Estimating Real and Nominal Term Structures using Treasury Yields, Inflation, Inflation Forecasts, and Inflation Swap Rates

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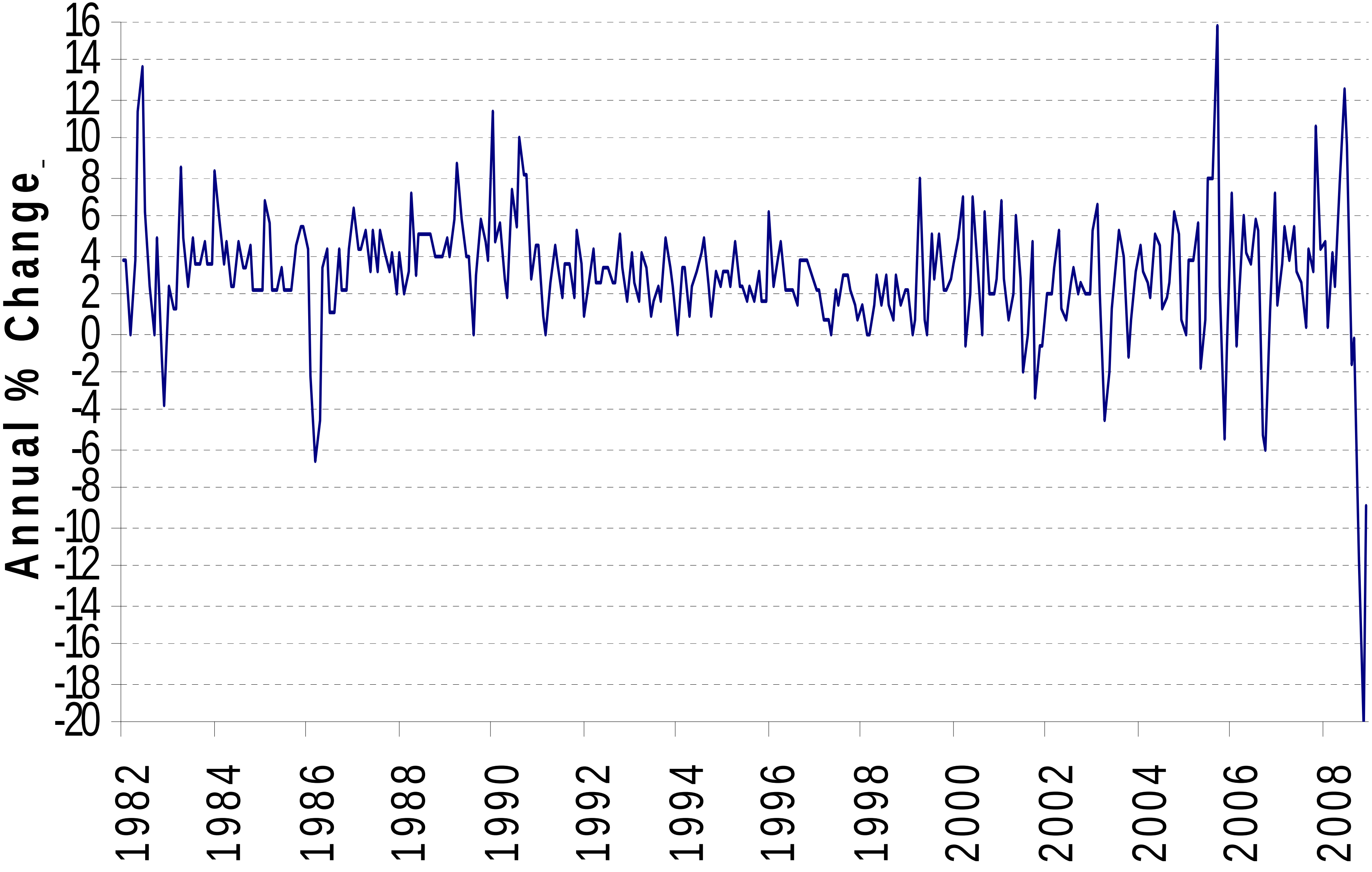
Consumer Price Index Growth Rates Each Month, Jan 1982 – Dec 2008*



* Annualized growth rates for each month

Motivation

- The volatility of inflation has increased in recent years.
  - The U.S. Consumer Price Index (CPI) grew 5.4 % for the year ending in July 2008. Since then it has declined at an 8.6 % annual rate.
  - Recession along with rising Federal budget deficits and money creation will make inflation forecasting challenging.
- Greater inflation uncertainty is likely to heighten the popularity of inflation-indexed bonds and inflation derivatives.

Purpose of This Paper

- Develop a model of the term structure of real and nominal interest rates that captures different economic environments with changing volatilities.
- Value inflation-indexed bonds and inflation derivatives.
- Present a technique for estimating the model's parameters.
- Provide insights into the characteristics of real interest rates, inflation, and real and inflation risk premia that may be helpful to policymakers (central banks), investors, and firms.

Preview of Results

- Our equilibrium model of real and nominal term structures permits inflation, expected inflation, inflation's 'central tendency,' and real interest rates to follow imperfectly correlated processes.
- Extending Heston and Nandi (2003), these processes display stochastic (GARCH) volatilities.
- We obtain closed-form solutions for the values of nominal bonds, inflation-indexed bonds having an indexation lag, and inflation swaps.
- The model's parameters are estimated using nominal Treasury yields, survey forecasts of inflation rates, and inflation swap rates.

Preview of Results (continued)

- Empirically, permitting stochastic volatility is important for describing real interest rates and expected inflation.
- Real interest rates and expected inflation display strong mean-reversion. Inflation's central tendency does not.
- Real interest rate risk premia for a ten-year bond vary between 150 and 170 basis points.
- Inflation risk premia for a ten-year bond vary between 38 and 60 basis points.
- The model's implied inflation-indexed yields closely match TIPS yields during the 2004 to 2008 period.
- Stock market returns react negatively to the model's implied shocks to short and long run inflation.

The Model

- Four sources of uncertainty, $\epsilon_{j,t+\Delta t} h_{j,t}$, $j = 1, 2, .., 4$, drive real and nominal term structures.
- Each $\epsilon_{j,t+\Delta t} h_{j,t}$ has a corresponding market price of risk, $\phi_j h_{j,t}^2$.
- The volatility factors, $h_{j,t}$, $j = 1, ..., 4$ follow the Nonlinear Asymmetric GARCH model of Engle and Ng (1993).

$$h_{j,t+\Delta t}^2 - h_{j,t}^2 = \left[d_{j0} + d_{j1} h_{j,t}^2 + d_{j2} \left(\epsilon_{j,t+\Delta t} - d_{j3} h_{j,t} \right)^2 \right] \Delta t$$

Dynamics of the State Variables

- Let I_t be the date t CPI and π_t be the one-period rate of expected inflation. Then

$$\ln\left(I_{t+\Delta t}/I_t\right) = \pi_t \Delta t - \frac{1}{2} h_{1,t}^2 \Delta t + h_{1,t}\sqrt{\Delta t}\,\epsilon_{1,t+\Delta t}$$

- Let r_t be the one-period real interest rate and α_t be inflation's 'central tendency.' The dynamics of π_t, r_t, and α_t satisfy

$$\begin{aligned}
\pi_{t+\Delta t} - \pi_t &= \left[\alpha_t + a_1 r_t + a_2 \pi_t\right]\Delta t + \sqrt{\Delta t}\sum_{j=1}^{2}\beta_j h_{j,t}\epsilon_{j,t+\Delta t} \\
r_{t+\Delta t} - r_t &= \left[b_0 + b_1 r_t + b_2 \pi_t\right]\Delta t + \sqrt{\Delta t}\sum_{j=1}^{3}\gamma_j h_{j,t}\epsilon_{j,t+\Delta t} \\
\alpha_{t+\Delta t} - \alpha_t &= \left[c_0 + c_1 \alpha_t\right]\Delta t + \sqrt{\Delta t}\sum_{j=1}^{4}\rho_j h_{j,t}\epsilon_{j,t+\Delta t}
\end{aligned}$$

Bond Yields, Expected Inflation Rates, and Inflation Swap Rates

- Analytic solutions that are linear in the state variables π_t, r_t, α_t, and $h^2_{j,t}$, $j = 1, \ldots, 4$ are derived for multiperiod
  1. Yields on nominal Treasury bonds
  2. Yields on inflation-indexed bonds having an indexation lag (TIPS)
  3. Expectations of inflation rates
  4. Inflation swap rates (which equal the difference between nominal and inflation-indexed yields)
- Monte Carlo methods can be used to price other inflation derivatives (e.g., caps and floors).

Data

- The model's parameters are estimated using the following data observed on the first trading day of each month:

1. Zero-coupon nominal Treasury yields for maturities of 1, 3, and 6 months and 1, 2, 3, 5, 7, 10, and 15 years from Gurkaynak, Sack, and Wright (2007) over the period January 1982 to June 2008.
2. Survey forecasts of the CPI from Blue Chip Economic Indicators for up to eight future quarters (maximum two year horizon) over the period January 1982 to June 2008.
3. Survey forecasts of the CPI over the next 10 years from the Survey of Professional Forecasters from December 1991 to June 2008. (Quarterly frequency).
4. Inflation swap rates from Bloomberg for maturities of 2 to 10, 12, 15, 20, and 30 years over the period November 2003 to June 2008.

Data Choices and Model Identification

- Nominal Treasury yields reflect real rates, expected inflation, and real and inflation risk premia.
- Using survey forecasts of inflation helps to distinguish between real rates and expected inflation.
- However, survey forecasts do not incorporate risk premia.
- Inflation swaps do since the (continuously compounded) swap rate is the difference between yields on nominal and real bonds.

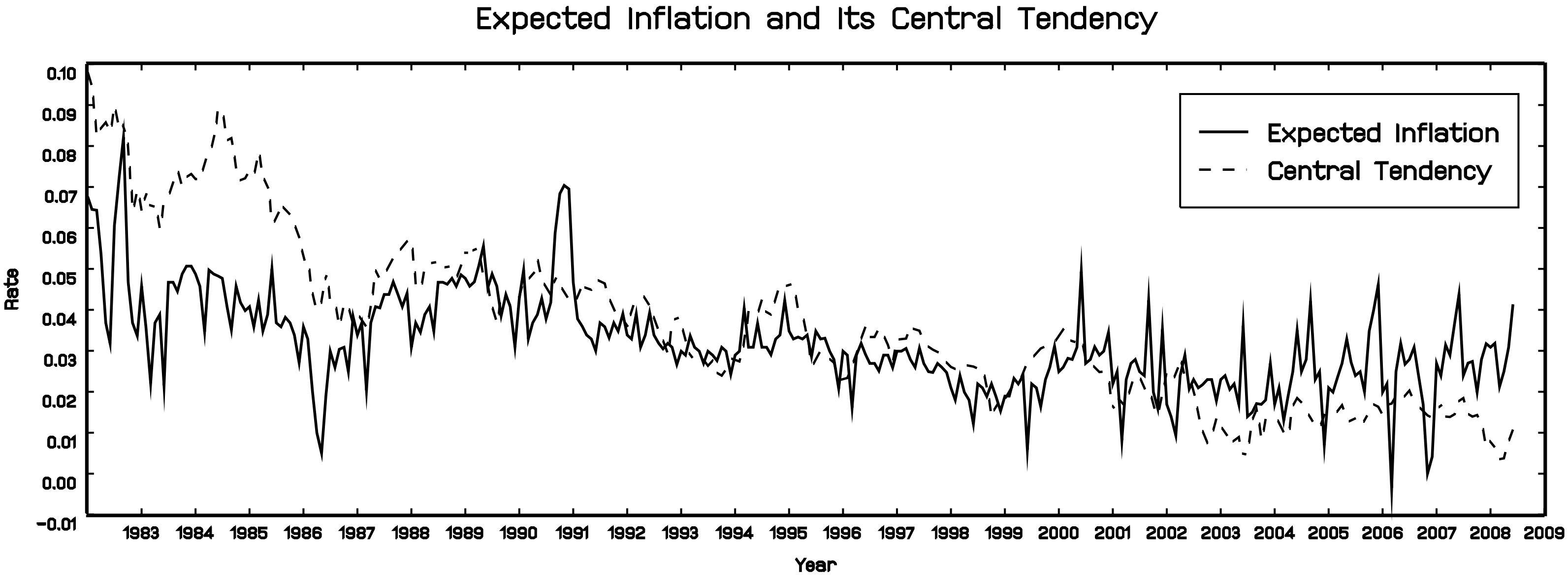
Estimation Technique

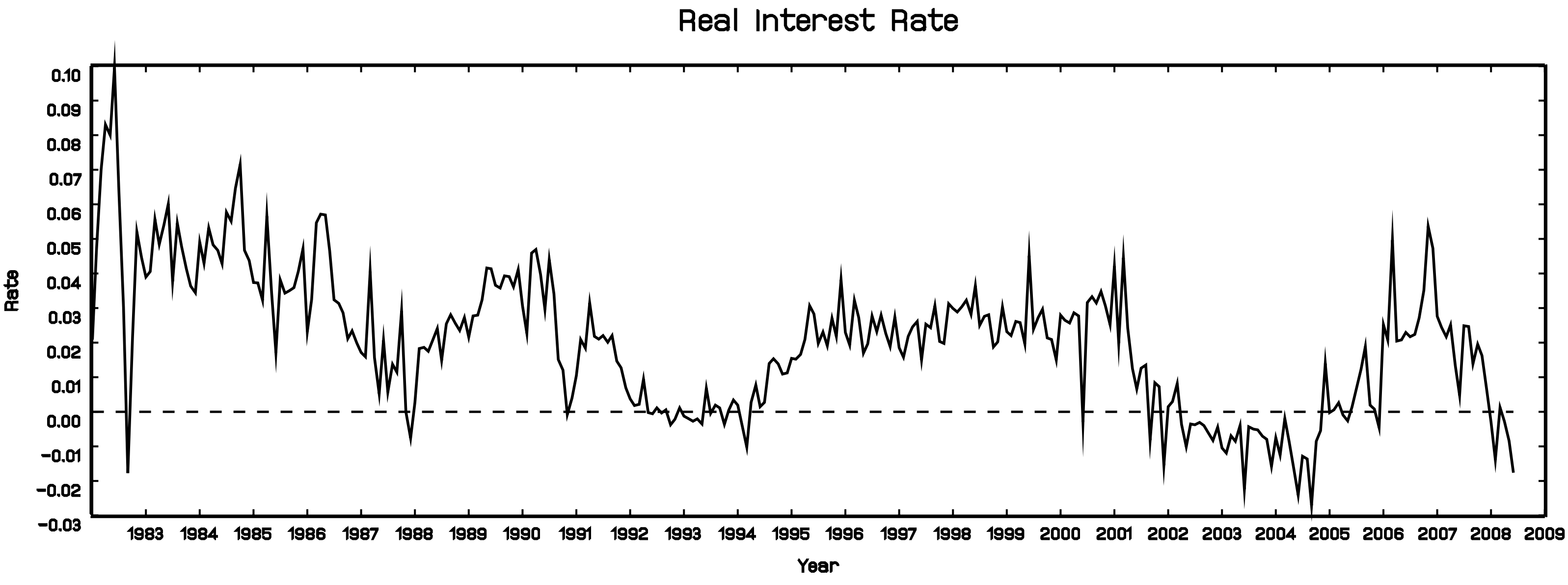
- We first obtain maximum likelihood estimates of the parameters of the one-month inflation process using data on inflation and survey expectations of one-month inflation.
- All other parameters are estimated by maximum likelihood assuming that observed Treasury yields, survey inflation forecasts, and inflation swap rates are equal to their model values plus measurement errors.

Empirical Results: Parameter Estimates and State Variable Dynamics

- Inflation displays significant GARCH effects.
- Allowing GARCH behavior for real rates and expected inflation is especially important.
- The unconditional means of inflation and the one-month real interest rate are $\overline{\pi} = 3.22\%$ and $\bar{r} = 1.57\%$.
- The standard deviations of measurement errors are 35, 39, and 27 basis points for Treasury yields, inflation rate forecasts, and inflation swap rates, respectively.
- One-month real interest rates and one-month expected inflation are highly negatively correlated.

Figure 2


Expected Inflation and Its Central Tendency
Expected Inflation
Central Tendency
Rate
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-0.01
1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009
Year


Real Interest Rate
Rate
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009
Year

Empirical Results: Term Structures and Risk Premia

- The steady state real risk premium equals 111, 156, 212, and 250 basis points at the 5-, 10-, 20-, and 30-year maturities, respectively.
- The steady state inflation risk premium equals 27, 51, 82, and 101 basis points at the 5-, 10-, 20-, and 30-year maturities, respectively.
- From 1982 to 2008, the real risk premium for a ten-year maturity bond varied from 150 to 170 basis points while the inflation risk premium for a ten-year maturity bond varied from 38 to 60 basis points.

Figure 5

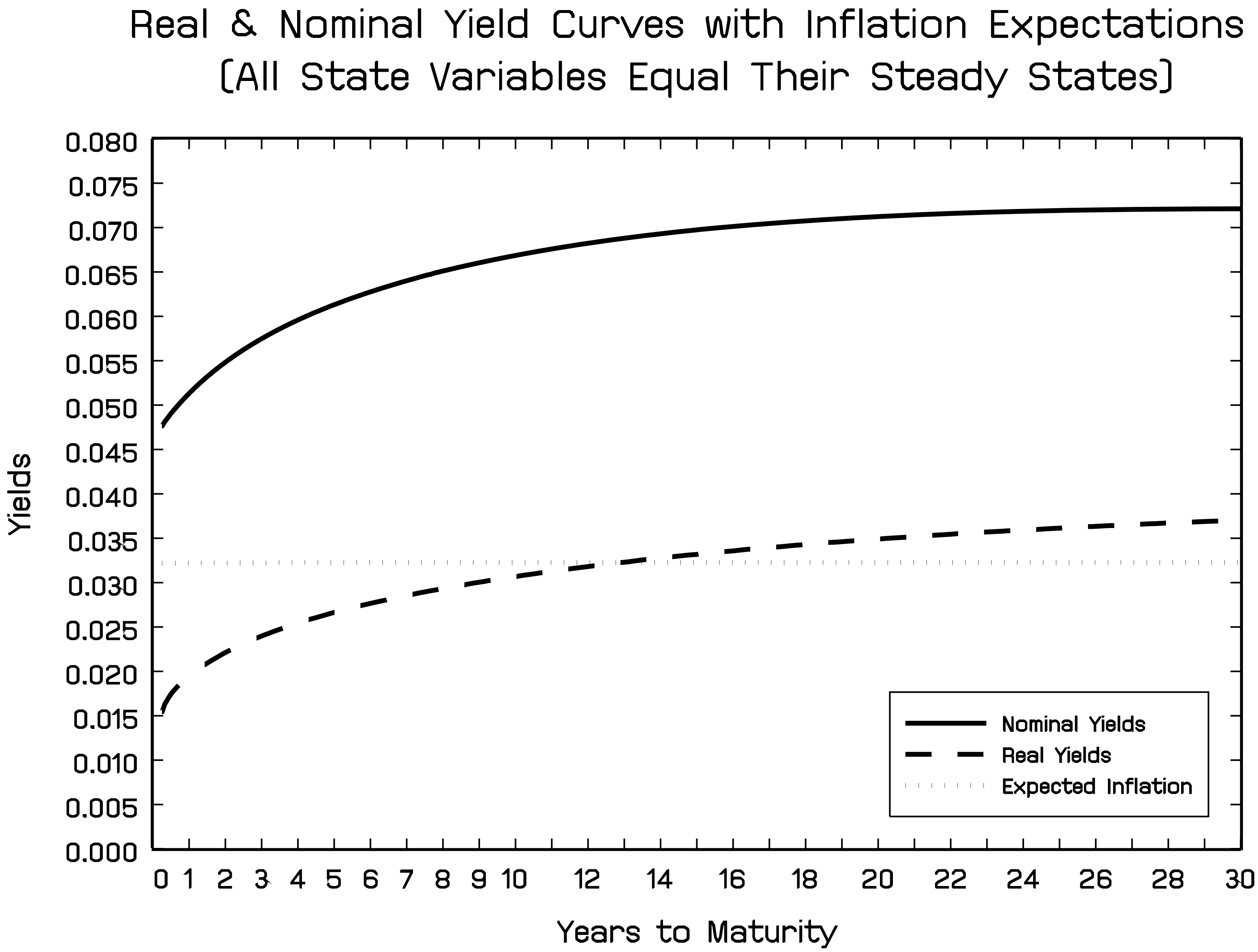

Real & Nominal Yield Curves with Inflation Expectations
(All State Variables Equal Their Steady States)
Yields
0.080
0.075
0.070
0.065
0.060
0.055
0.050
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0 1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22 24 26 28 30
Years to Maturity
Nominal Yields
Real Yields
Expected Inflation

Figure 6

Inflation-Indexed Yield Curves
1982 - 2008

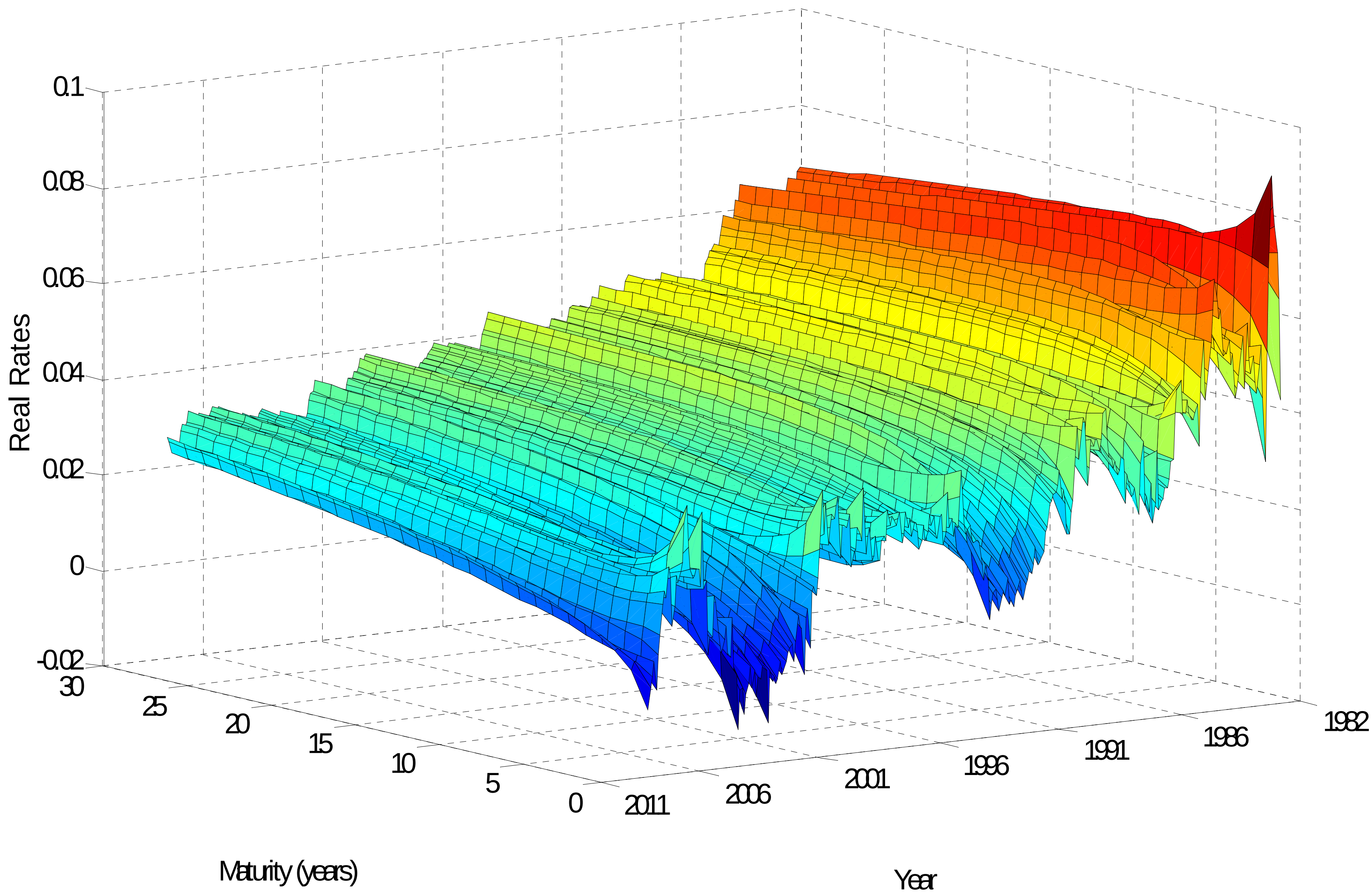
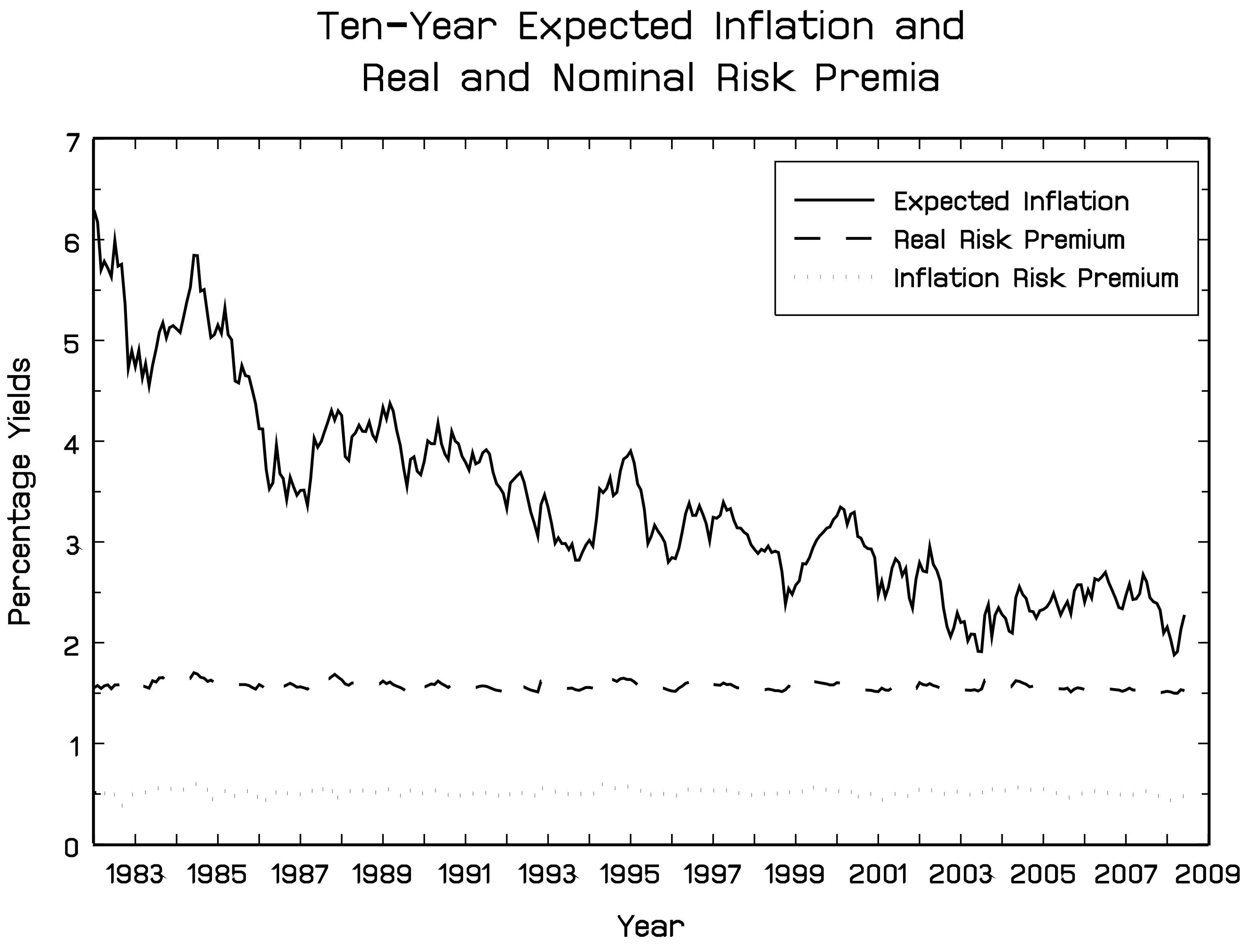


Figure 9

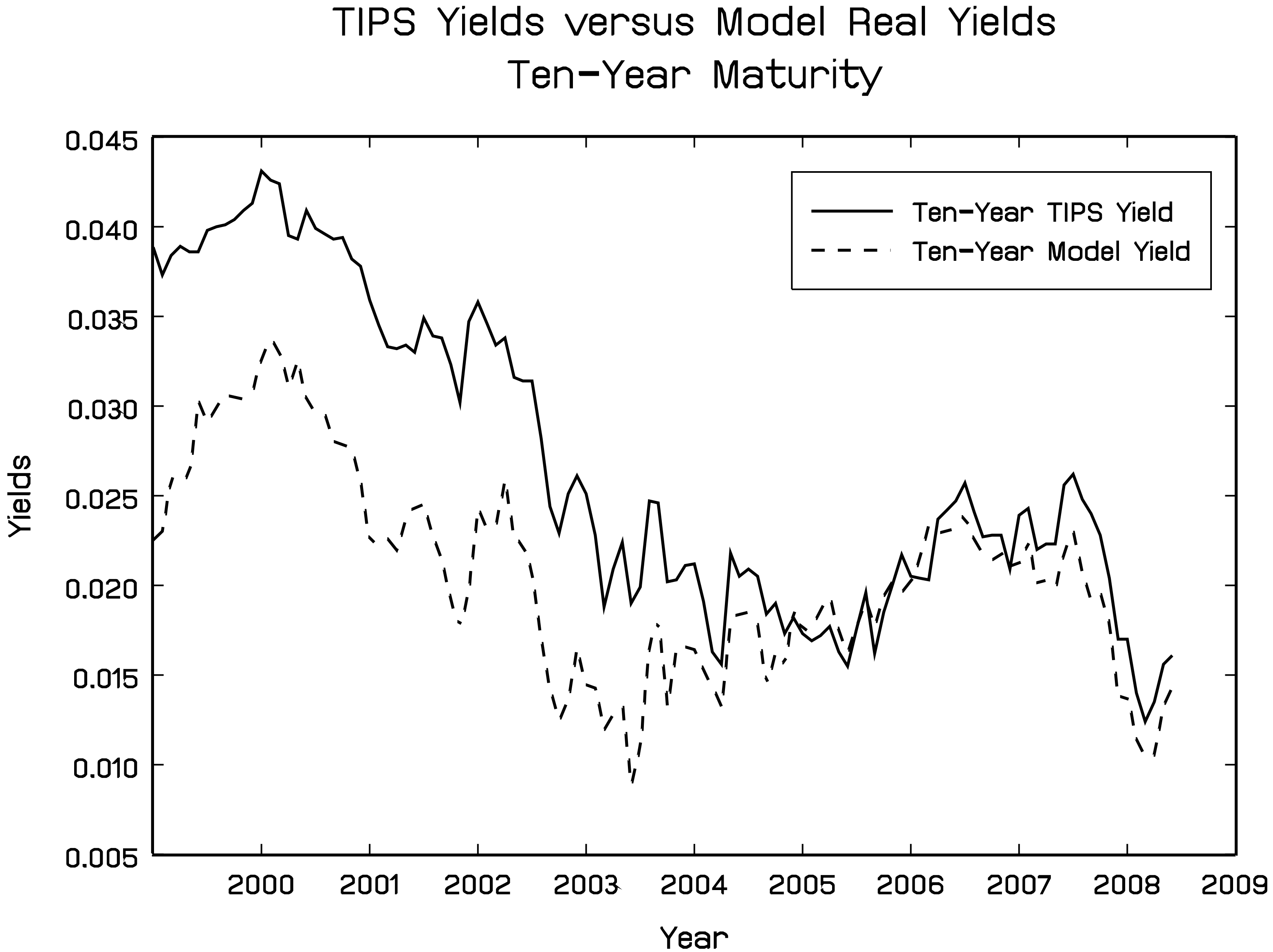


Ten-Year Expected Inflation and
Real and Nominal Risk Premia
Expected Inflation
Real Risk Premium
Inflation Risk Premium
Percentage Yields
0
1
2
3
4
5
6
7
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
2003
2005
2007
2009
Year

Empirical Results: Comparison to TIPS Yields

- In the spirit of an "out-of-sample" test, we compare our model's implied yields for TIPS to actual TIPS yields from Gurkaynak, Sack, and Wright (2008).
- Sack and Elsasser (2004), Shen (2006), and D'Amico, Kim, and Wei (2008) conclude that TIPS yields reflected a large "liquidity premium" prior to 2004.

Figure 12


TIPS Yields versus Model Real Yields
Ten-Year Maturity
Ten-Year TIPS Yield
Ten-Year Model Yield
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
Yields
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
Year

Empirical Results: Term Structure Shocks and Stock Returns

- The model is expanded to incorporate a fifth source of uncertainty capturing the independent component of stock index returns.
- A stock return process is estimated using monthly returns of the S&P500 from 1982 to 2008.
- Shocks to both short run and longer run inflation coincide with negative stock returns.

Conclusions

- By permitting a stochastic central tendency for inflation and GARCH processes for real rates and inflation, our model accounts for changing economic conditions.
- Although we allow a general correlation structure and stochastic volatilities, closed-form solutions for yields on nominal bonds and inflation-linked securities are possible.
- The estimated model produces realistic yield curves and risk premia.
- It matches TIPS yields during their more recent period of greater liquidity.
- Since inflation shocks coincide with negative stock market returns, inflation-linked securities may be an important component of investors' portfolios.