Abstract

A key empirical fact about household portfolios is that wealthier households hold a greater share of their portfolios in risky assets such as stocks. This paper offers a simple model of household portfolio choice that is consistent with this observation. The model consists of a two-person household within which each individual has a standard constant relative risk aversion (CRRA) utility function. Individual household members may have different levels of risk aversion. The model predicts that the share of risky assets in the household portfolio increases with wealth. It also predicts that the risk aversion of the spouse with a higher Pareto weight (which we interpret as bargaining power) determines the share of the risky asset in the household portfolio. The theoretical predictions are supported by empirical evidence from the Health and Retirement Study (HRS).
1 Introduction

A key empirical fact about household portfolios is that wealthier households hold a greater share of their portfolios in risky assets such as stocks. Several recent papers on household portfolio choice have focused on developing theoretical models that are consistent with this observation. The papers typically rely on assumptions on utility functions such as nonhomotheticity or include habit formation, ambiguity aversion, or learning in their framework.

This paper offers a simple alternative model that is also consistent with the observation that wealthier households hold a greater share of their portfolios in risky assets. The model consists of a two-person household within which each individual has a standard constant relative risk aversion (CRRA) utility function. Individual household members may have different levels of risk aversion.

Most papers solve household problems using a unitary framework, which treats the household as a single decision-making unit with one utility function. However, papers that do explicitly model household members with separate preferences have shown that this is an important consideration. For example, Browning (2000) and Mazzocco (2004) find that the allocation of resources within the household affects the consumption-savings decision when spouses differ in their preferences. Empirical estimates show that a majority of spouses do indeed differ in risk preferences (Barsky, Juster, Kimball, and Shapiro, 1997; Kimball, Sahm, and Shapiro, 2008).
This paper focuses on the household’s decision to allocate its savings between two assets – risky and risk-free. The literature on this choice is vast. The classic Merton-Samuelson framework, which models the household as a single agent with CRRA preferences, predicts that household portfolio choice is independent of wealth (Merton, 1969; Samuelson, 1969). Yet, empirical evidence suggests that the share of risky assets in the household portfolio increases with wealth (Bertaut and Starr-McCluer, 2002). Recent papers are able to match this theoretical fact under a variety of assumptions. Wachter and Yogo (2010) assume that households have nonhomothetic utility over basic and luxury goods and find that the share of risky assets in the household’s portfolio is higher for wealthier households. Achury, Hubar, and Koulovatianos (2012) obtain a similar result by introducing a Stone-Geary utility function with subsistence consumption in the Merton-Samuelson framework. Under the assumption of endogenous habit formation preferences, Polkovnichenko (2007) finds that among households with low to moderate wealth levels, the share of risky assets in their portfolio increases with wealth. This paper contributes to the literature by offering a simple alternative model in which the share of risky assets in the household portfolio increases in wealth while individual agents in the household have CRRA preferences.

This paper also contributes to the literature that relates individual risk preferences and bargaining power to household financial decisions. Most previous research on this topic has focused on the consumption-savings choice
(Browning, 2000; Lundberg, Startz, and Stillman, 2003). The problem arises because wives, who are on average younger and expected to live longer than their husbands, prefer to save more than their husbands. Browning (2000) uses a noncooperative bargaining model to show that the share of savings in the household portfolio depends on the distribution of income between spouses. Lundberg, Startz, and Stillman (2003) provide empirical support for this argument, showing that household consumption falls after the husband retires (and presumably loses bargaining power). The theoretical model in this paper predicts that the risk aversion of the spouse with a higher Pareto weight (which we interpret as greater bargaining power) determines the share of risky assets in the household portfolio.

The predictions of the model are tested using data from the Health and Retirement Study (HRS). The HRS is a longitudinal study that has surveyed older Americans every other year since 1992. The HRS includes detailed information on household portfolios and a series of questions that can be used to infer respondents’ risk aversion. It also includes questions on who makes the major family decisions within the household. Elder and Rudolph (2003) utilize this information and find that decisions are more likely to be made by the household member with more financial knowledge, more education, and a higher wage. Friedberg and Webb (2006) use the HRS to empirically investigate the effect of bargaining power on household portfolio

\footnote{The HRS is sponsored by the National Institute of Aging (grant number NIA U01AG009740) and is conducted by the University of Michigan.}
allocation. They find that households tend to invest more heavily in stocks as the husband’s bargaining power increases. Using education as our measure of bargaining power, we find support for their result in cases where the husband is the less risk-averse spouse.

2 Theoretical Framework

The theoretical framework is constructed under the assumption that household members cooperate and make efficient decisions. The economy consists of households with two agents, \( a \) and \( b \), who live for two periods. In the first period, each member of the household is endowed with wealth \( w_0^i, i = a, b \). The household can save using a risk-free asset, \( m \), that earns a certain return, \( r^m \), and a risky asset, \( s \), that earns a stochastic return, \( \tilde{r}^s(\theta) \), where \( \theta \) denotes the state of nature. Agents derive utility from consuming out of wealth a public good in periods 0 and 1, \( c_0 \) and \( \tilde{c}_1(\theta) \). The utility function of each agent, \( u^i \), is increasing, concave, and twice continuously differentiable.

Since the solution to the household problem is efficient, it can be obtained as the solution to the following Pareto problem. Given \( w_0 = w_0^a + w_0^b \), \( \tilde{r}^s \), and \( r^m \), the household chooses consumption, \( c_0 \) and \( \tilde{c}_1 \), savings in the risk-free asset, \( m_0 \), and savings in the risky asset, \( s_0 \), to solve

\[
\max_{c_0, \tilde{c}_1, m_0, s_0} \lambda \left[ u^a(c_0) + \beta^a E u^a(\tilde{c}_1) \right] + (1 - \lambda) \left[ u^b(c_0) + \beta^b E u^b(\tilde{c}_1) \right]
\]
subject to

\[ c_0 + m_0 + s_0 \leq w_0 \]
\[ \tilde{c}_1 \leq (1 + \tilde{r}_1^s)s_0 + (1 + r^m)m_0 \forall \theta. \]

Here \( \lambda \) denotes the Pareto weight or relative bargaining power of spouse \( a \) and \( \beta^i \) denotes the discount factor for each spouse.

Let \( x_0 = m_0 + s_0 \) denote total household savings and let \( \rho = \frac{s_0}{x_0} \) be the share of household savings invested in the risky asset. We can rewrite the above problem as

\[
\max_{x_0, \rho} \lambda [u^a(w_0 - x_0) + \beta^a E u^a ((1 + \tilde{r}_1^s)\rho x_0 + (1 + r^m)(1 - \rho)x_0)]
+ (1 - \lambda) [u^b(w_0 - x_0) + \beta^b E u^b ((1 + \tilde{r}_1^s)\rho x_0 + (1 + r^m)(1 - \rho)x_0)].
\]

Now assume that each agent has a constant relative risk aversion (CRRA) utility function of the form\(^2\)

\[
u^a(c_t) = \frac{c_t^{1-\gamma^a} - 1}{1 - \gamma^a} \text{ and } u^b(c_t) = \frac{c_t^{1-\gamma^b} - 1}{\delta(1 - \gamma^b)}.
\]

The optimal choice of savings and portfolio allocation, \( x_0^* \) and \( \rho^* \), are the

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\(^2\)The parameter \( \delta \) is required for accurate numerical simulation. Assume that \( \gamma^a < \gamma^b \). There is a threshold level of household wealth, \( \bar{w} \), above which “household risk aversion” is closer to \( \gamma^a \) and below which it is closer to \( \gamma^b \). The threshold can be set at an arbitrary value by changing the value of \( \delta \). The details are described later.
solution to the following first order conditions

\[
\frac{\lambda \delta \beta^a x_0^* \gamma^b - \gamma^a}{\beta^b (1 - \lambda)} \left\{ \frac{(w_0 - x_0^*)^{-\gamma^a}}{\beta^a x_0^{*\gamma^a}} + E \left[ (1 + \bar{r}_1^s) \rho^* + (1 + r^m)(1 - \rho^*) \right]^{-1 - \gamma^a} \right\} + \\
\frac{\lambda \delta \beta^a x_0^* \gamma^b - \gamma^a}{\beta^b (1 - \lambda)} \left\{ \frac{(w_0 - x_0^*)^{-\gamma^b}}{\beta^b x_0^{*\gamma^b}} + E \left[ (1 + \bar{r}_1^s) \rho^* + (1 + r^m)(1 - \rho^*) \right]^{-1 - \gamma^b} \right\} = 0 \quad (1)
\]

Before turning to the numerical solution to the problem, insights about household portfolio choice can be gained by deriving the expression for household risk aversion. Define the instantaneous utility function of the representative agent as

\[
V(w) = \max_c \lambda \frac{c^{1 - \gamma^a}}{1 - \gamma^a} + (1 - \lambda) \frac{c^{1 - \gamma^b}}{\delta(1 - \gamma^b)}
\]

subject to

\[
c \leq w.
\]

Household relative risk aversion is thus given by

\[
\gamma^{hh} \equiv -w \frac{V''(w)}{V'(w)} = \frac{\lambda \delta \gamma^a w^{-\gamma^a} + (1 - \lambda) \gamma^b w^{-\gamma^b}}{\lambda \delta w^{-\gamma^a} + (1 - \lambda) w^{-\gamma^b}}.
\]

\[3\text{This is common in the literature on heterogeneous risk preferences. See, for example, Dumas (1989) and Mazzocco (2003).}\]
The following results can be proved about household risk aversion (all proofs are in the Appendix):

**Lemma 1.**

a) As the relative bargaining power of the more (less) risk-averse spouse increases, household risk aversion increases (decreases).

b) An increase in wealth, \( w \), reduces household relative risk aversion.

**Lemma 2.** As household risk aversion increases (decreases), the solution to the household’s problem approaches the solution preferred by the more (less) risk-averse spouse.

The above lemmas lead to the paper’s two main propositions:

**Proposition 1.** As household wealth increases, the solution to the household’s problem approaches the solution most preferred by the less risk-averse spouse.

**Proposition 2.** As the relative bargaining power of a spouse increases, the solution to the household’s problem approaches the solution most preferred by that spouse.

Finally, observe that if spouse \( i \) can dictate his or her preferences (i.e., if \( \lambda = 0 \) or 1), the optimal share of the risky asset in the household portfolio is the solution to

\[
E \left\{ \left[ (1 + \tilde{r}^s_i)\rho^* + (1 + r^m_i)(1 - \rho^*) \right]^{-\gamma} (\tilde{r}^s_i - r^m_i) \right\} = 0. \tag{3}
\]
where \( i = a \) if \( \lambda = 1 \) and \( i = b \) if \( \lambda = 0 \). We know from the numerical solution to Equation (3) that as an individual’s risk aversion decreases, they prefer to hold a greater share of their portfolio in the risky asset.

Proposition 1 and 2 thus imply that an increase in wealth or an increase in the bargaining power of the less risk averse spouse will increase the share of risky assets in the household’s portfolio. These are the paper’s two key results. While the latter is intuitive, the former requires some explanation, particularly in light of the classic Merton-Samuelson result that the share of risky assets in a household’s portfolio is independent of its wealth.\(^4\) It turns out that this result is a special case of the result in this paper. If \( \delta = 1 \) and individual members of the household have equal Pareto weights, identical discount factors, \( \beta \), and identical coefficients of relative risk aversion, \( \gamma \), then (2) reduces to the following:

\[
E \left\{ \left[ (1 + \tilde{r}_1^s)\rho^* + (1 + r^m)(1 - \rho^*) \right] \gamma (\tilde{r}^s - r^m) \right\} = 0. \tag{4}
\]

Equation 4 represents the classic Merton-Samuelson result that the household’s choice of \( \rho \) is independent of its wealth. In general, however, wealth acts as a weight on the utilities of the spouses—as wealth increases, the weight shifts from the utility of the more risk averse spouse to that of the less risk averse spouse. This and other properties of the model can be seen through the numerical simulations in the next section and more formally via

\(^4\)Merton (1969); Samuelson (1969); also see Jagannathan and Kocherlakota (1996).
2.1 Numerical Simulation

Since the theoretical model cannot be solved analytically, we describe the properties of the model using numerical simulations. In particular, we are interested in how risk aversion, bargaining power, and wealth interact to determine optimal portfolio allocation, \( \rho^* \). To describe these relationships, we numerically solve Equations (1) and (2) to calculate \( \rho^* \) for various values of risk aversion, bargaining power, and wealth.

We assume throughout that the return on bonds, \( r^m \), is 1 percent and the return on risky assets, \( r^s_1 \), is either 27.03 percent, 13 percent, or \(-15.25\) percent with equal probability.\(^5\) Numerically realistic simulations also require choosing an appropriate value for \( \delta \). As mentioned earlier, given a threshold level of household wealth, \( \bar{w} \), \( \delta \) can be chosen such that household risk tolerance lies exactly between \( \gamma^a \) and \( \gamma^b \) at that level of wealth. The value of \( \delta \) for which this holds is \( \delta = \frac{1-\lambda}{\lambda} \bar{w}^{\gamma^a-\gamma^b} \).\(^6\) We abstract from differences in time preferences for the moment and assume that \( \beta^a = \beta^b = 0.95 \). Finally, we let \( \gamma^a = 4.8 \) and \( \gamma^b = 8.2 \).\(^7\)


\(^6\)This is obtained by solving \( \gamma^{hh} = \frac{\gamma^a + \gamma^b}{2} \). Alternately, \( \delta \) can be chosen so that the share of the household portfolio allocated to the risky asset lies exactly halfway between the most preferred allocations of spouse \( a \) and \( b \).

\(^7\)These are the values estimated for risk category I and III in the HRS; see Barsky, Juster, Kimball, and Shapiro (1997).
First, note that if the relative bargaining power of spouse \(a\) equalled 1, then the optimal portfolio allocation to the risky asset would be 48.4 percent. On the other hand, if the relative bargaining power of spouse \(b\) was 1, the optimal allocation would be 28.2 percent. The next section describes where the optimal household allocation lies between these two values depending on the wealth and the distribution of bargaining power within the household.

### 2.1.1 Wealth

To demonstrate the effect of wealth, assume that the relative bargaining power of both spouses is equal, that is, \(\lambda = 0.5\). Let \(w_0\), wealth in period 1, vary from \$1,000 to \$400,000. Figure 1 shows that the share of the household portfolio allocated to the risky asset increases with wealth. When household wealth is low, the allocation is closer to what the more risk averse spouse prefers and when household wealth is high, the allocation is closer to what the less risk averse spouse prefers.

### 2.1.2 Bargaining Power

To demonstrate the effect of bargaining power, we hold wealth constant and show how the portfolio allocation changes with relative bargaining power. Let \(w_0 = \$141,200\), which represents the mean financial assets of married couples in the 2000 HRS.\(^8\) Let \(\lambda\), which is spouse \(a\)’s relative bargaining power within the household, vary from 0 to 1. Figure 2 shows the result. The household

\(^8\)Assets in Individual Retirement Accounts (IRAs) are excluded here and in the analysis later because the HRS did not report what fraction of these were invested in risky assets.
portfolio allocation moves from spouse b’s to spouse a’s preferred allocation as spouse a’s bargaining power increases.

Finally, in Figure 3, both bargaining power and wealth are allowed to vary. Each line represents a different level of wealth. The figure shows that bargaining power matters most in the middle of the wealth distribution. For low levels of wealth, the portfolio allocation remains close to the more risk averse spouse’s preferred choice while for high levels of wealth it remains close to the less risk averse spouse’s preferred choice.

The theoretical model and simulations provide a number of empirically testable predictions. The next section describes the data used to test these predictions.
3 Data

The data comes from the 2000 wave of the HRS. The sample was restricted to married (or partnered) couples with both spouses in the data set, which yielded 6,279 married couples. Couples missing information about their years of education were dropped, after which 6,239 couples remained. A further 602 observations for whom financial wealth was zero were dropped. Those for whom risk aversion data was missing were dropped, after which 2,600 couples remained. Finally, household with negative wealth were dropped, yielding a final sample of 2,282 observations.

\[\text{We use an older wave of the survey to avoid the impact of recent recessions on household portfolios.}\]
The risk tolerance data comes from Kimball, Sahm, and Shapiro (2008), who constructed a cardinal measure of risk aversion using responses to a series of questions in the HRS about choosing between two jobs: one that paid the respondent their current income with certainty and the other that had a 50-50 chance of doubling their income or reducing it by a certain fraction.\textsuperscript{10}

Our variable of interest is the share of risky assets in household financial wealth. We define household financial wealth as the sum of the net value of assets in: 1) stocks, stock mutual funds, and investment trusts, 2) checking, savings, and money market accounts, 3) certificates of deposit (CDs), savings bonds, and Treasury bills, 4) bonds and bond funds and 5) other savings.

\textsuperscript{10}The data was downloaded from http://www-personal.umich.edu/~shapiro/data/risk_preference/.
The first category, hereafter referred to as “stocks,” defines risky assets.

The key variables of interest for the sample are described in Table 1. Mean household financial wealth is $158,500. Nearly 48% of households hold some part of this wealth in stocks. The average share of stocks is nearly 29%. The data supports the premise of the paper that spouses differ in risk aversion; the table shows that this is true of nearly 80% of couples in the sample. We use years of education as our measure of bargaining power.\textsuperscript{11} The table shows that over 70% of spouses differ in educational attainment and that husbands are slightly more likely to be more educated than wives.

The demographic characteristics of individual spouses are described in Table 2. Husbands are on average nearly four years older than wives. While their average level of education is nearly equal, husbands have nearly 16 years more work experience than wives and earn $7,700 more ($18,600 compared to $10,900).

4 Empirical Evidence

We now use the data to test the predictions of the theoretical model. To do so, we run regressions to measure the impact of wealth, risk aversion and bargaining power on the share of risky assets in the household portfolio. Based on the theory, we hypothesize that the share of risky assets in the

\textsuperscript{11}Another option would have been to use relative income shares within the household. However, a significant fraction of our sample is retired and reports no income. Education in progress is not an issue for this sample, and because completed education is highly correlated with income, we choose to use years of education in this instance.
Table 1: Descriptive Statistics for Couples (2000HRS, N=2,282)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Percentage/Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial Profile</strong></td>
<td></td>
</tr>
<tr>
<td>Financial wealth ($1,000)</td>
<td>158.5</td>
</tr>
<tr>
<td>Household income ($1,000)</td>
<td>74.8</td>
</tr>
<tr>
<td>Own risky assets (%)</td>
<td>47.50</td>
</tr>
<tr>
<td>Value of risky assets</td>
<td>90.8</td>
</tr>
<tr>
<td>Share of risky assets (%)</td>
<td>27.8</td>
</tr>
<tr>
<td><strong>Risk aversion</strong></td>
<td></td>
</tr>
<tr>
<td>Ratio of wife’s to husband’s risk aversion</td>
<td>1.1</td>
</tr>
<tr>
<td>Husband is less risk averse (%)</td>
<td>40.3</td>
</tr>
<tr>
<td>Spouses are equally risk averse (%)</td>
<td>21.7</td>
</tr>
<tr>
<td>Wife is less risk averse (%)</td>
<td>37.9</td>
</tr>
<tr>
<td><strong>Bargaining power</strong></td>
<td></td>
</tr>
<tr>
<td>Ratio of husband’s to wife’s education</td>
<td>1.05</td>
</tr>
<tr>
<td>Husband has more education (%)</td>
<td>38.6</td>
</tr>
<tr>
<td>Spouses have equal education (%)</td>
<td>30.2</td>
</tr>
<tr>
<td>Wife has more education (%)</td>
<td>31.2</td>
</tr>
</tbody>
</table>

Household portfolio will increase with household wealth. Further, we define a measure of household risk aversion and test whether the share of risky assets in the household portfolio increases as household risk aversion decreases. Finally, we examine the effect of the relative bargaining power of the two spouses on the household portfolio allocation.

We create an empirical parallel to the household risk aversion variable by defining

\[
\gamma^{hh} = \lambda \gamma^a + (1 - \lambda) \gamma^b
\]  

(5)
Table 2: Descriptive Statistics for Spouses (2000 HRS, N=2,282)

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Husband</th>
<th>Wife</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>64.6</td>
<td>60.9</td>
</tr>
<tr>
<td>Education</td>
<td>12.94</td>
<td>12.73</td>
</tr>
<tr>
<td>Years worked</td>
<td>42.8</td>
<td>27.1</td>
</tr>
<tr>
<td>Earning ($1,000)</td>
<td>18.6</td>
<td>10.9</td>
</tr>
<tr>
<td>White (%)</td>
<td>89.8</td>
<td>90.3</td>
</tr>
<tr>
<td>Retired (%)</td>
<td>61.2</td>
<td>41.4</td>
</tr>
</tbody>
</table>

where

$$\lambda = \frac{educ^a}{educ^a + educ^b}$$

Here $educ^a$ and $educ^b$ are the total years of education of each spouse. Note that consistent with the theory, as the bargaining power of spouse $a$ increases, the household risk aversion as defined in (5) approaches the risk aversion of spouse $a$.

Our main empirical model is

$$\rho_i = \beta_0 + \beta_1 \gamma^{hh} + \beta_2 \ln w_i + \epsilon_i. \quad (6)$$

We designate the husband to be spouse $a$ and the wife to be spouse $b$.

We compare the results of this model with two other models in which the risk aversion of only one spouse is included in the independent variables:

$$\rho_i = \beta_0 + \beta_1 \gamma^a_i + \beta_2 \ln w_i + \epsilon_i$$

$$\rho_i = \beta_0 + \beta_1 \gamma^b_i + \beta_2 \ln w_i + \epsilon_i$$
Finally, to account for the possibility that education affects household portfolio choice independently of its effect on bargaining power, we include controls for both spouses’ years of education:

$$\rho_i = \beta_0 + \beta_1 \gamma_{hi} + \beta_2 \ln w_i + \beta_3 educ_i^a + \beta_4 educ_i^b + \epsilon_i. \quad (7)$$

The results of the regressions are reported in Table 3.

The first point to note is that, consistent with the theory and with previous empirical literature, the share of risky assets in the household portfolio increases with wealth. A ten percent increase in household wealth is associ-
ated with an approximately 0.8 percentage point increase in the share of risky assets. The results also confirm the theoretical prediction that the share of risky assets in the portfolio increases as household risk aversion decreases. Moreover, we show that household risk aversion significantly affects portfolio choices, whereas the results are less conclusive when risk aversion of the spouses is considered separately. Note that the model with household risk aversion fits the data better than the models that only include risk aversion of one of the spouses. Finally, the results suggest that education levels matter independently of their effect on bargaining power. We find evidence that a higher level of education in the household is associated with a higher share of risky assets in household portfolio.

5 Conclusion

A key empirical fact about household portfolios is that wealthier households hold a greater share of their portfolios in risky assets. This paper offers a simple theoretical model that is consistent with this observation. The result is obtained by explicitly modeling the household as two distinct individuals. While each spouse has standard CRRA preferences, the model allows for the spouses to differ in their risk aversion. Modeling the household in this way also enables us to examine the role of bargaining power within the household. We find that as the bargaining power of the less risk averse spouse increases, the share of risky assets in the household portfolio also increases. We use
data from the 2000 HRS to confirm both results.

For the purposes of this paper, we assumed a cooperative bargaining model in which the Pareto optimal solution for the couple is always attained. It might be interesting to consider a non-cooperative framework. It is also important to note that we only used data from one wave of the HRS. However, the HRS is a longitudinal data set that is rich in financial and demographic information for both husbands and wives. It would be of interest to look at the impact that a change in bargaining power (say as a result of retirement) has on household portfolio allocation. Finally, it is important to acknowledge that the HRS collects data from a representative sample of older Americans. Thus, it may not be possible to generalize our findings to the U.S. population as a whole. While the HRS has detailed information on household decision-making and household portfolio composition, it may be worth analyzing other data that is representative of the U.S. population as a whole.
References


A Proofs

A.1 Proof of Lemma 1

Proof. a) Assume without loss of generality that spouse $a$ is less risk averse than spouse $b$, i.e., $\gamma^a < \gamma^b$. Then

$$\frac{\partial \gamma^{hh}}{\partial \lambda} = \frac{(1 - \lambda)\delta w^{-\gamma^a - \gamma^b} (\gamma^a - \gamma^b)}{(\lambda \delta w^{-\gamma^a} + (1 - \lambda) w^{-\gamma^b})^2} < 0.$$ 

Thus an increase in $\lambda$, the relative bargaining power of the less risk averse spouse, leads to a decrease in household risk aversion. A decrease in $\lambda$, i.e., an increase in $(1 - \lambda)$, the relative bargaining power of the more risk averse spouse, leads to an increase in household risk aversion.

b) The derivative of household risk aversion, $\gamma^{hh}$, with respect to wealth, $w$, is given by

$$\frac{\partial \gamma^{hh}}{\partial w} = -\lambda(1 - \lambda)\delta (\gamma^a - \gamma^b)^2 w^{-(\gamma^a + \gamma^b + 1)} \left(\lambda \delta w^{-\gamma^a} + (1 - \lambda) w^{-\gamma^b}\right)^2 < 0.$$ 

Thus household relative risk aversion decreases as wealth increases. \qed
A.2 Proof of Lemma 2

Proof. After algebraic manipulation, we can rewrite the first order conditions for the solution to the household’s problem as

\[ ZX(a) + X(b) = 0, \]

\[ ZY(a) + Y(b) = 0. \]

where

\[ Z = \frac{\beta^a \gamma^{hh} - \gamma^b}{\beta^b \gamma^a - \gamma^{hh}}, \]

\[ X(i) = \left\{ -\frac{(w_0 - x^*_0)^{-\gamma_i}}{\beta^i x^*_0 - \gamma_i} + E \left[ (1 + \tilde{r}_1^s)\rho^* + (1 + r^m)(1 - \rho^*) \right]^{1 - \gamma_i} \right\}, i = a, b, \]

\[ Y(i) = E \left\{ [(1 + \tilde{r}_1^s)\rho^* + (1 + r^m)(1 - \rho^*)]^{-\gamma_i} (\tilde{r}_s - r^m) \right\}, i = a, b. \]

Spouse a’s most preferred solution is chosen when \( \lambda = 1 \). If \( \lambda = 1 \), the first order conditions for the solution to the household’s problem are \( X(a) = 0 \) and \( Y(a) = 0 \). Spouse b’s most preferred solution is chosen when \( \lambda = 0 \). If \( \lambda = 0 \), the first order conditions for the solution to the household’s problem are \( X(b) = 0 \) and \( Y(b) = 0 \).

Suppose \( \gamma^a < \gamma^b \). Then

\[ \frac{\partial Z}{\partial \gamma^{hh}} = \frac{\gamma^a - \gamma^b}{(\gamma^a - \gamma^{hh})^2} < 0. \]
Thus as $\gamma^{hh}$ increases, $Z$ decreases, and the solution to the household problem gets closer to the preferred solution of spouse $b$, the more risk averse spouse.

Now suppose $\gamma^a > \gamma^b$. Then

$$\frac{\partial Z}{\partial \gamma^{hh}} = \frac{\gamma^b - \gamma^a}{(\gamma^a - \gamma^{hh})^2} > 0.$$  

Thus as $\gamma^{hh}$ increases, $Z$ decreases, and the solution to the household problem gets closer to the preferred solution of spouse $a$, the more risk averse spouse.

\[ \square \]

### A.3 Proof of Proposition 1

**Proof.** The proof follows directly from Lemma 1b and Lemma 2. \[ \square \]

### A.4 Proof of Proposition 2

**Proof.** The proof follows directly from Lemma 1a and Lemma 2. \[ \square \]