How Riskless Is “Riskless” Arbitrage?

Roman Kozhan
University of Warwick

Wing Wah Tham
Erasmus University Rotterdam

6th Annual Central Bank Workshop
on the Microstructure of Financial Markets
October, 2010
Arbitrage

With recent technological advances in financial markets, there is a dramatic increase in algorithmic high-frequency trading.

One of the most widely-used strategies of algo traders is high-frequency arbitrage between convertible assets.

Examples: CIP, triangular arbitrage, put-call parity.

Important characteristic is that they are virtually riskless:

- different from convergence trading
- can be exploited immediately without outlay of endowment
From academic perspective ...

teoretically such arbitrage opportunities may arise but should not persist in an efficient market

Why should such arbitrage persist when it is not regarded as risky?

PUZZLE?

No, there is a limit to arbitrage
In this paper...

- We propose and provide theoretical and empirical support that arbitrage opportunities can persist because of the uncertainty of completing a profitable arbitrage portfolio.

- This uncertainty arises due to the crowding trade effect as competing arbitrageurs impose negative externality of each other.

- We call this execution risk and it increases with the number of competing arbitrageurs, market illiquidity and inventory costs.
Example
Example
Example
In Equilibrium...

\[ A = \sum_{i=1}^{l} \phi_i \left( 1 - P_{i|n_i,k,\pi} \right) \]

- \( A \) – level of mispricing
- \( \phi_i \) – costs of missing the i-th leg
- \( P_{i|n_i,k,\pi} \) – probability of getting the best price in the market i
- \( k \) – number of competing arbitrageurs
- \( \pi \) – probability of participation

We consider two types of costs \( \phi_i \):

- illiquidity costs
- inventory costs
In Equilibrium...

\[ A = \sum_{i=1}^{I} \phi_i \left( 1 - P_{i|n_i,k,\pi} \right) \]

\( A \) – level of mispricing

\( \phi_i \) – costs of missing the i-th leg

\( P_{i|n_i,k} \) – probability of getting the best price in the market i

\( k \) – number of competing arbitrageurs

\( \pi \) – probability of participation

We consider two types of costs \( \phi_i \):

- illiquidity costs
- inventory costs
Empirical Implications

- “Riskless” arbitrage opportunities are not eliminated instantly in financial markets

- Existence of competing arbitrageurs induces potential losses in arbitraging

- These losses increase with the number of competing arbitrageurs

- Size of arbitrage deviations increases with market illiquidity and cost of inventory
Data

Triangular arbitrage: GBP/USD/EUR
Data comes from Reuters D3000 trading system
Tick-by-tick high-frequency limit order book
Sample period: from 2 Jan 2003 to 30 Dec 2004
We account for bid-ask spreads and brokerage fees

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of profitable clusters</td>
<td>44,166</td>
</tr>
<tr>
<td>Average arbitrage profit</td>
<td>1.56 bps</td>
</tr>
<tr>
<td>Average arbitrage duration</td>
<td>0.77 sec</td>
</tr>
</tbody>
</table>
Controlling for Latency

We control for latency – arbitrage profit is still economically and statistically significant

We compute average time between order arrival and removal from the system

<table>
<thead>
<tr>
<th>Year</th>
<th>EUR/USD</th>
<th>GBP/USD</th>
<th>EUR/GBP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>0.037</td>
<td>0.034</td>
<td>0.035</td>
</tr>
<tr>
<td>2004</td>
<td>0.033</td>
<td>0.031</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Arbitrage profit is computed after the average delay

<table>
<thead>
<tr>
<th></th>
<th>Without Latency</th>
<th>With Latency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Profit (GBP)</td>
<td>6,265,896.07</td>
<td>2,438,758.95</td>
</tr>
<tr>
<td>Mean Profit (bps)</td>
<td>1.56**</td>
<td>0.63**</td>
</tr>
<tr>
<td>Standard deviation (bps)</td>
<td>1.92</td>
<td>2.07</td>
</tr>
<tr>
<td>t-stat (profit without latency=profit with latency)</td>
<td>66.0</td>
<td></td>
</tr>
</tbody>
</table>
Arbitrage profits: Simulation

We simulate a trading game to:

- see the effect of crowding trade effect
- create ideal environment free of any other impediments

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\pi = 0.1$</th>
<th>$\pi = 0.2$</th>
<th>$\pi = 0.3$</th>
<th>$\pi = 0.4$</th>
<th>$\pi = 0.5$</th>
<th>$\pi = 0.6$</th>
<th>$\pi = 0.7$</th>
<th>$\pi = 0.8$</th>
<th>$\pi = 0.9$</th>
<th>$\pi = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.323</td>
<td>0.619</td>
<td>0.881</td>
<td>1.120</td>
<td>1.328</td>
<td>1.498</td>
<td>1.647</td>
<td>1.763</td>
<td>1.846</td>
<td>1.902</td>
</tr>
<tr>
<td>4</td>
<td>0.295</td>
<td>0.496</td>
<td>0.598</td>
<td>0.598</td>
<td>0.490</td>
<td>0.271</td>
<td>-0.064</td>
<td>-0.523</td>
<td>-1.109</td>
<td>-1.828</td>
</tr>
<tr>
<td>6</td>
<td>0.263</td>
<td>0.364</td>
<td>0.284</td>
<td>0.015</td>
<td>-0.448</td>
<td>-1.128</td>
<td>-2.023</td>
<td>-3.140</td>
<td>-4.495</td>
<td>-6.108</td>
</tr>
<tr>
<td>8</td>
<td>0.229</td>
<td>0.223</td>
<td>-0.049</td>
<td>-0.610</td>
<td>-1.472</td>
<td>-2.657</td>
<td>-4.178</td>
<td>-6.046</td>
<td>-8.271</td>
<td>-10.85</td>
</tr>
<tr>
<td>10</td>
<td>0.197</td>
<td>0.077</td>
<td>-0.401</td>
<td>-1.271</td>
<td>-2.558</td>
<td>-4.288</td>
<td>-6.477</td>
<td>-9.125</td>
<td>-12.25</td>
<td>-15.86</td>
</tr>
<tr>
<td>12</td>
<td>0.164</td>
<td>-0.102</td>
<td>-0.865</td>
<td>-2.168</td>
<td>-4.055</td>
<td>-6.549</td>
<td>-9.675</td>
<td>-13.44</td>
<td>-17.89</td>
<td>-23.04</td>
</tr>
<tr>
<td>14</td>
<td>0.127</td>
<td>-0.231</td>
<td>-1.154</td>
<td>-2.693</td>
<td>-4.892</td>
<td>-7.781</td>
<td>-11.38</td>
<td>-15.71</td>
<td>-20.80</td>
<td>-26.64</td>
</tr>
</tbody>
</table>

Arbitrageurs lose money because of crowding trade effect
We proxy inventory risk

EXECUTED

holding inventory for 60 sec

PROFIT OR LOSS

EXECUTED

NOT EXECUTED
Arbitrage deviation is proportional to market illiquidity and inventory risk

\[ A = a_0 + a_1 \cdot \phi_{GB/US} + a_2 \cdot \phi_{EU/US} + a_3 \cdot \phi_{EU/GB} + a_4 \cdot Tr.Vol + a_5 \cdot TED \]

<table>
<thead>
<tr>
<th>( \Delta_{GB/US} )</th>
<th>( \Delta_{EU/US} )</th>
<th>( \Delta_{EU/GB} )</th>
<th>( \lambda_{GB/US} )</th>
<th>( \lambda_{EU/US} )</th>
<th>( \lambda_{EU/GB} )</th>
<th>IC_{GB/US}</th>
<th>IC_{EU/US}</th>
<th>IC_{EU/GB}</th>
<th>Tr.Vol</th>
<th>TED</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1837 (90.2)</td>
<td>0.0521 (41.3)</td>
<td>0.0044 (3.00)</td>
<td></td>
<td></td>
<td></td>
<td>1.5964 (6.91)</td>
<td>-0.000029 (-2.30)</td>
<td>24.52</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.2904 (41.6)</td>
<td>0.1321 (33.4)</td>
<td>0.0912 (6.19)</td>
<td>1.3579 (5.13)</td>
<td>-0.000010 (-0.71)</td>
<td>8.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0132 (3.75)</td>
<td>0.0239 (8.46)</td>
<td>0.0266 (6.51)</td>
<td>1.0298 (3.87)</td>
<td>-0.000031 (-2.16)</td>
<td>1.97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1734 (88.9)</td>
<td>0.0512 (40.2)</td>
<td>0.0043 (2.94)</td>
<td></td>
<td></td>
<td></td>
<td>0.0015 (0.48)</td>
<td>0.0017 (0.69)</td>
<td>0.0139 (3.87)</td>
<td>1.5261 (6.67)</td>
<td>-0.000028 (-2.15)</td>
<td>24.59</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.2896 (39.6)</td>
<td>0.1184 (31.3)</td>
<td>0.0753 (5.13)</td>
<td>0.0097 (2.84)</td>
<td>0.0134 (4.93)</td>
<td>0.0187 (4.77)</td>
<td>1.0732 (4.12)</td>
<td>-0.000014 (-1.01)</td>
<td>9.52</td>
</tr>
</tbody>
</table>
Main Implications

1. “Riskless” arbitrage can be very risky!

2. Risk comes from crowding trade when arbitrageurs compete for scarce liquidity.

3. Competition is not always “good” for market efficiency – crowding effect with arbitrageurs imposing negative externality on each other.

4. It is also important for any other correlated algorithmic trading.
Thank You!