How riskless is “riskless” arbitrage? *

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Abstract

In this paper, we challenge the notion that exploiting “riskless” arbitrage is riskless. We show that if rational agents face uncertainty about completing their arbitrage portfolios, then arbitrage is limited even in markets with perfect substitutes and convertibility. We call this phenomenon “execution risk in arbitrage exploitation.” In our model, this risk arises from the crowding effect of competing arbitrageurs entering the same trade and inflicting negative externalities on each other. We demonstrate that the cost of illiquidity and holding inventory are potential negative externalities. Our empirical results provide evidence that support the relevance of execution risk in arbitrage.

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1. Introduction

The concept of arbitrage is one of the cornerstones of financial economics. Arbitrage can be classified into textbook "riskless" arbitrage and risky arbitrage, see Mitchell, Pulvino and Stafford (2002) and Liu and Timmermann (2009). "Riskless" arbitrage is defined as the simultaneous sales and purchases of identical assets, which ensures that arbitrageurs require no outlay of personal wealth, but need only set up simultaneous contracts such that the revenue generated from the selling contracts pays the costs of the buying contracts. Contrary to a risky arbitrage strategy, it does not rely on convergence trading where one bets that the future price difference between two assets with similar, but not identical, characteristics will narrow.[1]

Several recent papers show empirically that there are "riskless" arbitrage opportunities, such as deviations from triangular arbitrage parity, covered interest parity, put-call parity, and cross-listed security parity. Akram, Rime and Sarno (2008), Fong, Valente and Fung (2008), and Marshall, Treepongkaruna and Young (2008) find exploitable arbitrage opportunities in the foreign exchange (FX) market. Gagnon and Karolyi (2009) examine cross-listed and domestic stock pairs from 39 countries and find there are price discrepancies. Lamont and Thaler (2003) and Ofek, Richardson and Whitelaw (2004) find deviations from put-call parity.[2] Often, these mispricings are not exploited immediately and durations of these arbitrage opportunities are relatively short. With the recent proliferation of technological advancements and as the speed of trading approaching light speed in recent decade, exploitation of these arbitrage opportunities becomes increasing popular and generally relies on high frequency trading.[3] While there are many papers that document the existence of risky arbitrage opportunities and the associated risks of exploiting them, exploiting "riskless" arbitrage is widely recognized to be riskless.[4] In

[1] An example of convergence trading is exploitation of the mispricing of dual-listed companies (DLCs), which are defined as companies that have combined their operations. An important characteristic of DLC arbitrage is that the underlying shares are not convertible into each other. Risky arbitrage positions are kept open until prices converge. Hence, arbitrageurs with wealth constraints are concerned with noise trader, fundamental, and synchronization risk in convergence trading. References include De Jong, Rosenthal and Van Dijk (2009) and Froot and Dabora (1999) among many others.

[2] Explanations for the existence of these deviations are often based on institutional frictions like regulatory and short-selling constraints, taxes, and direct transaction costs.

[3] High frequency trading (HFT) strategies include cross-asset arbitrage, electronic market making, liquidity detection, short-term statistical arbitrage and volatility arbitrage, see Tabb, Iati and Sussman (2009).

[4] References include Shleifer and Vishny (1992), De Long, Shleifer, Summers and Waldmann (1990), Shleifer and Summers (1990), Abreu and Brunnermeier (2002), Barberis and Thaler (2003), and Schultz and Shive

this paper, we challenge this conventional belief and show that execution risk creates limits to "riskless" arbitrage.

The study of risk involving high frequency arbitrage can be important for practitioners and policy makers. Practitioners will be interested in the trading risk of high frequency arbitrage as high frequency trading is quickly becoming an important part today's financial market. High frequency trading is estimated to account for an annual aggregated profit of about U.S. 21 billion according to a report by Tabb et al. (2009) from TABB group. Moreover, risks involving high frequency arbitrage trading strategies affects market quality and it is attracting the concerns of financial regulators. Committee for European Securities Regulators takes particular interest in high frequency trading strategies and call for evidence on microstructure issues in the European equity markets (Ref:CESR/10-142). On 13th January 2010, the US Securities and Exchange Commission (SEC) issued a Concept Release on Equity Market Structure (Release No. 34-61358; File No. S7-02-10) seeking comments on the current market structure, high frequency trading and undisplayed liquidity. In particular, SEC seeks for broad comments on how high frequency trading strategies, such as arbitrage trading strategies, use by proprietary firms affects market quality.

From the academic's perspective, the existence and persistence of "riskless" arbitrage opportunities suggest the existence of limits to arbitrage. In this paper, we propose a new limit to arbitrage, one which exists because of the uncertainty of completing a profitable arbitrage portfolio due to the crowding effect of arbitrageurs competing for the scarce supply of the required assets. We call this "execution risk in arbitrage exploitation," which depends on the degree of competition among arbitrageurs, the cost of market illiquidity and the cost of holding inventory.[5] The intuition is simple: Consider 2 arbitrageurs competing to form a long-short arbitrage portfolio of two identical but mispriced assets, A and B, each with only 1 available unit at the profitable price. Then, exploiting this arbitrage can be risky as one of the arbitrageurs will

(2009) among others.

[5]The execution risk in arbitrage exploitation discussed in this paper is different from execution risk in trading. Execution risk in trading is often related to the timing uncertainty about when limit orders will execute while execution risk in arbitrage exploitation is concerned about the uncertainty in executing a profitable arbitrage strategy. See Parlour (1998) and Engle and Ferstenberg (2006) among many others for discussions about execution risk in trading.

fail at acquiring/shorting A and B at a profitable price and might incur liquidity cost (walking up the limit order book) or inventory cost (unwanted inventory from successfully acquiring A but unsuccessful from shorting B). We provide both theoretical and empirical support for this new limit, which is applicable to all asset markets. In contrast to previous limits to arbitrage, the mechanism we present does not rely on convergence trading, taxes, regulatory, and short-selling constraints. This risk can be particularly important in explaining the persistence and in exploiting deviations of "riskless" arbitrage opportunities.

To illustrate the relevance of execution risk, we build a simple model to capture the process by which arbitrageurs trade when they observe a violation of an arbitrage parity in a market in which identical assets can be converted into each other. Each arbitrageur in our model maximizes her trading profits, taking into account direct transaction costs (quoted bid-ask spread), execution risk, and anticipated actions of other competing arbitrageurs. In contrast to the common notion that competition improves price efficiency, competition among arbitrageurs limits efficiency in our model as they inflict negative externalities on each other through the indirect transaction cost of trading (liquidity cost) and the cost of holding inventory.

We empirically test our hypotheses by using a set of reliable and detailed limit order book data from a widely traded and liquid electronic trading platform of the spot FX market. To isolate execution risk from the existing impediments to arbitrage, our study focuses on triangular arbitrage exploitation in major currency pairs, because triangular arbitrage is a non-convergence trading based arbitrage and is not subject to taxes, regulatory, or short-selling constraints.

We find that even after we account for bid-ask spread (direct transaction cost) and latency cost, there are triangular arbitrage opportunities in the FX market and that these opportunities are not exploited instantly. This finding provides initial support for the existence of risk in non-convergence trading based arbitrage. If arbitrageurs do not exploit arbitrage opportunities, then either it might not be optimal for them to do so immediately, or it is because they are not being compensated appropriately for their risk exposure.

We examine the relation between the size of the arbitrage deviation and market illiquidity. We find a statistically significant relation in which the deviation is positively correlated with measures of market illiquidity. These results support the work of Roll, Schwartz and Subrahmanyam (2007), who argue that market liquidity plays a key role in moving prices to eliminate arbitrage opportunities. However, we argue that the economic reason behind this relation, in the case of "riskless" arbitrage, is the presence of execution risk. We also perform an economic evaluation of arbitraging strategies in a Monte Carlo study and find that arbitrageurs might incur losses in textbook "riskless" arbitraging.[6] Their losses worsen as the number of competing arbitrageurs increases. In addition, we study the the cost of holding inventory through the profit and loss distribution of arbitrage exploitation failures. We find a statistically significant and positive relation between arbitrage deviation and cost of inventory.

The role of competing arbitrageurs and market illiquidity on the existence of convergence-trading arbitrage opportunities has been studied recently by, among others, Stein (2009), Kondor (2009), and Oehmke (2009). Kondor (2009) studies the impact of competition on arbitrage. Oehmke (2009) investigates the role of strategic arbitrageurs, trading with liquidity frictions, on the speed of price convergence. However, these papers, like many others that examine limits to arbitrage, focus on convergence trading and the use of wealth constraints that cause arbitrageurs to unwillingly unwind their positions as prices diverge. Although the models in these papers are important in explaining the divergence of convergence trading based mispricings like DLCs, they do not explain the delayed elimination of "riskless" arbitrage such as triangular arbitrage and cross-listed stocks. The main difference between the mechanism of our model and those in Oehmke (2009) and Kondor (2009) is the effect of crowded trade on the elimination of arbitrage. Efficiency increases with the number of arbitrageurs in their models, while excess competition for scarce supply of arbitrage assets prevents immediate elimination of mispricing in our model.

The crowding effect in our model is similar to that of Stein (2009), where he proposes a limit to arbitrage that stems from the uncertainty about the number of arbitrageurs entering the same trade. His mechanism relies on arbitrage strategies with no fundamental anchor in contrast to

[6] Liu and Longstaff (2004) also find that an arbitrage portfolio experiences losses before the convergence date.

standard model of arbitrage where arbitrageurs know the fundamental value of the asset.[7] He argues that, in the event with fundamental anchor, the price mechanism mediates congestion, and there is no danger of the market becoming overcrowded with arbitrageurs. In contrast, we show that overcrowding of arbitrageurs can be risky even in exploiting mispricings with fundamental anchor. Our mechanism relies on the fact that financial assets can be indivisible because of minimum trade size restrictions. This implies that overcrowding of arbitrageurs competing for limited supply of arbitrage assets can incur losses as profits can no longer be shared among arbitrageurs in the case of infinitely divisible assets.

Kleidon (1992), Kumar and Seppi (1994) and Holden (1995) highlight the importance of execution risk in index arbitrage under stressful market conditions (crash of October 1987), where execution risk can be a concern due to trading on stale (lagged) prices.[8] Their mechanism is different from ours as the "paper environment" of NYSE in 1987 and software inadequacy to cope with cancelation or replacement of limit orders that contribute to stale orders and prices make exploitation of index arbitrage risky.

We contribute to the literature on limits to arbitrage by focusing on "riskless" arbitrage and challenging the assumption that exploiting non-convergence arbitrage is riskless. To the best of our knowledge, this is the first paper to address the risk of supposedly "riskless" arbitrage. Our paper also contributes to the market microstructure literature, since we demonstrate, by using the limit order book, the importance of how trading frictions and competition impede financial market efficiency. We also provide an alternative liquidity-based theory for impediments to arbitrage, thus supporting the empirical papers that relate "riskless" arbitrage deviations to market illiquidity. While earlier studies focus on the cost of inventory on market makers but not on arbitrageurs, we contribute to the literature as the first study of the effect of the cost of inventory on arbitrage activities. With an extremely detailed and reliable limit order and transaction data set, we believe our paper is among the first to empirically study high frequency

[7] Trading strategies with no fundamental anchor imply that arbitrageurs do not observe the fundamental value and do not base their demands on an independent estimate of fundamental value.

[8] Index arbitrage is the exploitation of mispricings between cash index and futures, can be considered a form of convergence trading, which is susceptible to shortselling constraints, fundamental, noise trader, and synchronization risk. Other references on index arbitrage among many others, include Brennan and Schwartz (1988), Kumar and Seppi (1994), Sofianos (1993), and Neal (1996).

arbitrage and the relation among arbitrage deviations, cost of inventory and market liquidity using liquidity measures derived from full limit order book information. Our study also have important implications for crowded trading, such as highly correlated algorithmic trades, see Chaboud, Chiquoine, Hjalmarsson and Vega (2010), and crowded unwinding of positions during the recent credit crisis, see Khandani and Lo (2007), in financial markets.

The remainder of the paper is organized as follows. In the next section, we present the model and discuss the equilibrium of the model. In Section 3, we briefly discuss triangular arbitrage in the FX market and review the related literature. In Section 4, we describe the data and empirically test the hypotheses derived from the model. Section 5 assesses the economic value of arbitrage activities in the presence of competing arbitrageurs. Finally, Section 6 concludes.

2. A simple model of execution risk in arbitrage

In this section we begin with an overview of our model and a detailed discussion of its implications.

2.1. Markets and assets

We consider a setup, where there are I assets indexed by $i \in \{1, 2, \ldots, I\}$ that are traded in I segmented markets. We assume there exists a portfolio, RP, consisting of all assets from the set $\{2, \ldots, I\}$, which has a payoff structure and dividend stream identical to asset 1. For simplicity, this portfolio includes long and short positions of one unit in each asset denoted by the vector $[w_2, \ldots, w_I]$. w_i takes the value of 1 if it is a long position and -1 if it is a short position in asset i. We assume that there are no short sale constraints in the market.

Assumption 1. There is perfect convertibility between asset 1 and portfolio RP.

Convertibility here is defined as the ability to convert one unit of asset 1 to one unit of portfolio RP. An example of such a setup is the FX market where a currency can be bought directly (asset 1) or indirectly (portfolio) vis-a-vis other currencies. However, this does not apply to DLCs as these assets are not convertible into each other. Although a DLC consists of two listed companies with different sets of shareholders sharing the ownership of one set of operational businesses, a shareholder holding a share of e.g. Royal Dutch NV cannot convert it into shares in Shell Transport and Trading PLC. Inconvertibility of assets with identical payoff structures and risk exposure will imply that any exploitation of mispricing will rely on convergence trading. With Assumption 1, traditional impediments to arbitrage like fundamental risk, noise trader risk and synchronization risk will be absent in our setup.

2.2. Traders

Following Kondor (2009), we assume that there are I groups of local traders, who operate only within their own corresponding markets. For example, local trader group 1 operates only in market 1 and local trader group 2 operates only in market 2. We assume each group of traders can only trade assets in their own market. There are groups of liquidity traders who trade the asset for exogenous reasons to the model. Liquidity is offered by these traders in the limit order book (LOB) through posting orders. Asymmetric demands and income shocks to these local traders may cause transient differences in the demand for assets in each market. This captures the idea that similar assets can be traded at different prices until arbitrageurs eliminate the mispricing.

In addition to the local traders, we also assume the existence of k competing risk-neutral arbitrageurs. These arbitrageurs can trade across all markets and exploit any existing mispricing.[9] We assume all exploitations are conducted via simultaneous sales and purchases of identical assets with no requirement of any outlay of personal endowment. Arbitrageurs will use market orders to ensure the simultaneity of sales and purchases of mispriced assets. We also consider the implications of using limit orders, which is discussed later in Section 2.7. For simplicity, we assume:

Assumption 2. All arbitrageurs can only buy one unit of each asset needed to form the arbitrage portfolio.

Allowing arbitrageurs to buy more than one unit of asset will only exacerbate execution risk. We denote the set of all arbitrageurs in the market by $\mathcal{K} = \{1, \ldots, k\}$ and the set of all competitors of arbitrager j for $j \in \{1, \ldots, k\}$ by $\mathcal{K}_{-j} = \mathcal{K} \setminus \{j\}$.

[9] Allowing for strategic liquidity traders or differential ability, speed of trading and risk aversion of arbitrageurs would generate interesting insights but not the overall understanding of the main issues examined in this paper.

2.3. *Limit order book*

We assume that all participants in our limit order setup have access to a publicly visible electronic screen, which specifies a price and quantity available at that price. There is no cost in posting, retracting or altering any limit orders at any time except in the middle of a trade execution. All participants are able to see details (all prices and depths) of the demand and supply schedules of the LOB. Many financial exchanges impose a minimum trade size restriction which implies that traded assets are treated as indivisible goods in practice. As such, we assume that all prices are placed in a discrete grid in the LOB and the minimum trade size is one unit.

We assume there are only two layers in our discrete demand and supply schedules. The first layer consists of the best bid and ask prices and the quantities available at this prices. The best bid and ask prices of asset i are denoted by p_i^b and p_i^a respectively. The corresponding quantities available at the best bid and ask prices of asset i are denoted by n_i^b and n_i^a. The next best available bid price of the asset is $p_i^b - \Delta_i^b$ and the next best ask price is $p_i^a + \Delta_i^a$ at the second layer. Δ_i^b and Δ_i^a are the price differences between the best and second best prices for demand and supply schedules respectively. As a simplifying assumption, prices of all assets at the second layer are assumed to be available with infinite supply.[10] The modeled structure of the LOB can be visualized in Figure 1.

Insert Figure 1 here

2.4. *Arbitrage deviation*

As defined earlier, portfolio RP consists of all assets from the set $\{2, \ldots, I\}$. The best price at which one unit of portfolio RP can then be bought is

$$P^a = \sum_{i=2}^{I} w_i p_i (w_i) ,$$

where $p_i (w_i) = p_i^b$ if $w_i = -1$ and $p_i (w_i) = p_i^a$ if $w_i = 1$. The best price at which one unit of portfolio RP can be sold is P^b. Since portfolio RP and asset 1 have identical payoff structures,

[10]Incorporating more than two layers in the demand and supply schedules, which increases the cost of liquidity, will only exacerbate execution risk

dividend streams and risk exposure, they should have the same price. Taking the transaction costs into account, a mispricing occurs if:

$$P^a < p_1^b \qquad \text{or} \qquad P^b > p_1^a,$$

and it can be exploited by arbitrageurs.[11] We define the magnitude of the mispricing then as

$$A = \max\left\{0, P^b - p_1^a, p_1^b - P^a\right\}.$$

Similar to Holden (1995) and Abreu and Brunnermeier (2002), the arbitrage deviation arises from a liquidity or a demand shock to the liquidity providers.[12]

2.5. Arbitrage strategies

Professional arbitrageurs frequently compete against each other in exploiting any observable arbitrage opportunities in financial markets. With limited and scarce supply of the required assets available to form an arbitrage portfolio, we assume that there exists an excess demand for these assets among competing arbitrageurs such that:

Assumption 3. $\max\left\{n_i^a, n_i^b\right\} < k$ for each $i = 1, \ldots I$.

Since arbitrageurs can only purchase one unit of each required asset, we assume that there are always more arbitrageurs than the maximum number of available units of assets.[13] This assumption is made for exposition purposes; execution risk exists as long as there are shortages of supply in at least one of the required assets. Market orders are preferred by arbitrageurs over limit orders because of the advantage of immediacy. With the enormous technological advances in trading tools over recent years, algorithmic trading is widely used in exploiting

[11] Note that only one of the inequalities $P^b - p_1^a > 0$ or $p_1^b - P^a > 0$ is true under the assumption of positive bid-ask spreads ($p_1^a > p_1^b$ and $P^a > P^b$).

[12] There is a literature that attempts to explain the existence of arbitrage opportunities. For example, Titman (1985) points out the relevance of tax clientele effect; Amihud and Mendelson (1980) provides an avenue of how inventory management can cause the existence arbitrage opportunities; Kumar and Seppi (1994) argues how heterogeneous information sets and different speeds of information processing can cause arbitrage opportunities to exist.

[13] Allowing arbitrageurs to purchase more than one unit of each required asset will only exacerbate execution risk.

arbitrage opportunities.[14] These algorithmic trades lead to almost simultaneous exploitation of arbitrage opportunities by large numbers of professional arbitrageurs in the financial market. Thus, arbitrageurs who want to trade upon observing any mispricing are assumed to submit their market orders simultaneously.[15]

Assumption 4. All arbitrageurs have the same probability of executing their market orders at the best available price when they submit market orders simultaneously.

For example, if there were three arbitrageurs vying for one available unit of asset at the best available price, the probability of an arbitrageur successfully acquiring this asset will be one-third. Arbitrageurs who are unsuccessful in acquiring the required asset at the best available price will execute their market orders at the next best available price. These prices are then either $p_i^a + \Delta_i^a$ for buy trade (or $p_i^b - \Delta_i^b$ for sell trades) for asset i. Thus, the penalty for missing a buy trade at the best price in one of required assets i is Δ_i^a.

In this circumstance, the arbitrageur will be left with a payoff of $A - \Delta_i^a$.[16] The worst situation an arbitrageur could face is one in which she fails to acquire all the required assets at the best available price. Her payoff at this instant will be $A - \sum\limits_{i=1}^{I} \Delta_i (w_i)$ where $\Delta_i (w_i) = \Delta_i^b$ if $w_i = -1$ and $\Delta_i (w_i) = \Delta_i^a$ if $w_i = 1$. We assume that her payoff in the worst scenario is negative,

$$A - \sum_{i=1}^{I} \Delta_i (w_i) < 0.$$

All arbitrageurs have two possible strategies upon observing an arbitrage opportunity, "to trade"or "not to trade". An arbitrageur who chooses not to trade will have a payoff of zero. We also assume that all information, arbitrageurs' strategies, preferences and beliefs are common knowledge.

[14] Algorithmic trading is the practice of automatically transacting based on a quantitative model. For work on algorithmic trading, see: Hendershott and Moulton (2007) and Hendershott, Jones and Menkveld (2007).

[15] In reality, actions of arbitrageurs need not take place literally at the same moment. Our assumption of simultaneity is valid as long as arbitrageurs have no knowledge of the queue position of their market orders.

[16] Let there be a mispricing, such that $A = p_1^b - P^a > 0$, and an arbitrageur failing to get the best price in market i. If $w_i = 1$, then the profit of the arbitrageur will be: $p_1^b - \sum\limits_{\iota=2}^{i-1} w_\iota p_\iota(w_\iota) - p_i^a - \Delta_i^a - \sum\limits_{\iota=i+1}^{I} w_\iota p_\iota(w_\iota) = p_1^b - P^a - \Delta_i^a = A - \Delta_i^a$.

2.6. Equilibrium

Given the model described above, arbitrageurs will choose whether to participate in exploiting arbitrage opportunities of a particular deviation size, A. Arbitrageurs seek to maximize their expected payoffs and trade only when these payoffs are positive. The expected payoff of an arbitrageur is given by the following proposition.

Proposition 1. *If the probability of getting the best price for asset i for arbitrageur j is $\mathbf{P}_i^j$, then her expected payoff $E\left(U^j\right)$ is given by*

$$E\left(U^j\right) = A - \sum_{i=1}^{I} \Delta_i\left(w_i\right)\left(1 - \mathbf{P}_i^j\right). \tag{1}$$

Proof. See appendix.

Equation (1) shows that an arbitrageur's expected payoff depends on the price slippage $\Delta\left(w\right)$ and $\left(1 - \mathbf{P}_i^j\right)$, which captures the probability of trader j not getting the best price in market i. In this case, the arbitrageur faces a loss due to price slippage of $\Delta_i\left(w_i\right)$ so the term $\Delta_i\left(w_i\right)\left(1 - \mathbf{P}_i^j\right)$ represents the expected loss for asset i. This loss arises from the execution risk of competing against other arbitrageurs for the observed mispricing between the two identical assets. The severity of these losses or the cost of execution failure increases with $\Delta_i\left(w_i\right)$, which increases with market illiquidity. This demonstrates that competition for scarce supply of assets and market illiquidity exacerbate execution risk when exploiting arbitrage opportunities. Hence, the expected payoff is the difference between the observed mispricing A and the expected loss due to execution risk.

In the case of full participation by all the arbitrageurs in exploiting the arbitrage opportunity with simultaneous market order submission, the probability of trader j executing a market order at the best price for asset i is

$$\mathbf{P}_{i|n_i(w_i),k}^j = \frac{n_i}{k},$$

where $n_i(w_i)$ denotes the quantity available at the best price and k denotes the number of competing arbitrageurs. The subscripts on $\mathbf{P}_{i|n_i,k}^j$ highlight the role of $n_i(w_i)$ and k in affecting the success of executing a best price market order. As the number of competing arbitrageurs increases, the probability of success converges to zero. As the breadth of asset i increases, an arbitrageur is more likely to execute her best price market order.[17] In this case, the expected loss of arbitrageur j is

$$E\left(L^j\right) = \sum_{i=1}^{I} \Delta_i\left(w_i\right)\left(1 - \frac{n_i(w_i)}{k}\right). \tag{2}$$

If $A \geq E\left(L^j\right)$, it is optimal for the trader to use the strategy "trade" and to receive a positive payoff. As the number of arbitrageurs increases, the expected payoff $E\left(U^j\right)$ converges to $A - \sum_{i=1}^{I} \Delta_i\left(w_i\right) < 0$, so the arbitrageurs are expected to suffer losses with increasing competition.

We assume that arbitrageurs adopt mixed strategies in their arbitrage strategies, where they participate in the market but with only a probability of exploiting any mispricing. We denote the probability of participation of arbitrageur $j \in \{1, \ldots, k\}$ by $\pi_j \in [0, 1]$. For a mixed strategy profile $\Pi = (\pi_1, \ldots, \pi_k)$, we use a standard notation $\Pi_{-j} = (\pi_1, \ldots, \pi_{j-1}, \pi_{j+1}, \ldots, \pi_k)$ to denote a strategy profile of all arbitrageurs other than j. We add the subscript, Π_{-j}, in the notation $\mathbf{P}^j_{i|n_i,k,\Pi_{-j}}$ to underline its dependence on strategies of trader j's opponents and denote $\bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}} = 1 - \mathbf{P}^j_{i|n_i,k,\Pi_{-j}}$.

The expected payoff of arbitrageur j in the case of mixed strategies is $\pi_j E\left(U^j|\Pi_{-j}\right)$, where according to Proposition 1,

$$E\left(U^j|\Pi_{-j}\right) = A - \sum_{i=1}^{I} \Delta_i\left(w_i\right) \bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}} \tag{3}$$

is the expected payoff of arbitrageur j playing pure strategy "trade" while her opponents use mixed strategies Π_{-j}.

Proposition 2. *For a given mixed strategy profile* $\Pi = (\pi_1, \ldots, \pi_k)$*:*

(i) the probability $\mathbf{P}^j_{i|n_i,k,\Pi_{-j}}$ *decreases monotonically with the number of participating arbitrageurs* k*;*

(ii) the expected profit $\pi_j E\left(U^j|\Pi_{-j}\right)$ *of arbitrageur* j *decreases monotonically with the number*

[17]Breadth of an asset is defined as the quantity available at the best price.

of participating arbitrageurs k.

Proof. See appendix.

The more arbitrageurs participate in the market, the smaller is the probability that someone will manage to complete all the necessary transactions to cash the profit out. This leads to the situation where most of them might face losses and these losses increase with competition.

By Nash's theorem, there exists a mixed strategy profile Π that forms a Nash equilibrium for the above game. The following proposition characterizes the mixed strategy equilibria of the game.

Proposition 3. *If a mixed strategy profile $\Pi = (\pi_1, ..., \pi_k)$ with $\pi_j \in (0, 1)$ is a Nash equilibrium of the game, then $\pi_j = \pi_{j'} = \pi$ for each j, j' in $\mathcal{K}$ and the observed arbitrage deviation is a linear function of the differences between the best and the second best prices on the corresponding markets*

$$A = \sum_{i=1}^{I} \Delta_i (w_i) \bar{\mathbf{P}}_{i|n_i,k,\pi}; \tag{4}$$

Proof. See appendix.

The above proposition states that risk-neutral arbitrageurs demand an expected payoff of zero and have an identical probability of participation, π, in a mixed strategy equilibrium. Since strategies of all arbitrageurs are identical, we will drop all superscript j to simplify the notation.

2.7. Limit orders and inventory cost

So far, we have only included the cost of liquidity as a penalty for unsuccessful arbitrageurs. Cost of liquidity is especially relevant when market orders are used in exploiting arbitrage. However, arbitrageurs might consider using limit orders to avoid liquidity cost instead. There is inventory risk if an arbitrageur uses limit orders. An example of this is when an arbitrageur successfully acquires two currency pairs but fails in the third currency pair in triangular arbitrage. She will be left with unwanted inventory of the successful acquisitions and the risk of large price movements. Thus, the crowded trades of arbitrageurs can inflict negative externality on each other through liquidity and inventory cost. Hence, we present the equilibrium profit in a more general form

$$A = \sum_{i=1}^{I} \phi_i \left(w_i\right) \bar{\mathbf{P}}_{i|n_i,k,\pi}, \tag{5}$$

where ϕ_i is a aggregate liquidity and inventory costs.

Equation (5) shows that the magnitude of the arbitrage deviation is associated with the execution risk for each of the I number of asset in an arbitrage portfolio. The total execution risk compensation or the arbitrage deviation can be seen as the sum of individual compensation for execution risk for each individual asset. Each of these individual components depends on the cost of execution failure, $\phi_i \left(w_i\right)$, and the failure probability of executing the best price market orders, $\bar{\mathbf{P}}_{i|n_i,k,\pi}$. Thus, the optimal probability of participation is also a function the arbitrage deviation, the breadth of the asset supply, the number of existing arbitrageurs.

In an efficient market, every arbitrage opportunity should be eliminated instantly. However, in order to eliminate the observed arbitrage deviation, it is necessary to execute in aggregate all available units of at least one of the assets $\{1, ..., I\}$. We denote the minimum breadth of all assets by $\underline{n} = \min_{i \in \mathcal{I}} \{n_i(w_i)\}$. In equilibrium, if arbitrageurs adopt mixed strategies (i.e., the arbitrage opportunity is not large enough to make the full participation optimal), they will participate with probability $\pi < 1$ in exploiting the observed mispricing. In this case, the probability that the arbitrage opportunity disappears immediately from the market is equal to the probability that $\underline{n}$ or more arbitrageurs out of k decide to compete (which can be described by the binomial distribution). Thus, we have established that

Proposition 4. *In the mixed strategy equilibrium, the probability that the arbitrage opportunity disappears immediately is*

$$P = \sum_{s=\underline{n}}^{k} \left(\begin{array}{c} k \\ s \end{array} \right) \pi^s (1 - \pi)^{k-s} < 1. \tag{6}$$

This result shows that under competition for scarce supply of the assets required for the arbitrage portfolio, the arbitrage opportunity might remain in the market for some time. In the face of the execution risk, arbitrageurs might all decide not to participate with some positive probability. Although we do not model the duration of the mispricing explicitly, we can see from Proposition 4 that the probability of immediate arbitrage elimination is smaller than one.

2.8. Some remarks on crowding effect

Stein (2009) studies the effect of competition on market efficiency when arbitrageurs inflict negative externality on one another. He introduces the mechanism of crowded trade effect where an arbitrageur is uncertain in real time about the number of competing arbitrageurs using the same model and taking the same position as her. The crowded trade effect in Stein (2009) relies on the assumption that individual arbitrageur has imperfect information about the aggregate arbitrage capacity. Moreover, the trading strategy in his model is one with no fundamental anchor where arbitrageurs are uncertain about the fundamental value of the asset. With these ingredients, he argues that competition not necessarily improves market efficiency. He also argues that there will not be crowding effect if arbitrageurs observe the fundamental value. Both assumptions are crucial in his model. If there is a fundamental anchor and the asset is infinitely divisible, each individual arbitrageurs will adjust her demand accordingly to accommodate competition such that they will share any profit. This price mechanism mediates congestion and there will be no danger of overcrowding.

However, it is often observed in practice that financial assets are indivisible because of minimum trade size restriction. By accounting for the restriction of minimum trade size in financial markets, there will be insufficient units of assets for all arbitrageurs to have a positive profit. Thus, competition among arbitrageurs for these indivisible assets creates excess demand for these assets. Under the assumption of competition for indivisible goods, it is not guaranteed that all arbitrageurs will always be able to make money. Taking all these into account in our model, we argue that competition can creates frictions to market efficiency even when the mispricing is directly observable to all arbitrageurs.

2.9. Main implications

Our model has three main implications. First, in the presence of competition, an arbitrageur faces execution risk in acquiring her arbitrage portfolio at the best price. This risk stems from the uncertainty in acquiring the portfolio at a profitable price because of competition for the scarce supply of profitable arbitrage portfolios. From Proposition 3, arbitrageurs demand a compensation for the execution risk and will participate in arbitrage activities with certainty only if the observed mispricing exceeds the expected loss given in Equation (2). Otherwise, arbitrageurs do not employ pure strategy "trade". In this case, arbitrageurs adopt mixed strategies with some positive probability of participation and the mispricing might not be exploited immediately, which is explicitly claimed in Proposition 4. This is consistent with the existing literature on limits to arbitrage, which suggests that arbitrage opportunities persist because exploiting them can be risky. However, the nature of risk arbitrage in the current literature relies on the existence of convergence trading while the driver of our risk is arbitrage competition. Thus, we argue that non-convergence arbitrage opportunities might persist because of execution risk.

Secondly, execution risk in arbitrage worsens with increasing number of competing arbitrageurs. This is because the failure probability of acquiring the arbitrage portfolio at a profitable price increases with the number of competing arbitrageurs. Thus, arbitrageurs incur more losses with increasing competition. This highlights the problem of infinite arbitrageurs' demands in a world of finite resources. The relevance of the number of competing arbitrageurs is analogous to the economic problem of scarcity, where not all the goals of society can be fulfilled at the same time with limited supply of goods. Increasing competition for limited numbers of exploitable arbitrage opportunities brings upon execution risk that prevents efficient elimination of asset mispricing.

Finally, the demanded compensation for execution risk in equilibrium increases with the market illiquidity and the cost of holding inventory. Our model provides a theoretical framework for recent empirical evidence on the relation between the deviation from the law of one price and market illiquidity (e.g. Roll et al. (2007), Deville and Riva (2007), Akram et al. (2008), Fong et al. (2008), Marshall et al. (2008), etc.). In equilibrium, we have shown that the cost of failure is related to the slope of the demand and supply schedules and the cost of inventory. The more illiquid is the market and the higher the inventory cost, the higher will the cost of failure be in acquiring an arbitrage portfolio at the best price. From the model, we establish the following testable hypotheses:

1. **"Riskless" arbitrage opportunities are not eliminated instantly in financial markets.**

2. **The existence of competing arbitrageurs induces potential losses in arbitraging.**

3. **These losses increase with the number of competing arbitrageurs.**

4. **The size of arbitrage deviations increases with market illiquidity and cost of inventory.**

We will test these hypotheses by examining the triangular arbitrage parity in FX market. Our study focuses on triangular arbitrage exploitation in major currency pairs, because triangular arbitrage is non-convergence trading based arbitrage and is not subject to taxes, regulatory, or short-selling constraints. In the next section, we will discuss about triangular arbitrage in the FX market and the relevant literature.

3. Triangular arbitrage in the FX market

In the FX market, price consistency of economically equivalent assets implies that exchange rates are in parity. They should be aligned so that no persistent risk-free profits can be made by arbitraging among currencies. Triangular arbitrage involves one exchange rate traded at two different prices, a direct price and an indirect price (vis-a-vis other currencies). Arbitrage profits might potentially be made by buying the lower of the two and selling the higher of the two simultaneously. Triangular arbitrage conditions ensure consistent pricing by arbitraging among the three markets. We denote $S\left(A/B\right)$ as the number of units of currency A per unit of currency B in the spot FX market. Arbitrageurs are often assumed to eliminate any price discrepancy if the inferred cross-rate between currencies A and B is known through the two currencies' quotes vis-a-vis the third currency C. Taking the transaction costs into account, the triangular no-arbitrage conditions are

$$S\left(A/B^{ask}\right) \geq S\left(C/B^{bid}\right) \times S\left(A/C^{bid}\right), \tag{7}$$

$$S\left(B/A^{ask}\right) \geq S\left(C/A^{bid}\right) \times S\left(B/C^{bid}\right). \tag{8}$$

Any deviation from Equation (7) or (8) would represent a textbook riskless arbitrage opportunity.[18]

The literature on triangular arbitrage supports the existence and persistence of non-convergence arbitrage opportunities. Aiba, Takayasu, Marumo and Shimizu (2002) find exploitable arbitrage opportunities that last about 90 minutes a day in the FX market using transaction data between yen-dollar, dollar-euro and yen-euro for the period January 25, 1999 to March 12, 1999. Marshall et al. (2008) finds the existence of exploitable arbitrage opportunities using one year binding quote data from EBS and argues that these opportunities are monies left on the table to compensate arbitrageurs for their service in relieving market-maker's order imbalance. Lyons and Moore (2009) model how direct trades and trades through vehicle currencies affect the revelation of information. They assumes that arbitrage traders account for their price impact of trades when they exploit triangular arbitrage deviations. Our model and findings support their assumption but further elaborate on how competition, cost of inventory and liquidity might deter exploitation of arbitrage deviations. Our paper contributes to this literature theoretically and empirically by emphasizing on the role of crowding trades, cost of inventory and latency cost in FX arbitrage with a detailed set of limit order book data.

4. Data sources and preliminary analysis

While most previous research uses data during the early implementation of electronic platforms, we use tick by tick data from the Reuters trading system Dealing 3000 for three currency pairs: US dollar per euro, US dollar per pound sterling, pound sterling per euro (hereafter EUR/USD, GBP/USD, and EUR/GBP, respectively) after year 2000. Our sample period runs from January

[18] The conditions state that what is bought cannot be sold at a higher price immediately.

2, 2003 to December 30, 2004. We choose a sample period from 2003 to 2004 to control for settlement risk.[19]

The Bank for International Settlement (BIS, 2004) estimates that trades in these currencies constitute up to 60 percent of the FX spot transactions, 53 percent of which are interdealer trades. These trading amounts indicate that our data represent a substantial part of the FX market.[20]

The data we analyze excludes all weekends and holidays. The advantage of this data set is the availability of volume in all limit orders, which allows one to reconstruct the full limit orderbook, without making any ad-hoc assumptions. For each limit order, the data set reports the currency pair, unique order identifier, price, order quantity, hidden quantity (D3000 function), quantity traded, order type, transaction identifier of order entered or removed, status of market order, entry type of orders, removal reason, time of orders entered and removed. The data time stamps are accurate to one-thousandth of a second. The minimum trade size in Reuters trading system Dealing 3000 is 1 million units of the base currency. This extremely detailed data set makes it easier for us to reconstruct the limit order book, and enables us to track all types of orders submitted throughout the day and to update the limit order book for all entries, removals, amendments, and trade executions.

4.1. Summary statistics

In this section, we report the preliminary statistics of arbitrage deviations and clusters (sequences) of profitable triangular arbitrage deviations. We define a cluster as at least one consecutive triangular arbitrage deviation. The duration of a cluster will simply be the elapsed

[19] Settlement risk is the risk that a settlement in a transfer system does not take place as expected. Generally, this happens because one party defaults on its clearing obligations to one or more counterparties. As such, settlement risk comprises credit risks. It arises when a counterparty cannot meet an obligation for full value on due date and thereafter because it is insolvent. Thus, arbitrage opportunities can exist because of settlement risk. See Lindley (2008) for the latest survey on settlement risk in FX market. In 2002, Continuous Linked Settlement (CLS) Bank went into operation, settling transactions involving seven currencies: the US dollar, euro, yen, pound sterling, Swiss franc, Canadian dollar and Australian dollar. Although traditional corresponding bank settlements are still being used in FX market, they constitute only minority amount of trades in FX market as dealers seek to reduce settlement risk. This is in line with the introduction of CLS into the FX market. Conversations with FX dealers who engage with triangular arbitrage activities indicate that settlement risk is not a major concern for our sample period.

[20] See Osler (2008), Lyons (2001) and Rime (2003) for a more detailed survey about the institutional features of the FX market.

time required for exchange rates to revert to no arbitrage values, after a deviation has been identified. In our sample, there are 40,166 arbitrage clusters. We identify a round-trip arbitrage opportunity as follows:

1. Record the latest best bid and best ask prices in the limit order book for the three currency pairs in our portfolio.

2. Identify if an arbitrage opportunity exists. Check this by buying one unit of currency 1 (e.g. GBP/USD) and buying currency 2 (e.g. EUR/GBP). This is equivalent to selling USD for GBP and using the GBP from sales to purchase EUR. Thus, our net position will be short USD/ long EUR, which we will compare against the rate for currency 3 (e.g. EUR/USD). We will buy currency 3 to obtain an arbitrage profit if the rate is lower than our current position. If the rate is higher than our current position, we will rerun this exercise by selling currency 1 (e.g. GBP/USD) and selling currency 2 (e.g EUR/GBP). We then sell currency 3 and check if we have a positive profit. All purchases and sales are carried out at the relevant ask and bid prices, respectively.

Table 1 reports the summary statistics for transaction and limit order data. On average a limit order arrives every 1.31, 1.05, and 1.71 seconds for EUR/GBP, EUR/USD, and GBP/USD, respectively.[21] This is much quicker than the quote arrival rate of 15-20 seconds reported by Engle and Russell (1998) and Bollerslev and Domowitz (1993). The increase in trading activities in the FX market is attributed to the recent growth of electronic trading platforms, which enables large financial institutions to set up more comprehensive trading facilities for the increasing numbers of retail investors. The average bid-ask spreads during the arbitrage cluster are 1.03, 2.13, and 2.07 pips for EUR/GBP, EUR/USD, and GBP/USD, respectively, indicating that D3000 is a very tight market as highlighted in Tham (2009).

The average slopes of the demand schedules are 31.37, 85.73, and 68.37 basis point per billion of currency trade for EUR/GBP, EUR/USD and GBP/USD, respectively. The average slopes

[21] A pip, which stands for "price interest point", represents the smallest fluctuation in the price of a currency. Depending on the context, normally one pip is 0.0001 in the case of EUR/USD and GBD/USD. GBP/EUR is displayed in a slightly different way from most other currency pairs in that although one pip is worth 0.0001, the rate is often displayed to five decimal places. The fifth decimal place can only be 0 or 5 and is used to display half pips.

of the supply schedules are 36.41, 99.07, and 74.60 basis point per billion of currency trade for EUR/GBP, EUR/USD and GBP/USD, respectively.[22] These summary statistics indicate that the currency pairs of interest are traded on a highly liquid market.

Insert Table 1 here

Table 2 presents the preliminary statistics on the deviations from triangular arbitrage parity. Panel A of Table 2 shows that the mean of the average arbitrage profit within a cluster is about 1.56 pips with a standard deviation of 1.92. Thus, *on average*, triangular arbitrage is profit making after accounting for transaction costs (i.e. net of bid and ask spread and 0.2 pip trade fees).[23] Furthermore, the associated t-statistics in Panel B suggest that the deviations are statistically significant. The average duration of a cluster of arbitrage opportunities is 0.77 seconds, indicating that the market eliminates profitable deviations rather quickly. The standard deviation of the duration is about 1.54 seconds. The sizable difference between the mean and standard deviations of the duration indicates that the durations are not exponentially distributed. This suggests that there are market conditions (i.e. low market liquidity) where the duration of the arbitrage clusters is persistently high. The average number of limit orders and trades within a cluster is 4.36.

Insert Table 2 here

Overall, the preliminary evidence shows that there are potential profitable arbitrage opportunities. These opportunities are small in relative number to the total number of limit orders, but they are sizeable.

5. Empirical results

We first test the validity of a common textbook arbitrage assumption that arbitrage opportunities are eliminated instantly from the market. Next, we carry out an economic evaluation

[22] We construct the slope of the demand and supply curves in our limit order book using the two best bid and ask prices and the associated depth at these prices.

[23] There are costs involved in obtaining a Reuters trading system, but given that market participants are bank dealers who participate in the foreign exchange market for purposes other than arbitrage these costs are sunk costs to a bank who wishes to also pursue arbitrage.

of arbitrage strategies with competing arbitrageurs to study the potential profit and loss for arbitrageurs. Finally, we test for the relation between market illiquidity, cost of inventory and triangular arbitrage deviations.

5.1. Instant elimination of arbitrage opportunities

The preliminary analysis has identified the existence of triangular arbitrage opportunities, which confirms findings by Aiba et al. (2002), Aiba, Takayasu, Marumo and Shimizu (2003) and Marshall et al. (2008). However, the finance literature often assumes that these opportunities will be eliminated instantly by arbitrageurs in the market. We first revisit the hypothesis that arbitrage opportunities are eliminated instantly given the implications of our model.

Hypothesis 1: "Riskless" arbitrage arbitrage opportunities are not eliminated instantly in the financial markets.

We test Hypothesis 1 by investigating if the duration of the deviations is statistically difference from zero. The associated t-statistics in Panel B of Table 2 suggest that the durations of the arbitrage clusters are statistically different from zero. Although the statistical result rejects the null hypothesis of immediate elimination of arbitrage opportunities, the null hypothesis of a zero duration arbitrage cluster is in fact unrealistic. Arbitrage opportunities will probably be eliminated in an efficient market by the next incoming trade or limit order, which very often takes more than a fraction of a second.

To account for this, we test the null hypothesis by splitting the arbitrage clusters into two groups. The first group consists of arbitrage clusters that are consistent with a textbook arbitrage example in that arbitrage opportunities in this group are eliminated by any next incoming order (market order, limit order and cancelation), i.e. clusters in this group have only one profitable triangular arbitrage deviation. The remaining clusters fall into the second group where market participants deliberate on their participation in the market to exploit the observed arbitrage opportunity. We call this the risky arbitrage.

Insert Table 3 here

Table 3 reports the mean, median, t-statistics of the durations for the textbook and risky arbitrage. A typical text book arbitrage has an average duration of about 0.12 second while the risky arbitrage takes an average of 1.15 seconds to be eliminated from the market. The duration of risky arbitrage also has a larger standard deviation of 1.82 seconds. The results from testing the statistical difference between the duration of textbook and risky arbitrage in Table 3 reject the hypothesis that triangular arbitrage is eliminated instantly in the FX market.

5.2. Data latency

Data latency is defined as the time delay experienced when data is sent from one point to another. Although the time stamp of our data is accurate to one-thousandth of a second, the time it takes for the electrical signal of an order to propagate down a wire to the time it is displayed on Reuters platform might be the reason behind the observations of arbitrage opportunities. Hence, to investigate the role of data latency, we study the time difference between the time a market order enters the system and the time this market order is transacted. The shorter the time difference, the lower the latency. Table 4 reports the preliminary statistics of the data latency for the three currency pairs. The average latency ranges between 34-37 milliseconds in 2003. The latency drops to the range of 31-33 milliseconds in 2004. The drop in latency is not surprising and reflects on the improvements of the trading platforms in the FX market.

To measure the cost of this latency risk in arbitrage, we tabulate the average in the changes of arbitrage deviations from the time an arbitrage is observed to 37 milliseconds later.[24] This measure provides us with an estimate of the average loss of an arbitrageur due to data latency. Measuring the latency cost is important as it allows us to study if the arbitrage deviations in our data is simply the compensation for latency risk. If the existence of the arbitrage opportunities is caused by data latency, we would expect our measure for latency cost to be greater than or equal to the average arbitrage deviation or the average duration of our clusters to be less than 34 milliseconds.

Table 5 reports the mean and standard deviation of profit from exploiting arbitrage opportunities with and without accounting for latency cost. The result indicates the importance of accounting for latency cost as the total arbitrage profit decreases after controlling for latency

[24] We have chosen 37 milliseconds because it is the largest of average latency range across the currency pairs in our sample

cost. However, the remaining arbitrage profit is significantly large and we reject the hypothesis that existence of triangular arbitrage is only due to data latency. The t-statistics indicates that arbitrage profits net of latency cost is still positive and statistically significant. The corresponding t-statistic is given in Table 5. Arbitrage opportunities are therefore not exploited immediately in financial markets as postulated in most textbooks. This conclusion is consistent with the work of De Long et al. (1990), Shleifer and Vishny (1992), Abreu and Brunnermeier (2002) and Kondor (2009) where they argue that exploiting arbitrage opportunities is risky.

Hypothesis 2: The existence of competing arbitrageurs induces potential losses in arbitraging.

Given that we do not know the actual number of arbitrageurs in the market, we investigate this hypothesis using a Monte Carlo backtesting exercise based on the theoretical model. Moreover, this exercise allows us to record the economic value of exploiting triangular arbitrage with different number of competing arbitrageurs using market data. It also demonstrates the relevance of execution risk in arbitraging by closely mimicking the trading behavior of arbitrageurs and studying the risk and return of their trading strategies.

The exercise is set up with k arbitrageurs competing for limited supplies of three currency pairs ($I = 3$) required to construct a profitable arbitrage portfolio. These competing arbitrageurs trade on three currency pairs in the spot FX market and are assumed to be able to see the whole limit order book. Thus, arbitrageurs have full information about the price and quantity available. The trading strategy of these arbitrageurs is to maximize their profits from the deviation of the three currency pairs from triangular parity.[25] When an arbitrage opportunity arises, all arbitrageurs observe it and compete to obtain the arbitrage profit. In order to do this, they will need to complete a full round of buying and selling of the three currencies in the three different markets. The individual demand d is assumed to be equal to one unit, hence the total demand $D = d \times k$. They place all three orders simultaneously using market orders at the best prices. Whether their demand for a particular currency is fulfilled at the best available price, will depend on the demand of the competitors and the supply at the best price. For all

[25] Please see Section 4. for explanations of deviations from triangular parity condition.

arbitrageurs to walk away with a profit, the minimum quantity available at the best price for each currency in the arbitrage portfolio has to be at least k. If there are more arbitrageurs than the quantity available for one of the currencies and the probability of participation is one, each arbitrageur has a probability of $\mathbf{P} = \frac{n_1^a}{k}$ to get the currency at the best price, where n_1^a is the quantity available at the best ask price.

In the Monte Carlo exercise, we first generate the number of participating arbitrageurs for some participating probability. We use a Bernoulli distribution to determine if an arbitrageur is participating and tabulate the total number of participating arbitrageurs, $|S| + 1$. We then determine whether an arbitrageur gets her currency i at the best price using the success probability of $\mathbf{P} = \frac{n_i^a}{|S|+1}$. Thus, some arbitrageurs will be unsuccessful in acquiring all the required currencies to form a profitable triangular arbitrage portfolio. These arbitrageurs are then assumed to complete the remaining legs of the arbitrage transactions at the next best price or sell their excess inventory at the best available price, whichever has the least loss.[26] This problem worsens with increasing number of arbitrageurs and higher individual demands (that is larger than one unit). Our starting and base currency is GBP. There will be some residual position exposures in the exercise because we assume that trades can only be carried out in multiples of one million units of the base currency. We trade out these residual positions at the market prevailing prices and convert them back to GBP at the end of the day.

The sample size of our data is 2 years and we will repeat the Monte Carlo exercise across the sample 1000 time with different number of arbitrageurs varying from 2 to 16. Thus, arbitrageurs will only attempt to eliminate arbitrage opportunities when they observe deviations from the parity condition of one basis point. We have also allowed the arbitrageurs to employ mixed strategies where they participate with some probability. Triangular arbitrage opportunities with transaction costs are identified for three bid and ask cross-rates for three currencies.

[26] Excess inventory exists when an arbitrageur fails to buy all three currencies at the best prices. For example, if an arbitrageur only manages to buy or sell currency 1 and 2 at the best available prices but misses out on currency 3 because of excess demand, she is left with an open position consisting of currency 1 and 2. She can either complete the third leg (currency 3) at the next available price or re-sell and re-buy currency 1 and 2 (losing out on transaction costs) to close her position. We assume she closes her position using the strategy with the best payoff (the payoff can still be positive if the next best price of currency 3 still yields a positive profit).

GBP/USD, EUR/USD and EUR/GBP are the currency pairs used. Bid and ask prices for the three currency pairs are obtained from the reconstructed limit order book. An arbitrage opportunity exists if the purchase of EUR/USD and the sale of GBP/USD (which is short EUR / long GBP) is lower than the sale of EUR/GBP. An arbitrage opportunity also exists if the sale of EUR/USD and the purchase of GBP/USD the purchase is lower than the purchase of EUR/GBP. If an arbitrage opportunity exists, it can only take either one the above ways unless the bid-ask spread is negative. All purchases and sales are carried out using the ask and bid price, respectively. As in Akram et al. (2008), arbitrage opportunities with inter-limit order duration longer than two minutes are not considered. Moreover, arbitrage opportunities, which are not immediately eliminated from the market or with a duration of more than a second, are only exploited once at the very first moment the arbitrage conditions are violated. Note that this profit is achieved net of any bid-ask spread costs. In our setup, we try to provide the best possible scenario to allow the competing arbitrageurs to profit from the arbitrage.

Table 6 presents the mean and standard deviation of profits and losses of arbitrageurs. It shows that arbitrageurs have positive profits when there are only two participating players in the market. The most significant profit is when an arbitrageur has participation probability of one. The arbitrageur would have a handsome average profit of about two million GBP across our sample period. As probability of participation drops, the profit of arbitrageurs in the case of two participants decreases.

In order to emphasize the importance of execution risk we increase number of competing arbitrageurs in the market. Arbitrageurs record negative profits already for four participants if they all participate with probability one. If arbitrageurs were to adopt a participation probability of one in the market with four participants, they will incur an average loss of about 1.8 million GBP across the two year sample period. This is in sharp contrast with respect to the two million GBP profit an arbitrageur would have made in the market with only two participating arbitrageurs. The results clearly demonstrate that arbitrageurs can incur losses in the presence of other competing arbitrageurs as suggested by our model. With an average breadth of about three million across each currency pair (see Table 1) and a setting of eight arbitrageurs, each with a demand of one million unit, it is clear that there is excess demand for the arbitrage portfolio. The loss of arbitraging increases as it becomes more difficult to complete the arbitrage portfolio at the desired price. However, they still have positive profits if they adopt a mixed strategy with a probability of participation of less than 20 percent. The results illustrate that a mixed strategy ($\pi < 1$) might be preferred over a pure strategy ($\pi = 1$) in some circumstances.

Using the time series of liquidity measures from the limit order book, we compute and study the time series of equilibrium model-implied probability of participation and probability of arbitrage elimination. The implied probability of participation π is computed as a solution of the equation $A = \sum_{i=1}^{I} \Delta_i \left(w_i\right) \bar{\mathbf{P}}_{i|n_i,k,\pi}$, where A is the average observed arbitrage deviation during a cluster, $\Delta_i \left(w_i\right)$ is an average difference between the best and the second best prices of the corresponding exchange rate during the arbitrage cluster, $n_i(w_i)$ is an average breadth of the corresponding exchange rate during the arbitrage cluster. The probability $\bar{\mathbf{P}}_{i|n_i,k,\pi}$ is computed according to Equation (12).

Figure 2 shows the time series plots of the probability of participation for two, eight, and sixteen arbitrageurs. It shows that the probability of participation for arbitrageurs is often below one. The implied probability of participation decreases with more competing arbitrageurs. The decrease is due to arbitrageurs reducing their participation in the presence of execution risk.

One implication is that arbitrage opportunities might not be eliminated immediately. The implied probability of arbitrage elimination is computed according to Equation (6) for different numbers of competing arbitrageurs k based on real data. We use the implied probability of participation as the input for this formula. Figure 3 shows the time series plots of the implied probability of arbitrage elimination. It can be easily observed that there are many circumstances where the probability of elimination is below one. Such circumstances coincide with the excess demand for the arbitrage assets and illiquidty of these assets.

Hypothesis 3: Losses increase with increasing number of competing arbitrageurs.

Table 6 demonstrates how execution risk increases as the number of competing arbitrageurs increases. This can be seen by the increase in magnitude of losses in a strategy with a probability of participation of one. The losses increase to an incredible 32 million GBP when there are 16 arbitrageurs. Figure 4 presents the plot of arbitrageurs' profits and losses with respect to the number of arbitrageurs when the strategy has a probability of participation of one. It shows that losses monotonically increasing with increasing number of arbitrageurs. With the increasing use of algorithmic trading in arbitrage in recent years and hundreds of competing arbitrageurs in the real world, our results show that arbitrage can be a very risky business because of execution risk.

We further test this hypothesis by estimating the following linear regression:

$$PL = x_0 + x_1 \times k. \tag{9}$$

where PL is the average profit and loss of k number of arbitrageurs in our backtesting. The results are shown in Table 7. The estimates and the t-statistics show that all the estimated parameters are highly significant. The results report a strong negative relation between the average profit and losses and the number of competing arbitrageurs. The negative relation remains for different probabilities of participation.

In summary, the Monte Carlo backtesting exercise using the LOB demonstrates the importance of execution risk. Arbitrageurs are found to incur losses in the presence of competition. These losses increase with the number of competing arbitrageurs. Thus, our results are in favor of the stated hypotheses and consistent with the model predictions.

5.3. Arbitrage deviations, cost of inventory and market illiquidity

In equilibrium, there is a positive relation between execution risk and market illiquidity. Execution risk is more severe in illiquid markets as the impact of trade on prices increases. A recent and growing body of literature points out that market liquidity can affect financial asset prices (see, inter alia, Stoll (1978), O'Hara and Oldfield (1986), Kumar and Seppi (1994), Chordia, Roll and Subrahmanyam (2002) and references therein). More specifically, Roll et al. (2007) shows that market illiquidity affects deviations from the law of one price in the US stock market.

We use two proxies for market illiquidity to verify this relation in our data. The first proxy, which we denote by Δ_i, is the average differences between the best and the second best prices within the arbitrage cluster. The second proxy λ_i is the slope of the corresponding side of the limit order book, averaged over the cluster.

We also hypothesize that there is a relation between arbitrage deviation and cost of inventory. We construct a measure using the distribution of profit and loss of an arbitrageur if she fails to complete the arbitrage portfolio and remove her excess inventory after 10, 60 and 120 seconds to study the impact of inventory cost on arbitrage deviation. We assume that an arbitrageur only execute two legs out of three at the best available quote across our sample. Since she has failed to complete the arbitrage portfolio (completion of the third leg), she has an unwanted inventory and she clears the unwanted inventory by trading it away at the cheapest way after 10, 60 or 120 seconds later. She might incur a cost or make a profit depending on the movement of the exchange rate in the next few seconds. The cost of holding inventory is captured by the standard deviation of the profit and loss distribution. The higher the standard deviation of the profit and loss distribution, the higher the cost of holding inventory. Our measure of inventory risk can also be seen as the price risk 10, 60 and 120 seconds immediately after the arbitrage has ended.

Table 8 reports the descriptive statistics of the profit and loss distribution when the corresponding exchange rate serves as a missed leg. We find a positive(negative) mean that indicates that an arbitrageur makes(loses) money across our sample for holding the inventory in 10, 60 or 120 seconds. More importantly, we observe that the standard deviations are economically large for all exchange rates and three holding horizons. For instance, standard deviation of profit/loss of the arbitrageur who systematically misses EUR/USD exchange rate and clears the inventory after 60 seconds is 4.3750 basis points. The result indicate the execution risk and cost of holding inventory can be a crucial friction to the elimination of arbitrage opportunities.

To further investigate the role of the inventory cost, we compute the hourly standard deviations of the profit and loss distribution and denote this variable by IC_i.

Hypothesis 4: The size of arbitrage deviations increases with market illiquidity and cost of inventory.

We test this hypothesis by first estimating the linear regression

$$\begin{aligned} A &= a_0 + a_1 \times illiq_{GBP/USD}(w_{GBP/USD}) + a_2 \times illiq_{EUR/USD}(w_{EUR/USD}) + a_3 \times illiq_{EUR/GBP}(w_{EUR/GBP}) \\ &+ b_1 \times IC_{GBP/USD}(w_{GBP/USD}) + b_2 \times IC_{EUR/USD}(w_{EUR/USD}) + b_3 \times IC_{EUR/GBP}(w_{EUR/GBP}) \qquad (10) \\ &+ c_1 \times Tr.Vol + c_2 \times TED, \end{aligned}$$

where $illiq_i$ is a measure of illiquidity in the market i, IC_i is a measure of cost of inventory in the market i, TED is TED spread and $i = GBP/USD$, EUR/USD, EUR/GBP. We control for counterparty risk in our regression using TED spread, defined as a measure of the premium on Eurodollar deposit rates in London relative to the U.S. Treasury, as a widening TED spread reflects the risk of default on interbank loans (counterparty risk) is increasing. From our model, arbitrage deviation has a positive relation with the number of competing arbitrageurs. Consistent with Hypothesis 2, increasing competing arbitrageurs, who exert negative externalities each other, increases potential losses in arbitraging. Thus, controlling for the number of competing arbitrageurs in our regression provides a robustness check on earlier results regarding Hypothesis 2. Since arbitrage activities is found to be positive related to trading volume, see Cornelli and Li (2002), we control for the effect of the number of arbitrageurs in our regression, denoted by $Tr.Vol$, using the hourly trading volumes of the corresponding arbitrage cluster normalized by the total daily trading volume. a_0 can be interpreted as the compensation for monitoring the market in the spirit of Grossman and Stiglitz (1976) and Grossman and Stiglitz (1980) on the risk premium paid to a risk adverse arbitrageur for the uncertainty of the execution risk. We estimate the parameters using generalized method of moments (GMM, Hansen (1982)) with a Newey-West correction for autocorrelation and heteroscedasticity.

The results are shown in Table 9. We first test the relation between market illiquidity and arbitrage deviation by regressing the observed deviation against Δ_i. The estimates and t-statistics show that all the estimated parameters are significantly different from zero, with p-

values less than 0.0001. The results establish a positive statistical relations between the market liquidity and arbitrage deviations. For robustness, we further test the relation between market illiquidity and arbitrage deviation by regressing the observed deviation against the slopes of demand and supply schedules of the three currency pairs. The estimates and t-statistics show that all the estimated parameters are positive and significantly different from zero, with p-values less than 0.0001. The results indicate that the evidence is strongly in support of our hypothesis. The conclusion is congruent with Roll et al. (2007) in that the more liquid the markets, the smaller the deviations from the law of one price. However, we argue that the economic reason behind this relation, in the case of non-convergence arbitrage, is the presence of execution risk. As the price impact of trade increases with market illiquidity, the cost of execution failure due to crowded trade increases as well.

We also include the cost of inventory in our regression and results are reported in Table 9. The estimates and t-statistics show that all the estimated parameters are significantly different from zero, with p-values less than 0.0001. There is a positive and statistically significant relation between cost of holding inventory and arbitrage profits. The significance of illiquidity remains despite the inclusion of the cost of inventory variables and control variables. Consistent with our earlier results on Hypothesis 2, we find that the number of arbitrageurs has a positive relation to arbitrage deviation. As increasing number of arbitrageurs increases the potential loss in arbitraging, arbitrageurs demands for higher compensation for execution risk in equilibrium. The impact of TED spread in all of our regression is either statistically insignificance or economically negligible. This indicates that the role of counterparty risk is small in our sample and the reason might be due to introduction of continuous linked settlement that reduces settlement failure and counterparty risk. Overall, the result demonstrates that crowded trade is an important limits of arbitrage if it imposes negative externality on other competing arbitrageurs. We find that this negative externality in "riskless" arbitrage is associated with the cost of liquidity and cost of inventory.

6. Conclusion

The paper challenges the conventional view that exploitation "riskless arbitrage" is riskless. We presents a new impediment to arbitrage, which we call execution risk. The distinctive feature of execution risk compared to previous limits to arbitrage is that it is neither related to the existence of behavioral traders nor the convergence uncertainty of mispriced securities. The risk is associated with the crowding effect of arbitrageurs competing for indivisible assets necessary to form their arbitrage portfolio. Arbitrage can be risky if traders imposes negative externality on each other. As a result, there can be significant and long departures from efficient prices. Hypotheses from our theoretical model are supported by our empirical evidence that such arbitrage opportunities are not eliminated from the market instantly, even after accounting for transaction and latency cost. Economic evaluation of various arbitrage strategies show that arbitrageurs suffer from losses in the presence of other competing arbitrageurs. These losses increase with the increasing number of competing arbitrageurs. We demonstrate that the cost of liquidity and holding inventory are potential negative externalities imposed on each other by arbitrageurs.

Tables and figures

Figure 1: Structure of Limit Order Book

The figure represents the structure of the limit order book in the market i. There are only two layers in the discrete demand and supply schedules. The first layer consists of the best bid and ask prices and the quantity available at these prices. p_i^b and p_i^a are the best bid and ask prices of asset i, respectively. n_i^b and n_i^a denote the corresponding quantity available at the best bid and ask prices. The next best available bid price of the asset is $p_i^b - \Delta_i^b$ and the next best ask price is $p_i^a + \Delta_i^a$ at the second layer. We assume that prices of all assets at the second layer are available in infinite supply.

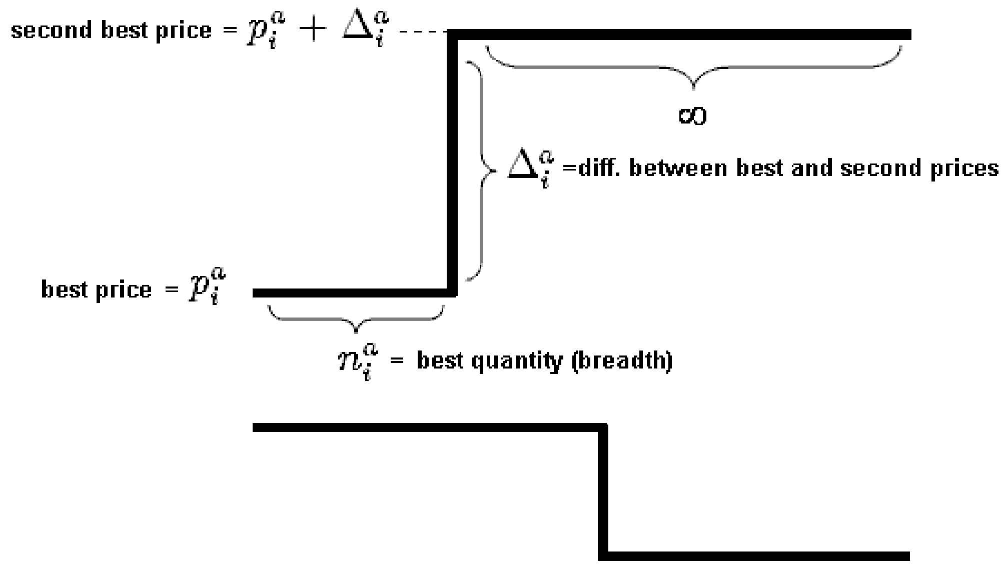
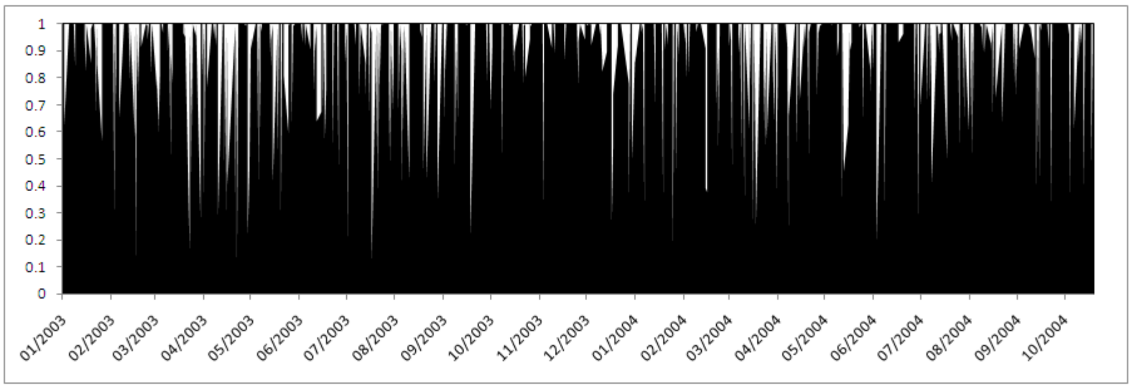


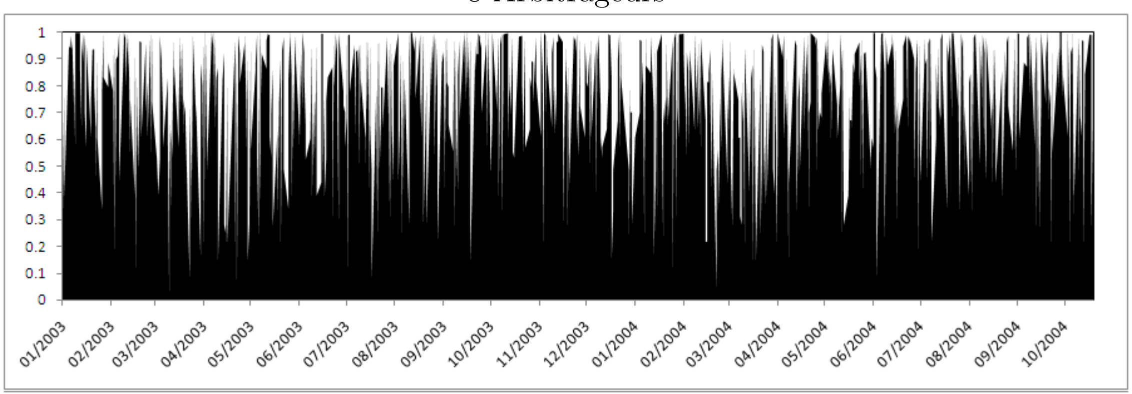
Figure 2: Implied Probability of Participation

The figure presents time series of implied probability of participation based on real data for different number of competing arbitrageurs k. We compute the probability of participation π as a solution of the equation $A = \sum_{i=1}^{I} \Delta_i(w_i)\,\bar{\mathbf{P}}_{i|n_i,k,\pi}$, where A is the average observed arbitrage deviation during a cluster, $\Delta_i(w_i)$ is an average difference between the best and the second best prices of the corresponding exchange rate during the arbitrage cluster, $n_i(w_i)$ is an average breadth of the corresponding exchange rate during the arbitrage cluster. We compute the probability $\bar{\mathbf{P}}_{i|n_i,k,\pi}$ according to Equation (12). All depth and breadth of the market are based in the reconstructed limit order book.

2 Arbitrageurs



8 Arbitrageurs



16 Arbitrageurs

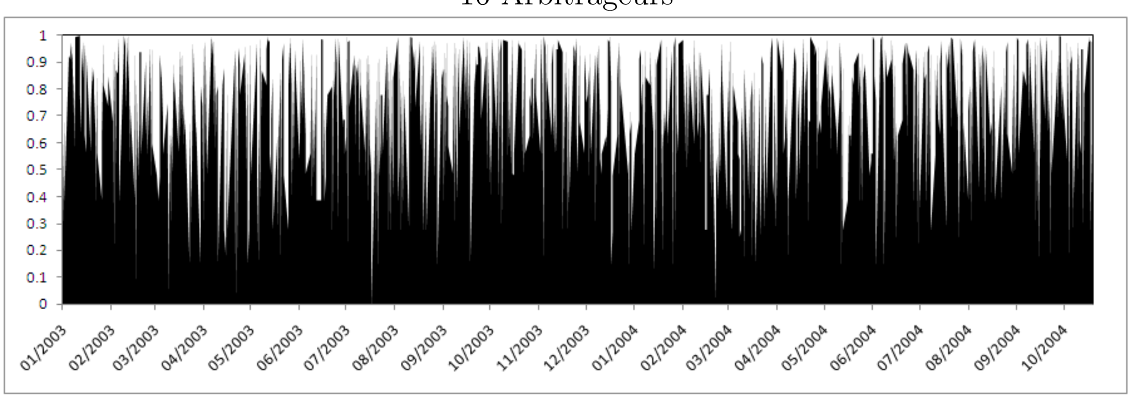


Figure 3: Implied Probability of Arbitrage Elimination

The figure presents time series of implied probability that the observed arbitrage opportunity will be eliminated instantly. We compute this probability according to Equation (6) for different numbers of competing arbitrageurs k based on real data. The probability of participation π is computed as a solution of the equation $A = \sum_{i=1}^{I} \Delta_i(w_i)\,\bar{\mathbf{P}}_{i|n_i,k,\pi}$, where A is the average observed arbitrage deviation during a cluster, $\Delta_i(w_i)$ is an average difference between the best and the second best prices of the corresponding exchange rate during the arbitrage cluster, and $n_i(w_i)$ is an average breadth of the corresponding exchange rate during the arbitrage cluster. We compute the probability $\bar{\mathbf{P}}_{i|n_i,k,\pi}$ according to Equation (12). All depth and breadth of the market are based in the reconstructed limit order book.

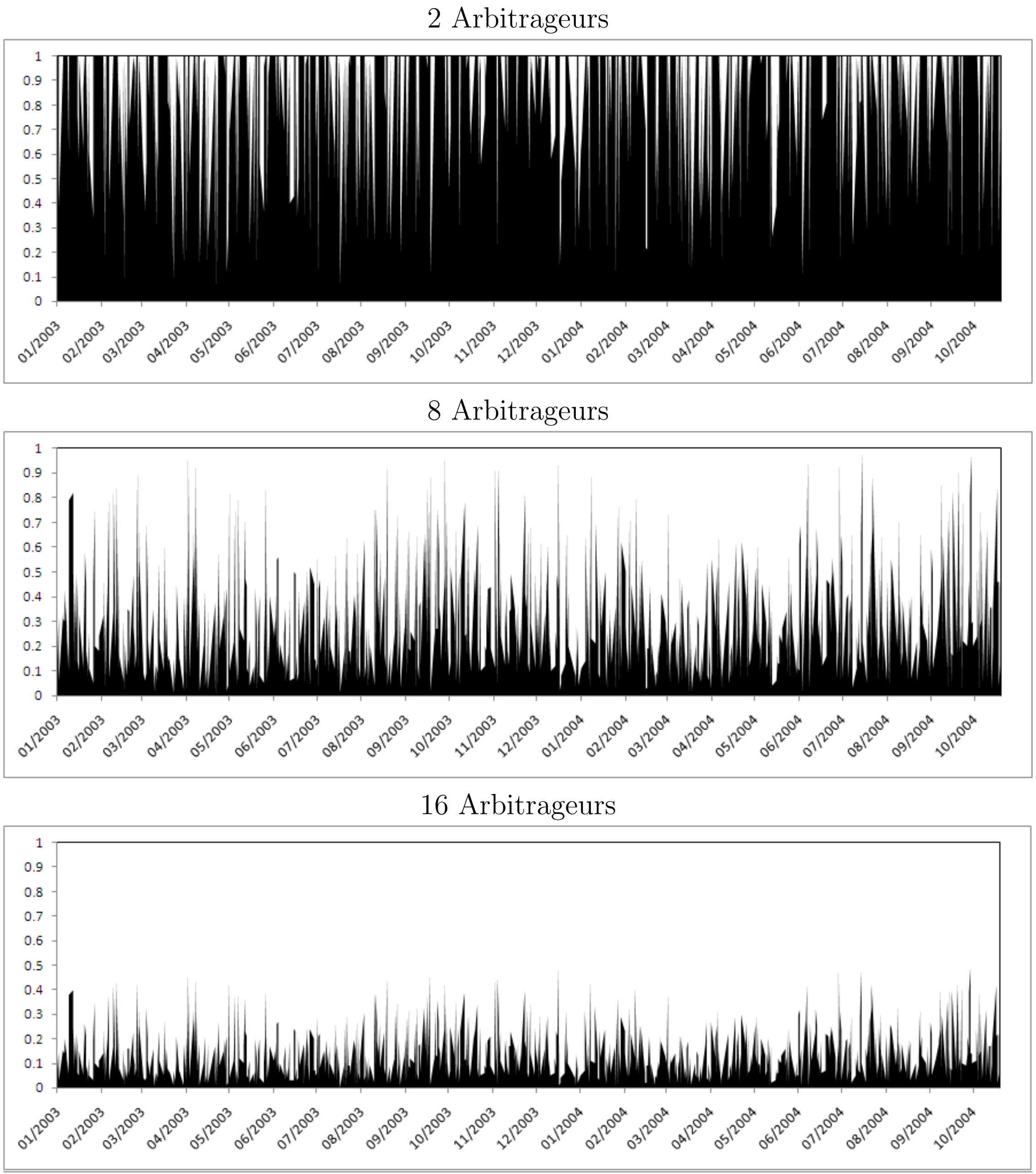


Figure 4: Expected Profit Compared to the Number of Arbitrageurs

The figure presents the relation between the mean of arbitrageurs' profits and losses and the number of arbitrageurs in the market. Profits are in million GBP. The arbitrage strategy is based on the triangular arbitrage condition. Arbitrageurs can participate in exploiting arbitrage opportunities when they observe any deviation. We assume that arbitrageurs use only pure strategies and that they will participate when the observed deviation exceeds 1 pip threshold. They will not participate if the deviation is below the threshold. All depth and breadth of the market are based on the reconstructed limit order book. The sample period is from January 2, 2003 to December 30, 2004.

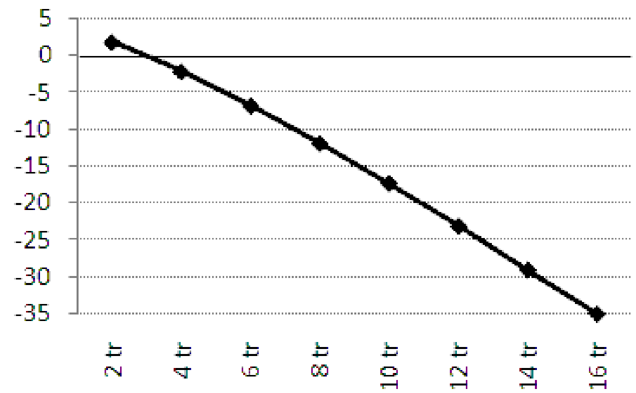


Table 1: Preliminary Data Analysis: Liquidity

The table provides descriptive statistics on the average inter-limit order duration (in seconds), average bid-ask spread (in pips), average slope of the demand and supply schedule in basis points per billion of the base currency, the average depth (in million of base currency) and the average difference between the best and the second best prices (in pips) for the EUR/USD, GBP/USD and EUR/GBP exchange rates. The sample period is from January 2, 2003 to December 30, 2004.

Exchange rate	EUR/GBP	EUR/USD	GBP/USD
Average inter-limit order duration	1.31	1.05	1.71
Average bid-ask spread	1.03	2.13	2.07
Average slope of demand schedules	31.37	85.73	68.37
Average slope of supply schedules	36.41	99.07	74.60
Average depth of demand schedules	41.29	29.46	45.08
Average depth of supply schedules	49.68	32.80	48.78
Average breadth of demand schedules	3.33	2.79	2.72
Average breadth of supply schedules	3.25	2.88	2.78
Best price minus second best price: demand schedules	1.12	2.82	3.11
Best price minus second best price: supply schedules	1.28	3.40	2.86

Table 2: Preliminary Data Analysis: Arbitrage Deviations

The table presents summary statistics on the deviations from triangular arbitrage parity. We identify triangular arbitrage opportunities by comparing the bid and ask prices for each set of three currencies. An arbitrage opportunity exists if there is a mismatch among these three currencies. Panel A reports the mean and standard deviation of arbitrage deviation (in pips), average trade duration (in seconds) and the numbers of arbitrage opportunities in an arbitrage cluster. Panel B presents results for the statistics tests we use to determine if the mean is statistically different from zero. We test the null hypothesis of no difference from zero using the t-test. The sample period is from January 2, 2003 to December 30, 2004.

	Panel A						
	Mean	Std. Dev	Min	Q1	Median	Q3	Max
Average arbitrage deviation (pips)	1.56	1.92	1	1.15	1.35	1.65	94.2
Arbitrage duration (seconds)	0.77	1.54	0.01	0.04	0.35	1.01	96.5
Number of arbitrage in a cluster	4.36	4.96	1	1	3	6	88
	Panel B						
t-stat (average deviation =zero)			162.84				
t-stat (arbitrage duration =zero)			100.21				
Number of profitable clusters			40,166				

Table 3: Duration of Arbitrage Clusters

The table presents descriptive statistics and t-statistics of the duration of arbitrage clusters. We identify triangular arbitrage opportunities by comparing the bid and ask prices for each set of three currencies. An arbitrage opportunity exists if there is a mismatch between these three currencies. The textbook arbitrage column reports statistics of arbitrage clusters that are eliminated by any next incoming order (market orders, limit orders and cancelation), indicating that clusters in this group have only one profitable triangular arbitrage deviation. The textbook arbitrage column comprises arbitrage clusters that are consistent with a textbook example of an efficient elimination of arbitrage opportunities. The remaining clusters fall into the risky arbitrage group, in which market participants deliberate on the participation of exploiting the observed arbitrage opportunity. By using t-statistics we compares if the mean of textbook arbitrage duration is statistically different from the mean of risky arbitrage duration. The sample period is from January 2, 2003 to December 30, 2004.

	Textbook Arbitrage	Risky Arbitrage
Mean	0.12	1.15
Standard deviation	0.36	1.82
Median	0.03	0.77
Number of clusters	15,001	25,165
t-stat (textbook arbitrage dur. = risky arbitrage dur.)		86.97

Table 4: Descriptive Statistics for Measure of Data Latency

The table provides descriptive statistics on the latency measures on Reuters D3000 trading platforms across 2003 and 2004. We measure data latency as the difference between the time a market order enters the system and the time this market order is transacted. Latency is measured in seconds. The sample period is from January 2, 2003 to December 30, 2004.

Exchange rate	Mean	Std. Dev	Min	Q1	Median	Q3	Max
2003 Year							
EUR/USD	0.037	0.031	0.020	0.030	0.030	0.040	2.790
GBP/USD	0.034	0.028	0.020	0.030	0.030	0.040	2.520
EUR/GBP	0.035	0.028	0.010	0.030	0.030	0.040	2.980
2004 Year							
EUR/USD	0.033	0.036	0.010	0.020	0.030	0.040	2.400
GBP/USD	0.031	0.034	0.010	0.020	0.030	0.030	2.800
EUR/GBP	0.032	0.036	0.010	0.020	0.030	0.030	2.310

Table 5: Latency Costs

The table presents descriptive statistics and t-statistics of the profits from exploiting triangular arbitrage deviations by a monopolistic arbitrageur (no competition). We identify triangular arbitrage opportunities by comparing the bid and ask prices for each set of three currencies. An arbitrage opportunity exists if there is a mismatch between these three currencies. "Without Latency" column reports statistics of arbitrage profits under assumptions that all observed arbitrage deviations are exploited immediately. "With Latency" column provides statistics of arbitrage profits under the assumption that the arbitrageur executes transaction only 37 milliseconds after the arbitrage cluster appeared. The sample period is from January 2, 2003 to December 30, 2004.

	Without Latency	With Latency
Total Profit	6,265,896.07	2,438,758.95
Mean	1.56	0.63
Standard deviation	1.92	2.07
t-stat (average profit = zero)	162.8	60.9
Number of clusters	40,166	
t-stat (profit without latency=profit with latency)	66.0	

Table 6: Arbitrage Strategy Average Profits

The table presents the mean and standard deviation (in parentheses) of arbitrageurs' profits and losses in the presence of execution risk. Profits are in million GBP. The arbitrage strategy is based on the triangular arbitrage condition. Arbitrageurs can participate in exploiting arbitrage opportunities when they observe any price discrepancy higher than 1 basis point. We allow each arbitrageur to trade one million GBP. We repeat the exercise with different number of participating arbitrageurs in the market. Arbitrageurs use a mixed strategy under which they participate with probability π. We compute the values in the table based on 1000 simulations. All depth and breadth of the market are based on the reconstructed limit order book. The sample period is from January 2, 2003 to December 30, 2004.

Nr. Traders	$\pi = 0.1$	$\pi = 0.2$	$\pi = 0.3$	$\pi = 0.4$	$\pi = 0.5$	$\pi = 0.6$	$\pi = 0.7$	$\pi = 0.8$	$\pi = 0.9$	$\pi = 1.0$
2	0.323	0.619	0.881	1.120	1.328	1.498	1.647	1.763	1.846	1.902
	(0.0566)	(0.0750)	(0.0850)	(0.0932)	(0.0934)	(0.0924)	(0.0879)	(0.0774)	(0.0582)	(0.0224)
4	0.295	0.496	0.598	0.598	0.490	0.271	-0.064	-0.523	-1.109	-1.828
	(0.0556)	(0.0761)	(0.0841)	(0.0928)	(0.0964)	(0.0926)	(0.0938)	(0.0838)	(0.0733)	(0.0561)
6	0.263	0.364	0.284	0.015	-0.448	-1.128	-2.023	-3.140	-4.495	-6.108
	(0.0546)	(0.0747)	(0.0875)	(0.0951)	(0.0983)	(0.1007)	(0.1003)	(0.0984)	(0.0960)	(0.0922)
8	0.229	0.223	-0.049	-0.610	-1.472	-2.657	-4.178	-6.046	-8.271	-10.85
	(0.0559)	(0.0745)	(0.0876)	(0.0969)	(0.1037)	(0.1070)	(0.1144)	(0.1175)	(0.1258)	(0.1278)
10	0.197	0.077	-0.401	-1.271	-2.558	-4.288	-6.477	-9.125	-12.25	-15.86
	(0.0563)	(0.0755)	(0.0903)	(0.1007)	(0.1103)	(0.1186)	(0.1281)	(0.1406)	(0.1551)	(0.1706)
12	0.164	-0.102	-0.865	-2.168	-4.055	-6.549	-9.675	-13.44	-17.89	-23.04
	(0.2530)	(0.3378)	(0.3856)	(0.4135)	(0.4248)	(0.4215)	(0.4020)	(0.3677)	(0.3127)	(0.2282)
14	0.127	-0.231	-1.154	-2.693	-4.892	-7.781	-11.38	-15.71	-20.80	-26.64
	(0.0567)	(0.0774)	(0.0948)	(0.1105)	(0.1289)	(0.1459)	(0.1695)	(0.1973)	(0.2263)	(0.2568)
16	0.091	-0.394	-1.551	-3.446	-6.123	-9.619	-13.96	-19.16	-25.23	-32.14
	(0.0564)	(0.0782)	(0.0973)	(0.1163)	(0.1374)	(0.1650)	(0.1917)	(0.2238)	(0.2601)	(0.2978)

Table 7: Arbitrage Strategy Average Profits Compared to the Number of Traders

The table presents the regression estimates x_1 of Equation (9), $PL = x_0 + x_1 \times k$. We also report the t-statistics (in parentheses) and R^2 coefficients. k is the number of competing arbitrageurs and PL is the average profit of these arbitrageurs from the backtesting exercise. PL is in million GBP. We assume that arbitrageurs use mixed strategies with the probability of participation given by π. We repeat the backtesting exercise 1000 times. PL is the average profit across the repeated exercise. All limit orders, depth and breadth of the market are based in the reconstructed limit order book. The sample period is from January 2, 2003 to December 30, 2004.

$\pi = 0.1$	$\pi = 0.2$	$\pi = 0.3$	$\pi = 0.4$	$\pi = 0.5$	$\pi = 0.6$	$\pi = 0.7$	$\pi = 0.8$	$\pi = 0.9$	$\pi = 1.0$
-0.01664	-0.0731	-0.1762	-0.3312	-0.5416	-0.8094	-1.1378	-1.5267	-1.9773	-2.4892
(-73.63)	(-54.23)	(-42.74)	(-39.03)	(-36.47)	(-35.06)	(-34.37)	(-33.91)	(-33.63)	(-33.42)
99.89%	99.80%	99.67%	99.61%	99.55%	99.51%	99.49%	99.48%	99.47%	99.47%

Table 8: Descriptive Statistics for Profit/Loss from Holding Inventory

The table provides descriptive statistics on the profit and loss from holding inventory when an arbitrageur fails to complete the arbitrage portfolio. We compute profit based on assumption that the arbitrageur only executes two legs out of three at the best available quote across our sample. The unwanted inventory is cleared by trading it away in the cheapest way after 10, 60 or 120 seconds later. The exercise is repeated three times for each of the exchange rate. The sample period is from January 2, 2003 to December 30, 2004.

Exchange rate	Mean	Std. Dev	Min	Q1	Median	Q3	Max
			10 seconds				
EUR/USD	0.7125	3.4422	-53.739	-0.7981	0.4427	1.8798	145.09
GBP/USD	0.3775	2.1080	-43.945	-0.5661	0.4226	1.2503	74.999
EUR/GBP	0.0851	1.9993	-64.695	-0.8114	0.0870	1.0798	42.674
			60 seconds				
EUR/USD	0.5574	4.3750	-90.865	-1.4003	0.3693	2.1944	106.95
GBP/USD	0.3638	3.3881	-72.001	-1.2152	0.4030	1.8631	89.732
EUR/GBP	-0.0099	2.8333	-26.951	-1.3874	0.0000	1.3418	50.164
			120 seconds				
EUR/USD	0.4590	5.1570	-121.18	-1.9191	0.3013	2.6516	62.597
GBP/USD	0.3718	4.2768	-45.866	-1.6734	0.3903	2.3585	104.34
EUR/GBP	-0.0400	3.4924	-33.323	-1.8139	0.0000	1.7620	43.271

Table 9: The Effect of Inventory Risk on Arbitrage Deviation

The table lists coefficient estimates from regression of arbitrage deviation on measures of liquidity, inventory risk and proxy of arbitrageurs competition. The sample consists of 40,166 triangular arbitrage clusters among EUR/USD, GBP/USD and EUR/GBP exchange rates appeared between January 2, 2003 to December 30, 2004. We identify triangular arbitrage opportunities by comparing the bid and ask prices for each set of three currencies. An arbitrage opportunity exists if there is a mismatch between these three currencies. We use every arbitrage opportunity cluster where deviation is larger than 1 basis point. A denotes the average arbitrage deviation size within the arbitrage cluster. Variables $\Delta_{GBP/USD}$, $\Delta_{EUR/USD}$ and $\Delta_{EUR/GBP}$ are average difference between the best and second best prices of the corresponding exchange rates within each cluster. $\lambda_{GBP/USD}$, $\lambda_{EUR/USD}$ and $\lambda_{EUR/GBP}$ are the average slopes of the demand or supply schedules calculated as difference between the best and the second best prices divided by the quantity of the best price of the corresponding exchange rate. We use either demand or supply of the limit order book that corresponds to the necessary transaction to exploit the arbitrage opportunity (depends on whether a purchase or sales of the direct currency price is involved in the transaction). $IC_{GBP/USD}$, $IC_{EUR/USD}$ and $IC_{EUR/GBP}$ are standard deviation of profit (or loss) from systematically missing the corresponding exchange rate and clearing the inventory after 120 seconds. We use IC variables as proxies for inventory risk. TR.VOL denotes aggregated hourly trading volumes across three markets of the corresponding arbitrage cluster normalized by the total daily trading volume. This variables proxies the number arbitrageurs in the market. TED is a TED-spread, define as the spread between the three-month interest rate banks charge each other (in the euro-dollar market) over the three-month Treasury bills, which is used as a control variable to eliminate the potential impact of counter-party risk. T-statistics are given in parentheses and are adjusted for autocorrelation.

$\Delta_{GBP/USD}$	$\Delta_{EUR/USD}$	$\Delta_{EUR/GBP}$	$\lambda_{GBP/USD}$	$\lambda_{EUR/USD}$	$\lambda_{EUR/GBP}$	$IC_{GBP/USD}$	$IC_{EUR/USD}$	$IC_{EUR/GBP}$	TR.VOL	TED	$Adj.R^2$
0.1837	0.0521	0.0044							1.5964	-0.000029	24.52
(90.2)	(41.3)	(3.00)							(6.91)	(-2.30)	
			0.2904	0.1321	0.0912				1.3579	-0.000010	8.79
			(41.6)	(33.4)	(6.19)				(5.13)	(-0.71)	
						0.0132	0.0239	0.0266	1.0298	-0.000031	1.97
						(3.75)	(8.46)	(6.51)	(3.87)	(-2.16)	
0.1734	0.0512	0.0043				0.0015	0.0017	0.0139	1.5261	-0.000028	24.59
(88.9)	(40.2)	(2.94)				(0.48)	(0.69)	(3.87)	(6.67)	(-2.15)	
			0.2896	0.1184	0.0753	0.0097	0.0134	0.0187	1.0732	-0.000014	9.52
			(39.6)	(31.3)	(5.13)	(2.84)	(4.93)	(4.77)	(4.12)	(-1.01)	

Appendix

Proof of Proposition 1

We will prove the statement of the proposition by induction. Let us first check that the statement is true for $I = 2$. If an arbitrageur j wins the competition and gets the best prices in both markets, she earns the profit A. The probability of winning the best prices in both markets is $\mathbf{P}_1^j \mathbf{P}_2^j$ because of the independence of the two markets. The probability of the arbitrageur failing to get the best price in the market 1 (in the market 2) but winning the best price in the market 2 (in the market 1) is $\mathbf{P}_2^j \left(1 - \mathbf{P}_1^j\right)$ (respectively, $\mathbf{P}_1^j \left(1 - \mathbf{P}_2^j\right)$). She earns, in this case, the profit $A - \Delta_1(w_1)$ ($A - \Delta_2(w_2)$, respectively). With the probability $\left(1 - \mathbf{P}_1^j\right)\left(1 - \mathbf{P}_2^j\right)$, the arbitrageur fails to get best prices in both markets and earns $A - \Delta_1(w_1) - \Delta_2(w_2)$. It is easy to check that the expected profit of the arbitrageur from the "trade"strategy is

$$
\begin{aligned}
E\left(U^j\right) &= A\mathbf{P}_1^j \mathbf{P}_2^j + \left(A - \Delta_1(w_1)\right) \mathbf{P}_2^j \left(1 - \mathbf{P}_1^j\right) + \left(A - \Delta_2(w_2)\right) \mathbf{P}_1^j \left(1 - \mathbf{P}_2^j\right) \\
&+ \left(A - \Delta_1(w_1) - \Delta_2(w_2)\right) \left(1 - \mathbf{P}_1^j\right) \left(1 - \mathbf{P}_2^j\right) \\
&= A - \Delta_1(w_1) \left(1 - \mathbf{P}_1^j\right) - \Delta_2(w_2) \left(1 - \mathbf{P}_2^j\right).
\end{aligned}
$$

We assume that the statement of the proposition is satisfied for $I - 1$ markets, that is, $E\left(U^j\right) = A - \sum_{i=1}^{I-1} \Delta_i(w_i) \left(1 - \mathbf{P}_i^j\right)$. Let us show that the corresponding statement is also true in the case of I markets.

Let us denote by $\mathcal{I}$ the set $\{1, ..., I\}$ of the markets indices and let J be a non-empty subset of $\mathcal{I}$. Similarly to the case of two markets, arbitrageur j earns the observed profit A if she gets the best prices in all I markets. If she fails to get the best prices in each market from J and gets the best prices in all the rest $\mathcal{I} \setminus J$ markets, her payoff will be $A - \sum_{i \in J} \Delta_i(w_i)$. The probability of the event that the trader fails exactly in each of J markets and wins the best prices all other markets is equal to

$$
\prod_{i \in J} \bar{\mathbf{P}}_i^j \cdot \prod_{i \in \mathcal{I} \setminus J} \mathbf{P}_i^j.
$$

The expected payoff of the arbitrageur is equal to the weighted sum of all possible payoffs where the weights are the corresponding probabilities.

The expected value of the profit is

$$\begin{aligned} E(U^j) &= \sum_{J\in 2^{\mathcal{I}}}\left(\left(A-\sum_{i\in J}\Delta_i(w_i)\right)\cdot\prod_{i\in J}\left(1-\mathbf{P}_i^j\right)\cdot\prod_{\iota\in\mathcal{I}\setminus J}\mathbf{P}_\iota^j\right) \\ &= A\sum_{J\in 2^{\mathcal{I}}}\left(\prod_{i\in J}\left(1-\mathbf{P}_i^j\right)\cdot\prod_{\iota\in\mathcal{I}\setminus J}\mathbf{P}_\iota^j\right)-\sum_{J\in 2^{\mathcal{I}}}\left(\left(\sum_{i\in J}\Delta_i(w_i)\right)\cdot\prod_{i\in J}\left(1-\mathbf{P}_i^j\right)\cdot\prod_{\iota\in\mathcal{I}\setminus J}\mathbf{P}_\iota^j\right) \\ &= A-\sum_{J\in 2^{\mathcal{I}}}\left(\left(\sum_{i\in J}\Delta_i(w_i)\right)\cdot\prod_{i\in J}\left(1-\mathbf{P}_i^j\right)\cdot\prod_{\iota\in\mathcal{I}\setminus J}\mathbf{P}_\iota^j\right). \qquad (11)\end{aligned}$$

In the above expression we used the equality

$$\sum_{J\in 2^{\mathcal{I}}}\prod_{i\in J}\left(1-\mathbf{P}_i^j\right)\cdot\prod_{\iota\in\mathcal{I}\setminus J}\mathbf{P}_\iota^j=\prod_{i=1}^{I}\left((1-\mathbf{P}_i^j)+\mathbf{P}_i^j\right)=1.$$

Let us decompose the last sum of the equality (11) into the term containing $\Delta_I(w_I)$ and not containing $\Delta_I(w_I)$. This gives

$$\begin{aligned} E(U^j) &= A-(1-\mathbf{P}_I^j)\sum_{J\in 2^{\mathcal{I}\setminus\{I\}}}\left(\left(\Delta_I(w_I)+\sum_{i\in J}\Delta_i(w_i)\right)\cdot\prod_{i\in J}\left(1-\mathbf{P}_i^j\right)\cdot\prod_{\iota\in\mathcal{I}\setminus J}\mathbf{P}_\iota^j\right) \\ &- \mathbf{P}_I^j\sum_{J\in 2^{\mathcal{I}\setminus\{I\}}}\left(\left(\sum_{i\in J}\Delta_i(w_i)\right)\cdot\prod_{i\in J}\left(1-\mathbf{P}_i^j\right)\cdot\prod_{\iota\in\mathcal{I}\setminus J}\mathbf{P}_\iota^j\right) \\ &= A-\Delta_I(w_I)(1-\mathbf{P}_I^j)-(1-\mathbf{P}_I^j)\left(\sum_{i=1}^{I-1}\Delta_i(w_i)(1-\mathbf{P}_i^j)\right)-\mathbf{P}_I^j\left(\sum_{i=1}^{I-1}\Delta_i(w_i)(1-\mathbf{P}_i^j)\right) \\ &= A-\sum_{i=1}^{I}\Delta_i(w_i)(1-\mathbf{P}_i^j).\end{aligned}$$

Q.E.D.

Proof of Proposition 2

(i) Let $2^{\mathcal{K}_{-j}}$ denotes a family of all subsets of the set $\mathcal{K}_{-j}$ of all opponents of arbitrager j. S is a subset of this family, such that $S\in 2^{\mathcal{K}_{-j}}$, where $|S|$ denotes the number of elements in S. $S\in 2^{\mathcal{K}_{-j}}$ means that all opponents of trader j from the subset S participate in the market with certainty and the rest $\mathcal{K}_{-j}\setminus S$ opponents do not.

The expression for the probability $\bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}}$ of failing to get the best price in the market i by trader j can be derived from the law of total probability. Consider the set of mutually exclusive and exhaustive events X_S with $S \in 2^{\mathcal{K}_{-j}}$ each of which means that all opponents of trader j from the subset S participate in the market with certainty and the rest $\mathcal{K}_{-j} \setminus S$ opponents do not. The probability of the event X_S occurring is equal to

$$\mathbf{P}(X_S) = \prod_{s \in S} \pi_s \prod_{s \in \mathcal{K}_{-j} \setminus S} (1 - \pi_s)$$

and the probability of failing to get the best price in the market i for trader j *conditional* on X_S is

$$\bar{\mathbf{P}}^j_{i|n_i,|X_S|} = \begin{cases} 0, & |S| \leq n_i(w_i) - 1 \\ 1 - \frac{n_i(w_i)}{|S|+1}, & |S| > n_i(w_i) - 1 \end{cases}$$

since there are only $|S|$ opponents are in the market. By the law of total probability we get

$$\bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}} = \sum_{S \in 2^{\mathcal{K}_{-j}}} \bar{\mathbf{P}}^j_{i|n_i,|S|} \cdot \mathbf{P}(X_S) = \sum_{S \in 2^{\mathcal{K}_{-j}}} \prod_{s \in S} \pi_s \prod_{s \in \mathcal{K}_{-j} \setminus S} (1 - \pi_s) \bar{\mathbf{P}}^j_{i|n_i,|S|}. \tag{12}$$

Let us now add one more arbitrageur $k+1$ into the market who plays her mixed strategy π_{k+1}. The new set of arbitrageurs is now denoted by $\mathcal{K}' = \{1, \ldots, k+1\}$. Let the new mixed strategy profile be $\Pi' = \{\pi_1, \ldots, \pi_k, \pi_{k+1}\}$. In order to prove the second statement of the theorem, we need to show that $\bar{\mathbf{P}}^j_{i|n_i,k+1,\Pi'_{-j}} > \bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}}$ for each $j \in \mathcal{K}$ and $i \in \mathcal{I}$.

Let us decompose the sum in Equation 12 into the part with subsets S containing the arbitrageur $k+1$ and not containing her.

$$\begin{aligned}
\bar{\mathbf{P}}^j_{i|n_i,k+1,\Pi'_{-j}} &= \sum_{S \in 2^{\mathcal{K}_{-j}}} \prod_{s \in S \cup \{k+1\}} \pi_s \prod_{s \in \mathcal{K}_{-j} \setminus S} (1 - \pi_s) \bar{\mathbf{P}}^j_{i|n_i,|S|+1} + \sum_{S \in 2^{\mathcal{K}_{-j}}} \prod_{s \in S} \pi_s \prod_{s \in \mathcal{K}'_{-j} \cup \{k+1\} \setminus S} (1 - \pi_s) \bar{\mathbf{P}}^j_{i|n_i,|S|} \\
&= \pi_{k+1} \sum_{S \in 2^{\mathcal{K}_{-j}}} \prod_{s \in S} \pi_s \prod_{s \in \mathcal{K}_{-j} \setminus S} (1 - \pi_s) \bar{\mathbf{P}}^j_{i|n_i,|S|+1} + (1 - \pi_{k+1}) \sum_{S \in 2^{\mathcal{K}_{-j}}} \prod_{s \in S} \pi_s \prod_{s \in \mathcal{K}'_{-j} \setminus S} (1 - \pi_s) \bar{\mathbf{P}}^j_{i|n_i,|S|} \\
&> \pi_{k+1} \sum_{S \in 2^{\mathcal{K}_{-j}}} \prod_{s \in S} \pi_s \prod_{s \in \mathcal{K}_{-j} \setminus S} (1 - \pi_s) \bar{\mathbf{P}}^j_{i|n_i,|S|} + (1 - \pi_{k+1}) \sum_{S \in 2^{\mathcal{K}_{-j}}} \prod_{s \in S} \pi_s \prod_{s \in \mathcal{K}'_{-j} \setminus S} (1 - \pi_s) \bar{\mathbf{P}}^j_{i|n_i,|S|} \\
&= \bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}}.
\end{aligned}$$

The strict inequality appears due to Assumption 3.

(ii) The statement is a consequence of (i) and Equation 3.

Q.E.D.

Proof of Proposition 3

Let the mixed strategy profile Π form a Nash equilibrium of the game. The expected profit of trader j is given by $\pi_j E\left(U^j|\Pi_{-j}\right)$. If $E\left(U^j|\Pi_{-j}\right) > 0$, it contradicts to the definition of Nash equilibrium since trader j can always choose a strategy $\pi'_j > \pi_j$. This will lead to $\pi_j E\left(U^j|\Pi_{-j}\right) < \pi'_j E\left(U^j|\Pi_{-j}\right)$. On the other hand, condition $E\left(U^j|\Pi_{-j}\right) < 0$ can not be true in equilibrium as the strategy "not to trade"with $\pi^j = 0$ provides better off for the trader. Therefore, in equilibrium, zero profit condition $E\left(U^j|\Pi_{-j}\right) = 0$ holds.

Let us consider a 2×2 subgame played by two arbitrarily chosen traders j and j'. The subgame has a form

		TRADER j'	
		0	1
TRADER j	0	$0, 0$	$0, y$
	1	$y, 0$	z, z

y denotes the payoff of the trader choosing "trade" ($\pi = 1$) and the other trader in the subgame choosing "not trade" ($\pi = 0$) while the remaining $k - 2$ arbitrageurs stick to the mixed strategy profile $\Pi_{-j,j'}$. z denotes the payoff of traders j and j' when they both choose to "trade". According to Proposition 2, the payoff z is smaller than y as there is one more participating opponent j'. In equilibrium, each trader must be indifferent between using "trade" or "not trade" strategies, so

$$\begin{aligned} \pi_j z + (1 - \pi_j)\, y = 0 \\ \pi_{j'} z + (1 - \pi_{j'})\, y = 0 \end{aligned},$$

which implies $(\pi_j - \pi_{j'})\,(z - y) = 0$. Since $z - y > 0$, we get $\pi_j = \pi_{j'}$. As arbitrageurs j and j' were chosen arbitrarily, this implies that all traders use the same mixed strategy.

Furthermore, Proposition 1 and zero profit condition imply

$$A = \sum_{i=1}^{I} \Delta_i(w_i) \bar{\mathbf{P}}_{i|n_i,k,\pi}.$$

Q.E.D.

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