A Model of Money and Credit, with Application to the Credit Card Debt Puzzle*

Irina A. Telyukova
University of Pennsylvania

Randall Wright
University of Pennsylvania

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Abstract

Many individuals simultaneously have significant credit card debt and money in the bank. The so-called credit card debt puzzle is, given high interest rates on credit cards and low interest rates on bank accounts, why not pay down this debt? Economists have gone to some lengths to explain this. As an alternative, we present a natural extension of the standard model in monetary economics to incorporate consumer debt, which we think is interesting in its own right, and which shows that the coexistence of debt and money in the bank is no puzzle.

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1 Introduction

A large number of households in the U.S. simultaneously have significant credit card debt and a significant amount of money in checking and savings accounts. Although there are many ways to measure this, a simple summary statistic is that 27% of U.S. households in 2001 had credit card debt in excess of $500, and over $500 in checking, savings and brokerage accounts. This is the so-called credit card debt puzzle: given 14% interest on credit card debt and 1 or 2% on bank accounts, why not pay down the debt? "Such behavior is *puzzling*, apparently inconsistent with no-arbitrage and thus inconsistent with any conventional model." (Gross and Souleles 2001, emphasis added).

Economists have gone to elaborate lengths to explain this phenomenon. For example, some people assume that consumers cannot control themselves (Laibson et al. 2000); others assume they cannot control their spouses (Bertaut and Haliassos 2002; Haliassos and Reiter 2003); still others hypothesize, counter to the facts, that all such households are on the verge of bankruptcy (Lehnert and Maki, 2001). We show that one does not have to resort to such extremes. This is not to say that these ideas have no merit, but simply that standard theory is not inconsistent with the observation that households simultaneously have substantial debt and money in the bank. By standard theory we mean modern monetary economics. These models are designed to study liquidity. They predict that agents may hold assets with low rates of return if they are liquid – i.e. if they have use as a medium of exchange.[1]

Our hypothesis accounts for the credit card debt puzzle in the following way. Households need money – more generally, liquid assets – for situations where credit cannot be used. The obvious and standard examples include taxis, cigarettes, and so on, although increasingly credit cards can be used for some of these, but we want to emphasize that there are also some big-ticket examples. For instance, usually rent or mortgage payments cannot be made by credit card. Thus, even if a household is revolving credit card debt, it needs to have money in the bank in order to meet these obligations. According to the Consumer Expenditure Survey, the median household that holds both debt and liquidity revolves $3,800 of credit card debt, has about $3,000 in the bank, and spends $1,993 per month on goods purchased with liquid assets

[1] Some version of this idea is in all of search-based monetary theory going back to Kiyotaki and Wright (1989).

(see Telyukova 2005). Moreover, according to the U.S. Statistical Abstract, 77% of consumer transactions in 2001 were done in liquid assets.

We develop a micro-founded model of monetary exchange to formalize these ideas. While we build upon recent work, and in particular Lagos and Wright (2005), we need to extend existing models to incorporate consumer credit, since the typical model in this literature does not have anything along these lines. This seems like a natural and interesting extension of modern monetary theory in its own right, and also allows us to argue that coexistence of consumer debt and money in the bank is not a puzzle. Whether this approach is able to account quantitatively for salient aspects of the data is the subject of ongoing research (Telyukova 2005).[2]

2 The Basic Model

We build on Lagos and Wright (2005), hereafter LW. That model gives agents periodic access to a centralized market, in addition to the decentralized markets where, due to various frictions, money is essential for trade as in the typical search-based model. Having some centralized markets is interesting for its own sake, and also makes the analysis more tractable than in much of the literature on the microfoundations of money.[3] We will extend this framework along several dimensions.

We now describe the basic physical environment. In this section, we consider a special case; later on, we will generalize the model. Time is discrete and there is a [0,1] continuum of infinitely-lived agents. There is one general consumption good at each date that is nonstorable and perfectly divisible. Agents can produce the good in each period using labor as an input. There is also money in this economy, an object that is storable and perfectly divisible; it is intrinsically worthless but potentially has use as a medium of exchange. The money supply is

[2] To be concrete, there are several facets to the credit card debt puzzle. For example, in addition to having debt and liquidity in their portfolios at the same time, we observe households persistently revolving debt, something we do not address. In Telyukova (2005), the current model is generalized to account for key empirical features of the credit card debt puzzle and to assess the validity of the theory quantitatively.

[3] See Molico (1997), Green and Zhou (1998), Camera and Corbae (1999), Zhou (1999) or Zhu (2003). A framework related to LW is described in Shi (1997). Earlier models, like Shi (1995) or Trejos and Wright (1995), were also tractable, but only because money was assumed to be indivisible.

fixed for now at M, but see below.

Although we frame the discussion as though agents literally use money to transact, it is now well known how to recast the model with agents depositing money into bank accounts and paying for goods using checks or debit cards. This is discussed in detail in He, Huang and Wright (2005). This is relevant for our purposes because what we have in mind is not necessarily cash, per se, but liquid assets generally. So when we say "agents carry money" in what follows, one should interpret this liberally as "agents hold liquid assets" or "have money in the bank".

In LW each period is divided into two subperiods. In one, say the morning, there is a centralized (frictionless, Walrasian) market. In the other, say the evening, there is a decentralized market where agents meet anonymously according to a random bilateral-matching process, which makes a medium of exchange essential. After each evening's meeting of the decentralized market, the next morning agents can consume, produce, and adjust their money holdings in the centralized market. Under the assumption of quasi-linear utility, it turns out that all agents will take the same amount of money out of the centralized and into the next decentralized market, which is a big simplification.

There is no role for credit in LW. Credit is not possible in the decentralized market, and not necessary in the centralized market. It is not possible in the decentralized market because of the assumption that agents are anonymous, which is needed to make money essential, and it is not necessary in the centralized market because of the assumption that all agents can produce, which is needed to make the distribution of money degenerate. Our idea is to introduce an intermediate subperiod, say afternoon, where at random some agents want to consume but cannot produce and vice-versa. There is a centralized market in this subperiod, where agents may use either credit or cash. This allows us to introduce consumer credit while maintaining both a role for a medium of exchange and the simplicity of LW.[4]

All agents want to consume in subperiod $s = 1$, and $u_1(x_1)$ is their common utility function. Only a random, and not necessarily the same, subset want to consume in $s = 2, 3$, and conditional on this, $u_s(x_s)$ is their utility function. All agents are able to produce in subperiod $s = 1$, and

[4]We will generalize this below, but having three subperiods is sufficient to make the basic point. Berentsen, Camera and Waller (2005a) also have a third subperiod, but it is a second round of decentralized exchange, and hence there is no possibility of credit (but see also Berentsen, Camera and Waller 2005b).

the disutility of working h_1 hours is linear, $c_1(h_1) = h_1$. A random subset are able to produce in subperiods $s = 2, 3$, and conditional on this, the disutility of working is some general convex function $c_s(h_s)$.When they can produce, all agents can transform labor one-for-one into output, $x_s = h_s$.[5] Let x_s^* denote efficient level of production – i.e. the solution to $u_s'(x_s^*) = c_s'(x_s^*)$. Let β_s be the discount factor between s and the next subperiod.

Generally, an individual's state variable is (m_{ts}, b_{ts}), denoting money and debt in period $t = 1, 2, ...$, subperiod $s = 1, 2, 3$, but we drop the t subscript when there is no risk of confusion. Let $W_s(m_s, b_s)$ be the value function in subperiod s. The value of money in the centralized markets at $s = 1, 2$ is ϕ_s; that is, $p_s = 1/\phi_s$ is the nominal price of the consumption good in subperiod s. There is no ϕ_3 since there is no centralized market at $s = 3$, although there will be an implicit price defined by bilateral trades in the decentralized market. Similarly, the real interest rate in the centralized markets at $s = 1, 2$ is r_s, but there is no r_3. Our covention for notation is as follows: if you bring debt b_s into subperiod $s = 1, 2$ you owe $(1 + r_s)b_s$.

The plan now is to consider each subperiod in turn. After this, we put the markets together and describe equilibrium.

2.1 Subperiod 1

In the morning, there is a standard centralized market. Given the state (m_1, b_1), agents solve[6]

$$\begin{aligned} W_1(m_1, b_1) &= \max_{x_1, h_1, m_2, b_2} \{u_1(x_1) - h_1 + \beta_1 W_2(m_2, b_2)\} \\ \text{s.t. } x_1 &= h_1 + \phi_1(m_1 - m_2) - (1 + r_1)b_1 + b_2. \end{aligned}$$

where x_1 is consumption, h_1 is labor, (m_2, b_2) gives the money and debt taken into subperiod 2, ϕ_1 is the value of money and r_1 is the interest due in subperiod 1 (of course this interest

[5] The real wage w is constant, and normalized to 1, because implicitly we have firms with a linear technology; nothing of substance changes if we introduce more general firms and determine w endogenously (Aruoba and Wright 2003).

[6] To rule out Ponzi schemes, one normally imposes a credit limit $b_j \leq \bar{B}$, either explicitly or implicitly. We do not need this, because we are going to explicitly impose that agents pay off all debts in subperiod 1 each period. Due to quasi-linearity, this will not be a binding constraint, in the sense that it does not affect agents' payoffs, just like the usual no-Ponzi- scheme conditions are not binding. Of course this assumes agents are at interior solutions; see LW for conditions on fundamentals to guarantee that this is valid in these types of models.

does not have to be paid now, and can be rolled over into b_2). Substituting h_1 from the budget constraint into the objective function, we have the Bellman equation

$$W_1(m_1, b_1) = \max_{x_1, m_2, b_2} \left\{ u_1(x_1) - [x_1 + \phi_1(m_2 - m_1) + (1 + r_1)b_1 - b_2] + W_2(m_2, b_2) \right\}.$$

The first-order conditions are

$$1 = u_1'(x_1) \quad (1)$$

$$\phi_1 = \beta_1 W_{2m}(m_2, b_2) \quad (2)$$

$$-1 = \beta_1 W_{2b}(m_2, b_2). \quad (3)$$

The first condition implies $x_1 = x_1^*$ for all agents. The other two determine (m_2, b_2), independent of x_1 and (m_1, b_1), a generalization of one of the basic results in LW. As long as W_2 is strictly concave, there is a unique solution. It is simple to check that the conditions to guarantee strict concavity in m in LW also apply here, so we can use this to conclude $m_2 = M$ for all agents. However, W_2 is actually linear in b_2, which means we cannot pin down b_2 for any individual.

This is no surprise with a perfectly competitive credit market and quasi-linear utility: given the equilibrium interest rates (see below), agents are indifferent to working a little more today and less tomorrow, so for any one of them we can raise h_1 and lower b_2. We cannot do this in the aggregate, of course, since average labor input $\bar{h}_1$ must equal total output x_1^*. We resolve this payoff-irrelevant indeterminacy by focusing on symmetric equilibria, in the sense that when two agents have the same set of solutions to a maximization problem, they choose the same one. Nothing of substance hinges on this; other equilibria are payoff equivalent, and observationally equivalent at the aggregate level. It simply means that we have $b_2 = \bar{b}_2$ for all agents.

Aggregating budget equations across agents,

$$\bar{x}_1 = \bar{h}_1 + \phi_1(\bar{m}_1 - \bar{m}_2) - (1 + r_1)\bar{b}_1 + \bar{b}_2.$$

We have $\bar{h}_1 = \bar{x}_1 = x_1^*$ and $\bar{m}_1 = \bar{m}_2 = M$, clearly, and $\bar{b}_1 = 0$ because average debt must be 0. Hence in equilibrium $b_2 = \bar{b}_2 = 0$ for all agents. This simply says that in equilibrium agents settle all past debts in subperiod 1; they are happy to do so, given quasi-linear utility. Hence we have[7]

$$m_2 = M \text{ and } b_2 = 0 \text{ for all agents.}$$

To close the analysis of subperiod 1, we have the envelope conditions

$$\begin{aligned} W_{1m}(m_1, b_1) &= \phi_1 && (4) \\ W_{1b}(m_1, b_1) &= -(1 + r_1). && (5) \end{aligned}$$

So W_1 is linear in (m_1, b_1).

2.2 Subperiod 2

In the afternoon, some agents want to consume but cannot produce, and vice-versa. To ease the presentation, assume these events are i.i.d., and each period a measure $\pi \leq 1/2$ of agents want to consume but cannot produce, and the same measure π can produce but do not want to consume (to reduce the notation, we assume that no agent does both, but this is easy to relax). Feasibility requires $x_2^C = h_2^P$, where x_2^C is the consumption of those who want to consume and h_2^P the production of those able to produce (it is also easy to relax the assumption that there is the same measure of producers and consumers). The expected value of entering the subperiod 2 centralized market is

$$W_2(m_2, b_2) = \pi W_2^C(m_2, b_2) + \pi W_2^P(m_2, b_2) + (1 - 2\pi) W_2^N(m_2, b_2),$$

where W_2^C, W_2^P and W_2^N are the value functions for a consumer, a producer and a nontrader.

For a nontrader,

$$\begin{aligned} W_2^N(m_2, b_2) &= \max_{m_3, b_3} \beta_2 W_3(m_3, b_3) \\ \text{s.t. } 0 &= \phi_2(m_2 - m_3) - (1 + r_2) b_2 + b_3, \end{aligned}$$

where r_2 interest due on debt brought into the subperiod (which again can be rolled over to the next subperiod). Note that although a nontrader neither consumes not produces, he can adjust

[7] The condition $b_2 = 0$ is the aforementioned payoff-irrelevant condition that also rules out Ponzi schemes; hence we do not need any additional credit constraints.

his portfolio.[8] One can rewrite

$$W_2^N(m_2,b_2)=\max_{m_3}\ \beta_2 W_3\left[m_3,\phi_2(m_3-m_2)+(1+r_2)b_2\right],$$

which implies the solution (m_3^N,b_3^N) satisfies

$$W_{3m}(m_3^N,b_3^N)=-\phi_2 W_{3b}(m_3^N,b_3^N) \tag{6}$$

plus the budget equation. The envelope conditions are

$$\begin{aligned} W_{2m}^N(m_2,b_2) &= \beta_2 W_{3m}(m_3^N,b_3^N) && (7)\\ W_{2b}^N(m_2,b_2) &= \beta_2(1+r_2)W_{3b}(m_3^N,b_3^N). && (8)\end{aligned}$$

For a consumer,

$$\begin{aligned} W_2^C(m_2,b_2) &= \max_{x_2,m_3,b_3}\{u_2(x_2)+\beta_2 W_3(m_3,b_3)\}\\ \text{s.t. } x_2 &= \phi_2(m_2-m_3)-(1+r_2)b_2+b_3.\end{aligned}$$

One can rewrite

$$W_2^C(m,b)=\max_{m_3,b_3}\left\{u_2\left[\phi_2(m_2-m_3)-(1+r_2)b_2+b_3\right]+\beta_2 W_3(m_3,b_3)\right\},$$

which implies the solution (x_2^C,m_3^C,b_3^C) satisfies

$$\begin{aligned} \phi_2 u_2'(x_2^C) &= \beta_2 W_{3m}(m_3^C,b_3^C) && (9)\\ -u_2'(x_2^C) &= \beta_2 W_{3b}(m_3^C,b_3^C) && (10)\end{aligned}$$

plus the budget equation. Using these, we write the envelope conditions as

$$\begin{aligned} W_{2m}^C(m_2,b_2) &= \phi_2 u_2'(x_2^C)=\beta_2 W_{3m}(m_3^C,b_3^C) && (11)\\ W_{2b}^C(m_2,b_2) &= -(1+r_2)u_2'(x_2^C)=(1+r_2)\beta_2 W_{3b}(m_3^C,b_3^C). && (12)\end{aligned}$$

For a producer,

$$\begin{aligned} W_2^P(m_2,b_2) &= \max_{h_2,m_3,b_3}\{-c_2(h_2)+\beta_2 W_3(m_3,b_3)\}\\ \text{s.t. } 0 &= h_2+\phi_2(m_2-m_3)-(1+r_2)b_2+b_3.\end{aligned}$$

[8] Hence, it might appear that calling these agents nontraders is inaccurate, but we will see that in fact they do not trade in equilibrium.

We rewrite

$$W_2^P(m_2, b_2) = \max_{m_3, b_3} \{-c_2 [\phi_2(m_3 - m_2) - b_3 + (1 + r_2)b] + \beta_2 W_3(m_3, b_3)\}$$

and the solution (h_2^P, m_3^P, b_3^P) satisfies

$$\phi_2 c_2'(h_2^P) = \beta_2 W_{3m}(m_3^P, b_3^P) \tag{13}$$

$$-c_2'(h_2^P) = \beta_2 W_{3b}(m_3^P, b_3^P) \tag{14}$$

plus the budget equation. The envelope conditions are

$$\begin{aligned} W_{2m}^P(m_2, b_2) &= \phi_2 c_2'(h_2^P) = \beta_2 W_{3m}(m_3^P, b_3^P) \\ W_{2b}^P(m_2, b_2) &= -(1 + r_2) c_2'(h_2^P) = (1 + r_2)\beta_2 W_{3b}(m_3^P, b_3^P). \end{aligned}$$

We cannot conclude here that (m_3, b_3) is independent of (m_2, b_2), as we could for subperiod 1, where we had (m_2, b_2) independent of (m_1, b_1), since we are not necessarily assuming quasilinearity here. Thus, if x_2^C depends on (m_2, b_2) then so will (m_3^C, b_3^C), in general, unless u_2 is linear, and if h_2^P depends on (m_2, b_2) then so will (m_3^P, b_3^P), in general, unless c_2 is linear. This actually does not make a big difference in equilibrium, since we already established that $(m_2, b_2) = (M, 0)$ for all agents. But even if, e.g., all consumers choose the same (x_2, m_2, b_2), so far we have nothing to say about the comparison of (m_3, b_3) across consumers, producers and nontraders.

In any case, we can combine envelope conditions to write

$$W_{2m}(m_2, b_2) = \beta_2 \pi [W_{3m}(m_3^C, b_3^C) + W_{3m}(m_3^P, b_3^P) + \tfrac{1-2\pi}{\pi} W_{3m}(m_3^N, b_3^N)] \tag{15}$$

$$W_{2b}(m_2, b_2) = \beta_2 (1 + r_2) \pi [W_{3b}(m_3^C, b_3^C) + W_{3b}(m_3^P, b_3^P) + \tfrac{1-2\pi}{\pi} W_{3b}(m_3^N, b_3^N)] \tag{16}$$

Again, note that the continuation value $W_3(m_3, b_3)$ does not depend on whether one was a consumer, producer or nontrader in subperiod 2, except inasmuch as this affects the state (m_3, b_3).

2.3 Subperiod 3

In the evening, agents enter the decentralized market where trade occurs via anonymous bilateral meetings. Here, we assume that trading is done via bargaining, but this is not restrictive: we could, instead, model this market using price taking or price posting, as in Rocheteau and Wright (2005), or using auctions, as in Kircher and Galenianos (2006). Because of anonymity, you cannot use credit: I will not take your promise for payment tomorrow because I understand that you can renege without fear of punishment (Kocherlakota 1998; Wallace 2001).[9] I will, however, take cash. I will also take a check in the version of the model laid out in He, Huang and Wright (2005), because there a check is a claim on your bank and not you personally (think of travellers' checks). Similarly, I would take a debit card, which is tantamount to cash. So you can pay with money, or with money in the bank, but you cannot get credit.[10]

Before presenting the value functions, consider a single-coincidence meeting, where one agent wants to consume and the other can produce. Call the former agent the *buyer* and the latter the *seller*. They bargain over the amount of consumption for the buyer x_3 and labor by the seller h_3, and also a dollar payment d from to the former to the latter. Since feasibility implies $x_3 = h_3$ we denote their common value by q. If (m_3, b_3) is the state of a buyer and $(\tilde{m}_3, \tilde{b}_3)$ the state of a seller, the outcome satisfies the generalized Nash bargaining solution,

$$(q, d) \in \arg\max \; S(m_3, b_3)^{\theta} \tilde{S}(\tilde{m}_3, \tilde{b}_3)^{1-\theta} \text{ s.t. } d \leq m_3, \tag{17}$$

where the constraint says the buyer cannot transfer more cash than he has, θ is the bargaining power of the buyer, and

$$\begin{aligned} S(m_3, b_3) &= u_3(q) + \beta_3 W_{1,+1}(m_3 - d, b_3) - \beta_3 W_{1,+1}(m_3, b_3) \\ \tilde{S}(\tilde{m}_3, \tilde{b}_3) &= -c_3(q) + \beta_3 W_{1,+1}(\tilde{m}_3 + d, \tilde{b}_3) - \beta_3 W_{1,+1}(\tilde{m}_3, \tilde{b}_3) \end{aligned}$$

[9] A question may arise why agents in the third subperiod cannot trade claims to the good in the first-subperiod centralized market. We assume that these claims, even if they were issued by some entity in the centralized market, could be counterfeited by agents in the decentalized market, while money can never be counterfeited. This rules out the use of these claims, as sellers in the DM would not accept them.

[10] Since the model is highly stylized, it is not clear exactly what a credit card might be; whatever it is, it cannot be used in this market. Although it is perhaps good to be agnostic about this, it is also good to have a story. One story is that agents can produce fake credit cards. Note that fake debit cards are not possible, because these involve instant settlement using money that is already on deposit at the bank. The same is true for travellers' checks, or personal checks that are verified using a check card. The point is that although we think the model captures nicely the distinction between cash and credit, one has to work a little to use it to discuss credit cards and other means of payment.

are the surpluses. A +1 in the subscript denotes next period. Using (4) and (5), the surpluses simplify to

$$\begin{aligned} S(m_3, b_3) &= u_3(q) - \beta_3 \phi_{1,+1} d \\ \tilde{S}(\tilde{m}_3, \tilde{b}_3) &= -c_3(q) + \beta_3 \phi_{1,+1} d \end{aligned}$$

Now the following result is a straightforward generalization of LW; see the Appendix for the the proof.

Lemma 1. $\forall (m_3, b_3)$ *and* $(\tilde{m}_3, \tilde{b}_3)$*, the solution to the bargaining problem is*

$$q = \begin{cases} g^{-1}(\beta_3 m_3 \phi_{1,+1}) & \text{if } m_3 < m_3^* \\ q^* & \text{if } m_3 \geq m_3^* \end{cases} \quad \text{and } d = \begin{cases} m_3 & \text{if } m_3 < m_3^* \\ m_3^* & \text{if } m_3 \geq m_3^* \end{cases} \tag{18}$$

where q^* *solves* $u_3'(q^*) = c_3'(q^*)$*, the function* $g(\cdot)$ *is given by*

$$g(q) = \frac{\theta u_3'(q) c_3(q) + (1-\theta) u_3(q) c_3'(q)}{\theta u_3'(q) + (1-\theta) c_3'(q)}, \tag{19}$$

and $m_3^* = g(q^*)/\beta_3 \phi_{1,+1}$.

Clearly, the bargaining solution (q, d) depends on the buyer's money holdings m_3, but on no other element of (m_3, b_3) or $(\tilde{m}_3, \tilde{b}_3)$; hence we write $q = q(m_3)$ and $d = d(m_3)$ from now on. Of course, q and d at t also depend on ϕ_1 at $t+1$, but this is left implicit in the notation. We argue in the Appendix that, as in LW, $m_3 < m_3^*$ in any equilibrium. So from Lemma 1, buyers always spend all their money m_3 and receive $q = g^{-1}(\beta_3 m_3 \phi_{1,+1})$. Notice $g' > 0$; thus, if the buyer brings an additional dollar to this market, the terms of trade change according to $\partial q / \partial m_3 = \beta_3 \phi_{1,+1} / g'(q) > 0$ and $\partial d / \partial m_3 = 1$.

Define

$$z(q) = \frac{u_3'(q)}{g'(q)}. \tag{20}$$

As is usual in this type of model, it is useful to make the assumption that $z'(q) < 0$ (see e.g. Rocheteau and Wright 2005). This assumption is not completely standard, as it involves third derivatives of utility.[11] We will not dwell on this here, except to say that conditions on preferences

[11] Since g depends on u_3' and c_3', g' depends on u_3'' and c_3'', and the monotonicity of u'/g' depends on third derivatives.

to guarantee that the assumption holds can be found in LW, and also to note that it always holds (for any preferences) when $\theta \approx 1$, because at $\theta = 1$, $g(q) = c_3(q)$ and $z(q) = u_3'(q)/c_3'(q)$.

This completes the analysis of bargaining in a single-coincidence meeting. Let σ denote the probability of such a meeting – i.e. the probability that two agents meet, one wants to consume the other's good but cannot produce, and the other can produce but does not want to consume.[12] Given the above results, the value function $W_3(m_3, b_3)$ satisfies the Bellman equation

$$\begin{aligned} W_3(m_3, b_3) &= \sigma \{u_3[q(m_3)] + \beta_3 W_1[m_3 - d(m_3), b_3]\} \\ &\quad + \sigma \mathbb{E} \{-c_3[q(\tilde{m}_3)] + \beta_3 W_1[m_3 + d(\tilde{m}_3), b_3]\} \\ &\quad + (1 - 2\sigma)\beta_3 W_1[m_3, b_3], \end{aligned} \tag{21}$$

where $\mathbb{E}$ is the expectation of $\tilde{m}_3$ (the money holdings of other agents, which may be nondegenerate even though all agents carry the same amount of money out of subperiod 1, because they may leave subperiod 2 with different amounts, depending on whether they are consumers, producers or nontraders).

Differentiating (21), using the linearity of W_1 derived in (4) and (5), and $q'(m_3) = \beta_3 \phi_{1,+1}/g'(q)$, we have

$$W_{3m}(m_3, b_3) = \beta_3 \phi_{1,+1} \{\sigma z [q(m_3)] + 1 - \sigma\} \tag{22}$$

$$W_{3b}(m_3, b_3) = -\beta_3 (1 + r_{1,+1}), \tag{23}$$

where $z(q)$ is given by (20). Here (22) gives the marginal value of money in the decentralized market as a weighted average of the values of using it in the decentralized market and of carrying it forward to the next subperiod. According to (23), the marginal value of debt is simply the value to rolling it over into subperiod 1 at $t + 1$, since credit is not adjusted in the decentralized market.

[12] In general, in the standard version of the model with many specialized commodities, one interprets a single-coincidence meeting as one where one agent wants what the other can produce, but not vice-versa. Since there is only one good in this paper, we interpret it instead as a meeting where one agent wants to consume but cannot produce, while the other can produce but does not want to consume. In the standard model, it is easy to allow some double-coincidence meetings, where both agents want what the other can produce. Here we can similarly allow some agents to both consume and produce in subperiod 3 – in which case they do not even need a meeting – but it adds little of interest.

2.4 Equilibrium

We now define formally an equilibrium. The definition is relatively standard, except that there is no market-clearing condition for $s = 3$: since all trade is bilateral in this market, it clears automatically. For similar reasons there are no market prices in subperiod 3: the terms of trade are defined implicitly via the bargaining solution. Also, to reduce notation we describe every agent's problem at $s = 2$ in terms of choosing (x_2, h_2, m_3, b_3), although it is implicit that for producers $x_2^P = 0$, for consumers $h_2^C = 0$, and for nontraders $x_2^N = h_2^N = 0$. Also to reduce notation, we do not index individual objects by an agent's identity i or his state, although agents in different states generally make different decisions.[13]

Definition 1. *An* equilibrium *is a set of (possibly time-dependent) value functions* $\{W_s\}$, $s = 1, 2, 3$, *decision rules* $\{x_s, h_s, m_{s+1}, b_{s+1}\}$, $s = 1, 2$, *bargaining outcomes* $\{q, d\}$, *and prices* $\{r_s, \phi_s\}$, $s = 1, 2$, *such that:*

1. *Optimization: In every period, for every agent,* $\{W_s\}$, $s = 1, 2, 3$, *solve the Bellman equations,* $\{x_s, h_s, m_{s+1}, b_{s+1}\}$, $s = 1, 2$, *solve the maximization problem, and* $\{q, d\}$ *solve the bargaining problem.*

2. *Market clearing: In every period,*

$$\bar{x}_s = \bar{h}_s,\ \bar{m}_{s+1} = M,\ \bar{b}_{s+1} = 0,\ s = 1, 2$$

   where for any variable y, $\bar{y} = \int y^i di$ *denotes the aggregate.*

Definition 2. *A steady state equilibrium is an equilibrium where the endogenous variables are consatnt across time periods (although not generally across subperiods within a period).*

[13] We do not include the distribution of the state variable in the definition of equilibrium, but it is implicit: given an initial distribution $F_1(m, b)$ at the start of subperiod 1, the decision rules generate $F_2(m, b)$; then the decision rules at $s = 2$ generate $F_3(m, b)$; and the bargaining outcome at $s = 3$ generates $F_{1,+1}(m, b)$. Also, we only consider interior equilibria, in the sense that $h > 0$, and if there is an upper bound for labor, say H, then $h < H$. This is important because we cannot impose Inada conditions on utility of labor in the centralized market, since utility is quasilinear in labor. It is possible instead to impose conditions on primitives to guarantee interiority; see LW.

We are mainly interested in equilibira where money is valued, and hence where it is valued in all subperiods in every period, by the usual logic (if it is not valued at some point, then it is never valued).

Definition 3. *A monetary equilibrium is an equilibrium where, in every period,* $\phi_s > 0$*,* $s = 1, 2$*, and* $q > 0$*.*

We now establish strong results about steady-state equilibria (we relax the steady state condition below). First, however, recall that in equilibrium we impose that in subperiod 1, if two agents have multiple solutions for b_2 they choose the same one, which means $b_2 = 0$. As we discussed above, due to quasi-linear utility, there are other equilibria but they are payoff-equivalent for individuals and observationally equivalent at the aggregate level. In these other equilibria, b_2 and h_1 may be different for individuals, although not at the aggregate level, but all other variables will be identical for every individual to what is described below.[14]

Theorem 1. *In any steady state monetary equilibrium:*

1. *At* $s = 1$*, all agents choose* $x_1 = x_1^*$*,* $m_2 = M$*,* $b_2 = 0$*, and*

$$h_1 = h_1(m_1, b_1) = x_1^* - \phi_1(m_1 - M) + (1 + r_1)b_1,$$

which implies $\bar{h}_1 = x_1^*$*.*

2. *At* $s = 2$*,*

   consumers choose $x_2 = x_2^*$*,* $m_3 = M$ *and* $b_3 = x_2^*$*;*

   producers choose $h_2 = x_2^*$*,* $m_3 = M$ *and* $b_3 = -x_2^*$*;*

   nontraders choose $m_3 = M$ *and* $b_3 = 0$*.*

3. *At* $s = 3$*, in every trade* $d = M$ *and* q *solves*

$$1 + \frac{\rho}{\sigma} = \frac{u_3'(q)}{g'(q)} = z(q), \tag{24}$$

where ρ *is the rate of time preference defined by* $\frac{1}{1+\rho} = \beta_1\beta_2\beta_3$*.*

4. *Prices are given by:*

$$\begin{aligned} r_1 &= \frac{u_2'(x_2^*) - \beta_2\beta_3}{\beta_2\beta_3},\ r_2 = \frac{\rho - r_1}{1 + r_1}, \\ \phi_1 &= \frac{g(q)}{\beta_3 M},\ \text{and } \phi_2 = \frac{\phi_1\left[\sigma z(q) + 1 - \sigma\right]}{1 + r_1}. \end{aligned}$$

[14] Recall that we also focus only on equilibria with interior solutions for h_s.

Proof: To begin, insert the envelope condition for W_{3b} from (23) into the first order conditions for consumers and producers with respect to b_3, (10) and (14), to get

$$u_2'(x_2^C) = \beta_2\beta_3(1+r_{1,+1}) \tag{25}$$

$$c_2'(h_2^P) = \beta_2\beta_3(1+r_{1,+1}). \tag{26}$$

Hence, $u_2'(x_2^C) = c_2'(h_2^P)$, and $x_2^C = h_2^P = x_2^*$. Similarly, insert the envelope condition for W_{3m} from (22) in to the first order conditions for consumers and producers with respect to m_3, (9) and (13), to get

$$\phi_2 u_2'(x_2^C) = \beta_2\beta_3\phi_{1,+1}\left\{\sigma z\left[q(m_3^C)\right] + 1 - \sigma\right\} \tag{27}$$

$$\phi_2 c_2'(h_2^P) = \beta_2\beta_3\phi_{1,+1}\left\{\sigma z\left[q(m_3^P)\right] + 1 - \sigma\right\} \tag{28}$$

Given $z(q)$ is decreasing and $q(m)$ is increasing for all $m < m_3^*$, and since $x_2^C = h_2^P = x_2^*$, we conclude that $m_3^C = m_3^P$.

Similarly, inserting the envelope conditions (23) and (22) into the first order condition for a nontrader,

$$\phi_{1,+1}\left\{\sigma z\left[q(m_3^N)\right] + 1 - \sigma\right\} = \phi_2(1+r_{1,+1}) \tag{29}$$

Exactly the same condition results from combining (25) and (27) for a consumer, or (26) and (28) for a producer. Hence, we conclude $m_3^N = m_3^C = m_3^P = M$, and everyone carries the same amount of money into subperiod 3. From the budget equations, this means debt is given by

$$\begin{aligned} b_3^C &= x_2^* + (1+r_2)b_2 \\ b_3^P &= -x_2^* + (1+r_2)b_2 \\ b_3^N &= (1+r_2)b_2. \end{aligned}$$

This completes the description of subperiod 2. Moving back to subperiod 1, clearly (1) implies $x_1 = x_1^*$. Inserting the envelope conditions for W_2 and W_3 into the first order conditions (2) and (3) for m_2 and b_2, we have

$$\phi_1 = \beta_1\beta_2\beta_3\phi_{1,+1}\{\sigma z[q(M)] + 1 - \sigma\} \tag{30}$$

$$1 = \beta_1\beta_2\beta_3(1+r_2)(1+r_{1,+1}), \tag{31}$$

where we have used in the first case that W_{3m} depends on m_3 but not b_3, and everyone has the same $m_3 = M$. Notice that (31) is an arbitrage condition between r_2 and $r_{1,+1}$: if it does not hold there is no solution to the agents' problem at $s = 1$; and if it does hold then any choice of b_2 is consistent with optimization. Hence we can set $b_2 = 0$ in any equilibrium. On the other hand, (30) implies

$$(1+\rho)\frac{\phi_1}{\phi_{1,+1}} = \sigma z[q(M)] + 1 - \sigma. \tag{32}$$

In steady state this implies (24).

The only things left to determine are the prices. We get r_1 from (25) with $x_2 = x_2^*$, and then set r_2 in terms of r_1 to satisfy the arbitrage condition (31). Given q, Lemma 1 tells us $\phi_1 = g(q)/\beta_3 M$, and (29) gives

$$\phi_2 = \frac{\phi_1[\sigma z(q) + 1 - \sigma]}{(1+r_1)}.$$

This completes the proof. ■

We now consider the relative rates of return on money and debt. First notice that condition (29) gives equality of values of a dollar taken out in liquid assets and a dollar taken out on credit: the value of a dollar in liquid assets is equal to its rate of return *plus the liquidity premium* that a consumer would get from spending it in the decentralized market. Suppose now that we do not consider the liquidity premium, and consider only the pure rate or return on money relative to debt. When a consumer in subperiod 2 at t makes a purchase, for every unit of the good he buys, his debt goes up by 1 unit. In subperiod 1 at $t+1$, he pays it off (principal plus interest) in the amount $1 + r_{1,+1}$. Hence, the interest rate on consumer debt is $r_{1,+1}$. In contrast, a dollar is worth ϕ_2 units of consumption in subperiod 2 at t, and worth $\phi_{1,+1}$ units of consumption in subperiod 1 at $t+1$. Hence the rate of return on money over the same period is $\phi_{1,+1}/\phi_2$. We now show that the rate of return on money is strictly less than the return on debt. That is, our model generates rate of return dominance, and therefore, since the same consumer is holding credit card debt and cash, our model generates the observation that has been called the credit card debt puzzle. In the following argument we leave the time subscript $+1$, even though we are (for now) focusing on steady states, because it facilitates the economic intuition, as in the above discussion.

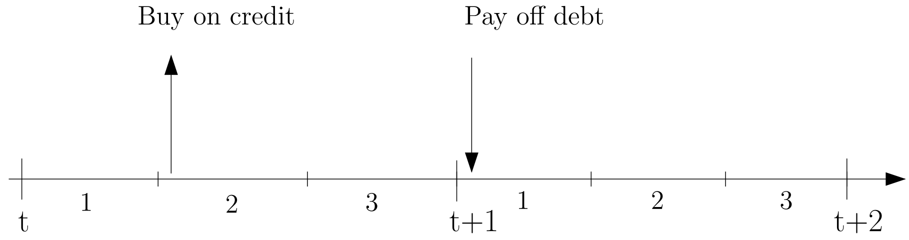

Figure 1: Computing Relative Rates of Return in Theorem 2

Theorem 2. *(Rate of Return Dominance) In any steady state monetary equilibrium,*

$$\frac{\phi_{1,+1}}{\phi_2} < 1 + r_{1,+1}.$$

Proof: By (29),

$$\frac{\phi_{1,+1}}{\phi_2} = \frac{1 + r_{1,+1}}{1 - \sigma + \sigma z(q)}.$$

Hence we have the desired result if $1 - \sigma + \sigma z(q) > 1$, or $z(q) > 1$. But by (24), in steady state monetary equilibrium $z(q) = 1 + \rho/\sigma > 1$. ■

We can, of course, also consider the relative returns on money and debt over an entire period. From subperiod 1 at t to subperiod 1 at $t + 1$, the return on money is 1 in steady state, while the cost of debt taken out in subperiod 1 is $(1 + r_2)(1 + r_{1,+1})$. We readily get from (31) that $(1 + r_2)(1 + r_{1,+1}) > 1$ as long as $\beta_1\beta_2\beta_3 < 1$, so the rate of return dominance holds across an entire period as well. Since we have established, however, that noone borrows in subperiod 1, this result is of less interest.

To summarize, note that point 2 in theorem 1, and theorem 2, give that in equilibrium the observation known as the credit card debt puzzle arises. Consumers in subperiod 2 choose $m_3 = M > 0$ and $b_3 = x_2^* > 0$, so in their portfolios we observe coexistence of debt and liquid asset holdings, even though the cost of debt is strictly higher than the rate of return on money. The reason consumers do this is because of the liquidity premium that holding money provides by giving them purchasing power in the decentralized market.

3 Discussion

The analysis is easily extended along a number of dimensions. First, in any equilibrium, and not just in any steady state equilibrium, essentially everything in the previous Theorem is true, except (32) does not reduce to (24). However, we can insert the bargaining solution $q = g^{-1}(\beta_3 m_3 \phi_{1,+1})$ from Lemma 1, which holds at every date in any equilibrium, to get

$$(1+\rho)\frac{g(q)}{g(q_{+1})} = \sigma z(q_{+1}) + 1 - \sigma. \tag{33}$$

A monetary equilibrium is now any (bounded, positive) solution $\{q_t\}$ to this difference equation.

Now there typically exist many (bounded, positive) solutions to (33), and hence many non-steady-state monetary equilibria, as is standard. However, in all of these equilibria, most of the results in Theorem 1 still hold: in every period we still have $x_1 = x_1^*$ and $b_2 = 0$, $x_2^C = x_2^*$, $b_3^C = x_2^*$, $h_2^P = x_2^*$, $b_3^P = -x_2^*$, and $b_3^N = 0$ for all agents, and at the aggregate level we still have $\bar{h}_1 = x_1^*$. Also, r_1 and r_2 are the same at every date as given in the Theorem, although ϕ_1 and ϕ_2 vary over time when q does. This is an another example of the dichotomy discussed in Aruoba and Wright (2003): in the LW framework, one can solve for the real allocations in the centralized and decentralized markets independently.[15]

Second, suppose $M_{+1} = (1+\gamma)M$, that is, money supply is changing over time at constant rate γ. Then we cannot have a steady state as defined above, but it is natural to look for an equilibrium where all real variables are constant, including q and real balances ϕM. Inserting this into (32), we have

$$(1+\rho)(1+\gamma) = \sigma z[q(M)] + 1 - \sigma.$$

Indeed, if we use the Fisher equation for the nominal interest rate, the left hand side is simply $1+i$, and so we have

$$1 + \frac{i}{\sigma} = z(q). \tag{34}$$

Hence, q is decreasing in i and, therefore, γ. But again, this does not affect the real allocation in the centralized market. Note also, that as is standard in these models, Friedman rule is the lower bound on inflation, and it is also the welfare-maximizing policy. Comparing the above

[15] There are some extensions of the model where the dichotomy does not hold; see Aruoba, Waller and Wright (2005).

condition with (24), it is clear that with money supply growth, $z(q) \geq 1$, which weakens the strict inequality we get in the case with constant money supply. At the Friedman rule, the relative returns of the two assets are equated, which means that, as is to be expected, the rate of return dominance is maintained only away from the Friedman rule.

Third, it should be clear that it is no puzzle that some agents carry debt and cash simultaneously. In particular, consumers in subperiod $s = 2$ choose positive money and debt holdings. Thus, we get co-existence, within any period, of liquid assets and consumer debt in their portfolios. However, in this model, agents do not roll over debt across periods: they pay it off at $s = 1$ in each the period. Nonetheless the model captures the idea in a straightforward way that agents may have high-interest consumer debt but not want to part with their liquid assets: they simply need the latter should they want to consume, in $s = 3$, when they cannot use credit.

Finally, this model may seem special in structure, but we think that it captures a general idea that since consumers may find themselves in situations when they want to consume but cannot use credit or get instant access to additional income, they will endogenously choose to keep some of their wealth in liquid assets even at the cost of not paying down high-interest debt. One of the ways that the model may seem special is the particular sequencing of markets: market 1 is centralized with no double-coincidence problem, market 2 has a double-coincidence problem, but since it is centralized, credit is possible, and market 3 has a double-coincidence problem and is sufficiently decentralized that credit is not available. In the next section we sketch a generalization that relaxes these strong assumptions and shows that the main results still hold.

4 A General Model

In this section, we pursue two generalizations. First, we allow any number n of subperiods per period, instead of only 3, and n may even change from period to period. Second, we allow the decentralized market to be open in all but the first subperiod, simultaneously with the centralized market, and have agents randomly transiting between markets: δ_s is the probablitiy that an agent in the centralized market at s will find himself in the decentalized market at $s + 1$.This means that when the centralized market meets in any subperiod, agents will have

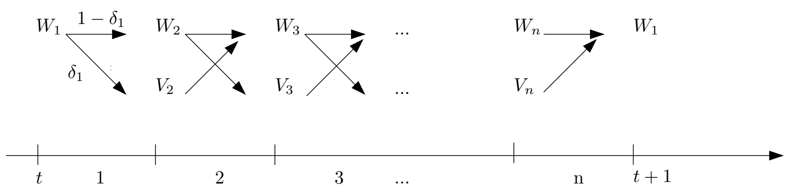

Figure 2: Market Structure of the Simultaneous-Market Model

a demand for liquidity because they may not be able to use credit next subperiod. The only special assumptions we make are: (a) at the end of subperiod n, all agents transit back to the centralized market to start the next period there ($\delta_n = 0$), but this is mainly for convenience and is not critical; and (b) if an agent is in the decentalized market at s, then he returns to the centralized market at $s+1$ with probability 1.[16]

At the beginning of period t, during subperiod 1, the centralized market is the usual frictionless market from LW, with no double-coincidence problem. Again, here agents are assumed to have preferences that are nonlinear in consumption, but linear in labor, and everyone can produce output 1 for 1 with their own labor. In each of the subsequent subperiods $s \in \{2, ..., n\}$, the centralized market is in the spirit of the market at $s = 2$ presented before: we retain the assumption that at least some of the consumers cannot produce. Here, the ability to produce is determined by a stochastic shock on productivity, denoted by ω_s, where the subscript $s \in \{2, ..., n\}$ stands for the subperiod, and the t subscript is omitted. We have already assumed above $\varepsilon_1 = 1$.

The decentralized market that is open simultaneously during any subperiod (but the first) is the standard LW decentralized market, where agents meet randomly, bargain, and trade bilaterally. The meetings are, as always, anonymous, so credit cannot be used in this market.

As before, let us associate the value function W_s with the centralized market in subperiod s, and the value function V_s with the decentralized market in subperiod s. Figure 2 demonstrates the market structure that we have in mind.

[16]This assures that agents in the decentralized market are willing to spend all their money – they know that next period they will be back in the centralized market where they can get more. We borrow this idea from Williamson (2005).

4.1 Subperiod 1

In the first subperiod, everyone solves the same problem, with the usual variables taken as states:

$$\begin{aligned} W_1(m_1, b_1) &= \max_{x_1, h_1, m_2, b_2} \{U_1(x_1) - h_1 + \beta_1(1-\delta_1)\mathbb{E}W_2(\omega_2, m_2, b_2) + \beta_1\delta_1 V_2(m_2, b_2)\} \\ s.t.\, x_1 &= h_1 + \phi_1(m_1 - m_2) + b_2 - (1+r_1)b_1 \end{aligned}$$

From this, we get the following first-order conditions:[17]

$$U_{1x}(x_1) = 1 \tag{35}$$

$$\beta_1(1-\delta_1)\mathbb{E}W_{2m}(\omega_2, m_2, b_2) + \beta_1\delta_1 V_{2m}(m_2, b_2) = \phi_1 \tag{36}$$

$$\beta_1(1-\delta_1)\mathbb{E}W_{2b}(\omega_2, m_2, b_2) + \beta_1\delta_1 V_{2b}(m_2, b_2) = -1. \tag{37}$$

The envelope conditions are

$$W_{1m}(\omega_1, m_1, b_1) = \phi_1 \tag{38}$$

$$W_{1b}(\omega_1, m_1, b_1) = -(1+r_1) \tag{39}$$

So we have, again, linearity of the first-subperiod value function W_1 in (m_1, b_1). Also, everyone chooses the same x_1 as the solution to (35). Finally, (m_2, b_2) is determined as the solution to (36) and (37), independent of (m_1, b_1), which does not appear in these conditions, as long as these conditions can be solved. This is the same as a standard LW model, and indeed collapses to such a model when $\delta_1 = 1$. As in the LW framework ,we must look to future periods in order to solve (36) and (37).

4.2 Subperiod $s \in \{2, ..., n\}$

In any subperiod $s \in \{2, ..., n\}$, agents in the centralized market solve the following problem, where the current realization of the shock ε_s is also taken as a state:

$$\begin{aligned} W_s(\omega_s, m_s, b_s) &= \max_{x_s, h_s, m_{s+1}, b_{s+1}} \{U_s(x_s, h_s) + \beta_s(1-\delta_s)\mathbb{E}W_{s+1}(\omega_{s+1}, m_{s+1}, b_{s+1}) \\ &\qquad + \beta_s\delta_s V_{s+1}(m_{s+1}, b_{s+1})\} \\ s.t.\ x_s &= \omega_s h_s + \phi_s(m_s - m_{s+1}) + b_{s+1} - (1+r_s)b_s \end{aligned}$$

[17] Note that just as before, we do not impose an explicit credit limit, but the assumption that everyone returns to the first-subperiod LW centralized market at the beginning of each period again rules out Ponzi schemes, as in the first model.

At the end of subperiod n, everyone must return to the standard LW centralized market again, where preferences are quasilinear. The formulation of the problem at $s = n$ is therefore a special case, where $\delta_n = 0$.

The first-order conditions of this problem in the centralized market are:

$$\begin{aligned} \omega_s U_{sx}(x_s, h_s) + U_{sh}(x_s, h_s) &= 0 & (40) \\ \beta_s(1-\delta_s)\mathbb{E}W_{s+1,m}(\omega_{s+1}, m_{s+1}, b_{s+1}) + \beta_s\delta_s V_{s+1,m}(m_{s+1}, b_{s+1}) &= \phi_s U_{sx}(x_s, h_s) & (41) \\ \beta_s(1-\delta_s)\mathbb{E}W_{s+1,b}(\omega_{s+1}, m_{s+1}, b_{s+1}) + \beta_s\delta_s V_{s+1,b}(m_{s+1}, b_{s+1}) &= -U_{sx}(x_s, h_s) & (42) \end{aligned}$$

Note that (35) from subperiod 1 is a special case of (40), where $\omega_1 = 1$ and $U_{1h}(x_1, h_1) = -1$. Similarly, (36) and (37) are special cases of (41) and (42) where $U_{1x}(x_1, h_1) = 1$. The envelope conditions are

$$\begin{aligned} W_{sm}(\omega_s, m_s, b_s) &= \phi_s U_{sx}(x_s, h_s) & (43) \\ W_{sb}(\omega_s, m_s, b_s) &= -(1+r_s)U_{sx}(x_s, h_s). & (44) \end{aligned}$$

which generalize (38) and (39). It is not at all obvious at this stage that W_s is linear in (m_s, b_s), or that all agents choose the same

Meanwhile, agents who are in the decentralized market in any subperiod s solve a Nash bargaining problem, where for simplicity we assume take-it-or-leave-it offers by buyers. Then, d_s must still satisfy $d_s = m_s$ as before, while q_s will solve

$$c(q_s) = \beta_s\mathbb{E}W_{s+1}[\omega_{s+1}, m_s + d_s, (1+r_s)b_s] - \beta_s\mathbb{E}W_{s+1}[\omega_{s+1}, m_s, (1+r_s)b_s] \tag{45}$$

The value function in this market, given that seller's surplus will be 0, is

$$\begin{aligned} V_s(m_s, b_s) &= \sigma\left\{u(q_s) + \beta_s\mathbb{E}W_{s+1}\left[\omega_{s+1}, m_s - d_s, (1+r_s)b_s\right]\right\} \\ &\quad + (1-\sigma)\beta_s\mathbb{E}W_{s+1}\left[\omega_{s+1}, m_s, (1+r_s)b_s\right] \end{aligned}$$

Thus, the envelope conditions are:

$$\begin{aligned} V_{sm}(m_s, b_s) &= (1-\sigma)\beta_s\mathbb{E}W_{s+1,m}\left[\omega_{s+1}, m_s, b_s(1+r_s)\right] + \sigma u'(q_s)q'(m_s) & (46) \\ V_{sb}(m_s, b_s) &= \beta_s(1+r_s)\mathbb{E}W_{s+1,b}\left[\omega_{s+1}, m_s, b_s(1+r_s)\right]. & (47) \end{aligned}$$

We now use first-order and envelope conditions of the centralized and decentralized markets in subperiod s to characterize the solution of the model. We proceed backwards, starting at subperiod n. Recall that $\delta_n = 0$. Thus, the first-order conditions for money and debt in the centralized market are

$$\begin{aligned}\phi_n U_{nx}(x_n, h_n) &= \beta_n W_{1,+1,m}(m_{1,+1}, b_{1,+1}) \\ U_{nx}(x_n, h_n) &= -\beta_n W_{1,+1,b}(m_{1,+1}, b_{1,+1})\end{aligned}$$

and using the envelope conditions (38) and (39), we get

$$\phi_n U_{nx}(x_n, h_n) = \beta_n \phi_{1,+1} \tag{48}$$

$$U_{nx}(x_n, h_n) = \beta_n(1 + r_{1,+1}). \tag{49}$$

The envelope conditions for subperiod n are, predictably:

$$W_{nm}(\omega_n, m_n, b_n) = \phi_n U_{nx}(x_n, h_n) \tag{50}$$

$$W_{nb}(\omega_n, m_n, b_n) = -(1 + r_n) U_{nx}(x_n, h_n) \tag{51}$$

Observe that combining (50)-(51) with (48)-(49) gives that the subperiod-n centralized-market value functions are again linear in the current portfolio variables. Finally, turning to the subperiod-n decentralized market, using (45), the linearity of W_n in m as given by (38), and the fact that $d_s = m_s \forall s$, we get consumption in the decentralized market in this subperiod as $c[q_n(m_n)] = \beta_n \phi_{1,+1} m_n$, which implies

$$q_n'(m_n) = \frac{\beta_n \phi_{1,+1}}{c'(q_n)}. \tag{52}$$

In any subperiod prior to n, agents can go from the centralized market to either the centralized or the decentralized market. The first-order conditions for money and debt in subperiod $n-1$ become, after using envelope conditions of both markets and simplifying:

$$\begin{aligned}\phi_{n-1} U_{n-1,x}(x_{n-1}, h_{n-1}) &= \beta_{n-1}(1 - \delta_{n-1})\beta_n \phi_{1,+1} \\ &\quad + \beta_{n-1}\delta_{n-1}[(1 - \sigma)\beta_n \phi_{1,+1} + \sigma u'(q_n) q'(m_n)] \\ U_{n-1,x}(x_{n-1}, h_{n-1}) &= \beta_{n-1}\beta_n(1 + r_{1,+1})(1 + r_n)\end{aligned}$$

The envelope conditions for the centralized market in $n-1$ are as in any subperiod s, so we do not reproduce them here. We now use (52) to simplify the first-order condition above to

$$\begin{aligned}\phi_{n-1}U_{n-1,x}(x_{n-1},h_{n-1}) &= [1-\delta_{n-1}+\delta_{n-1}(1-\sigma)]\beta_{n-1}\beta_n\phi_{1,+1} \\ &\quad + \delta_{n-1}\sigma\beta_{n-1}\beta_n\phi_{1,+1}\frac{u'(q_n)}{c'(q_n)}\end{aligned}$$

We can continue back in this fashion, which allows us to draw some general conclusions about the characterization of the problem in any subperiod s. First, the first-order conditions with respect to b_{s+1} give

$$U_{sx}(x_s,h_s) = \beta_s\beta_{s+1}...\beta_n(1+r_{s+1})...(1+r_n)(1+r_{1,+1}) \tag{53}$$

This makes clear that in any subperiod, the solution of the agent's problem is independent of his previous portfolio and shock history. Moreover, we can use this to derive the following characterization of the decentralized-market problem:

$$c[q_s(m_s)] = \phi_{s+1}\beta_s\beta_{s+1}...\beta_n(1+r_{s+1})(1+r_{s+2})...(1+r_{1,+1})m_s, \tag{54}$$

which can be used to simplify the first-order condition with respect to m_s. Putting this simplification aside for the moment, we have the first-order condition with respect to money holdings generalized as follows:

$$\phi_s U_{sx}(x_s,h_s) = \beta_s(1-\delta_s)\phi_2 U_{s+1,x}(x_{s+1},h_{s+1}) \tag{55}$$

$$+\beta_s\delta_s[(1-\sigma)\beta_{s+1}\phi_{s+2}U_{s+2,x}(x_{s+2},h_{s+2})+\sigma u'(q_{s+1})q'_{s+1}(m_{s+1})] \tag{56}$$

The decision on how much money to hold depends on the probabilities of participating in the decentralized market in any of the subperiods that follow s. Using (53) we observe that this decision does not depend on the current portfolio holding, since $U_{s+1}(\cdot), U_{s+2}(\cdot)$ and $q_{s+1}(\cdot)$ are all independent of (m_s, b_s). This means that agents will all carry equal amounts of money, in anticipation of possibly finding themselves in the decentralized market, and will transact in the centralized market using credit only.

Thus, we have shown that neither the sequential setup of subperiods and markets initially presented, nor the number of subperiods in a period are restrictive. Even if centralized and decentralized markets operate simultaneously, with each agent having an idiosyncratic path through them, and with any number of subperiods in a period, we retain the essence of the credit card debt puzzle in the model. Since we allow any number of subperiods within a period, one could interpret this by thinking of a period as a month consisting of 30 or 31 day-long subperiods. For 30 days, an agent can accumulate debt whenever he transacts in the centralized market, and spend his cash whenever he finds himself in the decentralized market. On the first day of the following month, he receives his paycheck and pays off his “credit card bill" in full, but during the month, we observe the co-existence of the two assets in a household portfolio, just as we did in the first version of the model, and the agent, in addition, revolves the debt for the entire duration of the month.

5 Conclusion

In this paper, we re-visit the issue of co-existence of assets with differing returns, motivated by the credit card debt puzzle - the empirical fact that many U.S. households simultaneously hold significant credit card debt and significant liquid accounts in the bank. We build on recent monetary literature with micro-foundations, in particular Lagos and Wright (2005), to introduce the option of credit in trade. Taking seriously frictions that arise in some trades, we show that existence of markets where the use of credit is precluded by anonymity and other features of the environment will induce households to hold money even in the presence of other assets that dominate it in return. We extend the basic model in several ways and show that this co-existence is robust to many such extensions. We consider our theoretical contributions interesting in their own right, in the context of the literature where rate of return dominance is a classic problem. We also show that the credit card debt puzzle itself need not be puzzling once we take certain trade frictions seriously.

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Appendix A Properties of the General Nash Bargaining Solution in the Decentralized Market

In this Appendix we do several things. First we derive the bargaining solution given in Lemma 1. The necessary and sufficient conditions for (17) are

$$\theta\left[\beta_3\phi_{1,+1}d-c_3\left(q\right)\right]u_3'\left(q\right) = (1-\theta)\left[u_3\left(q\right)-\beta_3\phi_{1,+1}d\right]c_3'\left(q\right) \tag{57}$$

$$\begin{aligned}\theta\left[\beta_3\phi_{1,+1}d-c_3\left(q\right)\right]\beta_3\phi_{1,+1} &= (1-\theta)\left[u_3\left(q\right)-\beta_3\phi_{1,+1}d\right]\beta_3\phi_{1,+1} \\ &\quad -\lambda\left[u_3\left(q\right)-\beta_3\phi_{1,+1}d\right]^{1-\theta}\left[\beta_3\phi_{1,+1}d-c_3\left(q\right)\right]^{\theta}\end{aligned} \tag{58}$$

where λ is the Lagrange multiplier on $d \leq m_3$. There are two possible cases: If the constraint does not bind, then $\lambda = 0$, $q = q^*$ and $d = m^*$. If the constraint binds then q is given by (57) with $d = m_3$, as claimed.

We now argue that $m_3 < m_3^*$. First, as is standard, in any equilibrium $\phi_{1,+1} \leq (1+\rho)\phi_1$; this just says the nominal interest rate i is nonnegative. In fact, again as is standard, although we allow $i \to 0$, we only consider equilibria where $i > 0$, so that $\phi_{1,+1} < (1+\rho)\phi_1$. Now suppose $m_3 > m_3^*$ at some date for some agent. Since the bargaining solution tells us he never spends more than m_3^*, he could reduce m_3 by reducing h_1 at t, then increase h_1 at $t+1$ so that he need not change anything else. It is easy to check that this increases utility, so $m_3 > m_3^*$ cannot occur in any equilibrium.

Hence $m_3 \leq m_3^*$. To show the strict inequality, suppose $m_3 = m_3^*$ for sme agent. Again he can reduce h_1 at t and carry less money. If he is a buyer in subperiod 3, he gets a smaller q, but the continuation value is the same since by the bargaining solution he still spends all his money. If he does not buy then he can increase h_1 at $t+1$ so that he need not change anything else. It is easy to check that the net gain from carrying less money is positive, exactly as in LW.