The views expressed here are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.
The credit crisis began in Aug. 2007 in the interbank market, as shown in this graph of the spread of 3-month Libor over Treasuries. Our estimation sample period ends in July 2008 during the period in which the Fed’s liquidity operations were being sterilized.
The Federal Reserve initially responded to these signs of stress on December 12, 2007 with two central bank liquidity operations:

- the establishment of reciprocal swap lines with the ECB and the SNB, and
- the creation of the Term Auction Facility (TAF), whose first auction was held on December 17, 2007.

**Question of interest:**
Were these operations successful in lowering Libor rates?

This is a challenging question because of:

- the need to account for variations in the risk-free rate and
- the need to account for variations in credit and liquidity risk.
Motivation: Spreads on 3m AA-rated financial debt

Other bank rates moved also, as in graph of the spreads of 3-month AA-rated US banks and financials over Treasuries. Similar pattern in this market, which has similar credit risk, but different market characteristics.
Motivation: 3-month AA-rated financial debt over Libor

This graph shows the spread of 3-month AA-rated US financials over Libor from 1995 through January 2009.

Given that the credit rating is common at AA, the differences must be caused by other factors (most likely liquidity concerns).
Two prior studies using event study methodologies:

- Taylor & Williams (2009) — no significant effect of the TAF on spreads
- McAndrews, Sarkar & Wang (2008) — TAF has helped ease strains in the interbank market

These studies account for:

- variations in the risk-free rate using short-term OIS rates and
- variations in credit risk using bank-related CDS spreads.

Our approach:

- use the entire Treasury curve to control for risk-free rates and
- use the term structure of financial corporate debt to control for credit risk.

Key assumption:
Libor rates have credit risk characteristics in common with senior unsecured AA-rated debt issued by US financial firms.
Overview: Our modeling approach

We model the yields in question using an arbitrage-free, dynamic Nelson-Siegel (AFNS) model that encompasses:

- the Treasury yield curve
  — 3 factors as per Christensen, Diebold, Rudebusch (2007)
- corporate debt curves for AA-rated banks & financial firms
  — 2 factors as per Christensen & Lopez (2007)
- Term LIBOR rates
  — 1 factor as per Feldhütter and Lando (2006)

This six-factor model estimated over our sample period suggests:

- Central bank liquidity operations, timed with the first TAF auction, altered the dynamics of the model-implied LIBOR factor and thus directly affected the interbank market.
- By how much? Our counterfactual analysis suggests 3-m LIBOR spread would have been 70 bp higher, on average.
Six-factor AFNS model: Treasury yields

Treasury yield curve data:
Zero-coupon Treasury yields at maturities of:
— 3m, 6m, 1y, 2y, 3y, 5y, 7y and 10y
(Source: BOG as per Gürkaynak et al. (2007))

Christensen, Diebold & Rudebusch (2007) propose an AFNS model with 3 factors:

\[ y^T_t(\tau) = L^T_t + \left( 1 - e^{-\lambda^T_{T\tau}} \right) S^T_t + \left( \frac{1 - e^{-\lambda^T_{T\tau}}}{\lambda^T_{T\tau}} - e^{-\lambda^T_{T\tau}} \right) C^T_t + \frac{A^T(\tau)}{\tau} . \]

where

- \( y^T_t(\tau) = \) Treasury yield for maturity \( \tau \) at time \( t \)
- \( L^T_t = \) Treasury level factor at time \( t \)
- \( S^T_t = \) Treasury slope factor at time \( t \)
- \( C^T_t = \) Treasury curvature factor at time \( t \)
- \( A^T(\tau) = \) CDR yield-adjustment
Six-factor AFNS model: Financial corporate yields

Financial corporate bond yield curve data:
Zero-coupon yields from Bloomberg at same maturities
Categories: U.S. financials, AA and A-rated
U.S. banks, AA, A and BBB-rated

Christensen & Lopez (2008) suggest a 5-factor AFNS model.

\[ y_{t}^{i,c}(\tau) = \left(1 + \alpha_{LT}^{i,c}\right)L_{t}^{T} + \left(1 + \alpha_{ST}^{i,c}\right)\left(1 - \frac{e^{-\lambda_{T}^{T}\tau}}{\lambda_{T}^{T}\tau}\right)S_{t}^{T} \]

\[ + \left(1 + \alpha_{ST}^{i,c}\right)\left(1 - \frac{e^{-\lambda_{S}^{T}\tau}}{\lambda_{S}^{T}\tau} - e^{-\lambda_{S}^{T}\tau}\right)C_{t}^{T} \]

\[ + \alpha_{0}^{i,c} + \left(\alpha_{LS}^{i,c}\right)L_{t}^{S} + \left(\alpha_{SS}^{i,c}\right)\left(1 - \frac{e^{-\lambda_{S}^{S}\tau}}{\lambda_{S}^{S}\tau}\right)S_{t}^{S} + \frac{A_{i,c}(\tau)}{\tau}, \]

where

- \( y_{t}^{i,c}(\tau) \) = (sector,rating) yield for maturity \( \tau \) at time \( t \)
- \( L_{t}^{S} \) = credit risk level factor at time \( t \)
- \( S_{t}^{S} \) = credit risk slope factor at time \( t \)
- \( A_{i,c}(\tau) \) = CDR yield-adjustment
LIBOR yield curve data:
LIBOR rates for 3, 6 and 12 months; AA-rated bank panel
We use AA-rated financials as the base rate because the AA-rated bank data does not encompass sample period.

\[ y_t^{Lib}(\tau) = \left(1 + \alpha_{LT}^{Fin,AA}\right)L_t^T + \left(1 + \alpha_{ST}^{Fin,AA}\right)\left(\frac{1 - e^{-\lambda T\tau}}{\lambda T\tau}\right)S_t^T + \alpha_0^{Fin,AA} + \alpha^{Lib} + \left(1 + \alpha_{ST}^{Fin,AA}\right)\left(1 - e^{-\lambda T\tau}/\lambda T\tau - e^{-\lambda T\tau}\right)C_t^T + \left(\alpha_{LS}^{Fin,AA}\right)L_t^S + \left(\alpha_{SS}^{Fin,AA}\right)\left(1 - e^{-\lambda S\tau}/\lambda S\tau\right)S_t^S + \left(1 - e^{-\kappa_{Lib}^Q T\tau}/\kappa_{Lib}^Q T\tau\right)X_t^{Lib} + \frac{A^{Lib}(\tau)}{\tau}, \]

where
- \( y_t^{Lib}(\tau) = \) LIBOR yield for maturity \( \tau \) at time \( t \)
- \( X_t^{Lib} = \) LIBOR-specific factor at time \( t \)
- \( A^{Lib}(\tau) = \) CDR yield-adjustment
Estimation and model specification

We use the Kalman filter since the model is linear and Gaussian. Also, the Kalman filter can easily handle the missing data for the AA-rated banks before September 2001.

Measurement equation:
\[
y_t = \begin{pmatrix} y^c_t \\ y^T_t \\ y^\text{Lib}_t \end{pmatrix} = \begin{pmatrix} A^c \\ A^T \\ A^\text{Lib} \end{pmatrix} F_t + \begin{pmatrix} B^c \\ B^T \\ B^\text{Lib} \end{pmatrix} + \varepsilon_t
\]

Transition equation:
\[
F_t = (I - \exp(-K^P))\mu^P + \exp(-K^P)F_{t-1} + \eta_t
\]

where
- $F_t$ is the (6x1) vector of the factors
- $\mu^P$ is the (6x1) mean vector
- $K^P$ is the (6x6) mean-reversion matrix.
Our primary specification focus is on $K^P$, which governs how the model’s 6 factors interact.

\[
\begin{pmatrix}
\kappa_{11}^P & \kappa_{12}^P & \kappa_{13}^P & \kappa_{14}^P & \kappa_{15}^P & \kappa_{16}^P \\
\kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P & \kappa_{24}^P & \kappa_{25}^P & \kappa_{26}^P \\
\kappa_{31}^P & \kappa_{32}^P & \kappa_{33}^P & \kappa_{34}^P & \kappa_{35}^P & \kappa_{36}^P \\
\kappa_{41}^P & \kappa_{42}^P & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P & \kappa_{46}^P \\
\kappa_{51}^P & \kappa_{52}^P & \kappa_{53}^P & \kappa_{54}^P & \kappa_{55}^P & \kappa_{56}^P \\
\kappa_{61}^P & \kappa_{62}^P & \kappa_{63}^P & \kappa_{64}^P & \kappa_{65}^P & \kappa_{66}^P
\end{pmatrix}
\begin{pmatrix}
L_t^S \\
S_t^T \\
L_t^T \\
C_t^T \\
X_t^{Lib}
\end{pmatrix}
\]
Our primary specification focus is on $K^P$, which governs how the model’s 6 factors interact.

\[
\begin{pmatrix}
\kappa_{11}^P & \kappa_{12}^P & \kappa_{13}^P & \kappa_{14}^P & \kappa_{15}^P & \kappa_{16}^P \\
\kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P & \kappa_{24}^P & \kappa_{25}^P & \kappa_{26}^P \\
\kappa_{31}^P & \kappa_{32}^P & \kappa_{33}^P & \kappa_{34}^P & \kappa_{35}^P & \kappa_{36}^P \\
\kappa_{41}^P & \kappa_{42}^P & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P & \kappa_{46}^P \\
\kappa_{51}^P & \kappa_{52}^P & \kappa_{53}^P & \kappa_{54}^P & \kappa_{55}^P & \kappa_{56}^P \\
\kappa_{61}^P & \kappa_{62}^P & \kappa_{63}^P & \kappa_{64}^P & \kappa_{65}^P & \kappa_{66}^P \\
\end{pmatrix}
\begin{pmatrix}
L_t^S \\
S_t^T \\
L_t^T \\
C_t^T \\
X_t^{Lib}
\end{pmatrix}
\]

Assumption #1:
As in Feldhütter and Lando (2006) and to sharpen our focus on the interbank market, we assume the Libor-specific factor is independent of the other factors.
Our primary specification focus is on $K^P$, which governs how the model’s 6 factors interact.

Assumption #2:
Drawing on Christensen and Lopez (2008), we assume that $L_t^T$ is independent and does not affect the credit risk factors.
Our primary specification focus is on $K^P$, which governs how the model’s 6 factors interact.

\[
\begin{pmatrix}
\kappa_{11}^P & \kappa_{12}^P & 0 & \kappa_{14}^P & \kappa_{15}^P & 0 \\
\kappa_{21}^P & \kappa_{22}^P & 0 & \kappa_{24}^P & \kappa_{25}^P & 0 \\
0 & 0 & \kappa_3^P & 0 & 0 & 0 \\
\kappa_{41}^P & \kappa_{42}^P & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P & 0 \\
\kappa_{51}^P & \kappa_{52}^P & \kappa_{53}^P & \kappa_{54}^P & \kappa_{55}^P & 0 \\
0 & 0 & 0 & 0 & 0 & \kappa_{66}^P
\end{pmatrix}
\begin{pmatrix}
L_t^S \\
S_t^T \\
L_t \\
S_t^T \\
C_t^T \\
X_t^{Lib}
\end{pmatrix}
\]

**Assumption #3:**

Drawing on Christensen and Lopez (2008), we assume that $L_t^T$ and $C_t^T$ affect $S_t^T$, but not each other and that $S_t^T$ does not affect the other two factors.
Our primary specification focus is on $K^P$, which governs how the model’s 6 factors interact.

\[ \begin{pmatrix}
  \kappa_{11}^P & \kappa_{12}^P & 0 & \kappa_{14}^P & \kappa_{15}^P & 0 \\
  \kappa_{21}^P & \kappa_{22}^P & 0 & \kappa_{24}^P & \kappa_{25}^P & 0 \\
  0 & 0 & \kappa_{33}^P & 0 & 0 & 0 \\
  \kappa_{41}^P & \kappa_{42}^P & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P & 0 \\
  \kappa_{51}^P & \kappa_{52}^P & 0 & 0 & \kappa_{55}^P & 0 \\
  0 & 0 & 0 & 0 & 0 & \kappa_{66}^P
\end{pmatrix} \begin{pmatrix}
  L_t^S \\
  S_t^S \\
  L_t^T \\
  S_t^T \\
  C_t^T \\
  X_t^{Lib}
\end{pmatrix} \]

Assumption #4:

Drawing on Christensen and Lopez (2008), we assume no feedback from $L_t^S$ to $C_t^T$ and no feedback from $S_t^S$ to $S_t^T$. 
Our preferred specification of $K^P$ is then:

$$
\begin{pmatrix}
\kappa_{11}^P & \kappa_{12}^P & 0 & \kappa_{14}^P & \kappa_{15}^P & 0 \\
\kappa_{21}^P & \kappa_{22}^P & 0 & \kappa_{24}^P & \kappa_{25}^P & 0 \\
0 & 0 & \kappa_{33}^P & 0 & 0 & 0 \\
\kappa_{41}^P & 0 & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P & 0 \\
0 & \kappa_{52}^P & 0 & 0 & \kappa_{55}^P & 0 \\
0 & 0 & 0 & 0 & 0 & \kappa_{66}^P \\
\end{pmatrix}
\begin{pmatrix}
L_t^S \\
S_t^S \\
L_t^T \\
S_t^T \\
C_t^T \\
X_t^{Lib} \\
\end{pmatrix}
$$

Based on the likelihood ratio test relative to the specification with the independent Libor factor,

$$
LR = 2 \times (180, 128.91 - 180, 125.11) = 7.60;
$$

i.e., we cannot reject the null hypothesis that these additional zero restrictions are reasonable.
Empirical results: the LIBOR factor

In the paper, we present the full set of estimation results and discover several areas of interest and further research. Here, we focus most directly on the implications for the interbank market.

The Libor-specific factor changes direction sharply after the first TAF auction, dropping below its historical mean and 2-s.d. band.
Does this drop constitute a structural break in the Libor factor?

We can test this hypothesis within the Kalman filter by imposing different parameters before and after that date for the Libor factor in the transition equation.

Before Dec. 21, 2007,  
\[ \psi_{\text{pre}}^{\text{Lib}} = (\kappa_{66}^P, \theta_{\text{Lib}}^P, \sigma_{\text{Lib}}, \kappa_{\text{Lib}}^Q, \alpha_{\text{Lib}}) \]

Afterward,  
\[ \psi_{\text{post}}^{\text{Lib}} = (\tilde{\kappa}_{66}^P, \tilde{\theta}_{\text{Lib}}^P, \tilde{\sigma}_{\text{Lib}}, \tilde{\kappa}_{\text{Lib}}^Q, \tilde{\alpha}_{\text{Lib}}) \]

For the null hypothesis that no break occurred, the LR test value is 52.14, which is highly significant under \( \chi^2(5) \).

So, the data supports that the dynamics of the Libor factor were modified after the first TAF auction, suggesting that central bank liquidity operations had an effect.
Empirical results: counterfactual

What was the magnitude of this effect?

We address this question within our model by "turning off" the Libor factor; that is, we keep the Libor factor constant at its long-term mean over the full sample.
Empirical results: counterfactual

What was the magnitude of this effect?

The counterfactual 3m-Libor spread rose to 200 basis points just before the Bear Stearns rescue.
Empirical results: counterfactual

What was the magnitude of this effect?

We address this question within our model by "turning off" the Libor factor; that is, after Dec. 21, 2007, we keep the Libor factor constant at its long-term mean.

The average difference between the observed and counterfactual 3-month Libor spread to Treasuries from Dec. 21, 2007 through July 25, 2008 was 71.2 basis points.

Again, suggesting that central bank liquidity operations had the effect of lowering interbank lending rates.
Summary and conclusions

What effect have central bank liquidity operations had on interbank lending rates?

We focus on the crisis period from the first TAF auction through July 2008, a period in which these operations were sterilized and did not entail unconventional monetary policy actions.

Using a 6-factor AFNS model that incorporates Treasury yields, financial corporate bond yields, and Libor rates, our empirical results suggest:

- the TAF auctions significantly affected the dynamics of the interbank market, as shown by the structural break in the behavior of our model-implied Libor factor, and
- these operations kept the Libor rate roughly 70 basis points lower than it could have been in their absence.
Empirical results: postscript

Updating the estimation results through January 2009, the Libor specific factor has dropped even further.
Updating the estimation results through January 2009, the counterfactual spread has moved higher, averaging 300 bp since October.
If you have any questions or comments, please contact me at

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