Discussion of Ares, Carapella, Maziero, and Weber:
A Model of Banknote Discounts

François R. Velde

fvelde@frbchi.org

Federal Reserve Bank of Chicago

Paper

The paper starts from a set of observations about discounts on banknotes of various banks in different locations in antebellum US.

1. local banknotes at par with each other
2. foreign banknotes at a discount, depending on the location of origin
3. discounts were asymmetric
4. foreign notes were discounted higher when they were not being redeemed
5. local notes discounted to specie when they were not being redeemed

random matching model to capture these facts.

Simplified version of the model

Two types of agents: buyers/consumers (holders of money) and sellers/producers

- sellers produce for buyers, become consumers

Two colors of agents (in same location): red people and blue people

Two types of money: red money and blue money

- with probability α_R, red holders of red money redeem it for Q_R
- with probability α_B, blue holders of blue money redeem it for Q_B
- blue holders can't redeem red money and vice-versa

With same probabilities, red (resp. blue) producers receive a red (resp. blue) note in exchange for q_R (resp. q_B).

model description (2)

The model is otherwise completely symmetric (number of red and blue agents, quantity of red and blue money).

Agents have tastes over different varieties, prob. of double coincidence is 2π.

Preferences: $u(q)$ over consumption, $-q$ over production ($u(0) = 0$, $u'(0) = +\infty$, $u'(+\infty) = 0$, $u(q^*) = q^*$).

Buyers make take-it-or-leave-it (TIOLI) offers.

Buyers don't trade with buyers.

Red buyers make offer x_i, blue buyers make offer z_i, $i \in \{R, B\}$.

Value functions

red people have V_i, blue people have W_i, with $i \in \{0, R, B\}$

Buyers:

$$\begin{aligned}
rV_R &= (1-\alpha_R)\pi(\max\{\lambda[u(x_R)-V_R]\} + \max\{\lambda[u(z_R)-V_R]\}) + \alpha_R \\
rV_B &= \pi(\max\{\lambda[u(x_B)-V_B]\} + \max\{\lambda[u(z_B)-V_B]\}) \\
rW_R &= \pi(\max\{\lambda[u(x_R)-W_R]\} + \max\{\lambda[u(z_R)-W_R]\}) \\
rW_B &= (1-\alpha_B)\pi(\max\{\lambda[u(x_B)-W_B]\} + \max\{\lambda[u(z_B)-W_B]\}) + \alpha_B
\end{aligned}$$

Sellers:

$$\begin{aligned}
V_0 &= (1-\alpha_R)\max\{\lambda(V_R - x_R)\} + \alpha_R(V_R - q_R) \\
W_0 &= (1-\alpha_B)\max\{\lambda(W_B - z_B)\} + \alpha_R(W_B - q_B)
\end{aligned}$$

Equilibrium

We look for a monetary equilibrium with both currencies : all λs are 1 (remember to check incentive condition)

TIOLI implies $V_0 = W_0 = 0$, $x_i = V_i$, $z_i = W_i$.

Rewrite buyers' value functions as:

$$\begin{aligned}
rx_R &= (1-\alpha_R)\pi[u(x_R)+u(z_R)-2x_R]+\alpha_R \\
rx_B &= \pi[u(x_B)+u(z_B)-2x_B] \\
rz_R &= \pi[u(x_R)+u(z_R)-2z_R] \\
rz_B &= (1-\alpha_B)\pi[u(x_B)+u(z_B)-2z_B]\})+\alpha_B
\end{aligned}$$

Four equations in four unknowns
(remember to check $x_i < q^*$, $z_i < q^*$, $u(x_i) > z_i$, $u(z_i) > x_i$).

Symmetric case

$\alpha_R = \alpha_B$

$$
\begin{aligned}
rx_R &= (1-\alpha)\pi[u(x_R) + u(z_R) - 2x_R] + \alpha \\
rx_B &= \pi[u(x_B) + u(z_B) - 2x_B] \\
rz_R &= \pi[u(x_R) + u(z_R) - 2z_R] \\
rz_B &= (1-\alpha)\pi[u(x_B) + u(z_B) - 2z_B]\}) + \alpha
\end{aligned}
$$

$x_R = z_B$, $x_B = z_R$ candidate solution (two equations, two unknowns).

If α not too large, there is an equilibrium.

In this equilibrium, discount on money i among people j are equal:

$$1 - \frac{x_R}{x_B} = 1 - \frac{z_B}{z_R}$$

Asymmetric case

$\alpha_R \neq \alpha_B$

Back to four equations:

$$
\begin{aligned}
x_R &= \gamma_R z_R + \alpha_R \\
z_R &= \frac{\pi}{r+2\pi}[u(x_R) + u(z_R)]
\end{aligned}
$$

$$
\begin{aligned}
z_B &= \gamma_B x_B + \alpha_B \\
x_B &= \frac{\pi}{r+2\pi}[u(x_B) + u(z_B)]
\end{aligned}
$$

But two separate systems for $\{x_R, z_R\}$ and $\{x_B, z_B\}$ (is this really about location?)

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Two equations in two unknowns z_R and x_B:

$$\begin{aligned} z_R - \kappa u(z_R) &= u(\gamma_R z_R + \alpha_R) \\ x_B - \kappa u(x_B) &= u(\gamma_B x_B + \alpha_B) \end{aligned}$$

Asymmetry and discount

In general, $z_R \neq x_B$.

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As α_R increases from $\alpha_R = \alpha_B$ (the symmetric case), what is the effect on the discount?

Ambiguous, partly due to the double impact of higher α_R (direct and through γ_R): greater pay-off for note, but fewer opportunities to trade.

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Variant: treat α_i as a dividend, paid to all note-holders. $\alpha_R \nearrow \Longrightarrow z_R \nearrow \Longrightarrow x_R \nearrow$ by more ($\gamma > 1$).

x_B and z_B unchanged: $1 - \frac{z_B}{z_R}$ rises by less than $1 - \frac{x_R}{x_B}$: blue notes are *less* discounted by red people than red notes by blue people.

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Variant: treat α_i as a dividend, paid to all note-holders.

The dividend will also capture suspensions, etc.

Source of asymmetric discounts

Asymmetric discounts caused by asymmetric α: but what is α?

"it was costly for nonbankers to go to local banks to obtain banknotes or to redeem banknotes."

Since discounts were the same for all banks in one location, the cost was the same as well.

Empirical validation: is there a relation between the size of the discount on notes of a given location and observable characteristics of that location (relative size of banking sector?)

Other Questions

- buyers can’t trade red notes against blue notes: Why not? Wouldn’t they want to?
- (related) discounts are not properly discounts, but ratios of market prices.
- so-called “bankers” play no interesting role here.
- not clear that the model is about locations. Introduce physical locations (with moving costs)
- suspensions as "steady states"?

Another example of asymmetry