

Money, Banking, and Monetary Policy*

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Abstract

One important function of banks is to issue liabilities, like demand deposits, that are relatively safe and also liquid (usable as means of payment). We introduce risk of theft and a safe-keeping role for banks into monetary theory. This provides a general equilibrium framework for analyzing banking in historical and contemporary contexts. The model can generate concurrent circulation of cash and bank liabilities as media of exchange (inside and outside money), and yields novel policy implications. For example, negative nominal interest rates are feasible, and for some parameters optimal; for other parameters, strictly positive rates (inflation above the Friedman Rule) are optimal.

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Genuine banks are distinguished from other kinds of financial intermediaries by the readily transferable or ‘spendable’ nature of their IOUs, which allows those IOUs to serve as a means of exchange, that is, money. ... Commercial bank money today consists mainly of deposit balances that can be transferred either by means of paper orders known as checks or electronically using plastic ‘debit’ cards. George Selgin , *Banking*.

1 Introduction

Banks perform many functions in modern economies, but one very important function is to issue liabilities, like demand deposits, that are relatively safe and also relatively liquid. Putting money in the bank obviously reduces the risk that it will get lost or stolen without necessarily hindering too much its use as a means of payment. Moreover, using something other than cash reduces other risks, since one may be able to “stop payment” with a check or credit card, e.g., if a purchase turns out to be flawed or fraudulent. While these points may be obvious, this does not mean they are uninteresting or unimportant for our understanding of money and banking. Yet they have been all but ignored in the literature.¹

¹See Gorton and Winton (2003) for a survey of mainstream banking theory. This literature has nothing to say about the medium of exchange function of money, let alone the relation between cash and bank liabilities in performing this function. Monetary theory along the lines of Kiyotaki and Wright (1989) is all about determining media of exchange endogenously, but typically has nothing that resembles banks. The few papers that do try to integrate banking and modern monetary theory include Cavalcanti and Wallace (1999a, b), Cavalcanti, Erosa and Temzilides (1999, 2005), Andolfatto and Nosal (2003), Wallace (2005), Berentsen, Camera and Waller (2005), Li (2006) and Chiu and Meh (2006); some other papers in an slightly older tradition of monetary theory are cited in He, Huang and Wright (2005). None of this work discusses safekeeping. The only work that does speak to the issue is Kahn, McAndrews and Roberds (2005), and Kahn and Roberds (2005), but they use a very different framework, and take the opposite point of view: they assume the use of cash *reduces* risk since it minimizes exposure to dangers such as “identity theft.” While this is a fine point, we think most people would agree that walking around with large quantities of cash is risky – or at least they should agree it is worth considering this case.

In a previous attempt to rectify this state of affairs (He, Huang and Wright 2005), we introduced a risk of theft into a microfounded model of monetary exchange based on search theory. This allowed us to study the role of banks as institutions that provide safekeeping plus liquidity in a setting where there is an endogenous role for a means of payment in the first place. A significant drawback with that analysis, however, is that we started with a rather crude model of monetary exchange. As in all simple first-generation search models (e.g. Kiyotaki and Wright 1993) we adopted the assumption that money is indivisible and agents can only hold at most 1 unit. While one may find this unsatisfying for a number of reasons, perhaps the main limitation is that it is impossible to discuss many interesting aspects of monetary policy, especially the effect of the inflation or nominal interest rates on banking and on the interaction between currency and bank liabilities as means of payment.

The goal of this paper is to bring the integration of banking and monetary theory into the 21st century by reconsidering these basic ideas in the recent generation of search models. This allows us to discuss issues and derive results going well beyond anything in the earlier work, and provides a genuine general equilibrium framework in which to formalize venerable ideas about how modern banks came to be in the first place. Although we do not want to dwell on history too much (see the previous paper for a few more details), it may be helpful to review the story about safekeeping and early banking. As told in general reference books, “The direct ancestors of modern banks were ... the goldsmiths. At first the goldsmiths accepted deposits merely for *safe keeping*; but early in the 17th century their deposit receipts were *circulating in place of money* and so became the first English

bank notes.” (Encyclopedia Britannica 1954, vol. 3, p. 41, emphasis added).

More specialized sources echo this view. “By the restoration of Charles II in 1660, London’s goldsmiths had emerged as a network of bankers ... Some were little more than pawn-brokers while others were full service bankers. The story of their system, however, builds on the financial services goldsmiths offered as fractional reserve, note-issuing bankers. In the 17th century, notes, orders, and bills (collectively called demandable debt) acted as *media of exchange that spared the costs of moving, protecting and assaying specie.*” (Quinn 1997, p. 411-12, emphasis added). “The crucial innovations in English banking history seem to have been mainly the work of the goldsmith bankers in the middle decades of the seventeenth century. They accepted deposits both on current and time accounts from merchants and landowners; they made loans and discounted bills; *above all they learnt to issue promissory notes and made their deposits transferrable by ‘drawn note’ or cheque.*” (Joslin 1954, p. 168, emphasis added).²

While to us this history is fascinating, there are also contemporary issues for which our analysis is relevant. A body of work surveyed by Boyd and Champ (2003) e.g. discusses many empirical findings concerning the relation between inflation (or interest) rates and financial markets, includ-

²Safekeeping was also crucial for earlier episodes in banking history, going back to those some consider history’s first bankers, the Templars (Weatherford 1997; Sanello 2003). However, the goldsmiths seem to be the first banks whose liabilities circulated as media of exchange. Previously, payments typically involved transferring funds from one account to another and “generally required the presence at the bank of both payer and payee” (Kohn 1999). Moreover, “In order that bank credit may be used as a means of payment, it is clearly quite essential that some convenient procedure should be instituted for assigning a banker’s debt from one creditor to another. In the infancy of deposit banking in mediaeval Venice, when a depositor wanted to transfer a sum to someone else, both had to attend the bank in person. In modern times the legal doctrine of negotiable instruments has been developed ... The document may take either of two forms: (1) a cheque, or the creditor’s order to the bank to pay; (2) a note or the banker’s promise to pay.” (*Encyclopedia Britannica* 1941, vol. 3, p. 44).

ing the banking sector.³ While we do not attempt to address any of these observations directly here, we think the framework provides a step in the right direction: if one is ever to make sense of empirical results on the relation between inflation, or interest rates, and banking, or financial markets more generally, it might be useful to have a theory that tried to integrate these markets into monetary economics more carefully. This is arguably true because it is by now well known that taking microfoundations seriously can make a big difference in monetary economics for both qualitative and quantitative results (see e.g. Lagos and Wright 2005).

The rest of the paper can be summarized as follows. In Section 2 we present basic assumptions. In Section 3 we study the case with exogenous risk of theft and no banks, and show how the value of money depends on this risk. We show that it is possible in equilibrium to have *negative* nominal interest rates, although there is a lower bound. In fact, in this model it is optimal to go to the lower bound, which means deflation in excess of the Friedman Rule, $i = 0$. In Section 4 we endogenize the risk associated with cash, still with no banks. In this version of the model, depending on parameters, it may or may not be possible to have negative nominal rates, but it will never be optimal: the optimal interest rate is either $i = 0$ or $i > 0$. The reason that some inflation in excess of the Friedman Rule may be optimal is that in equilibrium it reduces the risk associated with cash.

³Here are some examples of what we have in mind. Very low nominal interest and inflation rates are associated with low levels of investment and growth, but permanently higher rates above certain threshold also adversely affect growth. One interpretation is that high inflation leads to a low supply of funds and more credit rationing, and also reduces the incentive of firms to accumulate internal funds, causing them to rely more on external financing, and increasing informational frictions in financial market. Inflation is also negatively correlated with the ratio of bank assets or liquid liability to GDP when it is below 15%, but for inflation is above 15%, the relationship disappears although there is a fixed drop in banking development indicators.

In Section 5 we introduce banks when risk is exogenous. We show that generically agents either put all or none of their money in the bank, so we cannot get the concurrent circulation of multiple means of payment: bank liabilities drive cash out of circulation (or vice versa) whenever bank operating costs are small (big). The optimal policy is again $i < 0$, basically because appreciation in the value of money helps offset banking costs. In Section 6 we endogenize risk in the model with banks. Now we can easily generate the concurrent circulation of multiple means of payment. In this case, the optimal policy is generically either $i < 0$ or $i > 0$. This is interesting because usually the Friedman Rule is extremely robust: $i = 0$ is the optimal policy in a wide variety of monetary models and a variety of situations. In Section 7 we briefly discuss some additional issues and conclude.

2 Basic Assumptions

A $[0, 1]$ continuum of agents live forever in discrete time. Borrowing from Lagos and Wright (2005), hereafter referred to as LW, we assume that each period has two markets: a centralized market denoted CM, and a decentralized market denoted DM. The CM is a frictionless (Walrasian) market that can be specified quite generally, although for ease of presentation we assume two goods, consumption x and labor ℓ , and quasi-linear utility $U(x) - \ell$, with $U' > 0 \geq U''$. The DM is characterized by several frictions designed to generate a role for media of exchange. First, there is a standard *double-coincidence* problem, formalized here by assuming that agents specialize in production and consumption, and that they meet and trade bilaterally.

In particular, when two agents i and j meet in the DM, the following is true: i wants something j can produce but not vice versa with probability

σ ; j wants something i can produce but not vice versa with probability σ ; and neither wants what the other produces with probability $1 - 2\sigma$, where $\sigma \in (0, \frac{1}{2})$.⁴ If i produces q units of his output for j , the latter gets utility $u(q)$ and the former suffers disutility $c(q)$, where we assume $u', c' > 0$, $u'' < 0$, $c'' \geq 0$, $u(0) = c(0) = 0$, and $u(Q) = c(Q) = 0$ for some $Q > 0$. Denote by q^* the the efficient amount of production and consumption, i.e. the solution to $u'(q) = c'(q)$.

To generate a role for a medium of exchange we still need to have imperfect enforcement of credit, so that i cannot simply offer j an IOU in exchange for his output, promising to pay in the next CM where he can acquire the funds by e.g. supplying labor. The standard approach is to assume some form of *anonymity* – basically, j does not know who i is, which means i could renege on his promise without fear of repercussion. This makes some tangible medium of exchange essential, as in most modern monetary economics.⁵ In addition, we add another friction: the possibility that money can be stolen in the DM. Hence, agents in the CM may choose to deposit their cash into a bank account, which is assumed for simplicity to be perfectly safe, and can still be used to make DM payments.

We emphasize that the assumption that agents can use the liabilities of a bank as a means of payment does not conflict with the assumption that these agents are anonymous in some bilateral meetings. A seller may be willing to accept a claim on a bank from a person he does not know, and hence to whom he would never extend private credit, if he knows the bank.

⁴It would add nothing but notation, and so we do not bother, to allow some double-coincidence meetings, where i and j both want what the other can produce.

⁵See Kocherlakota (1998), Wallace (2001) or Corbae et al. (2003) for formal discussions of anonymity and the essentiality of a medium of exchange.

There are many examples in the real world that correspond more or less to what we have in the model. Travellers's Checks are a leading case; debit cards constitute a modern incarnation. The key point is that these can be used as means of payment almost as easily as cash – i.e. they are very liquid – and they are extremely safe.⁶

Agents do their banking in the CM, and trade using either cash or bank liabilities in the DM. However, paying with bank liabilities entails a fee ϕ . Banks have a resource cost $a > 0$ per dollar on deposit, since safekeeping is not free. Also, they are required legally to keep a fraction ρ of deposits on reserve, while the rest can be out on loan. In the benchmark model we assume $\rho = 1$; in this case banks must keep all deposits in the vault, and hence competition implies $\phi = a$. Let M be the aggregate stock of money at the start of the CM, which evolves over time according to $\hat{M} = (1 + \pi)M$ where we use \hat{z} for the value of any generic variable z next period. In steady state, π is the inflation rate. The government budget is $\pi M + T = pG$, where T is a lump sum nominal tax and G is exogenous government consumption.

3 Exogenous Theft

In this section we study the model where the risk of holding money is exogenous: there is simply a fixed fraction of the population $\lambda \in (0, 1)$ that are thieves and try to rob you if you meet them in the DM. With probability

⁶American Express Travellers's Checks in particular are safe for two reasons: first, people are less interested in stealing them (than cash) because in order to spend them one has to match the signature of the original purchaser; and second, if they are lost stolen, American Express refunds your money. Of course, not all bank liabilities are perfectly safe – bank notes e.g. were historically about as easy to steal as coins. And bank failures or bank robberies do occur. It seems obvious, however, that carrying a Travellers' Check, a certified check, a bank card with a PIN number, etc. is typically less risky than carrying a bundle of currency.

$\gamma \in (0, 1)$ they succeed, and with probability $1 - \gamma$ they fail and walk away empty handed. Thieves do not produce or consume anything in the DM, but act just like honest agents in the CM. We first study the case where the only asset is money – i.e. for now there are no banks. Let $W_j(m)$ and $V_j(m)$ be the value functions for type j entering the CM and the DM with money m , where $j = t, h$ indicates a thief or an honest person. Let β be the discount factor between the DM and the next CM, and δ the discount factor between the CM and the next DM, with $\delta\beta < 1$. See Figure 1.

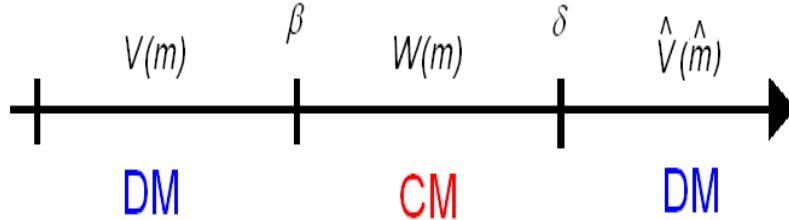


Figure 1: Timing

We first describe the CM problem. For type j

$$\begin{aligned}
 W_j(m) &= \max_{x, h, \hat{m}} \left\{ U(x) - \ell + \delta \hat{V}_j(\hat{m}) \right\} & (1) \\
 \text{s.t. } px &= w\ell + m - \hat{m} - T.
 \end{aligned}$$

where \hat{m} is money taken out of the CM and hence into the DM next period, \hat{V} is the DM value function next period, w is the nominal wage, and p is the price level. We only consider equilibria where $p < \infty$ at every date – i.e. monetary equilibria.⁷ Without loss in generality we can set the real wage to 1 by assuming a linear technology and normalizing labor productivity to

⁷We do not provide a formal definition of equilibrium, but it should be clear what we are after: a solution to the CM problem (1), and a solution to the DM bargaining problem discussed below (the proof of Lemma 3 gives more details).

unity, so $w = p$; it is easy to allow a concave technology but this adds little except notation for the current application.

A thief has no reason to bring money into the DM. Hence the solution to his problem is $\hat{m}_t = 0$, $x_t = x^*$ where $U'(x^*) = 1$, and $\ell_t = x^* - m/p + T/p$. For an honest person, who may choose to bring money into the DM in order to consume, the solution to his problem satisfies $\delta \hat{V}'_h(\hat{m}_h) = 1/p$, $x_h = x^*$ and $\ell_h = x^* - (m - \hat{m})/p + T$.⁸ Notice that \hat{m} does not depend on m ; hence every honest person chooses the same $\hat{m}_h = \bar{m}$, where $\bar{m} \equiv M/(1 - \lambda)$. The result that all agents of the same type choose the same \hat{m} , regardless of their history, follows from quasi-linear CM utility, and is what keeps the analysis tractable. Another convenient feature is that the CM value functions are linear: $W'_h(m) = W'_t(m) = 1/p \forall m$.

Consider now the DM. For a thief, who in equilibrium carries no money into this market,

$$V_t(0) = \lambda \beta W_t(0) + (1 - \lambda) [\gamma \beta W_t(\bar{m}) + (1 - \gamma) \beta W_t(0)]. \quad (2)$$

The first term is his payoff to meeting another thief, which occurs with probability λ , and implies he goes to the next CM empty handed. The second term is his payoff to meeting an honest person, which occurs with probability $1 - \lambda$, in which case with probability γ he is successful and goes to the CM with \bar{m} , while with probability $1 - \gamma$ he is not successful and goes to the CM empty handed. Since W_t is linear, we have $V_t(0) = \beta W_t(0) + (1 - \lambda) \gamma \beta \bar{m}/p$, which will come in handy later.

⁸These results assume interior solutions for ℓ , which we can guarantee as in LW with suitable restrictions on preferences. They also assume the second order condition holds for \hat{m} ; we check below that this is true in any equilibrium.

For an honest person,

$$\begin{aligned}
V_h(m) &= \lambda[\gamma\beta W_h(0) + (1 - \gamma)\beta W_h(m)] & (3) \\
&+ (1 - \lambda)\sigma [u(q) + \beta W_h(m - d)] \\
&+ (1 - \lambda)\sigma [-c(\bar{q}) + \beta W_h(m + \bar{d})] \\
&+ (1 - \lambda)(1 - 2\sigma)\beta W_h(m).
\end{aligned}$$

The first term is the payoff to meeting a thief. The second term is the payoff to meeting an honest person whose output you like and buying q units for d dollars, while the third term is the payoff to meeting an honest person and selling \bar{q} units for \bar{d} dollars (as we will see, q and d depend on the money holdings of the agent in question, m , while \bar{q} and \bar{d} depend on the money holdings of the representative agent to whom he sells, \bar{m}). The final term is the payoff to meeting an honest person but not trading.

Before proceeding further, we need to determine the terms of trade. While several options are available, we use the generalized Nash bargaining solution, with $\theta > 0$ denoting the bargaining power of the buyer.⁹ Thus, an honest buyer with m and an honest seller with \bar{m} settle on the pair (q, d) that maximizes

$$[u(q) + \beta W_h(m - d) - \beta W_h(m)]^\theta [-c(q) + \beta W_h(\bar{m} + d) - \beta W_h(\bar{m})]^{1-\theta}$$

subject to the constraint $d \leq m$, since obviously the buyer cannot pay more than he has. Using $W_j'(m) = 1/p$, this problem reduces to maximizing

$$[u(q) - \beta d/p]^\theta [-c(q) + \beta d/p]^{1-\theta} \quad (4)$$

⁹This is the assumption in the original LW model. Elsewhere in the literature, people have studied versions with alternative bargaining solutions (Rocheteau and Waller 2005), Walrasian price taking or price posting (Rocheteau and Wright 2005), and, in versions that allow some multilateral matches, auctions (Galenianos and Kircher 2006); any of these would work for our purposes.

subject to $d \leq m$. This immediately implies that the solution does not depend on \bar{m} and depends on m iff the constraint binds.

We now give the following generalized versions of results seen in simpler models using the LW structure:

Lemma 1 *Given the CM price level p , the solution to the bargaining problem in (4) is*

$$q = \begin{cases} g^{-1}(\beta m/p) & \text{if } m < m^* \\ q^* & \text{if } m \geq m^* \end{cases} \quad \text{and } d = \begin{cases} m & \text{if } m < m^* \\ m^* & \text{if } m \geq m^* \end{cases}$$

where q^* solves $u'(q^*) = c'(q^*)$, $g(\cdot)$ is given by

$$g(q) \equiv \frac{\theta c(q)u'(q) + (1 - \theta)u(q)c'(q)}{\theta u'(q) + (1 - \theta)c'(q)},$$

and $m^* = g(q^*)p/\beta$.

Proof: It is easy to verify that the proposed solution maximizes the objective in (4) subject to $d \leq m$. ■

Lemma 2 *Let $L(q) \equiv u'(q)/g'(q) - 1$. Then*

$$\begin{aligned} m > m^* &\Rightarrow V'_h(m) = \frac{\beta}{p} (1 - \lambda\gamma) \\ m < m^* &\Rightarrow V'_h(m) = \frac{\beta}{p} [1 - \lambda\gamma + (1 - \lambda)\sigma L(q)]. \end{aligned}$$

Proof: Use the implicit function theorem and Lemma 1. ■

While the above results are standard, given the recent monetary theory literature, the next set of results are perhaps a little more surprising. First, let i be the nominal interest rate between two meetings of the CM, which by the Fisher equation satisfies $1 + i = \hat{p}/p\delta\beta$ because \hat{p}/p is inflation and with quasi-linear utility the real interest rate is pinned down by $1 + r = 1/\delta\beta$. In the standard model, without theft, monetary equilibrium exists iff $i \geq 0$. Here is the generalization:

Lemma 3 $i \geq i^*$ is necessary and sufficient for the existence of monetary equilibrium, where $i^* \equiv -\lambda\gamma$.

Proof: To show necessity, consider problem (1). The derivative of the objective function with respect to \hat{m} is $\delta\hat{V}'_h(\hat{m}) - 1/p$, where we get $\hat{V}'_h(\hat{m})$ by updating $V'_h(m)$ in Lemma 2 one period. If $(1 - \lambda\gamma)\delta\beta/\hat{p} > 1/p$ then $\delta\hat{V}'_h(\hat{m}) - 1/p > 0$ for all $\hat{m} > \hat{m}^*$ and the problem has no solution. We conclude that in any equilibrium we must have $\hat{p}/p\delta\beta \geq 1 - \lambda\gamma$. This is equivalent to $i \geq i^*$.

To show sufficiency, we consider steady states where q and m/p are constant. Also, to ease the presentation we assume $\theta < 1$. Now for all $\hat{m} > \hat{m}^*$, we have $\delta\hat{V}'_h(\hat{m}) - 1/p \leq 0$ as long as $i \geq i^*$; and for all $\hat{m} < \hat{m}^*$, we have in steady state

$$\delta\hat{V}'_h(\hat{m}) - \frac{1}{p} = \frac{\delta\beta}{\hat{p}} [1 - \lambda\gamma + (1 - \lambda)\sigma L(q)] - \frac{1}{p}. \quad (5)$$

Straightforward analysis implies that for \hat{m} near \hat{m}^* , the right hand side of (5) is strictly negative. This means that the solution to the CM problem must be $\hat{m} < \hat{m}^*$. We are done if we can find an $m \in (0, \hat{m})$ solving the first order condition $\delta\hat{V}'_h(\hat{m}) - 1/p$, where the second order condition holds.

For $m \in (0, \hat{m})$, the first order condition can be rewritten using the Fisher equation and Lemma 2 as

$$L(q) = \frac{i + \lambda\gamma}{(1 - \lambda)\sigma}. \quad (6)$$

The second order condition reduces to $V''_h < 0$, or equivalently $L'(q) < 0$, by Lemma 2. Now $L(0) = \infty$ under standard conditions, and $L(q^*) < 0$ by routine calculation. By continuity, there exists $q \in (0, q^*)$ satisfying (6), which means the first order condition holds, such that $L'(q) < 0$, which

means the second order condition holds. Once we have q , Lemma 1 gives $m = pg(q)/\beta$, where $m < m^*$ since $q < q^*$ (Lemma 1). Then $p = \beta\bar{m}/g(q)$ is determined to clear the market with $\bar{m} = M/(1-\lambda)$. The other CM variables (x_j, ℓ_j) are determined as discussed above. This completes construction of a steady state equilibrium. ■

In the proof of Lemma 3 we assumed $\theta < 1$, but the case $\theta = 1$ is not much harder.¹⁰ More substantively, $L(q)$ defined in Lemma 2 represents a *liquidity premium* on cash, which (6) sets equal to the holding cost per period: the nominal rate i plus risk factor $\lambda\gamma$, times the expected number of periods until one spends the money, $1/(1-\lambda)\sigma$. Notice in fact that (6) must hold in *any* equilibrium, and not just any steady state, since it follows simply from the problem of choosing \hat{m} in the CM. Because $L'(q) < 0$, q increases as we lower i towards i^* . One can also show q is increasing in θ . We know that $q \leq q^*$, with strict inequality except when $\theta = 1$ and $i = i^*$, and hence we maximize expected utility by raising q as high as possible, which means $i = i^*$. At i^* we have $q = \hat{q}$, where $L(\hat{q}) = 0$, and $\hat{q} \leq q^*$ with strict equality iff $\theta = 1$. Hence, we do not get the first best q^* at the optimal i^* unless buyers have all the bargaining power.

Proposition 1 *With exogenous theft and no banks, there exists a monetary equilibrium iff $i \geq i^* = -\lambda\gamma$. In equilibrium $\partial q/\partial i < 0$, the optimal policy is $i = i^*$, and it implies $q = \hat{q}$, where $\hat{q} \leq q^*$ with strict equality iff $\theta = 1$.*

Proof: Follows directly from the discussion in the text. ■

¹⁰The only difficulty when $\theta = 1$ is that while the limit as $\hat{m} \nearrow \hat{m}^*$ of (5) is still negative it is only strictly negative when $i > i^*$. If $\theta = 1$ and $i = i^*$ the CM problem is solved at $\hat{m} = \hat{m}^*$, but also at any $\hat{m} > \hat{m}^*$ since (5) is 0 for all $\hat{m} \geq \hat{m}^*$. This nuisance indeterminacy can be eliminated if we assume $i > i^*$ and consider only the limit as $i \searrow i^*$ (i.e. we only consider equilibria at $i = i^*$ that are the limit of equilibria as $i \searrow i^*$).

The fact that nominal rates can be negative is unusual in monetary theory, but not so surprising given that cash is risky. The usual arbitrage argument to rule it out is that one could borrow a dollar today, pay back $1 + i$ tomorrow, and have money left over if $i < 0$. But now the dollar might get stolen! Arbitrage here says only that nominal rates cannot be *too* negative: $i \geq -\lambda\gamma$. The exact expression depends on specific modeling assumptions, but the bigger point is that one has to take into account the probability that any arbitrage attempt will go wrong. Empirically, $i < 0$ is not uncommon – an obvious example is American Express Travelers’ Checks, but any demand deposit with a zero or low nominal interest rate and positive fees (e.g. a charge per check) effectively pays $i < 0$.

4 Endogenous Theft

We now endogenize the decision to be a thief, and hence the risk of using cash. Now, we cannot have $\lambda = 1$ in any monetary equilibrium, since no one will work to acquire cash when all that happens is that it gets stolen. Therefore we look for equilibria with $\lambda \in [0, 1)$. To this end, let $\Delta(m) = W_h(m) - W_t(m)$, and note that Δ actually does not depend on m , since both W_h and W_t are linear with slope $1/p$. Then an equilibrium requires, in addition to the conditions discussed in Section 3,

$$\lambda = 0 \text{ if } \Delta > 0 \text{ and } \Delta = 0 \text{ if } \lambda \in (0, 1). \quad (7)$$

Given what we learned in the previous section, we can write the equilibrium payoffs in the DM as

$$\begin{aligned} V_t(0) &= \beta W_t(0) + \beta(1 - \lambda)\gamma \frac{m}{p} \\ V_h(m) &= \beta W_h(0) + (1 - \lambda)\sigma [u(q) - c(q)] + \beta(1 - \lambda\gamma) \frac{m}{p}. \end{aligned}$$

Similarly, in the CM,

$$\begin{aligned} W_t(m) &= U(x^*) - x^* - \frac{T}{p} + \frac{m}{p} + \delta \hat{V}_t(0) \\ W_h(m) &= U(x^*) - x^* - \frac{T}{p} + \frac{m}{p} + \delta \hat{V}_h(\hat{m}) - \frac{\hat{m}}{p}. \end{aligned}$$

Updating V_t and V_h one period, inserting these into W_t and W_h , and imposing in steady state, we have

$$\begin{aligned} (1 - \delta\beta) W_t(0) &= U(x^*) - x^* - \frac{T}{p} + \delta\beta(1 - \lambda)\gamma \frac{\hat{m}}{\hat{p}} \\ (1 - \delta\beta) W_h(0) &= U(x^*) - x^* - \frac{T}{p} + \delta(1 - \lambda)\sigma [u(q) - c(q)] \\ &\quad + \delta\beta(1 - \lambda\gamma) \frac{\hat{m}}{\hat{p}} - \frac{\hat{m}}{p} \end{aligned}$$

Using these, as well as the bargaining solution $\beta m/p = g(q)$ and the Fisher equation, we can simplify Δ to arrive at

$$\Delta \simeq (1 - \lambda)\sigma [u(q) - c(q)] - (i + \gamma)g(q), \quad (8)$$

where the notation $A \simeq B$ means A and B have the same sign. There are two possible types of equilibria. One possibility is $\lambda \in (0, 1)$, which requires $\Delta = 0$, and therefore

$$\lambda = 1 - \frac{(i + \gamma)g(q)}{\sigma [u(q) - c(q)]}. \quad (9)$$

The other possibility is $\lambda = 0$, which requires $\Delta \geq 0$. In either case we still have to satisfy equilibrium condition (6) for q from Section 3.

To fix ideas, consider an example with $\theta = 1$, which means $g(q) = c(q)$, and functional forms $u(q) = q^\alpha$ and $c(q) = q$ (general results follow below). Consider first equilibria with $\lambda > 0$. Then (9) and (6) imply:

$$\lambda = \frac{\alpha\gamma - (1 - \alpha)(\sigma + i)}{\gamma - \sigma(1 - \alpha)} \quad (10)$$

$$q = \left[\frac{\sigma(1 - \alpha)}{\gamma} \right]^{\frac{1}{1 - \alpha}} \quad (11)$$

The solution to (11) $q = \tilde{q}$ is independent of i . Also, $\lambda > 0$ iff $i < i_0 \equiv \alpha\gamma/(1-\alpha) - \sigma$, and $\lambda < 1$ iff $i > -\gamma$ which is not binding since $i \geq i^* = -\lambda\gamma$ (Lemma 3 applies here). Hence, equilibrium with $\lambda \in (0, 1)$ exists iff $i < i_0$.

Now consider equilibria with $\lambda = 0$, which means

$$q = \left[\frac{\alpha\sigma}{\sigma + i} \right]^{\frac{1}{1-\alpha}}. \quad (12)$$

An equilibrium with $\lambda = 0$ requires $\Delta \geq 0$, which holds iff $i \geq i_0$. Note that $q = q_i$ is decreasing in i in this equilibrium.

Summarizing, we have two generic cases.

Case (i) $\sigma(1-\alpha) > \alpha\gamma$. Then $i_0 < 0$, and equilibrium exists iff $i \geq i^* = 0$. In this equilibrium $\lambda = 0$, and (12) gives $q = q_i$ as a decreasing function of i . Given $\theta = 1$, in this example, we get $q_i = q^* = \alpha^{1/(1-\alpha)}$ at $i = 0$; more generally we get $q_i = \hat{q}$ at $i = 0$.

Case (ii) $\sigma(1-\alpha) < \alpha\gamma$. Then $i_0 > 0$. An equilibrium with $\lambda = 0$ and $q = q_i$ given by (12) exists iff $i \geq i_0$, and an equilibrium with $\lambda > 0$ and $q = \tilde{q}$ given by (11) exists iff $i \in (i^*, i_0)$. Now $i^* = -\gamma\lambda$ is endogenous, but it is easy to check that $\lambda \nearrow 1$ as $i \searrow i^*$, and hence $i^* = -\gamma$. Since \tilde{q} is independent of i when $\lambda > 0$, (10) implies that λ is linearly decreasing in i in this equilibrium.

Figure 2 depicts case (ii). The solid curve for q shows $q = q_i$ from (12) for $i > i_0$ and $q = \tilde{q}$ from (11) for $i < i_0$. The curve for λ comes from (9). Notice that as i falls below i_0 we do not increase q , we simply increase λ , and hence the risk associated with using money. Hence reducing i below i_0 clearly lowers welfare, or average utility. It does so because it hurts trade at the extensive margin by reducing the number of honest agents, with no change at the intensive margin because \tilde{q} does not vary with i when $i \in [i^*, i_0]$.

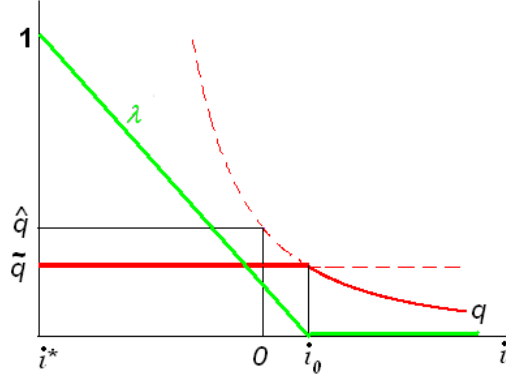


Figure 2: Equilibrium with Endogenous Risk

Hence the optimal policy is $i = i_0$. If we could force $\lambda = 0$ then we would want to reduce i to 0; but $\lambda = 0$ is endogenous.¹¹

We now show that these results actually do not depend on the parametric specification at all. In general, if $\lambda > 0$ then we can solve (9) for λ and insert it into (6) to derive

$$L(q) = \frac{\sigma[u(q) - c(q)] - \gamma g(q)}{\sigma g(q)}, \quad (13)$$

as long as $i > -\gamma = i^*$ (since we divide by $i + \gamma$ to get this). At $i = i^*$, we have $\lambda = 1$ and $q = \hat{q}$ where $L(\hat{q}) = 0$. The solution \tilde{q} to (13) does not depend on i , and so λ is linearly decreasing in i by (9). There is a value for i given by

$$i_0 = \frac{\sigma [u(\tilde{q}) - c(\tilde{q})]}{g(\tilde{q})} - \gamma \quad (14)$$

such that at i_0 we have $\lambda = 0$.

¹¹One can derive several other comparative static results; e.g. if we increase σ to make honest economic activity more attractive or decrease γ to make criminal activity less attractive, λ falls, and i_0 falls, allowing us to further decrease i and increase welfare.

We know from the example that i_0 can be positive or negative. If $i_0 < 0$ then $i^* = 0$, equilibrium exists iff $i \geq 0$, and $\lambda = 0$.¹² And if $i_0 > 0$, the situation is qualitatively exactly as in Figure 2. Summarizing these findings:

Proposition 2 *With endogenous theft and no banks, there is a value of i_0 , that can be positive or negative, with the following properties. If $i_0 < 0$ then equilibrium exists iff $i \geq 0$, and it implies $\lambda = 0$ and $\partial q/\partial i < 0$. In this case welfare is maximized at $i = 0$. If $i_0 > 0$ then equilibrium exists with $\lambda = 0$ iff $i \geq i_0$ and it implies $\partial q/\partial i < 0$, while equilibrium exists with $\lambda > 0$ iff $i^* \leq i < i_0$ where now $i^* = -\gamma$, and it implies $q = \tilde{q}$ independent of i . In this case, welfare is maximized at $i = i_0$.*

Proof: Follows directly from the discussion in the text.

Perhaps the most interesting part of these results is that welfare can be maximized at $i = i_0 > 0$. It is interesting because the Friedman Rule $i = 0$ is so robust in monetary economics. Now in the previous section we already found it was optimal to have $i = i^* < 0$, but this is in the *same spirit* as the Friedman Rule – drive inflation and nominal interest rates as low as possible. Here it can be optimal to have *more* inflation than the Friedman Rule. Curiously enough, the reason that a little bit of inflation may be good is that it keeps people honest.

5 Banks and Exogenous Theft

It is time to allow agents in the CM to put money in the bank. The model here is quite different from the one in He et al. (2005), which is nonconvex

¹²More formally, suppose $i_0 < 0$ and there is an equilibrium with $i < 0$. We require $i \geq -\gamma\lambda$, and hence $\lambda > 0$. Using (9), $i \geq -\gamma\lambda$ reduces to $(i + \gamma)L(q) \geq 0$. By definition, $i_0 = \sigma L(q)$ is the value of i that makes $\lambda = 0$; hence the previous inequality reduces to $(i + \gamma)i_0/\sigma \geq 0$, which contradicts $i_0 < 0$.

due to indivisible money. In that setup, some agents put money in the bank, others do not, and equilibrium determines the fraction of each. In the current convex model, all agents put a fraction b of their money in the bank. For now, assume 100% required reserves, and a banking cost of a dollars per dollar deposited. The equilibrium fee charged depositors is $\phi = a$ (so that the real cost does not depend on the absolute price level). We assume with no loss in generality that the cost and fee are paid in the next DM. Other than being perfectly safe, bank deposits are just like money.¹³

A thief's CM problem is unchanged. His DM payoff is

$$V_t(0) = \lambda\beta W_t(0) + (1 - \lambda) [\gamma\beta W_t(\bar{m} - \bar{b}\bar{m}) + (1 - \gamma)\beta W_t(0)],$$

given the representative honest agent now carries $(1 - \bar{b})\bar{m}$ in cash and has $\bar{b}\bar{m}$ in the bank. An honest person's CM problem becomes

$$W_h(m) = \max_{x, \hat{m}, \hat{b}} \left\{ U(x) - x + \frac{m - T - \hat{m}}{p} + \delta \hat{V}_h(\hat{m}, \hat{b}) \right\},$$

after substituting the budget equation, where m in this expression is money after paying fees to the bank. The constraint in the DM is still $d \leq m$, the bargaining solution is still given by Lemma 1, and the DM payoff is

$$\begin{aligned} V_h(m, b) &= \lambda [\gamma\beta W_h(bm - bma) + (1 - \gamma)\beta W_h(m - bma)] \\ &\quad + (1 - \lambda)\sigma [u(q) + \beta W_h(m - d - bma)] \\ &\quad + (1 - \lambda)\sigma [-c(\bar{q}) + \beta W_h(m + \bar{d} - bma)] \\ &\quad + (1 - \lambda)(1 - 2\sigma)\beta W_h(m - bma). \end{aligned}$$

¹³Of course, we do not need deposits to be perfectly safe, only safe *relative* to cash. In principle, the framework could accommodate bank failures or bank robbery, or could allow agents to steal your checkbook with some probability.

Differentiating with respect to m , we get the generalized version of Lemma 2,

$$\begin{aligned} m > m^* &\Rightarrow V_{hm} = \frac{\beta}{p} [1 - \lambda\gamma(1 - b) - ab] \\ m < m^* &\Rightarrow V_{hm} = \frac{\beta}{p} [1 - \lambda\gamma(1 - b) - ab + (1 - \lambda)\sigma L(q)]. \end{aligned}$$

The straightforward generalization of Lemma 3 implies a necessary and sufficient condition for equilibrium is $i \geq i^*$ where now $i^* = -\lambda\gamma(1 - b) - ab$.

Given $i > i^*$, the first order condition for \hat{m} implies

$$L(q) = \frac{i + \lambda\gamma(1 - b) + ab}{(1 - \lambda)\sigma}, \quad (15)$$

generalizing (6). To determine b , notice that

$$\hat{V}_{h2} \simeq \gamma\lambda - a. \quad (16)$$

Hence the first order condition from the CM implies $b = 1$ if $\gamma\lambda > a$ and $b = 0$ if $\gamma\lambda < a$.¹⁴

Therefore, we cannot get $b \in (0, 1)$ except in the nongeneric case $\lambda\gamma = a$. So we cannot have concurrent circulation of multiple means of payment in this version of the model: for big a banking is not viable and for small a cash is driven from circulation. In particular, for small a , all the money is in bank vaults, and all payments in the CM are made using deposits. This may or may not be the wave of the future for modern economies, but in any case, the formal result hinges on there being nothing to adjust to get $b \in (0, 1)$ endogenously. One might think λ should adjust. For example, if $b = 1$ then there is no cash in the DM, and it does not make much sense to

¹⁴Intuitively, because of quasi-linear utility, agents put all their money in the bank iff the cost a is less than the risk factor $\lambda\gamma$.

be a thief. Hence, in the next section, we endogenize λ . First, we summarize the above results.

Proposition 3 *With exogenous theft and banks, if $\lambda\gamma > a$ then $b = 1$, and if $\lambda\gamma < a$ then $b = 0$. In either case, equilibrium exists iff $i \geq i^* = -\lambda\gamma(1 - b) - ab$, and it implies $\partial q/\partial i < 0$. The optimal policy is $i = i^*$.*

Proof: Follows directly from the discussion in the text.

6 Banks and Endogenous Theft

Generalizing the expressions in Section 4, we have

$$\begin{aligned} (1 - \delta\beta) W_t(0) &= U(x^*) - x^* - \frac{T}{p} + \delta\beta(1 - \lambda)\gamma(1 - b)\frac{\hat{m}}{\hat{p}} \\ (1 - \delta\beta) W_h(0) &= U(x^*) - x^* - \frac{T}{p} + \delta(1 - \lambda)\sigma[u(q) - c(q)] \\ &\quad + \delta\beta[1 - \lambda\gamma(1 - b) - ab]\frac{\hat{m}}{\hat{p}} - \frac{\hat{m}}{p}, \end{aligned}$$

and the analog of (8) is

$$\Delta \simeq (1 - \lambda)\sigma[u(q) - c(q)] - [i + \gamma(1 - b) + ab]g(q). \quad (17)$$

There are three possibilities for equilibrium: $b = 1$, $b = 0$, and $b \in (0, 1)$. But we cannot have $b = 1$ as long as $a > 0$: if $b = 1$ then no one holds cash, which leads to $\lambda = 0$, which means $b = 1$ cannot be a best response. Hence we restrict attention to $b \in [0, 1)$.

Consider equilibrium with $b = 0$, which is a best response iff $\lambda\gamma \leq a$, by (16). In this case we determine q and λ exactly as in Section 4. In particular, to repeat the relevant parts of Proposition 2: $\lambda \in (0, 1)$ implies $q = \tilde{q}$ is independent of i ; $\lambda \in (0, 1)$ iff $i < i_0$; if $i_0 < 0$ then $i^* = 0$ and

$\lambda = 0$; and if $i_0 > 0$ then $i^* = -\gamma$, in which case $\lambda > 0$ for $i \in [i^*, i_0)$ and $\lambda = 0$ for $i \geq i_0$. Given this, we check the best response condition. If $\lambda = 0$ then $\lambda\gamma \leq a$, and $b = 0$ is obviously a best response. The only nontrivial case is $i_0 > 0$ and $i \in [i^*, i_0)$, where $\lambda \in (0, 1)$ is given by (9). In this case $\lambda\gamma \leq a$ iff $i \geq i_1$, where

$$i_1 \equiv \frac{(\gamma - a)\sigma[u(\tilde{q}) - c(\tilde{q})]}{\gamma g(\tilde{q})} - \gamma = \left(1 - \frac{a}{\gamma}\right) i_0 - a. \quad (18)$$

We conclude that when $i_0 > 0$ and $i \in [i^*, i_0)$, $b = 0$ is an equilibrium iff $i \geq i_1$.

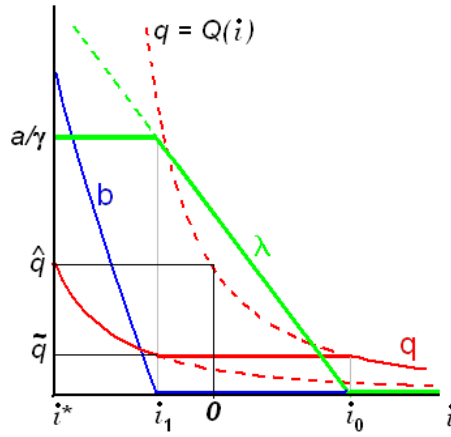


Figure 3: Equilibrium with Endogenous Risk and Banks

This is shown in Figure 3, where as i decreases from i_0 towards i_1 , $q = \tilde{q}$ stays fixed and λ increases from 0 to a/γ , exactly as in Figure 2. At $i = i_1$, however, something new and interesting happens. If i decreases further, we get $\lambda\gamma > a$ and the best response becomes $b = 1$. But $b = 1$ cannot be an equilibrium, so what must happen is the following: when i falls below i_1 , we stay at $\lambda = a/\gamma$ so that $b \in (0, 1)$ is a best response; and b adjusts so that

$\lambda = a/\gamma$ is a best response. Hence, equilibrium for $i \in [i^*, i_1)$ is determined by three conditions: (i) $b \in (0, 1)$ is a best response, which means $\lambda = a/\gamma$; (ii) $\lambda \in (0, 1)$ is a best response, which means $\Delta = 0$, or

$$\lambda = 1 - \frac{[i + ab + \gamma(1 - b)]g(q)}{\sigma[u(q) - c(q)]}; \quad (19)$$

and (iii) q satisfies the usual condition (15).

The interesting new outcome here is $b \in (0, 1)$, where banks operate and DM transactions use both deposits and money.¹⁵ To discuss it further, assume $a < \gamma$ (because $0 < b < 1$ requires $\lambda = a/\gamma < 1$). Since $i^* = -a$, in this case, (18) implies $i_1 \in (i^*, i_0)$. Now, given $i \in [i^*, i_1)$, this equilibrium exists, and has a nice recursive structure: first set $\lambda = a/\gamma$; then (15) reduces to

$$L(q) = \frac{\gamma(i + a)}{\sigma(\gamma - a)}, \quad (20)$$

which can be solved for $q = q_i$ with $\partial q_i / \partial i < 0$; then finally (19) can be solved for

$$b_i = \frac{i + \gamma}{\gamma - a} - \frac{\sigma[u(q_i) - c(q_i)]}{\gamma g(q_i)}. \quad (21)$$

Notice $b_i = 1 - \sigma[u(\bar{q}) - c(\bar{q})] / \gamma g(\bar{q}) < 1$ at i^* and $b_i = 0$ at $i = i_1$, as shown in Figure 3.

We summarize what we know as follows.¹⁶

¹⁵In fact, each buyer makes every purchase using both, but this stands in for the idea that agents make some purchases with one and some purchases with the other. To get this in the model, formally, we would need to relax assumption that they make only one purchase in each DM, or change things so that they sometimes spend less than their total resources in the DM, say because the utility of a seller's output is random (it depends on who you meet), as has been done in several other applications of the basic LW framework. If one wanted to pursue this it might also be useful to add a fixed cost each time deposits are used for a payment.

¹⁶The only thing we do not know for certain about this version of the model is, in equilibrium with $b \in (0, 1)$, do we have $\partial b / \partial i < 0$? We could not prove this generally, although it was always true in examples. Consider e.g. $\theta = 1$, $u(q) = q^\alpha$ and $c(q) = q$

Proposition 4 *With endogenous theft and banks, there is an i_0 with the following properties. If $i_0 < 0$ then monetary equilibrium exists iff $i \geq 0$, and it implies $\lambda = b = 0$ and $\partial q/\partial i < 0$. If $i_0 > 0$ then there is an $i_1 \in (i^*, i_0)$, where $i^* = -a$ in this case, such that the following is true. Equilibrium with $\lambda = b = 0$ exists iff $i > i_0$ and it implies $\partial q/\partial i < 0$; equilibrium with $\lambda \in (0, 1)$ and $b = 0$ exists iff $i \in (i_1, i_0)$ and it implies $q = \tilde{q}$ independent of i and $\partial \lambda/\partial i < 0$; and equilibrium with $\lambda \in (0, 1)$ and $b \in (0, 1)$ exists iff $i \in (i^*, i_1)$ and it implies $\partial q/\partial i < 0$ and $\lambda = a/\gamma$ independent of i .*

Proof: Follows from the discussion in the text.

Having characterized the equilibrium set, we turn to welfare. When $i_0 \leq 0$, we have $i^* = 0$ and $\lambda = 0$ in any equilibrium; in this case there is no roles for banks and welfare is maximized at $i = 0$. Consider therefore $i_0 > 0$, as in Figure 3. Suppose we first consider maximizing welfare over the range $[i_1, \infty)$. Then as in Section 4, the solution is obviously i_0 , since in the range $[i_1, i_0]$, q is constant and λ is decreasing in i , while in $[i_0, \infty)$, λ is constant and q is decreasing in i . Now consider maximizing welfare over $[i^*, i_1]$. In this range, λ is constant while q_i and b_i depend on i , and since banking is socially costly, the optimal policy is not so obvious. In this range, we must therefore calculate welfare explicitly.

Our welfare criterion \mathcal{W} is the CM payoff of an agent holding the average amount of money M . In equilibrium with $\lambda, b \in (0, 1)$, we show in the

(which is the case actually shown in Figure 3). Then in the equilibrium with $b \in (0, 1)$ we have

$$q = \left[\frac{(\gamma - a)\alpha\sigma}{(i + a)\gamma + (\gamma - a)\sigma} \right]^{\frac{1}{1-\alpha}} \quad \text{and} \quad b = 1 - \frac{(1 - \alpha)[(i + a)\gamma + (\gamma - a)\sigma]}{\alpha\gamma(\gamma - a)}.$$

Appendix that

$$\mathcal{W} = \left(1 - \frac{a}{\gamma}\right) \left\{ \sigma [u(q) - c(q)] - ag(q) \frac{i + \gamma}{\gamma - a} \right\}. \quad (22)$$

The first term in braces gives the expected surplus from DM trade, while the second term gives the resource cost of banking. Similarly, in equilibrium with $b = 0$ and $\lambda \in (0, 1)$ we have

$$\mathcal{W} = \left\{ \frac{(i + \gamma)g(q)}{\sigma [u(q) - c(q)]} \right\}^2 \quad (23)$$

and in equilibrium with $b = \lambda = 0$ we have

$$\mathcal{W} = \sigma [u(q) - c(q)]. \quad (24)$$

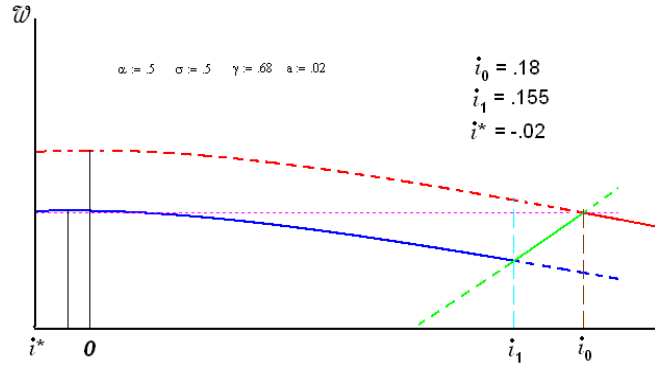


Figure 4: Welfare maximized at $i \in (i^*, 0)$.

In Figure 4, the upper curve corresponds to strategies $b = \lambda = 0$, the lower curve to $\lambda, b \in (0, 1)$, and the one the middle to $b = 0$ and $\lambda \in (0, 1)$. The solid portions of the curves give \mathcal{W} when equilibrium exists with these strategies, and the dashed portions show what welfare would be with these strategies even though they do not constitute equilibrium. It is clear that \mathcal{W}

is either maximized globally at $i = i_0$, or at some point $i \in [i^*, i_1]$. In fact, we show in the Appendix that the maximum over the range $[i^*, i_1]$ occurs at $i < 0$, but it could be either $i = i^*$ or $i \in (i^*, 0)$. In Figure 4, the global maximum occurs at $i \in (i^*, 0)$, but it is easy to construct examples where it occurs at i_0 . We summarize as follows.

Proposition 5 *With endogenous theft and banks, if $i_0 < 0$ then the optimal policy is $i = 0$, and if $i_0 > 0$ then the optimal policy may be either $i \in [i^*, 0)$ or $i = i_0 > 0$.*

Proof: Follows from the discussion in the text.

We think $i_0 > 0$ is most interesting, of course, and recall from Section 4 that it is easy for this to occur in examples. In this case, we never want to run the Friedman Rule, but either $i < 0$ or $i > 0$. Which of these is globally optimal depends on parameters. In words, the disadvantage of $i = i_0 > 0$ is that q is very low, but the advantage is that it implies $\lambda = b = 0$, which is good because it eliminates both criminals and bankers! Moving outside the confines of the formal analysis, the idea is that if we set inflation too low we may encourage undesirable behavior (in the model, crime), which is not only unproductive, but, in general equilibrium, diverts resources to combat such behavior (in the model, banking).

7 Conclusion

We have developed a model where there is an essential role for media of exchange, the choice of which is endogenous. Agents may use cash but this is relatively risky; they may use bank liabilities but this is costly; or they may use some of each. We think this theory is interesting because it is based on

the actual factors underlying the historical development of modern banking, going back at least to the goldsmiths in England. Additionally, it generates some novel predictions concerning monetary policy. It is feasible to have negative nominal interest rates, and for some parameter values, $i < 0$ is in fact optimal. For other parameter values, it is optimal to have $i > 0$. It is certainly not true that the Friedman Rule is optimal, in general, which is certainly different from most of monetary economics.

Having a model with multiple (endogenous) means of payment seems useful to the extent that one is interested in payments systems generally, and especially if one is interested in empirical or policy issues such as those mentioned in the Introduction related to the impact of inflation on banking and on the competition between money and other assets. Having said this, we understand our set up is simplistic. The idea was to take a first cut at modeling money and banking in a logically consistent framework; alternative environments should be considered. For example, one could replace the brutality of theft with the subtlety of lemons problems, as in the money models of Williamson and Wright (1994), Trejos (1997), and Berentsen and Rocheteau (2004). Monetary policy would still impact on incentives in interesting ways, but the analysis may appear more ‘modern’ if couched in terms of private information rather than crime.

Other extensions worth considering include reducing the reserve ratio below $\rho = 1$ and deriving the money multiplier, as we did in the earlier paper. It would be more complicated in this model, however, because it is less straight forward to derive a demand for loans with divisible money; hints as to how to proceed are contained in Berentsen et al. (2005) and Chiu and Meh (2006). A nice thing about $\rho < 1$ is that we get $\phi < a$,

since banks earn revenue from loans as well as fees, and we even get $\phi < 0$ (interest on checking) when a and ρ are small. The problem when $\phi < 0$, of course, is that we do not get $b < 1$, since bank liabilities are as liquid as money, safer, and yield a higher return. This is fine if one wants to capture a ‘cashless economy.’ But one could also add some feature to make bank liabilities less liquid – say, some agents do not accept checks because ... We are now getting into serious issues in monetary theory that will have to wait for additional research.

Appendix

Here we verify some claims made in Section 6. First we derive the expressions for welfare.

In equilibrium with $\lambda, b \in (0, 1)$, we have $W_h(M) = W_t(M)$, and we compute

$$\begin{aligned} (1 - \delta\beta) W_t(M) &= (1 - \delta\beta) W_t(0) + (1 - \delta\beta) \frac{M}{p} \\ &= U(x^*) - x^* + \delta\beta(1 - \lambda)\gamma(1 - b) \frac{\hat{m}}{\hat{p}} + (1 - \delta\beta) \frac{m(1 - \lambda)}{p} \\ &= U(x^*) - x^* + \frac{M}{p} [\delta\beta\gamma(1 - b) + 1 - \delta\beta + \pi], \end{aligned}$$

using $T = -\pi M$, $M = m(1 - \lambda)$, and $m/p = \hat{m}/\hat{p}$. Using $m/p = g(q)/\beta$ and the Fisher equation, this becomes

$$(1 - \delta\beta) W_t(M) = U(x^*) - x^* + \delta(1 - \lambda)g(q) [i + \gamma(1 - b)].$$

After inserting $\lambda = a/\gamma$, eliminating b using (21), and performing routine simplifications, we arrive at

$$(1 - \delta\beta) W_t(M) = U(x^*) - x^* + \delta \left(1 - \frac{a}{\gamma}\right) \left\{ \sigma [u(q) - c(q)] - ag(q) \frac{i + \gamma}{\gamma - a} \right\}.$$

In the text, we use $\mathcal{W} = [(1 - \delta\beta) W_t(M) + x^* - U(x^*)]/\delta$ since we can neglect constants. This yields (22); (23) and (24) are similar.

We now verify the optimal policy over $[i^*, i_1]$ is $i < 0$. Suppose we maximize (22) over $[i^*, i_1]$ by choosing (q, i) subject to (20). Using the constraint to eliminate i , the problem becomes

$$\max_q \mathcal{W} = \left(1 - \frac{a}{\gamma}\right) \left\{ \sigma [u(q) - c(q)] - ag(q) \left[\frac{\sigma}{\gamma} L(q) + 1 \right] \right\}.$$

Differentiating, we get

$$\begin{aligned}
\frac{\partial \mathcal{W}}{\partial q} &\simeq \sigma(u' - c') - a\left[\frac{\sigma}{\gamma}L(q) + 1\right]g' - ag\frac{\sigma}{\gamma}L' \\
&= \sigma(u' - c') - a\frac{\sigma}{\gamma}(u' - g') - ag' - ag\frac{\sigma}{\gamma}L' \\
&= \sigma(u' - c') - \sigma(u' - g') + \left(\sigma - a\frac{\sigma}{\gamma}\right)(u' - g') - ag' - ag\frac{\sigma}{\gamma}L' \\
&= \sigma(g' - c') + (i + a)g' - ag' - ag\frac{\sigma}{\gamma}L' \\
&= \sigma(g' - c') + ig' - ag\frac{\sigma}{\gamma}L'.
\end{aligned}$$

One can show $g' > c'$ and $L' < 0$ in equilibrium. Hence, $i \geq 0$ implies $\partial \mathcal{W} / \partial q > 0$, and therefore $i \geq 0$ implies $\partial \mathcal{W} / \partial i < 0$.

Finally, we verify that we cannot say generally whether the optimal policy over $[i^*, i_1]$ is i^* or $i > i^*$. Consider the example with $\theta = 1$, $c(q) = q$ and $u(q) = q^\alpha$. After computing equilibrium explicitly, we have

$$\frac{\partial \mathcal{W}}{\partial q} \simeq i - a\frac{\sigma}{\gamma}(\alpha - 1) \left[1 + \frac{\gamma(i + a)}{\sigma(\gamma - a)} \right].$$

If $(1 - \alpha)\sigma < \gamma$ then \mathcal{W} is maximized at $i > i^*$; else it is maximized at i^* .

References

- Andolfatto, D. and E. Nosal “A Theory of Money and Banking,” mimeo, 2003.
- Aruoba, S. and R. Wright “Search, Money and Capital: A Neoclassical Dichotomy,” *Journal of Money, Credit and Banking* 35 (2004), 1085-105.
- Berentsen, A., G. Camera and C. Waller “Money and Banking,” mimeo, 2005.
- Berentsen, A. and G. Rocheteau “Money and Information,” *Review of Economic Studies* 71 (2004), 915-944.
- Boyd J., and B. Champ “Inflation and Financial Market Performance: What Have We Learned in the Last Ten Years?” Working Paper 03-17, Federal Reserve Bank of Cleveland, 2003.
- Cavalcanti, R., A. Erosa and T. Temzilides “Private Money and Reserve Management in a Random Matching Model,” *Journal of Political Economy* 107 (1999), 929-945.
- Cavalcanti, R., A. Erosa and T. Temzilides “Liquidity, Money Creation and Destruction, and the Returns to Banking,” *International Economic Review*, 2005.
- Cavalcanti, R. and N. Wallace “Inside and Outside Money as Alternative Media of Exchange,” *Journal of Money, Credit, and Banking* 31 (1999a), 443-457.
- Cavalcanti, R. and N. Wallace “A Model of Private Bank-Note Issue,” *Review of Economic Dynamics* 2 (1999b), 104-136.
- Corbae, D., T. Temzilides and R. Wright “Directed Matching and Monetary Exchange,” *Econometrica* 71 (2003), 731-56.
- Chiu, J. and C. Meh “Money and Banking in the Market for Ideas,”

mimeo, 2006.

Davies, G., *A History of Money From Ancient Times to the Present Day*, 3rd. ed. Cardiff: University of Wales Press, 2002.

Galenianos, M. and P. Kircher “Dispersion of Money Holdings and Inflation,” mimeo, 2006.

Gorton, G. and A. Winton “Financial Intermediation,” in G. Constantinides, M. Harris and R. Stulz, eds., *Handbook of the Economics of Finance*, Amsterdam: North Holland, 2003, Chapter 8.

He, P., L. Huang and R. Wright “Money and Banking in Search Equilibrium,” *International Economic Review* 46, 2005, 637-70.

Joslin, D. M. “London Private Bankers, 1720-1785,” *Economic History Review* 7 (1954), 167-86.

Kahn, C., J. McAndrews and W. Roberds “Money is Privacy,” *International Economic Review* 46, 2005, 377-404.

Kahn, C., and W. Roberds “Identity Theft,” mimeo 2005.

Kiyotaki, N. and R. Wright “On money as a medium of exchange,” *Journal of Political Economy* 97 (1989), 927-954.

Kiyotaki, N. and R. Wright “A Search-Theoretic Approach to Monetary Economics,” *American Economic Review* 83 (1993), 63-77.

Kohn, M. “Early Deposit Banking,” Working Paper 99-03, Department of Economics, Dartmouth College, 1999.

Lagos, R. and R. Wright “A Unified Framework for Monetary Theory and Policy Evaluation,” *Journal of Political Economy* 113 (2005), 463-84.

Li, Y. “Currency and Checking Deposits as Means of Payment,” mineo, 2006.

Quinn, S. “Goldsmith-Banking: Mutual Acceptance and Interbanker Clear-

ing in Restoration London,” *Explorations in Economic History* 34 (1997), 411-42.

Rocheteau, G. and C. Waller “Bargaining and Money,” mimeo, 2005.

Rocheteau, G. and R. Wright “Money in Competitive Equilibrium, in Search Equilibrium, and in Competitive Search Equilibrium,” *Econometrica* 73 (2005), 175-202.

Sanello, F. *The Knights Templars: God’s Warriors, the Devil’s Bankers*. Lanham, MD: Taylor Trade Publishing, 2003.

Trejos, A. “Search, Bargaining, Money and Prices under Private Information,” *International Economic Review* 40 (1997), 679-695.

Wallance, N. “From Private Banking to Central Banking: Ingredients of a Welfare Analysis,” *International Economic Review* 46, 2005, 619-636.

Weatherford, J. *The History of Money*. New York: Crown Publishers, 1997.

Williamson, S. and R. Wright “Barter and Monetary Exchange Under Private Information,” *American Economic Review* 84 (1994), 104-123.