The Optimal Inflation Target in an Economy with Limited Enforcement

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Abstract

We formulate the central bank's problem of selecting an optimal long-run inflation rate as the choice of a distorting tax by a planner who wishes to maximize discounted stationary utility for a heterogeneous population of infinitely-lived households in an economy with constant aggregate income. Households are divided into cash agents, who store value in currency alone, and credit agents who have access to both currency and loans. The planner's problem is equivalent to choosing inflation and nominal rates consistent with a resource constraint, and with an incentive constraint that ensures credit agents prefer the superior consumption-smoothing power of loans to that of

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currency. We show that the optimum implied rate of inflation is positive, and the optimum implied nominal interest rate is higher than the inflation rate, if the social welfare function weighs credit agents no more than their population fraction.

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JEL codes: E31, E42, E58.

1 Introduction

1.1 Overview

Central bankers have comfort zones for long-run inflation and nominal interest rates which deviate substantially from the prescriptions of economic theory. For example, Federal Reserve Chairman Bernanke has stated a preference for core inflation in the one-to-two percent annual range, in general agreement with the more explicit inflation targets of the European Central Bank, the Bank of England, the Reserve Bank of New Zealand and other institutions. This target is typically achieved with a nominal interest rate near five percent.

Economic theory, on the other hand, calls for an inflation target that is consistent with the Friedman rule of a zero nominal interest rate. That inflation target is minus the growth rate of real income in life cycle economies (Freeman (1993), Abel (1987)), or minus the sum of the rate of time preference plus an adjustment for income growth in representative household economies (Friedman (1969), Foley and Sidrauski (1969), Woodford (1990)).

Why do central banks prefer low inflation rates to outright deflation? One argument is that deflation subsidizes the holding of money at the expense of deposits and loans, causing disintermediation and a weakening of financial markets, as in Smith (2002). Another argument concerns the impact of the zero bound on nominal interest rates in environments where the central bank is committed to lower interest rates when economic activity weakens, as suggested by Summers (1991); for an analysis see Eggertsson and Woodford (2003).

In this paper we take the disintermediation argument seriously and use it in reverse: if a small deflation hurts asset markets, then a small inflation may help them. We explore an economy in which moderate inflation loosens debt constraints, deepens financial markets and improves the ability of asset-trading households to smooth consumption. At the same time, inflation imposes a distortionary tax on money-trading households which works in the opposite direction. The central bank must choose the inflation rate to balance improvements in financial markets with deadweight losses from inflation.

1.2 What we do

We analyze an endowment economy with constant aggregate income, populated by a continuum of infinitely-lived households whose income shares fluctuate over time. There are two asset markets, for currency and consumption loans. Households are exogenously divided into two groups, called *cash* agents and *credit* agents. Members of the first group are anonymous and can store value only in currency of which they hold nonnegative amounts. Members of the second group can participate in either market subject to endogenous participation or debt constraints that successfully deter default: This group may hold assets in positive or negative amounts. Default is punished with perpetual exclusion from the loan market but still permits households to take long positions in currency.

In this environment, deflation raises the payoff from using money and makes default more attractive for borrowers. That, in turn, tightens the participation constraint (lowers debt limits) and weakens the loan market. Conversely, inflation raises debt limits and deepens the loan market up to the point where constraints cease to bind.

1.3 Main results

We formulate the central bank's problem of selecting an optimal long-run inflation rate as the choice of a distorting tax by a benevolent central planner who wishes to maximize a convex combination of discounted utilities for cash and credit agents, subject to a participation constraint that keeps credit agents from renouncing the loan market and switching to currency.

When aggregate income is constant, the deflation required by the Friedman rule turns out to be an infeasible choice for any planner who assigns positive weight to credit households. Deflation subsidizes currency at the expense of consumption loans, and increases the payoff from cash-holding above the payoff to loan-trading, leading credit agents to default on their loans and forcing the credit market to shut down.

At the other end of possible inflation targets, an inflation rate higher than the minimum required to slacken debt constraints is equivalent to a distortionary income transfer from lower-welfare cash agents to higher-welfare credit agents. Planners who do not assign extraordinarily high weight to credit agents will reject inflation rates above the value needed to slacken debt constraints on credit agents.

If the relative weight of credit households in the social welfare function is above zero and less than or equal to their population weight, we show that the optimum rate of inflation is positive and the associated optimum nominal interest rate is larger than the inflation rate. We interpret these findings to be consistent with the comfort zones articulated by some of the world's leading central banks.

1.4 Recent related literature

Several recent papers in the monetary theory literature have themes related to the ones in this paper.[1] The central theme in much of this literature is the infeasibility of the Friedman rule for monetary policy in economic environments with broadly defined private information like hidden action, hidden information, lack of commitment, or search frictions. In all of these environments, households are able to evade the taxes required to implement

[1]The intellectual origins of this literature date back to Bewley (1980) who showed that the Friedman rule is inconsistent with competitive equilibrium in an exchange economy with uninsurable idiosyncratic risks when currency is the only store of value.

the Friedman rule by witholding information about their type or by simply defaulting on their tax or loan obligations.

The Friedman rule typically turns out to violate truth-telling or participation constraints in economies with private information and related frictions. Small positive amounts of inflation, on the other hand, help relax these constraints by strengthening incentives to repay loans, lowering the real rate of interest, or by encouraging the use of credit at the expense of currency.

An early example of this line of work is Aiyagari and Williamson (2000), who study an environment with unobservable random endowments in which financial intermediaries sell debt contracts to households. These authors find that an increase in inflation raises the penalty for default but they do not define an optimum rate of inflation.

Optimum inflation is well-defined in a recent paper by Berentsen, Camera, and Waller (2007) who study the role of credit in the search-theoretic framework of Lagos and Wright (2005). They analyze an environment with search frictions in which money is essential for exchange and financial intermediaries cannot enforce the repayment of loan contracts; they can only refuse future credit to defaulters. An increase in inflation again raises the penalty for default because it lowers the payoff to using money. Berentsen, *et al.* (2007) show that the optimal rate of inflation is positive if the rate of time preference is less than the fraction of sellers in the total population of agents.

A related result appears in Andolfatto (2007), who looks at the search model of money without credit. Here, the Friedman rule is feasible and optimal if agents are sufficiently patient, infeasible otherwise. In the latter case, the optimum rate of inflation is again positive. Broadly similar conclusions are reached by Ragot (2006), who studies a two-period life cycle growth model with money in the utility function, and private information about the technology of intermediate goods production. Only producers of intermediate goods are constrained in this environment; households are not.

Deviatov and Wallace (2007) is a computational study of a Lagos and Wright (2005) environment with features similar to the ones emphasized in the present paper. In particular, an exogenous fraction of agents are monitored and hence have known histories, while the remainder are anonymous; in addition, aggregate productivity has periodicity two, resembling the alternating endowment process in the present paper. Defaulters in credit arrangements become anonymous agents. The optimal monetary policy is relatively complicated and takes incentive constraints into account as in the present paper, but the analysis is not concerned with an optimal inflation target as is present paper.[2]

The key difference between recent literature and our paper is that we study monetary policy in economies with *public information and complete markets* in which money is a store of value and limited enforcement is the only friction allowed. An optimum inflation target in our class of environments is associated with a constrained optimum allocation achieved by a planner who can only extract voluntary taxes from households. This constrained optimum duplicates the competitive equilibrium outcome at an efficient steady state.[3]

2 Environment

We describe the optimal rate of long-run inflation and analyze the associated optimal consumption plans in an endowment economy populated by four types of infinitely-lived household types, indexed by $i = 0, 1, 2, 3$. Household types 0 and 1 have mass $\lambda/2$ each, and households 2 and 3

[2] The idea that an increase in inflation may deter activity in certain sectors of the economy, and through this effect produce desirable consequences in the economy as a whole, is a theme that has been analyzed from other points of view. For example, Huang, He, and Wright (2006) study banking in an environment where money is essential for exchange, and in addition theft is possible. Here banks have an additional safekeeping role; positive inflation may then be desirable because it taxes thieves.

[3] We leave for a future paper the question of how a market economy reaches this desirable steady state. An earlier paper of ours, Antinolfi, Azariadis, and Bullard (2006), shows that economies with limited enforcement typically have many Pareto-ranked equilibrium outcomes, and that active monetary policy may serve as an equilibrium selection device.

have mass $1 - \lambda/2$ each, where $0 \leq \lambda \leq 1$. Individual income shares fluctuate deterministically and total income is constant. Time is discrete and is denoted by $t = 0, 1, 2, ...$ Agents have identical preferences given by

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right) \tag{1}$$

with $0 < \beta < 1$. Individual endowments and income shares are periodic,[4] given by,

$$\left(\omega_t^0, \omega_t^1\right) = \left(\omega_t^2, \omega_t^3\right) = \begin{cases} (1+\alpha, 1-\alpha) & \text{if } t = 0, 2, ... \\ (1-\alpha, 1+\alpha) & \text{if } t = 1, 3, ... \end{cases} \tag{2}$$

with $\alpha \in (0, 1)$. This endowment pattern means that type 0 and type 1 agents have negatively correlated income shares, as do agents 2 and 3. We introduce a critical difference between these two agent-pairs: We call agents 0 and 1 "Kehoe-Levine-type" or *credit* agents, and agents 2 and 3 "Bewley-type" or *cash* agents. Cash agents are anonymous households who may only use currency to smooth income fluctuations, as in Bewley (1980). No claims can be enforced on them or by them. Credit agents may enter into loan arrangements to smooth consumption subject to endogenous debt limits that give them proper incentives to repay, as in Kehoe and Levine (1993).

Incentives to repay loans are strongest, and debt limits are highest, when the payoff to default is lowest. We assume that credit agents who default are forever excluded from the loan market and must instead use money as a store of value. Clearly, the payoff to default at any point in time depends on future inflation rates.

The government acts as a benevolent central planner who chooses a constant inflation tax at which cash agents can trade currency across periods, and directly selects consumption vectors for credit agents who may

[4] In a growing economy, individual incomes need not be negatively correlated but income *shares* must be. This simple deterministic endowment process is the degenerate case of a stochastic economy with two Markovian states and a zero probability of remaining in the same state. Markovian endowments with two states are a straightforward extension. The assumption of two states or dates has obvious geometric advantages, but it is not innocuous where policy is concerned. We discuss this point further in the conclusion.

either accept their allocations or behave like cash agents in perpetuity. The inflation target in this economy is similar to an optimal tax subject to an incentive constraint as understood by Mirrlees (1971). When the government chooses a positive rate of inflation, it imposes a tax on cash agents and confers two benefits on credit agents: a transfer of resources from the cash sector as well as a reduction in the default payoff which brings about higher debt limits. Inflation, up to a point, deepens the credit market.

3 Inflation targeting as a planning problem

3.1 Overview

We now analyze inflation targeting as the solution to a particular stationary equal-treatment planning problem in which similar households are allocated similar consumption bundles independently of time. Allocations depend on household type only. To begin with, we suppose that the planner knows the following data:

(*a*) The common utility function and common income process of all households.

(*b*) The ability to identify agents $i = 0$ and $i = 1$, that is, to recognize all credit agents as well as their current income.

However,

(*c*) The planner does not know the current income of cash households and cannot discriminate between agent types $i = 2$ and $i = 3$.

A complete list of feasible actions for the planner and households is as follows.

(*i*) No household can be forced to pay a tax or surrender any part of its endowment against its will.

(*ii*) Any cash agent may purchase from the planner a non-negative stock of enforceable IOU's. Each IOU costs one unit of current consumption and pays off $1/\pi$ units of consumption next period. We call the constant parameter $\pi > 0$ the implied "inflation factor."

(iii) Given the tax rate $1-\frac{1}{\pi}$, cash agents choose the amount of IOU's they wish to buy from the planner. One option is zero IOU's, which amounts to autarky, that is, to consuming one's own endowment in perpetuity.

(iv) The planner collects all revenues from the "inflation tax," and asks all credit agents to surrender their endowments in return for a binding commitment to allocate forever $c_H > 0$ units of consumption to each high income credit agent, and $c_L > 0$ units of consumption to each low income credit agent. The planner's overall commitments to credit agents cannot exceed the combined endowment of that group plus the net revenue from the implied inflation tax. We call the marginal rate of substitution $u'(c_H) / [\beta u'(c_L)]$ the implied "real interest yield."

(v) Credit households reserve the right to reject the planner's proposal and behave instead like cash households. This includes the option to remain autarkic in perpetuity.

Next we describe the planning problem in three steps:

- The monetary authority sets a constant inflation factor π, or a tax rate $1-\frac{1}{\pi}$.

- Given π, high income cash agents choose a periodic consumption vector $(x_H, x_L) \geq 0$ to maximize stationary discounted utility

$$\frac{1}{1-\beta^2}\left[u(x_H) + \beta u(x_L)\right] \tag{3}$$

subject to

$$x_H \leq 1+\alpha, \tag{4}$$

$$x_H + \pi x_L = 1 + \alpha + \pi(1-\alpha), \tag{5}$$

and

$$u(x_H) + \beta u(x_L) \geq u(1+\alpha) + \beta u(1-\alpha). \tag{6}$$

The first inequality restricts excess demand for goods by high income cash agents to be nonpositive, and purchases of IOU's from the planner to be nonnegative. The second relation is a two-period budget constraint which assumes that credit households completely use up the planner IOU's or "money balances" to smooth consumption in low income dates. The third inequality allows households who dislike the announced inflation rate to renounce forever the use of IOU's and consume their endowments in perpetuity.

- Let $x_H(\pi)$ and $x_L(\pi)$ solve the previous problem. Given π, the planner now chooses consumption values $(c_H, c_L) \geq 0$ for credit households to maximize the equal-treatment welfare function

$$\frac{1}{1-\beta^2}\left[u(c_H) + u(c_L)\right] \tag{7}$$

of the credit community, subject to the resource constraint

$$\lambda(c_H + c_L) + (1-\lambda)\left[x_H(\pi) + x_L(\pi)\right] = 2, \tag{8}$$

and the participation constraint

$$u(c_H) + \beta u(c_L) \geq u\left[x_H(\pi)\right] + \beta u\left[x_L(\pi)\right]. \tag{9}$$

Equal treatment of high income and low income households means that the discounted utilities are weighted equally. High income households are given the infinite periodic consumption vector $(c_H, c_L, ...)$ with payoff

$$\frac{u(c_H) + \beta u(c_L)}{1-\beta^2}.$$

Low income households consume the periodic vector $(c_L, c_H, ...)$ with discounted value

$$\frac{u(c_L) + \beta u(c_H)}{1-\beta^2}.$$

The welfare function in equation (7) is a linear combination of these two discounted utilities with each group's weight equal to $1/(1+\beta)$. In addition, note that the resource constraint equates aggregate consumption with aggregate income. In other words, the planner allocates to the credit group the combined endowment of all credit households *plus* current revenue from IOU's just issued *minus* the redemption value of IOU's sold last period. Finally, the participation constraint ensures that high income credit agents prefer the planner's proposed allocation to using planner IOU's, that is, prefer to smooth their consumption through "credit" rather than through "money."

- If $c_H(\pi)$ and $c_L(\pi)$ solve the previous problem for a given $\pi > 0$, the planner selects the stationary implied inflation factor π to maximize the social welfare function

  $$\begin{aligned}\mathcal{W}(\pi, \delta) = \delta \{u[c_H(\pi)] + u[c_L(\pi)]\} \\ + (1-\delta)\{u[x_H(\pi)] + u[x_L(\pi)]\}.\end{aligned}$$

  This social welfare function assigns equal weights to members of the same group but potentially different weights to different groups. In particular, it weighs each credit community member by $\delta/(1+\beta)$, where $\delta \in (0,1)$, and cash community member by $(1-\delta)/(1+\beta)$. A strictly utilitarian welfare function, like the one used by Deviatov and Wallace (2007), would have equal weights for all households, that is, $\delta = \lambda$.

3.2 Optimum inflation without incentive constraints

To build up intuition, we solve the planner's problem outlined in section 3.1, ignoring for the moment the incentive constraints laid out in equations (6) and (9). As a first step we allow lump-sum income transfers from cash agents to credit agents which permits us to also ignore the cash agents' budget constraints (4) and (5). All the planner has to do is maximize the social welfare function

$$\mathcal{W}(\pi, \delta) = \delta [u(c_H) + u(c_L)] + (1-\delta)[u(x_H) + u(x_L)] \tag{10}$$

subject to the economy's resource constraint

$$\lambda (c_H + c_L) + (1-\lambda)(x_H + x_L) = 2. \tag{11}$$

The obvious solution is $(c_H, c_L, x_H, x_L) = (c^*, c^*, x^*, x^*)$ where c^* and x^* solve the following pair of equations:

$$\delta u'(c) = (1-\delta) u'(x)$$

$$\lambda c + (1-\lambda) x = 1.$$

We call this solution the *first best*. The implied optimal inflation and nominal interest rates can be inferred from the consumption Euler equation for the two household types, that is, from

$$\frac{\beta R^N}{\pi} = 1 \tag{12}$$

$$\frac{\beta}{\pi} = 1. \tag{13}$$

The first-best allocation is thus supported by Friedman's rule, that is, by $(\pi, R^N) = (\beta, 1)$.

Suppose next that the planner cannot impose a lump-sum tax on any agent but must instead use inflation or deflation and redistribute the resulting seigniorage from one group to another. Inflation is a proportional tax on the excess supply of goods by high income cash agents; it transfers resources from cash to credit households. Deflation does the exact opposite. The planner must now choose (π, c_H, c_L) to solve the problem outlined in section 3.1 subject to all constraints except (6) and (9). We call this outcome the *second best*.

To understand the optimum rate of inflation at the second best allocation, we examine the two polar cases $\delta = 1$ and $\delta = 0$. The first case, which assigns no welfare weight to the cash-using community, leads the planner to select that value of π which minimizes the consumption of that community. The maximum possible amount of seigniorage is transferred to the credit community, and the consumption of credit agents is smoothed completely.

Define the *maximal seigniorage inflation factor* from

$$\tilde{\pi} = \arg\min_{\pi \geq 1} \left[x_H(\pi) + x_L(\pi) \right] > 1. \tag{14}$$

Then the planner sets $(\pi, c_H, c_L) = (\tilde{\pi}, \tilde{c}, \tilde{c})$ where $\tilde{c}$ can be read from the resource constraint

$$2\lambda\tilde{c} + (1-\lambda)\left[x_H(\tilde{\pi}) + x_L(\tilde{\pi})\right] = 2. \tag{15}$$

In addition, $c_H = c_L$ implies $\beta R^N = \tilde{\pi}$. The second best allocation turns out to be supported by high rates of inflation and nominal interest, that is

$$\left(\pi, R^N\right) = (\tilde{\pi}, \tilde{\pi}/\beta). \tag{16}$$

At the other extreme, $\delta = 0$ describes a society in which the planner cares about the cash community only. This planner will deflate the economy in order to reduce the aggregate consumption of credit households, pushing the inflation factor as close to zero as possible. That is obvious from Figure 1 below, which superimposes the budget constraint of the high income cash household against social indifference curves that turn out to be symmetric about the diagonal.

A utilitarian social welfare function with $\delta = \lambda$ represents a sensible compromise between the extremes just described. A planner endowed with a utilitarian social welfare function will choose a *modified Friedman rule* that combines mild deflation with a small positive interest rate to guarantee smooth consumption for credit agents. The following result is proved in the Appendix.

Theorem 1 *The second best optimum allocation under a utilitarian social welfare function satisfies* $(c_H, c_L, x_H, x_L) = (c^{**}, c^{**}, x_H(\pi^{**}), x_L(\pi^{**}))$. *It is supported by a modified Friedman rule for some inflation factor* $\pi^{**} \in (\beta, 1)$, *and a nominal yield such that* $R^N = \pi^{**}/\beta > 1$.

Proof. See Appendix. ■

4 The role of incentive constraints

4.1 Basic assumptions

We suppose in what follows that the incentive constraints are restrictive enough to rule out both the first-best and the second-best allocations de-

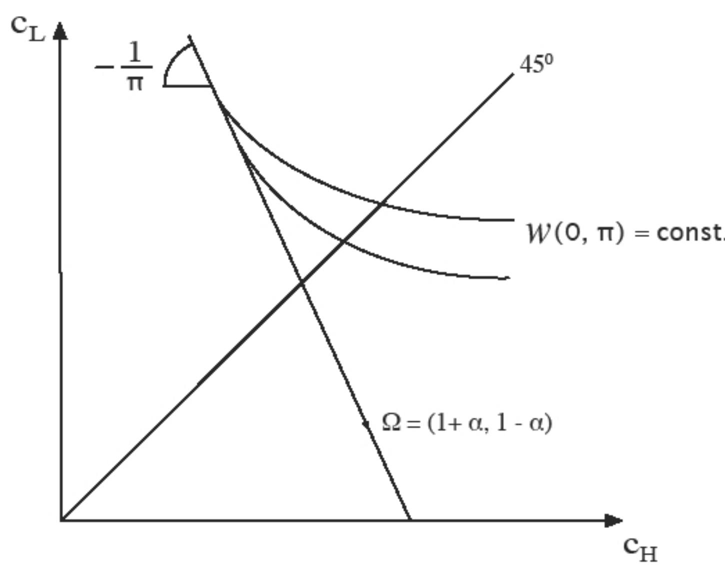

Figure 1: Social indifference curves for $\delta = 0$.

scribed in the previous section, and defeat the planner's desire to smooth completely the consumption profile for both the credit and the cash community. Define $y(\pi)$ to be the combined consumption of a pair of high income and low income credit agents. This consumption is maximal when the implied inflation factor π is equal to the maximal seigniorage inflation factor $\tilde{\pi}$. From the resource constraint we obtain

$$c_H + c_L = y(\pi) \equiv \frac{1}{\lambda}\left[2 - (1-\lambda)\left(x_H(\pi) + x_L(\pi)\right)\right].$$

Our key assumptions are these:

A1. $\bar{R} \equiv \frac{u'(1+\alpha)}{\beta u'(1-\alpha)} < 1$,

A2. $u(1+\alpha) + \beta u(1-\alpha) > (1+\beta)\, u(1)$, and

A3. $(1+\beta)\, u\left[\frac{y(\tilde{\pi})}{2}\right] > u\left[x_H(\tilde{\pi})\right] + \beta u\left[x_L(\tilde{\pi})\right].$

Assumptions **A1** and **A2** state that individual income shares are neither very stable nor highly variable. In particular, **A1** asserts that autarky is an allocation with a low implied rate of interest $\bar{R}$ and therefore cannot be a constrained efficient allocation for the credit community.[5] Geometrically, we require the initial endowment point $\Omega = (1+\alpha, 1-\alpha)$ in Figure 2 to lie below the tangency point G on the budget line $c_H + c_L = 2$. This assertion is innocuous. It means that the income variability parameter α is large relative to the consumer's rate of time preference if α is the same for all households. If, however, α should vary across households, then autarky is a low interest rate equilibrium whenever the rate of time preference is small relative to the *largest* α in the population.[6] Roughly speaking, **A1** amounts to asserting that there is at least one household in the economy whose income share fluctuates more than three or four percent per year.

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[5]On this point, see Alvarez and Jermann (2000).

[6]That is so because an autarkic allocation is decentralized as a competitive equilibrium by assigning a zero debt limit to all agents except the one with the lowest income growth rate, and by a competitive interest factor that exactly matches the autarkic marginal rate of substitution of the household with the lowest income growth rate.

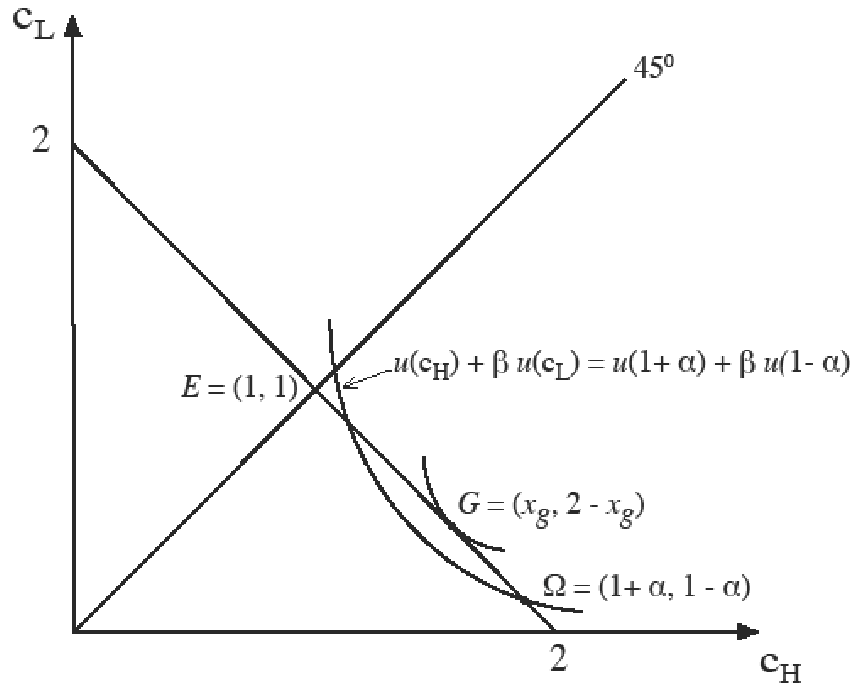

Figure 2: Assumptions *A*1 and *A*2.

The next assumption, **A2**, rules out plans that combine perfect consumption smoothing for credit agents with a zero rate of inflation, which would decentralize the golden rule allocation for cash agents. Zero implied inflation means no transfers of income between groups. Perfect consumption smoothing for credit agents is achieved by the allocation $c_H = c_L = 1$ whose payoff is below autarky by assumption **A2**. In Figure 2, the flat-consumption allocation point E lies below the indifference curve that goes through the initial endowment point Ω.

This assumption, too, is empirically innocuous: It holds automatically for values of α near zero. If α were to vary across households, assumptions **A1** and **A2** would assert that income shares are nearly constant for some agents and quite variable for others. But, since we have only one endowment profile in the entire economy, we need to assume that income shares are neither too smooth nor too variable. That is what is embodied in assumptions **A1** and **A2**.

It is worth noting that assumption **A2** is inconsistent with the Friedman rule for reasons similar to those advanced by Aiyagari and Williamson (2000), Berentsen, *et al.*, (2007), and Andolfatto (2007). Any constant, resource-feasible consumption plan $(c_H, c_L, x_H, x_L) = (c^*, c^*, 2 - c^*, 2 - c^*)$ will be vetoed by high-income credit agents who will refuse to pay the implied deflation tax.

The last assumption is a bit more controversial. It claims that credit agents can achieve perfectly smooth consumption albeit at higher rates of inflation. **A3** asserts that it is within the power of the central planner, and of the central bank, to lower the rate of return facing users of cash to the point where the incentive constraint on credit users becomes slack. **A3** states that allocations with perfectly smooth consumption, $c_H = c_L = y(\pi)/2$, are feasible at the maximum seigniorage rate of inflation and also at lower rates. For all of these implied inflation rates, the payoff from credit use exceeds the payoff from cash use. Figure 3 illustrates.

Let

$$v(\pi) \equiv u[x_H(\pi)] + \beta u[x_L(\pi)]$$

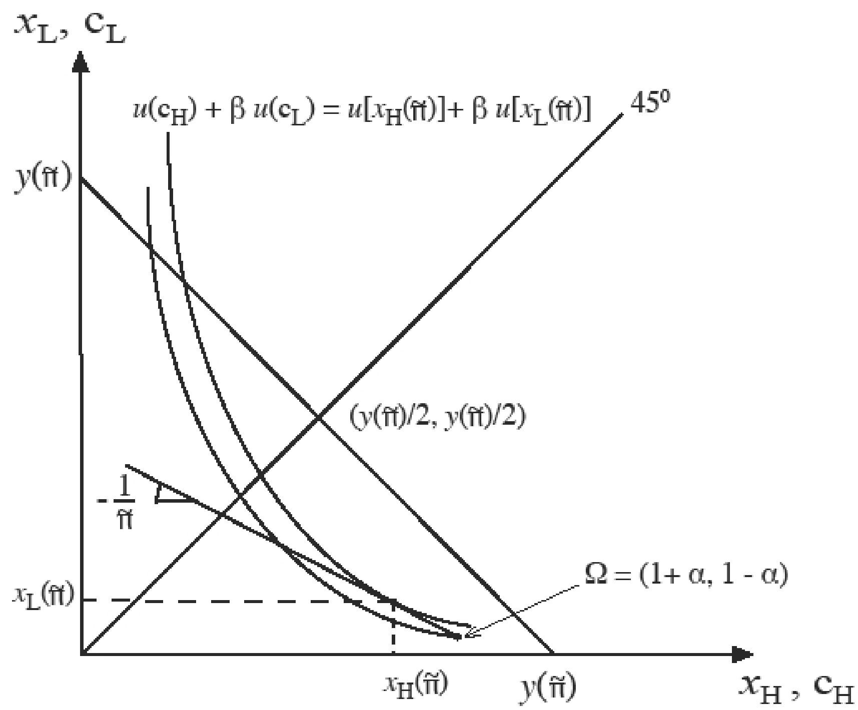

Figure 3: Assumption *A*3.

denote the two period payoff to any high income household using planner IOU's or "money." Then, for any isoelastic utility function $u : \mathbb{R}_+ \to \mathbb{R}$ for which c_H and c_L are gross substitutes, the seigniorage function $y(\pi)$ is continuous, positive, and increasing in π for all $\pi \in (1, \tilde{\pi})$; positive and decreasing in π for all $\pi \in (\tilde{\pi}, 1/\bar{R})$; and zero at $\pi = 1$ and $\pi = 1/\bar{R}$. The demand for money by cash agents vanishes at $\pi = 1/\bar{R}$ as households switch to autarky.

Assumption **A3**, together with the continuity of the function $y(\pi)$, guarantees the existence of an inflation factor $\bar{\pi}$ in the open interval $(1, \tilde{\pi})$ for which

$$(1+\beta)\, u\left[y(\bar{\pi})/2\right] = v(\bar{\pi}). \tag{17}$$

High income credit households are indifferent between cash and credit at $\pi = \bar{\pi}$, and the participation constraint (9) becomes slack when inflation reaches that value. In a decentralized economy, debt constraints will cease to bind, and the loan market will smooth consumption perfectly, when inflation is in the closed interval $[\bar{\pi}, \tilde{\pi}]$.

Figure 4 illustrates the relationship between credit rationing and inflation by graphing the payoffs to "credit" and "money" users when the credit community enjoys constant consumption. These payoffs are exactly equal at $\pi = \bar{\pi}$ and again at some higher $\pi_m \in (\tilde{\pi}, 1/\bar{R})$. Discounted utility $v(\pi)$ from the use of money is a monotonically decreasing function of the inflation tax π for any π less than $1/\bar{R}$. When π reaches or exceeds $1/\bar{R}$, the rate of return on money falls below the implied yield on autarky, and the demand for money as a store of value vanishes altogether.

Constant consumption for credit households rises as the inflation factor increases from 1 to $\tilde{\pi}$, then falls as π increases further from $\tilde{\pi}$ to $1/\bar{R}$. Seigniorage dries up at that point, and $c_H = c_L = 1$ for all $\pi \geq 1/\bar{R}$. However, for implied inflation factors in the interval $(\bar{\pi}, \pi_m)$, "credit" pays off more than "money." This means that imposing the incentive or participation constraint (9) on credit households improves the planner's ability to smooth the consumption profile of the credit community.[7]

4.2 Inflation and social welfare

We are now ready to deal with the complete planning problem described in Section 3.1. Our strategy is to show that the social welfare function $\mathcal{W}(\pi, \delta)$:

- Is continuously differentiable for all $\pi \geq 1$;
- Is undefined for $\pi < 1$ because deflation contradicts the participation constraint (9);
- Increases rapidly in π at $\pi = 1$;
- Decreases in π for all $\pi \in [\bar{\pi}, 1/\bar{R}]$ if $\delta \leq \lambda$;
- Is constant for π larger than $1/\bar{R}$.

These properties guarantee the existence of an optimum inflation factor

$$\pi^{\star}(\delta) = \arg \max_{\pi \in [1, 1/\bar{R}]} \mathcal{W}(\pi, \delta) \geq 1, \ > 1 \text{ if } \delta > 0.$$

The appendix contains a proof of the following result.

Lemma 2 *Define $\mathcal{W}_{\pi}(\pi, \delta) = \partial \mathcal{W} / \partial \pi$. Then (a) $\mathcal{W}_{\pi}(\pi, \delta) < 0 \ \forall (\pi, \delta) \in [\bar{\pi}, \tilde{\pi}] \times [0, \lambda]$, and (b) $lim_{\pi \to 1} \mathcal{W}_{\pi}(\pi, \delta) = +\infty$ when π converges from above.*

The intuition for part (*a*) is fairly simple. For any $\pi > \bar{\pi}$, assumption **A3** says that smoothing the consumption of credit households is consistent with the participation constraint. To raise π above $\bar{\pi}$ does not improve the ability of the planner to smooth the consumption of the credit community any further. Doing so merely transfers income from the cash community,

[7] The planner may in principle attempt to smooth the credit community's consumption vector by transfers of "money," that is, by selling IOU's to credit agents. However, non-negative IOU balances are of no help to rationed low income credit agents who wish to go short or to unconstrained high income credit agents for whom money is a lower-yielding store of value than loans.

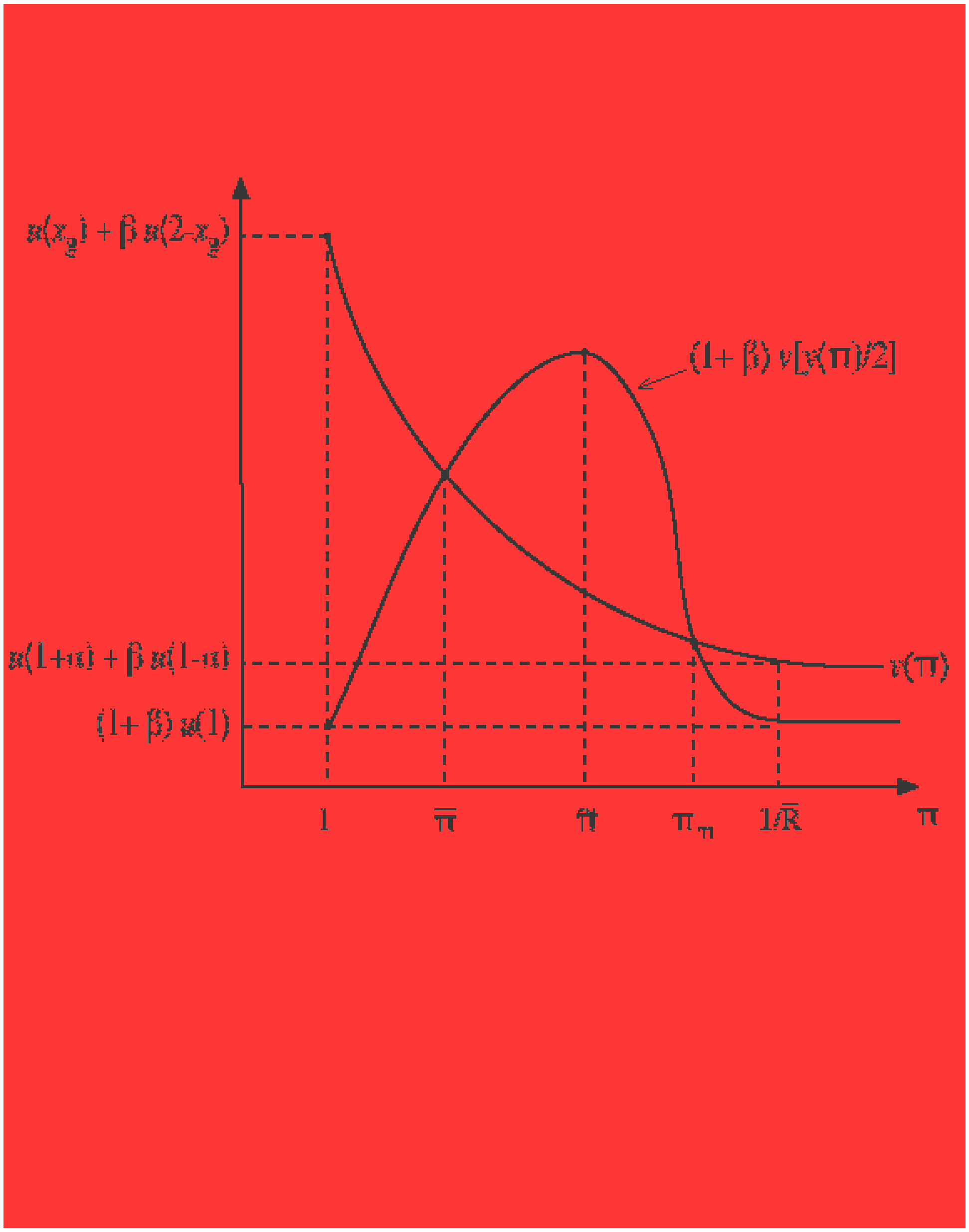

Figure 4: Inflation and credit rationing.

who are consuming less than two units of total income, to the credit community who are consuming more. This transfer will reduce social welfare except in cases where the favored credit households are extraordinarily important to the central planner, that is, when $\delta > \lambda$.

Part (*b*) can be understood in a similar way. At very small positive rates of inflation, the aggregate consumption of each community is proportional to its population weight and, by assumption **A2**, c_H is substantially different from c_L. A tiny increase in the inflation tax transfers a tiny amount of resources between two groups with roughly the same marginal utility of income. This insignificant transfer would have essentially no impact on the social welfare function except that it lowers the discounted utility of money for the credit community, allowing the central planner to dramatically smooth the consumption vector (c_H, c_L). The reason for this improvement is that the planner is able to set up at a zero inflation rate a credit market which cannot function under deflation.

Next we prove, again in the appendix, Lemma 3.

Lemma 3 *$\mathcal{W}(\pi, \delta)$ is not defined for $\pi < 1$. It is decreasing in π for $\pi \in (\tilde{\pi}, 1/\bar{R})$ and constant for $\pi \geq 1/\bar{R}$.*

The key part of Lemma 3 is understanding why the reduced-form social welfare function $\mathcal{W}(\pi, \delta)$, defined at the end of Section 3.1, does not exist for $\pi < 1$ or, equivalently, why deflation violates the participation constraint for high income credit households. Deflation means that each high income cash household will consume a vector (x_H, x_L) such that $x_H + x_L > 2$, attaining a point above the budget line $x_H + x_L = 2$. The corresponding high income credit household will consume (c_H, c_L) such that $c_H + c_L < 2$, reaching a point below the previous budget line. The outcome of any deflation is that money has a higher payoff than credit.[8]

The main result of this section, which follows directly Lemma 2 and Lemma 3, is stated below.

[8]Money is less useful as an asset in a growing economy which permits the planner to let the inflation rate drop below zero when the growth rate of aggregate income is positive. See Section 5 for details.

Theorem 4 *Suppose assumptions* **A1**, **A2**, *and* **A3** *hold, and* $0 < \delta \leq \lambda$. *Then the optimum inflation factor is* $\pi^{\star}(\delta) > 1$ *and the associated nominal interest yield,* $R^N \in (\pi^{\star}(\delta), \pi^{\star}(\delta)/\beta)$, *is even higher.*

Figure 5 uses Lemmas 2 and 3 to illustrate the planner's SWF for some welfare weight $\delta \in (0, \lambda)$ and any inflation factor $\pi \geq 1$. Assumption **A3** generates large improvements in the planner's consumption smoothing power from relatively small inflation rates. As inflation goes up, these improvements taper off, and after the optimum value $\pi^{\star}(\delta)$, they are negated by the deadweight loss of the inflation tax.

A related result in a search-theoretic framework is Proposition 5 in Berentsen, *et al.* (2007) where the optimum inflation rate is $\bar{\pi}$, that is, the rate at which borrowers become unrationed. This result obtains when the rate of time preference is less than the population fraction of agents who are selling consumption goods for money.

5 Extensions and conclusions

What factors should a benevolent, independent central bank consider when it sets a long run inflation target? Summers (1991) has expressed the view that the zero lower bound on nominal interest rates dictates an inflation target above zero. This paper suggests that a very different mechanism may be at work. In particular, Theorem 4 shows that, for an economy with limited enforcement, constant aggregate income and no collateral, the inflation target should strike a balance between the deadweight loss from inflation and the potential improvement in credit market conditions.

How does economic growth affect inflation targets? Suppose, for example, that all the endowments described in equation (2) are multiplied by a growth factor $g \geq 1$, and that the utility function is isoelastic, that is,

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}$$

where $\gamma \geq 0$, and $\beta_g \equiv \beta g^{1-\gamma} < 1$. In this growing economy the mathematical structure of the planning problem, defined in Section 3.1, remains

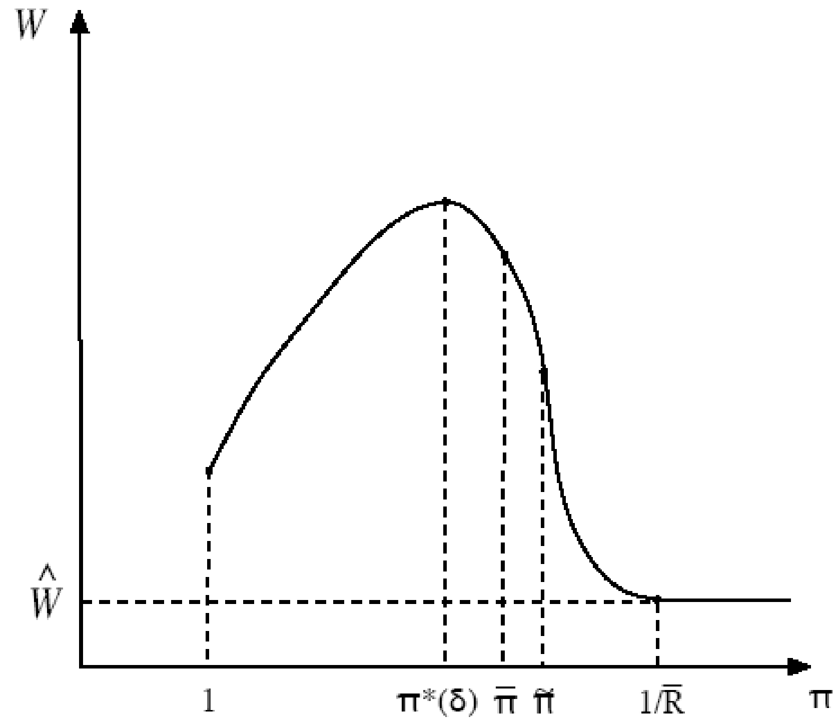

Figure 5: Inflation and social welfare.

the same if we replace the original discount factor β with a modified β_g and the original inflation factor π with the modified inflation factor $\pi_g \equiv g\pi$.

For any utility function with $\gamma \leq 1$ (which implies gross substitutability of intertemporal consumption goods), increases in g effectively raise the planner's patience and slacken the incentive constraints. We conjecture that this increase in effective patience will allow the planner to smooth consumption better at any given rate of inflation, and will lessen the need to subsidize the loan market at the expense of the currency market. The outcome should be a lower inflation target $\pi^\star$ for any given welfare weight δ. This conjecture is easily verified for the logarithmic utility function with $\gamma = 1$. In this case, the planner's effective discount rate remains at β and by Theorem 4 the optimum inflation rate should be $\pi_g = \pi^\star(\delta)$ or $\pi = \pi^\star(\delta)/g$. In other words, the sum of the inflation target plus the growth rate is a constant independent of the growth rate itself.

We also conjecture that collateral borrowing should have an effect on inflation targets similar to that of higher growth rates. Collateral improves the ability of credit agents to smooth consumption in a state of default by combining long positions in currency with short positions in collateralized loans. This will raise the payoff to default for cash agents and reduce the debt limits on non-collateral loans. Total borrowing, however, should improve as income becomes better collateral, and so will the planner's ability to smooth consumption without relying too much on the intermediating effect of higher inflation.[9]

We expect the opposite conclusions to obtain when the variability of individual income shares, as measured by the parameter α, goes up. This change should raise the payoff from market participation and relax debt constraints. Nevertheless, some of this additional idiosyncratic risk will have to be borne by credit agents in the form of higher consumption variability. The appropriate response of the central planner in this situation

[9] A calibration exercise in Ragot (2006) suggests that the optimum annual inflation rate is only 1.5 percent if 10 percent of intermediate goods producers are rationed, but rises to 4 percent when rationing affects 50 percent of those producers.

is likely to be a higher inflation target, that is, an attempt to subsidize the credit mechanism at the expense of money holding.

The main conclusion of this paper is that independent central banks will set low positive inflation targets in economies that possess highly developed financial markets. This finding seems to be broadly consistent with the comfort zones articulated by some of the world's leading central bankers. Less fortunate societies with relatively undeveloped asset markets will choose higher inflation targets to improve credit market performance. Slower growth tends to raise inflation targets, and the highest targets should be expected from stagnating economies with poorly developed financial institutions.

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A Proof of Theorem 1

The planner chooses (π, c_H, c_L) to maximize the utilitarian SWF

$$\mathcal{W}(\pi, c_H, c_L, \lambda) = \lambda \left[u(c_H) + u(c_L)\right] + \\ (1-\lambda)\left[u(x_H(\pi)) + u(x_L(\pi))\right]$$

subject to the resource constraint (8) and the definitions of $x_H(\pi), x_L(\pi)$ from equations (3), (4), and (5). The solution will clearly satisfy $c_H = c_L = c$. Using the resource constraint, we rewrite the SWF in the form

$$\mathcal{W}(\pi, \lambda) = 2\lambda u\left[\frac{2-(1-\lambda)(x_H + x_L)}{2\lambda}\right] + \\ (1-\lambda)\left[u(x_H) + u(x_L)\right].$$

Denoting $\mathcal{W}_\pi = \partial\mathcal{W}/\partial\pi$, we differentiate the SWF with respect to π and obtain

$$\frac{\mathcal{W}_\pi(\pi, \lambda)}{1-\lambda} = -u'(c)\left[x'_H(\pi) + x'_L(\pi)\right] + \\ u'(x_H)\, x'_H(\pi) + u'(x_L)\, x'_L(\pi)$$

where $u'(x_H) = (\beta/\pi)\, u'(x_L)$ is the consumption Euler equation of the cash group. Therefore,

$$\frac{\mathcal{W}_\pi(\pi, \lambda)}{1-\lambda} = -u'(c)\left(x'_H + x'_L\right) + u'(x_L)\left(\frac{\beta}{\pi}x'_H + x'_L\right). \tag{18}$$

Next we show that $\mathcal{W}$ is an increasing function of π at $\pi = \beta$, and a decreasing one for all $\pi \geq 1$. Since $\mathcal{W}$ is continuous in π, the intermediate value theorem implies that it attains a maximum in the interval $(\beta, 1)$. To check this, we note from (18) that

$$\frac{\mathcal{W}_\pi(\beta, \lambda)}{1-\lambda} = \left[x'_H(\beta) + x'_L(\beta)\right]\left[u'(x_L) - u'(c)\right]$$

where $x_H(\beta) = x_L(\beta) > 1 > c$ from the budget constraints, and $x'_H(\pi) + x'_L(\pi) < 0$ for all π, as shown by Figure 1. It follows that $\mathcal{W}$ is increasing in π at $\pi = \beta$.

Continuing along this line of argument, we observe that β/π is less than or equal to β for any $\pi \geq 1$, and $x'_H(\pi) > 0$ for all π if dated consumption goods are normal. Therefore, for any $\pi \geq 1$, we have

$$\frac{\mathcal{W}_\pi(\pi, \lambda)}{1-\lambda} \leq -u'(c)\left(x'_H + x'_L\right) + u'(x_L)\left(\beta x'_H + x'_L\right). \tag{19}$$

Next, we differentiate the budget constraint in equation (5) and obtain

$$x'_H = 1 - \alpha - x_L - \pi x'_L. \tag{20}$$

Substituting (20) into (19) yields

$$\begin{aligned}\frac{\mathcal{W}_\pi(\pi, \lambda)}{1-\lambda} \leq {} & -u'(c)\left[1 - \alpha - x_L + (1-\pi)x'_L\right] + \\ & u'(x_L)\left[\beta(1-\alpha-x_L) + (1-\beta\pi)x'_L\right]. \end{aligned} \tag{21}$$

Here, for any $\pi \geq 1$, the budget constraints and the consumption Euler equation for cash agents jointly imply $c > 1 > x_L$ and $1 - \alpha - x_L < 0$. Therefore, equation (21) leads to

$$\begin{aligned}\frac{\mathcal{W}_\pi(\pi, \lambda)}{1-\lambda} &\leq [x_L - (1-\alpha)]\left[u'(c) - \beta u'(x_L)\right] \\ &\quad - x'_L(\pi)\left[(1-\pi)u'(c) - (1-\beta\pi)u'(x_L)\right] \\ &\leq [x_L - (1-\alpha)](1-\beta)u'(x_L) \\ &\quad - x'_L(\pi)u'(x_L)[1 - \pi - 1 + \beta\pi]\end{aligned}$$

because $u'(c) < u'(x_L)$. Continuing,

$$\begin{aligned} \frac{\mathcal{W}_\pi(\pi,\lambda)}{1-\lambda} &\leq (1-\beta)\, u'(x_L)\left[x_L - (1-\alpha) + \pi x'_L\right] \\ &= -(1-\beta)\, u'(x_L)\, x'_H(\pi) \end{aligned}$$

by equation (20). Since $x'_H(\pi)$ is positive for all π, $\mathcal{W}_\pi(\pi,\lambda) < 0$ for all $\pi \geq 1$. This completes the proof.

B Proof of Lemma 2

Part (*a*). The Lemma is trivial for large $\pi \in [\tilde{\pi}, 1/\bar{R}]$. We focus on $\pi \in [\bar{\pi}, \tilde{\pi}]$. Now note that the derivative

$$\mathcal{W}_\pi(\pi,\delta) = \delta\left[u'(c_H)\, c'_H + u'(c_L)\, c'_L\right] + (1-\delta)\left[u'(x_H)\, x'_H + u'(x_L)\, x'_L\right]$$

can be written as

$$\mathcal{W}_\pi(\pi,\delta) = \delta u'\left[y(\pi)/2\right] y'(\pi) + (1-\delta)\, u'(x_L)\left[\frac{\beta}{\pi} x'_H + x'_L\right]$$

because $y(\pi)/2 = c_L = c_H$ and $u'(x_H) = (\beta/\pi)\, u'(x_L)$. Continuing, recall that $x'_H > 0$ by gross substitutes, $\beta/\pi < 1$ by assumption, and $x'_H + x'_L < 0$ because seigniorage is increasing in the interval $[1, \tilde{\pi}]$. Therefore, $(\beta/\pi)\, x'_H < x'_H$ and

$$\begin{aligned} \mathcal{W}_\pi(\pi,\delta) &< \delta u'(c_L)\, y'(\pi) + (1-\delta)\, u'(x_L)\left(x'_H + x'_L\right) \\ &= \delta u'(c_L)\left[-\frac{1-\lambda}{\lambda}\left(x'_H + x'_L\right)\right] + (1-\delta)\, u'(x_L)\left(x'_H + x'_L\right) \\ &= -\left(x'_H + x'_L\right)\left[\frac{\delta(1-\lambda)}{\lambda} u'(c_L) - (1-\delta)\, u'(x_L)\right] \\ &< -\left(x'_H + x'_L\right)\left[(1-\delta)\, u'(c_L) - (1-\delta)\, u'(x_L)\right] \end{aligned}$$

since $\delta \leq \lambda$. Therefore,

$$\frac{-\mathcal{W}_\pi}{\left(x'_H + x'_L\right)(1-\delta)} < u'(c_L) - u'(x_L). \tag{22}$$

Note next that $\beta < \pi$ implies $x_L < x_H$, $c_L = c_H$ by assumption, and also $c_L + c_H > 2 > x_L + x_H$ for all $\pi \in (\bar{\pi}, \tilde{\pi})$. It follows that $c_L > x_L$ and therefore that the right hand side of inequality (22) is negative. From this and the fact that $x'_H + x'_L < 0$ we infer that $\mathcal{W}_\pi(\pi, \delta) < 0$ for all $\pi \in (\bar{\pi}, \tilde{\pi})$ and all $\delta \in (0, \lambda]$.

Part (*b*). Assumption **A3** asserts that the central planner cannot set $c_H = c_L$ for any $\pi \in (1, \bar{\pi})$ without violating the participation constraint (9). For any π in that interval, the planner will smooth consumption as much as the participation constraint allows, choosing $c_H(\pi)$ to be the smallest solution to the equation

$$\begin{aligned} u(c_H) + \beta u[y(\pi) - c_H] &= v(\pi) \\ &\equiv u[x_H(\pi)] + \beta u[x_L(\pi)], \end{aligned} \tag{23}$$

where $c_L(\pi) = y(\pi) - c_H(\pi)$. Differentiate (23) with respect to π and obtain

$$c'_H(\pi) = \frac{v'(\pi) - \beta u'(c_L) y'(\pi)}{u'(c_H) - \beta u'(c_L)}. \tag{24}$$

Note also that, at $\pi = 1$, we have

$$\begin{aligned} c_H(1) &= x_H(1), \\ c_L(1) &= x_L(1), \\ c_H(1) + c_L(1) &= x_H(1) + x_L(1) = 2. \end{aligned}$$

Next we compute

$$\begin{aligned} \mathcal{W}_\pi(1, \delta) = u'(x_H(1)) \left[\delta c'_H(1) + (1-\delta) x'_H(1)\right] + \\ u'(x_L(1)) \left[\delta c'_L(1) + (1-\delta) x'_L(1)\right] \end{aligned}$$

where

$$u'(x_H(1)) = \beta u'(x_L(1)).$$

Continuing we obtain

$$\begin{aligned}\frac{\mathcal{W}_\pi(1,\delta)}{u'(x_L(1))} &= \beta\left[\delta c_H'(1)+(1-\delta)x_H'(1)\right]+ \\ &\quad \delta\left[y'(1)-c_H'(1)\right]+(1-\delta)x_L'(1) \\ &= Q+(\beta-1)\delta c_H'(1)\end{aligned}$$

where

$$Q\equiv\beta(1-\delta)x_H'(1)+\delta y'(1)+(1-\delta)x_L'(1).$$

Note now that

$$c_H'(1)=\lim_{\pi\searrow 1}c_H'(\pi)=-\infty$$

by equation (24) because

$$v'(1)-\beta u'(c_L(1))y'(1)<0$$

as the sum of two negative terms, and

$$u'(c_H(1))=\beta u'(c_L(1)).$$

Therefore $\lim_{\pi\searrow 1}\mathcal{W}_\pi(1,\delta)=+\infty$. This completes the proof.

C Proof of Lemma 3

The proof of this lemma is straightforward as shown in Figure 5. Note, however, that for $\pi>1/\bar{R}$ the payoff to money is just autarky. Therefore, we have

$$\begin{aligned}\mathcal{W}(\pi,\delta) &= \delta\left[u(\hat{x})+u(2-\hat{x})\right]+(1-\delta)\left[u(1+\alpha)+\beta u(1-\alpha)\right] \\ &\equiv \hat{W},\end{aligned}$$

where $\hat{x}\in(1,1+\alpha)$ is the smallest solution to the equation

$$u(x)+\beta u(2-x)=u(1+\alpha)+\beta u(1-\alpha).$$