A Quantitative Model of International Lending of Last Resort*

Pedro Gete†

This Draft: August 2016

Abstract

I analyze banking crises in a quantitative double-decker model of financial frictions. Small open economies have deeper crises than closed or large economies because the risk-free rate is unaffected. An international lender of last resort (LOLR), even if it induces an increase in banks’ leverage, benefits these economies. For the levels of liquidity support documented by Laeven and Valencia (2013), pools of small economies are sustainable LOLRs only if they have many uncorrelated countries or large initial levels of reserves. A country with ample reserves like China can be a sustainable international LOLR. However, model simulations suggest that China may have overreached its capacity to be a LOLR.

Keywords: Banking Crises, China, Financial Frictions, Lender of Last Resort

JEL Classification: E4, E5, F3, G2

*I appreciate the comments of James Albrecht, Oliver Bernal, Eduardo Cavallo, Harris Dellas, Behzad Diba, Pablo D’Erasmo, Martin Evans, Andres Fernandez, Hiro Ito, Brian Madigan, Virgiliu Midrigan, Alberto Martin, Givi Melkadze, Diego Herrera, Paolo Porchia, Andrew Powell, Joan Prats, Vincenzo Quadrini, Cesar Tamayo and seminar participants at the Inter-American Development Bank (IDB) and Georgetown University. I thank the IDB for financial support. Chuqiao Bi provided excellent research support.

†Georgetown University, GCER and IE Business School. Email: pg252@georgetown.edu. Phone: (+1) 202-687-5582. Address: 37th and O Sts., NW. Washington DC, 20057, USA.
1 Introduction

Banking crises are recurrent events that disrupt interbank markets and can be mitigated if there is a lender of last resort (LOLR). Many countries cannot act as a LOLR because their financial sector borrows in foreign currency (usually in dollars).\textsuperscript{1} During the last financial crisis the U.S. Federal Reserve acted as the international lender of last resort. It provided over half a trillion dollars to 14 foreign central banks (the credit-swap lines were the largest of all Fed programs implemented during the crisis). Moreover, private foreign banks with U.S. banking licenses were the main borrowers of the other Fed lending program.\textsuperscript{2,3}

Cechetti (2014) argues that the Fed’s international lending of last resort did not target countries needing dollars. Instead, the Fed’s international lending targeted countries of interest to the U.S. Wikileaks revealed that multiple countries applied for U.S. support but were rejected (Prasad 2014). The U.S. Dodd-Frank Act deliberately restricts the ability of the Fed to provide lending of last resort in future crises (Fisher 2016).

In this paper I analyze a quantitative model of lending of last resort. I study what types of international LOLR arrangements are sustainable. I also use the model to study the ability of China to be the international LOLR. Since 2008 China has entered into more than 50 bilateral agreements that can be used to obtain lending of last resort. At the end of 2015, China accounted for 85 percent of all global swap lines (Lagarde 2016). Reportedly, Argentina, Pakistan and Venezuela have already used China’s facilities to borrow renminbis and convert them into dollars. Aizenman et al. (2015) describe the strategy as a bundling of finance dealing (lending, swap-lines and trade credit) in tandem with outward FDI to promote a new type of Chinese-outward mercantilism. China seems to attach minimal conditions to its loans. The absence of conditionality may make China a popular LOLR because countries have been reluctant to use lending of last resort programs from the IMF to avoid the stigma and conditionality attached to them (see Allen and Moessner 2015, Cecchetti 2014 or Landau 2014).

The model has costly-state verification frictions between entrepreneurs and banks, and between banks and their depositors and lenders. Entrepreneurs’ borrowing costs are related to banks’ borrowing costs. This relationship is appealing to the study of last resort policies that subsidize banks’ borrowings to benefit the whole economy.

\textsuperscript{1}Areas as Latin America are de facto mostly dollarized (Corbo 2001 and Salvatore 2001). Moreover, dollar-denominated debt keeps increasing rapidly. In 2014, non-U.S. debt issuers had $6.04 trillion in outstanding bonds, up nearly fourfold since 2008 (Talley and Trivedi 2014).

\textsuperscript{2}Private foreign banks with U.S. banking licenses took over 70% of the Fed’s discount window loans and about 65% of the loans from the Term Auction Facility and the Commercial Paper Funding Facility (Broz 2013).

\textsuperscript{3}The Federal Reserve implemented currency swap agreements with Australia, Brazil, Canada, Denmark, England, Euro Area, Japan, Korea, Mexico, New Zealand, Norway, Singapore, Sweden, and Switzerland.
A banking crisis starts with a transfer from the equity of the domestic banks to the domestic households. For example, when households default on mortgage debts, banks’ equity falls. This pure financial shock does not imply per se the depletion of real resources like in a real shock. However, it does trigger the following mechanism that through financial frictions leads to lower output: when banks’ equity decreases, banks’ borrowing costs increase because their risk of default is higher and lenders account for this in their rates. Banks pass their higher borrowing costs to the entrepreneurs. In response, entrepreneurs reduce their borrowings and their purchases of capital causing lower capital prices, less investment, lower labor supply and a decrease in output. This reaction reflects the negative financial accelerator of Bernanke et al. (1999).

Small open economies have deeper banking crises than closed or large economies because their inability to affect the risk-free rate exacerbates the increase in interest rate spreads that triggers the negative financial accelerator. This insight connects with Hall (2011), which states that, in closed economies, frictions that prevent the adjustment of the interest rate cause deeper recessions.

Lending of last resort is a policy that allows banks to borrow from the LOLR at a cost below the market rate during a banking crisis. In case of a bank’s default there is a pecking order with banks’ private lenders in priority position, then the LOLR and finally the banks’ shareholders. Lower borrowing costs for domestic banks result in less significant increases in the borrowing costs for entrepreneurs and less contraction in their purchases of capital. Consumption, investment, employment and output fall by less when the economy has access to lending of last resort. However, lending of last resort generates moral hazard. The lenders of the banks understand that the LOLR will mitigate both the banks’ default and the fall in the price of capital (which serves as collateral in the loans), and lower the cost of banks’ borrowings encouraging banks to leverage more. For banking crises and liquidity support calibrated following Laeven and Valencia (2013), the benefits from lending of last resort are larger than the costs associated with the moral hazard.

Pools of small economies do not look like feasible arrangements for lending of last resort, because, for the levels of liquidity support documented by Laeven and Valencia (2013), they need at least 30 uncorrelated countries, or large initial levels of reserves, to have small probabilities of failure. A country with ample reserves like China can be a sustainable international LOLR. I input into the model the ratio of China’s foreign reserves to the total GDP of the countries that have signed lending agreements with China during the last nine years. Model simulations show that, as long as China receives some compensation from the insured countries, it is able to provide the levels of liquidity support documented by Laeven and Valencia (2013) with low
probabilities of failure.

Model simulations suggest that China may have overreached its capacity to be a LOLR. In 2009, China’s reserves were 50% of the total GDP of countries with lending agreements. By 2015, the rapid expansion in the number of agreements had brought the ratio to 15%. Moreover, the GDP correlations between the countries and China are high, hindering China’s ability to provide insurance.

Obstfeld et al. (2009) expressed concern that the scale of reserves needed to backstop financial crises in emerging markets surpassed the resources of the multilateral organizations and all but the largest reserve holders in the world. My results confirm that the largest reserve holders can play the role of LOLR. Thus, the recent Chinese initiatives seem to benefit many countries since self-insurance against domestic financial instability is one of the main drivers of reserve accumulation (Aizenman and Lee 2007, Obstfeld et al. 2010), but this accumulation comes at substantial costs (see for example Reinhart et al. 2016 or Rodrik 2006). Aizenman et al. (2011 and 2015) confirm that long-lasting LOLR agreements lead to lower reserve accumulation.

As discussed by Gertler and Kiyotaki (2015), there are two different approaches to capturing the interaction between banking distress and the real economy. One approach pioneered by Diamond and Dybvig (1983) focuses on liquidity mismatches and bank runs. This has been the most popular approach to studying international LOLR in qualitative models like for example Goodhart and Huang (2000). The second, more recent, approach emphasizes how the depletion of bank capital hinders a bank’s ability to intermediate funds and causes an economic downturn. This is the approach that this paper follows. So far, this approach has been used in studies of credit policy in closed economies, such as, Del Negro et al. (2016), Gertler and Karadi (2011) and Gertler et al. (2012). In these last two papers the shock that triggers the crisis is a real shock, an exogenous reduction in the quality of the capital stock. In Del Negro et al. (2016) the shock is a reduction in the resaleability of private paper.

To my knowledge only Akinci and Queralto (2014) and Kollmann et al. (2013) have studied open economy models with banks and credit policy. Akinci and Queralto (2014) focus on a model with pecuniary externalities and a subsidy on equity issuance which the government finances by levying a tax on bank assets. Kollmann et al. (2013) study an open economy New Keynesian model with government support for banks and find that this is an effective tool for stabilizing output.

The model that I analyze is related to Fernandez and Gulan (2015). Fernandez and Gulan

---

4See Corsetti et al. (2006) for a global games analysis of the trade-off between liquidity provision and debtor moral hazard in international crises.
(2015) show that a small open economy model with a costly-state verification friction between entrepreneurs and lenders explains the business cycle dynamics of non-financial leverage and interest rates. Here I expand the setup to a double costly-state verification friction to capture banks’ leverage and analyze policies that reduce banks’ borrowing costs. There are few quantitative double-decker model of financial frictions. Hirakata et al. (2013) and Elenev et al. (2016) analyze closed economy models with similar mechanisms to introduce banks’ default.

The paper proceeds as follows. Section 2 describes the model. Section 3 analyzes banking crises and lending of last resort policies. Section 4 studies the sustainability of an international LOLR and the case of China. Section 5 concludes. The appendices contain the optimality conditions of the model, the numerical algorithm and the data sources.

2 Model

First, I present the benchmark country. It consists of households, firms, entrepreneurs and banks. Then, I discuss lending of last-resort arrangements in a multi-country setting. The model is real with consumption serving as numeraire. Only consumption goods are tradable.

2.1 Households

There is a continuum of homogeneous households who maximize expected utility over consumption \( C_t \) and hours worked \( H_t^H \),

\[
\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(C_t, H_t^H) .
\] (1)

Households own and build the physical stock of capital that accumulates according to

\[
K_t = (1 - \delta)K_{t-1} + I_{n,t},
\] (2)

where \( I_{n,t} \) is investment net of adjustment costs,

\[
I_{n,t} = \left( 1 - \frac{\phi}{2} \left( \frac{I_{g,t}}{I_{g,t-1}} - 1 \right)^2 \right) I_{g,t},
\] (3)

and \( I_{g,t} \) is investment before the investment adjustment costs. The parameter \( \phi \) controls these
costs that ensure that the price of capital \( Q_t \) differs from one outside the steady state. Fluctuations in investment change the price of capital and this change leads to balance sheet effects that generate a financial accelerator mechanism.

Households sell the new capital \( K_t \) while they repurchase the capital sold last period that remains, \((1-\delta)K_{t-1}\). Households can borrow (or save) \( B_{H,t} \) at the gross risk-free rate \( R_w \). There is a convex cost of borrowing \((\psi > 0)\) to ensure a well-defined steady state. The households’ budget constraint is:

\[
C_t + I_{g,t} + R_w B_{H,t-1} = W_t H_t^H + B_{H,t} - \frac{\psi}{2} B_{H,t}^2 + Q_t I_{n,t} + \Pi_{E,t} + \Pi_{B,t}. \tag{4}
\]

\( \Pi_{E,t} \) and \( \Pi_{B,t} \) are dividends paid by entrepreneurs and banks. The representative household chooses \( C_t, H_t^H, I_{g,t} \) and \( B_{H,t} \) to maximize (1) subject to (2), (3) and (4).

\[2.2 \text{ Firms} \]

The representative firm hires labor \( H_t \) at wage \( W_t \), and it rents capital \( K_{t-1} \) at rate \( r_t \). Firms’ profits are:

\[
\Pi_t = K_{t-1}^{1-\alpha} H_t^\alpha - W_t H_t - r_t K_{t-1}. \tag{5}
\]

In equilibrium, the firm’s labor demand equals the labor supply from the households \( (H_t^H) \), from the entrepreneurs \( (H^E) \), and from the banks \( (H^B) \),

\[
H_t = H_t^H + H^E + H^B. \tag{6}
\]

Perfect substitution across the three sources of labor implies a common wage. The labor hours of entrepreneurs and banks are constant over time. It is a mechanism to ensure new inflows into their net wealth and avoid that entrepreneurs and banks can end with zero equity. The results are the same if alternatively, I assume transfers from the households to entrepreneurs and banks’ equity.

\[2.3 \text{ Entrepreneurs} \]

There is a continuum with a mass of one of entrepreneurs whose aggregate equity is \( N_t^E \) and whose aggregate borrowings from the domestic banks is \( B_t^E \). Every period entrepreneurs
invest their equity and borrowings in purchasing capital:

\[ Q_tK_t = N_t^E + B_t^E. \] (7)

Entrepreneurs’ capital serves as collateral.

Next period, entrepreneurs will rent at price \( r_{t+1} \) the capital purchased at \( t \), and will sell the undepreciated capital. The rate of return per unit of capital will be

\[ R_{t+1}^E = \frac{r_{t+1} + Q_{t+1}(1 - \delta)}{Q_t}. \] (8)

After each entrepreneur borrows and buys capital she receives an idiosyncratic i.i.d. shock \( \omega_E \) such that \( K_t \) units of capital generate \( \omega_E K_t \) units of effective capital. Thus, an entrepreneur with shock \( \omega_E \) will have \( \omega_E R_{t+1}^E Q_t K_t \) as return on her effective capital. The \( \omega_E \) shocks generate profitable and unprofitable entrepreneurs. Banks can only observe the \( \omega_E \) shocks when the loan is due at \( t + 1 \) and after paying a proportional bankruptcy cost \( \mu_E \). The \( \omega_E \) shocks have a lognormal cumulative density function \( F_E(\omega_E) \) with parameters that satisfy \( \mathbb{E} [\omega_E] = 1 \).

Entrepreneurs’ borrowing rate between \( t \) and \( t + 1 \) is \( R_{E,t+1}^L \), which, like in Bernanke et al. (1999), is state-contingent such that entrepreneurs absorb the aggregate risk.\(^5\) Entrepreneurs with idiosyncratic realizations below the threshold \( \tilde{\omega}_{E,t+1} \) default and their assets are seized by the banks. The default threshold is the entrepreneur whose assets equal in value her debt:

\[ \tilde{\omega}_{E,t+1} R_{E,t+1}^L K_t = R_{E,t+1}^L B_t^E, \] (9)

where \( \tilde{\omega}_{E,t+1} \) and \( R_{E,t+1}^L \) are endogenously determined below.

At the end of each period, entrepreneurs pay the share \( (1 - \gamma_E) \) of aggregate profits as dividends to the households,\(^6\)

\[ \Pi_{E,t} = (1 - \gamma_E) \left[ \int_{\tilde{\omega}_{E,t}}^{\infty} [\omega_E R_t^E Q_{t-1} K_{t-1} - R_{E,t}^L B_{t-1}^E] dF_E(\omega_E) \right]. \] (10)

Entrepreneurs’ aggregate equity in period \( t \) is the sum of the retained earnings and the labor

---

\(^5\)This modelling device does not affect the results and thus I followed the workhorse model of financial frictions.

\(^6\)In the literature following Bernanke et al. (1999), \( (1 - \gamma_E) \) is usually referred to as a “death rate” of entrepreneurs. Fernandez and Gulan (2014) argue for an interpretation as dividend payments.
income:
\[ N_t^E = \gamma_E \left[ \int_{\omega_{E,t}}^{\infty} \omega_E R_t^E Q_{t-1} K_{t-1} - R_{t-1}^E B_t^E \right] dF_E(\omega_E) + W_t H^E. \] (11)

### 2.4 Contract between entrepreneurs and banks

Using (9) to replace the lending rate, the financial contract between entrepreneurs and banks solves for \( K_t \) and \( \hat{\omega}_{E,t+1} \) to maximize entrepreneurs’ expected profits:

\[
\max_{\{K_t, \hat{\omega}_{E,t+1}\}} \mathbb{E}_t \int_{\hat{\omega}_{E,t+1}}^{\infty} \left[ (\omega_E - \hat{\omega}_{E,t+1}) R_{t+1}^E Q_t K_t \right] dF_E(\omega_E) \quad (12)
\]

subject to the participation constraint of the banks:

\[
\int_{\hat{\omega}_{E,t+1}}^{\infty} \hat{\omega}_{E,t+1} R_{t+1}^E Q_t K_t dF_E(\omega_E) + (1 - \mu_E) \int_{0}^{\hat{\omega}_{E,t+1}} \omega_E R_{t+1}^E Q_t K_t dF_E(\omega_E) = R_t^B B_t^E. \quad (13)
\]

The first integral in (13) is the banks’ revenue from the entrepreneurs who repay their debts. The second integral is the value (net of bankruptcy costs) of the assets seized from the entrepreneurs that default. The right-hand-side of (13) specifies that banks participate as long as they cover the required rate of return \( R_t^B \) on the funds that they lend \( B_t^E \). In Bernanke et al. (1999), banks’ cost of funds is the risk-free rate since banks hold no capital and cannot default. Next, I specify how \( R_t^B \) is determined when I relax those two assumptions.

### 2.5 Banks

There is a continuum with mass one of domestic banks. They lend \( B_t^E \) financed with their equity \( N_t^B \), and with borrowings \( B_t^B \) from deposits and from international financial markets,\(^8\)

\[
B_t^E = N_t^B + B_t^B. \quad (14)
\]

The banks’ revenue from lending is \( R_t^B B_t^E \). I assume that \( R_t^B B_t^E \) is unevenly distributed across banks such that some banks cannot repay their borrowings. That is, banks are subject to idiosyncratic i.i.d. shocks \( \omega_B \) such that \( \omega_B R_t^B B_t^E \) is the effective return on assets for a bank

\(^7\)All entrepreneurs have the same leverage ratio because entrepreneurs are risk-neutral and their technology is constant returns-to-scale. Thus, it is equivalent to solve the problem of each entrepreneur and then to aggregate than to solve directly the aggregate problem as I do here.

\(^8\)In the model these two lenders to the banks are indistinguishable because both lenders take into account the risk of banks’ default and both receive the same return.
with shock $\omega_B$. These shocks capture that some banks hold high quality loans while others hold low quality loans. The $\omega_B$ shocks are lognormally distributed with cumulative density function $F_B(\omega_B)$ with mean one, $\mathbb{E}[\omega_B] = 1$. The $\omega_B$ shocks are not observable when the banks are borrowing.

Denoting the banks’ endogenous borrowing rate as $R^L_{B,t}$, the banks’ default threshold ($\tilde{\omega}_{B,t}$) is the bank whose assets equal in value its debt:

$$R^L_{B,t} B_t^B = \tilde{\omega}_{B,t} R^E_t B_t^E.$$  \hspace{1cm} (15)

Bank’s borrowing rate $R^L_{B,t}$ is determined by the participation constraint of the banks’ lenders:

$$\int_{\tilde{\omega}_{B,t}}^{\infty} R^L_{B,t} dF_B(\omega_B) + (1 - \mu_B) \int_0^{\tilde{\omega}_{B,t}} \omega_B R^E_t B_t^E dF_B(\omega_B) = R_w B_t^B.$$ \hspace{1cm} (16)

That is, the banks’ lenders are guaranteed a risk-free return $R_w$. These lenders invest $B_t^B$ in the continuum of banks ($\omega_B$ is not observable ex-ante). The first integral in (16) is the revenue from the banks repaying $R^L_{B,t}$, the second integral is the revenue (net of bankruptcy cost $\mu_B$) from those banks that default. Equation 16 implies that, as long as there is positive probability of the banks’ default, then banks will borrow at some positive spread relative to the risk-free rate ($R^L_{B,t} - R_w > 0$).

Moreover, the participation constraint for the banks’ shareholders requires that banks’ expected profits cover the opportunity cost of banks’ equity, that I assume is the risk-free rate:

$$\int_{\tilde{\omega}_{B,t}}^{\infty} [\omega_B R^E_t B_t^E - R^L_{B,t} B_t^B] dF_B(\omega_B) = R_w N_t^B.$$ \hspace{1cm} (17)

Equations 16 and 17 pin down the banks’ borrowing rates $R^L_{B,t}$, and banks’ required rate of return $R^E_t$. These equations connect the entrepreneurs and banks’ borrowing costs. When the banks’ borrowing costs are higher, their lending rates to the entrepreneurs are also higher to ensure that the banks’ lenders and equity holders get an expected return $R_w$.

To avoid that the banks accumulate enough equity such that they do not need to borrow, at the end of each period, banks pay a fraction $(1 - \gamma_{B,t})$ of their profits as dividends:

$$\Pi_{B,t} = (1 - \gamma_{B,t}) \left[ \int_{\tilde{\omega}_{B,t-1}}^{\infty} [\omega_B R^E_{t-1} B_{t-1}^E - R^L_{B,t-1} B_{t-1}^B] dF_B(\omega_B) \right].$$ \hspace{1cm} (18)
Banks’ aggregate equity $N_{B,t}$ is the sum of past retained profits and labor income:

$$N_{B,t} = \gamma_{B,t} \left[ \int_{\omega_{B,t-1}}^{\infty} \left[ \omega_B R_{t-1}^B B_{t-1}^E - R_{B,t-1} B_{t-1}^B \right] dF_B(\omega_B) \right] + W_t H^B. \quad (19)$$

### 2.6 Banking crises

To generate banking crises I assume that the banks’ dividend rate $\gamma_{B,t}$ is a stochastic variable. That is, there are exogenous transfers from banks’ equity to the households. For example, households default on mortgages not included in the model. These shocks are pure financial shocks which do not imply the depletion of real resources. They only reallocate wealth between banks and households. The transfer is lump-sum so it does not distort the households’ decisions. I assume that the shocks follow a stationary AR(1) process and, to ensure that $\gamma_{B,t} \in (0, 1)$, I assume

$$\gamma_{B,t} = 1 - \frac{1}{\exp (x_t)}, \quad (20)$$

$$x_t = \rho x_{t-1} + \exp (\varepsilon_t), \quad (21)$$

$$\varepsilon_t \overset{i.i.d.}{\sim} N(0, \sigma). \quad (22)$$

At steady state, when $\varepsilon_t = 0$, then $\gamma_B = 1 - \frac{1}{\exp (\frac{1}{\sigma})}$. I calibrate these processes to match the banking crises reported by Laeven and Valencia (2013).

### 2.7 Lending of last resort

In a financial crisis the banks can also borrow $B_t^G$ from a lender of last resort (LOLR) that can be the domestic government or an international lender as specified below. When there is lending of last resort, the domestic banks finance their loans to the entrepreneurs ($B_t^E$) with their own equity ($N_t^B$), with borrowings from the private sector ($B_t^B$) and from the LOLR ($B_t^G$). That is, (14) becomes

$$B_t^E = B_t^B + N_t^B + B_t^G. \quad (23)$$

There is a pecking order between the three sources of banks’ funds. Private lenders are the first to be paid. Banks with assets $\omega_{B,t} R_{t}^B B_t^E$ below the debt due to the private lenders ($R_{B,t}^L B_t^B$) default, and these lenders seize the assets. The threshold for banks’ default on private lenders is $\tilde{\omega}_{B,t}$ defined by

$$\tilde{\omega}_{B,t} R_{t}^B B_t^E = R_{B,t}^L B_t^B. \quad (24)$$
Remark that equation 24 functions as if there is no LOLR, and thus the participation constraint 16 still holds for the private lenders of the banks.

LOLR’s debt is the second in line to be repaid. That is, among those banks able to repay their private lenders, those banks unable to repay $R^L_t B_t^C$ to the LOLR will default, and the LOLR seizes their assets. The banks’ default threshold on LOLR is $\bar{\omega}_{B,t}$ defined as

$$\bar{\omega}_{B,t} R^B_t B^E_t - R^L_t B^B_t = R^L_t B^G_t. \quad (25)$$

The pecking-order implies that more banks default on the LOLR than on the private lenders ($\hat{\omega}_{B,t} > \hat{\omega}_{B,t}$) since the two default thresholds are connected:

$$\bar{\omega}_{B,t} = \hat{\omega}_{B,t} \frac{R^L_t B^G_t}{R^B_t B^E_t}. \quad (26)$$

The banks’ shareholders receive the earnings from those banks able to pay both the private lenders and the LOLR. Thus, when there is lending of last resort the participation constraint (17) for banks’ shareholders becomes

$$\int_{\hat{\omega}_{B,t}}^{\bar{\omega}_{B,t}} \left( \omega_B R^B_t B^E_t - (R^L_t B^B_t + R^L_t B^G_t) \right) dF_B(\omega_B) = R_w N^B_t. \quad (27)$$

The law of motion for banks’ equity (19) becomes:

$$N^B_t = \gamma_{B,t} \left[ \int_{\hat{\omega}_{B,t-1}}^{\bar{\omega}_{B,t}} \left( \omega_B R^B_{t-1} B^E_{t-1} - (R^L_{B,t-1} B^B_{t-1} + R^L_{B,t-1} B^G_{t-1}) \right) dF_B(\omega_B) \right] + W_t H^B. \quad (28)$$

Like Gertler and Karadi (2011), I model LOLR as a policy rule

$$B^G_t = \begin{cases} \chi \left( \frac{R^L_{B,t} - R^L_{B,ss}}{R^L_{B,ss}} \right) B^B_t & \text{if } R^L_{B,t} > R^L_{B,ss} \\ 0 & \text{if } R^L_{B,t} \leq R^L_{B,ss} \end{cases}. \quad (29)$$

That is, only when the banks’ costs of borrowing from the private lenders ($R^L_{B,t}$) are higher than their steady state level ($R^L_{B,ss}$) then the LOLR lends ($B^G_t > 0$). The parameter $\chi > 0$ controls the size of last resort lending. I calibrate $\chi$ to match the liquidity support documented by Laeven and Valencia (2013).

The LOLR incurs losses ($\Omega_t < 0$) because it is lending below the market rate ($R^L_G < R^L_{B,t}$).
These losses $\Omega_t$ are the difference between the LOLR’s cost of funds, that I assume is the risk-free rate $R_w B_t^G$, and the revenue obtained from the domestic banks:

$$\Omega_t = \int_{\omega_{B,t}}^{\infty} R^L_G B^G_t dF_B(\omega_B) + (1 - \mu_B) \int_{\omega_{B,t}}^{\Omega_{B,t}} \left( \omega_B R^B_t B^E_t - R^L_{B,t} B^G_t \right) dF_B(\omega_B) - R_w B_t^G. \quad (30)$$

The first integral is the revenue from the banks repaying the LOLR, the second integral captures the assets of those banks that can pay the private lenders but default on the LOLR.

### 2.8 Single-country LOLR

I refer to "single-country LOLR" when the lender of last resort in country $i$ is financed with taxes from the households of country $i$. For example, during the 2008 financial crisis, households in countries like Ireland, Portugal or Spain paid higher taxes to support their domestic financial systems until the EU allowed the European Stability Mechanism and the ECB to exert as LOLR (Santos 2014, Zeissler et al. 2015). Formally, in a banking crisis the households’ budget constraint (4) adds the cost of the lending of last resort ($-\Omega_t > 0$) to become:

$$C_t + I_{g,t} + R_w B_{H,t-1} - \Omega_t = W_t H_t^H + B_{H,t} - \frac{\psi}{2} B_{H,t}^2 + Q_t I_{n,t} + \Pi_{E,t} + \Pi_{B,t}. \quad (31)$$

### 2.9 International LOLR

I refer to "international LOLR" when the lender of last resort is a pool of countries that starts with some endowment of reserves $M_0$ and charges a participation premium ($p$) every period to each country in the pool. Assuming that past reserves return the risk-free rate $R_w$, and that there are $N$ countries paying the participation premium, then the reserves of the international LOLR evolve as

$$M_t = R_w M_{t-1} + Np + \sum_{n=1}^{N} \Omega_{nt}. \quad (32)$$

That is, the reserves of the pool are the sum of the return on the past reserves, plus the insurance premiums paid by the countries in the pool ($Np$), minus the sum of the losses incurred with the countries in the pool ($\Omega_{nt} \leq 0$ are the losses incurred in country $n$). These losses are zero

---

9We can also interpret the LOLR as credit guarantees. That is, the LOLR guarantees lenders that they will receive a return $R_w$ on their funds even if they are lending at a rate $R^L_G < R^L_{B,t}$. The LOLR pays for the losses associated with the guarantee.
for country $n$ if in period $t$ the country $n$ is not in a banking crisis. The international LOLR fails if $M_t < 0$. That is, the LOLR lacks the resources to fulfill its role as LOLR.

To evaluate the sustainability of LOLR arrangements, I use as participation premium the maximum amount that a country would be willing to pay for access to the international LOLR. This premium is the rate at which the households of the country obtain the same expected utility between the autarky single-country LOLR discussed in Section 2.8 and the international LOLR. That is,

$$
\mathbb{E}[u(C_{p,t}, H_{p,t}^H)] = \mathbb{E}[u(C_{T,t}, H_{T,t}^H)],
$$

(33)

where $C_{p,t}$ and $H_{p,t}^H$ are consumption and labor supply when the country pays $p$ to belong to the international LOLR, while $C_{T,t}$ and $H_{T,t}^H$ are consumption and labor supply when the country finances the LOLR with taxes. The premium has to be paid every period while the taxes only need to be paid when there is a crisis.

### 3 Banking crises and lending of last resort

In this section I calibrate the model and compare a banking crisis in a small open economy and in a closed economy. Then, I study lending of last resort in a small open economy. Appendix B discusses in detail the numerical algorithm.

#### 3.1 Calibration

One period in the model is one quarter. For the utility function, I use GHH preferences,

$$
U(C_t, H_t^H) = \frac{\left[C_t - \theta^{-1} (H_t^H)^{\theta}\right]^{1-\gamma} - 1}{1 - \gamma}.
$$

(34)

This utility function is popular in quantitative versions of small open economies because it ensures that labor supply does not depend on the level of consumption.

I divide the parameters into two groups. Some parameters are chosen exogenously following standard values in the literature. Some other parameters are calibrated such that the model matches some empirical targets. Table 1 contains the exogenously assigned parameters: (i) a Frisch wage elasticity of labor supply of $\frac{1}{\theta-1} = 1.67$; (ii) a risk aversion parameter $\gamma = 2$; (iii) the share of labor in output $\alpha = 0.6$; (iv) the depreciation rate $\delta = 0.025$; (v) a discount factor $\beta = 0.99$ that generates a 1% quarterly interest rate in steady state; (vi) the default cost of
entrepreneurs $\mu_E = 0.12$ is the same value used by Bernanke et al. (1999). It is consistent with the range of costs of closing a business reported in the World Bank Doing Business database (from 6.46% for small open developed countries to 16.08% for developing countries); (vii) the bond adjustment cost in households’ portfolio is set to a small number, $\psi = \frac{7}{105}$. This adjustment cost is a technical device to have a well defined steady-state and does not affect the quantitative results.

Insert Table 1 about here

Table 2 has the endogenous parameters that generate the following annualized targets reported in Table 3: (i) A spread between banks’ borrowing costs and the international risk-free rate of 1.23%. In the data, the spread between interbank loans and U.S. government debt (the TED spread) is usually between 1% and 3%. The average JP Morgan emerging market spread is between 2% and 3%; (ii) Default rate of entrepreneurs of 5.14%, which is close to the 5.1% estimated by Fernandez and Gulan (2015) and in the range of the bankruptcy rates estimated by Claessens and Klapper (2005); (iii) Default rate of domestic banks of 3.25%, which is consistent with the 2% - 6% reported by IMF (2007); (iv) Equity share for entrepreneurs of 55.59%, which is within the range of 50% to 75% estimated by Ağca et al. (2013), and it is also consistent with the equity share of 58.5% for non-financial firms estimated by Fernandez and Gulan (2015) for 12 emerging economies; (v) Equity share for banks is 11.47%, which is within the range of 7% - 13% from Costa et al. (2014); (vi) A dividend-to-equity ratio of entrepreneurs of 3.59%, which is within the range of the average dividend-to-equity ratios estimated by Fernandez and Gulan (2015) for emerging markets; (vii) and (viii) A ratio of return on capital to risk-free rate of 1.04 and a ratio of consumption-to-output of 0.73, which are close to the values estimated by Fernandez and Gulan (2015) for these ratios (1.03 and 0.746, respectively); (ix) The median duration of a crisis is 4 years, like in Laeven and Valencia (2013);10 (x) The median output losses (conditional on being in a crisis) are 7% per year. In Laeven and Valencia (2013) the median output losses in 3 years is 23.2% of pre-crisis GDP, which is equivalent to 7.73% per year. (xi) The median of liquidity support (as % of banks’ liabilities) in Laeven and Valencia (2013) is 9.6% while in the model is 9.57%.

Insert Tables 2 and 3 about here

10I define a crisis when annual output is more than 10% below its steady state level in a given year or in the sum of two or three years of downward dynamics. Three years is the time range used by Laeven and Valencia (2013) to compute output losses during a crisis.
3.2 Banking crises in a small open economy

Figure 1 compares a banking crisis in a small open economy (SOE) and in a closed economy (GE). In the closed economy the risk-free rate is endogenous (it is the rate that equates households’ deposits with banks’ borrowings) and can change with the domestic shock \( R_{wt} \). However, by definition, in the small economy the risk-free rate is exogenous and constant \( R_w \). The exogenous shock is the same in both economies. It is a redistribution of wealth from the banks to the households (a decrease in \( B_{t}^{;t} \) to capture, for example, households defaulting on mortgages not included in the model). It is a pure financial shock that does not reduce output per se but that will trigger a mechanism leading to lower output. Figure 1 illustrates the mechanism: lower banks’ equity increases the risk of banks’ default and banks’ lenders price it with higher spreads. This increase in spreads leads to higher borrowing costs for banks and entrepreneurs and both reduce their borrowings. Capital purchases and capital prices decrease. This decrease leads to less investment, lower output, a fall in the stock of capital, less employment and even lower output, like in the classical negative financial accelerator of Bernanke et al. (1999).

Figure 1 highlights that the banking crisis is deeper in the small open economy than in the closed economy. This result is due to the risk-free rate not reacting to the negative shock. In the closed economy, the reduction in banks’ equity that triggers the banking crisis leads to lower risk-free rates because the crisis encourages higher households’ savings. The reduction in risk-free rates partially compensates for the increase in banks’ borrowing spreads. Thus, in the closed economy banks’ lending rates increase by less, banks and entrepreneurs’ have more access to credit, the negative financial accelerator is mitigated and the fall in output is smaller.

3.3 Lending of last resort

When there is lending of last resort during the banking crisis, banks are able to borrow from the LOLR at cheaper rates than in the markets. Moreover, in case of default there is a pecking order and the LOLR is paid after the private lenders of the banks. Therefore, with lending of last resort, the private lenders of the banks face a smaller risk of banks’ default and increase by less their lending spreads. As a consequence, lending of last resort mitigates the increase in banks and entrepreneurs’ borrowing costs and the drop in their borrowings. Investment, the price of capital, employment, wages, consumption and output fall by less when there is a LOLR. Figure 2 plots these dynamics. It plots the same shock of Figure 1 but now
comparing a small economy with lending of last resort and another without it.

Insert Figure 2 about here

Figure 3 shows that the existence of a LOLR creates moral hazard. The figure plots the average banks’ leverage in the stationary distribution of the model for different values of the parameter $\chi$ that controls the size of the lending of last resort in equation (29).\(^{11}\) The stronger the support that the LOLR commits (a larger parameter $\chi$) the higher is the leverage of the banks. For example, with no LOLR ($\chi = 0$) the ratio of banks’ assets-to-equity is around 8.7, it increases to 10 for the levels of lending of last resort calibrated in Section 3.1. This moral hazard happens because the private lenders of the banks know that in a crisis the LOLR will reduce banks’ default probability and the value of the collateral will fall by less. Thus, these lenders are more willing to finance the banks and therefore the banks attain higher leverage.

Insert Figure 3 about here

Table 4 compares the average of the stationary distribution with and without single-country LOLR for the benchmark calibration. By single-country LOLR I mean that the lender of last resort is financed with taxes from the households as discussed in Section 2.8. Table 4 shows the tradeoff illustrated in Figures 2 and 3: conditional on a crisis, lending of last resort mitigates the negative financial accelerator. However, it induces larger crises because banks have higher leverage when there are lending of last resort policies. Table 4 reports that the economy with lending of last resort has on average higher output, employment, consumption and households’ utility than the economy without it. However, banking crisis happen more often (that is, banks spreads are more often above their steady state, $R_{B,t}^L > R_{B,ss}^L$), and last longer, in the economy with lending of last resort because for the same negative shock banks’ spreads increase more when banks’ leverage is higher.

Insert Table 4 about here

\(^{11}\)To obtain the stationary distribution I simulate the model many times following the algorithm discussed in Section B.
4 Sustainability of an international LOLR

This section analyzes the sustainability of an international lender of last resort that starts with some endowment of reserves contributed by the participating countries, and charges a participation premium every period to each country in the pool. I compare different cross-country correlations of the banking crises, different number of participating countries and different initial levels of reserves. Then I ask whether a country with ample reserves like China can be a sustainable international LOLR.

4.1 Probability of failure of an international LOLR

Figures 4 and 5 analyze the probability of failure of the international LOLR for different cases. In all of them the analysis is based on the benchmark calibration and an horizon of 100 years. The international LOLR fails when $M_t < 0$ in equation 32, in that case the disbursements due to the countries in crisis are larger than the sum of the inflows from new participation premiums and the existing stock of reserves.

Figure 4 focuses on the case when the initial level of reserves is zero. That is, countries do not make any contribution of reserves to join the LOLR pool. However, they pay the participation premiums defined in (33). Figure 4 has two main results: 1) Without initial reserves, for the levels of liquidity support documented by Laeven and Valencia (2013), pools of small economies are sustainable LOLR only if they have many uncorrelated countries. For example, we need at least 30 uncorrelated countries for the probability to be below 5%. 2) If the shocks are correlated then adding new countries does not help to reduce the probability of failure. The Law of Large Numbers fails to bring benefits from pooling risks for highly correlated shocks. In fact, Figure 4 shows that the higher the correlations, the more likely is the pool to fail.

Although the exercises are different, it is interesting to compare the results of Figure 4 with those from Callen et al. (2015). Callen et al. (2015) find that carefully chosen pools of as few as seven countries in distant regions with negatively correlated outputs can provide nearly perfect risk sharing (consumption growth in each country equals poolwide output growth). As the number of countries in the pool increases, hedging opportunities diminish since countries tend to have positively correlated shocks. Figure 4 shows that, even with uncorrelated shocks, the number of countries cannot be below 10 for the LOLR to be sustainable. Adequate hedging
through an international LOLR of the risks associated with banking crises requires at least 30 countries.

Figure 5 plots the case when countries do make contributions of reserves to join the LOLR pool and in addition pay the participation premiums defined in (33). There are several results to highlight: 1) A country alone needs to have a large ratio of reserves-to-GDP (above 20%) to be completely sure that it will be able to be LOLR to its financial sector in a crisis. Bussière et al. (2016) report that the average level of reserves-to-GDP in non-advanced countries was below 20 until 2007, since then many countries boosted their reserves and the average level is closer to 25. 2) If the cross-country shocks are uncorrelated (Panel A) then pooling with an international LOLR allows to dramatically reduce the need to contribute reserves. For example, an international LOLR formed by more than 15 uncorrelated countries, each contributing reserves about 10% of GDP and paying the participation premiums, basically eliminates the risk of default. 3) For highly correlated cross-country shocks (Panel B) there are no gains from pooling and the levels of initial reserves need to be as high as when the country self-insures. These results suggest that LOLR through regional agreements among small correlated countries are unlikely to be feasible.

Insert Figure 5 about here

4.2 An application to China

In this subsection I evaluate whether it is feasible that a country with a large stock of reserves becomes the international LOLR. This country would receive benefits from being the LOLR that could be explicit (that is, collect insurance premiums) or implicit (for example, political influence or trade benefits). The insured countries would not need to contribute reserves, just to pay the participating premium.

The natural candidate to become the international LOLR is China because it is the country with the largest stock of foreign reserves. Figure 6 plots the dynamics of these reserves.

Insert Figure 6 about here

Since December 2008 China has entered into more than 30 bilateral currency swap agreements. Moreover, China has created an even larger network of lending agreements through its development and export-import banks. Aizenman et al. (2015) describe the strategy as a bundling of finance dealing (lending, swap-lines and trade credit) to promote a new type of Chinese-outward
mercantilism. Table 5 summarizes the evolution of these agreements.

China’s lending agreements have multiple goals: facilitate settlement in renminbi, promote trade and also serve as a source of liquidity as a lender of last resort. For example, in 2013 Pakistan reportedly borrowed an equivalent of US$ 600 million to avert a domestic crisis (later it received a US$6.6 billion loan from the IMF). In a similar move, in 2014 Argentina drew $2.7 billion upon its swap line with China to combat a shortage of dollar funding. China does not provide dollar liquidity but both Pakistan and Argentina were able to convert renminbi to dollars in the offshore market. Venezuela has also relied on China’s lending of last resort. Contrary to the IMF, China seems to attach minimal conditions to its loans.

Panel A in Figure 7 plots both the evolution of the number of countries with which China has a lending agreement, and the average correlation of output between China and the countries with which it has signed agreements. As pointed out by Aizenman et al. (2015), the correlation is relatively high because China has given preference to countries with natural resources or whose economies are strategic for the Chinese economy. The correlation has fallen as more countries have signed agreements. Panel B of Figure 7 plots the foreign reserves of China as % of the GDP in the pool of countries with an agreement with China. This ratio has decreased over time since China’s foreign reserves have been flat or decreasing since 2012 while the number of agreements keeps increasing. Even so, in 2015, if we exclude the Euro area from the covered countries, China has foreign reserves that are around 15% of the GDP of the pool of countries with an agreement.

Figure 8 uses the model to simulate the likelihood that China fails to be a sustainable LOLR. I calibrate the initial level of reserves in equation 32 to match the foreign reserves of China as percentage of poolwide GDP reported in Figure 7B excluding the Euro area. The simulation assumes that the cross-country correlation is 0.91, this is the average from Figure 7A. Countries pay the participation premium defined in (33). China’s reserves evolve as in equation 32. Reserves are invested at the international risk-free rate. Inflows are the participation premiums

---

13 Morelli et al. (2015) show that during the recent financial crisis the U.S. lending was directed towards those countries more important for the stability of the U.S. financial system.
14 The calibration may be conservative in this regard because it is based on CRRA preferences with a risk aversion of two. This is a standard value in macro models but fails to generate the risk premiums implicit in asset prices.
collected from the insured countries. Outflows are the lending of last resort subsidies provided to the countries in banking crises.

Figure 8 compares the likelihood of failure over different time periods. The longer the time period, the more likely that a bad shock in multiple countries arrives and China fails as LOLR. Figure 8 shows that the probability of failure is not even 5% if China is able to extract the maximum economic benefit from their lending support. Thus, given its large stock of foreign reserves, it seems that China can be a sustainable LOLR.

However, model simulations reported in Figure 9 suggest that China may have overreached its capacity to be a LOLR. The top panel plots average annual excess returns over the risk-free rate (and its standard deviation) in a 100 years period for the different sizes of the insured pool reported in Figure 7. Figure 9 shows that as China signed new agreements with countries with positive and high output correlations, the expected excess return from being the LOLR decreases and its volatility increases. The bottom panel shows the distribution of annual excess returns when 52 countries are insured, like in 2015, excluding the Euro countries. That is, China reserves cover 15% of the insured GDP. We see that most of the probability distribution is with positive excess returns, but there is a significant probability that China’s reserves are lost while being LOLR and excess returns are very negative.

I interpret this result as suggesting that it may not be optimal for China to further expand the number of countries with which it has lending agreements as long as these countries have high positive correlations between themselves and with China.

---

15If \( M_0 \) are the initial reserves, and the level of reserves 100 years later is \( M_{100} \). Then I define the annual gross return \( R_{LOLR} \) for China from being a LOLR for 100 years as

\[
\max \{ M_{100}, 0 \} = M_0 (R_{LOLR})^{100}.
\]

The max operator ensures limited liability. That is, China’s maximum losses are its initial reserves. Then, the annual excess return is

\[
R_{LOLR} - R^4_w
\]

where \( R^4_w \) is the annual risk-free rate. Reserves evolve as in equation 32 with insured countries paying the participation premium defined in Section 2.9.
5 Conclusions

This paper studied banking crises and lending of last resort in a quantitative double-decker model of financial frictions. Banking crises are deeper when risk-free rates do not react to mitigate the increase in banks and entrepreneurs’ borrowing spreads. Lending of last resort seems beneficial overall, even if it induces moral hazard through higher leverage. For the levels of liquidity support documented by Laeven and Valencia (2013), pools of small countries do not seem to be feasible lenders of last resort as they require too many uncorrelated countries, or large contributions to the initial stock of reserves, to be sustainable. However, an economy with a large stock of reserves like China appears to be a sustainable international LOLR. Thus, through the lenses of the model, the recent Chinese initiatives to increase its clout as an international LOLR seem beneficial and long-lasting.
References


Aizenman, J., Cheung, Y.-W. and Ito, H.: 2015, International reserves before and after the global crisis: Is there no end to hoarding?, *Journal of International Money and Finance*.


Garcia-Herrero, A. and Xia, L.: 2013, China’s RMB bilateral swap agreements: What explains the choice of countries?


Hirakata, N., Sudo, N. and Ueda, K.: 2013, Is the net worth of financial intermediaries more important than that of non-financial firms?


Lagarde, C.: 2016, Accelerating reforms to establish a risk prevention system - A view from the IMF.


Online Appendices-Not-For-Publication

A. First Order Conditions

A1 Households

The households maximize:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(C_t, H_t^H)$$  \hspace{1cm} (A1)

subject to

$$K_t = (1 - \delta)K_{t-1} + I_{n,t},$$  \hspace{1cm} (A2)

$$I_{n,t} = \left(1 - \frac{\phi}{2} \left(\frac{I_{g,t}}{I_{g,t-1}} - 1\right)^2\right) I_{g,t},$$  \hspace{1cm} (A3)

$$C_t + I_{g,t} + R_w B_{H,t-1} = W_t H_t^H + B_{H,t} - \frac{\psi}{2} B_{H,t}^2 + Q_t I_{n,t} + \Pi_{E,t} + \Pi_{B,t}.$$  \hspace{1cm} (A4)

Rearranging equations A2, A3 and A4, the households’ budget constraint can be written as:

$$C_t + I_{g,t} + R_w B_{H,t-1} = W_t H_t^H + B_{H,t} - \frac{\psi}{2} B_{H,t}^2 + Q_t \left(1 - \frac{\phi}{2} \left(\frac{I_{g,t}}{I_{g,t-1}} - 1\right)^2\right) I_{g,t} + \Pi_{E,t} + \Pi_{B,t}.$$  \hspace{1cm} (A5)

The first order conditions are:

$$u_1(C_t, H_t^H) \left(1 - \psi B_{H,t}\right) = \beta \mathbb{E}_t \left[u_1(C_{t+1}, H_{t+1}^H) R_w\right],$$

$$-\frac{u_2(C_t, H_t^H)}{u_1(C_t, H_t^H)} = W_t,$$

$$1 + Q_t \left(\frac{\phi \left(I_{g,t-1} - 1\right) \left(I_{g,t} \left(I_{g,t-1} - 1\right)^2\right)}{1 - \frac{\phi}{2} \left(I_{g,t} - 1\right)^2}\right) = \beta \mathbb{E}_t \left[u_1(C_{t+1}, H_{t+1}^H) Q_{t+1} \phi \left(I_{g,t+1} H_{t+1}^H - 1\right) \left(I_{g,t+1} - I_{g,t}\right)^2\right].$$
A2 Firms

The firm’s problem is:

$$\max_{\{H_t, K_{t-1}\}} K_{t-1}^{1-\alpha} H_t^{\alpha} - W_t H_t - r_t K_{t-1}$$

The first order conditions are:

$$W_t = \alpha \left( \frac{K_{t-1}}{H_t} \right)^{1-\alpha},$$

$$r_t = (1 - \alpha) \left( \frac{H_t}{K_{t-1}} \right)^{\alpha}.$$ 

A3 Contract between entrepreneurs and banks

It is convenient to follow Bernanke et al. (1999) and define the functions

$$\Gamma_i (\bar{\omega}_{i,t}) = \int_{\bar{\omega}_{i,t}}^{\infty} \bar{\omega}_{i,t} dF_i(\omega_i) + \int_0^{\bar{\omega}_{i,t}} \omega_i dF_i(\omega_i),$$

$$G_i (\bar{\omega}_{i,t}) = \int_0^{\bar{\omega}_{i,t}} \omega_i dF_i(\omega_i), \text{ for } i = E, B.$$

Assuming

$$\mathbb{E}_t (\omega_E) = \mathbb{E}_t (\omega_B) = 1, \forall t,$$

we can rewrite the contract between entrepreneurs and banks as:

$$\max_{\{K_t, \bar{\omega}_{E,t+1}\}} \mathbb{E}_t [[1 - \Gamma_E (\bar{\omega}_{E,t+1})] R_{t+1}^E Q_t K_t]$$

subject to

$$R_{t+1}^E Q_t K_t [\Gamma_E (\bar{\omega}_{E,t+1}) - \mu G_E (\bar{\omega}_{E,t+1})] = R_t^B (Q_t K_t - N_t^E).$$

The first order conditions are:

$$\mathbb{E}_t [[1 - \Gamma_E (\bar{\omega}_{E,t+1})] R_{t+1}^E] + \lambda_t [\Gamma_E (\bar{\omega}_{E,t+1}) - \mu E G_E (\bar{\omega}_{E,t+1})] R_{t+1}^E - R_t^B] = 0,$$

$$\mathbb{E}_t [-\Gamma'_E (\bar{\omega}_{E,t+1}) R_{t+1}^E] + \lambda_t [\Gamma'_E (\bar{\omega}_{E,t+1}) - \mu E G'_E (\bar{\omega}_{E,t+1})] R_{t+1}^E = 0.$$
A4 Other equations

Using the definition of $\bar{\omega}_{B,t}$ from (15) to substitute $R_{B,t}^L B_t^B$, the participation constraint for the banks’ shareholders becomes:

$$R_t^B B_t^E [1 - \Gamma_B (\bar{\omega}_{B,t})] = R_w N_t^B.$$ 

For the banks’ lenders the participation constraint can be written as:

$$R_t^B B_t^E [\Gamma_B (\bar{\omega}_{B,t}) - \mu_B G_B (\bar{\omega}_{B,t})] = R_w B_t^B.$$ 

The net worth and dividends paid by the entrepreneurs become:

$$N_t^E = \gamma_E R_t^E Q_{t-1} K_{t-1} [1 - \Gamma_E (\bar{\omega}_{E,t})] + W_t H^E,$n$$

$$\Pi_t^E = (1 - \gamma_E) R_t^E Q_{t-1} K_{t-1} [1 - \Gamma_E (\bar{\omega}_{E,t})].$$

The net worth and dividends paid by the banks can be written as:

$$N_t^B = \gamma_{B,t} R_{t-1}^B B_{t-1}^E [1 - \Gamma_B (\bar{\omega}_{B,t-1})] + W_t H^B,$n$$

$$\Pi_t^B = (1 - \gamma_{B,t}) R_{t-1}^B B_{t-1}^E [1 - \Gamma_B (\bar{\omega}_{B,t-1})].$$

A5 Model definitions

These are the model counterparts to the moments reported in Table 3. This Table reports annualized data while the model is calibrated at quarterly frequency.

(1) Banks’ borrowing spread: $(R_B^L)^4 - (R_w)^4$.

(2) Annualized default rate of entrepreneurs: $4F_E(\bar{\omega}_E)$.

(3) Annualized default rate of banks: $4F_B(\bar{\omega}_B)$.

(4) Ratio of equity-to-assets of entrepreneurs: $\frac{N^E}{QK}$. And for banks: $\frac{N^B}{B^E}$.

(5) Return on capital over risk-free rate: $\left(\frac{R^E}{R_w}\right)^4$.

(6) Dividend-to-equity ratio of entrepreneurs: $\frac{\Pi^E}{N^E}$.

(7) Consumption-to-output ratio: $\frac{C}{Y}$.
B. Numerical appendix

The model has a large number of state variables and the analysis of the sustainability of the international LOLR requires solving the model many times for many countries. I use the following algorithm that adapts a first-order perturbation approach and applies it in a piecewise fashion, like models with occasionally binding constraints as Guerrieri and Iacoviello (2015).\textsuperscript{16} The algorithm is slightly different for impulse responses than for simulations. This is to avoid linearizing the utility function in the simulations, and because, like Gertler and Karadi (2011), the impulse responses are conditional on shocks that trigger lending of last resort. The algorithm is as follows:

1) In equation 29, assume $B_t^G = \chi \left( \frac{R_{t,t}^L - R_{t,t,ss}^L}{R_{t,ss}^L} \right)$, and use first-order perturbation to compute the matrices $A_{LOLR}$, $B_{LOLR}$, $C_{LOLR}$ and $D_{LOLR}$ that determine the state space representation of the system when the lending of last resort is operating:

$$s_t = A_{LOLR}s_{t-1} + B_{LOLR}\varepsilon_t, \quad (A5)$$
$$x_t = C_{LOLR}s_{t-1} + D_{LOLR}\varepsilon_t. \quad (A6)$$

Where $\varepsilon_t$ is the banking crisis shock defined in Section 2.6, $s_t$ is the vector of states and $x_t$ is the vector of controls.

2) Use $(A5)$ – $(A6)$ to compute the responses reported in Figures 1 and 2 conditional on a negative shock that triggers lending of last resort, that is, $R_{B,t}^L \leq R_{B,ss}^L$.

3) In equation 29, assume $B_t^G = 0$, and compute the matrices $A_{noLOLR}$, $B_{noLOLR}$, $C_{noLOLR}$ and $D_{noLOLR}$ that determine the state space representation of the system when the lending of last resort is not operating:

$$s_t = A_{noLOLR}s_{t-1} + B_{noLOLR}\varepsilon_t, \quad (A7)$$
$$x_t = C_{noLOLR}s_{t-1} + D_{noLOLR}\varepsilon_t. \quad (A8)$$

4) For a given vector of shocks $\{\varepsilon_t\}_{t=0}^T$ and a given $s_{t-1}$, every period, use $(A5)$, $(A6)$ and $\varepsilon_t$ to compute $s_t$ and $x_t$ except if this leads to $R_{B,t}^L \leq R_{B,ss}^L$. If $R_{B,t}^L \leq R_{B,ss}^L$, then compute $s_t$ and $x_t$ using $(A7)$, $(A8)$ and $\varepsilon_t$. This piecewise algorithm ensures that the lending of last resort only applies to banking crisis as defined by equation 29. Moreover, the algorithm ensures that agents decide as if there is lending of last resort since the default is to compute the policy

\textsuperscript{16}In fact, I checked my results using Guerrieri and Iacoviello (2015) Occbin toolbox and obtained the same results.
functions using (A5) and (A6).

5) In the simulations, I start the model from the steady-state \( s_{-1} = s_{SS} \) and I delete the first 100 periods to remove the impact of the initial conditions. Table 3 averages the results of 100,000 simulations, each with a length of 400 quarters. For the remaining simulations that involve multiple countries, I computed 10,000 simulations.

6) To compute welfare and the participation premium defined in Section 2.9, I input the time series of consumption \( (C_t) \) and hours worked \( (H^H_t) \) obtained from step 5 into the non-linear equation 34. I used the same vector of shocks \( \{\varepsilon_t\}_t^T \) for the case with and without LOLR.

7) For a given simulation, I define failure of the international LOLR when \( M_t < 0 \) during the 400 quarters. The evolution of \( M_t \) follows (32). I average over the total number of simulations to compute the probability of failure of the international LOLR.
C. Data Sources

The data of foreign exchange reserves that Figure 6 plots are from the People’s Bank of China (PBOC). The GDP data used in Figures 7 to 9 come from the World Bank.

I merged three databases to construct the set of countries with lending programs with China reported in Table 5:\footnote{For countries in several databases, like Argentina and Brazil, Table 5 records the date of the first agreement.} 1) Countries that have signed bilateral currency swap agreements according to the PBOC.\footnote{http://www.pbc.gov.cn/huobizhengceersi/214481/214511/214541/2967384/2016040615334732261.pdf} These countries are Albania, Argentina, Armenia, Australia, Belarus, Brazil, Canada, Chile, European Central Bank, Hong Kong, Hungary, Iceland, Indonesia, Kazakhstan, Malaysia, Mongolia, New Zealand, Pakistan, Qatar, Republic of Korea, Russia, Singapore, South Africa, Sri Lanka, Suriname, Switzerland, Tajikistan, Thailand, Turkey, Ukraine, United Arab Emirates, United Kingdom, Uzbekistan.\footnote{Although the PBOC records the agreement with South Korea in 2009 it seems the agreement was already in place in 2008 according to Garcia-Herrero and Xia (2013) and news outlets like the China Daily.} 2) The China-Latin America Finance Database compiled by Inter-America Dialogue.\footnote{http://www.thedialogue.org/map_list/} This database contains data for Argentina, Bahamas, Barbados, Bolivia, Brazil, Costa Rica, Ecuador, Guyana, Jamaica, Mexico, Peru, Trinidad and Tobago, Venezuela. 3) Brautigam and Gallagher (2014) database on agreements with African countries. These countries are Angola, Democratic Republic of Congo, Equatorial Guinea, Ethiopia, Ghana, Nigeria, Republic of Congo, Sudan, Zimbabwe.
### Table 1: Exogenous parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Frisch elasticity</td>
<td>$\frac{1}{\theta-1}$</td>
<td>1.67</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Labor share in production</td>
<td>$\alpha$</td>
<td>0.6</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$R_w$</td>
<td>$\frac{1}{\beta}$</td>
</tr>
<tr>
<td>Bond adjustment cost</td>
<td>$\psi$</td>
<td>$\frac{7}{10^5}$</td>
</tr>
<tr>
<td>Entrepreneur’s default cost</td>
<td>$\mu_E$</td>
<td>0.12</td>
</tr>
<tr>
<td>Description</td>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-----------</td>
<td>-------</td>
</tr>
<tr>
<td>Std. dev. of entrepreneurs’ shocks</td>
<td>$\sigma_E$</td>
<td>0.34</td>
</tr>
<tr>
<td>Std. dev. of banks’ shocks</td>
<td>$\sigma_B$</td>
<td>0.05</td>
</tr>
<tr>
<td>Banks’ default cost</td>
<td>$\mu_B$</td>
<td>0.35</td>
</tr>
<tr>
<td>Entrepreneurs’ labor supply</td>
<td>$H^E$</td>
<td>0.5</td>
</tr>
<tr>
<td>Banks’ labor supply</td>
<td>$H^B$</td>
<td>0.125</td>
</tr>
<tr>
<td>Entrepreneurs’ dividend</td>
<td>$1 - \gamma_E$</td>
<td>0.035</td>
</tr>
<tr>
<td>Autocorrelation of bank equity shock</td>
<td>$\rho$</td>
<td>0.689</td>
</tr>
<tr>
<td>Std. dev. of banks’ equity shock</td>
<td>$\sigma$</td>
<td>3.3</td>
</tr>
<tr>
<td>Investment adjustment cost</td>
<td>$\phi$</td>
<td>0.21</td>
</tr>
<tr>
<td>LOLR parameter</td>
<td>$\chi$</td>
<td>19</td>
</tr>
</tbody>
</table>

Note: See Section 3.1 for the calibration strategy.
<table>
<thead>
<tr>
<th>Moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steady State</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Banks’ borrowing spread</td>
<td>1% – 3%</td>
<td>1.23%</td>
</tr>
<tr>
<td>Default rate of entrepreneurs</td>
<td>5.1%</td>
<td>5.14%</td>
</tr>
<tr>
<td>Default rate of banks</td>
<td>2% – 6%</td>
<td>3.25%</td>
</tr>
<tr>
<td>Ratio of equity-to-assets of entrepreneurs</td>
<td>50 – 75%</td>
<td>55.59%</td>
</tr>
<tr>
<td>Ratio of equity-to-assets of banks</td>
<td>7% – 13%</td>
<td>11.47%</td>
</tr>
<tr>
<td>Return on capital over risk-free rate</td>
<td>1.03</td>
<td>1.04</td>
</tr>
<tr>
<td>Dividend-to-equity ratio of entrepreneurs</td>
<td>3.76%</td>
<td>3.59%</td>
</tr>
<tr>
<td>Consumption-to-output ratio</td>
<td>0.746</td>
<td>0.73</td>
</tr>
<tr>
<td><strong>Simulations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median duration of a crisis</td>
<td>4 years</td>
<td>4 years</td>
</tr>
<tr>
<td>Median output losses per year</td>
<td>7.73%</td>
<td>7%</td>
</tr>
<tr>
<td>Median liquidity support (as % of banks’ liabilities)</td>
<td>9.6%</td>
<td>9.57%</td>
</tr>
</tbody>
</table>

Note: See Section 3.1 for details.
Table 4: Comparing averages of stationary distributions with and without LOLR

<table>
<thead>
<tr>
<th>% Change from No-LOLR to LOLR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
</tr>
<tr>
<td>Households’ employment</td>
</tr>
<tr>
<td>Wage</td>
</tr>
<tr>
<td>Capital</td>
</tr>
<tr>
<td>Price of capital</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Households’ per period utility</td>
</tr>
<tr>
<td>Median duration of a crisis</td>
</tr>
<tr>
<td>Likelihood of triggering LOLR ((R_{B,t}^L &gt; R_{B,ss}^L))</td>
</tr>
</tbody>
</table>

Note: See Section 3.3 for details.
<table>
<thead>
<tr>
<th>Year of the agreement</th>
<th>2007 and before</th>
<th>2008</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angola</td>
<td>Argentina</td>
<td>Democratic Rep. of Congo</td>
<td>Belarus</td>
</tr>
<tr>
<td>Brazil</td>
<td>Equatorial Guinea</td>
<td>Ethiopia</td>
<td>Ecuador</td>
</tr>
<tr>
<td>Ghana</td>
<td>Jamaica</td>
<td>South Korea</td>
<td>Indonesia</td>
</tr>
<tr>
<td>Nigeria</td>
<td>Republic of Congo</td>
<td></td>
<td>Peru</td>
</tr>
<tr>
<td>Venezuela</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2010</th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iceland</td>
<td>Bahamas</td>
<td>Australia</td>
</tr>
<tr>
<td>Singapore</td>
<td>Mongolia</td>
<td>Guyana</td>
</tr>
<tr>
<td></td>
<td>Pakistan</td>
<td>Turkey</td>
</tr>
<tr>
<td></td>
<td>Uzbekistan</td>
<td>Ukraine</td>
</tr>
<tr>
<td></td>
<td>Thailand</td>
<td>United Arab Emirates</td>
</tr>
<tr>
<td></td>
<td>Zimbabwe</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albania</td>
<td>Canada</td>
<td>Armenia</td>
</tr>
<tr>
<td>Euro Area</td>
<td>Qatar</td>
<td>Barbados</td>
</tr>
<tr>
<td>Hungary</td>
<td>Russia</td>
<td>Chile</td>
</tr>
<tr>
<td>Mexico</td>
<td>Sri Lanka</td>
<td>Costa Rica</td>
</tr>
<tr>
<td>Trinidad and Tobago</td>
<td>Switzerland</td>
<td>South Africa</td>
</tr>
<tr>
<td>United Kingdom</td>
<td></td>
<td>Suriname</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tajikistan</td>
</tr>
</tbody>
</table>

Note: See Appendix C for data sources
Figure 1. Banking crises in closed and small open economies. The panels plot the responses (deviation from the steady state) to a negative equity shock to the banking sector. Each panel compares a small open economy (SOE) with a closed economy (GE).
Figure 2. Banking crises in small open economies with and without lending of last resort. The panels plot the responses (deviation from the steady state) to a negative equity shock to the banking sector. Each panel compares a small open economy with and without lending of last resort (LOLR).
Figure 3. Banks’ assets-to-equity ratio for different levels of lending of last resort. This figure plots the asset-to-equity ratio of the banks, in the stationary distribution, for different levels of the parameter $\chi$ that controls the intensity of the lending of last resort in equation (29). The case $\chi = 0$ is the case with no LOLR.
Figure 4. Probability that the international LOLR fails for different correlations of shocks and different number of participating countries. The figure assumes that participating countries pay the participation premium defined in Section 2.9. The LOLR starts with no initial reserves. The probability of failure is computed over a 100 years period.
Figure 5. Probability that the international LOLR fails for different initial levels of reserves and different number of participating countries. Both panels assume that participating countries pay the participation premium defined in Section 2.9. The cross-country correlation of the shocks is zero in the top panel and 0.9 in the bottom panel. The probabilities of failure are computed over a 100 years period.
Figure 6. China’s foreign exchange reserves. Data source: People’s Bank of China.
Figure 7. Evolution of China’s lending programs. The top panel plots the number of countries with which China has signed lending agreements, and the average correlation over the 2000-2014 period between China’s GDP and the GDP of the countries with signed agreements at a given date. Section 4.2 has more details. The bottom panel plots China’s foreign exchange reserves as a % of the total GDP of the countries that have signed lending agreements with China. One line excludes the Euro area. For data sources see Appendix C.
Figure 8. Probability China would fail as international LOLR for different sizes of the insured pool. This figure plots the probability that the international LOLR fails when the reserves of the LOLR are calibrated to match China’s agreements reported in Figure 7, excluding the Euro area. Each line plots the probability of failure over different time horizons. Insured countries pay the participation premium defined in Section 2.9.
Figure 9. Expected excess returns for China as international LOLR. The top panel plots average annual excess returns over the risk-free rate (and its standard deviation) for the different sizes of the insured pool reported in Figure 7. The bottom panel shows the distribution of annual excess returns for the case in which 52 countries are insured (China reserves cover 15% of the insured GDP). Annual excess returns are computed as $R_{LOLR} - R^4_w$ where $R^4_w$ is the annual risk-free rate and $R_{LOLR}$ is the annual return for the LOLR on a 100 years period defined in footnote 15.