

# Systemic Risk and Liquidity in Payment Systems

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## Abstract

We study liquidity and systemic risk in high-value payment systems, such as for the settlement of accounts receivable and payable among industrial firms, and interbank payment systems. Flows in high-value systems are characterized by high velocity, meaning that the total amount paid and received is high relative to the stock of reserves. In such systems, banks and firms rely heavily on incoming funds to finance outgoing payments, necessitating a high degree of coordination and synchronization. We use lattice-theoretic methods to solve for the unique fixed point of an equilibrium mapping and conduct comparative statics analyses on changes to the environment.

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# 1 INTRODUCTION

The complex web of transactions in an economic system generate the corresponding flow of payments that settle the ensuing obligations. The interlocking claims and obligations generate the potential for liquidity spillovers where the ability to discharge my own obligations in a timely manner depends on the extent to which others discharge theirs to me.

A good example is the cash management by a firm that tries to balance the outflows of cash on the firm's accounts payable with the inflows of cash from its accounts receivable. If the firm suffers a liquidity shock so that it finds itself running low on its cash holdings, one way it can conserve cash is to delay payments on its accounts payable, in the hope that the continued inflows from its receivables rebuild its cash holdings. However, my accounts payable are the accounts receivable of my supplier firms, and the delay in payments will have an adverse impact on the liquidity positions of those supplier firms. If some firms delayed their payments, this delay would lead to a deterioration in the liquidity position of their supplier firms and a delay in the payment of these supplier firms, which then has a further impact on their own supplier firms, and so on. From a system perspective, not all firms can conserve liquidity by delaying payments. The consequence of the attempt by all firms to conserve liquidity by delaying payments would be gridlock, and a breakdown in all flows within the system.

The drastic fall in output and the breakdown of complex production relationships in the countries of the former Soviet Union after 1991 has been attributed to the arrears in accounts payable that built up among the newly decentralized firms as they attempted to delay payments to their suppliers, while demanding payment from their customers. Blanchard and Kremer (1997) coined the term "disorgani-

zation” to describe such a phenomenon, and document the drastic falls in GDP that results. For Russia, GDP fell by 50% from 1989 to 1997. Other former Soviet republics fared even worse. The system-wide implications of a disruption to smooth flows are amply demonstrated in this episode.

A shorter, but perhaps an even sharper episode of the systemic implications of the gridlock in payments came in the interbank payment system following the 9/11 attacks. The interbank payment system processes very large sums of transactions between banks and other financial institutions. Moreover, one of the reasons for the large volumes of flows is due to the two-way flow that could potentially be netted between the set of banks. That is, the large flows leaving bank *A* is matched by a similarly large flow into bank *A* over the course of the day. However, the fact that the flows are not exactly synchronized means that payments flow backward and forward in gross terms, generating the large overall volume of flows.

The nettable nature of the flows allows a particular bank to rely heavily on the inflows from other banks to fund its outflows. McAndrews and Potter (2002) note that banks typically hold only a very small amount of cash and other reserves to fund their payments. The cash and reserve holdings of banks amount to only around 1% of their total daily payment volume. The rest of the funding comes from the inflows from the payments made by the other banks. To put it another way, one dollar held by a particular bank at the beginning of the day changes hands around one hundred times during the course of the day. Such high velocities of circulation have been necessitated by the trend toward tighter liquidity management by banks, as they seek to lend out spare funds to earn income, and to calculate fine tolerance bounds for spare funds.

There is, however, a drawback to such high velocities that come from the fragility of overall payment flows to disruptions to the system, or a small step

change in the desired precautionary balances targeted by the banks. After the 9/11 attacks, banks attempted to conserve liquidity and raise their precautionary cash balances as a response to the greater uncertainty. Given the high velocity of funds, even a small change in target reserve balances can have a marked effect on overall payment volumes, and this is exactly what happened after 9/11. McAndrews and Potter (2002) give a detailed account of the events in the U.S. Fedwire payment system following the September 11th attacks.

Our paper addresses the issue of liquidity in a flow system, whether it be the liquidity of an interbank payment system, or the flows of cash among firms in a production economy. The focus is on the interdependence of the agents in the system, and the manner in which equilibrium payments are determined and how the aggregate outcome changes with shifts in the parameters describing the environment. In keeping with the systemic perspective, we model the interdependence of flows and show how the equilibrium flows correspond to the (unique) fixed point of a well-defined equilibrium mapping. The usefulness of our approach rests on the fact that our model abstracts away from specific institutional details, and rest only of the robust features of system interaction. The comparative statics exercise draws on methods on lattice theory, developed by Topkis (1978) and Milgrom and Roberts (1994). Additionally, the relatively simple nature of our framework allows numerical simulations based on rules of thumb behavior that have good empirical foundations, and which generate outcome paths under “normal” circumstances that mimic well the observed patterns.

However, much more important than the ability to mimic the observed patterns in the data is the ability to conduct “what if” type exercises where we can address the counterfactual questions of how the payment system would behave in the face of specific changes in the environment, or the failure of one or more nodes in the

system. The fact that the aggregate outcome takes account of the spillover effects gives us hope of capturing the potential consequences of systemic failure in the real world.

The outline of the paper is as follows. In the next section we introduce a theoretical framework for the role of interlocking claims and obligations in a flow system. An application to the interbank payment system then follows. Section 3 briefly reviews the US payment system paying special attention to the Fedwire Funds Service. Section 4 presents numerical simulations based on a stylised payment system which allows us to better understand the response of payment systems to disruptions in payments and operational changes in the system. Section 5 concludes.

## 2 THE MODEL

There are  $n$  agents in the payment system, whom we will refer to as “institutions”. Every member of the payment system maintains an account to make payments. This account contains all balances including its credit capacity.

Institutions in a payment system rely heavily on incoming funds to make their payments. Let us denote by  $y_t^i$  the time  $t$  payments institution  $i$  sends to other members in the payment system. These payments are increasing in the total funds  $x_t^i$  institution  $i$  receives from other members during some period of time (from  $t - 1$  to  $t$ ). We do not need to impose a specific functional form on this relationship. In particular, we will allow each institution to respond differently to incoming funds. The only condition we impose is that each institution only pays out a proportion of its incoming funds. Formally, it entails that transfers do not decrease as incoming payments rise and that its slope is bounded above by 1 everywhere. Then, outgoing

transfers made by institution  $i$  at time  $t$  are given by:

$$y_t^i = f^i(x_t^i, \theta_t)$$

where  $\theta_t = (b_t, c_t)$  and  $b_t$  represents the profile of balances  $b_t^i$  and  $c_t$  is the profile of remaining credit  $c_t^i$ . Outgoing payments made by institution  $i$  will depend on incoming funds, which in turn depends on all payments sent over the payment system. Then, for every member in the payment system we have:

$$y_t^i = f^i(x_t^i(y_{t-1}), \theta_t) \quad i = 1, \dots, n$$

This system can be written as:

$$y_t = F(y_{t-1}, \theta_t)$$

where  $y_t = [y_t^1, y_t^2, \dots, y_t^n]^\top$  and  $F = [f^1, f^2, \dots, f^n]^\top$ .

The task of determining payment flows in a financial system thus entails solving for a consistent set of payments - that is, solving a fixed point problem of the mapping  $F$ . We will show that our problem has a well-defined solution and that the set of payments can be determined uniquely as a function of the underlying parameters of the payment system. We will organise the proof in two steps. Step 1 shows the existence of at least one fixed point of the mapping  $F$ . We will show uniqueness in Step 2. Before we proceed, we revise some concepts which will help us formalise our argument.

**Lemma 1.** (*Mean Value Theorem*) *Let  $f$  be a function which is differentiable on*

a closed interval  $[a, b]$ . Then, there exists a point  $c \in (a, b)$  such that

$$f(b) - f(a) = f'(c)(b - a)$$

A *complete lattice* is a partially ordered set  $(Y, \leq)$  which satisfies that every non-empty subset  $S \subseteq Y$  has both a least upper bound (join)  $\sup(S)$  and a greatest lower bound (meet)  $\inf(S)$ . In our payments setting, we can define a complete lattice  $(Y, \leq)$  as formed by a non-empty set of outgoing payments  $Y$  and the binary relation  $\leq$ . Every subset  $S$  of the payment flows  $Y$  has a greatest lower bound (flows are non-negative) and a least upper bound which we will denote by  $\bar{y}_i$ .  $\bar{y}_i$  represents the maximum flow of payments institution  $i$  can send through the payment system. This condition can be understood as a maximum flow capacity due to some technological limitations of the networks and communication systems used by the institutions to receive and process transfer orders. We have:

$$Y = [0, \bar{y}_1] \times [0, \bar{y}_2] \times \dots \times [0, \bar{y}_n]$$

The relation  $\leq$  formalises the notion of an ordering of the elements of  $Y$  such that  $y \leq y'$  when  $y_i \leq y'_i$  for all the components  $i$  and  $y_k < y'_k$  for some component  $k$ . Having introduced the Mean Value Theorem and the concept of complete lattice we now turn to our two-step proof.

**Step 1.** *There exists at least a fixed point of the mapping  $F$ .*

**Lemma 2.** *(Tarski (1955) Fixed Point Theorem) Let  $(Y, \leq)$  be a complete lattice and  $F$  be a non-decreasing function on  $Y$ . Then there are  $y^*$  and  $y_*$  such that  $F(y^*) = y^*$ ,  $F(y_*) = y_*$ , and for any fixed point  $y$ , we have  $y_* \leq y \leq y^*$ .*

In our payments problem,  $(Y, \leq)$  is a complete lattice and since outgoing payments made by institution  $i$  do not decrease as incoming funds rise, i.e.  $f^i$  is a non-decreasing function, then  $F = [f^1, f^2, \dots, f^n]^\top$  is non-decreasing on  $Y$ . Our setting hence satisfies the conditions of the Tarski's Theorem and as a result there exists *at least* one fixed point of the mapping  $F$ . Moreover, in Step 2 we will show that the fixed point is unique. This result is summarized in Theorem 1.

**Step 2.** *There exists a unique fixed point of the mapping  $F$ .*

**Theorem 1.** *There exists a unique profile of payments flows  $y_t$  that solves  $y_t = F(y_{t-1}, \theta_t)$ .*

**Proof.**

$F$  is a non-decreasing function on a complete lattice  $(Y, \leq)$ . Then, by Tarski's Fixed Point Theorem (Lemma 2),  $F$  has a largest  $y^*$  and a smallest  $y_*$  fixed point. Let us consider, contrary to Theorem 1, that there exist two distinct fixed points such that  $y_i^* \geq y_{i*}$  for all components  $i$  and  $y_k^* > y_{k*}$  for some component  $k$ . Denote by  $x_i^*$  the payments received by institution  $i$  evaluated at  $y_i^*$  and by  $x_{i*}$  the payments received by institution  $i$  evaluated at  $y_{i*}$ . By the Mean Value Theorem (Lemma 1), for any differentiable function  $f$  on  $[x_{i*}, x_i^*]$ , there exists a point  $z \in (x_{i*}, x_i^*)$  such that

$$f(x_i^*) - f(x_{i*}) = f'(z)(x_i^* - x_{i*})$$

We have assumed that the slope of the outgoing payments is bounded above



by 1 everywhere ( $\frac{\partial f^i}{\partial x_i} < 1$  everywhere). Hence,

$$\left\{ \begin{array}{l} y_1^* - y_{1*} = f^1(\bar{y}_1, x_1^*) - f^1(\bar{y}_1, x_{1*}) \leq x_1^* - x_{1*} \\ y_2^* - y_{2*} = f^2(\bar{y}_2, x_2^*) - f^2(\bar{y}_2, x_{2*}) \leq x_2^* - x_{2*} \\ \vdots \\ y_k^* - y_{k*} = f^k(\bar{y}_k, x_k^*) - f^k(\bar{y}_k, x_{k*}) < x_k^* - x_{k*} \\ \vdots \\ y_n^* - y_{n*} = f^n(\bar{y}_n, x_n^*) - f^n(\bar{y}_n, x_{n*}) \leq x_n^* - x_{n*} \end{array} \right.$$

Re-arranging the previous system of equations we get

$$\left\{ \begin{array}{l} x_{1*} - y_{1*} \leq x_1^* - y_1^* \\ x_{2*} - y_{2*} \leq x_2^* - y_2^* \\ \vdots \\ x_{k*} - y_{k*} < x_k^* - y_k^* \\ \vdots \\ x_{n*} - y_{n*} \leq x_n^* - y_n^* \end{array} \right.$$

Summing across we have

$$\sum_{i=1}^n x_{i*} - \sum_{i=1}^n y_{i*} < \sum_{i=1}^n x_i^* - \sum_{i=1}^n y_i^*$$

so that the total value of the balances including credit capacity is strictly larger under  $x^*$ , which is impossible. Therefore, there cannot exist two distinct fixed points and as a result  $y^* = y_*$ .

□

Although uniqueness is relevant to our analysis of payment systems, our key insights stem from the comparative statics results due to Milgrom and Roberts (1994).

## 2.1 COMPARATIVE STATICS

**Theorem 2.** *Let  $y_t^*(\theta_t)$  be the unique fixed point of the mapping  $F$ . If for all  $y_t \in Y$ ,  $F$  is increasing in  $\theta_t$ , then  $y_t^*(\theta_t)$  is increasing in  $\theta_t$ .*

**Proof.**

Let  $F$  be monotone non-decreasing and  $Y$  a complete lattice. From Tarski's Fixed Point Theorem (Lemma 2) and Theorem 1 there exists a unique fixed point  $y_t^*(\theta_t)$  of the mapping  $F$ . For the simplicity of the argument, let us suppress the subscript  $t$ . Define the set  $S(\theta)$  as

$$S(\theta) = \{y | F(y, \theta) \leq y\}$$

and define  $y^*(\theta) = \inf S(\theta)$ . Since  $F$  is non-decreasing in  $\theta$ , the set  $S(\theta)$  becomes more exclusive as  $\theta$  increases. Hence,  $y^*(\theta)$  is a non-decreasing function of  $\theta$ . Formally, if  $F$  is increasing in  $\theta$ , then for  $\theta' > \theta$ ,  $F(\theta') > F(\theta)$  and

$$S(\theta') = \{y | F(y, \theta') \leq y\} \subset S(\theta)$$

Thus,

$$y^*(\theta') = \inf S(\theta') > \inf S(\theta) = y^*(\theta)$$

Therefore, if  $F$  is increasing in  $\theta$ , the fixed point  $y^*(\theta)$  is increasing in  $\theta$  too.

□

### 3 PAYMENT SYSTEMS

Payment and securities settlement systems are essential components of the financial systems and vital to the stability of any economy. A key element of the payment system is the interbank payment system that allows funds transfers between entities. Large-value (or wholesale) funds transfer systems are usually distinguished from retail systems. Retail funds systems transfer large volumes of payments of relatively low value while wholesale systems are used to process large-value payments. Interbank funds transfer systems can also be classified according to their settlement process. The settlement of funds can occur on a net basis (net settlement systems) or on a transaction-by-transaction basis (gross settlement systems). The timing of the settlement allows another classification of these systems depending on whether they settle at some pre-specified settlement times (designated-time (or deferred) settlement systems) or on a continuous basis during the processing day (real-time settlement systems).

A central aspect of the design of large-value payment systems is the trade-off between liquidity and settlement risk. Real-time gross settlement systems are in constant need of liquidity to settle payments in real time while net settlement systems are very liquid but vulnerable to settlement failure<sup>1</sup>. In the last twenty

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<sup>1</sup>Zhou (2000) discusses the provision of intraday liquidity by a central bank in a real-time gross settlement system and some policy measures to limit the potential credit risk.

years, large-value payments systems have evolved rapidly towards greater control of credit risk<sup>2</sup>.

In the United States, the two largest large-value payment systems are the Federal Reserve Funds and Securities Services (Fedwire) and the Clearing House Interbank Payments System (CHIPS). CHIPS, launched in 1970, is a real-time, final payment system for US dollars that uses bi-lateral and multi-lateral netting to clear and settle business-to-business transactions. CHIPS is a bank-owned payment system operated by the Clearing House Interbank Payments Company L.L.C. whose members consist of 46 of the world's largest financial institutions. It processes over 300,000 payments on an average day with a gross value of \$1.5 trillion.

Fedwire is a large-dollar funds and securities transfer system that links the twelve Banks of the Federal Reserve System<sup>3</sup>. The Fedwire funds transfer system, which we will discuss in more detail below, is a real-time gross settlement system, developed in 1918, that settles transactions individually on an order-by-order basis without netting. The average daily value of transactions exceeded \$2 trillion in 2005 with a volume of approximately 527,000 daily payments. Settlement of most US government securities occurs over the Fedwire book-entry security system, a real-time delivery-versus-payment gross settlement system that allows the immediate and simultaneous transfer of securities against payments. More than 9,100 participants hold and transfer US Treasury, US government agency securities and securities issued by international organisations such as the World Bank. In 2005 it processed over 89,000 transfers a day with an average daily value of \$1.5 trillion.

Figure 1 depicts the evolution of the average daily value and volume of transfers

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<sup>2</sup>Martin (2005) analyses the recent evolution of large-value payment systems and the compromise between providing liquidity and settlement risk. See also Bech and Hobijn (2006) for a study on the history and determinants of adoption of real-time gross settlement payment systems by central banks across the world.

<sup>3</sup>See Gilbert et al. (1997) for an overview of the origins and evolution of Fedwire.

sent over CHIPS and Fedwire.

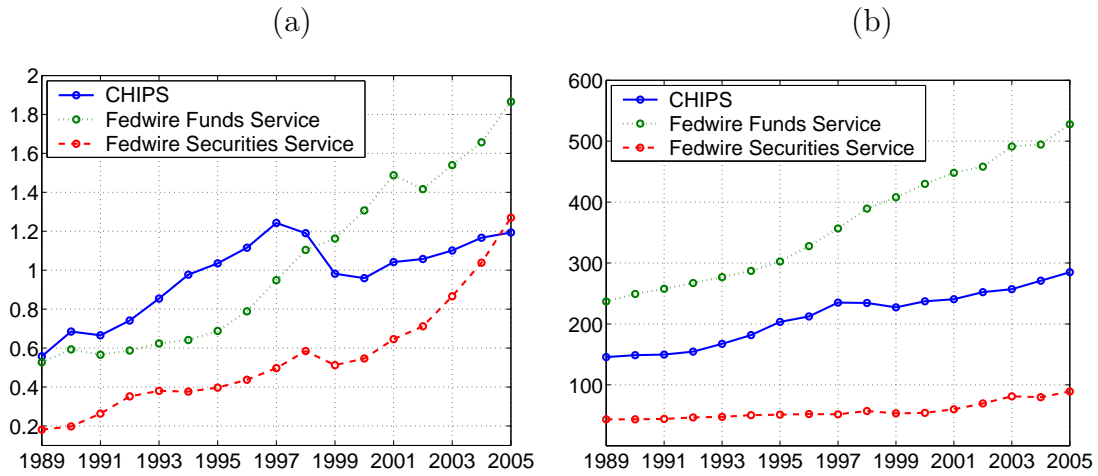


Figure 1: Average daily value (a) (\$ trillion) and volume (b) (thousands) of transactions over CHIPS, Fedwire Funds Service and Fedwire Securities Service, 1989-2005. Source: The Federal Reserve Board and CHIPS.

### 3.1 FEDWIRE FUNDS SERVICE

Fedwire Funds Service, owned and operated by the Federal Reserve Banks, is an electronic payment system that allows participants to make same-day final payments in central bank money. An institution that maintains an account at a Reserve Bank can generally become a Fedwire participant. Approximately 9,400 participants are able to initiate and receive funds transfers over Fedwire. When using the Fedwire Funds Service, a sender instructs a Federal Reserve Bank to debit its own Federal Reserve account for the amount of the transfer and to credit the Federal Reserve account of another participant.

The Fedwire Funds Service operates 21.5 hours each business day (Monday through Friday), from 9.00 p.m. Eastern Time (ET) on the preceding calendar

day to 6.30 p.m. ET<sup>4</sup>. It was expanded in December 1997 from ten hours to eighteen hours (12:30 a.m. - 6:30 p.m.) and again in May 2004 to accommodate the twenty-one and a half operating hours. This change increased overlap of Fedwire's operating hours with foreign markets and helped reduce foreign exchange settlement risk.

A Fedwire participant sending payments is required to have sufficient funds, either in the form of account balance or overdraft capacity, or the payment order may be rejected. The Federal Reserve imposes a minimum level of reserves, which can be satisfied with vault cash<sup>5</sup> and balances deposited in Federal Reserve accounts, neither of which earn interest. A Fedwire participant may also commit itself or be required to hold balances in addition to any reserve balance requirement (clearing balances). Clearing balances earn no explicit interest but implicit credits that may offset the cost of Federal Reserve services. Fedwire participants thus tend to optimise the size of the balances in their Federal Reserve accounts<sup>6</sup>.

When an institution has insufficient funds in its Federal Reserve account to cover its debits, the institution runs a negative balance or *daylight overdraft*. Daylight overdrafts result because of a mismatch in timing between incoming funds and outgoing payments (McAndrews and Rajan (2000)). Each Fedwire participant may establish (or is assigned) a maximum amount of daylight overdraft known as *net debit cap*<sup>7</sup>. An institution's net debit cap is a function of its capital measure. Specifically, it is defined as a cap multiple times its cap capital measure, where

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<sup>4</sup>A detailed description of Fedwire Funds Service operating hours can be found at [www.frbservices.org/Wholesale/FedwireOperatingHours.html](http://www.frbservices.org/Wholesale/FedwireOperatingHours.html).

<sup>5</sup>Vault cash refers to U.S. currency and coin owned and held by a depository institution.

<sup>6</sup>Bennett and Peristiani (2002) find that required reserve balances in Federal Reserve accounts have declined sharply while vault cash applied against reserve requirements has increased. They argue that reserve requirements have become less binding for US commercial banks and depository institutions.

<sup>7</sup>Appendix A briefly reviews the evolution of net debit caps and describes the different cap categories and associated cap multiples.

the cap multiple is determined by the institution's cap category. An institution's capital measure varies over time while its cap category does not normally change within a one-year period. Each institution's cap category is considered confidential information and hence it is unknown to other Fedwire participants (Federal Reserve (2005), Federal Reserve (2006d)).

In 2000 the Federal Reserve Board's analysis of overdraft levels, liquidity patterns, and payment system developments revealed that although approximately 97 percent of depository institutions with positive net debit caps use less than 50 percent of their daylight overdraft capacity, a small number of institutions found their net debit caps constraining (Federal Reserve (2001)). To provide additional liquidity, the Federal Reserve now allows certain institutions to pledge collateral to gain access to daylight overdraft capacity above their net debit caps. The maximum daylight overdraft capacity is thus defined as the sum of the institution's net debit cap and its collateralised capacity.

To control the use of intraday credit, the Federal Reserve began charging daylight overdraft fees in April 1994. The fee was initially set at an annual rate of 24 basis points and it was increased to 36 basis points in 1995<sup>8</sup>. At the end of each Fedwire operating day the end-of-minute account balances are calculated. The average overdraft is obtained by adding all negative end-of-minute balances and dividing this amount by the total number of minutes in an operating day (1291 minutes). An institution's daylight overdraft charge is defined as its average overdraft multiplied by the effective daily rate (minus a deductible). Table 3 presents an example of the calculation of a daylight overdraft charge. An institution incurring daylight overdrafts of approximately \$3 million every minute during a Fedwire operating day would face an overdraft charge of \$6.58.

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<sup>8</sup>Fedwire operates 21.5 hours a day, hence the effective annual rate is 32.25 basis points ( $36 \times \frac{21.5}{24}$ ) and the effective daily rate is 0.089 basis points ( $32.25 \times \frac{1}{360}$ ).

At the end of the operating day, a Fedwire participant with a negative closing balance incurs overnight overdraft. An overnight overdraft is considered an unauthorized extension of credit. The rate charged on overnight overdrafts is generally 400 basis points over the effective federal funds rate. If an overnight overdraft occurs, the institution will be contacted by the Reserve Bank, it will be required to hold extra reserves to make up reserve balance deficiencies and the penalty fee will be increased by 100 basis points if there have been more than three overnight overdraft occurrences in a year. The Reserve Bank will also take other actions to minimise continued overnight overdrafts (Federal Reserve (2006a)).

## 4 AN EXAMPLE OF PAYMENT SYSTEM

Consider a network of four banks. Each bank sends and receives payments from other members of the payment system. The payment system opens at 9.00 p.m. on the preceding calendar day and closes at 6.30 p.m. Every bank begins the business day with a positive balance at its central bank account and may incur daylight overdrafts to cover negative balances up to its net debit cap. For simplicity we assume initial balances and net debit caps of equal size. The expected value of bank  $i$ 's outgoing payments equals the expected value of its incoming funds to guarantee that no bank is systematically worse off. Each member of the payment system is subject to idiosyncratic shocks which determine its final payments.

Following McAndrews and Potter (2002) we define outgoing transfers as a linear function of the payments a bank receives from all other banks. Specifically, at every minute of the operational day, bank  $i$  pays at most 80 percent of its cumulative receipts and a proportion of its reserves and credit capacity (which we fix at 10 percent of the bank's net debit cap). We assume banks settle obligations whenever



they have sufficient funds. When the value of payments exceeds 80 percent of a bank's incoming funds and 10 percent of its net debit cap, payments are placed in queue. Queued payments are settled as soon as sufficient funds become available<sup>9</sup>.

When banks use more than 50 percent of their own daylight overdraft capacity<sup>10</sup>, they become concerned about liquidity shortages and reduce the value of their outgoing transfers. Inspired by McAndrews and Potter's estimates of the slope of the reaction function of banks during the September 11, 2001, events, we assume that banks would then pay at most 20 percent of their incoming funds. In that case, bank  $i$  may face one of two possible scenarios. Its balance may become positive (it has been receiving funds from all other banks according to the 80 percent rule while it has been paying out only 20 percent of its incoming transfers). The "episode" would be over and bank  $i$  would return to normal conditions. However, it may also be possible that despite reducing the amount of outgoing payments its demand for daylight overdraft continues to rise. Bank  $i$  would incur negative balances up to its net debit cap. At that time, it would stop using intraday credit to make payments and any incoming funds would be devoted to settle queued payments and to satisfy outgoing transfers at the 20 percent rate per minute.

We first introduce the baseline model and then we analyse disrupted conditions and potential implications of different policies.

## Baseline Model

We consider a payment system as the one just described above. The results of our first set of simulations are summarised in Figure 2. Each bank starts the

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<sup>9</sup>To avoid excessive fluctuations we consider that if at any time bank  $i$ 's use of reserves and credit capacity is below the 10 percent threshold, bank  $i$  will devote its spare capacity to settle queued payments. Otherwise, payments will remain in queue.

<sup>10</sup>According to a Federal Reserve Board's review, in 2000, 97 percent of depository institutions with positive net debit caps use less than 50 percent of their daylight overdraft capacity (Federal Reserve (2001)).

operating day with a positive balance in their central bank accounts (Figure 2(a)). The value of payments by time of the day is depicted in Figure 2(b). It follows the pattern of the value of transactions sent over the Fedwire Funds Service<sup>11</sup>. Before 8 a.m. there is almost no payment activity, from then on the value of payments increases steadily and it peaks around 4.30 p.m. and again around 5.15 p.m. The market closes at 6.30 p.m.

In this exercise, we set net debit caps equal to 100. None of the four banks' balances falls below  $-50$  (50 percent of their cap) and hence during this business day every bank pays at most 80 percent of received funds (Figure 2(f)). Just after 1 p.m. bank *C* starts running negative balances and thus incurring daylight overdrafts (Figure 2(c)). Overdrafts peak at 5.10 p.m., shortly after bank *C* places some payments in queue (Figure 2(e)). These queued payments are settled at the next minute. After that bank *C* begins receiving more payments than payment orders. At 5.25 p.m. it runs a positive balance and ends the day with a positive balance (its closing balance more than doubles its opening balance).

Banks *A*, *B* and *D* also incur daylight overdrafts and delayed payments (Figures 2(c) and 2(e)). Banks *A* and *D* reach the end of the operating day with positive balances while *B* runs a negative closing balance and it will need to "sweep" deposits from another account to its account at the central bank to avoid an overnight overdraft charge (Figure 2(a)).

### **Defaulting Bank Model**

Let us consider a scenario where one bank (Bank *A*) suffers a severe adverse shock and only pays 20 percent of the funds it receives (and up to 10 percent of its net debit cap per minute). Banks *B*, *C* and *D* initially behave as in the baseline

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<sup>11</sup>See McAndrews and Rajan (2000) (Chart 3) and Coleman (2002) (Chart 1).

model. Results are summarised in Figure 3.

Just before 1 a.m., banks  $B$ ,  $C$  and  $D$  begin running negative balances (Figure 3(a)) and incurring daylight overdrafts (Figure 3(c)). Around 10.15 a.m. their daylight overdrafts exceed half of their caps (Figure 3(a)) and they start paying out at most 20 percent of the funds they receive (Figure 3(f)). At noon, banks  $B$ ,  $C$  and  $D$ 's daylight overdrafts reach their net debit caps (Figure 3(c)), they start placing payments in queue (Figure 3(e)) and total payments are finally disrupted (Figure 3(b)). It is interesting to highlight the size of the banks' balances compared to the baseline model (Figures 2(a) and 3(a)), the disruption of payments (Figure 3(b)) and the enormous use of intraday credit (approximately 500 times larger than overdrafts in the baseline model. Figures 2(d) and 3(d)).

### **Timing Miscoordination Model**

We are also interested in the consequences of a miscoordination in the timing of payments. In the baseline model we assumed banks perfectly synchronise their payment activity, i.e., the value of the payments made by every bank exhibited the same pattern. Now we consider that banks experience a five-minute delay with respect to each others. Figure 4 summarises our findings.

Bank  $A$  starts sending payments first.  $B$  begins five minutes after  $A$ , then  $C$  after  $B$  and  $D$  will be the last one. As a result, there will be a mismatch in timing between the settlement of payments owned and the settlement of payments due. Initially, bank  $A$  makes more payments that it receives (Figure 4(a)) and hence it incurs daylight overdrafts (Figure 4(c)). The bank that pays first will demand the largest amount of intraday credit. Then, the bank after the first one and so on (Figure 4(c)). This pattern persists across business days (simulations).

Banks  $A$  and  $B$  end the operating day with a negative balance while banks

$C$  and  $D$  run positive closing balances. We could think this is a consequence of the time mismatch. However, this is not the case. In this exercise, payments are delayed but the expected value of outgoing funds and incoming payments are still equal. A different business day is presented in Figure 5.

A miscoordination in payment thus induces an increase in the size of balances at the central bank accounts (Figures 2(a) and 5(a)) and a more intense use of the intraday credit (Figures 2(d) and 5(d)). Also, some payments are placed in queue (Figure 5(e)) and the some banks reduce the relation between outgoing and incoming transfers (Figure 5(f)).

### **Multiple Synchronisation Model**

To economise on the use of intraday credit, a potential operational change in settlement systems which is being considered (Federal Reserve (2006b)) is the possibility of developing multiple settlement periods. An example of such policy could be the establishment of two synchronisation periods, one late in the morning and then another early in the afternoon peak, during which banks would not be charge for any overdrafts (McAndrews and Rajan (2000)).

In our exercise (Figure 6) we conclude that introducing multiple synchronisation periods does not alter significantly the size of banks' balances at their central bank accounts or the ratio between outgoing and incoming funds. On the contrary, it reduces the use of daylight overdraft (Figures 2(d) and 6(d)) and the amount of payments in queue (Figures 2(e) and 6(e)).

### **Morning Synchronisation Model**

We now analyse the consequences of having one synchronisation period late in the morning rather than the existing one (late in the afternoon). The main

difference lies in the use of daylight overdraft. Figure 7 summarises our findings.

Let us focus on the bank, which ended the business day in the baseline model with a negative balance (Figure 2(a)): bank  $B$ . In the morning synchronisation model, bank  $C$  faces a similar scenario but shifted in time (Figure 7(a)). After the peak in payment activity, bank  $C$  runs a negative balance. It incurs daylight overdraft until the end of the operating day. As a result, banks will prefer to synchronise close to the end of the business day (Figures 2(d) and 7(d)).

## 5 CONCLUDING REMARKS

The focus of the paper has been on the role of liquidity in a flow system. We have argued for the importance of the interdependence of the flows in high-value payment systems. High-value payment systems, such as the settlement of accounts receivable and payable among industrial firms, or the interbank payment systems that constitute the backbone of the modern financial system, link banks or firms together into a tightly knit system. Banks and firms rely heavily on incoming funds to make their payments and as such, their ability to execute payments will affect other participants' capability to send out funds. Changes in outgoing transfers will affect incoming funds and incoming funds changes will affect outgoing transfers. The loop thus created may generate amplified responses to any shocks to the high-value payment system.

We have drawn from the literature on lattice-theoretic methods to solve for the unique fixed point of an equilibrium mapping in high-value payment systems. Using numerical simulations based on simple decision rules which replicate the observed data on the Fedwire payment system in the U.S., we have performed comparative statics analyses on changes to the environment of this payment system.

Our framework also allowed simulations of counterfactual “what if” scenarios of disturbances that allow calibration of the scale of shocks that lead to gridlock and systemic breakdown, as well as the role of the central bank in mitigating such risks.

## A NET DEBIT CAPS

In 1985, the Federal Reserve Board developed a payment system risk policy on risks in large-dollar wire transfer systems. The policy introduced four categories of limits (net debit caps) on the maximum amount of daylight overdraft credit that the Reserve Banks extended to depository institutions: *high*, *above average*, *average* and *zero*. In 1987 a new net debit cap (*de minimis*) was approved. It was intended for depository institutions that incur relatively small overdrafts. The Board incorporated a sixth cap class (*exempt-from-filing*) and modified the existing *de minimis* cap multiple in 1990. The *de minimis* cap multiple was then increased in 1994 when daylight overdraft fees were introduced<sup>12</sup>. A brief summary<sup>13</sup> of the actual cap categories and their associated cap multiples for maximum overdrafts on any day (single-day cap) and for the daily maximum level averaged over a two-week period (two-week average cap) are presented in Tables 1 and 2.

## B EXAMPLE DAYLIGHT OVERDRAFT CHARGE CALCULATION

Table 3 contains an example of the calculation of a daylight overdraft charge (Federal Reserve (2006d)).

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<sup>12</sup>For a comprehensive study of the history of Federal Reserve daylight credit see Coleman (2002). See also Federal Reserve (2005).

<sup>13</sup>For a detailed reference, see Federal Reserve (2006c).

Table 1: Brief definition of cap categories.

Cap Category	Chosen by institutions that	Requirements
<i>High</i> <i>Above Average</i> <i>Average</i>	Regularly incur daylight overdrafts in excess of 40 percent of their capital.  They are referred to as “self-assessed”.	Self-assessment of own creditworthiness, intraday funds management, customer credit and operating controls and contingency procedures. Each institution’s board of directors must review the self-assessment and recommend a cap category at least once in each twelve-month period.
<i>De minimis</i>	Incur relatively small daylight overdrafts.	Board-of-directors resolution approving use of daylight credit up to de minimis cap at least once in each 12-month period.
<i>Exempt-from-filing</i>	Only rarely incur daylight overdrafts.	Exempt from performing self-assessments and filing board-of-directors resolutions.
<i>Zero</i>	Do not want to incur daylight overdrafts and associated fees. A Reserve Bank may assign a zero cap to institutions that may pose special risks.	

Table 2: Net debit cap multiples of capital measure.

Cap Category	Single Day	Two-week Average
<i>High</i>	2.25	1.50
<i>Above Average</i>	1.875	1.125
<i>Average</i>	1.125	0.75
<i>De minimis</i>	0.4	0.4
<i>Exempt-from-filing</i> <sup>a</sup>	$\min\{\$10 \text{ million}, 0.2\}$	$\min\{\$10 \text{ million}, 0.2\}$
<i>Zero</i>	0.0	0.0

<sup>a</sup>The net debit cap for the *exempt-from-filing* category is equal to the lesser of \$10 million or 0.20 multiplied by a capital measure.



Table 3: Daylight Overdraft Charge

Example of Daylight Overdraft Charge Calculation <sup>a</sup>
<p><b>Policy parameters</b>            Official Fedwire day = 21.5 hours            Deductible percentage of capital = 10%            Rate charged for overdrafts = 36 basis points (annual rate)</p> <p><b>Institution's parameters</b>            Risk-based capital = \$50 million            Sum of end-of-minute overdrafts for one day = \$4 billion</p> <p><b>Daily Charge calculation</b>            Effective daily rate = <math>.0036 \times (21.5/24) \times (1/360) = .0000089</math>            Average overdraft = <math>\\$4,000,000,000 / 1291 \text{ minutes} = \\$3,098,373</math>            Gross overdraft charge = <math>\\$3,098,373 \times .0000089 = \\$27.58</math>            Effective daily rate for deductible = <math>.0036 \times (10/24) \times (1/360) = .0000042</math>            Value of the deductible = <math>.10 \times \\$50,000,000 \times .0000042 = \\$21.00</math>  <b>Overdraft charge = <math>27.58 - 21.00 = \\$6.58</math></b></p>

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<sup>a</sup>Federal Reserve (2006d)

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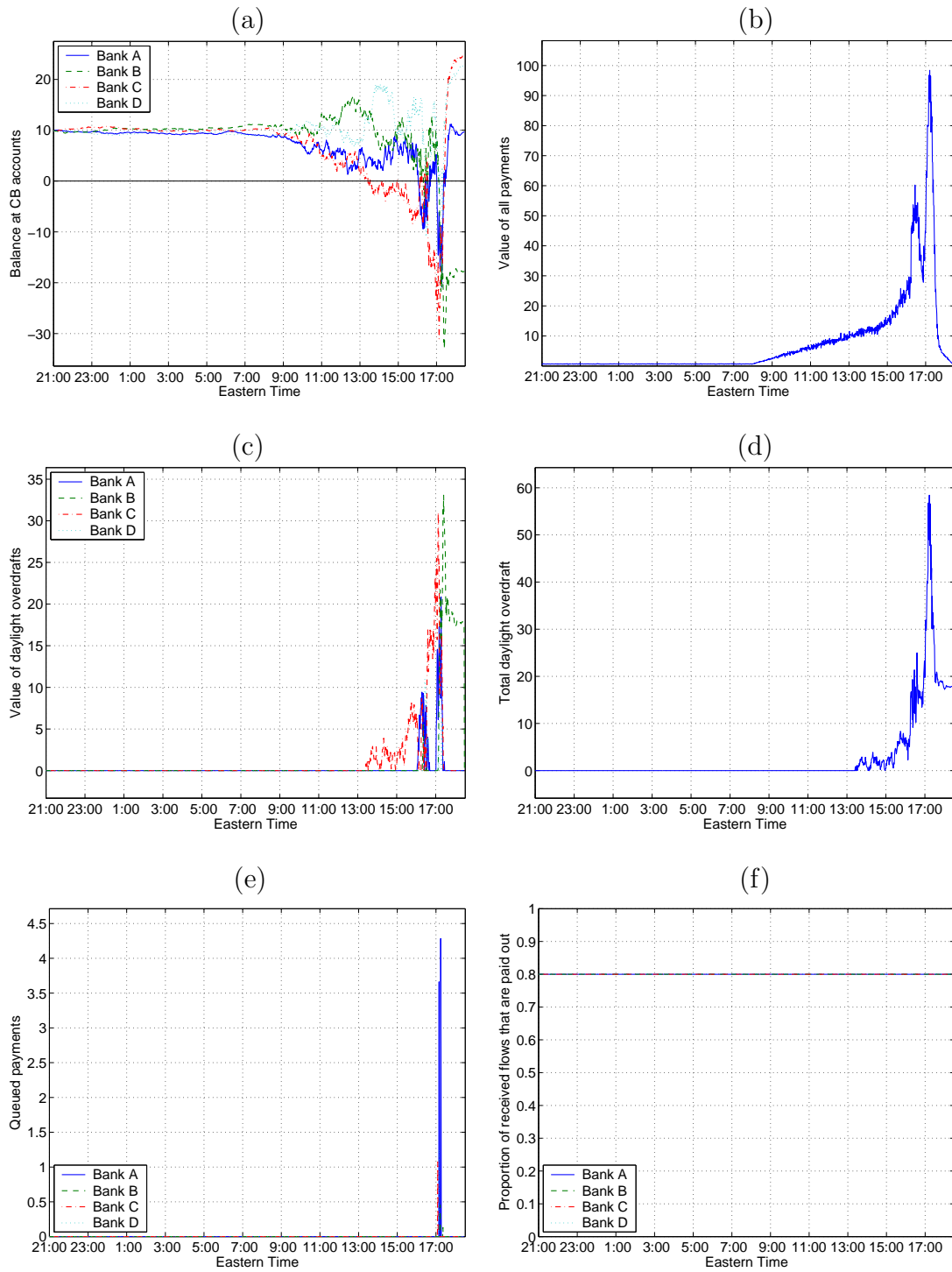


Figure 2: BASELINE MODEL

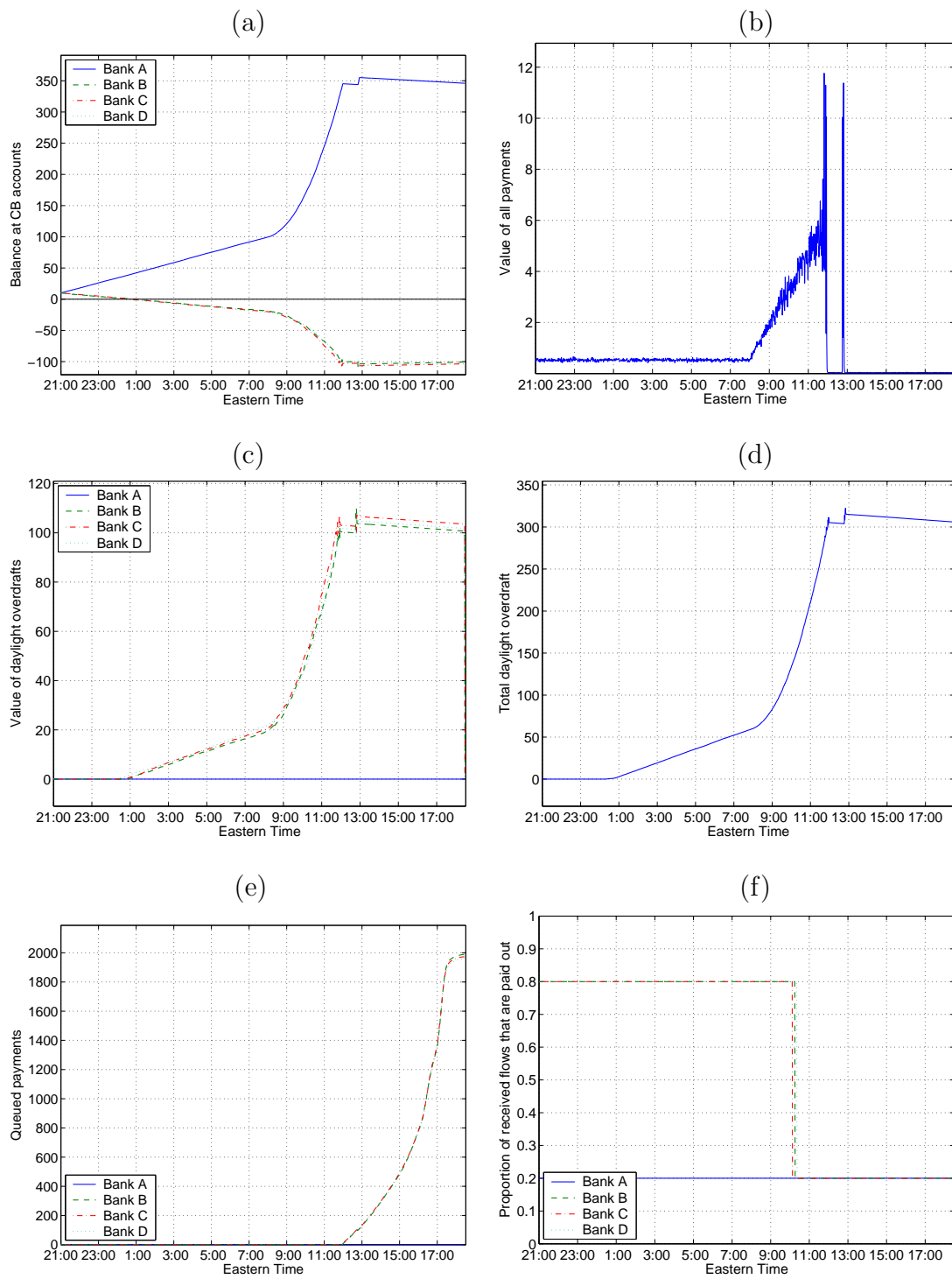


Figure 3: DEFAULTING BANK MODEL

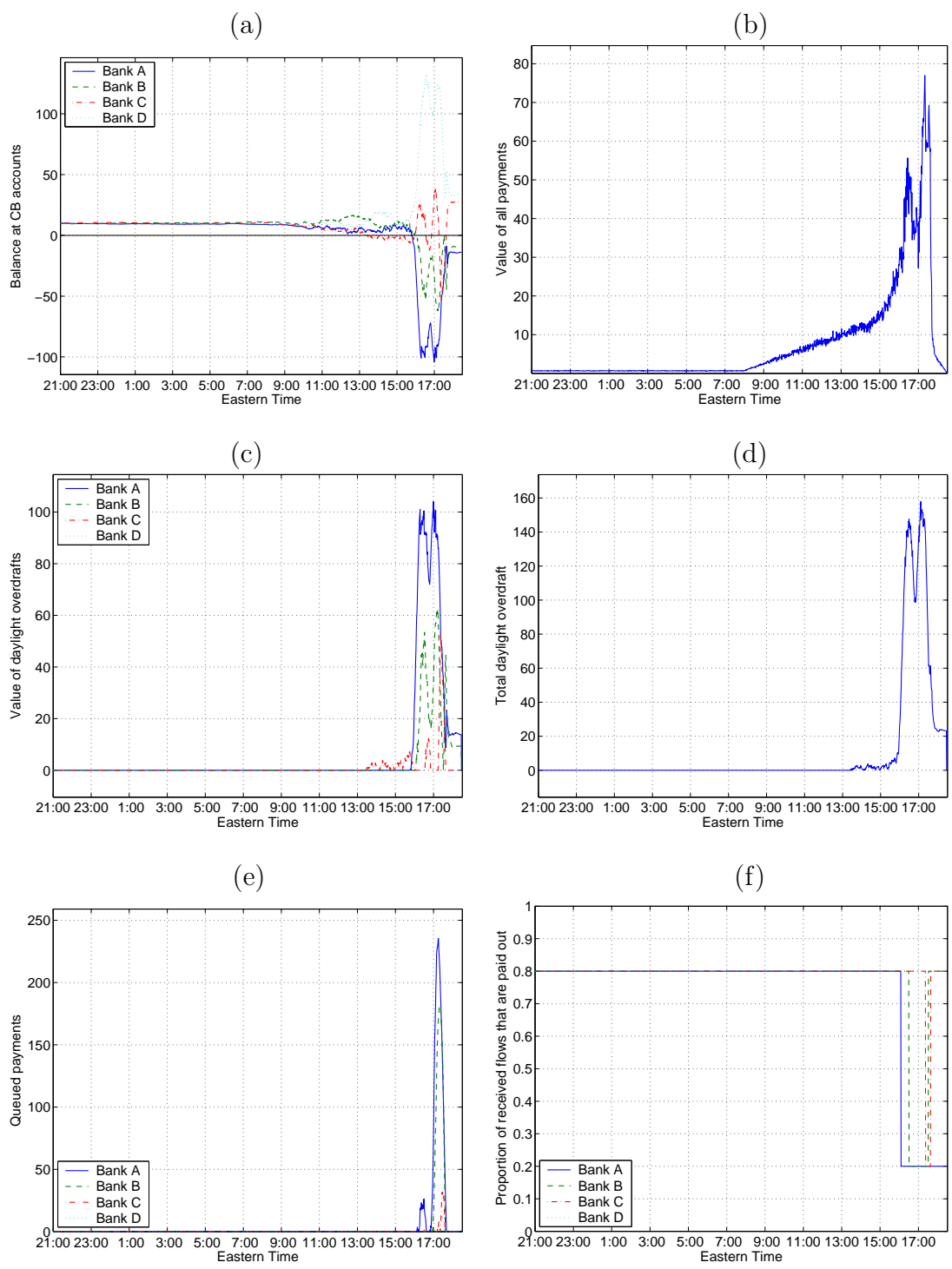


Figure 4: TIMING MISCOORDINATION MODEL

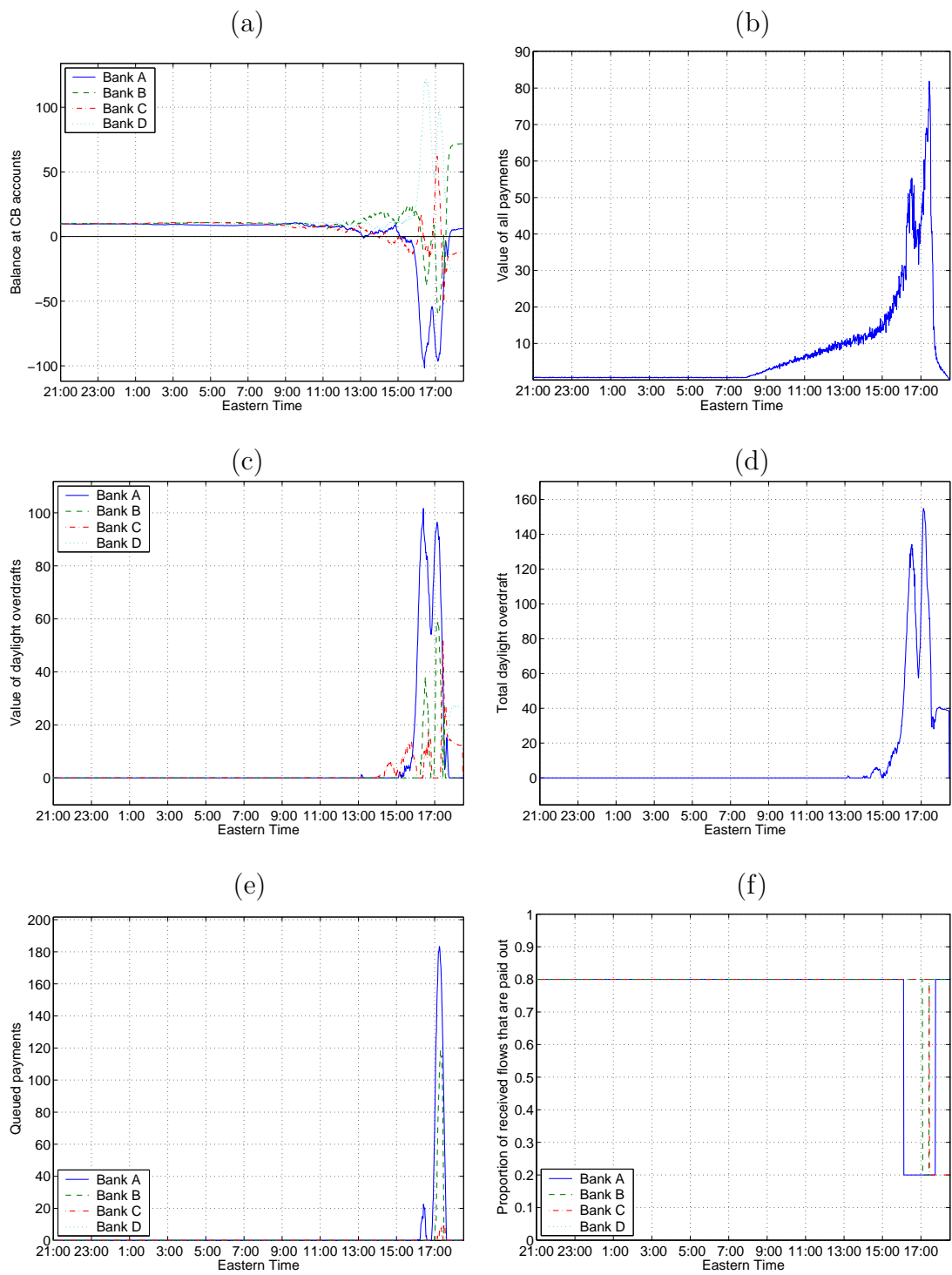


Figure 5: TIMING MISCOORDINATION MODEL (DIFFERENT BUSINESS DAY)

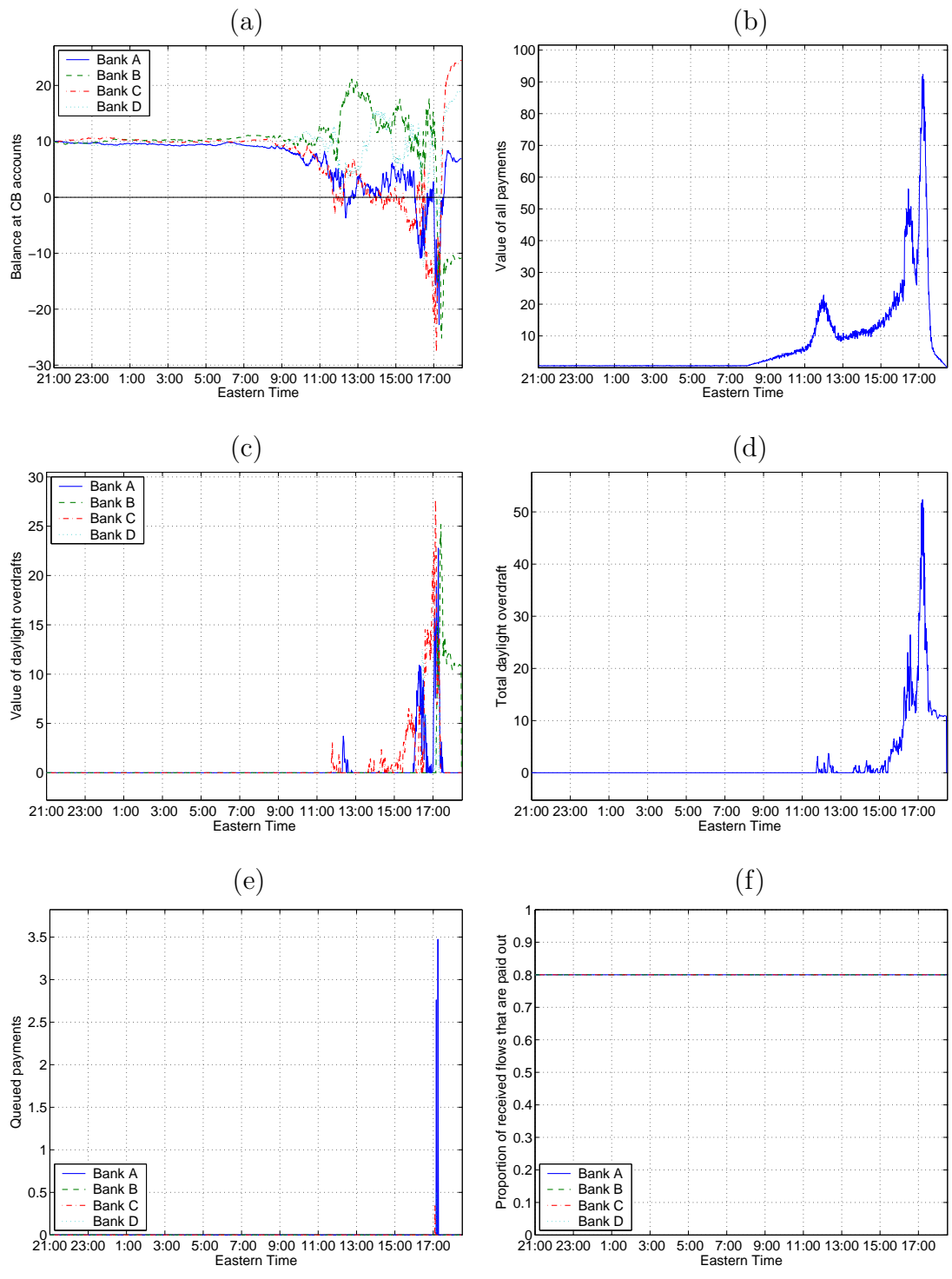


Figure 6: MULTIPLE SYNCHRONISATION MODEL



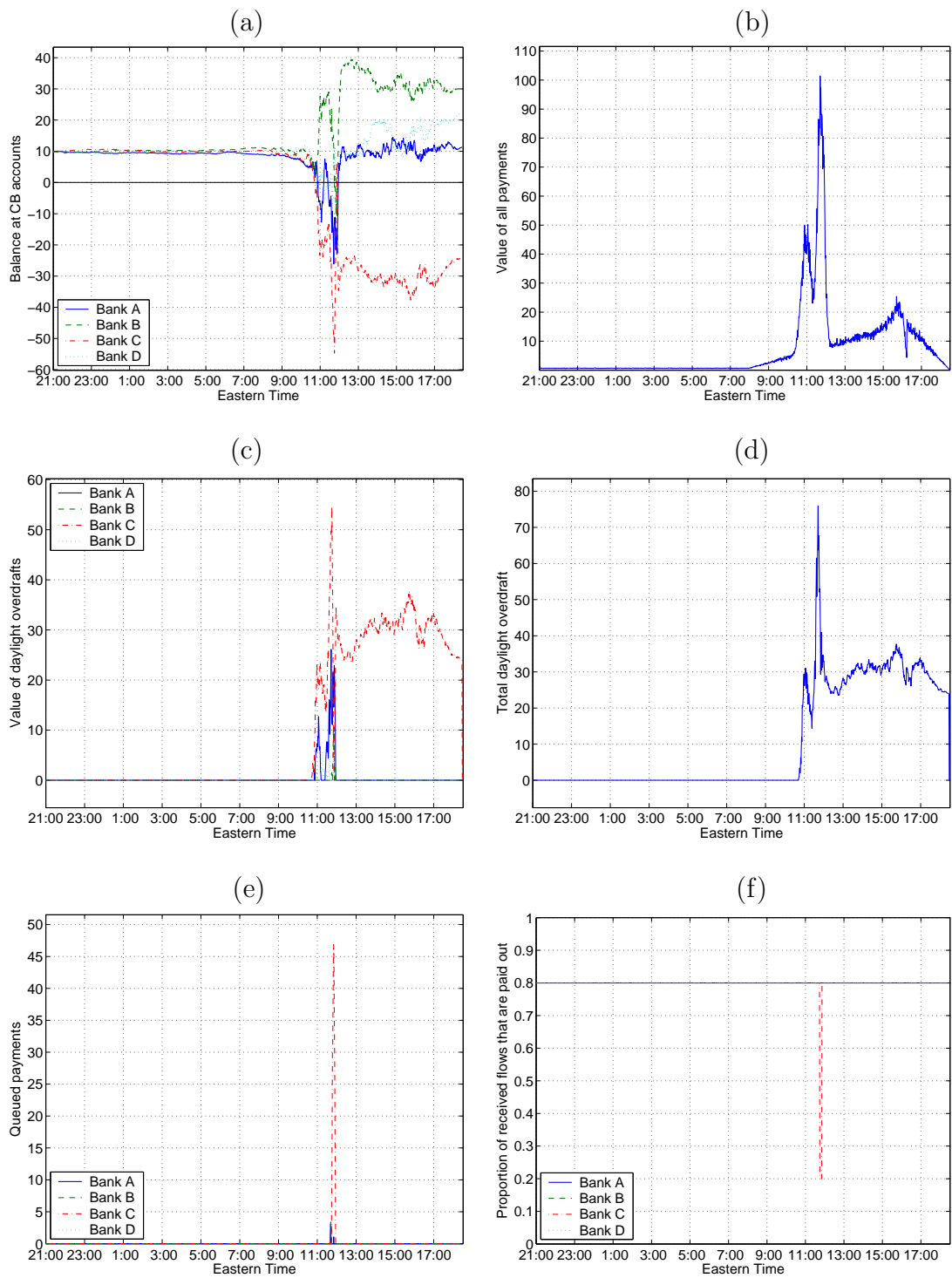


Figure 7: MORNING SYNCHRONISATION MODEL