Repo Runs

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Abstract

This paper develops a model of financial institutions that borrow short-term and invest into long-term marketable assets. Because these financial intermediaries perform maturity transformation, they are subject to potential runs. We endogenize the profits of the intermediary and derive distinct liquidity and collateral conditions that determine whether a run can be prevented. We then examine the microstructure of repo and similar markets in more detail and show how our conditions apply. The sale of assets can help to eliminate runs under some conditions, but because of cash-in-the-market pricing, this becomes impossible in the case of a general market run. In this case, our original solvency and collateral conditions are again relevant for the stability of financial institutions.

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1 Introduction

This paper develops a model of financial institutions funded by short-term borrowing and holding marketable assets. We show that such institutions are subject to the threat of runs similar to those faced by commercial banks and study the conditions under which runs can occur. We argue that profits are a key stabilizing element against runs, endogenize the profits, and derive distinct liquidity and collateral conditions for such institutions. Both conditions must hold for runs to be avoided. We also ask whether the sale of marketable assets can help prevent runs. Asset sales may help if the price of assets is sufficiently high. However, as more institutions try to sell assets, their prices decline, limiting the amount that can be raised. In the limit, asset sales are completely ineffective. Indeed, if all borrowers try to sell assets, no institution is in a position to purchase them and the borrowers find themselves in the same situation as if their assets were not marketable.

Our framework is general and could be used to study several types of financial institutions that use short-term borrowing as a main source of financing. Such institutions include money market mutual funds, hedge funds, off-balance sheet vehicles including asset-backed commercial paper (ABCP) conduits, and structured investment vehicles (SIVs). We apply our model to large securities dealers who use the tri-party repo market as a main source of financing. This market is particularly interesting because of the key role it played during the Great Financial Crisis of 2007-09. It played a role in the collapse of Bear Stearns, which was triggered by a run of its creditors and customers, analogous to the run of depositors on a commercial bank.[2] This run was surprising, however, in that Bear Stearns's borrowing was largely secured – that is, its lenders held collateral to ensure repayment even if the company itself failed. However, given the illiquidity of markets in mid-March, creditors may have lost confidence that they could recoup their loans by selling the collateral. Many short-term lenders declined to renew their loans, driving Bear to the brink of default (Bernanke 2008). More generally, as noted by the Task Force on Tri-Party Repo Infrastructure (2009), "Tri-

[2] See Duffie (2010) for more details on the dynamics that can lead to the failure of a dealer bank.

party repo arrangements were at the center of the liquidity pressures faced by securities firms at the height of the financial crisis". The creation of the primary dealer credit facility (PDCF) provided a backstop for the tri-party repo market.

We show how the settlement rules in the tri-party repo market can affect the fragility of dealers. We also compare the organization of the tri-party repo market, where there is no first-come-first-serve constraint, with bilateral repos, money market mutual funds, and traditional banks, where such a constraint plays a key role.

We develop a framework to study the fragility of dealers who hold marketable securities funded by short-term collateralized liabilities, building on the theory of commercial bank instability developed by Diamond and Dybvig (1983), Qi (1994), and others. In our view, there are important similarities between the fragility of commercial banking and securities trading. Our main goal is to exhibit and model these similarities, and to highlight the fundamental differences between securities dealers that borrow in the repo market against marketable securities as collateral and commercial banks that borrow unsecured deposits and hold nonmarketable loan portfolios.

A key contribution of our paper is to endogenize profits of dealers and show how profits are important to reduce financial fragility. Dealers have the choice between funding securities with their own cash or with short-term debt. We derive a dynamic participation constraint under which dealers will prefer to fund their operations with short-term debt and show that this condition implies that dealers make positive profits in equilibrium. These profits can be used to forestall a run and thus serve as a systemic buffer. If current profits are insufficient to forestall a run, dealers can boost current cash flows at the expense of future profits by distorting their investment strategy. We derive two constraints that can be interpreted as "liquidity" and "collateral" constraints and that are sufficient to prevent a run.

While traditional banks hold opaque assets that are difficult to liquidate to meet withdrawals, securities dealers hold marketable assets that can potentially be sold to generate cash. We show that the ability to sell assets can help a dealer forestall a run. Whether asset sales can help, however, depends on various factors, including the market price of assets, that we identify in

Section 6. Healthy dealers are willing to pay for assets up to the opportunity cost of their funds. As more assets are sold, the price of assets declines, and it becomes more difficult for a distressed dealer to raise cash. If several dealers simultaneously are in distress and attempt to sell their assets at the same time, cash-in-the market pricing limits this option further, as in Allen and Gale (1994) or Acharya and Yorulmazer (2008). In the limit, in the case of a market-wide run, no dealer is available to buy assets, and our liquidity and collateral conditions are necessary and sufficient to rule out market-wide runs.

Our theory is based on a dynamic rational expectations model with multiple equilibria. However, unlike in conventional models of multiple equilibria, not "everything goes" in our model. The theory pins down under what conditions individual institutions are subject to potential self-fulfilling runs, and when they are immune to such expectations. Since the intermediaries in our model are heterogenous and the liquidity and solvency conditions are specific to each institution, the theory makes predictions about individual institutions, and equilibrium is consistent with observations of some institutions failing and others surviving in case of changing market expectations.

An important economic function of the tri-party repo market, and of repo markets more generally, is to perform maturity transformation. An overnight repo is a short-term liability that is backed by a long-term asset, in the form of a security. Tri-party investors lend overnight repo and have access to their funds every morning, even if the securities that back the repos are not liquid. In "normal" times, maturity transformation is possible because there is a large number of tri-party lenders with largely independent needs for cash. On a given day, an individual lender may decide to "withdraw" its funds from the tri-party repo market by not rolling over the overnight loan. But in the aggregate, the amount of cash available in tri-party repos in our model will be stable by the law of large numbers. This is what happened in the market until 2007.

The maturity transformation provided by tri-party repo contracts resembles, in many ways, the maturity transformation achieved by commercial banks. Banks offer demand deposit contracts that allow the depositors to obtain their funds whenever they want. Yet, banks typically hold long-term assets. The decision of a depositor not to withdraw her funds from the bank is similar to the decision of a repo lender to reinvest. The bank can provide a demand deposit contract because it knows that depositors are unlikely to all withdraw their funds at the same time, but it is nevertheless vulnerable to coordination failures. We show that the same vulnerability can arise in other arrangements performing maturity transformation. In fact, the kind of strategic complementarities that can lead to runs in our model have also been found empirically in other types of intermediaries, notably mutual funds (see Chen, Goldstein, Jiang, 2010).

Conceptually, our theory of banking differs from that of Diamond and Dybvig (1983) in one important aspect. In Diamond and Dybvig (1983), deposit contracts are collective insurance devices for risk-averse households. In our framework, dealers interact with financial investors such as pension funds, money market funds and other institutions, for whom risk-aversion is probably not the right, and certainly not a robust, assumption. We therefore do not place restrictions on investor preferences except for monotonicity. The *raison d'être* of banking in our model are fixed costs as in Diamond (1984). The creation, management, and marketing of securities is a specialized activity that requires the payment of fixed costs. Delegating these activities to a dealer is more efficient than having them performed by many small investors separately. Since this theory of delegation is standard, we do not develop it in this paper, and simply assume that the technology is only operated by dealers.

Our paper is complementary to Gorton and Metrick (2009), who point out the similarity between traditional bank runs and repo market instability. In particular, they argue that Repo rates, collateral, and other features of "securitized banking", as they call it, have counterparts in commercial banking. However, Gorton and Metrick (2009) do not propose a formal model of securitized banking and thus cannot identify the determinants of profits, liquidity, and solvency that are at the core of our analysis.[3] They document a large increase in haircuts for some repo transactions and argue that

[3]Shleifer and Vishny's "Unstable Banking" (2009) formalizes some elements of securitized banking, but focusses mostly on the spillover of irrational investor sentiments into the securitized loan market.

the rise in margins is akin to a run on the repo market. Their data does not include the tri-party repo market. Available data for the tri-party repo market, however, suggests that margins in the tri-party repo market did not increase much during the crisis, if at all. It appears that some tri-party repo investors prefer to stop financing a dealer rather than increase margins to protect themselves (see Task Force on Tri-Party Repo Infrastructure (2009) and Copeland, Martin, Walker, 2010). This is consistent with our model of expectations-driven runs and in contrast to the type of margin spirals invoked by Gorton and Metrick (2009) and described in Brunnermeier and Pederson (2009).

The remainder of the paper proceeds as follows. Section 2 describes our model. Section 3 characterizes steady states without runs. In particular, we derive the dealers' dynamic participation constraint in this section and show that profits are positive. Section 4 studies the dealers' ability to withstand runs in terms of liquidity. Section 5 considers the fragility of different market microstructures and derives collateral constraints. Section 6 studies asset sales and Section 7 the case of a market wide run. Section 8 discusses an extension of the model. Section 9 concludes.

2 The Model

2.1 Framework

We consider an economy that lasts forever and does not have an initial date. At each date t, a continuum of mass N of "young" investors is born who live for three dates. Investors are born with an endowment of 1 unit of goods, that they can invest at date t and have no endowment thereafter. Investors' preferences for the timing of consumption are unknown when born at date t. At date $t + 1$, investors learn their type. "Impatient" investors need cash at date $t + 1$, while "patient" investors do not need cash until date $t + 2$. The information about the investors' type and age is private, i.e. cannot be observed by the market. Ex ante, the probability of being impatient is α. We assume that the fraction of impatient agents in each generation is also α (the Law of Large Numbers).

The timing of the investors' needs of cash is uncertain because of "liquidity" shocks. In practice, capital market investors, such as money market mutual funds, may learn about longer term investment opportunities and wish to redeploy their cash, or they may need to generate cash to satisfy sudden outflows from their own investors. We do not model explicitly what investors do with their cash in the event of a liquidity shock and, for the remainder of the paper, simply assume that they value it sufficiently highly to want to withdraw it from the repo market at the given point in time.[4] Their utility from getting payments (r_1, r_2) over the two-period horizon can therefore simply be described by

$$U(r_1, r_2) = \begin{cases} u_1(r_1) & \text{with prob. } \alpha \\ u_2(r_2) & \text{with prob. } 1-\alpha \end{cases} \tag{1}$$

with u_1 and u_2 strictly increasing.[5]

Everybody in the economy has access to a one-period storage technology, which can be thought of as cash and returns 1 for each unit invested.

The economy is also populated by M infinitely-lived risk-neutral agents called dealers and indexed by $i \in \{1, ..., M\}$. Dealers have no endowments of their own but access to an investment technology, which we think of as investment in, and possibly the creation of, securities. These investments are illiquid in the sense that they cannot be liquidated instantaneously and require managerial expertise and other scarce resources. In terms of costs, a dealer i who wants to invest in securities in a given period must pay a fixed operating cost $f_i \geq 0$ per period, where $f_1 \leq ... \leq f_M$. Hence, dealers can be heterogenous in terms of their cost structure.

The dealers' investments are subject to decreasing returns, which we

[4]This assumption is as in Diamond and Dybvig (1983). As we shall show in the next section, together with a no-arbitrage assumption it implies that dealers are funded short-term. This argument is different from that of Diamond and Rajan (2001) who argue that short-term liabilities are a way to provide incentives to bankers who cannot commit to repay the proceeds of their investments to depositors. Kashyap, Rajan, and Stein (2008) also emphasize the role of short-term liabilities to provide incentives. For a critical assessment see Admati et al (2010).

[5]We do not assume the traditional consumption-smoothing motive of the Diamond-Dybvig literature (concave u_t), which would make little sense in our context.

model simply by assuming that there is a limit beyond which the investment provides no returns. Hence, investing I^t units at date t yields

$$\begin{cases} RI^t & \text{if } I^t \leq \overline{I} \\ R\overline{I} & \text{if } I^t \geq \overline{I} \end{cases} \tag{2}$$

with $R > 1$ at date $t+2$ and yields nothing at date $t+1$.[6] To simplify things, we assume that the return on these investments is riskless and verifiable. Investment returns can only be realized by the dealer who has invested in the asset, because dealers have a comparative advantage in managing their security portfolio. Other market participants only realize a return of γR from these assets, with $\gamma < 1$.

Dealers borrow the endowment of young investors to purchase, or invest in, securities. [7] To make the model interesting, we must assume that the total investment capacity $M\overline{I}$ strictly exceeds the investors' amount of cash available for investment, N. Without this assumption, there would be no competition among dealers for borrowing short-term cash from investors. Dealers could extract all the surplus from investors by simply offering to repay the storage return of 1 each period, and there would be no instabilities or runs. Instead of the condition $M\overline{I} > N$, we assume the slightly stronger condition

$$\overline{I} > \frac{N}{M-1} \tag{3}$$

which implies that no dealer is pivotal. Hence, even if one dealer fails, there will still be competition for investor funds.

If dealer i in period t invests I_i^t, borrows b_i^t from young investors, repays r_{1i}^t after one period or r_{2i}^t after two periods, impatient investors do not roll over their loans when middle-aged, but patient investors do, then the dealer's expected cash flow at date τ is

$$\pi_i^\tau = RI_i^{\tau-2} + b_i^\tau - \alpha r_{1i}^{\tau-1} b_i^{\tau-1} - (1-\alpha) r_{2i}^{\tau-2} b_i^{\tau-2} - I_i^\tau - f_i \tag{4}$$

[6]To our knowledge, the need to assume such capacity constraints (or more generally, decreasing returns) in dynamic models of liquidity provision has first been pointed out by van Bommel (2006).

[7]We can think of these or other securities as serving as collateral for these loans. But since dealers' returns are verifiable, these loans are not subject to information problems on the dealers' side. For now, we can therefore simply ignore the issue of collateral.

The dealer's objective at each time t then is to maximize the sum of discounted expected cash flows $\sum_{\tau=t}^{\infty} \beta^{\tau} \pi_i^{\tau}$, where $\beta < 1$. In order to make the problem interesting, we assume that dealers are sufficiently patient and long-term investment is sufficiently profitable:

$$\beta^2 R > 1. \tag{5}$$

Given the investors' preferences in (1), there is no scope for rescheduling the financing from investors. Hence, if $\pi_i^t < 0$ the dealer is bankrupt, unless he is able to borrow from other dealers.

3 Steady-state without runs

As a benchmark, this section characterizes steady-state allocations in which in each period young investors lend their cash to dealers and withdraw their funds precisely at the time of their liquidity shocks. We assume that the Law of Large Numbers also holds at the level of the dealer: each period the realized fraction of impatient investors at each dealer is α. Hence, in every period, each dealer obtains loans from young investors, and repays a fraction α of middle-aged investors and all remaining old investors. Thus there is no uncertainty about dealers' cash flows, and each dealer's realized cash flow is equal to his expected cash flow (4). Furthermore, since dealers' returns are verifiable, collateral does not play a role in steady state. There is only cash changing hands, and the underlying collateral is arbitrary, if there is any.

Each period, dealers compete for investors' funds. Since dealers have a fixed investment capacity, they cannot make unconditional interest rate offers, but must condition their offers on the amount of funds they receive. The simplest market interaction with this feature is as follows.[8]

1. Dealers offer contracts $(r_{1i}^t, r_{2i}^t, Q_i^t) \in \mathbb{R}_+^3$, $i = 1, ..., M$.

2. Investors $j \in [0, N]$ choose a dealer i or none at all $(i = 0)$.

[8] Our analysis in this section would be unchanged if we assumed a competitive lending market, with competitive interest rates r_1 and r_2. Explicit interest rate competition only becomes relevant in the later analysis of runs.

Here, $r^t_{\tau i}$ is the (gross) interest payment offered by dealer i on τ-period borrowing and Q^t_i the maximum borrowing for which this offer is valid. Total borrowing by the M dealers then is $(b^t_1, ..., b^t_M) \in \mathbb{R}^M_+$, with $b^t_i \leq Q^t_i$ for $i = 1, ..., M$ and $\sum b^t_i \leq N$ (all in per-capita terms).

A steady state without a run is a collection of $(r_{1i}, r_{2i}, b_i, I_i)$ for each dealer i, where b_i is borrowing and $I_i \leq \overline{I}$ investment per dealer, no dealer would prefer another borrowing and investment policy and no investor another lending policy, given the behavior of all others.[9]

Lemma 1 *For each i with $b_i > 0$, $r_{2i} = r^2_{1i}$.*

Proof: Clearly, $r_{2i} \geq r^2_{1i}$, because otherwise investors would strictly prefer to never roll over their loans, regardless of their type. Patient middle-aged investors would withdraw their funds and then invest again with young investors. Suppose that this inequality is strict. In this case, an impatient middle-aged investor will optimally roll over the loan and at the same time borrow the amount $r_{1i} + \varepsilon$ on the market at interest rate $r_{1i} - 1$. He can then claim back r_{2i} from the dealer one period later and repay his one-period loan $(r_{1i} + \varepsilon) r_{1i}$ which is feasible and profitable if $\varepsilon > 0$ is sufficiently small.

The proof is based on a simple no-arbitrage argument. It is different from the classical argument by Jacklin (1987) in the context of the Diamond-Dybvig (1983) model, because investors in our context do not have access to the long-term investment technology. It is also different from the argument by Qi (1994), who assumes strict concavity of the investors' utility. In our market context, the no-arbitrage argument is natural and sufficient.[10]

Lemma 2 *$r_{1i} = r_{1j}$ for all dealers i, j with $b_i, b_j > 0$.*

[9] The bound Q_i plays no role in steady state, because it only binds out of equilibrium. We therefore ignore it in the description of the steady state, where it can be thought of as being set to $Q_i = \overline{I}$.

[10] In a market context, "early dyers" (as the Diamond-Dybvig literature calls them) do not die, and are perfectly able to transact after their liquidity shock.

Proof: Suppose that $r_{1i} < r_{1j}$ for some i, j with $b_i, b_j > 0$. Let $\mathcal{J}_i$ be the set of all dealers k with $r_{1k} > r_{1i}$ and $b_k > 0$. All $k \in \mathcal{J}_i$ must be saturated, i.e. have $b_k = Q_k$ (otherwise investors from i would deviate). Hence, any dealer $k \in \mathcal{J}_i$ can deviate to $r_{1k} - \varepsilon$ for $0 < \varepsilon < r_{1k} - r_{1i}$ and strictly increase his profit.

By Lemma 2, we can denote the single one-period interest rate quoted by all active dealers by $r = r_1$. Then the steady-state budget identity of dealer i is

$$RI_i + b_i = I_i + \alpha r b_i + (1-\alpha) r^2 b_i + f_i + \pi_i \tag{6}$$

where the left-hand side are the total inflows per period and the right-hand side total outflows. Clearly, if $R > 1$, the higher is I_i the higher are profits.[11] We do not concern ourselves with showing how a steady state with $I_i > 0$ would emerge if there were a startup period. But under our assumption (5) that dealers are sufficiently patient, it is clear that dealers have an interest in building up investment if investment costs are not too high (as we will assume below).

We now characterize the steady states in which dealers invest by a sequence of simple observations.

Lemma 3 *If $r > 1$, total steady-state repo borrowing is maximal: $\sum_{i=1}^{M} b_i = N$.*

Proof: The total supply of loanable funds is inelastically equal to N in each period if $r > 1$. The scarcity constraint (3) implies that there is a dealer who invests less than full capacity, $I_i < \overline{I}$. Suppose that $\sum_{i=1}^{M} b_i < N$. If i makes strictly positive profits, he strictly increases his profits by setting $Q_i = \overline{I}$ and thus attracting more funds. If i makes zero profits, he can make strictly

[11] The literature has not always been clear about the distinction between investment capacity ($\overline{I}$ in our model) and per capita borrowing (N/M). In particular, the implicit assumption that $\overline{I} = N/M$ in Qi (1994), Bhattacharya and Padilla (1996) and Fulghieri and Rovelli (1998) is not necessary, and may even ignore interesting dynamic features. See van Bommel (2006) for an excellent discussion.

positive profits by reducing his interest rate marginally, setting $Q_i = \overline{I}$, and attracting the previously idle supply of funds.

Lemma 4 *If $\pi_i > 0$, steady-state investment of dealer i is maximal: $I_i = \overline{I}$.*

Proof: Suppose the lemma is wrong. The dealer can then increase investment slightly at any date t by using his own cash. By condition (5), this yields a strict increase in discounted profits.

Lemma 5 *If there exists a dealer i with $\pi_i > 0$ and $b_i > 0$ then steady-state interest rates satisfy*

$$(1-\alpha)\beta^2 r^2 + \alpha\beta r \leq 1 \tag{7}$$

Proof: For each unit of cash that a dealer borrows and invests at date t, he pays back αr in $t+1$, generates returns R in $t+2$ and pays back $(1-\alpha)r^2$ in $t+2$. Hence, his expected discounted profits on this one unit is $\beta^2(R-(1-\alpha)r^2) - \beta\alpha r$. Alternatively he could invest his own cash. The discounted profits from not borrowing the one unit and rather investing his own money is $\beta^2 R - 1$. If the dealer borrows in steady state ($b_i > 0$) and has funds of his own ($\pi_i > 0$), this cannot be strictly better, which implies (7).

Lemma 6 *Suppose that (7) holds as a strict inequality. If $I_i > 0$, then $b_i = \overline{I}$.*

Proof: The dealer's expected profit from borrowing and investing 1 unit is $\beta^2(R-(1-\alpha)r^2) - \beta\alpha r$. If (7) is strict, then

$$\beta^2(R-(1-\alpha)r^2) - \beta\alpha r > \beta^2 R - 1 \tag{8}$$

which is strictly positive by (5). Hence, the dealer strictly prefers to borrow rather than use his own funds, and if investing is profitable at all ($I_i > 0$), he will borrow up to the maximum.

Lemma 6 together with (3) implies that (7) cannot hold as a strict inequality, because then the demand for funds would exceed supply. Hence, if at least one investor makes positive profits, Lemma 5 implies that (7) must hold as an equality:

$$(1-\alpha)\beta^2 r^2 + \alpha\beta r = 1$$

We call this condition the dealers' "dynamic participation constraint". Basic algebra shows that its solution is $\overline{r} = 1/\beta > 1$. This makes sense: at the margin, dealers discount profits with the market interest rate. But it is interesting to note that $\overline{r}$ does not depend on the supply and demand characteristics R and α. Furthermore, the dynamic participation constraint implies that the marginal profit from borrowing is strictly positive. Hence, dealers make positive profits at this interest rate, if the fixed costs f_i are not too high.

As a benchmark we consider symmetric steady states, i.e. steady states in which $I_i = I$ and $b_i = b$ for all i. By Lemma 3, $b = N/M$. Using $r = 1/\beta$ in (4), if dealers make positive profits, Lemmas 4, 5, and 6 imply that profits are

$$\pi_i = (R-1)\overline{I} - \left(\frac{\alpha}{\beta} + \frac{1-\alpha}{\beta^2} - 1\right)\frac{N}{M} - f_i. \tag{9}$$

To make the analysis interesting we will assume that these profits are indeed strictly positive, i.e. that R, $\overline{I}$, or β are sufficiently large or f_i is sufficiently small for all i. All of these assumptions are reasonable and consistent with our previous assumptions.

Assumption: Period costs satisfy

$$f_M < (R-1)\overline{I} - \left(\frac{\alpha}{\beta} + \frac{1-\alpha}{\beta^2} - 1\right)\frac{N}{M} \tag{10}$$

Hence, even the dealer with the highest costs can make strictly positive profits in the symmetric steady state. Under Assumption (10) we can now show that steady states with strictly positive profits exist.

Proposition 1 *Assume that (10) holds. Then there exist steady states in which investors roll over their loans according to their liquidity needs and* *dealers make strictly positive profits. In all such steady states, $I = \overline{I}$ and $r = \overline{r}$.*

Proof: Lemma 4 implies that for all dealers with $\pi_i > 0$ $I_i = \overline{I}$. Lemma 5 implies that $r \leq \overline{r}$. Lemma 3, 6 and (3) then imply that $r = \overline{r}$. At this rate, dealer i is indifferent between borrowing and using his own cash π_i and thus finds it indeed optimal to borrow any positive amount b_i at which he makes non-negative profits. This is the case if $(R-1)\overline{I} - \left(\frac{\alpha}{\beta} + \frac{1-\alpha}{\beta^2} - 1\right) b_i - f_i \geq 0$, the left-hand side of which is strictly positive by (10) if b_i is not too large. Since $r > 1$ patient middle-aged investors find it indeed optimal to roll over their loans and young investors find it optimal to lend all their endowment.

The steady states identified in Proposition 1 will serve as a benchmark for the rest of the analysis. In these steady states dealers make strictly positive profits. For certain parameters, one can construct steady states in which dealers make zero profits, by choosing borrowing and investment levels (b_i, I_i) appropriately. Such steady states are knife-edge cases and not very interesting in the context of investment banking.[12]

The steady states of Proposition 1 all feature maximum investment and the interest rate $\overline{r}$, but dealers can differ in their short-term borrowing. In fact, dealer profits are strictly decreasing in b_i:

$$\pi_i = (R-1)\overline{I} - \left(\frac{\alpha}{\beta} + \frac{1-\alpha}{\beta^2} - 1\right) b_i - f_i. \tag{11}$$

Hence, to the extent that period profits act as a buffer against adverse shocks, as we show in the following sections, dealers with larger exposure to short-term borrowing will be more fragile.

An important and novel feature of the equilibrium of Proposition 1 is that condition (7) prevents competition from driving up interest rates to levels at which dealers make zero profits. The reason why profits from short-term borrowing are positive is intuitive (but not trivial): dealers must have an incentive to use their investment opportunities on behalf of investors instead

[12] By Lemma 5, such zero-profit steady states cannot exist if dealers have endowments of their own. It is also easy to see that no symmetric steady state with zero profits exists.

of using internal funds to reap those profits for themselves. This rationale of positive intermediation profits is different from the traditional banking argument of positive franchise values (e.g., Bhattacharya, Boot, and Thakor (1998), or Hellmann, Murdock and Stiglitz, (2000)), as it explicitly recognizes the difference between internal and external funds. Hence, the co-existence of internal and external funds and the internalization of all cash flows arising from them implies that financial intermediaries make positive profits.[13]

4 Runs without asset sales

In this section, we study the stability of dealers in the face of possible runs. We analyze this problem under the assumption that behavior until date t is as in Proposition 1 and ask whether a given dealer can withstand the collective refusal of all middle-aged investors to roll over their loans and of young investors to provide fresh funds.[14] In the next section we will describe the specific microstructure of the tri-party repo market and other institutions that can make such collective behavior of investors optimal and thus imply that the corresponding individual expectations are self-fulfilling.

The key question is how much cash the dealer can mobilize to meet the repayment demands by middle-aged investors in such a situation. At the beginning of the period, a dealer, on the asset side of his balance sheet, holds $R\overline{I}$ units of cash from investments at date $t-2$, as well as securities that will yield $R\overline{I}$ units of cash at date $t+1$. The dealer holds maturing loans on the liability side of his balance sheet. In this section, we assume that the dealer cannot sell his assets.

The dealer's obligations from maturing loans in case of a run are $(\overline{r} + (1-\alpha)\overline{r}^2)b_i$. If there is no fresh borrowing in the run and new investment is

[13] This is different from Acharya, Myers, and Rajan (forthcoming) where overlapping generations of bankers try to pass on the externality of debt.

[14] In our interpretation, a run is a situation in which the dealer is suddenly cut off from all short-term funding. In terms of our model, this means that middle-aged investors do not roll over their funding and new investors do not provide fresh funds. One can consider intermediate scenarios in which middle-aged investors all cut their funding, but young investors provide fresh funds. This seems less plausible, but the analysis would be similar, yielding similar but slightly weaker constraints for dealer survival.

maintained at the steady-state level $\overline{I}$, the run demand can be satisfied by the individual dealer if

$$(R-1)\overline{I} \geq (\overline{r} + (1-\alpha)\overline{r}^2)b_i - f_i \tag{12}$$

But more is possible. In the event of a run at date t, the cash position of the individual dealer who satisfies the run demand is

$$I_0 = R\overline{I} - (\overline{r} + (1-\alpha)\overline{r}^2)b_i - f_i \tag{13}$$

Clearly, if $I_0 < 0$ the dealer does not have the liquidity to stave off the run and is bankrupt. If $I_0 \geq 0$, but (12) does not hold, the dealer must adjust his borrowing or investment in order to survive the run. Since after a run in $t+1$ the dealer will have $R\overline{I}$ in cash and no debt to repay, he can resume his operations by investing I_0 at date t and continuing to invest thereafter. Because he makes strictly positive profits from borrowing by (8), he will borrow and invest as much as possible, until he has reached his investment capacity of $\overline{I}$.

In the appendix, we derive the condition under which the dealer can resume his full borrowing b_i at date $t+1$ by adjusting his investment over time. If this "rebounding constraint," (15) in the following proposition, is satisfied, the run has no impact on investors and the other dealers. This identifies situations in which runs are harmless and cannot be recognized in the data if they occur. If it is not satisfied, the dealer must scale down and some investors must lend to other dealers after the run. If (14) holds but (15) does not, runs can be staved off if they occur, but their consequences are recognized in the market.

The liquidity constraint, (14) in the following proposition, is obtained by simply writing out the condition $I_0 \geq 0$ from (13).

Proposition 2 *In steady state, a run on dealer i can be accommodated if and only if the dealer's liquidity constraint holds, i.e. if*

$$\beta^2 R\overline{I} \geq (1-\alpha+\beta)b_i + \beta^2 f_i. \tag{14}$$

Remark 1 *If*

$$\begin{aligned}\beta^2(R-1)\overline{I} \;\; &\geq \;\; \left[R(1-\alpha-\beta^2)+\alpha\beta+\frac{R-1}{R+1}(\beta(1+\beta)R-2(1-\alpha))\right]b_i \\ &\quad +\beta^2 R f_i \qquad (15)\end{aligned}$$

the dealer can resume his full borrowing of b_i after the run.

It can easily be seen that, if f_i is sufficiently small, both inequalities (14) and (15) are independent of each other and strictly stronger than the condition that steady-state profits π_i as given in (11) are positive. In particular, a dealer who makes positive profits in steady state may still fail in a run. The comparative statics of the liquidity constraint are simple and we collect them in the following proposition.

Proposition 3 *The liquidity constraint (14) is the tighter,*

- *the higher are the fixed costs f_i,*
- *the higher is short-term borrowing b_i,*
- *the lower is investment capacity $\overline{I}$,*
- *the lower is productivity R.*

Proposition 3 shows that if dealers have sufficient access to profitable investment ($\overline{I}$ large), if these investment opportunities are sufficiently profitable (R large), if their period fixed costs are sufficiently small (f_i small), or if they have sufficiently little exposure to short-term borrowing (b_i small), then dealers are more likely to stave off runs individually, only by reducing their borrowing or investment temporarily. In this case, unexpected runs cannot bring down dealers out of equilibrium. If in addition to (14) condition (15) holds, the dealer who is run on can resume his full borrowing b_i after the run. If condition (14) is violated, a run would bankrupt the individual dealer if he cannot sell his illiquid assets.

5 Fragility

In this section, we examine different microstructures that are associated with repo markets or other money markets. We ask whether runs can occur in each of the institutional environments we consider. We focus on the tri-party repo market, but we also examine bilateral repos, money market mutual funds, and traditional bank deposits. We derive a collateral constraint for each market and show that if and only if the liquidity constraint and the collateral constraint are violated, then a run can occur for the particular market structure.

We study unanticipated runs that arise from pure coordination failures. As discussed in the previous section, in a run at date t all investors believe that i) no middle-aged investors renew their funding to dealer i, so the dealer must pay $[r + (1 - \alpha)r^2]\, b_i$ to middle-aged and old investors, and ii) no new young investors lend to the dealer. We ask whether such beliefs can be self-fulfilling in a collective deviation from the steady state.

Since the Law of One Price holds in our model, a trivial coordination failure may induce all investors of a given dealer to switch to another dealer out of indifference. This looks like a "run", but is completely arbitrary. We will therefore assume that investors if indifferent lend to the dealer they are financing in steady state. Hence, in order for a collective deviation from the steady state to occur we impose the stronger requirement that the individual incentives to do so must be strict.

The first insight, which appplies to all institutional environments considered in this section, is simple but useful to state explicitly: a run cannot occur if a dealer is liquid in the sense of Proposition 2.

Lemma 7 *If a dealer satisfies the liquidity constraint (14), there are no strict incentives to run on this dealer.*

The proof is simple. In a run on this dealer, all middle-aged patient investors would be repaid in full regardless of what young investors do and without affecting the dealer's asset position. Hence, patient middle-aged and young investors are indifferent between lending to the dealer or to another one. By our assumption about the resolution of indifference, there is thus no reason to run in the first place. Intuitively, patient middle-aged investors would just "check on their money" before it is re-invested. Since the dealer has the money, such a check does not cause any real disruption, and the dealer may as well keep it until he invests into new securities.

5.1 The US tri-party repo market

This section briefly reviews the microstructure of the tri-party repo market and emphasize the key role played by the clearing bank.[15] In particular, we show that a practice called the "unwind" of repos leads to fragility in this market.

The clearing banks play many roles in the tri-party repo market: They take custody of collateral, so that a cash investor can have access to the collateral in case of a dealer default. They value the securities that serve as collateral. They make sure the specified margin is applied. They help dealers optimize the use of their securities as collateral. They settle transaction on the repos on their books.

In the US tri-party repo market, new repos are organized each morning, between 8 and 10 AM. These repos are then settled in the afternoon, around 5 PM, on the books of the clearing banks. For operational simplicity, because dealers need access to their securities during the day to conduct their business, and because some cash investors want their funds early in the day, the clearing banks "unwind" all repos in the morning. Specifically, the clearing banks send the cash from the dealers' to the investors' account and the securities from the investors' to the dealers' account. They also finance the dealers' securities during the day, extending large amounts of intraday credit. At the time when repos are settled in the evening, the cash from the overnight investors extinguished the clearing bank's intraday loan.

From the perspective of our theory, we can model the clearing bank as an agent endowed with a large amount of cash. By assumption, the clearing bank

[15] More details about the microstructure of the tri-party repo market can be found in Task Force (2010) and Copeland, Martin, and Walker (2010). The description of the market corresponds to the practice before the implementation of the 2010 reforms.

can finance the dealer only intraday. At each date, the clearing bank finances dealers according to the following intra-period timing, which complements the timing considered n the previous section.:

1. The clearing bank "unwinds" the previous evening's repos. For a specific dealer i this works as follows:

   (a) The clearing banks sends the cash amount $b_i \left[r_i + (1-\alpha)r_i^2\right]$ to all investors of dealer i, extinguishing the investors exposure to the dealer they invested in.

   (b) At the same time, the clearing bank takes possession of the assets the dealer has pledged as collateral.

   (c) In the process, the investors' loans to the dealer are reimbursed and the clearing bank finances the dealers temporarily, holding the assets as collateral for its loan.

2. Some of the the assets of a dealer mature (in steady state $\bar{I}$ assets, yielding $R\bar{I}$ in cash), allowing the dealer to repay some of its debt to the clearing bank.

3. The dealer offers a new repo contract (r_i, Q_i, κ_i), where κ_i is the amount of collateral the investor receives.

4. New and patient middle-aged investors decide whether to engage in new repos with a dealer.

5. If the dealer is unable to repay its debt to the clearing bank, then it is illiquid and must declare bankruptcy. Otherwise, the dealer continues.

For simplicity, we assume that the clearing bank extends the intraday loan to the dealer at a zero net interest rate. Also, since runs are zero probability events the clearing banks has no reason not to unwind repos.

In step 3, the dealer must specify the interest rate and the amount of collateral that the investors receive. We assume that all investors are willing to accept either securities maturing at $t+1$ or securities maturing at $t+2$ as collateral. Because these securities do not mature at the same date, they are valued differently. Yet, both dealers and investors discount the future at the same rate β. This comes, on the one hand, from the assumption on the dealer's intertemporal utility function and, on the other hand, from the fact that investors face an equilibrium interest rate of $\bar{r}-1 = 1/\beta - 1$. Hence, the maximum amount of collateral a dealer can pledge in steady state is $\bar{I}(1+\beta)$, in terms of securities maturing at $t+1$.

In steady state, the total amount of funds provided by investors per period is $b_i\left[1+(1-\alpha)\bar{r}\right] = b_i\left[1-\alpha+\beta\right]/\beta$. It follows that the maximum amount of collateral per unit borrowed that the dealer can offer is

$$\kappa_i \equiv \frac{\beta \bar{I}(1+\beta)}{b_i\left[1-\alpha+\beta\right]}. \tag{16}$$

Note that steady-state collateral depends on the dealer's borrowing b_i, hence is dealer-specific, whereas $r_i = \bar{r}$ as derived in Section 3. In response to the contract offer by the dealer, individual investors must compare their payoff from investing with the dealer in question to that from investing with another dealer. The latter decision yields the common market return $\bar{r}$,[16] the return from the former depends on what the other investors do. Table 1 shows the payoffs of the two decisions for the individual investor (rows), given what the other investors do (columns). If the investor engages in a repo with the dealer, the clearing bank will accept the cash, since it reduces its intraday exposure to the dealer, and give the investor assets that mature at date $t+1$. These are the only assets available in case of a run since the clearing bank will not let the dealer invest in new securities unless it obtains enough funding. Hence, in case of a run, an investor who agrees to provide financing receives securities that yield $\gamma R \kappa_i$ at date $t+1$ if the dealer defaults. If the dealer does not default, the investor gets a return of $\bar{r}$.

[16] This is obvious if the investor is the only one to deviate, because then he is negligible. If all investors of the dealer in question deviate, this follows from the slack in assumption (3).

	other investors	
	invest	don't
invest	$\overline{r}$	$\gamma R\kappa_i$
don't	$\overline{r}$	$\overline{r}$

Table 1: Payoffs in tri-party repo with unwind

Hence, investors will finance the dealer (resp., roll over their repo) iff $\overline{r} \leq \gamma R\kappa_i$,[17] which, by using (16), is equivalent to

$$\beta^2 R\bar{I} \geq \frac{1-\alpha+\beta}{\gamma(1+\beta)} b_i \tag{17}$$

If this condition does not hold, the collective decision not to lend to the dealer in question is self-enforcing. In this case, the yield from the securities pledged as collateral is so low that an investor who believes that nobody will invest with dealer i would also choose not to invest. Note that condition (17) is conservative, as it uses the maximum amount of collateral that the dealer can possibly post. In our model, collateral is indeterminate because dealer returns are verifiable and runs do not occur in equilibrium. If less collateral is posted than the maximum amount κ_i, the collateral condition (17) becomes stricter (is satisfied for a larger set of parameter values).

Combining the above results with those of the previous section yields the following prediction about the stability of the tri-party repo market.

Proposition 4 *In the tri-party repo market, a run on a dealer i can occur and bankrupt the dealer if and only if the dealer's liquidity constraint (14) and his collateral constraint (17) are both violated.*

It can easily be seen that the conditions (14) and (17) are independent - neither of the two implies the other. As for the liquidity constraint derived in Proposition 2, the comparative statics of the collateral constraint for the tri-party model are simple and we collect them in the following proposition.

[17] The weak inequality is due to the assumption that investors do not switch dealers if indifferent. If $\overline{r} = \gamma R\kappa_i$, there exists the trivial run equilibrium discussed at the beginning of this section.

Proposition 5 *The collateral constraint (17) is the tighter,*

- *the lower is the liquidation value of collateral* γ*,*
- *the higher is short-term borrowing* b_i*,*
- *the lower is investment capacity* $\overline{I}$*,*
- *the lower is productivity* R*.*

Hence, the comparative statics with respect to b_i, $\overline{I}$, and R are identical for both constraints. Both constraints are relaxed if dealers have sufficient access to profitable investment ($\overline{I}$ large), if these investment opportunities are sufficiently profitable (R large), or if they have sufficiently little exposure to short-term borrowing (b_i small). In this case, there is no reason for unexpected runs to occur on the investor side and they cannot bring down dealers if they occur out of equilibrium. In the opposite case, a run can be a self-fulfilling prophecy and bankrupt the dealer.

5.1.1 Coordination problem between the clearing bank and investors

The tri-party repo market is also vulnerable to another coordination problem, this time between the clearing bank and the investors. Suppose that just before step 1 the clearing bank comes to believe that at step 4 all investors will refuse to engage in repos with dealer i. In this case, the clearing bank will refuse to unwind if the loan it makes to the dealer, $b_i\left[r+(1-\alpha)r^2\right]$, exceeds the proceeds it could obtain from the assets, $R\bar{I}(1+\beta\gamma)-f_i$. This condition can be written as

$$b_i \leq \beta^2 \frac{(1+\gamma\beta)R\bar{I}-f_i}{1+\beta-\alpha}. \tag{18}$$

This condition is different from the condition for investors not to have a strict incentive to run. We can show that

$$\beta^2 \frac{(1+\gamma\beta)R\bar{I}-f_i}{1+\beta-\alpha} > \frac{\beta^2\left(R\bar{I}-f_i\right)}{[1+\beta-\alpha]} \Leftrightarrow R\bar{I}(1-\gamma) > f_i. \tag{19}$$

The difference occurs because the clearing bank has access to all the cash and the assets of the dealer. In contrast, the clearing bank can only pledge securities as collateral to its investors.

The flip side of this coordination problem is that investors may choose not to invest with dealer i if they believe that the clearing bank will refuse to unwind that dealer's repos the next morning.[18] In this case, the condition for investors to have a strict incentive to run is the same as in the case where investors believe other investors may not engage in repos.

5.2 Tri-party repo without unwind

To highlight the importance of the unwind mechanism for the fragility of the tri-party repo market, it is interesting to consider what happens to the game described in the previous section when there is no unwind. This case is similar to the tri-party repo markets in continental Europe. It is also similar to the US tri-party repo market once the recommendation of the Task Force are implemented.[19]

When there is no unwind, the timing of events intraday is as follows:

1. The dealer offers a new repo contract (r_i, Q_i, κ_i).
2. New and patient middle-aged investors decide whether to engage in new repos with a dealer.
3. If the dealer is unable to repay his debt to last period's repo investors, he is illiquid and must declare bankruptcy. Otherwise, the dealer continues.

From Lemma 7 it is enough to consider the case in which the dealer is illiquid after a run. The situation without the unwind facility differs in two

[18] Clearing banks are not obligated to unwind a dealer's repos. Failure to unwind the repos would almost certainly force the dealer into bankruptcy.

[19] In this paper, we do not model why the unwind may be necessary. As described in Task Force (2010) and Copeland, Martin, and Walker (2010), the unwind makes it easier for dealers to trade their securities during the day. Collateral management technologies, as are currently used in continental Europe and are being proposed in the US, allow dealers to have access to their securities even as investors remain collateralized.

important respets from the one with unwind. First, without the unwind, an individual investor is repaid $\overline{r}$ if and only if the dealer can repay everybody - otherwise the dealer is bankrupt and repays everybody less than the contractual payment. Second, in contrast to the case with unwind, new and middle-aged investors are in a different situation when there is no unwind. New investors hold cash while middle-aged investors hold a repo with the dealer, until the dealer is able to repay his claim.

In case of a run, an illiquid dealer is bankrupt. All middle-aged investors then keep their collateral and may obtain additional cash as unsecured creditors depending on the bankruptcy rules. This payment is independent of whether an individual investor has demanded to be repaid or has agreed to roll over his loan. Given the tie-braking rule assumed throughout this section, patient middle-aged investors therefore reinvest. This in turn induces young investors to invest with the dealer.

Lemma 8 *If middle-aged patient investors reinvest, investing is a dominant strategy for new investors.*

Proof. If middle-aged patient investors do not withdraw their funds, the dealer is liquid. The dealer then has enough assets that will mature in the future to satisfy all future claims by young agents who invest today. ■

Hence, when there is no unwind, the incentives of investors are modified so that they never have a strict incentive to run. In essence, this is because the overnight repo market is an institution that creates simultaneity: if a sufficiently large number of investors do not re-invest, there is bankruptcy and all current creditors (the middle-aged investors) are treated equally, regardless of their intention to withdraw funding. This eliminates fragility due to pure coordination failures.

Proposition 6 *When there is no unwind, there are no strict incentives to run on dealers.*

5.3 Bilateral repos

Our model can also be adapted to think about a dealer that finances securities through bilateral repos. Typically, bilateral repos have a longer term than tri-party repos. Hence, one period in our model should be thought of as representing a few weeks.[20]

To simplify the exposition of institutional details, we consider a dealer that funds "Fed-eligible" securities; securities that can be settled using the Fedwire Funds Service®. The settlement of Fed-eligible securities is triggered by the sender of securities. Once the instruction to send the securities have been received, reserves are automatically deducted from the Fed account of the institutions receiving the securities and credited to the Fed account of the institution sending the securities.

This procedure creates a "first come first serve" constraint. In the case of a run, investors who send the securities they hold as collateral early are more likely to receive cash than investors who send their securities late. With bilateral repos, the timing is as follows:

1. The dealer offers a new repo contract (r_i, Q_i, κ_i).

2. New and patient middle-aged investors decide whether to engage in new repos with a dealer.

3. Investors are repaid in the order in which they send back their collateral, until the dealer runs out of cash. From that point on, investors receive their collateral. Any investor who chooses to invest receives his collateral.

The analysis of the total amount of collateral available is as before. In steady state, the maximum amount of collateral a dealer can pledge is $\bar{I}(1+\beta)$, in terms of securities maturing at $t+1$. The total amount of funds provided by investors is $b_i\left[1+(1-\alpha)\frac{1}{\beta}\right]$. Hence, the maximum amount of collateral available per expected unit invested, κ_i, is unchanged from (16).

[20] Also, a dealer may choose to stagger the terms of its repos, so that only a small portion of these repos are due on any given day. Because of the distribution of investor liquidity needs, this cannot happen in our model. He and Xiong (2010) analyze the consequences of (exogenously determined) staggered short-term debt for the stability of financial institutions. In addition, the assumption that investors either invest or don't invest may not be as appropriate for bilateral repos. Indeed, Gorton and Metrick (2009) show that "partial withdrawals" occurred in the form of increased margins.

Because of Lemma 7 we only consider illiquid dealers:

$$R\overline{I} - \left(r + r^2(1-\alpha)\right) b_i - f_i < 0.$$

Hence, only a fraction

$$\varphi \equiv \frac{R\bar{I} - f_i}{b_i \left[r + (1-\alpha)r^2\right]} \in [0, 1]. \tag{20}$$

of middle-aged investors can decide not to renew their repos before the dealer becomes illiquid. With probability $1 - \varphi$, the investor gets securities. As before, we assume that investors who are able to obtain their cash back can invest it with another dealer.

	other investors	
	invest	don't
invest	$\overline{r}$	$\gamma R \kappa_i$
don't	$\overline{r}$	$\varphi\overline{r} + (1-\varphi)\gamma R \kappa_i$

Table 2: Payoffs in bilateral repos

Table 2 gives the payoff to an individual investor as a function of the collective behavior of all other investors. Comparing this table to Table 1 shows that the condition for a dealer to be runproof is the same as in the case of the tri-party repo market with unwind. Hence, the predictions for both markets are the same.

Proposition 7 *In bilateral repo markets, a run on a dealer i can occur and bankrupt the dealer if and only if the dealer's liquidity constraint (14) and his collateral constraint (17) are both violated.*

5.4 Money market mutual funds

In this section, we adapt our model to the case of money market mutual funds (MMMFs) that can offer shares at a fixed net asset value (NAV). These funds are also known as 2a-7 funds, named after SEC rule 2a-7. MMMFs offer their investors shares that can be redeemed at a fixed price, typically \$1. Positive returns by the fund increases the number of shares, without affecting the price. If the fund loses value, however, the number of shares cannot decrease. In such a case, the fund is said to have "broken the buck" and is liquidated. Investors' shares give them a pro rata claim on the proceeds from the liquidation of the assets.

The fixed NAV makes MMMFs similar to banks since, under most circumstances, investors can obtain their funds on demand at a fixed price. However, MMMFs don't hold capital and don't have access to the discount window. MMMFs invest mainly in marketable assets.

One way to think of a MMMF in our environment is as a borrower which sets $b_i = \bar{I}$ in steady-state. In this case all of the assets held by the fund are purchased with investor funds. Alternatively, we can think of a MMMF as being part of a larger financial institution, which we call a parent institution. In this case, the size of the MMMF is determined by b_i and investors have a claim only on the assets purchased with their funds. The assets purchased with funds from the parent institution, $\bar{I} - b_i$, could be used to rescue the MMMF but the MMMF investors do not have a formal claim on these assets.[21]

With MMMFs, the timing is as follows:

1. The MMMF offers a new contract (r_i, Q_i).
2. New and patient middle-aged investors decide whether to withdraw from the MMMF.
3. There is a first-come-first-serve constraint in that the first investors to withdraw can get cash until the MMMF runs out. At that time, the MMMF has broken the buck and the remaining investors get a claim on remaining assets (that mature the next period).

[21] Shilling, Serrao, Ernst, Kerle (2010) provide information on support of MMMF by parent institutions during the recent financial crisis and earlier episodes.

For simplicity, we assume that the parent institution can credibly commit to use the resources at its disposal to support a troubled MMMF.[22]

An MMMF is potentially illiquid if the withdrawals it faces, $b_i\left[r+(1-\alpha)r^2\right]$, exceeds the cash available to it and its parent company minus the fixed cost, $R\bar{I}-f_i$. The probability that a withdrawing investors is able to obtain cash is φ, as defined in the previous section. With probability $1-\varphi$, the investor is unable to withdraw early enough to obtain cash. The investor thus gets a claim on the assets of the MMMF and the parent company. The value of these assets, per unit invested in the MMMF, is given by

$$\mu \equiv \frac{\gamma R\bar{I}}{b_i\left[r+(1-\alpha)r^2\right]-R\bar{I}+f_i}.$$

This condition is the same as the condition the clearing bank considers when choosing to unwind, if it believes investors will not engage in repos with dealer i, which was derived in section 5.1.1. If $\mu \geq r$, then investors do not have a strict incentive to run on an MMMF. Rewriting this condition we get

$$b_i \leq \beta^2 \frac{(1+\gamma\beta)R\bar{I}-f_i}{1+\beta-\alpha}. \tag{21}$$

A version of proposition 4 holds for MMMF, but the condition for runproofness is given by (21) instead of (17). Simple algebra shows that $\mu > \kappa$ if and only if $R\bar{I}(1-\gamma) > f_i$.

Comparing κ and μ is delicate because some variables may not have the same interpretation in each case. Nevertheless, it is interesting to understand why these two expressions are different. The fixed cost f_i does not appear in κ because a dealer pledges only assets to investors, in steady state, and consumes any uninvested cash. In case of a run, the cash that would be consumed by the dealer is not available as the clearing bank keeps it to offset part of its intraday loan. In contrast, an MMMF, and its parent company, first provides all available cash and then divides any assets equally among investors.[23]

[22] New SEC rules would allow a fund to "suspend convertibility", but only to avoid fire sales when a fund needs to be liquidated. In such a case, the remaining depositors have a claim on the assets of the fund, as in our setup. Hence, this type of suspension of redemption cannot prevent runs on MMMFs.

[23] In the case of bilateral repos, middle-aged investors who return their repos early obtain

5.5 Traditional banks

The analysis for traditional banks is similar to the analysis for MMMF. If $b_i < \bar{I}$, then the assets $\left(\bar{I} - b_i\right)(1+\beta)$ can be thought of as the equity of the bank. One major difference between a MMMF and a bank is that the bank holds nonmarketable assets. Hence, we would think of γ as being very low in the case of a bank.

The timing is as follows

1. The bank offers a new deposit contract (r_i, Q_i).

2. New and patient middle-aged investors decide whether to withdraw from the bank.

3. There is a first-come-first-serve constraint in that the investors who withdraw early can get cash until the bank runs out. At that time, the bank is bankrupt and the remaining investors get a claim on remaining assets (that mature the next period).

The analysis and the payoff table is the same as in the case of a MMMF, as is the condition for runproofness. Hence, a version of proposition 4 also hold in that case. However, because γ is expected to be very small for traditional banks, we expect that condition (21) would never be satisfied.

6 Runs and Asset Sales

In this section, we introduce the possibility of asset sales as a reaction to a run. As in Section 4, we consider a situation where, after dealers have made contract offers, the investors of one dealer coordinate unexpectedly not to lend to this dealer anymore. The question is again: if behavior until date t is steady state as in Proposition 1, can the beliefs that all investors of a given dealer will refuse to roll over their loans at date t be self-fulfilling and bankrupt the dealer? For this to be the case, it is necessary that the dealer

cash while other investors get their collateral. A dealer may have additional assets but cannot pledge them to investors who return their repos late. If a dealer can sell those assets, as described in the next section, then the dealer may become liquid.

cannot mobilize enough cash to satisfy the run demand. As pointed out by, e.g., Shleifer and Vishny (1992), Acharya and Yorulmazer (2008), and Diamond and Rajan (2009), “fire sales", i.e. asset sales under distress, can mitigate this problem.

To investigate this possibility, consider a dealer, say i, at date t who holds assets that will yield $R\bar{I}$ at date $t+1$. We assume that in response to a run, the dealer can sell these assets to other dealers at some market price p. If the dealer under distress sells an amount A of assets, this improves his current liquidity by pA and reduces his cash at date $t+1$ by RA. Generalizing (13), his cash position after the run at date t therefore is

$$I_0 = R\bar{I} + pA - (\bar{r} + (1-\alpha)\bar{r}^2)b_i - f_i \tag{22}$$

If the dealer is to continue investing after date t the amount of assets he can sell is restricted by

$$R(\bar{I} - A) \geq f_i \tag{23}$$

Combining the condition $I_0 \geq 0$ with condition (23) and noting that not both can bind for the dealer to continue operating yields the new liquidity condition

$$R\bar{I} + p(\bar{I} - \frac{f_i}{R}) - (\bar{r} + (1-\alpha)\bar{r}^2)b_i - f_i > 0 \tag{24}$$

If p satisfies (24) the dealer will survive by selling a sufficient amount of assets, if not he will be bankrupt. Whether the dealer can raise enough cash through the asset sale depends on the cash in the market (Allen and Gale 1994), i.e. on the total amount of cash held by all other dealers. At the moment of the run, i.e. when the dealers have repaid their steady-state borrowing, received their new loans including the funds $b_i + (1-\alpha)\bar{r}b_i$ that have not gone to dealer i, but before they have invested their funds, this cash is

$$\begin{aligned} C &= b_i + (1-\alpha)\bar{r}b_i + \sum_{j\neq i}\left[R\bar{I} - (\alpha\bar{r} + (1-\alpha)\bar{r}^2 - 1)b_j - f_j\right] \\ &= N + (M-1)R\bar{I} - (\alpha\bar{r} + (1-\alpha)\bar{r}^2)(N - b_i) + (1-\alpha)\bar{r}b_i - \sum_{j\neq i} f_j \end{aligned}$$

This cash is sufficient to cover dealer i's missing amount

$$m_i \equiv -I_0 = (\bar{r} + (1-\alpha)\bar{r}^2)b_i + f_i - R\bar{I} \tag{25}$$

if and only if

$$M R \overline{I} - (\alpha \overline{r} + (1-\alpha)\overline{r}^2 - 1)N - f \geq 0 \tag{26}$$

where

$$f = \sum_{j=1}^{M} f_j$$

are the total transactions costs in the system.

By assumption (10), we have

$$M(R-1)\overline{I} - (\alpha \overline{r} + (1-\alpha)\overline{r}^2 - 1)N - f > 0 \tag{27}$$

Clearly (27) implies (26). Hence, there is enough cash in the market to satisfy the run demand.

The question is whether this cash can be mobilized in equilibrium. The benefits from mobilizing this cash are the asset returns in $t+1$, the cost is the foregone investment that yields benefits in $t+2$ and thereafter. The demand for cash is easily described. The dealer must raise m_i as given by (25). From (24), the proceeds from the asset sale will be sufficient to cover m_i if and only if

$$p > \frac{R m_i}{R\overline{I} - f_i} \equiv \underline{p}$$

Concerning the supply of cash, note first that, since the assets sold by the dealer yield only γR to outsiders next period, the demand for these assets, hence the supply of cash, will be 0 if $p > \beta\gamma R$. The full supply of liquidity is derived in the following lemma.

Lemma 9 *The supply of cash by all other dealers is*

$$S = \begin{cases} 0 & \textit{for } p > \beta\gamma R \\ \sum_{j\neq i} \pi_j + (1+(1-\alpha)\overline{r})b_i & \textit{for } \beta\gamma R > p > \frac{\gamma}{\beta} \\ \quad \sum_{j\neq i} \pi_j + (1+(1-\alpha)\overline{r})b_i \\ \qquad + \left(\frac{1}{R} + ... + \frac{1}{R^n}\right)\left(\sum_{j\neq i} \pi_j + b_i\right) & \textit{for } \left(\frac{1}{\beta^2 R}\right)^{n-1} \frac{\gamma}{\beta} > p > \max(\gamma, \left(\frac{1}{\beta^2 R}\right)^{n} \frac{\gamma}{\beta}) \textit{ for } n \geq \\ C & \textit{for } p \leq \gamma \end{cases}$$

Proof: If a dealer $j \neq i$ invests into his own assets this returns R at date $t+2$. Alternatively, he can buy the distressed dealer's assets, which returns $\gamma R/p$ in $t+1$, which he can store until $t+2$. The latter is strictly preferred if $p < \gamma$. Hence, if $p < \gamma$ there can be no dealer who invests in his own assets and all cash in the market is spent on dealer i's assets.

Consider $p < \beta\gamma R$. If all dealers $j \neq i$ invest $\overline{I}$ into their assets as in steady state they have a total of $\sum_{j\neq i} \pi_j + (1+(1-\alpha)\overline{r})b_i$ in cash. Investing this cash into the distressed dealer's assets yields a return of $\gamma R/p$ next period, which is strictly preferred to consuming the cash now. A dealer $j \neq i$ can also reduce his investment in his own assets below $\overline{I}$ and make up for the shortfall in $t+2$ by using profits that accrue in $t+2$. One unit at date t thus costs $\beta^2 R$ at date $t+2$. Hence, this investment distortion is optimal if $\beta^2 R < \beta\gamma R/p$, .i.e. if $p < \gamma/\beta$. In this case, all available cash at date $t+2$, $\sum_{j\neq i} \pi_j + b_i$, will be brought forward, which yields $(\sum_{j\neq i} \pi_j + b_i)/R$ at date t. Continuing for the periods $t+2k$ in this fashion, yields the recursion in the third line of S.

It is not optimal to bring cash from dates $t+2k+1$ forward to date t if $p > \gamma$.

From lemma 9, it is clear that if $\gamma \geq \underline{p}$, the sales of assets allows the dealer to accommodate the demand for cash in case of a run. However, if $\gamma < \underline{p}$, and if

$$(1-\alpha)\overline{r}b_i + \frac{R}{R-1}(\sum_{j\neq i} \pi_j + b_i) \leq m_i, \tag{28}$$

then assets sales cannot prevent a run because the price of the assets is too low. The right hand side of inequality (28) is small if the fixed costs faced by dealers are high, so that π_j, $j \neq i$, is small, and b_i is small. Hence, there are some parameter values for which the asset sales cannot prevent a run.

6.1 Interpretation

Most assets serving as collateral in the tri-party repo market are liquid, so we should expect γ to be close to 1. Hence, we can interpret the result of this section as suggesting that when markets are not stressed, dealers in the tri-party repo market can accommodate the demand that would arise from an idiosyncratic run. This is broadly consistent with the conventional wisdom before the financial crisis.

There are two cases, however, where we might expect γ to be low in the tri-party repo market. A low γ should be expected if a dealer uses relatively less liquid collateral to back its repos. In such a case, it will be more difficult for whoever tries to liquidate the collateral to obtain a high value. Anecdotal evidence suggests that the share of less liquid collateral in the tri-party repo market had been increasing before the crisis, maybe reaching 30 percent of the collateral in that market. This would have made dealers financing less liquid collateral more susceptible to runs.

A low γ may also apply if the quantity of a relatively liquid asset used as collateral in tri-party repos is so large that the market may not be able to absorb all the collateral in case of a dealer default. For example, Agency MBSs are considered liquid securities, but the amount of such securities financed in tri-party was so large that the market may not have been able to absorb them without some price effect.

It is also worth pointing out that our model probably overstates dealer's ability to accommodate the demand for cash in a run and the ability of other dealers to purchase assets. In our model, the share of repos held by old and impatient middle aged investors is close to half of all the repos made by a dealer.[24]. Hence, the demand for funds in the case of a run is about twice as large as the steady state demand. Anecdotal evidence suggests that the share of repos being rolled over in the tri-party repo market is much larger, probably over 80 percent. This would mean that the run demand is five time as large as the stead-state demand, which would be more difficult for a dealer to accommodate.

Our model could be adapted to increase the share of repos rolled over every period. For example, we could consider an economy in which agents lived longer lives and assets matured after more periods. In such an economy, the share of cash and maturing assets would be a smaller share of all assets. Similarly, the share of new and withdrawing investors, which must be equal in steady state, would represent a smaller fraction of the population of all investors. Hence, the demand for funds in case of a run would be much larger

[24]The exact share will vary depending on the parameters α and β

than the steady state demand, compared to the economy we consider. The share of unmatured assets that can be sold, compared to the available cash, would also be greater, increasing the fire sale effect.

7 Market Runs

As noted above, the more dealers are in trouble, the more assets troubled dealers are trying to sell and the fewer dealers are available to buy these assets. This puts pressure on the price of assets and it makes it less likely that a run can be avoided. In the extreme case of a market run, all dealers are facing a run demand and no dealer may be available to buy assets.

Consider a case where the dealers are not too heterogeneous, so that no dealer has b_i so low that it can accommodate its run demand and have enough cash to purchase assets from other dealers. In such a case dealers are in the same situation as if their assets were not marketable. Hence, the conditions of Proposition 2 are the relevant ones in order to evaluate the possibility of runs.

Condition (14) will differ for dealers depending on b_i and f_i. We can think of b_i as a measure of the dealer's leverage, since it indicates the amount of its financing coming from external sources. A dealer with a higher b_i is more leveraged. Proposition 2 shows that whether dealer i can accommodate the demand for cash from its investors depends on her fixed cost and her leverage. We define the critical leverage threshold function

$$\overline{b}_i(f_i) \equiv \frac{\beta^2(R\overline{I} - f_i)}{1 - \alpha + \beta} \geq 0,$$

which can be used to classify which dealers will fail in the case of a market run.

Proposition 8 *In the case of a market run, all dealers with leverage $b_i \leq \overline{b}_i(f_i)$ are able to accommodate the run demand and all dealers with leverage $b_i > \overline{b}_i(f_i)$ are forced into bankruptcy.*

Equivalently, we can instead define the critical cost threshold function

$$\overline{f}_i(b_i) \equiv \frac{\beta^2 R\overline{I} - (1 - \alpha + \beta)b_i}{\beta^2},$$

and we have the parallel result that in the case of a market run, all dealers with cost $f_i \leq \overline{f}_i(b_i)$ are able to accommodate the run demand and all dealers with leverage $f_i > \overline{f}_i(b_i)$ are forced into bankruptcy.

The critical leverage threshold function $\overline{b}_i(f_i)$ is the largest leverage b_i (as a function of dealer i's cost f_i) such that condition (14) in Proposition 2 holds for dealer i. Equivalently, the critical cost threshold function $\overline{f}_i(b_i)$ is the largest cost f_i (as a function of dealer i's leverage b_i) such that condition (14) in Proposition 2 holds for dealer i. While our theory is a theory of multiple equilibria and therefore cannot predict runs, Proposition 8 makes a prediction about the outcome of a market run: A firm that is comparatively weaker in terms of having greater leverage or cost structure will always fail whenever a comparatively stronger firm (with less leverage or cost structure) fails.

8 Extension: Liquidity provision

Access to a lender of last resort is a standard tool used to strengthen the banking sector in the face of financial fragility. Theoretical work has shown how access to a lender of last resort can prevent bank runs (see, for example, Allen and Gale 1998, Martin 2006, Skeie 2004). In the U.S., the broker dealers that rely on the tri-party repo market as a source of short-term funding did not have direct access to discount window. This lack of access to emergency liquidity proved destabilizing during the crisis and motivated the Federal Reserve to introduce the Primary Dealer Credit Facility (PDCF). Similar concerns about money market mutual funds, who represent an important share of investors in the tri-party repo market, motivated the creation of the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF), and the Money Market Investor Funding Facility (MMIFF). These facilities were created under section 13.3 of the Federal Reserve Act, which allows the Federal Reserve to lend to a variety of institutions under unusual and exigent circumstances. As such, these facilities are temporary.[25]

[25] The MMIFF expired on October 30, 2009. The Board of Governors approved extension of the AMLF and the PDCF through February 1, 2010.

The Task Force on Tri-Party Repo Infrastructure (2009) notes the need to "Consider establishing an industry-sponsored utility with the ability to finance the securities portfolio of a faltering or defaulted dealer and limit the associated stress on the market while their portfolio is liquidated." The model in our paper suggests that there would be benefits to the creation of a lender-of-last-resort facility for the tri-party repo market. The argument is similar to the case of banking. In case of a run, investors do not refuse to roll over their loans because they need cash, but because they are concerned about the default of the dealer and having to hold collateral that they might have to liquidate. As in Allen and Gale (1998), Martin (2006), or Skeie (2004), a lender of last resort could lend cash to the dealer taking securities as collateral. The cash could be used to pay all investors who do not roll over their loans. This would prevent the default of the dealer and allow it to manage the collateral until it matures. Knowing that the dealer will not default, investors no longer have to worry about having to hold or liquidate assets, so their incentive to run is reduced.

9 Conclusion

In this paper we study a model of short-term collateralized borrowing and the conditions under which runs can occur. Our framework resembles the dynamic model of banks studied in Qi (1994), but expands that model in a number of directions. We derive a dynamic participation constraint that must hold for dealers to agree to purchase securities on behalf of investors. Under this constraint, dealers will make profits that can be mobilized to forestall runs.

A key difference between traditional banks and modern financial intermediaries is that the former mainly hold opaque assets while the latter's assets are much more liquid and marketable. We study the role of marketable assets in preventing runs on these intermediaries. Without asset sales, runs can be forestalled by mobilizing sufficient liquidity and having sufficiently valuable collateral. This gives rise to two constraints that can be interpreted as a liquidity and a collateral constraint. The liquidity constraint guarantees that the necessary resources are available at the date the run occurs. The collateral constraint makes sure that investors want to continue collateralized lending instead of running for their money. This constraint critically depends on the microstructure of the market under consideration. A run can be prevented if either constraints is satisfied, meaning that the dealer is liquid or has enough collateral.

Next we consider the case where dealers can sell their assets. Because of cash-in-the-market pricing, the price of assets will depend on the quantity of assets supplied by troubled dealers and the demand for these assets by healthy dealers. In particular, the opportunity cost of funds for healthy dealers increases as they purchase more assets. As more dealers try to sell their assets, the price of the assets they sell will decline. In the limit, no dealers are available to purchase distressed assets, which takes the analysis back to the case of runs without the possibility of asset sales.

Our framework can be used to consider interesting policy questions related to the fragility of the tri-party repo funding mechanism. For example, Lehman's demise highlighted an important problem: There is no framework to unwind the positions of any large bank that deals in repo should it fail. Lehman required large loans from the Federal Reserve Bank of New York to settle its repo transactions (WSJ 2009). Our framework can be used to study a liquidation agent, as suggested in the Task Force on Tri-Party Repo Infrastructure (2009), that could be used to unwind the positions of a defaulting dealer.

10 Appendix

10.1 Proof of Remark 1

The liquidity constraint (14) has been derived in the text. In this appendix, we analyze how the dealer has to adjust his investment if he is to resume full borrowing of b_i from date $t+1$ on.

At date t, the dealer can invest less than the steady state level $\overline{I}$ in order to liberate cash to accommodate the run demand. This yields a lower return in $t+2$, but can still be consistent with continuing borrowing the full amount of b_i from investors at the steady state rate $\overline{r}$ in the future, because the dealer can make payments out of his date $t+2$ cash flows to cover the shortfall resulting from (a limited degree of) underinvestment. In the limit, the dealer can exhaust all of his profits at date $t+2$ and reduce investment in t correspondingly by π/R. In fact, he can carry this further. At date $t+2$ he can reduce investment below the steady state level to liberate cash that can be used to meet the shortfall resulting from a further reduction in investment at date t, etc. This way, the dealer can reduce investment in future periods $t+2k$, $k=1,2,...$, in order to shift profits forward to date t, which allows him to liberate more and more of the current cash to accommodate the run demand.

Lemma 10 *In response to a run, the optimal sequence of investments at dates $t+2k$, $k=0,1,2,...$, reduces profits to zero up to a certain period (which can be ∞), from which on investment is back to the steady state level $\overline{I}$.*

Proof: Suppose that there is a period $t+2k_0$ in which investment is smaller than $\overline{I}$ and the dealer makes positive profits. Then the dealer can reduce profits in $t+2k_0$ slightly by investing $\delta>0$ more, which yields $R\delta$ in $t+2k_0+2$. From then on he sticks to the former investment sequence. By (5), he is strictly better off. To complete the proof note that if there is a period in which the dealer can invest $\overline{I}$, then he can do so ever after and this is optimal.

The policy identified in Lemma 10 is a value reducing distortion of investment. But in fact, more is possible. The above investment strategy does not involve the dealer's behavior at dates $t + 2k + 1$, $k = 0, 1, 2, ...$ If the dealer sets an amount S aside out of date $t + 1$ - profits and stores it until $t + 2$, then he can reduce investment in t by S/R, by using S in $t + 2$ to cover the shortfall. By the same logic as above, the dealer can now increase the amount set aside in $t+1$ by reducing investment in $t+1$ and making up for the shortfall in $t + 3$ by using profits from $t + 3$, etc. As in Lemma 10, it is straightforward to show that the optimal strategy in periods $t + 2k + 1$ features maximum investment for as long as necessary to get back to the steady state level. By following this strategy, the dealer can again bring all future profits from periods $t + 2k + 1$ forward.

This shows that the dealer faces essentially two constraints. One is the liquidity constraint from (13), the other refers to the dealer's overall solvency, namely the total amount of funds, F_0, that the dealer can mobilize by reducing current and future investment. Whether or not the dealer can indeed pay out F_0 depends on his liquidity, $\max(0, I_0)$. Hence, the actual amount of cash the dealer can liberate in response to the run is $\min(max(0, I_0), F_0)$. We now derive the precise form of the solvency constraint.

Using Lemma 10 and the analogous argument for the sequence of investments at dates $t + 2k + 1$, we can generalize (13) and describe the optimal response to a run at date t recursively. To facilitate notation denote the dealer's net steady state repayments to investors by

$$n_i = (\alpha\overline{r} + (1 - \alpha)\overline{r}^2 - 1)b_i \tag{29}$$

The maximal sequence of investments, starting at date t, then is

$$\begin{aligned}
I_0 &= R\overline{I} - (\overline{r} + (1 - \alpha)\overline{r}^2)b_i - f_i = R\overline{I} - n_i - (1 + (1 - \alpha)\overline{r})b_i - f_i && (30)\\
I_1 &= R\overline{I} + b_i - S - f_i = R\overline{I} - n_i + (\alpha\overline{r} + (1 - \alpha)\overline{r}^2)b_i - S - f_i && (31)\\
I_2 &= RI_0 + b_i + S - \alpha\overline{r}b_i - f_i = RI_0 - n_i + (1 - \alpha)\overline{r}^2 b_i + S - f_i && (32)\\
I_3 &= RI_1 + b_i - (\alpha\overline{r} + (1 - \alpha)\overline{r}^2)b_i - f_i = RI_1 - n_i - f_i && (33)\\
I_4 &= RI_2 + b_i - (\alpha\overline{r} + (1 - \alpha)\overline{r}^2)b_i - f_i = RI_2 - n_i - f_i && (34)\\
&\text{etc.} && (35)
\end{aligned}$$

Writing out the two recursions in (30) - (35) and re-arranging yields

$$I_{2k} = R^{k-1}\left[RI_0 - \frac{R}{R-1}(n_i+f_i) + S + (1-\alpha)\overline{r}^2 b_i\right] + \frac{n_i+f_i}{R-1} \quad (36)$$

$$I_{2k+1} = R^{k}\left[I_1 - \frac{n_i+f_i}{R-1}\right] + \frac{n_i+f_i}{R-1} \quad (37)$$

for $k \geq 1$. Each of these two recursions comes to an end when $I_n \geq \overline{I}$. The sequences I_{2k} and I_{2k+1} satisfy

$$I_{2k} \nearrow \infty \Leftrightarrow RI_0 > \frac{R}{R-1}(n_i+f_i) - (1-\alpha)\overline{r}^2 b_i - S \quad (38)$$

$$I_{2k} \searrow -\infty \Leftrightarrow RI_0 < \frac{R}{R-1}(n_i+f_i) - (1-\alpha)\overline{r}^2 b_i - S \quad (39)$$

$$I_{2k+1} \nearrow \infty \Leftrightarrow I_1 > \frac{1}{R-1}(n_i+f_i) \quad (40)$$

$$I_{2k+1} \searrow -\infty \Leftrightarrow I_1 < \frac{1}{R-1}(n_i+f_i) \quad (41)$$

Because profits cannot be negative, the proposed investment strategy is infeasible if (39) or (41) hold. More generally, the dealer can satisfy the run demand and continue to borrow b_i after date t if and only if there exists an $S \geq 0$ such that (38) and (40) hold with weak inequality and the initial feasibility constraints $I_0 \geq 0$ and $I_1 \geq 0$ hold. Note that (40) implies $I_1 \geq 0$.

From (31), the maximum S consistent with (40) in its weak form is

$$\begin{aligned}\widehat{S} &= R\overline{I} + (\alpha\overline{r} + (1-\alpha)\overline{r}^2)b_i - (n_i+f_i) - \frac{1}{R-1}(n_i+f_i) \\ &= R\overline{I} - \frac{R}{R-1}(n_i+f_i) + (\alpha\overline{r} + (1-\alpha)\overline{r}^2)b_i \\ &> 0\end{aligned}$$

where the last inequality follows from the assumption that steady-state profits are positive.[26] Constraint (38) holds for some $S \geq 0$ consistent with (40) if and only if it holds for $\widehat{S}$. Inserting $\widehat{S}$ into (38) with weak inequality yields

$$RI_0 \geq \frac{2R}{R-1}(n_i+f_i) - R\overline{I} - (\alpha\overline{r} + 2(1-\alpha)\overline{r}^2)b_i \quad (42)$$

[26] Using (11), this assumption can be expressed as $(R-1)\overline{I} - n_i - f_i > 0$.

Substituting I_0 from (30) into (42), using $\overline{r} = 1/\beta$, and re-arranging yields

$$R\overline{I} \geq \frac{R}{R-1}(n_i + f_i) + \frac{1}{\beta^2(R+1)} \underbrace{\left(\beta^2 R + (1-\alpha)\beta R - \alpha\beta - 2(1-\alpha)\right)}_{m} b_i \tag{43}$$

Upon substituting for n_i and re-arranging, (43) is the rebounding constraint (15). Note that the coefficient m in (43) is strictly positive. Hence, (43) is strictly stronger than the condition that dealer i's steady state profits are positive, as noted after Proposition 2.

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