Optimal Central Counterparty Risk Management

Philipp Haene and Andy Sturm
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Optimal Central Counterparty Risk Management *

Philipp Haene\textsuperscript{a,}\textsuperscript{*} and Andy Sturm\textsuperscript{a}

\textsuperscript{a}Swiss National Bank, Oversight, Bundesplatz 1, 3003 Berne, Switzerland

Abstract

In order to protect themselves against the potential losses in case of a participant’s default and to contain systemic risk, central counterparties (CCPs) need to maintain sufficient financial resources. Typically, these financial resources consist of margin requirements and contributions to a collective default fund. Based on a stylized model of CCP risk management, this article analyzes the main factors affecting the trade-off between margins and default fund. The optimal balance between these two risk management instruments is found to depend on collateral costs, participants’ default probability, and the extent to which margin requirements are associated with risk-mitigating incentives. Given the increasing role of CCPs in financial markets in general and for financial stability in particular, these considerations are not only important for CCPs themselves, but also for financial regulators.

Key words: Central counterparty, margin requirements, default fund, financial stability, incentives.


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\textsuperscript{*} Corresponding author. email: philipp.haene@snb.ch. Tel: +41 31 327 06 85.

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1 Introduction

Central counterparties (CCPs) are a critical element of financial markets’ post-trade infrastructure. While originally introduced to absorb the counterparty risk for exchange-traded derivatives, they are increasingly used in cash markets and, most recently, for OTC (over-the-counter) derivatives.\(^1\) The spread of CCPs into new market segments is often welcomed as an important step in strengthening the resilience of the financial system and actively supported by financial regulators (see e.g. Federal Reserve Bank of New York 2008).

A CCP is an entity that interposes itself between trading partners to become a buyer to every seller and a seller to every buyer, thereby ensuring settlement even if one of the original trading partners fails to meet its obligations. Although the terms CCP and clearing house are frequently used synonymously, there is a conceptual distinction (see Committee on Payment and Settlement Systems (CPSS) 2003). While a CCP is narrowly defined as an entity taking over the counterparty risk in a financial transaction, a clearing house is defined more broadly as a central location or central processing mechanism through which financial institutions agree to exchange payment instructions or other financial obligations. This might include assuming the counterparty risk in a financial transaction, but not necessarily so. Since this article focuses on the management of counterparty risk, we use the term CCP in the remainder of this paper, keeping in mind that our findings are valid also for those clearing houses which assume the counterparty risk incurred in financial transactions.

While simplifying risk management for its participants, CCPs concentrate counterparty risk in a single entity. To avoid that CCPs themselves fail and become a source of systemic risk, they need a strong risk management framework. In particular, any CCP’s risk management needs to ensure that the CCP has at its disposal sufficient financial resources in order to cover the potential losses in case of a participant’s default. To achieve this, CCPs may rely on a variety of risk management instruments. Broadly speaking, these instruments can be grouped according to two principles: the defaulter-pays principle and the survivors-pay principle. The defaulter-pays principle is typically implemented by requiring each participant to provide collateral in form of margins to cover its current risk exposure. In case of a participant’s default, the CCP then relies on the margins provided by the defaulting party to cover potential losses. In contrast, the survivors-pay principle is typically implemented by establishing a pre-funded collective default fund. In case of a participant’s default, the CCP then relies on the default fund to cover any losses.

Some aspects of the risk management techniques applied by CCPs have been analyzed in great detail especially in the context of CCPs for exchange-traded

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\(^1\) For a historical overview of the role of CCPs see for example Kroszner (2000).
futures. In particular, various studies investigate the optimal size and calculation method for margin requirements, taking into account the potential exposure over one or more days. These studies typically draw on statistical models, optimization models, or option pricing models. For example, Kupiec (1994) and Kupiec and White (1996) evaluate the degree of risk protection from margin requirements based on different calculation methods. Other studies such as Gemmill (1994), Bates and Craine (1999) and Shanker and Balakrishnan (2005) analyze the combined adequacy of margins and other financial resources available in the case of default. An overview of these and other studies is provided by Knott and Mills (2002).

CCPs usually apply a combination of the defaulter-pays and the survivors-pay principles to cover the losses from a participant’s default. A common practice is to set margin requirements to cover the losses incurred should a participant default under normal market circumstances, while a default fund would account for the losses in excess of the margins. As a consequence, the default fund contributions would typically only be used if a participant defaults in highly volatile market conditions. While this seems to be common practice among CCPs, we are not aware of any in-depth analysis on whether such a combination of risk management instruments is optimal. Indeed, the existing literature provides only very limited guidance on the optimal balance between margins and default fund contributions. In this paper, we therefore investigate the factors determining the optimal risk management combination for a CCP and analyze under what conditions margin contributions, default fund contributions or a combination of these are advisable.

While additional margins and default fund contributions both increase the financial resources available to a CCP to cover potential losses in case of a participant’s default, their effectiveness in covering such a loss and their cost implications for participants differ. On the one hand, establishing a pre-funded collective default fund minimizes the individual contributions by mutualizing the loss from a default, similar to a default insurance. On the other hand, default fund contributions tend to be comparatively more costly than margins, as there is a positive probability that some fraction of a (non-defaulting) participant’s default contribution will be retained by the CCP to cover the losses caused by another participant’s default.

In addition, to the extent that margins and default fund contributions create different incentives for participants, this should be reflected in the design of a CCP’s risk management framework. Two different incentive mechanisms might be distinguished. First, to the extent that participants’ access to collateral is limited, margin requirements put a cap on trading positions and consequently on the maximum loss a participant can incur for the CCP (Hartzmark 1986, Hardouvelis and Kim 1995 and Gibson and Murawski 2008). Second, as in the case of other insurance mechanisms, a collective default fund might lead to
moral hazard problems by creating less incentives for a participant to avoid default, as the cost of the default is partially borne by the other participants through their default fund contributions (Kahn and Roberds 1998). At the same time, the need to deposit margins in relation to current risk exposures is likely to limit a participant’s leverage and risk-taking, thereby lowering its default probability. Therefore, ceteris paribus, increasing the relative importance of margin requirements in the CCP’s risk management framework might decrease both the likelihood of and the size of the potential loss in case of a participant’s default.

Taking into account the cost of collateral and the different incentives created by margin requirements and default fund contributions, this article analyzes how these two instruments should be combined in a CCP’s risk management framework. The results are relevant for CCPs, market participants and regulators. For CCPs, having an adequate balance between margin requirements and default fund contributions is essential, not only from a risk management perspective, but also for commercial reasons. Indeed, as competition between CCPs has become more fierce in recent years, in many market segments trading parties now have a choice regarding the CCP they want to use for clearing their trades. For market participants, the quality of the CCP’s risk management and its cost implications are critical factors to be taken into account when deciding whether to make use of CCP services, be that as a direct or indirect clearing member. Again, careful evaluation of these factors is particularly important if market participants have the choice between several CCPs. Finally, since CCPs are critical for the stability of the financial system as a whole, financial regulators also take a strong interest in CCP risk management. Indeed, even though the international standards for central counterparties published jointly by the Committee on Payment and Settlement Systems (CPSS) and the Technical Committee of the International Organization of Securities Commissions (IOSCO)(CPSS/IOSCO 2004) cover a broad range of issues, it is clear that regulators’ primary emphasis is on ensuring that CCPs apply sound risk management practices and that they have sufficient financial resources at their disposal. Moreover, the rapid spread of CCPs into new market segments and the increasing competition accentuates the importance of understanding the pros and cons of alternative risk management instruments, not least to avoid regulatory arbitrage.

The remainder of this article is organized as follows: Section 2 briefly reviews the role of CCPs and Section 3 discusses the risk management instruments they typically apply. Section 4 introduces a stylized model that allows to analyze how margins and default fund contributions should be combined in

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2 Although Kahn and Roberds (1998) focus on payment systems with deferred net settlement, their analysis can also be applied to the context of a CCP. For a description of the defaulter-pays and survivors-pay principles in the context of deferred net settlement payment systems see Bank of England (2005).
the CCP’s risk management framework, taking into account the different costs and incentives these instruments imply. Section 5 concludes and points out some areas of interest for future research.

2 The Role of Central Counterparties

A CCP interposes itself between the trading parties at the moment or right after they enter a transaction. The CCP assumes the obligations related to the transactions and guarantees their fulfilment. It thus becomes the buyer to every seller and the seller to every buyer.

For the trading parties, clearing via a CCP can provide various benefits (see e.g., Giordano 2002 and Ripatti 2004). First and foremost, the CCP eliminates counterparty risk vis-à-vis other trading parties. When concluding a transaction, the trading parties incur the risk of the other party defaulting and not fulfilling its obligations. This credit exposure typically exists from the point in time the transaction is concluded until the settlement of the resulting obligation(s). By interposing itself between the trading partners, the CCP assumes these counterparty risks and guarantees the fulfillment of the obligations, even in the case one of the trading partners defaults. A participant thus no longer has to worry about the solvency of all its trading partners but can focus on managing its exposure vis-à-vis a single counterparty, the CCP. This is particularly useful if trading takes place via an anonymous trading platform. Moreover, as CCPs typically enjoy very high credit standings, the involvement of a CCP may reduce capital charges. Finally, CCPs establish detailed default procedures, creating transparency and certainty on the process to be followed should one of the participants default.

Participants of a CCP may also benefit from multilateral netting. Especially if market participants have to provide each other collateral based on their open positions, multilateral (instead of bilateral) netting allows to reduce the collateral to be posted. Also, with multilateral netting, the number of obligations to be settled is significantly lower than in the case of bilateral or no netting at all. Not only allows this to reduce transaction fees and liquidity cost, but it also minimizes settlement-related principal and liquidity risk.

Finally, provided trading takes place on an electronic exchange or multilateral trading platform with anonymous trading (pre-trade anonymity), the use of a CCP allows to maintain anonymity also after a transaction is concluded (post-

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3 The benefits of multilateral netting compared to bilateral netting have been investigated in several studies. Baer, France and Moser (1996) show that the move from bilateral to multilateral netting is improving social welfare. Jackson and Manning (2007) quantify these benefits using a simulation approach.
3 Risk Management Instruments

As CCPs concentrate counterparty risks in a single entity, their failure could have systemic implications and lead to widespread disruptions in financial markets. CCPs are thus critical elements of the financial system and it is important that the risks related to their activities are adequately managed.

In general, the nature of (financial) risks that need to be managed depends on the type of markets or financial instruments for which the CCP offers its clearing services. That is, the risk profile of a CCP clearing cash markets such as equities or bonds is not the same as the risk profile of a CCP clearing exchange-traded or OTC derivatives. Of course, these differences need to be taken into account in the CCP’s risk management framework. For instance, if the financial instruments to be cleared via the CCP involve the settlement of mutual obligations (e.g. the delivery of bonds or equities against the payment of funds), the CCP may face principal risk, which is defined as the risk that it fulfils its contractual obligation while its counterparty fails to fulfil its obligation. To eliminate principal risk, the CCP’s risk management would thus have to ensure that transactions are settled via a delivery-versus-payment mechanism.

However, irrespective of the underlying financial instrument to be cleared, the main risk component for a CCP is replacement cost risk, which materializes if one of the participants defaults on its contractual obligations during the period between the time the trade was agreed and the time of final settlement. In that case, in order to be able to live up to its own obligations vis-à-vis the non-defaulting participants, the CCP has to enter into a replacement transaction which may be possible only on less favorable terms. The potential loss to the CCP (i.e. its risk exposure) is thus a function of (i) the defaulting participant’s open positions at the time of default and (ii) the (adverse) price movements that have taken place in the underlying financial instruments since the original contracts were entered (market risk). With regard to the management of replacement cost risk, CPSS/IOSCO (2004) suggest that CCPs should hold sufficient financial resources to withstand the default of at least their largest participant (in terms of risk exposure) in extreme but plausible market conditions. From a systemic risk perspective, this is the key requirement for any CCP.

To comply with this requirement, CCPs first need to assess their potential losses in case of a major participant’s default. Typically, such assessments
rely on stress-tests, which analyze the impact of a major participant’s default combined with extreme (unfavorable) movements in market prices. Both factors—the maximum exposure and the adverse price movements—can be based on historic observations or on theoretical assumptions. CCPs then need to ensure that their financial resources are sufficient to cover the identified potential stress losses. To do so, CCPs may rely on various risk management instruments, which can be broadly grouped according to two principles: the defaulter-pays principle—where the defaulting participant covers the loss—, and the survivors-pay principle—where the loss is covered by the remaining participants. The defaulter-pays principle is typically implemented by requiring participants to post margins in relation to the current risk exposure they incur for the CCP. That is, an increase in the participant’s open position or an increase in market volatility (or both) translate instantaneously into higher margin requirements. In contrast, the survivors-pay principle usually requires participants to make contributions to a pre-funded collective default fund. Individual default fund contributions are typically based on each participant’s (average) risk exposure over a longer time period. These contributions are thus less reactive to temporary shifts in current open positions or changing market volatility.

4 A Model of CCP Risk Management

In practice, CCPs tend to apply both margin requirements and default fund contributions, but to varying degrees. This raises the question whether there is an optimal balance between these risk management instruments, and if yes, by what factors this balance is affected. To answer these questions, this section introduces a stylized model of CCP risk management.

4.1 Assumptions

The model covers one period, which could be interpreted, for instance, as one day, one month or one year. The market place for which the CCP offers its services is covered by \( i = 1 \ldots n \) homogenous and risk-neutral financial institutions. The CCP is user-owned and user-governed, which ensures that the CCP’s risk management framework reflects the preferences of its participants.

A financial institution’s benefit from clearing its transactions via the CCP is \( B_i \). As discussed in Section 2, this benefit may arise from the reduction of counterparty risk and simplified risk management, multilateral netting and post-trade anonymity. Participants’ cost related to clearing their transactions via the CCP are threefold:
(1) Service fees $F_i$, intended to recover the CCP’s operational cost (including some return on capital). $F_i$ is assumed to be a fixed cost.

(2) Opportunity costs $C_i$ from posting collateral at the CCP in the form of margins, a default fund contribution, or both. The marginal opportunity cost of collateral is $\alpha \geq 0$.

(3) In case of another participant’s default, the remaining participants might lose some or all of their default fund contributions (but none of their margins). The associated expected cost is denoted $L_i$.

Financial institutions’ expected utility from participating in the CCP thus is

$$U_i = B_i - F_i - C_i - L_i.$$  \hspace{1cm} (1)

Below, we will assume that $B_i$ is sufficiently large, so that the participation constraint $U_i \geq 0$ is always satisfied. Moreover, since $F_i$ is a fixed cost and does not depend on the CCP’s risk management framework, the analysis will focus exclusively on the two other cost factors, $C_i$ and $L_i$.

Given participants’ trading strategy (which is not modeled explicitly), any participant defaults with some positive probability on its obligations vis-à-vis the CCP during the period under consideration. In case of a default, the CCP invokes the close-out netting procedure, which—depending on the defaulting participant’s positions and movement of market prices—may or may not result in positive replacement costs and thus a loss to the CCP. For any participant, the probability of a default which leads to positive replacement costs for the CCP is $\phi > 0$. Note that we ignore those defaults which do not result in a loss to the CCP (because the replacement cost is negative). Hence, the default probability may be larger than $\phi$. For simplicity, it is assumed that the replacement cost $\Lambda$ is uniformly distributed over the range $[0, \Lambda_{\text{max}}]$ and that at most one participant defaults per period.

The regulatory authority requires the CCP to have at its disposal sufficient financial resources $R$ to cover its losses in case of default of any of its participants in extreme but plausible circumstances. In other words, the regulator requires that $R \geq \Lambda_{\text{max}}$, which ensures that the CCP will always be able to fulfil its obligations vis-à-vis the non-defaulting participants.

Each participant is obliged to provide financial resources (i.e. collateral) to the CCP in the form of margins ($M_i$), pre-funded contributions to a default fund ($DF_i$), or both. Due to homogeneity the total default fund is $DF = \sum_i DF_i = n \cdot DF_i$. In case of participant $i$’s default, the CCP may have recourse to $M_i$ and $DF$ in order to cover its losses. The regulatory requirement can thus be written as

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\[ R = M_i + n \cdot DF_i \geq \Lambda_{\text{max}}. \] (2)

In case of a default by participant \( j \), the CCP covers the loss \( \Lambda \) in the following order:

1. The CCP realizes participant \( j \)’s margins \( M_j \).
2. If \( \Lambda > M_j \), the CCP realizes also participant \( j \)’s default fund contribution \( DF_j \).
3. Eventually, if \( \Lambda > M_j + DF_j \), the CCP will take recourse to other participants’ default fund contributions on a pro rata basis.

In case of participant \( j \)’s default, it is thus possible that some fraction of the (non-defaulting) participant \( i \)’s default fund contribution will be used by the CCP to cover its losses. The (expected) cost for participant \( i \) is captured by the term \( L_i \) in Equation (1).

As there is no reason for the CCP to hold more financial resources than required, it will be assumed that the regulatory requirement in Equation (2) holds with equality, in which case each participant’s default fund contribution can be written as a function of its margins, i.e. \( DF_i = \frac{\Lambda_{\text{max}} - M_i}{n} \). The CCP’s optimization problem can then be restated in terms of the following cost minimization problem:

\[
\min_{M_i} T_i(M_i) = C_i(M_i) + L_i(M_i), \quad \text{with } M_i \in [0, \Lambda_{\text{max}}],
\] (3)

and where \( T_i \) measures participant \( i \)'s total (expected) cost associated with the CCP’s risk management framework.

In the following, we will analyze how the CCP should set the level of margins (and hence implicitly also the size of the default fund), taking into account the trade-off between collateral costs \( (C_i) \) and the expected loss of individual default fund contributions \( (L_i) \). From the perspective of a participant, this trade-off can be summarized as follows: Low margin requirements (which imply a high default fund) allow to economize on collateral costs but go along with higher expected losses in case of another participant’s default; high margin requirements (which imply a low default fund) cause high collateral costs, but the expected losses in case of another participant’s default are lower or even zero.

As outlined in the introduction, one would expect that the optimal balance between margin requirements and default fund contributions can also depend
on the intensity of two incentive mechanisms associated with margin requirements. That is, to the extent that higher margins contribute to limiting participants’ risk exposures and/or to reducing their default probability, the CCP should increase its margin requirements (compared to the situation where these incentives are absent). However, to ease the exposition, subsection 4.2 first determines the optimal risk management framework for the case where incentives are absent (benchmark model). Subsection 4.3 then discusses to what extent the result is altered if the CCP’s risk management framework affects the probability of a default with positive replacement cost for the CCP and/or the maximum loss to the CCP in case of a default (extended model).

4.2 The Benchmark Model

In this section, it is assumed that participants’ trading and risk management behavior is not affected by the CCP’s risk management framework. Or, to put it differently, an individual participant’s probability of a default with positive replacement cost for the CCP (\( \phi \)) and the maximum loss to the CCP in case of default (\( \Lambda_{\text{max}} \)) do not depend on the CCP’s risk management framework (i.e. on the combination of margins and default fund contributions).

In order to determine the optimal balance between margins and default fund contributions, the terms \( C_i \) and \( L_i \) need to be specified in more detail. First, participant \( i \)'s opportunity costs of collateral increase linearly with the level of margins and default fund contributions to be posted at the CCP. In this respect, it is instructive to look at the implications of two extreme solutions. On the one hand, if the CCP relies solely on a default fund and thus \( M_i = 0 \), it is apparent from Equation (2) that each participant would have to make a contribution to the default fund of \( DF_i = \frac{\Lambda_{\text{max}}}{n} \) and thus \( C_i = \alpha \frac{\Lambda_{\text{max}}}{n} \). On the other hand, if the CCP’s risk management is based solely on margins and hence \( DF_i = 0 \), it follows that each participant would have to provide margins of size \( M_i = \Lambda_{\text{max}} \) and thus \( C_i = \alpha \Lambda_{\text{max}} \). There are of course an infinite number of intermediate solutions, but in general the opportunity costs for collateral can be written as

\[
C_i(M_i) = \alpha(M_i + DF_i) = \frac{\alpha}{n} [\Lambda_{\text{max}} + (n - 1)M_i].
\] (4)

Turning to the second term \( L_i \), note that participant \( i \)'s expected loss due to the default of any of the other \( n - 1 \) participants can be written as
\[ L_i = (n - 1) \phi LGD_i, \]

where \( \phi \) is the probability of a default resulting in positive replacement cost for the CCP and \( LGD_i \) is participant \( i \)'s expected loss of its default contribution given a default by any of the other participants. In order to determine \( LGD_i \), assume that participant \( j \) defaults. The expected residual loss to be shared between the non-defaulting participants—i.e. the expected loss to the CCP after subtracting the margins and default fund contributions by \( j \)—is equal to \( E(\Lambda) - (M_j + DF_j) \). Taking into account that \( E(\Lambda) = \frac{\Lambda_{\text{max}}}{2} \), \( M_j = M_i \), \( DF_j = DF_i \), and \( DF_i = \frac{\Lambda_{\text{max}} - M_i}{n} \), it can be shown that

\[
LGD_i(M_i) = \begin{cases} 
\frac{(n-2)\Lambda_{\text{max}}}{2n(n-1)} - \frac{M_i}{n} & \text{if } 0 \leq M_i < \frac{n-2}{2(n-1)} \Lambda_{\text{max}} \\
0 & \text{if } \frac{n-2}{2(n-1)} \Lambda_{\text{max}} \leq M_i \leq \Lambda_{\text{max}}.
\end{cases}
\]

In case of another participant’s default, the expected residual loss is thus decreasing in \( M_i \), reaching zero whenever margins are set sufficiently high.

Substituting Equations (4), (5) and (6) into (3), the CCP’s cost minimization problem thus becomes

\[
\min_{M_i} T_i(M_i) = \frac{\alpha}{n} [\Lambda_{\text{max}} + (n - 1)M_i] + \phi (n-1) \max \left[ 0, \frac{(n-2)\Lambda_{\text{max}}}{2n(n-1)} - \frac{M_i}{n} \right].
\]

The first order condition \( \partial T_i/\partial M_i = 0 \) then yields the optimal individual margin requirements

\[
M_i^* = \begin{cases} 
0 & \text{if } \alpha > \phi \\
\frac{n-2}{2(n-1)} \Lambda_{\text{max}} & \text{if } \alpha < \phi
\end{cases}
\]

as well as the optimal individual default fund contributions

\[
DF_i^* = \begin{cases} 
\frac{\Lambda_{\text{max}}}{n} & \text{if } \alpha > \phi \\
\frac{\Lambda_{\text{max}}}{2(n-1)} & \text{if } \alpha < \phi.
\end{cases}
\]
The two cases are illustrated in Figure 1. One may observe that irrespective of the specific parameter values for $\alpha$ and $\phi$, it is always beneficial to establish a default fund, whereas the exclusive use of margin requirements is never optimal. This can be explained by the fact that if margins are set above a certain level, the risk that default fund contributions are being retained to cover other participants’ losses becomes very small, which makes the collective default fund more attractive.

**Fig. 1. Optimal Margin Contribution – Benchmark Model**

The relative importance of the optimal default fund in the CCP’s risk management framework, in the following measured by the variable $df \equiv DF^* / \Lambda_{\max} \in [0, 1]$, depends on the specific parameter values. If the opportunity cost of collateral is larger than the default probability ($\alpha > \phi$), then $df = 1$ and the CCP should thus not collect any margins at all. In contrast, if $\alpha < \phi$, then $df = \frac{n}{2(n-1)}$ and the optimal balance between margins and default fund contributions depends on the number of clearing participants: The more participants, the smaller should the relative importance of the default fund be. As the number of participants increases, the risk of a default increases, even if the participants’ individual default probabilities remain unchanged. This increases the expected loss on default fund contributions and makes them less attractive. However, even with a large number of participants, the size of the default fund should never fall below 50% of the total required financial resources.

How do these theoretical results relate to existing CCPs’ risk management arrangements? As noted above, CCPs typically apply a combination of both

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$4$ If $\alpha = \phi$, there is a continuum of optimal solutions with $M_i^* \in \left[0, \frac{n-2}{2(n-1)} \Lambda_{\max}\right]$ and $DF_i^* \in \left[\frac{\Lambda_{\max}}{2(n-1)}, \frac{\Lambda_{\max}}{n}\right]$. 

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margin requirements and default fund contributions. Assuming that CCPs have chosen an optimal risk management framework, this would imply that the opportunity cost of collateral is lower than the perceived probability of a default with positive replacement cost for the CCP. For instance, assuming that opportunity cost of collateral is 5 basis points per annum, this would imply that the perceived default probability of participants over a one year horizon is larger than .0005.

4.3 The Extended Model

In this subsection, the model is expanded by taking into account that margins—as opposed to default fund contributions—may affect participants’ behavior via two different incentive mechanisms.

First, margins may limit the maximum loss the CCP needs to cover in case of a participant’s default. Indeed, higher margins make peak exposures more costly and, in addition, to the extent that participants face a collateral constraint, margins impose a cap on participants’ trading activity and hence their risk exposures. These assumptions are supported both theoretically and empirically by Hartzmark (1986), Hardouvelis and Kim (1995) and Gibson and Murawski (2007). One might argue that higher default fund contributions also increase the cost of trading and therefore reduce trading activity, but these contributions are typically tied to average risk exposures over a past period. They thus have only a limited bearing on peak trading activity and risk exposures.

In our model, the impact of higher margins on the maximum risk exposure is captured as follows:

\[ \tilde{\Lambda}_{max} = \Lambda_{max} - \delta M_i, \text{ with } \delta \geq 0 \text{ and } M_i \in \left[ 0, \frac{\Lambda_{max}}{1 + \delta} \right]. \]

\( \Lambda_{max} \) thus now defines an upper limit for the loss (namely if \( M_i = 0 \)), and the parameter \( \delta \) can be interpreted as measuring the intensity of this incentive mechanism: The larger \( \delta \), the stronger is the dampening effect of margins on participants’ risk exposure and on the maximum loss in case of a default. One may also note that it can never be optimal to collect margins higher than \( \frac{\Lambda_{max}}{1 + \delta} \), as this amount would always be sufficient to cover the maximum loss.

The second incentive mechanism takes into account that higher margins are likely to increase participants’ incentives to avoid default, thereby reducing default probability. Indeed, to the extent that the risk of default is an endogenous variable as argued by Kahn and Roberds (1998), too strong reliance on the collective default fund in the CCP’s risk management framework can raise
moral hazard issues and lead to higher default rates than is socially optimal. By internalizing the cost of default, higher margins reduce the incentives to free-ride on resources provided by other participants and may limit risk taking. Hence, increasing the relative importance of margins in the CCP’s risk management framework is expected to reduce participants’ default probability.\textsuperscript{5}

This incentive mechanism is captured by assuming that the probability of a default (with positive replacement cost for the CCP) $\tilde{\phi}$ depends on the level of margins as follows:

\[
\tilde{\phi}(M_i) = \phi - (\phi - \phi_{\min}) \frac{M_i}{\tilde{\Lambda}_{\max}}, \quad \text{with } \phi \geq \phi_{\min} > 0, M_i \in [0, \tilde{\Lambda}_{\max}]
\]

The parameter $\phi$ now defines an upper limit to the probability of a default resulting in positive replacement cost. A gradual increase in $M_i$ reduces the endogenous default probability $\tilde{\phi}$, which reaches the lower limit of $\phi_{\min}$ when margins are raised to $M_i = \tilde{\Lambda}_{\max}$. The spread between $\phi$ and $\phi_{\min}$ captures the intensity of this incentive mechanism. The larger the spread, the stronger the impact of margins on participants’ default probability.

To what extent do the two incentive mechanisms affect the optimal CCP risk management framework? First, note that the regulatory requirement now is

\[
R = M_i + nDF_i \geq \tilde{\Lambda}_{\max} = \Lambda_{\max} - \delta M_i.
\]  \hfill (10)

Maintaining the assumption that the regulatory requirement holds with equality, it follows that $DF_i = \frac{\Lambda_{\max} - (1+\delta)M_i}{n}$ and hence

\[
\tilde{C}_i(M_i) = \alpha(M_i + DF_i) = \frac{\alpha}{n}[\Lambda_{\max} + (n - \delta - 1)M_i].
\]  \hfill (11)

Moreover, the expected loss due to the default of any other participant now is

\[
\tilde{L}_i = (n - 1) \tilde{\phi} \tilde{LGD}_i,
\]  \hfill (12)

where

\textsuperscript{5} Gibson and Murawski (2007) point out that margin requirements may negatively affect participants’ wealth and welfare and hence, under specific circumstances, may increase a participant’s default probability. In our model, this effect is not taken into account.
\( \tilde{LD}_i(M_i) = \frac{1}{n-1}[E(\tilde{\Lambda}) - (M_i + DF_i)] \)
\[
= \begin{cases} 
\frac{(n-2)\Lambda_{\text{max}}}{2n(n-1)} - \frac{\delta(n-2)+2(n-1)}{2n(n-1)} M_i & \text{if } 0 \leq M_i < \frac{(n-2)\Lambda_{\text{max}}}{\delta(n-2)+2(n-1)} \\
0 & \text{if } \frac{(n-2)\Lambda_{\text{max}}}{\delta(n-2)+2(n-1)} \leq M_i.
\end{cases}
\tag{13}
\]

The cost minimization problem can thus be written as
\[
\min_{M_i} \tilde{T}_i(M_i) = \tilde{C}_i(M_i) + (n-1)\tilde{\phi} \tilde{LD}_i
= \frac{\alpha}{n}[\Lambda_{\text{max}} + (n - \delta - 1)M_i]
+ (n-1)\left[\phi - (\phi - \phi_{\text{min}})\left(\frac{M_i}{\Lambda_{\text{max}} - \delta M_i}\right)\right] \ldots \\
\max \left[0, \frac{(n-2)\Lambda_{\text{max}}}{2n(n-1)} - \frac{\delta(n-2)+2(n-1)}{2n(n-1)} M_i\right].
\tag{14}
\]

As it is not possible to derive closed-form solutions for \( M_i^* \) or \( DF_i^* \) from the first-order condition, we evaluate the results numerically. To facilitate the discussion and to compare the results with the benchmark model, it is again useful to look at the relative importance of the default fund in the CCP’s risk management framework, which now is \( \tilde{df} \equiv DF^*/\Lambda_{\text{max}} \in [0, 1] \). Depending on the values of the parameters \( \alpha \) and \( \phi \), the following four cases can be distinguished:

**Case 1:** \( \alpha \) is significantly larger than \( \phi \). Recall that in the benchmark model, for \( \alpha > \phi \), it would be optimal to rely solely on a collective default fund. Now, allowing for incentive effects, the maximum loss and the probability of default need to be highly sensitive to margin requirements in order to make margins attractive, i.e., \( \delta \) would need to be high and \( \phi_{\text{min}} \) significantly lower than \( \phi \). For realistic values of \( \delta \) and \( \phi_{\text{min}} \), however, the CCP should again rely exclusively on default fund contributions, i.e. \( \tilde{df} = 1 \).

**Case 2:** \( \alpha \) is slightly larger than \( \phi \). The sensitivity of the maximum loss (described by \( \delta \)) and the default probability (described by \( \phi - \phi_{\text{min}} \)) to an increase in margins can now compensate for the slightly higher cost of collateral relative to the default probability and make margin contributions in combination with a default fund worthwhile for the CCP’s risk management. As can be seen in the left panel of Figure 2, which shows \( \tilde{df} \) for various parameter combinations of \( \delta \) and \( \phi_{\text{min}} \), \( \tilde{df} \) drops quickly as \( \delta \) increases, while the effect is less pronounced for a reduction in \( \phi_{\text{min}} \). This can be explained by the opposing effects a reduction in \( \phi_{\text{min}} \) has on the optimal margin requirement. On the one hand, margin contributions reduce the total cost of risk management by
reducing the default probability and therefore margin contributions become more attractive. On the other hand, however, the lower default probability makes default fund contributions comparatively less costly, as the risk that some or all of the contributions are retained to cover losses from a participant’s default is also reduced. Overall, a reduction in \( \phi_{\text{min}} \) still increases the optimal amount of margins, but to a lesser (and declining) degree compared to an increase in \( \delta \).

**Case 3: \( \alpha \) is slightly smaller than \( \phi \).** As the sensitivity of the maximum loss to margins increases (i.e., an increase in \( \delta \)), the optimal absolute level of margins decreases, as the maximum loss is reduced. However, the optimal balance between margins and default fund contributions (and therefore also \( \tilde{df} \)) remains unchanged. An increase in the sensitivity of the default probability to margins (i.e. a decrease in \( \phi_{\text{min}} \)) leads to a higher \( \tilde{df} \). While this might seem counter-intuitive at first sight, it is again explained by the opposing effects a reduction in \( \phi_{\text{min}} \) has on the optimal margin requirement. On the one hand, margins reduce the total cost of risk management by reducing the default probability, which makes margins more attractive. On the other hand, however, the reduced default probability makes default fund contributions again less costly, as the risk that some or all of the contributions are retained to cover losses from a default is reduced. Overall, and as illustrated in the right panel in Figure 2, a reduction in \( \phi_{\text{min}} \) decreases the optimal amount of margins and increases \( \tilde{df} \). This is exactly the opposite result as in case 2, where a reduction in \( \phi_{\text{min}} \) was found to increase the optimal amount of margins. While this might seem counter-intuitive, it is explained by the fact that only by requiring margins the CCP can profit from a reduction in the default probability, which then again makes default fund contributions more attractive. Therefore, the negative effect of margin requirements on the default probability makes a balanced combination of margins and default fund contributions preferable for the CCP.

**Case 4: \( \alpha \) is significantly smaller than \( \phi \).** An increase in the sensitivity of the maximum loss to margins (i.e. an increase in \( \delta \)) reduces the optimal absolute level of margins, but leaves the optimal combination of margin requirements and default fund contributions unchanged. An increase in the sensitivity of the default probability to margins (i.e. a decrease in \( \phi_{\text{min}} \)) has no effect on the optimal balance nor the absolute level of margins and default fund contributions.

These results thus demonstrate that by taking into account incentive effects margins become more attractive in some—but not all—cases. In particular, it is again never found to be optimal for the CCP to rely solely on margin requirements. That is, even if the alleged incentive effects of margin requirements are important, it is always beneficial to establish some sort of collective default fund.
If the cost of collateral is significantly smaller or higher than the probability of a participant default (with a positive replacement cost for the CCP), incentives typically have no effect on the optimal combination of the risk management instruments. However, if cost of collateral and the probability of a participant default are fairly similar, incentives can determine the optimal balance between margins and the default fund. If the maximum loss decreases with an increase in margin requirements, the latter can become preferable even if the opportunity cost of collateral is equal or slightly higher than the probability of a participant default. If we take into account that margins reduce the probability of a participant default, a balanced combination of default fund contributions and margins becomes preferable. If the opportunity cost of collateral is smaller than the default probability, there is the counter-intuitive effect that an increase in the sensitivity of the default probability to margin requirements can actually reduce the optimal share of margins relative to the default fund. Indeed, in that case, even a small increase in margin requirements implies a significant reduction of the default probability, which in turn makes the default fund contributions comparatively cheaper.

In any case, provided that one or both of the alleged incentive mechanisms exist, the optimal balance between default fund contributions and margin requirements is similar to or lies in between the two corner solutions established for the benchmark model in Section 4.2. Moreover, it should also be noted that the total cost of collateral associated with the CCP’s optimal risk management framework is lower than in the benchmark model.
In this paper, we have analyzed the optimal combination of margin requirements and a collective default fund in a stylized CCP model. The model takes into account the total cost of collateral, participants’ default probability and the potential risk-mitigating incentives associated with margins.

We find that establishing a default fund is always optimal, and in some cases a sufficiently large default fund is even all it takes. The use of margin requirements—and hence a somewhat smaller default fund—is recommended if the opportunity cost of collateral is lower than the probability of a participant’s default (resulting in a positive replacement cost for the CCP). Moreover, the imposition of margin requirements becomes more attractive if they are associated with risk-mitigating incentives, e.g. if higher margins reduce participants’ default probability or if they put a cap on the maximum loss in case of a participant’s default.

In practice, CCPs typically combine margin requirements with a default fund. Against the background of our theoretical results, this could be attributed to two potential causes: It could mean that the opportunity cost of collateral is low compared to the perceived probability of a participant’s default; or it could imply that margins have (or are said to have) significant incentive effects which are taken into account in the CCP’s risk management framework. The two explanations are not mutually exclusive.

Our work has implications for CCPs as well as regulatory authorities. In particular, it suggests that the design of CCPs’ risk management frameworks should carefully weight the cost and benefits (including incentives effects) of different risk management instruments. Indeed, by striking the optimal balance between margin requirements and default fund contributions, CCPs can either enhance their resilience to financial shocks without imposing additional cost on their participants, or they can achieve a certain level of resilience at lower cost. At the same time, our results suggest that regulatory authorities should not only focus on the total level of financial resources available to a CCP, but also take into account how the various risk management instruments are combined and what kind of incentives they create.

There are a number of extensions to our stylized model which warrant further investigation. First, it would be interesting to allow for multiple defaults. Second, one might relax the risk-neutrality assumption by modeling risk-averse participants. In that case, we would expect the optimal balance between risk management instruments to shift towards higher margin requirements. Third, the assumption that the losses to the CCP in case of a participant’s default are uniformly distributed could be relaxed, for example by analyzing losses which
are positively skewed. Fourth, one might study how heterogeneity among participants (e.g. in terms of different trading volumes and hence risk exposures or in terms of different default probabilities) would affect optimal CCP risk management.

Finally, it should be stressed that while the model captures two particular incentive effects associated with margin contributions, it leaves aside other potential incentives. In particular, Knott and Mills (2002) argue that the mutualization of risk creates stronger incentives for clearing members to take an active interest in the CCP’s risk management, which could have beneficial effects on the CCP risk management standards and the quality of its participants. Contrary to our model, this would reduce the default probability when the share of default fund contributions in the CCP’s risk management increases. From a broader financial stability perspective, one might also mention that margins can have pro-cyclical effects. Indeed, as margins tend to increase in times of higher market volatility, they potentially add further stress to participants’ liquidity needs (Borio, Furfine and Lowe 2001). As default fund contributions are typically less pro-cyclical, taking into account this effect might also shift the optimal balance between the two risk management instruments in favor of a higher default fund.
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