The No Surcharge Rule and Card User Rebates:
Vertical Control by a Payment Network

by

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Abstract: The No Surcharge Rule (NSR) prohibits merchants from charging higher prices to consumers who pay by card instead of other means (‘cash’). We analyze the NSR’s effects when a card association faces local monopolist merchants. Importantly, end-users’ demand for the merchant’s product is elastic. The NSR raises card issuers’ profit and harms cash users and merchants, while overall welfare decreases if and only if the ratio of cash to card users is sufficiently small. If rebates to card users are not feasible, the NSR reduces total consumer surplus (of cash plus card users) and, if the cash market is sufficiently small, even card users lose. When rebates are feasible, the association will grant them and raise its merchant fee, benefitting total consumer surplus and overall welfare but harming cash users compared to no rebates. An increase in the merchant’s gross benefit from card rather than cash sales worsens the NSR’s effects on overall welfare and total consumer surplus if card user rebates are not feasible, but the reverse holds if rebates are feasible.

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1. Introduction

Transactions through electronic payment networks (EPNs) in the U.S. exceeded $1.7 trillion in 2002 and are growing rapidly.¹ Several practices in this important industry have attracted controversy and antitrust scrutiny.² One such practice involves constraints on the ability of merchants to set different prices depending on the means of payment employed, such as credit or debit cards, cash or checks. We examine these constraints as instruments of vertical control, assess their welfare effects, and show that their presence may explain the phenomenon of rebates and reward programs in payment markets.

Uniform price constraints were at various times imposed by law or by EPN rules that prohibited merchants from imposing surcharges (or adverse non-price terms) for card payments, even though merchants may face higher costs for card transactions due to EPN fees.³ Even in the absence of formal prohibitions, merchants are often reluctant to set different retail prices for different means of payment.⁴ We refer to all these limits as the No-Surcharge ‘Rule’ (NSR). Our analysis is relevant to assessing the desirability of laws or private regulations governing surcharging; for example, prohibitions on surcharges are banned in the United Kingdom and in Australia (Reserve Bank of Australia 2002). When such repeal of the NSR is not an option —

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¹ Credit cards and offline debit cards accounted for $1.5 trillion ($755 billion for Visa and $444 billion for Mastercard — both known as bank card associations — and the rest through proprietary networks such as American Express and Discover) and $203 billion was via online debit cards. Nilson Report, March & April 2003, issues 784 & 785; ATM Debit News EFT Data Book 2003. For a lucid description of the card industry see Hunt (2003).

² For example, there is debate over whether the joint setting of certain network fees by EPN-member banks (as in bank card associations and regional networks) is anti-competitive (Salop 1990; Carleton and Frankel 1995, 1995a; Evans and Schmalensee 1995, 1999). Also, a common EPN requirement is that merchants must accept all of an EPN’s cards (e.g. debit and credit cards) if they wish to accept any. This requirement was the target of a major lawsuit by Walmart and other retailers against Mastercard and Visa, alleging anti-competitive tying. Visa and Mastercard recently settled this suit, agreeing to relax the requirement and pay plaintiffs over $3 billion.

³ Surcharges on credit card transactions were prohibited by federal statutes from 1968 to 1985 and remain prohibited by some states (e.g., Florida). For a detailed history of the U.S. legislative and regulatory treatment of surcharges, see Chakravorti and Shah (2001). In the U.S., Visa long had its own no-surcharges rule which it relaxed recently. Mastercard currently prohibits its merchants from “surcharging” customers for credit purchases, though it allows cash “discounts” (www.mastercard.com/consumer/cust_serv.html). In some European countries, card associations prohibit both discounts and surcharges. (Rochet and Tirole 2002.)

⁴ According to one retailer survey, fewer than 1% of merchants offer cash discounts. Chain Store Age, Fourth Annual Survey of Retail Credit Trends, January 1994, section 2.
because merchants’ reluctance to surcharge derives from other characteristics of the trading environment — our analysis helps understand, for example, the welfare effects of an EPN’s granting of rebates to card users. Finally, the analysis is a necessary step towards evaluating card tying policies (see fn. 2 above), since such tying would have no force if merchant surcharges were unrestricted.

Any payment network intermediates between consumers and merchants. In a proprietary (or ‘closed’) network such as American Express, the same entity deals with, and sets the fees to, merchants and card users. In an association such as Visa (or ‘open network’ since membership is open to multiple banks), a typical transaction involves two different banks: the cardholder’s (‘issuer’) and the merchant’s (‘acquirer’). The issuer sets the fees to its cardholders (e.g., a per transaction fee or, more often, a rebate) and the acquirer sets the fee to its merchants (the ‘merchant discount’ – the transaction amount minus what the merchant receives from the acquirer). The association sets an interchange fee paid by the acquirer to the issuer, which of course affects their respective charges to merchants and cardholders. Formal economic analysis of the interchange fee was pioneered by Baxter (1983), who showed that the socially optimal fee must reflect the net benefits from card use on both sides of the transaction. However, in Baxter’s analysis the association is indifferent between any levels of the interchange fee, because issuers and acquirers are assumed perfectly competitive.

Analyses of the interchange fee under imperfect competition among association members came considerably later. Schmalensee (2002) studies how the fee affects marketing efforts (and pricing) by issuers and acquirers. Closer to our focus are Rochet and Tirole (2002), who also provide a review of recent literature. In their model, consumers have unit demands for transactions but heterogeneous private values of paying with cards versus the outside instrument, ‘cash’. Duopolist merchants are spatially differentiated and, given the interchange fee, choose simultaneously whether to accept cards and, then, set prices. Acquiring banks are assumed perfectly competitive, while issuers are imperfectly competitive.\(^5\) Issuers’ fee to cardholders is represented by a reduced form, decreasing function of the interchange fee, since a higher interchange fee lowers issuers’ net marginal cost of issuing cards. If merchants can freely

\(^5\) At least in the U.S., competition is viewed as stronger on the acquiring side. Rochet and Tirole (2002, p. 552) state that acquiring is “widely viewed as highly competitive,” citing Evans and Schmalensee (1999), while issuing “is generally regarded as exhibiting market power.”
surcharge for card transactions, the level of the interchange fee is irrelevant ("neutral") and card diffusion among consumers is socially too low, because of the imperfect competition among issuers. With a no surcharge rule, there exists an equilibrium in which both merchants accept cards, the net price falls to card and rises to cash users, and card diffusion either remains too low (if so, the NSR raises overall welfare) or becomes excessive (if so, welfare may rise or fall).

Rochet and Tirole – and all other literature to our knowledge – focus on consumers’ choice between cards and cash but hold the total quantity of transactions fixed. Though a sensible first approximation, this is obviously an abstraction. Our analysis is complementary, by considering elastic demand for transactions.

To focus on this dimension, we make two simplifications. First, we treat payment mode as exogenous: one group of consumers (of mass one) use only cards, while others (of mass $\alpha$) use only cash. Transactions demand per capita is the same for members of each group and is elastic. Thus, while the number of card users is exogenous in our model, the per capita quantities of card and cash transactions are determined endogenously. The assumption that some consumers must use cash can be justified partly on grounds that a sizeable portion of the population are ineligible for cards. Moreover, as shown by Rochet and Tirole (2002, Proposition 6), the key property that the NSR allows an EPN to tax cash users continues to hold when there is imperfect substitution between cash and cards. Our assumption that cardholders cannot use cash is strong but we invoke it for two reasons. The assumption introduces consumer benefits from cards in a simple (albeit extreme) way. More importantly, it captures the opposite polar case to the one assumed in existing literature. There, a merchant who refuses cards only loses transactions to another merchant who accepts cards (Rochet and Tirole, 2002) or converts all customers to using

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6 In Rochet and Tirole, this follows because of the assumptions of unit demands and that the market is always covered in the Hotelling competition between merchants.

7 We use ‘card’ to denote any electronic payment instrument, and ‘cash’ to denote the alternative means of payment. Also, we sometimes will refer to the EPN as the card company. We abstract from the credit role of some electronic payments instruments and focus solely on its payment function. Chakravorti and Emmons (2001) present a model where some consumers use cards for both functions while others use them only as a payment instrument, and investigate the presence of cross-subsidies under an NSR from the former to the latter.

8 About 24% of U.S. families do not hold cards of any kind (Federal Reserve Board, 2001, p. 25). Presumably a large fraction of these families cannot get cards. Evans and Schmalensee (1999) characterize non-card holders as being “on the economic fringes of society” (p. 87), with a median household income 50% below the overall average, and more than 40% of them with incomes below the government's estimated poverty line.
cash (Wright, 2003). In practice, a merchant refusing cards can lose some transactions entirely, most obviously because some cardholders are liquidity constrained or perhaps would only buy certain items with cards due to other bundled properties of the card such as the superior dispute-resolution protection. Reality is likely to lie between the two polar cases.

Our second simplifying assumption is that merchants are local monopolists, so we do not explore an important aspect studied by Rochet and Tirole, the interdependence between merchants’ decisions to accept or reject cards. In their model, imperfect merchant competition is the sole reason why under the NSR card use relative to cash use can be excessive (a merchant’s gain from accepting cards comes partly by diverting sales from the other merchant). In our model, the NSR has a different welfare-reducing aspect: it lowers per capita transactions by cash users due to elastic demands. The different environments lead to some interesting differences. For example, with the NSR, card use relative to cash can remain too low in Rochet and Tirole, but is always excessive in our model. Also, if rebates to card users are not feasible, then in our model the NSR can harm even card users.

Our analysis and results contrast more sharply with those of Wright (2003), who finds that when merchants are local monopolists (as in our model), the NSR lowers price to card users and leaves price to cash users unchanged. The absence of a subsidy from cash to card users hinges on his assumption of unit demand by cash users and is responsible for the strong prediction that the NSR unambiguously increases welfare. More typically, when per capita demand varies continuously as in our model, the effects of an NSR is to induce merchants (in part) to raise their price to cash users. Holding other fees constant, this effect induces a cross-subsidy from cash to cards. We illustrate that this cross-subsidization plays an important role in explaining the prevalence of commonly observed negative charges (that is, rebates) to card users.

Regarding the association members, we follow Rochet and Tirole and others in assuming that acquirers are perfectly competitive, and we consider two polar cases regarding issuers.

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9 Wright also considers homogeneous-products Bertrand competition among merchants. As expected, cross-subsidy from cash to cards is not possible there either, this time because merchants will specialize in accepting cash or cards.

10 In Wright’s model, consumers vary in their benefits from cards but have identical maximal value $v$ if using cash. With a local monopolist merchant able to surcharge for cards, price to cash users is $v$, and higher to card users. Under no surcharging and $v$ sufficiently large, the merchant sets a uniform price of $v$, because any higher price means losing the high margin on all cash transactions.
Section 6 analyzes almost perfect competition – homogeneous Bertrand issuers with prices set in discrete units. Our main model, however, is one of perfectly collusive issuers; with perfectly competitive acquirers, this is equivalent to a monopolist proprietary EPN that sets fees to the merchant and to cardholders. Given a monopolist merchant, this environment yields double marginalization if the merchant can surcharge card transactions. Hence, one might expect an NSR to increase total consumer surplus and overall welfare, as in optimal taxation (or Ramsey pricing) where inefficiency is reduced by using a broader tax base to reduce the tax rate. The (imperfect) analogy is that, holding constant the EPN’s fees, an NSR leads the merchant to set an intermediate uniform price for all transactions instead of a higher card price and lower cash price, which reduces misallocation in the mix of transactions. We show, however, that the analogy is flawed. Since the EPN is unregulated, allowing it to tax also non-card sales via the NSR leads the EPN to raise its merchant charge. If EPN rebates to cardholders are not feasible, the higher fee to the merchant can cause the price paid by card users also to rise.\textsuperscript{11} If cardholder rebates are feasible, the NSR can induce excessive use of cards relative to cash – reversing the no NSR bias. We show how the welfare tradeoffs depend on two parameters: the size of the cash users’ group relative to card users, our parameter $\alpha$, and the merchant’s additional benefit from handling a card rather than a cash transaction, captured by a parameter $b$.

Our analysis also highlights why the NSR differs from a well-known instrument of vertical control – maximum resale price maintenance (RPM) – that also enables a supplier to reduce the margin charged on its product by an imperfectly competitive downstream firm (Tirole, 1988). Maximum RPM only reduces the price of the targeted product, while the NSR squeezes the merchant’s margin indirectly, by requiring that the same price be charged for the other product (here, cash transactions), thereby causing that price to rise.

Finally, the analysis brings out sharply how the NSR, by constraining merchant pricing, breaks the EPN’s indifference between charging to merchants or to cardholders. As several authors have noted (and we show as well), if merchants can surcharge for card transactions, the division of the EPN’s charges is neutral – only the total charge matters. With the NSR, however, the EPN prefers to load its charges on merchants, in the case of a card association by charging a

\textsuperscript{11} In contrast, standard comparisons of uniform pricing vs. third-degree price discrimination by a monopolist (our merchant) find, under regularity conditions that are met here, that requiring a uniform price causes at least some price(s) to fall (Nahata et al., 1990; Malueg, 1992).
high interchange fee. Indeed, the EPN then prefers negative fees to card users – rebates. Such rebates are often viewed as reflecting the inability of an association to prevent its issuing banks from competing for card users and dissipating rents generated by high fees to merchants. Our analysis reveals a different possibility: rebates allow an EPN to better exploit a no-surcharging constraint on merchant pricing. Consistent with this interpretation, rebates have been offered also by proprietary networks, such as Discover, where a single entity sets all fees.\textsuperscript{12} In our model, the EPN grants rebates to card users so as to boost their demand and raises its fee to merchants knowing that they will absorb part of the increase, because under the NSR any price increase must apply equally to cash users.

This paper is organized as follows. Section 2 presents the model and shows that when the merchant can set different cash and card prices, the division of the EPN’s fees between card users and merchants (hence also the interchange fee) is neutral. Section 3 shows that with the NSR, the EPN strictly prefers to shift its charge away from card users. Section 4 addresses the case where rebates to card users are not feasible. If the cash group is relatively small, the NSR harms even card users. With a larger cash group, card users gain but aggregate consumer surplus is still lower than under no NSR. Total surplus, however, is higher if and only if the cash group is sufficiently large.

Section 5 allows for rebates. With the NSR, the EPN always grants rebates, and card users gain but cash users lose compared to no NSR. Aggregate consumer surplus (of cash plus card users) rises only if the cash group is sufficiently small whereas total surplus rises only if the cash market is sufficiently large. A larger merchant benefit from using cards instead of cash improves the effects of the NSR on both total surplus and overall consumer surplus when card user rebates are feasible – but the reverse occurs when rebates are not feasible.

Section 6 considers the case of Bertrand rather than collusive issuers. An NSR again will induce rebates, benefiting card users but harming cash users. Overall consumer surplus rises regardless of the relative sizes of the two groups. However, provided the merchant’s benefit from card use is not too large, overall welfare falls with the NSR. Section 7 concludes.

\textsuperscript{12} An alternative, but not inconsistent, explanation for Discover’s behavior is that it was more eager to attract cardholders than merchants.
2. The Model and Pricing With Surcharging

Consumers: We consider two types of consumers. Type $e$ consumers (‘card users’) hold cards from the EPN. They buy units of a good only by using cards; their mass is $I$. Type $c$ consumers buy units of a good using only an outside means of payment, call it cash. We assume that they do not have cards; their mass is $\alpha$. Consumers are otherwise identical and have quasilinear preferences from purchases of goods given by

$$U(p_e, q_e) = V(q_e) - p_e q_e, \quad U(p_c, q_c) = V(q_c) - p_c q_c,$$

where $V(N(\cdot)) > 0$, $V''(\cdot) < 0$.

Throughout, $q_j$ is the per capita number of transactions of a consumer of type $j = c, e$, and $p_j$ is the net price per unit of transaction paid by such a consumer. The net price paid by cash users equals the price charged by the merchant but the two prices may differ for card users: $p_e = p_e^M + t$, where $p_e^M$ denotes the price charged by the merchant to a card-using consumer and $t$ is the per unit charge (or rebate if $t < 0$) imposed by the EPN on card users. For each type of consumer, the (downward sloping) inverse demand function is given by $V(q_j) = p_j$.

Merchants: We assume that merchants are local monopolists who treat the above inverse demand curve as the demand for their product from each type of consumer. The marginal cost of providing a good to a cash consumer is assumed constant and is normalized to zero. The merchant may also gain a benefit, $b \geq 0$, from being paid by card instead of cash, reflecting potential savings on cash-handling costs. The size of $b$ affects the degree of distortion due to double marginalization under surcharging. The merchant is charged a per-unit fee $i$ by the EPN.

The merchant’s profit is $e p_e q_e$ from cash users and $p_e^M q_e - (i-t)q_e$ from card users, where quantities are given by $p_e = V(q_e)$ and $p_e^M = V(q_e) - t$. For given values of $i$ and $t$, the merchant’s problem can therefore be formulated as choosing a level of $x$ to solve

$$\max_x (V(q_e) - (i+t-b))x.$$

Observe that $i = t = b = 0$ yields the merchant’s problem vis-a-vis the cash market. Written this way, the term $i+t$ can be interpreted as the total tax imposed on the card market by the EPN and the term $i+t-b$ as the merchant’s net marginal cost in the card market. Thus, the merchant’s optimal quantity in the card market is only a function of the total $i+t-b$ and not of the composition of the charges. For given $(i, t)$, we denote the value of the merchant’s optimization problem by $\Pi^M(i,t;b)$. 
The merchant’s alternative to accepting cards is to serve the cash market alone, yielding a profit-maximizing per-capita level of transactions \( x_0 \) and total profit \( \alpha x_0 V(x_0) \). The merchant must be assured at least this amount in any equilibrium, that is, for any \((i,t)\),

\[
IP(i,t;b) \geq \alpha x_0 V(x_0).
\] (IR)

We refer to this as the ‘individual rationality’ or IR constraint.

**Electronic Payment Network:** Our model considers a profit-maximizing agent, the EPN, setting the charge to a merchant and, for most of the paper, also to card users. This model is most obviously interpreted as one of a proprietary card network. It also characterizes the behavior of a card association under two conditions: (a) acquiring banks are identical and competitive; and (b) issuing banks are identical and collude in pricing to card users. Condition (a) implies that variations in the interchange fee are fully passed through to the merchant discount, and the merchant discount is effectively set by the EPN’s issuing banks through their choice of interchange fee. Condition (b) implies that card user charges are chosen to maximize overall profits of issuing banks.\(^{13}\) We, therefore, suppress the distinction between the interchange fee and merchant discount, and simply view the EPN as setting the charge to merchants, \( i \), monopolistically. The timing of price setting is in a Stackelberg manner: that is, the EPN sets \( t \) and \( i \) and commits to this profile of prices and, given \( t \) and \( i \), the merchant sets her monopoly price. The EPN’s marginal cost of servicing a card transaction is assumed to be zero.

We assume that two-part tariffs are not available either to the EPN or to the merchant. What is important for our analysis is that the sequential monopoly environment between the card company and the merchant lead to some inefficient pricing at both the merchant and EPN levels.\(^{14}\) For simplicity, we assume that only linear pricing is feasible for each agent.

The first-order conditions from the merchant’s problem yields a derived inverse demand

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\(^{13}\) However, in Section 6 we analyze the other polar case of competition among issuer banks — card user fees are then set in a Bertrand fashion.

\(^{14}\) There are a variety of reasons why fully efficient two-part tariffs (or other nonlinear pricing) may not be achievable for the EPN to eliminate such double marginalization. A typical EPN has relationships with a vast number of merchants, and contracting costs could make merchant-specific, two-part tariffs prohibitively expensive. Furthermore, merchants aggregated together in a single market place, such as a mall, may be able to avoid most of the impact of a fixed fee by channeling all card purchases to a single merchant. Additionally, in the context of asymmetric information, for example with heterogeneous merchants, the optimal two-part tariff generally yields some surplus to the high demand merchant and pricing at levels above marginal cost.
curve for card transactions defined, implicitly, by

\[ i+t = V(x) + xV'(x) + b \]  

(1)

Therefore, the EPN maximizes \((i+t)x\) or

\[ \Pi^*(b) = \max_x (V(x) + xV'(x) + b) x \]  

(2)

Since \(x\) is a function of \(i+t\) but not \(i\) or \(t\) separately, the card company varies \(x\) by varying the sum of charges, \(i+t\). This leads immediately to the following well-known neutrality result:\n
**Proposition 1:** Suppose merchant surcharging for card transactions is allowed. Then equilibrium card transactions, merchant profit and EPN profit all depend only on the EPN’s total fee, \(i+t\), and not on \(i\) and \(t\) individually. That is, if \((i,t)\) maximizes the profits of the EPN, then so too does any pair \((i',t')\) where \(i' + t' = i + t\).

Since the sum, \(i+t\), can be viewed as a transactions tax, Proposition 1 echoes the familiar result that the effects of a tax are invariant to whether the obligation to pay the tax is placed on buyers or on sellers. However, the next section shows that, in the presence of an NSR, EPN profits will vary for a given \(i+t\) depending on the relative values of \(i\) and \(t\).

Many of our results will hold under fairly general conditions on consumer demand, \(V_N(C)\). The proofs and intuition, however, are more concisely conveyed under the simpler case of linear demand. Thus, the remainder of the paper restricts attention to this case:

**A1)** Consumer per capita inverse demand is \(V(x) = 1 - x\) and the merchant’s benefit from card use satisfies \(b < 1\).

The assumption of linear demand enables closed form solutions for most of the relevant variables in the analysis, and ensures the following properties hold — it is these properties rather than linear demand per se that our qualitative results rely on:

**P1) The merchant’s revenue is strictly concave in quantity and price and any increase in the merchant’s marginal cost in serving cards is not fully passed through to consumers when surcharging is possible.**

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15 This result was noted in Carleton and Frankel (1995). A generalization of the result and an explanation of the intuition underlying it can be found in Gans and King (2003).
P2) The EPN’s revenue function, $(x V'(x) + V(x))$ is strictly concave in quantity.

P3) With merchant surcharging of card transactions, the EPN sets a total fee, $i + t > b$, hence per capita transactions are lower for card users than for cash users: $q_c < q_e$.

In P3), the EPN’s optimal charge to a monopolist merchant is $i + t = (1 + b)/2 > b$ given $b < 1$. Thus, the merchant’s net marginal cost is positive for serving card users, but zero for cash users. Standard revealed preference arguments (e.g. Tirole, 1988, pp. 66-67) imply that as marginal cost rises, merchant quantity falls, hence per capita card transactions are lower than cash transactions. The restriction $b < 1$ is weak — the merchant’s added benefit from card use (e.g. saving on cash handling cost) is less than the consumers’ maximal willingness to pay for the good itself.

3. Under a No Surcharge Rule the EPN Prefers Lower Card User Charges

Suppose the EPN requires any merchant that accepts its card to charge no more to card users than to cash users, $p_e^M \leq p_c$. In this section we demonstrate that such a pricing constraint binds on the merchant whenever the EPN’s fee to card users is low enough. We also show that in contrast to the neutrality result when merchant surcharging is allowed, with a binding NSR the EPN prefers to offer a high merchant fee and a low card user fee.

Proposition 1 showed that, with surcharging allowed, EPN profits are constant for any given $i + t$. Nevertheless, a merchant’s cash and card prices will vary depending on how the EPN divides its aggregate ‘tax’ between the merchant and card users. By P3), the EPN’s optimal aggregate fee with no NSR will satisfy $i + t > b$, so the merchant faces a higher net marginal cost of serving card users than cash users; thus, if $t = 0$ a card user’s inverse demand is equal to that of a cash user, hence the merchant’s higher marginal cost dictates setting a higher price to card users ($p_e^M > p_c$). Proposition 2) below shows that the merchant also prefers to charge a higher price to card users if, instead, the EPN’s fee to cardholders is positive but below some

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16 Of course, a merchant may refuse and forgo card transactions. To understand the direction of EPN incentives under the NSR, in this section we examine the structure of EPN pricing assuming the merchant’s IR constraint does not bind. In later sections we address this constraint explicitly.
We can assume the profile of charges satisfies this condition pre-NSR since, by Proposition 1, only the sum of charges matter. Proposition 2ii) shows that if the NSR is accepted, the EPN would benefit. Proposition 2iii) shows that, under an NSR, the EPN has the incentive to continue raising the charge to the merchant and lowering the charge to card users.

**Proposition 2:** Fix \( i+t \) at the level \( k>b \) and define \( t^*(k) = V(x(k-b))-V(x_0) > 0 \). For any \((i,t)\), with \( i+t = k \), \( t < t^*(k) \):

i) When merchant surcharges are allowed, \( p_e^M > p_c \), implying that with this profile of fees the imposition of an NSR constrains merchant pricing;

ii) If an NSR is accepted, holding \((i,t)\) fixed, then cash purchases fall but card purchases, and thus EPN profits, rise;

iii) Provided the merchant continues to accept the NSR, a cut in the card user fee \( t \) and an equal rise in merchant fee \( i \) increases per capita card transactions and EPN profits.

The intuition for Proposition 2ii) is straightforward. With a binding NSR, the merchant will choose a uniform price between its pre-NSR card and cash prices: starting from a uniform price equal to \( p_e^M \), a small move towards \( p_c \) imposes a zero first-order loss in the card market while moving closer to the optimal cash price, and similarly starting from \( p_c^M \) and moving towards \( p_e^M \). Proposition 2ii) establishes that the EPN gains if a binding NSR is accepted: EPN profits rise at the pre-NSR charges \((i,t)\) and any departure from these price post-NSR, by revealed preference, would further benefit the EPN.

The intuition behind Proposition 2iii) is as follows. A cut in \( t \) and an offsetting increase in \( i \) would leave the EPN’s margin unchanged, and hence profit unchanged, only if card transactions remained unchanged. This in turn would only happen if the merchant raised her price to card users by the full increase in \( i \), since card users’ inverse demand shifts up by an amount equal to the fall in \( t \) (equivalently, to the increase in \( i \)). But since the NSR forces the merchant to charge

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\( ^{17} \) An implication of this observation is that, with a low cardholder fee (which Proposition 2 shows is desired by the EPN), we can formulate the no-surcharge rule mathematically as the inequality constraint, \( p_e^M \leq p_c \), even if, formally, the constraint is a ‘No Discrimination Rule’, that is, a uniform pricing rule rather than a no-surcharge on card use rule (which would be better captured by the constraint, \( p_c^M = p_c \)). Although credit card companies, for example, have historically imposed such rules on their merchant clients, an inequality constraint may obscure other reasons for merchant pricing constraints. Some merchants argue that even without a formal no-surcharge rule, social conventions make it very difficult for them to charge different prices for users of different means of payments. Proposition 2i) shows when the effects of the two constraints are the same.
the same price to cash users as to card users, and since the marginal cost of serving cash users has not changed, the merchant prefers to raise her uniform price by less than the full increase in \( i \) and accept a lower margin on card sales.\(^\text{18}\)

Proposition 2 shows that with the NSR, it becomes relevant how \( i+t \) is distributed: the EPN prefers a lower card user charge (provided the merchant still accepts). Rebates to card users -- negative \( t \) -- are often taken as evidence of the inability of a bank card association to control competition for card users by its member banks. Proposition 2iii) offers an alternative interpretation: rebates can be a pricing tactic designed to better exploit the power of the NSR.\(^\text{19}\)

Given the incentives for an EPN to raise \( i \) and reduce \( t \), what determines the floor on \( t \)? One limit may be institutional. For historical, practical or other reasons, rebates to card users may not be an option.\(^\text{20}\) Section 4 investigates the effects of the NSR when rebates are not possible. A priori, the binding constraint may be the non-negativity of \( t \), the merchant’s option to reject the NSR and serve only cash users, or both.\(^\text{21}\) In fact, as Proposition 3 illustrates, the non-negativity constraint always binds. Section 5 allows for rebates, showing that the EPN may be constrained either by the merchant’s IR constraint or by the need to ensure that the merchant will not price so high that cash users are driven out (a type of incentive compatibility constraint).

\(^{\text{18}}\) This argument also implies that, under the NSR, the total price to card users falls by more with a unit reduction in \( t \) than in \( i \). Let \( p \) denote the merchant’s price to card users, hence their total price is \( p + t \). Under surcharging, Proposition 1 implies that a unit reduction in \( i \) or in \( t \) yields the same change in \( p + t \):

\[
\frac{\partial (p+t)}{\partial i} = \frac{\partial p}{\partial i}, \quad \frac{\partial (p+t)}{\partial t} = \frac{\partial p}{\partial t}, \quad \text{so} \quad 1 + \frac{\partial p}{\partial i} = \frac{\partial p}{\partial t}.
\]

The NSR, however, dampens the merchant’s price response to a change in \( t \) or in \( i \) (\(|\frac{\partial p}{\partial i}\| \text{and} \|\frac{\partial p}{\partial t}\| \text{fall} \)), because the same price change must be made also in the cash market. Thus, cutting \( i \) yields a smaller reduction in the price to card users under the NSR than under surcharging (\( \frac{\partial p}{\partial i} \) is smaller), while cutting \( t \) yields a larger reduction under the NSR (\( \frac{\partial p}{\partial i} \) is negative but smaller in absolute value, so \( 1 + \frac{\partial p}{\partial i} \) is positive and larger): card users receive this cut directly, and (by \( P1 \)) the merchant responds by increasing \( p \) by less than with no NSR.

\(^{\text{19}}\) Gerstner and Hess (1991) obtain a similar effect in a somewhat different context. They consider a monopolist manufacturer selling to a monopolist retailer that faces two customer groups, low demanders and high demanders, where high demanders incur a higher transaction cost of using a rebate/coupon. In our model, the NSR plays roughly the same role as their differential transaction costs in motivating rebates.

\(^{\text{20}}\) The phenomenon of card user rebates is relatively recent. While credit cards date to the late 1960s/early 1970s, money-back rebates were first offered, by Discover, in 1986. Rebate cards only became common, however, in the early 1990s with the introduction of the GM Mastercard and other cards that offer reward points associated with co-branding partner companies (such as frequent-flier miles). See generally, Evans and Schmalensee (1999). By the late 1990s, roughly half of all credit volume were associated with rebates of various sorts. Faulkner and Gray (2000).

\(^{\text{21}}\) In \((i,t)\) space, under the NSR the merchant’s level sets have slope strictly less than \(-1\). Therefore, for any given \( k \), the line \( t = k - i \) eventually crosses the line given by \( I^{\text{NSR}}(i,t;b) = \alpha x_i \setminus \{ x_0 \} \). Thus, if the EPN holds \( i+t \) fixed and lowers \( t \), it eventually runs against the merchant IR constraint.
4. Equilibrium Under No Rebates

Let $i_0$ be the EPN’s optimal charge given $t = 0$ and (for the moment) ignoring the merchant’s IR constraint. Whether or not the IR binds depends on the relative size of the cash market, $\alpha$. If, at $(0, i_0)$, the IR does not bind, then, by Proposition 2, these prices are optimal for the EPN. If the IR is violated at these prices, in Proposition 3 we provide sufficient conditions under which setting $t = 0$ is still optimal for the EPN.

With an NSR, cash users and card users pay the same merchant price, $p$. Linear demand then implies that per capita cash consumption is $q_c = 1 - p$ and per capita card consumption is $q_c = 1 - p - t$. For any given $(i, t)$, the merchant then selects price to solve

$$I^{\text{NSR}}(i, t; b) = \max_x \alpha(p(1-p)+(p-i+b)(1-p-t),$$

yielding

$$p(i, t; b) = \frac{1+\alpha + i - b - t}{2(1+\alpha)}, \quad q_c(i, t; b) = \frac{1+\alpha - i + b + t}{2(1+\alpha)}, \quad q_e(i, t; b) = \frac{1+\alpha - i + b - t(1+2\alpha)}{2(1+\alpha)} \quad (3)$$

The EPN’s profit maximization problem when an NSR is imposed can now be expressed as

$$P^{\text{EPN}} : \max_{i, t} (i + t) q_e(i, t; b)$$

s.t. \hspace{1cm} I^{\text{NSR}}(i, t; b) \geq \alpha x_0 V(x_0) \hspace{1cm} (IR)$$

$$t \geq 0 \hspace{1cm} \text{(No Rebates)}.$$

The objective function of the EPN is concave in $(i, t)$ while the IR constraint is linear in $(i, t)$. Thus the Kuhn-Tucker first order conditions are sufficient.\(^{22}\)

**Proposition 3 (Prices):** Suppose card rebates are not feasible ($t \geq 0$). Under the NSR:

i) For any relative size of the cash market, $\alpha$, the EPN’s optimal fee implies $t = 0$ (no card fees), hence per capita card and cash transactions are equal.

ii) There exists $\alpha^*$ such that the EPN choice of $i$ is determined by the merchant’s IR constraint if and only if $\alpha > \alpha^*$.

iii) If $\alpha > \alpha^*$, then $i + t - b (= i - b)$ is independent of the merchant benefit $b$ and falls as $\alpha$ rises.

iv) The interchange fee, $i$, is higher than the total charge under surcharging for all $\alpha$; $i$ increases in $\alpha$ for $\alpha < \alpha^*$, otherwise it decreases in $\alpha$.

\(^{22}\) With the exception of 3iv) Proposition 3 can be shown to hold for demand functions much more general than linear demand.
For Proposition 4ii)c), define the change in total surplus when rebates are not feasible ($\Delta T^{SNR}$) to be total surplus under the NSR without rebates minus total surplus under no NSR.\textsuperscript{23}

**Proposition 4 (Quantities and Welfare):** Suppose an NSR is imposed but card user rebates are not feasible ($t \geq 0$). Compared to the equilibrium with no NSR,

i) If the cash market is small enough that the merchant IR does not bind ($x \leq x^*$), then:
   a) Cash users’ transactions and consumer surplus are lower;
   b) Card users’ transactions and consumer surplus are unchanged if $b = 0$ and lower if $b > 0$;

ii) If the merchant IR binds ($x > x^*$), then:
   a) Cash users’ transactions and consumer surplus are lower;
   b) Card users’ transactions and consumer surplus are higher if $x$ is sufficiently larger than $x^*$;
   c) Aggregate quantity ($q_c + x q_e$) and aggregate consumer surplus are lower for all $x$ and all $b$.
   d) $\Delta T^{SNR}$ rises in $x$ and falls in $b$. For $b=0$, $\Delta T^{SNR}=0$ at a value of $x$ above $x^*$.

Proposition 3i) shows that the EPN’s desire for lower card user fees and higher merchant fees illustrated in Proposition 2 outweighs the merchant’s preference for the reverse pattern. Even if the merchant’s IR constraint binds on the EPN before the EPN achieves its optimal fee pair, the EPN will choose to move down the merchant’s IR locus to set card user fees to zero: the EPN chooses its unconstrained optimum ($i_o, 0$) if the merchant’s IR is not binding at this point, otherwise it accepts a merchant fee lower than $i_o$ but maintains $t=0$. An implication is that if the non-negativity constraint is relaxed (rebates are allowed, as in Section 5) then the EPN under the NSR will set $t$ negative.

Proposition 3ii) shows that the merchant IR constraint binds if and only if the cash market is not too small. (With $b=0$, the IR binds if $x > x^* = 1/3$.) Intuitively, the merchant’s profit from serving only cash customers is proportional to the size of the cash market, therefore, as the latter increases, the EPN eventually must depart from its unconstrained optimal charges to maintain merchant participation.

\textsuperscript{23} With the exception of 4ii)c) Proposition 4 can be shown to hold for demand functions much more general than linear demand.
Under cost and demand conditions satisfied here, prohibiting third-degree price discrimination leads a monopolist to charge an intermediate uniform price. Sufficient conditions are that marginal cost be non-decreasing and that demands in the various markets be independent, each yielding a quasi-concave profit function (Nahata et al. 1990, Malueg 1992).

The quantity effects and the role of $b$ can be understood as follows. Under surcharging, the derived inverse demand function facing the EPN is $i(q) = b + (V''q + V')$, where $V''q + V'$ is the merchant’s decreasing marginal revenue function. With the NSR, the EPN faces $i^N(q) = b + (1+\alpha)(V''q + V')$. Recall that $x_0$ is the merchant’s monopoly output for zero marginal cost (the cash-market output with surcharging), hence: $V''(x_0)x_0 + V'(x_0) = 0$. Thus, $i^N(q)$ cuts $i(q)$ from above at $q=x_0$, $i=b$: $i^N(q) = i(q) = b$ at $q = x_0$, while $i^N(q) > i(q)$ at $q < x_0$ and $i^N(q) < i(q)$ at $q > x_0$. (Intuitively, $i=b$ makes the merchant’s net marginal cost of card transactions zero—as for cash—so the merchant would choose equal card and cash quantities $x_0$ under surcharging hence the NSR would have no effect. Card quantities $q < x_0$ correspond to the merchant facing higher marginal cost for card than for cash sales, hence the EPN can attain such quantities at a higher $i$ under the NSR because the merchant’s uniform price is then pulled down by the lower marginal cost on cash sales; conversely, $q > x_0$ requires cutting $i$ below $b$ by more under the NSR than under surcharging.) The equilibrium card quantity is where the EPN’s marginal revenue obtained from the relevant inverse demand function, $i^N(q)$ or $i(q)$, equals its marginal cost of zero: $MR = i(q) + i'(q)q = 0$ under surcharging, and $MR^N = i^N(q) + i^N'(q)q = 0$ under the NSR. Observe that $MR^N = (1+\alpha)MR - ab$. Thus, for $b = 0$, the card quantity is the same with surcharging or the NSR. For $b > 0$, $MR = 0$ implies $MR^N < 0$, so the EPN’s optimal card quantity is lower under the NSR. (Note that this will be true also if the EPN’s marginal cost were positive but not too large.)

IR Not Binding. One might have expected the NSR to raise card transactions by inducing the merchant to choose a uniform price that lies between its card and cash prices under surcharging. This indeed would occur if the EPN’s fees remain fixed. Instead, the NSR leads the EPN to adopt a merchant fee $i_0$ so much higher than its total fee $i+t$ under no NSR that card transactions remain unchanged with the NSR if $b=0$ and fall if $b>0$ (Proposition 4i)).

When $\alpha < \alpha^*$, the welfare consequences of the NSR are therefore stark. The NSR reduces even card transactions (leaving them unchanged only if $b=0$), thus harming card users. Since (per capita) cash transactions exceed card transactions under surcharging but are equal to them with the NSR, the NSR also reduces cash transactions. With all quantities falling, total surplus must fall. The merchant’s profit also falls since the NSR both leads to a higher total EPN charge and constrains the merchant’s pricing to consumers. The NSR in this case therefore benefits only the EPN at the expense of all other parties.

IR Binding. When the cash market is large enough that the merchant’s IR constraint binds on the EPN’s pricing, the NSR still reduces the merchant’s profit — since the merchant now loses all its surplus from dealing with the EPN — and cash transactions. However, if the cash market is sufficiently large, card transactions are higher with the NSR (Proposition 4ii)).

To see this, observe that with the NSR and no rebates ($t=0$) the merchant’s profit can be expressed as $-\frac{(1+\alpha)Q^2V'(Q)}{\alpha V'(x_0)}$, where $Q$ is the equal per-capital level of cash and card transactions. The IR constraint is therefore $-\frac{(1+\alpha)Q^2V'(Q)}{\alpha V'(x_0)} = \alpha x_0 V''(x_0)$ or

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24 Under cost and demand conditions satisfied here, prohibiting third-degree price discrimination leads a monopolist to charge an intermediate uniform price. Sufficient conditions are that marginal cost be non-decreasing and that demands in the various markets be independent, each yielding a quasi-concave profit function (Nahata et al. 1990, Malueg 1992).

25 The quantity effects and the role of $b$ can be understood as follows. Under surcharging, the derived inverse demand function facing the EPN is $i(q) = b + (V''q + V')$, where $V''q + V'$ is the merchant’s decreasing marginal revenue function. With the NSR, the EPN faces $i^N(q) = b + (1+\alpha)(V''q + V')$. Recall that $x_0$ is the merchant’s monopoly output for zero marginal cost (the cash-market output with surcharging), hence: $V''(x_0)x_0 + V'(x_0) = 0$. Thus, $i^N(q)$ cuts $i(q)$ from above at $q=x_0$, $i=b$: $i^N(q) = i(q) = b$ at $q = x_0$, while $i^N(q) > i(q)$ at $q < x_0$ and $i^N(q) < i(q)$ at $q > x_0$. (Intuitively, $i=b$ makes the merchant’s net marginal cost of card transactions zero—as for cash—so the merchant would choose equal card and cash quantities $x_0$ under surcharging hence the NSR would have no effect. Card quantities $q < x_0$ correspond to the merchant facing higher marginal cost for card than for cash sales, hence the EPN can attain such quantities at a higher $i$ under the NSR because the merchant’s uniform price is then pulled down by the lower marginal cost on cash sales; conversely, $q > x_0$ requires cutting $i$ below $b$ by more under the NSR than under surcharging.) The equilibrium card quantity is where the EPN’s marginal revenue obtained from the relevant inverse demand function, $i^N(q)$ or $i(q)$, equals its marginal cost of zero: $MR = i(q) + i'(q)q = 0$ under surcharging, and $MR^N = i^N(q) + i^N'(q)q = 0$ under the NSR. Observe that $MR^N = (1+\alpha)MR - ab$. Thus, for $b = 0$, the card quantity is the same with surcharging or the NSR. For $b > 0$, $MR = 0$ implies $MR^N < 0$, so the EPN’s optimal card quantity is lower under the NSR. (Note that this will be true also if the EPN’s marginal cost were positive but not too large.)
\[ - Q^2 V'(Q) = (\alpha(1+\alpha)x_0V'(x_0)). \]  

As the size of the cash market, \( \alpha \), increases, to satisfy (4) the EPN must induce an increase in \( Q \) (since concavity of the merchant’s revenue function in quantity implies merchant profit is increasing in \( Q \)), which requires cutting the merchant fee \( i \). As \( \alpha \to \infty \), \((\alpha(1+\alpha)x_0V'(x_0) \to x_0V'(x_0)\), so \( Q \) must approach \( x_0 \), the merchant’s cash market quantity under surcharging. The NSR therefore lowers cash transactions (since \( Q < x_0 \) except in the limit), but for sufficiently high \( \alpha \) it raises card transactions (since these are less than \( x_0 \) under surcharging, by P3)).

Total quantity is lower under the NSR (Proposition 4ii(c)). Given equal per-capita linear demands by card and cash users, imposing the NSR would leave total quantity unchanged only if the EPN’s total charge remained unchanged, but in fact the EPN raises its total charge (Proposition 3iv) to exploit the decreased elasticity of demand that it faces from the merchant, so total quantity falls. Overall consumer surplus, therefore, also must fall because of the following property:

**Lemma 1:** Consider any pair of prices \((p_c, p_e)\) to cash users and card users (where \( p_c \) includes any EPN charge \( t \)) that yield a fixed total quantity of transactions, \( \alpha q_c + q_e = k \). Then overall consumer surplus of cash and card users increases with the dispersion in per capita quantities \( |q_c - q_e| \).

Intuitively, identical linear demands for cash and card users imply that (a) total quantity is constant only if the weighted average price \( \alpha p_c + p_e \) is constant, and (b) dispersion in per capita quantities \( |q_c - q_e| \) is linear in \( |p_c - p_e| \). Overall consumer surplus, \( \alpha S(p_c) + S(p_e) \), is proportional to the consumer surplus of an individual who faces \( p_c \) with probability \( \alpha/(1+\alpha) \) and \( p_e \) with probability \( 1/(1+\alpha) \). Given that an individual’s consumer surplus \( S(p) \) is convex in price, any mean-preserving spread of the prices increases the expected surplus. Since the NSR with no rebates reduces the spread in per capita quantities (to zero) as well as total quantity, overall consumer surplus must fall.

Total surplus, however, can be higher with the NSR if the cash market is large enough. The efficiency gain comes because the lower total quantity of transactions is allocated more efficiently between cash and card users. To see this, consider \( b = 0 \), in which case the welfare maximizing allocation requires equal per capita card and cash quantities. The NSR achieves this,
while surcharging does not. If the cash market is sufficiently larger than the level where the merchant’s IR binds on the EPN (for \( b = 0 \), if \( \alpha > 1.53 > \alpha^* = 1/3 \)), then the benefit from improved allocation outweighs the harm from the reduction in total quantity so the NSR raises total surplus. Intuitively, a large cash market allows the NSR to curb the double marginalization that curtails card transactions under surcharging while introducing only a small distortion in the *per capita* quantity of cash transactions.

Finally, consider the role of \( b \), the merchant’s gross benefit from card rather than cash transactions. With surcharging, the distortion from double marginalization on card transactions increases with \( b \) (the gap between the efficient and actual card quantities is \( 3(1+b)/4 \)). Since in our model double marginalization is what motivates the NSR, one might expect the NSR to have a more favorable effect on total surplus the larger is \( b \). In fact, the NSR’s advantage over surcharging in raising card transactions is *smaller* the larger is \( b \); hence for total surplus, the best case for the NSR occurs when \( b = 0 \). Section 5 shows (and explains why) this conclusion is reversed when rebates to card holders are feasible; there, \( \Delta TS \) is *increasing* in \( b \).

### 5. Equilibrium When Rebates Are Feasible

Proposition 3i) shows that when the card user fee must be non-negative, the EPN cuts this fee to 0. Thus, this is no longer the equilibrium when rebates are feasible (\( t < 0 \)).

One obvious constraint on the EPN’s equilibrium charges remains the merchant’s IR, its option to reject the EPN and forgo card users as discussed earlier. In addition, a less evident constraint emerges when rebates are feasible: the merchant’s willingness to continue serving *cash* customers. With large enough card user rebates and a sufficiently small cash market, the monopoly price appropriate for card users alone will exceed the choke price of cash users, and the merchant under the NSR will choose this price instead of cutting price enough to serve also cash users.\(^{26}\) Such an outcome, however, clearly is not optimal for the EPN: since the merchant’s price to card users is then unaffected by cash users, the NSR loses its value. This issue does not arise with \( t \geq 0 \) — (per capita) inverse demand of cash users is then lower than that of card users, so any price that yields cash sales also yields card sales — but must be tackled under rebates.

Propositions 5 and 6 compare the equilibrium under the NSR with rebates to that under

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\(^{26}\) A monopolist that faces two markets but is prohibited from 3\textsuperscript{rd}-degree price discrimination will drop the low market if the dispersion in the demands is sufficiently large (Tirole 1988, p.139).
no NSR. Proposition 7 summarizes the incremental effect of rebates by comparing the equilibria under the NSR with and without rebates.

**Proposition 5 (Prices): Under an NSR with rebates feasible:**

i) For all $\alpha$, the EPN’s optimal choice involves granting rebates $(t<0)$;

ii) For low $\alpha$ ($<.22$ if $b=0$), the requirement that $(i, t)$ induce the merchant to continue to sell to cash customers is a binding constraint on the EPN; for high enough $\alpha$ (above approximately .18 if $b=0$) the IR constraint binds.

iii) When the IR binds, the sum of card user and merchant charges, $i+t$, is the same as the EPN’s optimal choice under surcharging ($(1+b)/2$). As $\alpha$ increases, $i$ falls and $t$ rises.

Proposition 5i) follows from Proposition 3i) for the case where rebates were not feasible. Figure 1 illustrates the case where the merchant’s IR constraint does not bind at $i_0$, the EPN’s optimal merchant fee conditional on $t=0$. Recall from Proposition 2 that, for fixed $i+t$, the EPN wishes to lower $i$ in the absence of other constraints. Thus, a movement down and to the right along the line $i+t = i_0$ (i.e., a cut in $t$ and an equal increase in $i$) raises EPN profit. Expression (4) yields the IR constraint. Given linear demand, this constraint is linear with slope steeper than -1. Point B in Figure 1 represents the intersection of the line $i+t$ with this manifold. The EPN’s solution is, then, to move down and to the right from $(i_0, 0)$ to B along the line $i+t = i_0$ then down the IR line until it reaches an EPN indifference curve that is tangent to the IR (point C in Figure 1). This point represents a lower total EPN charge, $i+t$, and a lower $t$ compared to $(i_0, 0)$. If, instead, the IR constraint binds at $t=0$ (IR cuts the horizontal axis at $i < i_0$), then with rebates the EPN immediately moves down and to the right along the IR manifold to a point of tangency. In both cases, therefore, $i+t$ is lower with rebates than in the NSR equilibrium under no rebates.

Turning to Proposition 5ii), if the cash market is sufficiently small then the floor on $t$ is not the merchant’s IR constraint but the need to induce the merchant to continue serving cash users; consequently, even with rebates feasible, the EPN cannot always fully extract the merchant’s surplus.

When the cash market is large enough that EPN charges are determined by the IR constraint, the total charge $i+t$ under the NSR is the same as under surcharging and is independent of the size of the cash market, $\alpha$, but the spread between $i$ and $t$ (which is irrelevant under surcharging) shrinks as $\alpha$ increases. (Proposition 5iii.) These results can be understood as follows. Recall the merchant’s choices as functions of $(i, t)$ given by expression (3). Note that $q_e$
is decreasing in \( i \) and \( t \), but the effect of \( t \) is stronger. (By contrast, \( i \) and \( t \) affect \( q_e \) symmetrically, since both operate on \( q_e \) only indirectly via the merchant’s price \( p \).) The EPN’s equilibrium fees are then (see Appendix),

\[
t^* = -\frac{1+b}{4(\alpha + \sqrt{\alpha^2 + 1})} \frac{b}{2}, \quad i^* = \frac{1+b}{2} - t^*
\]

The total charge when the merchant IR binds, \( t^* + i^* \), is therefore \((1+b)/2\), the same as under surcharging, but lower than under the NSR with no rebates. Under the NSR with rebates, card transactions increase more if the total EPN fee is cut through rebates than through cutting \( i \) (see fn.18). The EPN prefers to grant rebates and reduce the total charge as needed to satisfy the merchant’s IR because it gains enough from the increased transactions. As the cash market grows, the EPN continues meeting the IR with the same total charge (as opposed to cutting it under no rebates) but reducing the spread between \( i \) and \( t \): \( t^* \) rises with \( \alpha \) (smaller rebates) while \( i^* \) falls. The merchant benefits from this reduced spread because it gains the option of maintaining the same margin \( p-i \) on cards but at a price \( p \) closer to the cash market optimum.

The next Proposition describes the effects of the NSR with rebates on quantities and welfare, when the cash market is large enough that EPN charges are determined by the merchant IR constraint (\( \alpha > 0.22 \) if \( b=0 \)). Define the change in total surplus when rebates are allowed \((\Delta TS^n)\) to be total surplus under the NSR with rebates minus total surplus under no NSR. The change in aggregate consumer surplus \((\Delta CS^n)\) is defined analogously.

**Proposition 6 (Quantities and Welfare):** Suppose an NSR is imposed and rebates are feasible.

For \( \alpha \) large enough that the merchant IR binds, compared to the equilibrium with no NSR:

i) Cash users’ transactions and consumer surplus are lower;

ii) Card users’ transactions and consumer surplus are higher;

iii) Aggregate transactions \((q_e + \alpha q_c)\) are unchanged;

iv) \( \Delta TS^n \) rises in \( \alpha \). For \( b=0 \), it is positive if and only if \( \alpha > 1/3 \); 

v) \( \Delta CS^n \) falls in \( \alpha \). For \( b=0 \), it is negative if and only if \( \alpha > 1/3 \);

vi) \( \Delta TS^n \) and \( \Delta CS^n \) rise in \( b \).

Parts i)-iii) of Proposition 6 follow because the EPN’s total charge \( i+t \) is equal under the two regimes. Under surcharging, equilibrium quantities are invariant to how \( i+t \) is divided
between $i$ and $t$, in particular, the same quantities would arise if one set $(i^*, t^*)$ — the values that are optimal under the NSR. Imposing the NSR while charging $(i^*, t^*)$, however, constrains the merchant’s retail pricing, causing cash transactions to fall and card transactions to rise; total transactions remain the same because of the linearity of demand.

Now consider why $\Delta TS^r$ increases with the size of the cash market (Proposition 6iv). As $\alpha$ increases, the total quantity of transactions rises under both regimes but remains equal. Thus, the behavior of $\Delta TS^r$ hinges on the change in allocation of transactions between cash and card users. The efficient per-capita levels are 1 for cash and $1+b$ for cards. With surcharging, the cash quantity is $1/2$ and the card quantity is $(1+b)/4$, both independent of $\alpha$. Under the NSR, as $\alpha$ increases both quantities rise towards $1/2$. As $\alpha$ increases, therefore, the allocation improves only under the NSR, so $\Delta TS^r$ rises in $\alpha$. With zero merchant benefit from card use, $b=0$, $\Delta TS^r$ is positive if and only if $\alpha > 1/3$. The case of $b>0$ is discussed shortly.

By contrast, the change in aggregate consumer surplus when moving to the NSR with rebates declines in $\alpha$ (Proposition 6v). The narrowing of the gap between per-capita cash and card quantities as $\alpha$ increases under the NSR — but not under surcharging — is harmful to overall consumer surplus, by Lemma 1. For $b=0$, the NSR with rebates reduces overall consumer surplus if and only if $\alpha > 1/3$.

**Positive Merchant Benefit.** Moving from surcharging to the NSR with rebates, therefore, causes opposite changes in total surplus and overall consumer surplus if $b=0$: when $\Delta TS^r > 0$ ($\alpha > 1/3$), $\Delta CS^r < 0$. Thus, the increase in EPN profit always comes at least partly at the expense of consumers and the merchant (recall that the merchant loses for all $\alpha$). The case $b=0$, however, presents an overly negative picture of the NSR. When there are gross merchant benefits from card use, the NSR with rebates can increase both total surplus and overall consumer surplus. Since both $\Delta TS^r$ and $\Delta CS^r$ are increasing in $b$ (Proposition 6vi) and both equal 0 at $\alpha = 1/3$, for $b>0$ there will be an interval around $\alpha=1/3$ in which $\Delta TS^r$ and $\Delta CS^r > 0$. These results are illustrated in Figure 2.

The intuition for why $\Delta TS^r$ and $\Delta CS^r$ are increasing in $b$ is as follows, starting with total surplus. Since total transactions are equal under surcharging and under the NSR with rebates, the differential effect of $b$ under these regimes works via its effect on the mix of transactions. The efficient per-capita cash and card quantities are $1+b$ and 1. With surcharging, quantities are $1/2$ and $(1+b)/4$, hence $q_c - q_c = (b-1)/4$. Recalling that $b<1$, the card quantity is always lower than the cash quantity under surcharging, and the gap closes at the rate $\Delta b/4$. Under the NSR and
rebates, \( q_e \) is higher than with surcharging, and any such gap is socially more valuable the larger is \( b \). Moreover, as \( b \) increases the gap between card and cash quantities rises faster under the NSR than under surcharging and the gap under the NSR never exceeds the efficient gap, \( b \). Thus, an increase in \( b \) magnifies the allocation advantage of the NSR. Finally, since the gap \( |q_e - q_c| \) rises with \( b \) under the NSR but falls under surcharging, while total quantity is the same under both regimes, it follows that \( \Delta CS^r \) increases in \( b \) by Lemma 1.

Interestingly, the favorable impact of \( b \) on the NSR’s effect on total surplus is reversed when rebates are not feasible (Proposition 4ii)(d). With the NSR and no rebates, the EPN responds to an increase in \( b \) (in the range of \( \alpha \) where the merchant’s IR binds) by raising its charge to the merchant so as to leave the net marginal cost of card transactions, \( i+t-b \), unchanged (Proposition 3iii), and therefore quantities unchanged.\(^{27}\) Under surcharging, an increase in \( b \) leads the EPN to permit a reduction in \( i+t-b \), and therefore an increase in card transactions. Thus, an increase in \( b \) raises card transactions under surcharging but not under the NSR with no rebates. Under the NSR with rebates, however, the EPN responds to an increase in \( b \) by allowing card transactions to rise faster than under no NSR, causing \( \Delta TS^r \) to increase in \( b \).

Proposition 7 draws on previous results to compare the outcomes under the NSR if rebates are or are not feasible. For simplicity, we focus on the case when the cash market is large enough that the merchant’s IR determines EPN pricing with or without rebates.

**Proposition 7 (Rebates vs. No Rebates):** Suppose the NSR is imposed, the merchant’s IR binds, and rebates are feasible. Compared to the outcome under the NSR with no rebates:

i) Card users’ consumer surplus is higher, cash users’ consumer surplus is lower, and aggregate consumer surplus is higher with rebates.

ii) For relative sizes of the cash market \( \alpha \) that make the merchant’s IR constraint bind in both cases, total transactions and total surplus are higher with rebates.

The superiority of rebates for total output and overall welfare (Proposition 7ii) reflects the ability of rebates under the NSR (and only then) to more effectively reduce double marginalization than by relying just on cutting the merchant fee. Recall that with the NSR an increase in \( i \) by \( \Delta \) and an equal cut in \( t \) would lower the net price to card users, because the

\(^{27}\) Under the NSR and no rebates, when the merchant’s IR binds, per capita transactions \( Q \) are determined by (3), whose right hand side is independent of \( b \) — since the merchant’s outside option of serving only cash customers is independent of \( b \), so too is the profit, and thus quantity, \( Q \), that the EPN must leave to the merchant.
merchant would raise its uniform price by less than $\Delta$. (In (3), $\Delta p = (\Delta i - \Delta t)/(1 + \alpha) = 1/((1 + \alpha)$.)

In fact, the price to card users is even lower under rebates, because when cutting $t$ below 0 the EPN raises its merchant fee by less than the rebate amount. The lower aggregate EPN charge $i + t$ under rebates implies that total transactions increase. Aggregate consumer surplus therefore rises with rebates because total quantity is higher and per capita transactions are unequal under rebates but equal with no rebates which further benefits consumers (Lemma 1). Total surplus is therefore also higher with rebates: overall consumer surplus is higher, the EPN’s profit is higher (by revealed preference), and the merchant’s profit is the same (for values of $\alpha$ that make the IR bind under rebates or no rebates, the merchant loses the entire surplus from dealing with the EPN in either case).

Cash users, however, lose from the NSR even with no rebates to card users (Proposition 4), and lose further if such rebates are feasible. Granting rebates increases the inverse demand of card users, prompting the merchant to raise its retail price.\footnote{Gerstner and Hess (1991) cite empirical evidence that retailers indeed raise their prices in response to manufacturers’ granting of rebates to consumers.} In addition, when the EPN cuts $t$ below 0 it also raises $i$ somewhat, putting further upward pressure on the merchant’s price (see (3) where $p$ increases in $i$ and decreases in $t$, while $q_c$ does the reverse).

6. Competitive Card Issuers

To this point, our analysis applies most directly to the case of proprietary networks where the EPN is a single card issuer. Alternatively, it describes outcomes when despite multiple card issuers, the issuing industry behaves as if were maximizing issuing banks’ joint profits. How do the results change if the EPN is an association of competitive issuing banks? In this scenario, member banks issue the cards, and they, rather than the network, set most of the terms to cardholders, including prices (annual fee, interest rate, rebates). This section explores the effects of an NSR when the EPN is unable to control $t$.

A sequential/simultaneous game emerges. First, through their partnership with the EPN, banks set the merchant discount fee $i$ and commit to it. Merchants continue to set prices taking $i$ as given but recognizing that $t$ is determined through competition for card users by issuing banks. If bank member $W$ of the EPN is one of $m$ banks charging the lowest card user fee, it obtains
The Bertrand assumption implies \( t = -i \). Linear demand implies that, under surcharging, the merchant’s price is \( (1-b-2t)/2 \). This exceeds the cash price \( (1/2) \); that is, the NSR binds only if \( t < -b/2 \).

The theorem is shown for \( b = 0 \), however, given the continuity of the environment, quantity and welfare results will continue to hold for \( b \) small and positive. They may not hold for \( b \) large since, even with competitive issuers, there is then a significant bias away from cards under no NSR. (The efficient quantities are \( 1 \) for cash and \( 1+b \) for cards while the no NSR levels are \( 1/2 \) and \( (1+b)/2 \), so only the card underprovision rises with \( b \).)
Proposition 8: Assume \( b = 0 \). With perfectly competitive issuers, in the equilibrium under the NSR:

i) If \( \alpha < 1 \), the EPN sets \( i \) until merchants are just indifferent between selling to cash customers or not; if \( \alpha > 1 \), the merchant’s IR constraint binds;

ii) Cash transactions are lower than with no NSR, card transactions are higher, but total transactions are the same;

iii) For all values of \( \alpha \), overall consumer surplus is higher than with no NSR but merchant profit and total surplus are lower;

iv) In the limit as the mass of cash users becomes large, the per-capita cash quantity approaches the single monopoly level and the per capita card quantity approaches the competitive level.

Proposition 8i) illustrates that, with competitive issuers, the constraint that the EPN ensures that the merchant continues to serve the cash market binds for a larger size of the cash market (\( \alpha \leq 1 \) rather than \( \alpha \leq .22 \)). This is because the stronger tendency to offer rebates under competition among card issuers makes the option of pricing cash users entirely out of the market relatively more attractive to merchants. Total quantity remains the same as under no NSR (Proposition 8iii) because demand is linear and the total EPN fee remains the same (zero). Card and cash quantities therefore move in opposite directions because only card users get rebates.

Given the same total quantity and \( b=0 \), total surplus must fall under the NSR with rebates, since per capita quantities of cash and card users are then different, while efficiency calls for equal levels as occurs with competitive issuers and no NSR. This divergence of quantities only with the NSR implies, however, that overall consumer surplus rises (trapezoids argument).

Result 8iv) shows that as the cash market becomes large relative to cards, the NSR in conjunction with competitive rebates by card issuers succeed in eliminating the distortion in the pricing of card transactions due to the monopolist merchant. The merchant charges a uniformly high (monopoly) price to both card and cash users, but card users receive a rebate and therefore obtain a net price close to the competitive price. However, the net price to cash users is the (uniform) price charged by the merchant. When the cash market is large, the merchant’s price is driven by the cash market and thus will approach the simple monopoly level.
7. Conclusion

The complex cycle that makes up a typical payment network offers a rich field for economic analysis, with prices playing important roles at every link of the cycle. Our principal model analyzed the No Surcharge Rule as an imperfect instrument of vertical control by a card payment network (EPN) facing a merchant in an environment of double marginalization, where the merchant also serves outside consumers—‘cash’ users. By requiring the merchant’s card price to equal its cash price, the NSR leads the EPN to prefer a higher fee to the merchant and a lower fee to card users (whereas the EPN is indifferent to how it allocates its total fee when the merchant can set the card price independent of the cash price). Throughout, the NSR benefits the EPN but harms the merchant and cash users. Other welfare effects depend on the ratio of cash to card users, the merchant’s benefit from card versus cash transactions, and whether rebates (negative fees) to card users are feasible.

If rebates are not feasible, the EPN charges card users zero but raises its merchant fee above its no-NSR total fee. This increase in total fee reduces total transactions and aggregate consumer surplus; with a sufficiently small cash market, even card users pay more under the NSR. Despite the fall in total quantity, overall welfare increases if (and only if) the cash market is large enough, because the rise in per capita card quantity combats the pre-NSR bias from double marginalization, at the cost of a relatively small distortion in per capita cash quantity.

If rebates are feasible the EPN grants them, benefiting itself and card users while harming cash users and (weakly) the merchant. However, the EPN’s total fee is lower with rebates (as needed to maintain merchant participation), so total quantity is higher than under the NSR with no rebates, as are aggregate consumer surplus and overall welfare. Relative to no NSR, the NSR with rebates leaves total quantity unchanged but reverses the gap between the card and cash quantities from negative to positive. Overall welfare rises if and only if the cash market is sufficiently large, because the per capita cash distortion then is small (though a larger cash market makes the NSR less favorable to total consumer surplus). A larger merchant benefit from card compared to cash transactions increases the pre-NSR distortion from double marginalization in card pricing and improves the effects of the NSR with rebates on overall welfare and total consumer surplus; interestingly, the reverse occurs if rebates are not feasible.

Our emphasis has been on the impact of limits on merchant pricing flexibility when there exists some power over price. To highlight this effect, we assumed monopoly pricing at both the
merchant and EPN levels, but we conjecture that similar effects will arise whenever there remains a significant margin between price and marginal cost at both levels. We also analyzed a case where the EPN margin is almost zero, because the EPN’s card issuing banks behave as Bertrand competitors. In that case, pre-NSR there is no significant bias against cards (if merchant benefit from cards is low), so by encouraging card transactions at the expense of cash the NSR with rebates reduces welfare, though it increases overall consumer surplus. Our analysis abstracted away from consumers’ choice of the means of payment in order to focus on the impact on the level of transactions per consumer. Extensions of this research would include analyses of the effects of the ‘rule’ under a broader class of merchant market structures and with endogenous consumer choice of the means of payment (e.g., Rochet, 2003). Another direction will be to examine how the NSR influences the competition among rival payment networks both in pricing and in other practices such as the tying of multiple cards (e.g., Rochet and Tirole, 2003).
Appendix

Proof of Proposition 2: (i) Since merchant sales are decreasing in marginal cost, \(x(k-b) < x_0\) and therefore, \(V(x(k-b)) > V(x_0)\). Since \(t^* = V(x(k-b)) - V(x_0)\), for all \(t, i\) such that \(i + t = k\), \(x(k-b)\) remains constant and \(t < t^*\) implies \(V(x(k-b)) - t = p_c^M > V(x_0) = p_c^M\).

(ii) Consider a choice of \((q_c, q_e)\) that solves the merchant’s profit maximization problem with an NSR. Suppose that \(q_e < x(k-b)\). The pair \((x(k-b), q_c)\) is also feasible for the merchant since \(V(q_c) + t < V(q_e)\) by the concavity of \(V(\bullet)\). But the choice of \((q_c, q_e)\) over \((x(k-b), q_c)\) then implies that

\[
q_c (V(q_e) - k + b) > x(k-b) (V(x(k-b)) - k + b)
\]

which violates the definition of \(x(k-b)\). A similar proof shows \(q_c > x_0\). Now suppose \(q_c = x(k-b)\). The merchant’s first order condition with respect to \(q_e\) under the NSR constraint is

\[
q_c V(q_e) + V(q_e) - t + b - \lambda V(q_e)\]

where \(\lambda > 0\) is the multiplier on the constraint imposed by the NSR. Evaluating this expression at \(x(k-b)\) yields \(-\lambda V(x(k-b)) > 0\) since the first terms are the merchant’s first order condition with no NSR and equal zero at \(x(k-b)\). Therefore, merchant profits are strictly increasing in \(q_e\) at \(q_c = x(k-b)\).

(iii) Let \(f(p)\) be the demand curve of cash users with \(pf(p)\) concave. The demand curve of card users is \(f(p + t)\). Let \(i + t = k\) and let \(p\) denote the optimal (uniform) price charged by the merchant under an NSR when the card user fee is \(t\) (so \(i = k - t\)). Similarly, let \(p'\) denote the optimal uniform price charged by the merchant when the card user fee is \(t' < t\). Finally, for convenience, set \(t - t' = \Delta > 0\). By definition of \(p\), charging a price \(p\) under the fee profile, \((k-t, t)\) yields higher merchant profits than charging a price \(p' + t\). Note that this second price implies a net price to card users of \(p' + t\). Thus,

\[
af(p) + (p + t - (k-b))f(p + t) > a(p' + \Delta)f(p' + t) + (p' + t - (k-b))f(p' + t).
\]

Similarly, under the fee profile, \((k-t', t')\), \(p'\) raises more profits than charging a price \(p + \Delta\).

\[
af(p') + (p' + t - (k-b))f(p' + t) > a(p + \Delta)f(p + \Delta) + (p + t - (k-b))f(p + t).
\]

Adding the two inequalities and eliminating the common terms which denote revenues in the card market and dividing by \(a\), yields

\[
pf(p) - (p + \Delta) f(p + \Delta) > (p' - \Delta)f(p' - \Delta) - pf(p').
\]

Recall that \(p\) and \(p'\) are higher than the price which maximizes \(pf(p)\). Suppose that \(p' + t > p + t\). This implies \(p' - \Delta > p\). But this violates the assumption of concavity of \(pf(p)\) since the slope of the revenue function must become steeper as we move further to the right of the maximum point.
**Proof of Proposition 3: i)** When the IR constraint does not bind, the result follows from Proposition 2iii). Now suppose the IR binds and consider \((t,i)\) space. At \(t = 0\), and \(i\) such that the merchant IR curve binds, we show that the slope of the EPN level set is a lower negative number than the slope of the merchant IR curve which has slope with absolute value less than one. This implies that this point is a constrained maximum. (See Figure 3.)

![Figure 3](image)

Under an NSR, the Lagrangian representing the merchant’s profit maximization problem is

\[
L(q_e, q_c, \lambda; i, t) = \alpha q_e V(q_e) + q_e (V(q_e) - i + b) + \lambda (V(q_e) + t - V(q_e)),
\]

where \(\lambda > 0\) is the lagrangian on the No Surcharge constraint. The EPN’s profit function is given by

\[
\Pi = (i + t)q(t, i; b).
\]

Now consider the level sets of the merchant and the EPN in \((t,i)\) space. The slopes at \(t = 0\) are given by

\[
\frac{di^e}{dt}_{t=0} = \left(1 - \frac{\frac{\partial q_e}{\partial i} \frac{\partial q_e}{\partial t}}{\frac{\partial q_e}{\partial i} + Q}\right), \quad \frac{di^M}{dt}_{t=0} = \left(1 - \frac{\lambda}{Q}\right).
\]

Note that the denominator in the first expression is non-negative since the EPN’s profit function is quasi-concave along the line \(t=0\) and the fact that the IR constraint binds implies it is
constrained to select an \( i \) less than its unconstrained optimum. Equation (3) which provides the merchant’s optimal choice of \( q^* \) then implies

\[
\frac{\partial q_e}{\partial i} = \frac{1 + 2\alpha}{2(1 + \alpha)} < \frac{\partial q_e}{\partial i} = -\frac{1}{2(1 + \alpha)} < 0.
\]

The first order conditions for the merchant’s optimal choice of quantity imply that \( i = b + \lambda (1 + \alpha) / \alpha \). This yields

\[
i \left( \frac{\partial q_e}{\partial i} - \frac{\partial q_e}{\partial t} \right) = b \left( \frac{\alpha}{1 + \alpha} \right) + \lambda \geq \lambda
\]

and

\[
0 \leq i \frac{\partial q_e}{\partial i} + Q = b \frac{\partial q_e}{\partial i} \frac{\lambda}{2\alpha} + Q \leq Q - \frac{\lambda}{\alpha}.
\]

The first inequality comes because the IR is binding on the EPN and the second inequality follows because \( q_e \) is decreasing in \( i \). Combining these results yields

\[
\frac{i \left( \frac{\partial q_e}{\partial i} - \frac{\partial q_e}{\partial t} \right)}{i \frac{\partial q_e}{\partial i} + Q} \geq \frac{\lambda}{Q - \frac{\lambda}{\alpha}}.
\]

Now consider the level sets of the EPN and the merchant in \((t, i)\) space. Subtracting the second from the first yields, after substituting the inequality from above,

\[
\frac{d_i^e}{d_t} - \frac{d_i^M}{d_t} \geq \left( \frac{\lambda}{Q - \frac{\lambda}{\alpha}} \right) \left( \frac{\lambda}{Q} \right) \geq 0.
\]

Recalling that \( Q - \lambda / \alpha \) is positive, 2i) follows.

ii) Note that at \( t = 0 \), the first order conditions for the merchant’s problem imply that

\[
(1 + \alpha)(Q^2 V'' + QV') = (i-b) Q
\]

(Note that this implies that the merchant’s choice of \( Q \) is a strictly decreasing function of \( i-b \).)

Substituting in for \((i-b) Q\) the IR constraint is equivalent to
\[(1+\alpha) Q V(Q) - (i-b)Q = -(1+\alpha) Q^2 V''(Q) \geq \alpha x_0 V(x_0)\]

or (Equation (4) in the text)

\[- Q^2 V''(Q) \geq \alpha x_0 V(x_0)/(1+\alpha) . \] (1A)

The right side is increasing in \(\alpha\). Concavity of the merchant revenue function in quantity implies the left side is increasing in \(Q\). For low \(\alpha\), the constraint does not bind when the EPN selects its globally optimal \(Q\) at \((i_0,0)\). As \(\alpha\) rises, the constraint binds and the EPN must offer a successively higher \(Q\) (lower \(i\)) in order to induce the merchant to participate.

**iii)** The merchant choice of \(Q\) is strictly decreasing in \(i-b\), so, holding \(b\) fixed, 3ii) implies \(i\) decreasing in \(\alpha\). When the IR binds, \(Q\) is determined by (1A) which, in turn, determines \(i-b\).

**iv)** If the IR does not bind, then the EPN optimal choice of \(i\) is \((1+\alpha+b)/2\) which is increasing in \(\alpha\). If the IR binds, then the optimal choice of \(i\) is determined solely by the merchant’s IR constraint (at \(t=0\) and is given by

\[1+\alpha-\sqrt{\alpha+\alpha^2+b}\]

This is decreasing in \(\alpha\) for all \(\alpha\).

**Proof of Proposition 4: i)** Proposition 3 yields the optimal solution \(t = 0\) which implies that per capita cash and card purchases are the same. This gives the first order condition of the merchant,

\[i = b+(1+\alpha)( V(x)+xV'(x)).\]

Define \(q_{\alpha}= \text{argmax}_x x(b+(1+\alpha)(V(x)+xV'(x)))\) to be the quantity of card-user transactions which maximizes EPN profits with the NSR. Concavity of the EPN’s profit function implies this is unique. Note that \(q_0\) maximizes profits with no NSR. If \(b=0\), then the definition indicates that \(q_0 = \text{argmax}_x (1+\alpha) x((V(x)+xV'(x)))\) and so \(q_0\) also solves the EPN’s problem with the NSR. Thus, card transactions are unchanged with the NSR (and \(b=0\)) but cash transactions are lower (since they exceeded card transactions without the NSR). Now consider \(b>0\).

By definition,

\[q_0 (b+V(q_0)+q_0 V'(q_0)) \geq q_{\alpha}(b+V(q_{\alpha})+q_{\alpha} V'(q_{\alpha})),\]

and

\[q_0 (b+(1+\alpha)( V(q_0)+q_0 V'(q_0))) \leq q_{\alpha}(b+(1+\alpha)( V(q_{\alpha})+q_{\alpha} V'(q_{\alpha}))).\]

Subtract the two inequalities and divide by \(-\alpha\) to get

\[q_0 (V(q_0)+q_0 V'(q_0)) \leq q_{\alpha}(V(q_{\alpha})+q_{\alpha} V'(q_{\alpha})).\]

Suppose that \(q_{\alpha} > q_0\). Then \(b q_0 < b q_{\alpha}\) This implies

\[q_0 (b+V(q_0)+q_0 V'(q_0)) < q_{\alpha}(b+V(q_{\alpha})+q_{\alpha} V'(q_{\alpha})).\]
which violates the definition of $q_\theta$. The EPN first order conditions with the NSR, evaluated at $q_0$, indicates that EPN profits are strictly declining in quantity at that point:

$$\frac{\partial \pi^*(x)}{\partial x} \bigg|_{x=q_0} = b(1-(1+\alpha)) = -\alpha b < 0$$

so $q_\theta < q_0$ given $b > 0$. Thus, card transactions are lower with the NSR for $b > 0$ and so, too, are cash transactions.

ii) a) - b) The limit of the right side of (1A) as $\alpha$ becomes large is $x_0 \sqrt{V(x_0)}$ so $Q$ must approach $x_0$ before reaching the limit, though, $Q < x_0$ so cash users’ purchases and surplus fall with the NSR. P3) implies that eventually cardholder purchases and surplus are higher with the NSR.

c)-d) With an NSR and linear demand, merchant profit is $(1+\alpha)Q^2$ so (4) can be written $Q^2 = \alpha/(4(1+\alpha))$. With no NSR, the total quantity of transactions is $(1+b)/4 + \alpha/2$. For $\alpha > \alpha^*$, the IR constraint binds and determines $Q = (\alpha/(4(1+\alpha))^\frac{1}{2}$. The NSR thus raises total quantity if and only if,

$$(1+\alpha)\sqrt{\frac{\alpha}{4(1+\alpha)}} = \frac{1+b}{4} + \frac{\alpha}{2}$$

This is impossible so total quantity falls. The limit as $\alpha$ goes to infinity of the difference in total quantity is

$$\lim_{\alpha \to \infty} (1+\alpha)Q = -\alpha/2 - (1+b)/4 = \frac{1}{2} \lim_{\alpha \to \infty} (\sqrt{(1+\alpha)\alpha} - \alpha) - \frac{1+b}{4}$$

$$= \frac{1}{2} \lim_{\alpha \to \infty} \left( \frac{1+\alpha}{\alpha} \sqrt{1/\alpha} - 1 \right) - \frac{1+b}{4}$$

Applying L’Hopital’s Rule to the limit (the term in the limit is $1/2$) yields a value for the difference in total quantity as $\alpha$ goes to infinity is $-b/4$.

For any two pairs of per capita transactions, $(q_c, q_e), (Q_c, Q_e)$, and defining,

$$\Delta Q_c = Q_c - q_c, \Delta Q_e = Q_e - q_e, \Delta Q_T = a \Delta Q_c + \Delta Q_e$$

the change in total surplus when moving from the first outcome to the second is
Thus, let \( q_c = 1/2 \), \( q_e = (1+b)/4 \) be the per capita transactions when surcharging is allowed and \( Q = Q_c = Q_e \) the (common) per capita quantity under the NSR without rebates. The IR constraint yields \( Q^2 = 0.25 \alpha/(1+\alpha) \), so the limit as \( \alpha \) goes to infinity of \( Q \) is \( 1/2 \). Thus, as \( \alpha \) goes to infinity, \( \Delta Q_c = 0, \Delta Q_e = (1-b)/4, \Delta Q_T = -b/4 \) and we have

\[
\lim_{\alpha \to \infty} \Delta TS^{NR} = (1+b)^2/32 - b^2/4.
\]

So the limit is decreasing in \( b \). Furthermore, using the fact that \( Q \) and \( q_c \) are independent of \( b \) and \( \partial q_c/\partial b = 1/4 \), we have

\[
\partial \Delta TS^{NR}/\partial b = \partial q_c/\partial b + q_c, \quad \partial q_e/\partial b + Q - q_e - b \partial q_e/\partial b.
\]

Therefore,

\[
\partial(\partial \Delta TS^{NR}/\partial b)/\partial \alpha = \partial Q / \partial \alpha > 0.
\]

Direct computation shows that \( \Delta TS^{NR} \) is increasing in \( \alpha \) for \( b = 0 \). Thus, it is increasing in \( \alpha \) for all \( b > 0 \). Since, for \( \alpha \) large enough, \( b' > b \) implies \( \Delta TS^{NR}(\alpha, b') < \Delta TS^{NR}(\alpha, b) \) (the limit is decreasing in \( b \)), and since \( \partial \Delta TS^{NR}/\partial \alpha \) is increasing in \( b \), we have \( \Delta TS^{NR}(\alpha, b') < \Delta TS^{NR}(\alpha, b) \) for all \( \alpha \).

(Computations show that total surplus exceeds total surplus with no NSR at \( \alpha > 1.53 \) for \( b = 0 \).

The change in consumer surplus is

\[
\Delta CS = 0.5(Q_c + q_c)\Delta Q_T + 0.5((Q_e - Q_c) + (q_e - q_c))\Delta Q_e,
\]

and in this case, \( \Delta Q_T < 0, Q_e - Q_c = 0, \) and \( (q_e - q_c)\Delta Q_e < 0 \).

**Proof of Proposition 5:**

i) If \( \alpha < \alpha^* \), then the IR does not bind under no rebates and, by Proposition 2iii), the EPN increases profits by holding \( i \) fixed and lowering \( t \). If \( \alpha \geq \alpha^* \), from the Proof of Proposition 3i) the slope of the EPN indifference curve in \((i, t)\) space is steeper than the slope of the merchant’s IR curve at the constrained optimal solution, \( t=0 \). Thus, again, EPN profits are strictly higher as \((i, t)\) is varied by lowering \( t \) below zero and raising \( i \) so as to stay on the IR curve.

ii) With \( t < 0 \), the merchant’s demand curve for card transactions is strictly above the (per capita) cash demand curve. With sufficiently high rebates, the monopoly price from serving the card market exceeds the choke price for the cash market. In this case, the merchant’s profit function has two local maxima. If the merchant serves only the card market, the per capita card transaction is the same as with surcharging allowed but cash transactions are now zero. Repeating (3), if the merchant prices to serve both markets, the solution is

\[
p = \frac{1+\alpha + i - b - t}{2(1+\alpha)}, \quad q_c = \frac{1+\alpha - i + b + t}{2(1+\alpha)}, \quad q_e = \frac{1+\alpha - i + b - t(1+2\alpha)}{2(1+\alpha)}
\]

(4A)
The merchant chooses to serve both markets and thus selects prices and quantities as in (4A) if and only if

\[ i-t-b \leq \sqrt{1+\alpha} \]  

(5A)

Using the values for \( p, q_e, q_c \) from (4A) in (1A) gives the values of \((i,t)\) for the EPN when the IR binds. Maximizing with respect to \((i,t)\) yields

\[ t^* = \frac{1+b}{4(\alpha + \sqrt{\alpha(1+\alpha)})} - \frac{b}{2}, \quad i^* = \frac{1+b}{2} - t^* \]  

(6A)

Equation (6A) implies that (5A) is violated as \( \alpha \) gets small.

iii) Follows from Equation (6A).

**Proof of Proposition 6:** i)-iii) Follows from (4A) and noting that under surcharging, per capita cash quantity is \( 1/2 \) and card quantity is \( (1+b)/4 \).

iv)-v): Total surplus is affected by total quantity and by the differences in quantities. The NSR and linear demand imply \( q_e^{NSR} = q_e^{NSR} + t \). Utilizing Equation (2A) for the change in total surplus moving from surcharging to an NSR with rebates, along with the fact that total quantity is unchanged yields

\[ \Delta TS^{NSR} = b(-t-(1+b)/4)/2 \Delta Q_e = (7b+1+4t)\Delta Q_e/8. \]

The change in total surplus is positive if and only if \((7b+1+4t)>0\). It is increasing in \( \alpha \) if \((7b+1+4t)>0\) since \( \Delta Q_e \) and \( t \) are increasing in \( \alpha \). Note that if \( b=0 \), then \( TS^{NSR} - TS^{SUR} = 0 \) at \( \alpha = 1/3 \).

(At that point, \( t = -1/4 \) and \( q_e^{NSR} \) exceeds \( q_e^{NSR} \) by exactly the same amount that \( q_e^{SUR} \) exceeds \( q_e^{SUR} \).)

Given constant total quantity and linear demand, aggregate consumer surplus depends on the split between the types of consumers. Equation (3A) yields

\[ \Delta CS^R = -(q_e^{NSR} - q_e^{SUR})(q_e^{NSR} - q_e^{NSR}) + (q_e^{SUR} - q_e^{SUR})]. \]

Direct computation yields that \( \Delta CS^R \) is increasing in \( \alpha \) for all \( b \). Since \( \Delta TS^R \) depends on \( \alpha \) as \( \Delta CS^R \) depends on \( -\alpha \), we also have \( \Delta TS^R \) falls in \( \alpha \) for \( \alpha \) such that \((7b+1+4t)<0\).

vi) Direct computation shows that \( \Delta TS^R \) and \( \Delta CS^R \) rise in \( b \).
Proof of Proposition 7: i) Equation (4A) shows that $q_c$ falls as $i-t$ rises and $q_e$ rises if $i+t$ and $t$ fall. Propositions 3 and 5 reveal that compared to the NSR with no rebates, when rebates are feasible, $i+t$ and $t$ are lower and $i-t$ is higher. If the IR binds, then Propositions 4 and 6 imply total quantity is higher under the NSR with rebates and since, with no rebates, per capita quantities are always identical, $(q_e=q_c)$ and with rebates, $Q_e > Q_c$, Equation (3A) implies total consumer surplus must rise.

ii) Suppose that the IR binds at the optimal solution with $t=0$. The optimal solution with the $t \geq 0$ relaxed is at a point downward and to the right of this point. Part i) implies total consumer surplus rises. Revealed preference implies that EPN profits rise and, since we remain on the merchant’s IR curve, merchant profits stay the same. Thus, total surplus rises when the $t \geq 0$ constraint is relaxed from a point at which the IR constraint binds.

iii) Shown by computation.

Proof of Proposition 8: i): Solving the merchant participation constraint simultaneously with the constraint $t=-i$, yields

$$i^{\text{comp(IR)}} \leq \frac{1}{2} \sqrt{\frac{1+\alpha}{\alpha}}$$

The constraint that the merchant continue to be willing to serve the cash market is $t > i-(1+\alpha)^{3/2}$. Using $t=-i$, this yields a value

$$i^{\text{comp(IC)}} \leq \frac{1}{2} \sqrt{1+\alpha}$$

The lowest value for $i^{\text{comp}}$ is the binding constraint. The second one is lower than the first if and only if $\alpha<1$.

ii), iii): Use the quantity equations from Equation (4A) for per capita purchases and $t=-i$ to get $\alpha q_e = \alpha(0.5-i(1+\alpha))$ and $q_e=(0.5+\alpha i)(1+\alpha))$. Summing the two yields total quantity $(1+\alpha)/2$ which is independent of $i$ and is equal to the total quantity of purchases with competitive issuers and no NSR. Conditional on total quantity remaining constant, social surplus is maximized when the cash and non-cash quantities are the same. Any value of $t$ strictly less than zero along with the NSR, violates this condition, so social surplus must fall. Consumer surplus rises because, holding
total quantity fixed, the loss to cash consumers from the higher price is more than compensated by the gain to EPN consumers from the lower price. Using $q_e = (.5 + \alpha \frac{i}{1+\alpha})$ and letting $\alpha$ grow large yields $i$ approaches $1/2$, EPN quantity approaches $1$ and cash quantity approaches $1/2$. 

Figure 1
Figure 2
References


