a similar way and letting  $v_q = \frac{1}{3} \begin{pmatrix} 1 & 2 & 3 & 2 & 1 \end{pmatrix}$ ,

$$\begin{split} D &= \begin{bmatrix} (\mathbf{1}_{n_m \times 1} - \phi_{m1} - \dots - \phi_{mp_e}) \odot \mu_m \\ & \exists \mu_q \end{bmatrix}, \\ H &= \begin{bmatrix} H_g & H_f & \mathbf{0}_{n_m \times 5n_q} \\ H_g &= \begin{bmatrix} \iota_m & -\phi_{m1} \odot \iota_m & \dots & -\phi_{mp_e} \odot \iota_m & \mathbf{0}_{n_m \times (4-p_e)} \end{bmatrix}, \\ H_f &= \begin{bmatrix} \Lambda_m & -\phi_{m1} \odot \Lambda_m & \dots & -\phi_{mp_e} \odot \Lambda_m & \mathbf{0}_{n_m \times (4-p_e)n_f} \end{bmatrix}, \\ H_q &= \begin{bmatrix} \iota_q v_q & v_q \otimes \Lambda_q & v_q \otimes I_{n_q} \end{bmatrix}, \\ \Sigma_{e,t} &= \text{blk diag} \left( \text{diag}(\sigma_{em,t}^2), \mathbf{0}_{n_q \times n_q} \right), \\ F &= \text{blk diag} \left( F_g, F_f, F_q \right), \\ F_g &= \begin{bmatrix} \mathbf{1} & \mathbf{0}_{1 \times 4} \\ I_4 & \mathbf{0}_{4 \times 1} \end{bmatrix}, \\ F_f &= \begin{bmatrix} \Phi & \mathbf{0}_{n_f \times (5-p_f)n_f} \\ I_{4n_f} & \mathbf{0}_{4n_f \times n_f} \end{bmatrix}, \\ F_q &= \begin{bmatrix} \text{diag}(\phi_{q1}) & \dots & \text{diag}(\phi_{qp_e}) & \mathbf{0}_{n_q \times (5-p_e)n_q} \\ I_{4n_q} & \mathbf{0}_{4n_q \times n_q} \end{bmatrix}, \\ G &= \text{blk diag} \left( \begin{bmatrix} \mathbf{1} \\ \mathbf{0}_{4 \times 1} \end{bmatrix}, \begin{bmatrix} I_{n_f} \\ \mathbf{0}_{4n_f \times n_f} \end{bmatrix}, \begin{bmatrix} I_{n_q} \\ \mathbf{0}_{4n_q \times n_q} \end{bmatrix} \right), \\ \Sigma_{n,t} &= \text{blk diag} \left( \gamma_g^2, \text{diag}(\sigma_{f1}^2), \text{diag}(\sigma_{eq,t}^2) \right), \\ \mu_1 &= \mathbf{0}_{n_X \times 1}, \\ \Sigma_1 &= \tau_X I_{n_X}. \end{split}$$

A call to the simulation smoother produces a draw of  $\{X_t\}$  which we can use to recover the detrended

<sup>&</sup>lt;sup>12</sup>In the data it would start at  $t > p_e$ .

monthly-equivalent series,

$$\tilde{y}_t = y_t - \iota g_t = \mu + \Lambda f_t + e_t.$$

This takes care of missing data and simplifies the treatment of quarterly series.

(2) DRAW  $\mu$ . Construct

$$\begin{aligned} r_{\mu,t} &= \sigma_{et}^{-1} \odot \left\{ (\tilde{y}_t - \Lambda f_t) - \phi_1 \odot (\tilde{y}_{t-1} - \Lambda f_{t-1}) - \dots - \phi_{p_e} \odot (\tilde{y}_{t-p_e} - \Lambda f_{t-p_e}) \right\}, \\ R_{\mu,t} &= \operatorname{diag}(\sigma_{et}^{-1} \odot (\mathbf{1}_{n \times 1} - \phi_1 - \dots - \phi_{p_e})). \end{aligned}$$

We obtain a regression problem,

$$r_{\mu,t} = R_{\mu,t}\mu + u_{\mu,t}, \quad u_{\mu,t} \stackrel{iid}{\sim} N(0_{n\times 1}, I_n).$$

Define

$$\begin{split} \tilde{m}_{\mu} &= \tilde{P}_{\mu}^{-1} \left( P_{\mu} m_{\mu} + \sum_{t=1}^{0} rk + + PTR'_{\mu,t} r_{\mu,t} \right), \\ \tilde{P}_{\mu} &= P_{\mu} + \sum_{t=1}^{T} R'_{\mu,t} R_{\mu,t}. \end{split}$$

Then,

$$\mu|\theta_{\sim\mu}, Y_{1:T} \sim N(\tilde{m}_{\mu}, \tilde{P}_{\mu}^{-1}).$$

(3) DRAW  $\gamma_g$ . Construct

$$r_{\gamma_g,t} = g_t - g_{t-1}$$

We obtain the scale estimation problem,

$$r_{\gamma_{g},t} = \gamma_g \upsilon_{gt}, \quad \upsilon_{gt} \stackrel{iid}{\sim} N(0,1).$$

Define

$$\begin{split} \tilde{s}_g^2 &= \tilde{v}_g^{-1} \left( v_g s_g^2 + \sum_{t=1}^T r_{\gamma_g, t}^2 \right), \\ \tilde{v}_g &= v_g + T. \end{split}$$

Then,

$$\gamma_g | \theta_{\gamma_g}, Y_{1:T} \sim 1/\sqrt{\Gamma(\tilde{v}_g/2, 2/(\tilde{v}_g \tilde{s}_g^2))}.$$

(4) DRAW  $\Lambda$ . Construct

$$\begin{split} r_{\Lambda,t} &= \sigma_{et}^{-1} \odot \left\{ (\tilde{y}_t - \mu) - \phi_1 \odot (\tilde{y}_{t-1} - \mu) - \dots - \phi_{p_e} \odot (\tilde{y}_{t-p_e} - \mu) \right\}, \\ R_{\Lambda,t} &= \left\{ \left( f_t - \phi_1 \odot f_{t-1} - \dots - \phi_{p_e} \odot f_{t-p_e} \right)' \right\} \otimes \operatorname{diag} \left( \sigma_{et}^{-1} \right). \end{split}$$

We obtain the regression problem,

$$r_{\Lambda,t} = R_{\Lambda,t} \operatorname{vec}(\Lambda) + u_{\Lambda,t}, \quad u_{\Lambda,t} \stackrel{iid}{\sim} N(0_{nn_f \times 1}, I_{nn_f}).$$

Define

$$\begin{split} \tilde{m}_{\Lambda} &= \tilde{P}_{\Lambda}^{-1} \left( P_{\Lambda} m_{\Lambda} + \sum_{t=1}^{T} R'_{\Lambda,t} r_{\Lambda,t} \right), \\ \tilde{P}_{\Lambda} &= P_{\Lambda} + \sum_{t=1}^{T} R'_{\Lambda,t} R_{\Lambda,t}. \end{split}$$

Then,

$$\operatorname{vec}(\Lambda)|\theta_{\sim\Lambda}, Y_{1:T} \sim N(\tilde{m}_{\Lambda}, \tilde{P}_{\Lambda}^{-1}).$$

(5) DRAW  $\Phi$ . Construct

$$\begin{aligned} r_{\Phi,t} &= \sigma_{ft}^{-1} \odot f_t, \\ R_{\Phi,t} &= \begin{pmatrix} f_{t-1}' & \dots & f_{t-p_f}' \end{pmatrix} \otimes \operatorname{diag} \left( \sigma_{ft}^{-1} \right). \end{aligned}$$

We obtain the regression problem,

$$r_{\Phi,t} = R_{\Phi,t} \operatorname{vec}(\Phi) + u_{\Phi,t}, \quad u_{\Phi,t} \stackrel{iid}{\sim} N(0_{n_f^2 p_f \times 1}, I_{n_f^2 p_f}).$$

Define

$$\begin{split} \tilde{m}_{\Phi} &= \tilde{P}_{\Phi}^{-1} \left( P_{\Phi} m_{\Phi} + \sum_{t=1}^{T} R'_{\Phi,t} r_{\Phi,t} \right), \\ \tilde{P}_{\Phi} &= P_{\Phi} + \sum_{t=1}^{T} R'_{\Phi,t} R_{\Phi,t}. \end{split}$$

Then,

$$\operatorname{vec}(\Phi)|_{\theta_{\sim \Phi}}, Y_{1:T} \sim N(\tilde{m}_{\Phi}, \tilde{P}_{\Phi}^{-1}).$$

(6) DRAW { $\sigma_{ft}$ }. Construct

$$r_{\sigma_f,t} = f_t - \Phi_1 f_{t-1} - \dots - \Phi_{p_f} f_{t-p_f}.$$

We obtain a set of independent stochastic volatility models,

$$r_{\sigma_{f},t} = \sigma_{ft} \odot \varepsilon_{ft}, \quad \varepsilon_{ft} \stackrel{iid}{\sim} N(0_{n_f \times 1}, I_{n_f}).$$

Each entry of  $\{\sigma_{ft}\}$  is to be updated following Kim, Shephard, and Chib (1998) (and the refinement in Omori, Chib, Shephard, and Nakajima (2007)).

(7) DRAW  $\gamma_f$ . Construct

$$r_{\gamma_f,t} = \ln \sigma_{ft}^2 - \ln \sigma_{f,t-1}^2.$$

We obtain the scale estimation problem,

$$r_{\gamma_f,t} = \gamma_f \odot v_{ft}, \quad v_{ft} \stackrel{iid}{\sim} N(0_{n_f \times 1}, I_{n_f}).$$

Define

$$\begin{split} \tilde{s}_f^2 &= \tilde{\nu}_f^{-1} \odot \left( \nu_f s_f^2 + \sum_{t=1}^T r_{\gamma_f,t}^2 \right), \\ \tilde{\nu}_f &= \nu_f + T. \end{split}$$

Then,

$$\gamma_f | \boldsymbol{\theta}_{\sim \gamma_f}, \boldsymbol{Y}_{1:T} \sim 1/\sqrt{\Gamma_{n_f}(\tilde{v}_f/2, 2/(\tilde{v}_f \tilde{s}_f^2))}$$

(8) DRAW  $\phi$ . Construct  $e_t = \tilde{y}_t - \mu - \Lambda f_t$  and

$$\begin{split} r_{\phi,t} &= \sigma_{et}^{-1} \odot e_t, \\ R_{\phi,t} &= \sigma_{et}^{-1} \odot \left( \text{diag}(e_{t-1}) \quad \dots \quad \text{diag}(e_{t-p_e}) \right). \end{split}$$

We obtain the regression problem,

$$r_{\phi,t} = R_{\phi,t} \operatorname{vec}(\phi) + u_{\phi,t}, \quad u_{\phi,t} \stackrel{iid}{\sim} N(0_{np_e \times 1}, I_{np_e}).$$

Define

$$\begin{split} \tilde{m}_{\phi} &= \tilde{P}_{\phi}^{-1} \left( P_{\phi} m_{\phi} + \sum_{t=1}^{T} R_{\phi,t}' r_{\phi,t} \right), \\ \tilde{P}_{\phi} &= P_{\phi} + \sum_{t=1}^{T} R_{\phi,t}' R_{\phi,t}. \end{split}$$

Then,

$$\operatorname{vec}(\phi)|\theta_{\sim\phi}, Y_{1:T} \sim N(\tilde{m}_{\phi}, \tilde{P}_{\phi}^{-1}).$$

(9) DRAW { $\sigma_{et}$ }. Construct

$$r_{\sigma_{e,t}} = e_t - \phi_1 \odot e_{t-1} - \dots - \phi_{p_e} \odot e_{t-p_e}.$$

We obtain a set of independent stochastic volatility models,

$$r_{\sigma_{e},t} = \sigma_{et} \odot \varepsilon_{et}, \quad \varepsilon_{et} \stackrel{iid}{\sim} N(0_{n\times 1}, I_n).$$

Each entry of  $\{\sigma_{et}\}$  is to updated as Kim et al. (1998) (and the refinement in Omori et al. (2007)).

(10) DRAW  $\gamma_e$ . Construct

$$r_{\gamma_{e},t} = \ln \sigma_{et}^2 - \ln \sigma_{e,t-1}^2.$$

We obtain the scale estimation problem,

$$r_{\gamma_{e,t}} = \sigma_e \odot v_{et}, \quad v_{et} \stackrel{iid}{\sim} N(0_{n \times 1}, I_n).$$

Define

$$\begin{split} \tilde{s}_e^2 &= \tilde{v}_e^{-1} \odot \left( v_e s_e^2 + \sum_{t=1}^T r_{\gamma_e,t}^2 \right), \\ \tilde{v}_e &= v_e + T. \end{split}$$

Then,

$$\gamma_e | \theta_{\gamma_e}, Y_{1:T} \sim 1/\sqrt{\Gamma_n(\tilde{\nu}_e/2, 2/(\tilde{\nu}_e \tilde{s}_e^2))}.$$

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