Ratings Shopping and Asset Complexity: A Theory of Ratings Inflation

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Abstract

Many blame the recent financial market turmoil on ratings agencies. We develop an equilibrium model of the market for ratings and use it to examine popular arguments about the origins of and cures for ratings inflation. In the model, asset issuers can shop for ratings – observe multiple ratings and disclose only the most favorable – before auctioning their assets. When assets are simple, agencies' ratings are similar and the incentive to ratings shop is low. When assets are sufficiently complex, ratings differ enough that an incentive to shop emerges. Thus an increase in the complexity of recently-issued securities could create a systematic bias in disclosed ratings despite the fact that each ratings agency discloses an unbiased estimate of the asset's true quality. Increasing competition among agencies would only worsen this problem. Switching to a investor-initiated ratings system alleviates the bias, but could collapse the market for information.

Life can only be understood backwards; but it must be lived forwards.
Soren Kierkegaard (1813 - 1855)

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Most market observers attribute the recent credit crunch to a confluence of factors: lax screening by mortgage originators, improperly estimated correlation between bundled assets, market-distorting regulations, rating agency conflicts of interest, and a rise in the popularity of new asset classes whose risks were difficult to evaluate.[1] This paper investigates the mis-rating of structured credit products, widely cited as one contributor to the crisis. Our main objective is to critically examine two arguments about why ratings problems arose and show how combining the two could produce ratings bias that would be unanticipated by rational, but imperfectly informed, investors.

One argument focuses on asset issuers who shop around for the highest ratings. Former chief of Moody's, Tom McGuire, explains:[2]

> The banks pay only if [the ratings agency] delivers the desired rating. . . . If Moody's and a client bank don't see eye-to-eye, the bank can either tweak the numbers or try its luck with a competitor like S&P, a process known as *ratings shopping*.

While the issuer-initiated ratings system has been around since the 1970's, ratings bias only recently emerged as a concern. To argue that it took 30 years to detect the bias is to suggest that learning by financial market participants is unrealistically slow. This raises the question: Is it possible that ratings shopping previously had no or a small effect and that something about the credit market changed to amplify its effect?

A second argument about why credit derivatives were mis-rated attributes the problem to the increasing complexity of assets. As Mark Adelson testified to congress,[3]

> The complexity of a typical securitization is far above that of traditional bonds. It is above the level at which the creation of the methodology can rely solely on mathematical manipulations. Despite the outward simplicity of credit ratings, the

[1] See, for instance, page 1 of the Memorandum for the President from the President's Working Group on financial markets dated March 13, 2008.

[2] Quote from New York Times Magazine, "Triple-A-Failure," April 27, 2008. Other articles making similar arguments include "Why Credit-rating Agencies Blew It: Mystery Solved," available from http://robertreich.blogspot.com/2007/10/they-mystery-of-why-credit-rating.html, "Stopping the Subprime Crisis" New York Times, July 25, 2007, "When It Goes Wrong" The Economist, September 20, 2007, and "Credit and Blame" The Economist, September 6, 2007.

[3] Adelson: Director of structured finance research at Nomura Securities. Testimony before the Committee on Financial Services, U.S. House of Representatives, September 27, 2007. On January 26, 2008, The New York Times quoted the CEO of Moody's, saying " In hindsight, it is pretty clear that there was a failure in some key assumptions that were supporting our analytics and our models." He said that one reason for the failure was that the "information quality" given to Moody's, "both the completeness and veracity, was deteriorating." See also page 10 of the *Summary Report of Issues Identified in the Commission Staff's Examinations of Select Credit Rating Agencies*, United States Securities and Exchange Commission, July 8, 2008.

> inherent complexity of credit risk in many securitizations means that reasonable professionals starting with the same facts can reasonably reach different conclusions.

However, the credit market crisis was not generated by independent ratings errors. Only systematic upward ratings would produce a widespread rise in the prices of credit products. This raises the question: Is it possible that more dispersion in ratings can translate into higher ratings on average?

We show that the combination of an increase in asset complexity and the ability of asset issuers to shop for ratings can produce ratings inflation, even if each ratings agency produces an unbiased rating. We do not argue that the complexity of any given asset increased. Rather, the composition of assets being sold changed so that the more complex type of asset, the structured financial products, particularly those that were mortgage-backed, became more prevalent. For example, while under \$10 billion in structured finance collateralized debt obligations (CDO's) were distributed in 2000, nearly \$200 billion were issued in 2006 (Hu 2007).

The intuition behind our results is as follows: Each ratings agency issues an unbiased forecast of an asset's value. However, if the announced rating is the maximum of all realized ratings, it will be a biased signal of the asset's true quality. The more ratings differ, the stronger are issuers' incentives to selectively disclose (shop for) ratings. For simple assets, agencies issue nearly identical forecasts. Asset issuers then disclose all ratings because more information reduces investors' uncertainty and increases the price they are willing to pay for the asset. For complex assets, ratings may differ, creating an incentive to shop for the best rating. There is a threshold level of asset complexity such that once this threshold is crossed, shopping becomes optimal and ratings inflation emerges. Furthermore, the link between asset complexity and ratings shopping can work in both directions. An issuer who shops for ratings might want to issue an even more complex asset, to get a broader menu of ratings to choose from. This, in turn, makes shopping even more valuable.

Biased ratings affect securities prices if investors are unaware of the bias. If investors do not know that the complexity of assets has changed, then based on past data, they would rationally expect ratings to be unbiased, until they observed a sufficient amount of data to detect the bias.

It is always possible that there was no ratings bias and this episode is simply a bad draw. Because of the persistence in asset returns and the short sample history of many of the new credit derivative products, proving that their ratings were biased is a task that will become feasible only in the far future. However, the pattern of ratings suggests a relationship between asset complexity and over-optimistic ratings. Mason and Rosner (2007) document that complex CDOs have significantly higher default rates than simple corporate bonds with identical ratings.[4] Similarly, it was mortgage-backed securities whose underlying credit risk, correlation risk and pre-payment risk are notoriously difficult to assess, that experienced more widespread downgrades than assets based on other collateral types.[5]

Section 1 models a market for ratings with a few salient, realistic features. In reality, the two largest agencies, Moody's and S&P account for 80% of market share. Government regulation essentially inhibits free entry.[6] A ratings agency bases its rating primarily on publicly observable information, but has exclusive know-how about how to translate this information into a signal about the asset's return. When a structured credit product is issued, the issuer typically proposes an asset structure to an agency and asks them for a "shadow rating." This rating is private information between the agency and the issuer, unless the issuer pays the agency to make the rating official and publicize it. In the model, an asset issuer can purchase and make public one or two signals about the payoff of an asset. We call these signals "ratings." After choosing how many ratings to observe and which ones to make public, the issuer holds a menu auction for his assets. After each investor submits a menu of price-quantity pairs, the asset issuer sets the highest market-clearing price for his asset and all investors pay that price per share.

As a benchmark, section 2 solves this model with mandatory disclosure of all observed ratings. Section 3 solves the model with voluntary disclosure. Our main results are in section 4. If assets became more complex and harder to rate, the issuer is more likely to ratings shop, which creates bias in disclosed ratings. Furthermore, if an asset issuer can choose to make his asset more complex, then knowing he will shop for ratings can make more asset complexity desirable.

Section 5 uses our model to evaluate the effect of recently-proposed reforms. First, we consider whether only allowing investors-initiated ratings is viable or desirable. We modify the model to allow any investor to observe an asset's rating at a cost. To create the potential for ratings bias, we make a fraction of investors subject to investment-grade securities regulation, which only allows them to purchase assets whose ratings exceed a minimum threshold. Inflated

[4] Alan Greenspan acknowledged the greater complexity of CDOs in his May 2005 testimony, "the credit risk profile of CDO tranches poses challenges to even the most sophisticated market participants' and cautioned investors "not to rely solely on rating-agency assessments of credit risk."

[5] "Other collateral types that began to be securitized well after mortgages are far less complex. The first non-mortgage securitization was equipment leases, followed by credit cards and auto loans, and more recently, home equity, lease finance, manufactured housing, student loans, and synthetic structures. All of those types of collateral illustrate tranching structures that are measurably simpler than those for RMBS." (Mason and Rosner 2007)

[6] See Sylla (2001) and Coffee (2006) for overviews of the ratings industry.

ratings expand their investment possibility set. However, investors do not shop for ratings because they optimally use all available information in forming their bids. Thus, if the source of bias is not ratings agencies themselves, but is instead ratings shopping, then investor-initiated ratings are likely to be less biased. The downsides to investor-initiated ratings are that investors can free-ride on others' information and that the ratings market can easily collapse. If investors cannot overcome these problems, even biased ratings may be better than no information at all.

Another proposed reform is to allow more agencies to compete in providing ratings. For the case of issuer initiated ratings, we show that having more rating agencies would exacerbate the problem of bias because there will be more ratings to choose from. In the investor-initiated ratings model, increasing the number of agencies is inconsequential. Finally, we consider the logistical problems with mandatory disclosure laws.

A controversial assumption we maintain throughout is that agencies produce unbiased ratings. Appendix B explores a version of our model that relaxes this assumption. We do not deny that rating agencies may report biased ratings in an attempt to increase their business. Rather, our point is that even if ratings agency conflicts of interest are resolved, bias could continue to plague ratings.

Our contribution vis-a-vis existing literature As a theoretical contribution, out paper builds on three distinct literatures. First, we add an interaction between ratings and equilibrium asset prices to the literature on ratings agencies. Other papers that model ratings agencies are quite district from our work. Faure-Grimaud, Peyrache and Quesada (2007) identify circumstances under which the optimal renegotiation-proof contract between the rating agency and the firm results in the firm owning its rating. In their setup, the rating reveals the asset value perfectly and the price of the asset is exogenous. Farhi, Lerner and Tirole (2008) focus on other aspects of ratings, such as their transparency and coarseness. In Damiano, Li and Suen (2008), Bolton, Freixas and Shapiro (2008) and Becker and Milbourn (2008), a rating agency prefers to inflate its clients' ratings, but has some reputation cost of reporting a value far away from the asset's true, exogenous price. They investigate the equilibrium level of bias. In contrast, our model's rating agencies report the truth. We show that even if ratings agencies produce unbiased ratings, bias in disclosed ratings can still exist.

Second, we extend the literature on information in asset markets by modeling what information investors have access to. While most papers in this literature ask how some exogenous information structure affects asset prices and portfolios, there are a few that, like ours, consider endogenous information.[7] In each of these models, investors can acquire unbiased signals. Our model explores how asset issuers choose to disclose signals and the resulting signal bias.

Third, we augment the existing literature on sellers who provide information about their goods by considering how much information to provide.[8] In Shavell (1994), either the seller or a buyer can acquire information. Like our paper, Shavell studies equilibrium information acquisition under voluntary and mandatory disclosure regimes. But because Shavell's buyers know that sellers may be hiding information, any equilibrium without full disclosure unravels. In Jovanovic (1982), the fact that disclosure is costly prevents unraveling. In all these papers, there is a single signal to reveal or not. This precludes the possibility of shopping for ratings.

More broadly, our findings highlight the role that institutions, rules and market structure play in an industry that produces information. A central question in the mechanism design literature is what institutions are most desirable when information is asymmetric or dispersed. This paper asks the reverse question: What information do agents choose to observe or disclose in a given institution and market structure? As the recent crisis highlights, understanding the information provision is as important as understanding the institutions. When information production runs amok, large economic fluctuations can result.

1 A Model of an Asset Auction and a Market for Ratings

This is a static model of an *asset issuer*, who has many units of an asset to sell and a continuum of *investors* who want to buy those assets. The asset's value is unknown to the market participants. Information about the value of the asset is produced and sold by the *credit rating agencies.* The total supply of the asset is fixed and determined by the issuer. The market-clearing price is determined though a uniform price auction, where the sum of the bidders' bidding schedules determines the aggregate demand.[9] Investors choose their bidding functions so as to maximize their utility subject to the information that they have. What information they have depends on whether the rating is purchased by the issuer (in which case the issuer makes it public) or by the investors. Below we investigate how each of these different arrangements about who

[7]For an overview of the role of exogenous information in asset markets, see Brunnermeier (2001). For more recent work on this topic, see Banerjee (2007). Seminal works on endogenous information include Grossman and Stiglitz (1980) and Hellwig (1980). More recent work on investor-initiated information acquisition includes Barlevy and Veronesi (2000), Bullard, Evans and Honkapohja (2005) and Peress (2004).

[8]Seminal contributions in this literature include Grossman (1981) and Milgrom (1981). Benabou and Laroque (1992), Morgan and Stocken (2003) and more recently Bolton, Freixas and Shapiro (2007) analyze the conflict of interest between the buyer and seller in this environment.

[9]This auction is similar to the limit economy in Reny and Perry (2006), but we incorporate investor risk aversion and budget constraints.

purchases the information affects the quality and the amount of information available to market participants.

We now move on to describe a model where issuers pay for ratings. Later in the paper we analyze a version of this model where investors pay for ratings.

Assets There are two assets: The 'safe' asset offers riskless return r, and the risky asset pays u, which is normally distributed $u \sim N(\bar{u}, \sigma_u^2)$. The price of the riskless asset is 1. The price of risky asset is p, which is endogenous.

Investors A continuum of ex-ante identical investors has utility

$$U = -e^{-\rho(m_i r + qu)} \tag{1}$$

where ρ is the coefficient of absolute risk aversion and q_i and m_i are the number of risky and riskless asset shares investor i ends up with. Each agent is endowed with m_i^0 units of riskless asset, but can borrow and lend that asset freely at the riskless rate r. Hence each investor's budget constraint is

$$m_i + pq_i = m_i^0. \tag{2}$$

The Auction The price of the risky asset is determined in an auction. Each investor submits a bidding function that specifies the maximum amount that he is willing to pay for q units of the risky asset as a function of his information. These bid functions determine the aggregate demand. The auctioneer specifies a market clearing price p that equates aggregate demand and supply and each trader pays this price for each unit purchased (uniform price auction).

We now state the bid function and verify that it constitutes an equilibrium. Bids depend on each investor's information set $\mathcal{I}_i$, which includes information inferred from b being the price paid per unit.

$$b(q|\mathcal{I}_i) = \frac{E(u|\mathcal{I}_i) - q\rho V(u|\mathcal{I}_i)}{r}, \tag{3}$$

where $E(u|\mathcal{I}_i)$ and $V(u|\mathcal{I}_i)$ are the mean and variance of the risky asset's return, conditional on the investor's information. The price paid per unit is exogenous from each investor's perspective because he is infinitesimal compared to the rest of the market, implying that the price he faces is determined by other investor's bid functions, together with the aggregate supply. Therefore, a realized price b reveals information about others' bids, which in turn is partially informative about what they know.

Each bidder is infinitesimal, which implies that he takes the market clearing price as given. Thus, the bidding function (3) is the inverse demand function of a trader who seeks to maximize (1) subject to (2), taking p as given. It can be easily verified that the objective of this constrained maximization problem is concave in q so that the first order condition describes the optimal portfolio:

$$q_i = \frac{1}{\rho} V[u|\mathcal{I}_i]^{-1}(E[u|\mathcal{I}_i] - pr). \tag{4}$$

Because the above bidding function is an inverse demand function of (4), it is a best response given everyone else's bid function.

When issuers solicit a rating, they either disclose the rating to all investors or keep it private so that no investor observes it. Either way, investors have symmetric information $\mathcal{I}$. Integrating over the asset demand (4) and equating aggregate demand with the asset supply, delivers the equilibrium price

$$p = \frac{1}{r}(E[u|\mathcal{I}] - \rho Var[u|\mathcal{I}]x). \tag{5}$$

Ratings Agencies Credit ratings agencies produce ratings, which are noisy unbiased signals about the risky asset payoff u. We consider two rating agencies, because this is the simplest setting in which to illustrate our results.

We assume that a shadow rating θ is an unbiased signal about the payoff $\theta \sim N(u, \sigma_\theta^2)$, produced at marginal cost $\tilde{\chi}$. The issuer can choose to keep the rating private or to make it public information.[10] All rating agencies produce the same service. Since there is no quantity choice, firms compete in a Bertrand way and set price equal to marginal cost: $\tilde{\chi}$ for shadow ratings and $\chi + \tilde{\chi}$ for publicly-issued ratings.

Definition 1 *A more complex asset is one with more noise in its ratings: It has a higher σ_θ^2.*

Asset Issuer The issuer is endowed with x shares of the risky asset and his objective is to maximize expected profit. We first consider the case where the issuer initiates the rating. In this model, the issuer's expected profit is the price times quantity of the asset sold, minus the cost of observed and disclosed ratings:

$$\Pi = px - \tilde{s}\tilde{\chi} - s\chi, \tag{6}$$

[10]When a separation occurs, ratings agencies are not withholding the rating because the exact structured product they rated is rarely issued. Rather, the rating and asset structure are negotiated. Another agency may tell the issuer that structuring the security with slightly more low-risk assets will earn it the sought after rating. Then, this slightly modified security would be issued. See Mason and Rosner (2007) for a detailed description of this process.

where $\tilde{s}$ is the number of shadow ratings observed, including the ones eventually disclosed, and s is the number of publicly disclosed ratings.

Model Timing *Stage 1: Ratings Acquisition and Disclosure.* The issuer decides whether to obtain a rating or not. If he decides to do so, he visits one of the two ratings agencies. Upon obtaining a shadow rating he decides whether to obtain another shadow rating or not. If he does not, he decides whether to publish the obtained rating or not. If he decides to move on to obtain a shadow rating from the other agency, then he decides whether to disclose no, one, or both ratings.

Stage 2: Price Determination. An auction determines the market clearing price.

With voluntary disclosure, the asset issuer's decisions are summarized in Figure 1. After these stage-1 decisions are made, the asset auction takes place.

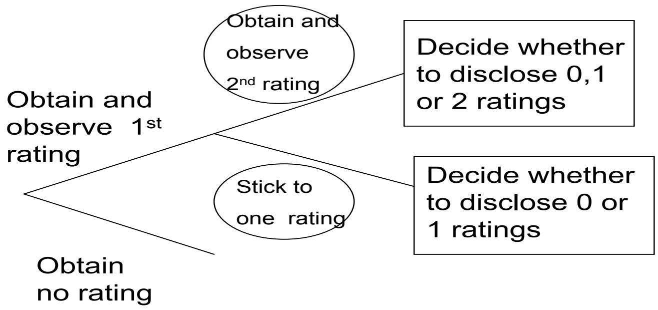

Figure 1: An asset issuer's decision tree.

2 The Benchmark: Mandatory Disclosure of Shadow Ratings

When asset issuers must choose how many ratings to acquire before observing the ratings and must disclose every rating observed, there is no opportunity for selection effects to bias the disclosed ratings. In order to investigate the ratings bias that ratings shopping generates, we first solve a mandatory disclosure model without any ratings bias as a benchmark. With mandatory disclosure, the only thing that the issuer decides is whether to obtain zero, one or two ratings. He must make that choice before observing the ratings.

The expected price of the asset when no rating is obtained is[11]

$$p_0 \equiv \frac{1}{r}(\bar{u} - \rho\sigma_u^2 x). \tag{7}$$

If the issuer initiates a rating, Bayes' law dictates that the expected value of the asset is $E[u|\theta] = (\sigma_u^{-2}\bar{u} + \sigma_\theta^{-2}\theta)/(\sigma_u^{-2} + \sigma_\theta^{-2})$ and the conditional variance of the asset will be $V[u|\theta] = 1/(\sigma_u^{-2} + \sigma_\theta^{-2})$. Since the issuer decides to acquire the rating before he knows its outcome, he considers the expected price

$$\bar{p}_1 \equiv \frac{1}{r}(\bar{u} - \rho V[u|\theta]x) = \frac{1}{r}\left(\bar{u} - \frac{\rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}}\right). \tag{8}$$

Thus, the difference in issuer utility from buying information is

$$\bar{\Pi}^M_{s=1} - \bar{\Pi}^M_{s=0} = \frac{\rho x^2}{r}\frac{1}{\sigma_\theta^2\sigma_u^{-4} + \sigma_u^{-2}} - \chi - \tilde{\chi}, \tag{9}$$

where $\bar{\Pi}^M_{s=0}$ ($\bar{\Pi}^M_{s=1}$) stands for the issuer's expected profits from obtaining no (respectively one) rating under mandatory disclosure. The issuer chooses to purchase a rating if (9) is non-negative.

The asset's expected price with two ratings is

$$\bar{p}_2 = \frac{1}{r}\left(\bar{u} - \frac{\rho x}{\sigma_u^{-2} + 2\sigma_\theta^{-2}}\right). \tag{10}$$

The issuer chooses to obtain two shadow ratings instead of one if

$$\bar{\Pi}^M_{s=2} - \bar{\Pi}^M_{s=1} = \frac{\rho x^2}{r}\frac{1}{\sigma_\theta^2\sigma_u^{-4} + 3\sigma_u^{-2} + 2\sigma_\theta^{-2}} - \chi - \tilde{\chi} > 0. \tag{11}$$

Note, that if (9) is positive, (11) will be too. Hence if the issuer profits from acquiring one shadow rating, he also profits from a second rating.

Comparative Statics with Mandatory Disclosure The incentive to acquire an extra shadow rating is increasing in the coefficient of risk aversion because information reduces the risk the investors face when they buy the asset. The more averse they are to this risk, the more information increases their value for the asset. The value of a rating is also increasing in the quantity of the asset offered because information has returns to scale. It is likewise decreasing in the risk-free rate, because when the excess return on the risky asset is lower, information about

[11]Some fixed-income securities are issued without ratings. Such unrated bonds are classified as junk bonds. Typically these are small net worth assets.

that asset is less valuable. Finally, the value of a rating is non-monotonic in asset complexity (σ_θ). The value of the first rating is always decreasing in complexity because a more complex asset is harder to rate and the resulting rating is less precise and thus less valuable ((9) is decreasing in σ_θ^2). The value of the second rating could also be lower for the same reason, or it could be higher because having less information from the first rating increases the marginal value of additional information: (9) is increasing in asset complexity if the variance of the asset's returns is high, relative to the complexity of the asset ($2\sigma_u^4 > \sigma_\theta^2$) and is decreasing otherwise.[12]

Ultimately, a model with complete disclosure of ratings by asset issuers and truthful reporting by ratings agencies cannot explain bias in ratings. We introduced it because it illustrates the mechanics of the solution. We now explore the more realistic voluntary disclosure case to understand where ratings bias might come from.

3 Solving the Voluntary Disclosure Model

The key trade-off an asset issuer faces is the following: Withholding the most negative ratings makes the asset appear more valuable to investors, while publicizing more ratings makes the asset less risky. In other words, disclosure lowers the conditional variance of the risky asset payoff u while ratings shopping increases its conditional mean. Both effects increase the price investors are willing to pay and thus increase the issuer's profit. We investigate which circumstances favor ratings shopping, as well as the resulting bias in the asset price.

For this discussion to be meaningful, we need to ensure that ratings bias is not irrelevant. In particular, sophisticated investors should not be able to infer the expected bias, subtract that expected bias from the announced rating, and neutralize its effects through their actions.

Assumption 1 *Investors do not correct for ratings selection bias: For every announced rating, they believe $\theta \sim N(u, \sigma_\theta^2)$. For unrated assets, they believe that $u \sim N(\bar{u}, \sigma_u^2)$.*

This assumption implies that investors are not able to make correct inferences from the rating agencies actions (the number of ratings they chose to disclose). The feature that players are unable to form the correct mapping about the informativeness of other people's actions is an important feature of the equilibrium notion of Eyster and Rabin (2005), and of Esponda (2008) and is also present in the analysis of DeMarzo, Vayanos and Zwiebel (2003) among others.

[12] $\frac{\partial(\Pi^M_{s=2}-\Pi^M_{s=1})}{\partial\sigma_\theta} = -\frac{2\sigma_\theta\sigma_u^{-4}-4\sigma_\theta^{-1}}{(\sigma_\theta^2\sigma_u^{-4}+3\sigma_u^{-2}+2\sigma_\theta^{-2})^2}$, is positive if $\frac{4}{\sigma_\theta} > 2\sigma_\theta\sigma_u^{-4}$ or $2 > \sigma_\theta^2\sigma_u^{-4}$ or $2 > \frac{\sigma_\theta^2}{\sigma_u^4}$ or $2(\sigma_u^2)^2 > \sigma_\theta^2$.

The assumption that investors believed ratings to be unbiased is consistent with our main argument that much of the bias was a recent phenomenon. Suppose investors did not observe asset complexity in a dynamic model, but knew that regime changes in complexity were possible. They would infer asset complexity and thus ratings bias from the past history of ratings and asset outcomes. Since the historical data came from mostly simple assets, investors would initially believe that assets are simple, that no ratings shopping is taking place, and that ratings are unbiased. Even after assets became more complex, this belief would persist until they observed a sufficiently long series of ratings and payoffs from the complex assets. Thus, with an unexpected change in asset characteristics, even rational investors would not have initially detected ratings bias.[13]

3.1 The disclosure decision

To solve the model, we start with the last decision and work backwards. We begin by considering an issuer who has already chosen how many ratings to acquire and is deciding how many to disclose. If the asset issuer chooses not to solicit any ratings, then the asset price is the same whether disclosure is voluntary or mandatory because there is no rating to disclose. But when the asset issuer solicits one or two shadow ratings, he faces the following choices.

Disclosure with two shadow ratings We first investigate the disclosure decision of an issuer who has acquired two shadow ratings. We call the higher rating $\bar{\theta}$, and the lower $\underline{\theta}$, so that $\bar{\theta} > \underline{\theta}$. We want to identify under which conditions the issuer will disclose none, one or both ratings. Since the asset issuer is always more inclined to announce a higher rating, disclosing one rating means disclosing $\bar{\theta}$.

We first compare the alternatives of disclosing one versus no ratings. If the issuer announces no ratings, the conditional mean and variance of the asset payoff are the unconditional mean and variance, $\bar{u}$ and σ_u^2. Therefore, the price of the asset is the same as in (7).

If the issuer announces rating $\bar{\theta}$, the price will be

$$p_1(\bar{\theta}) = \frac{1}{r}\left(\frac{\sigma_u^{-2}\bar{u} + \sigma_\theta^{-2}\bar{\theta} - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}}\right). \tag{12}$$

Let $\Pi_{D=d}$ stand for the issuer's profit from disclosing d ratings. Then the additional utility

[13] If investors are sophisticated and can perfectly account for how the issuers disclosure rule varies with asset complexity, we anticipate that unraveling would occur similar to that in Shavell (1994), where all undisclosed information is treated as if it is bad news. Thus, all ratings would be disclosed.

gained from disclosing information is

$$\Pi_{D=1}(\bar{\theta}) - \Pi_{D=0} = \left[p_1(\bar{\theta}) - p_0\right] x - \chi, = \frac{x}{r} \frac{\bar{\theta} - \bar{u} + \rho x \sigma_u^2}{1 + \sigma_u^{-2} \sigma_\theta^2} - \chi. \tag{13}$$

Having purchased at least one shadow rating, the asset issuer discloses that rating if (13) is positive. Let a be the value of $\bar{\theta}$ that causes (13) to be zero:

$$a = \frac{r(1 + \sigma_u^{-2} \sigma_\theta^2)}{x} \chi + \bar{u} - \rho x \sigma_u^2. \tag{14}$$

Since (13) is monotonically increasing in $\bar{\theta}$, the asset issuer discloses at least one rating if $\bar{\theta} \geq a$. In summary, when disclosure of ratings is voluntary, the issuer discloses one versus no ratings when the rating obtained is high enough.

Consider next the choice between disclosing one or two ratings. If the issuer discloses both $\bar{\theta}$ and $\underline{\theta}$, the asset price will be

$$p_2(\bar{\theta}, \underline{\theta}) = \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} (\bar{\theta} + \underline{\theta}) - \rho x}{\sigma_u^{-2} + 2\sigma_\theta^{-2}}. \tag{15}$$

The issuer prefers to disclose both ratings if

$$\begin{aligned} \Pi_{D=2}(\bar{\theta}, \underline{\theta}) - \Pi_{D=1}(\bar{\theta}) &= \left[p_2(\bar{\theta}, \underline{\theta}) - p_1(\bar{\theta})\right] x - \chi, \\ &= \frac{x}{r} \frac{\left(\sigma_\theta^{-2} \sigma_u^{-2} \left(\underline{\theta} - \bar{u}\right) + \sigma_\theta^{-4} (\underline{\theta} - \bar{\theta}) + \rho x \sigma_\theta^{-2}\right)}{\left(\sigma_u^{-2} + 2\sigma_\theta^{-2}\right) \left(\sigma_u^{-2} + \sigma_\theta^{-2}\right)} - \chi > 0. \end{aligned} \tag{16}$$

Since (16) is monotonically increasing in $\underline{\theta}$, the issuer discloses both ratings if $\underline{\theta} \geq b(\bar{\theta})$, where $b(\bar{\theta})$ is the value of $\underline{\theta}$ that equates (16) to zero when the highest rating is $\bar{\theta}$:

$$b(\bar{\theta}) = \frac{1}{(\sigma_\theta^{-2} \sigma_u^{-2} + \sigma_\theta^{-4})} \left(\chi \frac{r \left(\sigma_u^{-2} + 2\sigma_\theta^{-2}\right) \left(\sigma_u^{-2} + \sigma_\theta^{-2}\right)}{x} + \sigma_\theta^{-2} \sigma_u^{-2} \bar{u} + \sigma_\theta^{-4} \bar{\theta} - \rho x \sigma_\theta^{-2} \right). \tag{17}$$

In other words, the issuer prefers to shop, that is disclose only the highest of the two ratings, when $\underline{\theta}$ is much lower than $\bar{\theta}$, ($\underline{\theta} < b(\bar{\theta})$).

Finally, the issuer will disclose no ratings if no ratings are preferable to one rating and to two ratings. Both these conditions are satisfied if $\bar{\theta} < a$.[14]

[14] Zero ratings are preferred to two ratings when $\Pi_{D=0} - \Pi_{D=2}(\bar{\theta}, \underline{\theta}) > 0$, which happens if $\left[\frac{1}{r}(\bar{u} - \rho \sigma_u^2 x) - \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} (\bar{\theta} + \underline{\theta}) - \rho x}{\sigma_u^{-2} + 2\sigma_\theta^{-2}}\right] x + 2\chi > 0$. When $\bar{\theta} \leq a$, the highest possible sum for $\bar{\theta} + \underline{\theta} = 2\left(\frac{r(1 + \sigma_u^{-2} \sigma_\theta^2)}{x} \chi + \bar{u} - \rho x \sigma_u^2\right)$ (this is because by definition $\underline{\theta} \leq \bar{\theta}$). Substituting this sum in the above inequality, one can see that it is always satisfied. The details can be found in Technical Appendix: Computation Details for

In summary, the disclosure decision for an issuer that has acquired two shadow ratings is

$$\begin{array}{rc} \text{Disclose both ratings if} & \bar{\theta} \geq a \text{ and } \underline{\theta} \geq b(\bar{\theta}). \\ \text{Disclose highest rating if} & \bar{\theta} \geq a \text{ and } \underline{\theta} < b(\bar{\theta}). \\ \text{Disclose no ratings if} & \bar{\theta} < a. \end{array}$$

Disclosure with one shadow rating Suppose the asset issuer has acquired only 1 shadow rating. With a slight abuse of notation, we call that rating $\bar{\theta}$. The issuer prefers to disclose if (13) is positive (when $\bar{\theta} \geq a$) and does not disclose otherwise.

3.2 The acquisition decision

Now that we understand the issuer's disclosure decisions, we move back one node in the decision tree to study ratings acquisition. The issuer makes two decisions sequentially. First he decides whether to acquire the first shadow rating and then he decides whether to acquire the second one. Again we work backwards: We start with the decision of the issuer who already has one shadow rating and considers whether to obtain a second. We call θ_1 the first shadow rating observed and θ_2 the second.

The decision to acquire the second rating The decision depends on whether the first rating is high enough to disclose ($\theta_1 \geq a$).

Case 1: The first rating was high ($\theta_1 > a$). If the second draw is low relative to the first draw ($\theta_2 < b(\theta_1)$), then the issuer discloses only the first rating θ_1. If the second draw is sufficiently high to disclose and not so high that it makes the first rating no longer worthwhile to disclose ($b(\theta_1) < \theta_2 < \theta^*$), the issuer discloses both ratings. If the second rating is much higher than the first rating, the issuer discloses only the second rating. Thus, the expected price ($E(p_2|\theta_1 > a)$) contains three terms corresponding to these three possibilities:

$$\bar{p}^{(2)}(\theta_1|\theta_1 > a) = F(\underline{\theta}^c(\theta_1))p_1(\theta_1) + \int_{b(\theta_1)}^{\theta^*} p_2(\theta_1, \theta_2)f(\theta_2)d\theta_2 + \int_{\theta^*}^{\infty} p_1(\theta_2)f(\theta_2)d\theta_2, \quad (18)$$

where $p_1(\theta_1)$ and $p_2(\theta_1, \theta_2)$ are defined in (12) and (15), and where θ^* is the value of θ_2 such that $\theta_1 = b(\theta_2)$, implying that $\theta^* = b^{-1}(\theta_1)$.

the Disclosure Decision 2 versus 0. This observation allows us to conclude that the issuer will disclose no ratings when $\bar{\theta} < a$.

If $\theta_1 > a$ and the issuer sticks with one rating, his profit is

$$\Pi^{(1)}(\theta_1) = p_1(\theta_1)x - \chi - \tilde{\chi}.$$

If he obtains a second rating, his expected profit conditional on θ_1 is

$$\Pi^{(2)}(\theta_1|\theta_1 > a) = \bar{p}^{(2)}(\theta_1|\theta_1 > a)x - 2(\tilde{\chi} + \chi), \tag{19}$$

where $\bar{p}^{(2)}(\theta_1|\theta_1 > a)$ is given by (18). The expected benefit from acquiring the second rating depends on the difference in the asset price between disclosing two versus one ratings and on the difference in the price between disclosing θ_2 alone and disclosing θ_1 alone. Each difference is weighted by the probability that θ_2 takes on a value that makes the associated disclosure optimal.

$$\begin{aligned}
\Pi^{(2)}(\theta_1|\theta_1 > a) - \Pi^{(1)}(\theta_1) &= \int_{b(\theta_1)}^{\underline{\theta}^{c-1}(\theta_1)} \left[x\left(p_2(\theta_1,\theta_2) - p_1(\theta_1)\right) - \chi\right] f(\theta_2)d\theta_2 \\
&\quad + \int_{b^{-1}(\theta_1)}^{\infty} x\left(p_1(\theta_2) - p_1(\theta_1)\right) f(\theta_2)d\theta_2 - \tilde{\chi}, \\
&= \int_{b(\theta_1)}^{\underline{\theta}^{c-1}(\theta_1)} \left[\frac{x}{r}\frac{\left(\sigma_\theta^{-2}\sigma_u^{-2}\left(\theta_2 - \bar{u}\right) + \sigma_\theta^{-4}(\theta_2 - \theta_1) + \rho x \sigma_\theta^{-2}\right)}{\left(\sigma_u^{-2} + 2\sigma_\theta^{-2}\right)\left(\sigma_u^{-2} + \sigma_\theta^{-2}\right)} - \chi\right] f(\theta_2)d\theta_2 \\
&\quad + \int_{b^{-1}(\theta_1)}^{\infty} \left(\frac{x}{r}\left(\frac{\sigma_u^{-2}\bar{u} + \sigma_\theta^{-2}(\theta_2 - \theta_1) - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}}\right)\right) f(\theta_2)d\theta_2 - \tilde{\chi}.
\end{aligned} \tag{20}$$

where the second line uses (12) and (15) to substitute out p_1 and p_2.

Case 2: The first rating was low ($\theta_1 < a$). If the second rating is also low ($\theta_2 < a$), then the issuer will disclose no ratings. If the second rating is moderately high, it is possible that the issuer discloses both ratings, even though the first was too low to disclose on its own. If the second rating is high ($\theta_2 > b^{-1}(\theta_1)$), the issuer discloses only the second rating. The expected price ($E(p_2|\theta_1 < a)$) contains three terms that correspond to these three possibilities

$$\bar{p}^{(2)}(\theta_1|\theta_1 < a) = F(a)p_0 + \int_a^{b^{-1}(\theta_1)} p_2(\theta_1,\theta_2)f(\theta_2)d\theta_2 + \int_{b^{-1}(\theta_1)}^{\infty} p_1(\theta_2)f(\theta_2)d\theta_2, \tag{21}$$

where p_0 is given by (7).

If $\theta_1 < a$ and the issuer does not obtain a second rating, then he discloses no rating and earns profit $\Pi^{(1)}(\theta_1) = p_0 x$. If he obtains a second rating, his expected profit conditional on θ_1

is $\Pi^{(2)}(\theta_1|\theta_1 < a) = \bar{p}^{(2)}(\theta_1|\theta_1 < a)x - 2(\tilde{\chi} + \chi)$. Thus, the expected benefit from acquiring the second rating is

$$\Pi^{(2)}(\theta_1|\theta_1 < a) - \Pi^{(1)}(\theta_1)$$

$$= \int_a^{b^{-1}(\theta_1)} \left[(p_2(\theta_1, \theta_2) - p_0)\, x - 2\chi\right] f(\theta_2) d\theta_2 + \int_{b^{-1}(\theta_1)}^{\infty} \left[(p_1(\theta_2) - p_0)\, x - \chi\right] f(\theta_2) d\theta_2 - \tilde{\chi}$$

$$= \int_a^{b^{-1}(\theta_1)} \left(\frac{x\left((\theta_1 + \theta_2) - 2\bar{u} + 2\rho x \sigma_u^2\right)}{r(2 + \sigma_\theta^2 \sigma_u^{-2})} - 2\chi \right) f(\theta_2) d\theta_2 + \int_{b^{-1}(\theta_1)}^{\infty} \left(\frac{x(\theta_2 - \bar{u} + \rho x \sigma_u^2)}{r(1 + \sigma_u^{-2} \sigma_\theta^2)} - \chi \right) f(\theta_2) d\theta_2 - \tilde{\chi}. \quad (22)$$

Let Θ_2 denote the region of realizations of the first rating where the differences in (20) and in (22) are positive. For this region, the issuer will choose to obtain a second draw.

The decision to acquire the first rating When the issuer makes this decision he has only his prior information. He compares the expected profit from acquiring no rating, $\Pi^{(0)} = \frac{x}{r}(\bar{u} - \rho \sigma_u^2 x)$, with the expected profit from acquiring the first rating, $\Pi^{(1)}$. This profit is calculated anticipating four possible future acquisition and disclosure decisions: the first rating may be too low to disclose but may prompt the investor to acquire a second rating; the first rating may be high enough to disclose and still prompt the acquisition of a second rating; the first rating may be too low to disclose and may still deter the acquisition of a second rating; and finally, the first rating may be high enough to disclose and may deter the acquisition of the second rating.

$$\begin{aligned} \bar{\Pi}^{(1)} &= \int_{[-\infty,a]\cap\Theta_2} \Pi^{(2)}(\theta_1|\theta_1 < a) dF(\theta_1) + \int_{[\bar{\theta}^c,\infty]\cap\Theta_2} \Pi^{(2)}(\theta_1|\theta_1 > \bar{\theta}^c) dF(\theta_1) \\ & \int_{[-\infty,a]\cap\Theta_2^c} p_0 x dF(\theta_1) + \int_{[a,\infty]\cap\Theta_2^c} (p_1(\theta_1)x - \chi - \tilde{\chi}) dF(\theta_1) \end{aligned} \quad (23)$$

where p_0 is given by (7), p_1 is given by (12), and the formula for expected profits with two ratings $\Pi^{(2)}(\cdot)$ is in (19).

4 Main Results: Asset Complexity and Ratings Bias

So far we have analyzed the issuer's incentives to acquire shadow ratings under two regimes: the mandatory and the voluntary disclosure regime. For each of these regimes we identified cases where the issuer prefers to obtain none, one, or two shadow ratings. In each case, choices depend on the characteristics of the asset to be rated. One of those characteristics is how reliably an asset can be rated, an attribute we call asset complexity.

This section considers the interaction between greater asset complexity and the ratings shopping which creates ratings bias. There are two pieces to this interaction. First, we consider how an exogenous change in asset complexity affects the incentive to shop for ratings. Second, we show that ratings shopping can create an incentive to structure more complex securities.

4.1 How complexity affects the incentive to shop for ratings

Our argument is that ratings shopping arose as the nature of credit products and their market changed. For this to be a plausible explanation, we need to show that the benefit of ratings shopping in (16) is increasing in some parameter that was trending up at the time. This leads us to ask the following comparative statics question: Given two shadow ratings, what happens to the incentive to publicly disclose these ratings as an asset becomes more complex?

The effect of complexity turns out to be non-monotonic. For either very low or high asset complexity ($\sigma_\theta^2 \to 0$ or $\sigma_\theta^2 \to \infty$), ratings shopping always takes place. When $\sigma_\theta^2 \to \infty$ in (16), $\Pi_{D=1}(\bar{\theta}) - \Pi_{D=2}(\bar{\theta}, \underline{\theta})$ clearly converges to χ. When $\sigma_\theta^2 \to 0$ the same results follows from the fact that $\bar{\theta} - \underline{\theta}$ tends to zero. Since χ is positive, disclosing one rating is preferred to disclosing both. The intuition is that when asset complexity is small, ratings are precise. The extent to which publicizing a second rating reduces the risk of investing in the asset is too small to be worth the cost. When asset complexity is high, ratings become uninformative. Since investors know this, issuing multiple ratings has little price impact and is again, not worth the cost.

Numerical example We consider an asset issuer who has observed two shadow ratings and is deciding how many of these to disclose. The net benefit of disclosing a second rating is given by (16). Since this depends on the realized ratings, we first take an expectation over these realizations. For a normal variable with mean $\bar{u}$ and variance σ_θ^2, the expected highest and lowest order statistics out of a sample of two are $\frac{\sigma_\theta}{\pi} + \bar{u}$ and $-\frac{\sigma_\theta}{\pi} + \bar{u}$ (Kotz, Balakrishnan and Johnson 2000).[15]

Figure 2 shows a case where the expected profit of disclosing the second rating is lower, higher and then lower again compared to the expected profit from disclosing one rating. The vertical lines represent the two roots – levels of σ_θ that make the issuer indifferent between

[15] The expectation of $\bar{\theta}$, is the expectation of the highest order statistic out of a sample of two. From Kotz et. al. we know that this expectation for a standard normal distribution is given by $\frac{1}{\pi}$. Since here ratings are distributed by $N(\bar{u}, \sigma_\theta^2)$, this expectation is given by $\frac{\sigma_\theta}{\pi} + \bar{u}$. This follows from the fact that the expectation is linear in the mean and standard deviation.

disclosing 1 or 2 ratings. Ratings bias would arise in the first or third regions where the issuer chooses only to disclose the higher of the two observed ratings.

Not every set of parameters will generate this pattern. If both roots are not positive real numbers, then disclosing one or no ratings could be optimal. But in that case, it would never be optimal to disclose two ratings, for any level of asset complexity.

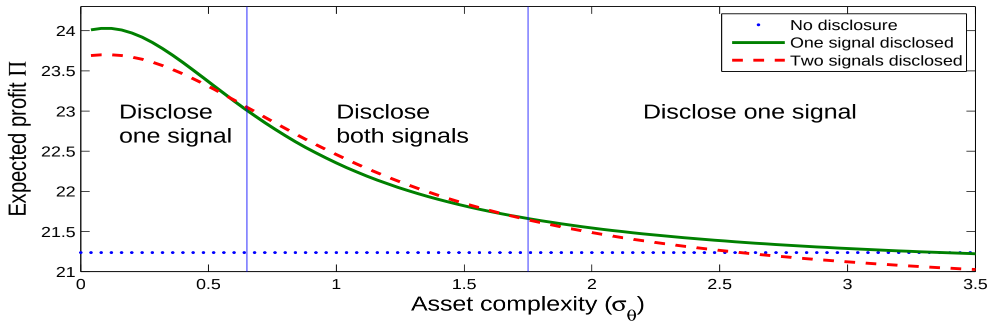

Figure 2: The average asset issuer's expected value of disclosing zero, one or two ratings, for various levels of asset complexity (σ_θ^2). Parameter values: $x = 2.5$, $r = 1.03$, $\sigma_u^2 = .5$, $\bar{u} = 10$, $\chi = 0.3$ and $\rho = 1$.

This example was constructed for the average signal realizations, in order to keep it simple. As asset complexity increases, there will be a distribution of signal outcomes. Some will prompt asset issuers to disclose them as ratings, others not. Thus, for a given set of parameter values we can calculate the probability of ratings shopping. Then, instead of a discrete change from no bias to bias, there is a continuous change from a low to a high probability of ratings shopping. The next set of results take into account the random nature of the realized ratings.

4.2 How complexity affects the demand for shadow ratings

Just like complexity had a non-monotonic effect on disclosure, it also has a non-monotonic effect on ratings acquisition. The reason that complexity's effect is non-monotonic can be seen by considering its limiting cases again. When complexity approaches zero, each rating is perfectly precise. Therefore, there is no benefit and only a cost to acquiring a second rating. When complexity approaches infinity, ratings are uninformative. Again, there is no benefit and only a cost to acquiring either a first or second rating. In between these two extremes, we know that there can be a value to acquiring more than one rating in order to shop for the best one. But if that is the case, then rising complexity must cause the net benefit of a second rating to rise and then fall.

A key effect of changing complexity is that it changes the distribution of $\bar{\theta}$ and $\underline{\theta}$. To incorporate this effect, we do a change of variables and express the expected benefit of acquiring a second rating ((20) and (22)) in terms of standard normal random variables. The partial derivative with respect to σ_θ^2 reveals that the effect of complexity on ratings acquisition is non-monotonic.[16]

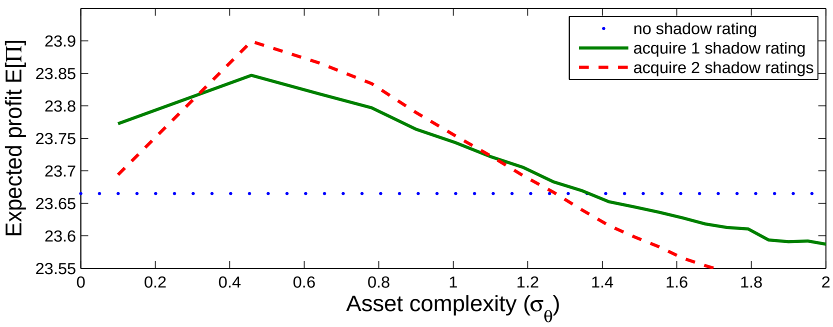

Figure 3: The asset issuer's expected value of obtaining zero, one or two shadow ratings for various levels of asset complexity (σ_θ^2). Parameter values are identical to those in figure 1, except that we take expectations over θ_1 and θ_2, instead of fixing their values. In addition, $\tilde{\chi} = 0.1$.

To see what effects arise for non-extreme parameter values, we return to our numerical example. Using the same parameters as in figure 2, figure 3 plots the asset issuer's expected profit from obtaining 0, 1 or 2 shadow ratings. As asset complexity rises, the number of shadow ratings acquired starts at one, rises to two, falls back to one, and eventually becomes zero.

4.3 How complexity affects ratings bias

The average percentage bias in ratings appears in Figure 4. The concurrent increase in asset complexity and ratings bias is the amalgamation of many different effects. First, it incorporates the disclosure decision in figure 3. As complexity rises, a firm with two average ratings discloses one then two then one rating again. But figure 3 obscures the fact that the issuer will not always draw average ratings. Therefore, there will always be some probability of ratings shopping whenever ratings are acquired. The second effect is that as complexity rises, two ratings become farther apart, on average. Thus the ratings bias generated by ratings shopping grows. The third effect is the change in ratings acquisition. As complexity grows from a low level, more issuers get a second rating, enabling them to choose the highest one to disclose.

When complexity becomes very large, bias plummets. This corresponds to the level of

[16]Details available upon request.

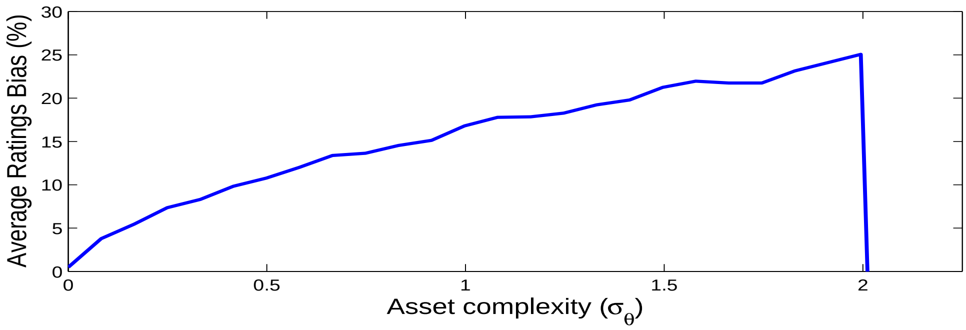

Figure 4: The percentage bias in ratings for various levels of asset complexity (σ_θ^2). Parameter values are identical to those in figure 2. Ratings bias is the average of all disclosed ratings, minus the true mean of the distribution from which the ratings are drawn. It is expressed as a percentage deviation from the true mean.

complexity where issuers no longer want to acquire any shadow rating because ratings contain too little information to be worth their cost. One might wonder why bias does not drop at $\sigma_\theta = 1.4$, where the the value of acquiring no ratings surpasses the value of one rating in figure 3. This is because the solid line is the value of acquiring one and only one rating. But choosing to acquire a first rating also gives the issuer the option value to acquire a second rating. This total value of acquiring a first shadow rating is surpassed by the value of observing no ratings at $\sigma_\theta = 2$. When no ratings are observed, ratings bias disappears.

4.4 When do asset issuers prefer more complex assets?

Consider an asset issuer who can choose whether to structure his asset as a simple security or a complex security. Both the simple and the complex security have the same payoff distribution, but the variance of the ratings σ_θ^2 is higher for the more complex asset. The issuer chooses his asset's complexity before it observing its shadow ratings.

Mandatory disclosure The price of the asset with no, one or two ratings is given by (7), (8) or (10). All three prices are either constant or decreasing in the complexity of the asset. Hence given the opportunity to design the asset, the issuer who is required to reveal every shadow rating he obtains prefers a less complex asset.

Voluntary disclosure The drawback that more complex assets are riskier to investors and are therefore less profitable for asset issuers is still present with voluntary disclosure. But that drawback may now be offset by the following benefit. If ratings are drawn from a higher-variance distribution, their maximum will be higher, on average; that makes ratings shopping more profitable. Complex assets offer a broader menu of ratings for the issuer to pick from. When this advantage is large, the asset issuer will prefer to make his asset more complex.

Whether higher asset complexity is desirable depends on parameter values. Figure 3 illustrates a case where for small levels of asset complexity, profits rise with added complexity. In this region with average ratings realizations, the issuer obtains only one shadow rating. But an issuer with a lower than average first rating acquires a second rating; if the two ratings are sufficiently far apart, he discloses the larger of the two. The fact that profit increases in σ_θ indicates a motive to design more complex assets, which exacerbates the bias in disclosed ratings.

5 Evaluating Policy Recommendations

This section uses the model framework to explore the pros and cons of three proposed reforms: switching to investor-initiated ratings, increasing competition in the market for ratings, and reforming risk-management regulation.

5.1 Investor-initiated ratings

One possible solution to the problem of ratings bias is to replace the system of issuer-initiated ratings with investor-initiated ratings. We show that even though some investors, those subject to the investment-grade securities regulations, would prefer biased ratings, they cannot shop for ratings. However, a investor-initiated market for ratings may provide too little or even no information.

5.1.1 A model of investor information acquisition

In this scenario, investors choose whether or not to buy a single rating. In order to ensure that prices are not perfectly revealing, we will modify the assumption that the supply is known and fixed. We will instead assume that the issuer of the asset is endowed with $x + \bar{x} = \int_i q_i^0 di$ shares of the asset. It is partly random: $x \sim N(0, \sigma_x)$. This randomness keeps investors from being able to free-ride on the information other investors know.

We also change the specification of prior beliefs slightly to make the expressions simpler: The asset payoff u is the sum of its rating θ, and the noise in the rating ϵ: $u = \theta + \epsilon$. Prior beliefs are that $\theta \sim N(u, \sigma_\theta^2)$ and $\epsilon \sim N(0, \sigma_\epsilon^2)$, where θ and ϵ are independent. Thus we can write the prior belief about the payoff as $u \sim (\bar{u}, \sigma_u^2)$, where $\sigma_u^2 = \sigma_\theta^2 + \sigma_\epsilon^2$.

Let λ denote the fraction of investors that decide to buy a rating. A rating costs χ. Since investors are ex-ante identical, either all investors buy information ($\lambda = 1$), all investors do not buy information ($\lambda = 0$) or all are indifferent. We use the indifference condition to solve for equilibria where $\lambda \in (0, 1)$. To do this, we need to calculate the expected utility of the informed and the uninformed investors. The first step is to derive their risky asset demands.

Since investors have the same utility function as in the previous section, their equilibrium asset demands satisfy the same first order condition (equation 4). However, the information structure has changed. An informed investor, one who has observed the asset's rating θ has posterior beliefs that the asset payoff is distributed $u \sim N(\theta, \sigma_\epsilon^2)$. Substituting this posterior mean and variance into the first order condition yields informed investors' asset demand:

$$q^I = \frac{1}{\rho}\sigma_\epsilon^{-2}(\theta - pr). \tag{24}$$

For uninformed investors, the asset demand q^U is more complicated. Uninformed investors learn something about θ from observing the asset price p. At the same time, the price depends on investors' demand. This is a fixed point problem. We need to solve for p and q^U jointly.

The price of the risky asset is determined by the market clearing condition

$$\lambda q^I + (1 - \lambda) q^U = x + \bar{x}. \tag{25}$$

Lemma 2 *The price of the risky asset is a linear function of the rating and the random component of the asset supply: $p = A + B\theta + Cx$.*

Uninformed investors combine their prior belief that $\theta \sim N(\bar{u}, \sigma_\theta^2)$ and their signal from the price $(p - A)/B \sim N(\theta, (C/B)^2 \sigma_x 2)$ to form their posterior belief: $\theta \sim N(\hat{\mu}, \sigma_{\theta|p}^2)$ where the posterior variance is

$$\sigma_{\theta|p}^2 \equiv V[\theta|\bar{u}, p] = \left[\sigma_\theta^{-2} + \left(\frac{B}{C}\right)^2 \sigma_x^{-2}\right]^{-1} \tag{26}$$

and the posterior mean is

$$\hat{\mu} = \sigma_{\theta|p}^2 \left[\sigma_\theta^{-2}\bar{u} + \left(\frac{B}{C}\right)^2 \sigma_x^{-2} \left(\frac{p - A}{B}\right)\right]. \tag{27}$$

Therefore, the uninformed investors' optimal portfolio is

$$q^U = \frac{1}{\rho} \frac{\hat{\mu} - pr}{\sigma^2_{\theta|p} + \sigma^2_\epsilon}. \tag{28}$$

Given informed and the uninformed investors' risky asset demand, we can calculate expected utilities. The argument of the utility function is risk aversion times wealth: $\rho q(u - pr) = (E[u] - pr)' V[u]^{-1}(u - pr)$. This is a product of correlated normal variables. To take expected utility, we need to know the expectation of its exponential. Appendix A.2 derives this expectation.

Lemma 3 *The ratio of informed investors' expected utility to uninformed investors' expected utility, before accounting for information cost, is*

$$\frac{E[U^I]}{E[U^U]} = \left(\frac{\sigma^2_\epsilon}{\sigma^2_\epsilon + \sigma^2_{\theta|p}} \right)^{1/2}.$$

This ratio is less than one because utility is negative. Informed investors' utility is higher when it is less negative and therefore smaller in absolute value.

In equilibrium, either there must be a corner solution $\lambda = \{0, 1\}$ or the expected net benefit of information $(E[U^I] - E[U^U])$ must equal the expected utility cost $-e^{\rho\chi} E[U^I]$. Thus, the condition for an interior equilibrium is $(\sigma^2_\epsilon + \sigma^2_{\theta|p})^{1/2} / \sigma_\epsilon = e^{\rho\chi}$.

5.1.2 Can investors acquire biased ratings?

Suppose that some subset of investors can only buy assets that are "investment grade." That means the asset's rating surpasses a threshold $(\theta > \theta^{INV})$. This group of investors then achieves higher expected utility if they obtain an upward-biased rating: Let Φ be the cumulative probability density function for the unbiased rating θ. Suppose that instead of θ, the ratings agency, with the knowledge of the investor, issues a rating $\theta + \epsilon$. Then with probability $\Phi(\theta^{INV} - \epsilon)$, the investor gets a rating that is too low, cannot invest, and gets no income from the risky security. With probability $1 - \Phi(\theta^{INV} - \epsilon)$, the investor can invest and chooses the optimal portfolio in equation (24). This informed investor's expected utility is proportional to

$$U^{bias} \propto - \int_{\theta^{INV} - \epsilon}^{\infty} E \left[\exp \left[-\frac{1}{2} (\theta - pr)^2 \sigma_\epsilon^{-2} \right] \right] d\theta. \tag{29}$$

Since the bias ϵ enters only in the bounds of integration and expands them, and since this is a negative number times an integrand which is strictly positive, ratings bias unambiguously increases expected utility.

Investor-initiated ratings bias has limited price impact If the investors shop for ratings, it is in order to find a ratings agency that gives the asset an investment-grade rating. But once the investor finds that the asset is in his feasible investment set, he should use all available information to determine the optimal bidding strategy for the asset. Since investors use all available information in forming their asset demands, the only price effect of ratings bias is to raise the price of some assets that are not truly investment grade to the level they would be at in the absence of regulation.

5.1.3 Downsides to investor-initiated ratings

Investor-initiated ratings can create two other problems due to information market externalities: information leakage and market collapse due to demand complementarity. Since information requires a fixed cost to discover and is free (or at least quite cheap) to replicate, efficiency dictates that a discovered piece of information should be distributed to every asset investor so that all investors benefit from lower asset payoff risk. Yet, when investors have to pay for ratings themselves, either no investors or too few investors may end up being informed.

Information leakage One reason why asset investors may decide not to buy information is that they can partially free-ride on the information others observe. The price of the asset will depend on what informed investors know. While the uninformed investors cannot literally observe the price before they bid, they can condition on the price when they submit their menu of bids. When deciding on the quantity they will associate with each realized price, the investor asks himself, "If that is the realized price, what would it tell me about what the informed investors have learned?" In this way, uninformed investors can use information contained in prices to free ride on what others have learned.

An increase in the number of informed investors λ reduces the posterior uncertainty of uninformed investors, conditional on the price level $\sigma_{u|p}$. Recall that the noise in the asset price about the rating θ is $(C/B)^2\sigma_x$. Having more informed investors (higher λ) reduces $(C/B)^2$ (equations (31) and (32) in appendix). This makes prices more informative (reduces $\sigma_{u|p}$ in equation (26)) and reduces the benefit of acquiring information (lemma 3). This is a form of strategic substitutability that makes it unlikely that asset investors will ever all choose to be informed.

Complementarity in information demand and market collapse Endogenizing the price of ratings so that what each investor pays $\chi(\lambda)$ depends on how many purchase the information introduces a complementarity that can collapse the market for information. Market collapse is when no investor buys a rating because no other investors are buying them. Such a collapse can arise in situations where the asset issuer would be willing to provide information to all investors.

Instead of assuming that ratings are provided at a fixed cost, we consider a profit-maximizing information-production sector. The sector has three crucial features: First, information can be produced with a fixed-cost technology. A rating θ can be discovered by any agent for a fixed cost c, which is the cost to the ratings agency of collecting information to construct the rating (measured per capita). Once the rating is determined, it is costless to replicate it and sell it to multiple investors. Each investor pays the ratings agency a price χ to observe their rating. Second, reselling purchased information is forbidden. The realistic counterpart to this assumption is intellectual property law that prohibits copying a publication and re-distributing it for profit. Third, there is free entry. Any agent can discover information at any time, even after other information producers have announced their prices χ. That information markets are competitive is crucial. The exact market structure is not.[17]

Lemma 4 *The equilibrium price for information $\chi(\lambda)$ is decreasing in the quantity of information sold λ. Specifically, $\chi(\lambda) = c/\lambda$.*

With an exogenous cost c, there is a unique equilibrium. But with the endogenous price for information in an information market, there can be three equilibria. If $\left((\sigma_u^{-2} + \sigma_\theta^{-2})/\sigma_u^{-2}\right)^{1/2} < e^{\rho c}$, then $\lambda = 0$ is an equilibrium. An investor who wants to acquire information must pay the entire fixed cost c for that information. This cost (in utility terms) exceeds the benefit of information. But it may also be the case that there are an additional two values of $\lambda \in (0, 1]$ such that $\left((\sigma_u^{-2} + \sigma_\theta^{-2})/\sigma_{u|p}^{-2}\right)^{1/2} = e^{\rho c/\lambda}$. The lower of the two solutions will be an unstable equilibrium, but the higher one will be stable. As λ increases, the cost of information drops, precipitously. At the same time, the information content of the asset's price $\sigma_{u|p}^{-2}$ rises gradually, which reduces the benefit of information.

The problem arises when the equilibrium is the no-information ($\lambda = 0$) one. No investors buy information because no other investors are buying information and if a given investor buys information by himself, he would have to bear the entire fixed cost c of discovering that

[17] Veldkamp (2006) analyzes Cournot and monopolistic competition markets for information. All three markets produce information prices that decrease in demand.

information. In such a situation, the biased information provided by the asset-issuer-initiated ratings could result in more accurate asset prices than no information at all.

5.2 Increasing competition among ratings agencies

Because market failures are often associated with a lack of free competition, many have suggested that regulatory barriers inhibiting entry of new ratings agencies be abolished. While this might cure some problems with ratings provision, it does not remedy ratings shopping. In fact, it would worsen the problem.

When the issuer shops for ratings, the more draws the issuer can observe before choosing a rating, i.e. the larger the number of rating agencies, the higher this bias will be.

Proposition 5 *If the asset issuer will disclose only the most favorable rating, then increasing the number of ratings agencies will (weakly) increase the bias of the disclosed rating.*

This result follows from the simple observation that the more rating agencies are available, the greater the possibilities of ratings shopping. Of course, having more ratings agencies does not ensure that an asset issuer will observe more shadow ratings. If not, then the bias will stay constant. However, if some issuers prefer to obtain more shadow ratings than what was previously available to them, increasing the number of agencies will increase the number of observed ratings and the bias from shopping for the best one. It is also possible that the price of shadow ratings falls due to higher competition, encouraging asset issuers to sample more ratings would increase ratings bias even more.

5.3 The effect of risk-management legislation

Another target for criticism in the ratings scandal has been the role that risk-management rules played. Many banks and pension funds are required to hold only investment-grade securities. These are assets who earn sufficiently high ratings from one of the nationally-recognized statistical ratings organizations (NRSRO's) (this group includes Moody's, S&P and Fitch). This rule puts an enormous amount of pressure on asset issuers to ensure their assets achieve this rating. Without it, the pool of potential investors is considerably smaller and the asset's prices will be considerably lower.

Repealing the investment-grade securities regulations alone will not solve the problem of ratings shopping. With investor-initiated ratings, the bias arose without the regulation. However, it is likely that this regulation further encouraged ratings shopping by increasing the payoff for acquiring a high rating, for a given level of ratings uncertainty.

5.4 Mandatory disclosure laws

Perhaps an obvious suggestion is to mandate disclosure of all ratings. While that is a cure in theory, in practice, it is difficult to regulate the transmission of information directly. For example, the line between informal advice and a rating can be easily blurred. Prohibiting a discussion of how various assets might be rated if they were issued could easily be ruled an infringement on free speech. An additional problem is that when undesirable ratings are proposed, the asset in question is frequently restructured. A tiny change in asset structure would make the previous rating no longer applicable and could effectively hide that rating.

6 Conclusions

Examining commonly-forwarded arguments about how ratings may have distorted credit derivatives prices exposed logical gaps. But it also suggested a coherent story about why ratings bias might have emerged and why investors could not use past data to detect it. Developing a model of the market for ratings where asset issuers can shop for ratings revealed circumstances where an increase in asset complexity could generate ratings bias. Solving the model also delivered additional insights. It revealed a feedback effect whereby an increase in asset complexity prompted ratings shopping, which gave issuers and incentive to structure even more complex assets. It also illustrated how more competition in ratings markets could make the distortions in ratings even more severe. An extension of the model where investors purchase ratings uncovered multiple equilibria. This taught us that a move to investor-initiated ratings is risky because markets for ratings may collapse, leaving investors with even less reliable information than before.

None of the policy options we examined were without their drawbacks. Yet, the model points to two possible solutions to the ratings bias problem. Investor-initiated ratings are a cure for bias. The problem of market collapse is mitigated when the investors are large players in their markets who find it valuable to purchase information, even if other investors do not. Since most complex credit products are purchased by institutional investors, rather than households, the large investor assumption might not be a bad one. The second possible solution is to have one ratings agency, a regulated monopoly, that rates every bond. If every asset has one and only one rating, shopping is not possible. Of course, regulated monopolies have less incentive to provide reliable information. However, the incentives for accuracy in the current near-duopoly market are already quite weak. Ultimately, the choice between these two options is a quantity versus quality choice. An investor-initiated system will produce less information, but will be more reliable.

Many markets supply information or certification services: academic testing services, appraisals or job head-hunters are just a few examples. (See also Bar-Isaac, Caruana and Cunat (2008).) Our paper raises the question: What determines the quality of the information produced? It points out that not only does the nature of the good being sold affect the information available about it, but also that the nature of the evaluated products may change to game the ratings system, possibly to a disastrous effect.

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A Technical Appendix: Proofs and Derivations

A.1 Computation Details for the Disclosure Decision 2 versus 0

The question is the following: Is it possible to have $\Pi_{D=2}(\bar{\theta},\underline{\theta})-\Pi_{D=0}>0$ when $\bar{\theta}\leq a\equiv\frac{r(1+\sigma_u^{-2}\sigma_\theta^2)}{x}\chi+\bar{u}-\rho x\sigma_u^2$? When the maximum of the two draws, that is $\bar{\theta}$, is below $a=\frac{r(1+\sigma_u^{-2}\sigma_\theta^2)}{x}\chi+\bar{u}-\rho x\sigma_u^2$, then the second highest, that is $\underline{\theta}$, will also be necessarily below $\frac{r(1+\sigma_u^{-2}\sigma_\theta^2)}{x}\chi+\bar{u}-\rho x\sigma_u^2$. Then, when $\bar{\theta}\leq a$, the highest possible sum for $\bar{\theta}+\underline{\theta}$ is $2a$. We now show that $\Pi_{D=2}(\bar{\theta},\underline{\theta})-\Pi_{D=0}>0$ cannot be satisfied even when $\bar{\theta}+\underline{\theta}$ is $2a$.

To see this, note that

$$
\begin{aligned}
&\left[\frac{1}{r}\frac{\sigma_u^{-2}\bar{u}+\sigma_\theta^{-2}2\cdot\left(\frac{r(1+\sigma_u^{-2}\sigma_\theta^2)}{x}\chi+\bar{u}-\rho x\sigma_u^2\right)-\rho x}{\sigma_u^{-2}+2\sigma_\theta^{-2}}-\frac{1}{r}(\bar{u}-\rho\sigma_u^2 x)\right]x-2\chi\\
=&\left[\frac{1}{r}\frac{\sigma_u^{-2}\bar{u}+\left(\frac{\sigma_\theta^{-2}2r(1+\sigma_u^{-2}\sigma_\theta^2)}{x}\chi+\sigma_\theta^{-2}2\bar{u}-\sigma_\theta^{-2}2\rho x\sigma_u^2\right)-\rho x}{\sigma_u^{-2}+2\sigma_\theta^{-2}}-\frac{1}{r}(\bar{u}-\rho\sigma_u^2 x)\right]x-2\chi\\
=&\left[\frac{1}{r}\frac{\sigma_u^{-2}\bar{u}+\left(\frac{\sigma_\theta^{-2}2r(1+\sigma_u^{-2}\sigma_\theta^2)}{x}\chi+\sigma_\theta^{-2}2\bar{u}-\sigma_\theta^{-2}2\rho x\sigma_u^2\right)-\rho x}{\sigma_u^{-2}+2\sigma_\theta^{-2}}-\frac{1}{r}(\bar{u}-\rho\sigma_u^2 x)\right]x-2\chi\\
=&\left[\frac{1}{r}\frac{\sigma_u^{-2}\bar{u}+\sigma_\theta^{-2}2\bar{u}-\sigma_\theta^{-2}2\rho x\sigma_u^2-\rho x}{\sigma_u^{-2}+2\sigma_\theta^{-2}}+\frac{\sigma_\theta^{-2}2r(1+\sigma_u^{-2}\sigma_\theta^2)}{x(\sigma_u^{-2}+2\sigma_\theta^{-2})}\chi-\frac{1}{r}(\bar{u}-\rho\sigma_u^2 x)\right]x-2\chi\\
=&\left[\frac{1}{r}\frac{\sigma_u^{-2}\bar{u}+\sigma_\theta^{-2}2\bar{u}-\sigma_\theta^{-2}2\rho x\sigma_u^2-\rho x}{\sigma_u^{-2}+2\sigma_\theta^{-2}}-\frac{1}{r}(\bar{u}-\rho\sigma_u^2 x)\right]x+\left(\frac{2(\sigma_\theta^{-2}+\sigma_u^{-2})}{(\sigma_u^{-2}+2\sigma_\theta^{-2})}-2\right)\chi\\
=&\left[\frac{1}{r}\left[\frac{\sigma_u^{-2}\bar{u}+\sigma_\theta^{-2}2\bar{u}}{\sigma_u^{-2}+2\sigma_\theta^{-2}}+\frac{1}{r}\frac{-\rho x\sigma_u^2(\sigma_\theta^{-2}2+\sigma_u^{-2})}{\sigma_u^{-2}+2\sigma_\theta^{-2}}\right]-\frac{1}{r}(\bar{u}-\rho\sigma_u^2 x)\right]x+\left(\frac{2(\sigma_\theta^{-2}+\sigma_u^{-2})}{(\sigma_u^{-2}+2\sigma_\theta^{-2})}-2\right)\chi\\
=&\left[\frac{1}{r}\left[\bar{u}-\rho\sigma_u^2 x\right]-\frac{1}{r}(\bar{u}-\rho\sigma_u^2 x)\right]x+\left(\frac{2(\sigma_\theta^{-2}+\sigma_u^{-2})}{(\sigma_u^{-2}+2\sigma_\theta^{-2})}-2\right)\chi\\
=&\left(\frac{2(\sigma_\theta^{-2}+\sigma_u^{-2})}{(\sigma_u^{-2}+2\sigma_\theta^{-2})}-2\right)\chi<0.
\end{aligned}
$$

A.2 Proof of Lemma 2

The linear price result is analogous to that in Grossman and Stiglitz (1980). We guess that prices take the linear form $p=A+B\theta+Cx$, determine what this price implies for risky asset demands, substitute those demand functions into the market clearing conditions and match coefficients to verify the hypothesis. Substituting asset demands (24) and (28) in to the market clearing condition (25), yields

$$\lambda\sigma_\epsilon^{-2}(\theta-pr)+(1-\lambda)\frac{\hat{\mu}-pr}{\sigma_{\theta|p}^2+\sigma_\epsilon^2}=\rho(\bar{x}+x)$$

Substituting in for $\sigma_{\theta|p}^2$ from (26) and $\hat{\mu}$ from (27) and collecting price terms reveals that the equilibrium price formula is linear in x and θ:

$$A=-\psi\left[\rho\bar{x}+(1-\lambda)\frac{\hat{\sigma}_{\theta|p}^2}{\hat{\sigma}_{\theta|p}^2+\sigma_\epsilon^2}\left(\frac{AB\sigma_x^{-2}}{C^2}-\sigma_\theta^{-2}\mu\right)\right] \tag{30}$$

$$B=\psi\lambda\sigma_\epsilon^{-2} \tag{31}$$

$$C=-\psi\rho \tag{32}$$

where $\psi=[\lambda\sigma_\epsilon^2 r+(1-\lambda)\frac{\hat{\sigma}_{\theta|p}^2}{\hat{\sigma}_{\theta|p}^2+\sigma_\epsilon^2}\frac{\lambda\sigma_\epsilon^{-2}\sigma_x^2}{\rho C}]^{-1}$.

A.3 Proof of Lemma 3

To compute the value of information, we proceed in three steps, using the law of iterated expectations:

$$E[e^{-\rho W}]=E[E[E[e^{-\rho W}|E[u],p]\;|p]]$$

Expected Utility for Uninformed Investor First, take the first expectation, over u. Using the formula for the mean of a log-normal, we get

$$E[u^U|pr] = -\exp[\frac{(\hat{\mu}-pr)^2}{\sigma_e^2+\sigma_{\theta|p}^2} - \frac{1}{2}\left(\frac{(\hat{\mu}-pr)}{\sigma_e^2+\sigma_{\theta|p}^2}\right)^2(\sigma_e^2+\sigma_{\theta|p}^2)]$$

$$E[u^U|pr] = -\exp[-\frac{1}{2}\frac{(\hat{\mu}-pr)^2}{\sigma_e^2+\sigma_{\theta|p}^2}]$$

There is a second expectation over pr that we haven't taken. It turns out we won't need to. So, we leave this line of argument here for now.

Expected Utility for Informed Investor For informed investors, following the same steps yields:

$$= E[e^{-\frac{1}{2}(\theta-pr)'\sigma_e^{-2}(\theta-pr)}]$$

where θ replaced $\hat{\mu}$ as the conditional mean and σ_e^2 is now the conditional variance. Next step: take expectation over θ, but not pr.

Now, this is a moment-generating-function of a quadratic normal (called a Wishart). General formula for multi-variate quadratic forms: If $z \sim N(0,\Sigma)$,

$$E[e^{z'Fz+G'z+H}] = |I-2\Sigma F|^{-1/2}\exp[\frac{1}{2}G'(I-2\Sigma F)^{-1}\Sigma G+H]$$

We need this more general form because $\theta - pr$ is not mean-zero, conditional on pr. It has mean $\hat{\mu} - pr$.

$$\rho W^I = \rho q^i(u-pr) = (\theta-pr)'\sigma_e^{-2}(u-pr)$$

The $\theta - pr$ in the informed investor's expected utility, is a r.v, conditional on pr. It's mean is $\hat{\mu} - pr$ and its variance is $var[\theta|pr] = \sigma_{\theta|p}^2$.

The mean-zero random variable in the moment-generating function formula is $\theta - \hat{\mu}$.

$$F = -\frac{1}{2}\sigma_e^{-2}$$

$$G' = -(\hat{\mu}-pr)\sigma_e^{-2}$$

$$H = -\frac{1}{2}(\hat{\mu}-pr)^2\sigma_e^{-2}$$

$$\Sigma = \sigma_{\theta|p}^2$$

Applying the formula:

$$E[U^I|rp] = -|I-2\sigma_{\theta|p}^2(-\frac{1}{2})\sigma_e^{-2}|^{-1/2}\exp[\frac{1}{2}(\hat{\mu}-pr)^2\sigma_e^{-4}(I+\sigma_{\theta|p}^2\sigma_e^{-2})^{-1}\sigma_{\theta|p}^2 - \frac{1}{2}(\hat{\mu}-pr)^2\sigma_e^{-2}]$$

Note that if we multiply numerator and denominator by σ_e^2, $(I+2\sigma_{\theta|p}^2(\frac{1}{2})\sigma_e^{-2})^{-1} = \frac{\sigma_e^2}{\sigma_e^2+\sigma_{\theta|p}^2}$.

$$= -(\frac{\sigma_e^2}{\sigma_e^2+\sigma_{\theta|p}^2})^{1/2}\exp[\frac{1}{2}(\hat{\mu}-pr)^2\sigma_e^{-2}(\sigma_e^{-2}\frac{\sigma_e^2}{\sigma_e^2+\sigma_{\theta|p}^2}\sigma_{\theta|p}^2 - 1)]$$

Canceling $\sigma_e^2\sigma_e^{-2}$ and rewriting 1,

$$= -(\frac{\sigma_e^2}{\sigma_e^2+\sigma_{\theta|p}^2})^{1/2}\exp[\frac{1}{2}(\hat{\mu}-pr)^2\sigma_e^{-2}(\frac{\sigma_{\theta|p}^2}{\sigma_e^2+\sigma_{\theta|p}^2} - \frac{\sigma_e^2+\sigma_{\theta|p}^2}{\sigma_e^2+\sigma_{\theta|p}^2})]$$

collecting terms in the numerator and setting $\sigma_{\theta|p}^2 - \sigma_{\theta|p}^2 = 0$,

$$= -(\frac{\sigma_e^2}{\sigma_e^2+\sigma_{\theta|p}^2})^{1/2}\exp[\frac{1}{2}(\hat{\mu}-pr)^2\sigma_e^{-2}(\frac{-\sigma_e^2}{\sigma_e^2+\sigma_{\theta|p}^2})]$$

note that $\sigma_e^{-2} * (-\sigma_e^2) = -1$.

$$= (\frac{\sigma_e^2}{\sigma_e^2+\sigma_{\theta|p}^2})^{1/2}\exp[-\frac{1}{2}\frac{(\hat{\mu}-pr)^2}{\sigma_e^2+\sigma_{\theta|p}^2}]$$

$$E[U^I|rp] = (\frac{\sigma_e^2}{\sigma_e^2+\sigma_{\theta|p}^2})^{1/2}E[U^U|rp]$$

The ratio of variances is a known quantity when information is acquired. All agents can infer the equilibrium strategies of other agents and deduce how much information will be revealed through the price level. Since the two expected utilities are related by a known constant, their unconditional expectations $E[U^I]$ and $E[U^U]$ must be proportional as well.

A.4 Proof of Lemma 4

Let $d_{it} = 1$ if agent i decides to discover information in period t and $d_{it} = 0$ otherwise. Let per capita demand for information with price χ_{it}, given all other posted prices χ_{-it}, be $I(\cdot, \cdot)$. Then the objective of the information producer is to maximize profit:

$$\max_{d_{it}, \chi_{it}} d_{it}(\chi_{it} I(\chi_{it}, \chi_{-it}) - c). \tag{33}$$

Suppose the equilibrium information price was above average cost $\chi > c/\lambda$. Then, an alternate supplier could enter the market with a slightly lower price, and make a profit. If a supplier set price below marginal cost, they would make a loss. This strategy would be dominated by no information provision. If there are two or more suppliers, then either price is above marginal cost, which can't be an equilibrium by the first argument, or both firms price at (or below) marginal cost, split the market, and make a loss, which is dominated by exit.

B Ratings Agencies' Incentive to Bias Ratings

Here we sketch a model where ratings agencies can choose to bias ratings if they find it profitable to do so. The agency's utility depends on its profits and a reputation cost that is a quadratic function of the distance between their forecast and the true asset payoff.

Suppose that once a rating is ordered, it is paid for. Then the rating agencies payoff is given by:

$$U^r = \chi + \tilde{\chi} - c - \alpha(\theta_i - u)^2,$$

where c stands for the cost of producing a rating. Suppose that the investor of a rating can observe the rating before purchasing it. Then, in this case the profit of the rating agency depends on the probability that the asset issuer purchases their rating $\pi(\theta_i)$ and the ratings price χ

$$U^r = \pi(\tilde{\theta}_i)(\chi + \tilde{\chi}) - \alpha(\tilde{\theta}_i - u)^2 - c. \tag{34}$$

When the investor of a rating (so far we have looked at issuers buying ratings) buys the highest of all ratings, than

$$\pi_i(\tilde{\theta}_i) = prob(\tilde{\theta}_i = \max_{j \in I} \tilde{\theta}_j).$$

There are two equivalent ways to model "ratings inflation:" One way is to assume that rating agencies draw a rating θ_i from an unbiased distribution, but they report a different rating $\tilde{\theta}_i = r(\theta_i)$. The other is to model rating agencies who draw from biased distributions. We sketch the first approach.

Assuming that all rating agencies use the same technology to produce ratings (draw ratings from the same distribution), and that all I rating agencies use the same monotonic (strictly increasing in θ_i) reporting strategy, then

$$\pi(\theta_i) = F^{I-1}(r^{-1}(\tilde{\theta}_i)),$$

and the payoff of rating agency is given by

$$U^r = F^{I-1}(r^{-1}(\tilde{\theta}_i))(\chi + \tilde{\chi}) - \alpha(\tilde{\theta}_i - u)^2 - c.$$

Heuristic derivation of the equilibrium reporting strategy:

First let G denote the distribution of the highest order statistic, namely $G = F^{I-1}$. We also use g to denote its density. Then U^r can be rewritten as:

$$U^r = G(r^{-1}(\tilde{\theta}_i))(\chi + \tilde{\chi}) - \alpha(\tilde{\theta}_i - u)^2 - c.$$

Maximizing this expression with respect to $\tilde{\theta}_i$ we get the following first order condition:

$$\frac{g(r^{-1}(\tilde{\theta}_i))(\chi + \tilde{\chi})}{r'(r^{-1}(\tilde{\theta}_i))} - 2\alpha(\tilde{\theta}_i - u) = 0$$

Now recalling that $\tilde{\theta}_i = r(\theta_i)$ the above first order condition can be rewritten as:

$$\frac{g(\theta_i)(\chi + \tilde{\chi})}{r'(\theta_i)} - 2\alpha(r(\theta_i) - u) = 0$$

which in turn can be rewritten as:

$$g(\theta_i)(\chi + \tilde{\chi}) = r'(\theta_i) \cdot 2\alpha(r(\theta_i) - u).$$

This is a first order differential equation. To make things more transparent for its solution, let $t = \theta_i$ and $r = y$, then it can be rewritten as

$$g(t)(\chi + \tilde{\chi}) = y'(t) \cdot 2\alpha(y(t) - u),$$

or

$$\begin{aligned} y'(t) &= \frac{g(t)(\chi + \tilde{\chi})}{2\alpha(y(t) - u)} \\ \frac{dy(t)}{dt} &= \frac{g(t)(\chi + \tilde{\chi})}{2\alpha(y(t) - u)} \\ dy(t) \cdot 2\alpha(y(t) - u) &= g(t)(\chi + \tilde{\chi}) \cdot dt \end{aligned}$$

integrating the left side with respect to y and the right side with respect to t we get that:

$$\int_y 2\alpha(y(t) - u) dy(t) = \int_t g(t)(\chi + \tilde{\chi}) \cdot dt + c$$

where c is a constant. The solution to this differential equation characterizes the optimal bias of the ratings agencies.