A Model of Central Bank Liquidity Provision

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**Policy Questions**

When a central bank provides liquidity through collateralized loans (e.g. intraday central bank liquidity, overnight liquidity facility) such as in the Canadian case through SLF or SPRA/SRA:

- How should it design its collateral policy?
- In particular, how should it determine its *haircut* policy?
WHAT ARE “HAIRCUTS”? 

Borrowing Constraint: \( L_t \leq A_t \psi_t (1 - h) \)

where \( L \): loans, \( A \): asset, \( \psi \): asset price, \( h \): haircut
Motivation

Research into haircut policy is motivated by the following questions

- What is the essential trade-off involved in setting haircuts?
- What are the equilibrium effects of changing haircuts?
- What are the welfare implications of collateralized lending policy?
- What are the key factors that determine an optimal haircut? (e.g. collateral types, borrowers, lending mechanism)
Motivation

Payment systems all transactions in most settlement systems are subject to “collateral-in-advance” constraints

liquidity provision Central banks need guidance for their collateral policy
Develop a Tractable Model of Liquidity Provision in a Settlement System

Four building blocks:

1. Portfolio choice: liquid vs illiquid assets
2. Uncertain liquidity needs $\Rightarrow$ CB liquidity provision
3. Potential for default $\Rightarrow$ Collateral requirement
4. Asset price uncertainty $\Rightarrow$ Haircuts
Findings

• A central bank liquidity facility is a portfolio of two types of insurance:
  1. Insurance against liquidity risk
  2. Insurance against downside risk of asset

• Setting a haircut involves a trade-off between:
  • Relax liquidity constraint of illiquid agents
  • Tighten liquidity constraint of liquid agents through:
    1. Lower value of liquid asset
    2. Increased opportunity cost of holding liquid asset
    3. Distortion of the portfolio choice

• The optimal haircut is higher when:
  • Default incentives and portfolio choices respond strongly to haircut change
  • Volatility of asset prices is higher
  • Unable to target lending to agents who really need liquidity
Model

- Time is discrete: \( t = 0, 1, 2, \ldots \)
- Continuum of infinitely lived agents
- Three consecutive sub-periods (denoted by \( s \)):
  - AM: centralized asset market (portfolio choice) \((s = 1)\)
  - DM: decentralized goods market (liquidity need) \((s = 2)\)
  - CM: centralized market (settlement) \((s = 3)\)
**Preference**

Period utility of an agent

\[ u(q^b_2) - q^s_2 - h_3, \]

where

- \( q^b_2 \): consumption of the DM goods when the agent is a buyer
- \( q^s_2 \): production of the DM goods when the agent is a seller
- \( h_3 \): production (net of consumption) of the CM goods
- \( \beta \): discount factor
Portfolio: Money and Asset

- $M_t$: liquid asset (e.g. fiat money/ bank reserves)
  - exogenous growth rate $\gamma$
- $A_t$: illiquid asset (e.g. claims to investment projects)
  - endowed with $A$ projects at the beginning of a period
  - each unit yields $\delta_t$ units of CM goods at the end of a period
  - $\delta$ is a random i.i.d. (over time and across owners) variable: $\delta_t \sim U(\bar{\delta}(1 - \varepsilon), \bar{\delta}(1 + \varepsilon))$, and with $\bar{\delta} < 1$
  - Price of asset: $\psi_s, s = 1, 2, 3$
Sub-Period 1: Asset Market AM

- An agent starts with \((m_1, A)\) and receives signal \(S \in \{H, L\}\):
  - \(H\): likely to become a buyer in the DM (high liquidity need)
  - \(L\): likely to become a seller in the DM (low liquidity need)
- Given the signal, agents trade in AM and make portfolio choice \((m_2, a_2)\)
- The signal turns out to be incorrect with a probability \(\theta < \frac{1}{2}\)
  - an agent with \(H\) signal will be a buyer with prob. \(\sigma^H = 1 - \theta\)
  - an agent with \(L\) signal will be a buyer with prob. \(\sigma^L = \theta\)
An agent starts with \((m_2, a_2)\)

- The trading status realizes: buyer or seller
- Trading subject to liquidity constraint (only \(m\) is accepted)
- Before trade, agents have access to central bank lending facilities:
  - Borrow a nominal loan \(l_2\) by posting asset as collateral
  - The loan has to be settled in the next CM
Sub-Period 3: Settlement CM

- An agent starts with \((m_3, a_3, l_2)\), and \(\delta_t\) is realized
- Agents decide whether to settle the loan \(l_2\) or to default
- Agents trade \(h_3\), and choose \(m_{+1}\) for next period
Reducing The Value of Holding Liquid Asset

MC of liquidity = MB of liquidity

$$\phi_3(1 + i) = \frac{1}{2}(\lambda^H + \lambda^L)$$

where

$$\lambda^H = \phi_3(1 + \sigma^H \Delta^H)$$

$$\lambda^L = \frac{\bar{\delta}}{\psi_1} \{1 + \sigma^L [\Delta^L(h) + S(h)](1 - h)\}$$

$$\Delta^j = u'(q^i) - 1$$

$$q^H = 2M\phi_3$$

$$q^L = 2A\bar{\delta}(1 - h)$$

$$h \downarrow \Rightarrow q^L \uparrow \Rightarrow \Delta^L \downarrow \quad \text{(relax L-type liquidity constraint)}$$

$$\Rightarrow \lambda^L \downarrow \Rightarrow \phi_3 \downarrow \Rightarrow q_H \downarrow \quad \text{(tighten H-type liquidity constraint)}$$
Increasing the Opportunity Cost of Holding Liquid Asset

Fisher’s equation

\[ 1 + i = \frac{\gamma}{\beta} \geq \frac{\sigma^L A}{\beta M} E[S(h)] + \frac{1}{\beta}, \]

where \( E[S(h)] = \frac{\bar{\delta}}{4\phi_3 \varepsilon} (\varepsilon - h)^2 \)

\[ h \downarrow \Rightarrow E[S(h)] \uparrow \text{(insure against downside risk)} \]
\[ \Rightarrow \gamma \uparrow \Rightarrow i \uparrow \Rightarrow \phi_3 \downarrow \Rightarrow q_H \downarrow \text{(tighten } H\text{-type liquidity constraint)} \]
Distorting Portfolio Choice

\[ \phi_3(1 + i) = \frac{1}{2}(\lambda^H + \lambda^L) \]

where

\[ \lambda^L = \frac{\bar{\delta}}{\psi_1} \{1 + \sigma^L(\Delta^L(h) + S(h))(1 - h)\} \]

\[ h \downarrow \Rightarrow \text{induce } H\text{-type to hold more illiquid asset } \Rightarrow q_H \downarrow \]

\[ \Rightarrow \psi_1 \uparrow \Rightarrow \phi_3 \downarrow \Rightarrow q_H \downarrow \text{ (tighten } H\text{-type liquidity constraint)} \]
CONCLUSION

We have developed a model of collateralized central bank lending and shown the:

1. Equilibrium effects of reducing haircuts
   
   \[ h \downarrow \Rightarrow \text{provide liquidity insurance} \]
   
   \[ h \downarrow \Rightarrow (i) \text{lower value of liquid asset} \]
   
   \[ \Rightarrow (ii) \text{increase opp. cost of holding liquid asset} \]
   
   \[ \Rightarrow (iii) \text{distort portfolio choice} \]

2. Optimal haircut is lower if
   
   - Downside risk of collateral is low [small (ii)]
   - Perfect enforcement [no (ii)] or exogenous default [small (ii)]
   - CB can target lending to agents really in need of liquidity [small (ii)]
   - Portfolio choice insensitive to haircut change [small (iii)]
   - It is an unanticipated, temporary cut in \( h \) [no (i), (iii), small (ii)]