Technical Appendix to “Has the Fed Stabilized the Price Level?”
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As discussed in our blog post, it is common to describe the Federal Open Market Committee’s (FOMC) setting of the federal funds rate with an interest-rate rule of the form:

\[ i_t = \varphi_\pi (\pi_t - \pi^*) + \varphi_y \Delta y_t + \rho i_{t-1} \]

where \( i_t \) denotes the federal funds rate, \( \pi_t \) is the inflation rate, \( \pi^* \) is the inflation target, \( \Delta y_t \) is GDP growth, \( \varphi_\pi \), \( \varphi_y \) are constant parameters, and \( \rho \) is the degree of interest-rate inertia. We omit a constant and an error term capturing unexpected movements in the interest rate. Using quarterly U.S. data from 1990:Q1 to 2013:Q2, simple regression analysis yields positive estimates of \( \varphi_\pi \) and \( \varphi_y \), and an estimate as high as 0.95 for \( \rho \). More sophisticated estimations typically yield estimates of \( \rho \) above 0.80 (see, for example, Smets and Wouters [2007]).

While such a policy rule formally describes the setting of conventional monetary policy as a function of inflation deviations, it is important to realize that a high degree of interest-rate inertia implies that monetary policy tends to stabilize price-level deviations. Indeed, as \( \rho \) approaches 1, we can rewrite the above policy rule as:

\[ i_t - i_{t-1} = \varphi_\pi (p_t - p_{t-1}^* - p_t^* + p_{t-1}^*) + \varphi_y (y_t - y_{t-1}) \]

where \( p_t \) is the logarithm of the price level, \( y_t \) is the logarithm of real GDP, and \( p_t^* \) denotes the price level associated with the inflation target and which evolves according to \( p_t^* = p_{t-1}^* + \pi^* \). For appropriate initial conditions, this policy rule can thus be simplified as follows:

\[ i_t = \varphi_\pi (p_t - p_t^*) + \varphi_y y_t. \]

Therefore, if the degree of interest-rate inertia is sufficiently large, then the central bank’s interest-rate policy approximately stabilizes the price level around the trend defined by the inflation target, even if it moves the interest rate in response to deviations of the inflation rate from its target. The degree of interest-rate inertia in the United States remains below 1 so that the Fed is not fully stabilizing the price level, but it does not appear to be far from it.

The FOMC announced that it had a “longer-run inflation goal” of 2 percent. It is interesting to note that stabilizing a long-run average inflation is akin to stabilizing the price level around a trend line. To see
this, remember that the average inflation gap over the $T$ periods following some initial date 0 can be written as

$$(\pi_1 + \cdots + \pi_T)/T - \pi^* = ((p_1 - p_0) + \cdots + (p_T - p_{T-1}))/T - \pi^* = (p_T - p_0)/T - \pi^*$$

Noting that the logarithm of the price-level trend starting from $p_0$ evolves according to $p_t^* = p_0 + t\pi^*$, we can rewrite the average inflation gap as

$$(\pi_1 + \cdots + \pi_T)/T - \pi^* = (p_T - p_0)/T - (p_T^* - p_0)/T = (p_T - p_T^*)/T$$

Thus, equating average inflation over the long run (that is, a distant horizon $T$) with its target is analogous to equating the price level with a trend line.