

Understanding the Decline in the Safe Real Interest Rate *

Robert E. Hall
Hoover Institution and Department of Economics,
Stanford University
National Bureau of Economic Research
rehall@stanford.edu; stanford.edu/~rehall

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Abstract

Over the past few decades, worldwide real interest rates have trended downward. The real interest rate describes the terms of trade between risk-tolerant and risk-averse investors. Debt pays off equally across contingencies at a given future date, so debt is valuable to risk-averse investors to smooth consumption across those contingencies. In an equilibrium with trade between investors who differ in attitudes toward risk, the risk-tolerant investors will borrow from the risk-averse ones, shifting the risk to those whose preferences favor taking on risk. In the case where investors have preferences that are additively separable in future states and in time, attitudes toward risk are heterogeneous among investors if they differ in the curvature of their utility kernels and differ in their beliefs about the probabilities of outcomes, especially adverse outcomes. If the composition of investors shifts toward those with higher curvature (higher coefficients of relative risk aversion) and toward investors who believe in higher probabilities of bad events, the real interest rate falls. The paper calculates likely magnitudes of the decline and presents evidence in favor of a shift in the composition of investors toward the more risk-averse. The downward trend in real interest rates is a significant problem for monetary policy but is helpful to heavily indebted countries.

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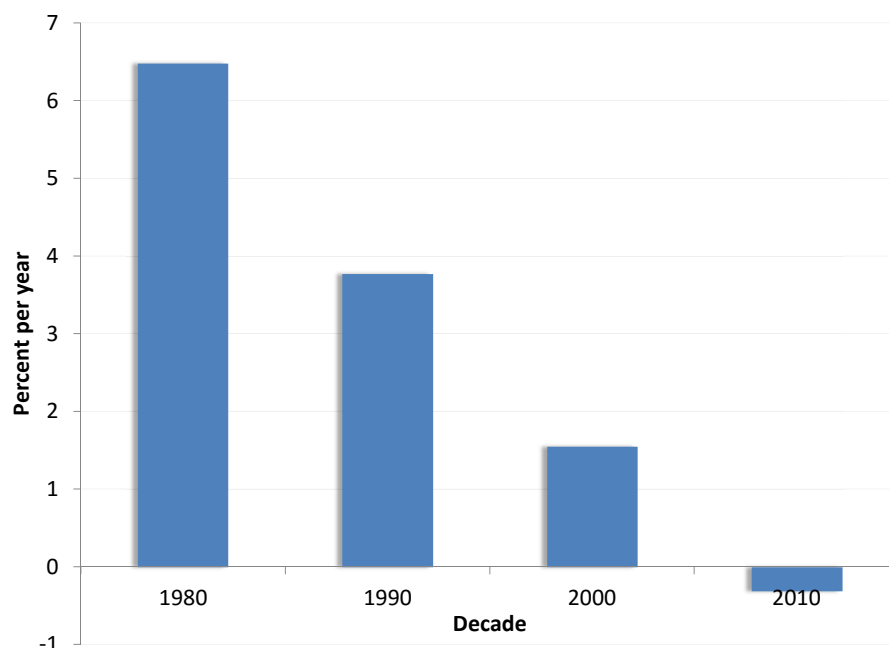


Figure 1: Decade Averages of the Real Interest Rate on 5-Year Treasury Notes

1 The Issues

Figure 1 shows the real interest rate on 5-year obligations of the United States Treasury, averaged over recent decades. Since the decade of the 1980s, the rate has declined monotonically, reaching a negative level in the current decade. Real rates on other safe obligations around the world have declined in parallel. The decline in real interest rates has been welcome in some respects, notably in reducing the burden of the national debt. But the decline has created a challenge for monetary policy, because it has lowered the headroom of monetary policy to offset an incipient recession. Central banks fear that in future recessions, even if mild, monetary policy's expansionary effect will be limited because the interest rate will be pinned at its effective lower limit.

Bernanke, Bertaut, DeMarco and Kamin (2011) laid out an explanation of the decline in the real rate based on the “global saving glut”, an idea that Bernanke had advanced in speeches before and during his service at the Federal Reserve. This explanation views the interest rate as a price that varies to equate flows of saving and investment. Ricardo J. Caballero (2008) offer a portfolio-based explanation based on an inability of some economies to create safe assets to fund risky real investments.

This paper takes a different approach. Building on Barro and Mollerus (2014), it pursues the implications of heterogeneity among investors who trade with each other in a risky environment. The investors start out with endowments of equal claims to the same risky investment. They trade claims in a capital market to take advantage of the gains to trade that exist among heterogeneous agents. In the post-trade equilibrium, risk-tolerant investors absorb more of the downward shocks and gain more from the upward shocks. The risk-averse investors, on the other hand, have more equal returns, losing less in bad times and gaining less in good times. In effect, the risk-tolerant investors sell insurance to the risk-averse ones. Safe debt-type claims play a role in the equilibrium by paying off the same amount in both good and bad times. Throughout, the paper embodies the principle of modern financial economics that securities are packages of the underlying fundamental risk factors of the economy.

The paper explores the implications of these patterns of trade for the implied safe real interest rate, the terms under which investors swap pre-specified non-contingent claims across time periods. The interest rate is a useful summary of the terms under which the risk-tolerant help the risk-averse stabilize returns, because providing a non-contingent return is a powerful tool for stabilizing returns. A growing fraction of wealth around the world in the hands of risk-averse investors could be the explanation of the declining safe real interest rate.

An upward trend in the proportion of wealth held by investors with higher coefficients of relative risk aversion is one potential explanation of the decline in the safe real interest rate. A second form of heterogeneity may also play a role. A rising share of wealth held by investors who believe in a higher probability of bad future outcomes raises the volume of trade in safe claims. Interest rates based on trade between risk-tolerant optimists and risk-averse pessimists can be substantially lower than occur with trade between investors with common beliefs about the distribution of returns. This paper joins many others in financial economics that emphasize heterogeneous beliefs.

An important dimension of heterogeneity is the relative size of the resources commanded by different types of investors. In the economy considered here, the interest rate is particularly low if the risk-tolerant investors own fewer resources than do the risk-averse. It follows that the real safe interest rate will trend downward if the resources in the hands of the risk-averse grow faster than those in the hands of the risk-tolerant.

2 Model

The model describes an endowment economy with two types of investor-consumers. They consume a variety of products, each involving the same physical good, but delivered in different time periods and different states of the world. The economy has complete Arrow-Debreu markets. Investors own endowments of the products. There are two types of investors. One type is risk-tolerant in two respects and the other is risk-averse. At the beginning of time, the investors make contingent trades whose general character is to transfer risk from the risk-averse to the risk-tolerant. The two types are equally impatient, so intertemporal trade is not an important feature of the economy. Both types believe in the same rate of growth of the endowment, so heterogeneity in beliefs about growth is not a factor in determining the real interest rate.

The products are numbered by the index i . Product $i = 1$ is immediate delivery of the good with certainty. This product serves as numeraire, with price $p_1 = 1$. The risk-tolerant investors consume c_i of product i and the risk-averse c_i^* . The preferences of the risk-tolerant are expressed in a utility function $U(c_1, \dots, c_N)$. They solve the problem

$$\max U(c_1, \dots, c_N) \text{ subject to } \sum_i p_i(c_i - \alpha y_i) = 0. \quad (1)$$

Here y_i is the economy's endowment of product i and α is the fraction of the endowment owned by the risk-tolerant investors. The outcome of the choice is a set of excess supply functions $d_i(p, y)$ describing the amount of product i that the risk-tolerant offer to trade. Variables without subscripts are vectors of the corresponding variables with subscripts. The risk-averse solve a similar problem and have excess supply functions $d_i^*(p, y)$. An equilibrium of the economy is a vector of prices p such that $d_i(p, y) + d_i^*(p, y) = 0$, for all products i . Such a vector exists, according to standard continuity principles—this economy satisfies all of the standard properties of general-equilibrium models in the Arrow-Debreu tradition.

This setup has no explicit role for the probability that the delivery of a given product occurs. Probabilities of states of the world are bundled into the utility functions. Nothing requires that risk-tolerant investors agree on the probabilities. Heterogeneity in beliefs about probabilities plays a large and growing role in the finance literature because it appears to help understand many features of the operation of financial markets. References to this and other branches of the relevant literature appear in a section at the end of the paper.

Under the assumption of state- and time-separable preferences, the utility function has the special form

$$U(c_1, \dots, c_N) = \sum_{i=1}^N \beta_{\tau(i)} \phi_i u(c_i). \quad (2)$$

Here $\tau(i)$ is the time period when product i is delivered, β_t is a time-weight describing impatience (if β_t declines with t), and ϕ_i is the belief of the investor about the probability. The probabilities sum to one within each time period:

$$\sum_{i \text{ such that } \tau(i)=t} \phi_i = 1. \quad (3)$$

The notation's asymmetric treatment of i and t saves a lot of double subscripts in what follows. The utility function of a risk-averse investor is

$$U^*(c_1^*, \dots, c_N^*) = \sum_{i=1}^N \beta_{\tau(i)}^* \phi_i^* u^*(c_i^*). \quad (4)$$

I make the assumption

$$\beta_t = \beta_t^* = \beta^t \quad (5)$$

to impose equal impatience on both investor types and to adopt the standard geometric pattern of discounting future utilities.

2.1 Variation of the endowment

Uncertainty arises in the economy entirely through differences in the endowment. I assume that, for each product i , a ternary event occurs that changes the endowment associated with i , y_i , to one of three values in the next period:

$$\{y_i(1 - \Delta_-), y_i, y_i(1 + \Delta_+)\}. \quad (6)$$

Thus each product except those in the terminal period has three successor products in the following period. The two types of investors agree on the set of values that the endowment can take—agreement is essential for meaningful trading.

There is one possible value of the endowment in the first period, normalized at one. There are three possible values in the second period, $\{1 - \Delta_-, 1, 1 + \Delta_+\}$. Among the nine products in the third period, there are six possible values,

$$\{1 - \Delta_-, 1, 1 + \Delta_+, (1 - \Delta_-)^2, (1 - \Delta_-)(1 + \Delta_+), (1 + \Delta_+)^2\}. \quad (7)$$

The set of values becomes correspondingly richer with each passing period.

2.2 Beliefs about probabilities

For each product in a given period, the risk-tolerant investors believe that the probabilities are π_- that the endowment will fall in the next period, π_+ that it will rise, and $\pi_0 = 1 - \pi_- - \pi_+$ that it will remain the same. The corresponding beliefs of the risk-averse investors are π_-^* that the endowment will fall, π_+^* that it will rise, and $\pi_0^* = 1 - \pi_-^* - \pi_+^*$. Both types of investors believe that the expected change in the endowment is zero, so

$$\pi_- \Delta_- = \pi_+ \Delta_+ \quad (8)$$

and

$$\pi_-^* \Delta_- = \pi_+^* \Delta_+. \quad (9)$$

Thus both types believe that the endowment is an untrended random walk, though they disagree about the probabilities of the changes in endowment. The risk-averse investors believe that the dispersion of the endowment fans out over time at a higher rate than the risk-tolerant investors believe.

The ternary setup in the process is necessary to accommodate heterogeneity in beliefs under the random-walk restriction. With a binary increment to the endowment, only a single pair of probabilities would be consistent with the random walk.

The other restriction is that none of the probabilities can be negative. There is a one-dimensional subspace of probabilities that satisfies the random walk and non-negativity. The set of probability beliefs satisfying all the constraints is

$$0 \leq \pi_- \leq \frac{\Delta_+}{\Delta_+ + \Delta_-} \quad (10)$$

and

$$\pi_+ = \frac{\Delta_-}{\Delta_+} \pi_-, \quad (11)$$

and similarly for π^* .

The probabilities π and π^* induce probability beliefs ϕ_i and ϕ_i^* on the products indexed by i . For example, the probabilities on the endowment values in period 3 listed earlier,

$$\{1 - \Delta_-, 1, 1 + \Delta_+, (1 - \Delta_-)^2, (1 - \Delta_-)(1 + \Delta_+), (1 + \Delta_+)^2\} \quad (12)$$

are

$$\{\pi_0 \pi_-, \pi_0^2, \pi_0 \pi_+, \pi_-^2, \pi_- \pi_+, \pi_+^2\}. \quad (13)$$

They sum to one, keeping in mind that three of them occur twice.

2.3 Utility kernel

I take the utility kernel for risk-tolerant investors to be

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad (14)$$

so marginal utility is $c^{-\gamma}$. Similarly, for risk-averse investors, marginal utility is $(c^*)^{-\gamma^*}$.

2.4 Solving the model

The problem is to find trades x_i and Arrow-Debreu prices p_i that satisfy

$$\beta^{\tau(i)} \phi_i c_i^{-\gamma} = p_i c_1^{-\gamma}, \quad (15)$$

$$c_i = \alpha y_i - x_i, \quad (16)$$

$$\beta^{\tau(i)} \phi_i^* (c_i^*)^{-\gamma^*} = p_i (c_1^*)^{-\gamma^*}, \quad (17)$$

$$c_i^* = (1 - \alpha) y_i + x_i, \quad (18)$$

and

$$\sum_i p_i x_i = 0. \quad (19)$$

Solution by Newton's method is straightforward and speedy if the sparsity of the Jacobian is taken into account. Some of the specifications result in poorly conditioned Jacobians, but all of the reported results are accurate to well beyond the number of digits in the tables.

3 The Safe Interest Rate

A safe (discount) bond is a package of Arrow-Debreu claims containing one unit of each of the products that are delivered in a particular time period. The package has probability one of delivering one unit. The sum of the prices of those products is the cost of a claim that yields one unit of output in a future period. It is the market price at origination of a pure discount bond that pays no coupon and makes a single unit payment in that future period. For a bond with maturity of m periods, the price is

$$P_m = \sum_{i \text{ such that } \tau(i)=m} p_i. \quad (20)$$

The interest rate in the sense of a per-period yield is

$$r_m = P_m^{-1/m} - 1. \quad (21)$$

Thus r_m traces out the yield curve for risk-free bonds in the economy.

Factors that raise P_m lower the safe interest rate. The value is

$$P_m = \sum_{i \text{ such that } \tau(i)=m} \pi_i \beta^m \left(\frac{c_i}{c_1} \right)^{-\gamma} = \mathbb{E} \beta^m \left(\frac{c_i}{c_1} \right)^{-\gamma} = \mathbb{E}^* \beta^m \left(\frac{c_i^*}{c_1^*} \right)^{-\gamma^*}. \quad (22)$$

Because the utility kernel with constant relative risk aversion belongs to the family of precautionary utility functions with convex marginal utility, $u'''(c) > 0$, Jensen's inequality implies that higher dispersion of consumption c_i raises the price of certain future output and lowers the risk-free interest rate. Forces that raise the dispersion of c_i and c_i^* will lower the interest rate.

3.1 The yield curve

The slope of the yield curve depends on how the dispersion of future consumption rises with futurity, m . The yield curve will be flat at level r if

$$P_m = \left(\frac{1}{1+r} \right)^m. \quad (23)$$

Thus, to generate a flat yield curve, the dispersion of consumption will be higher for maturity m relative to $m-1$ sufficiently to make P_m lower than P_{m-1} by the ratio $\frac{1}{1+r}$. To a reasonable approximation, this condition implies that the dispersion rises by a constant factor with each period of added maturity. A constant proportional increase in dispersion over time is a characteristic of a (geometric) random walk, so a flat yield curve goes with consumption that evolves as something like a random walk. The assumption that both types of investors believe that the endowment is a random walk turns out to imply that their consumption levels are very close to random walks.

3.2 Relation between debt and complete financial markets

In place of the Arrow-Debreu setup, where investors purchase contingent products at the beginning of time, one could study an equivalent setup where investors make contingent one-period contracts that are more like the financial contracts seen in the real world. Each contract would specify contingent payments. One is in units of product i , interpreted as the

purchase of a security. There are three others specified in units of the successor products, interpreted as the (possibly) random payoffs of the security in the succeeding period.

One of the reasons to consider the setup with one-period contracts is that the Arrow-Debreu setup appears to involve breathtaking issues of commitment, with all contracts made once-and-for-all at some mythical starting time. Putting aside the issues of finite lifetimes and yet-unborn investors and assuming that the one-period contracts are sufficiently detailed, an economy where investors wait to make contracts until the time when they become operative will have the same rational-expectations equilibrium as the Arrow-Debreu equilibrium.

To keep the notation simple, I will drop the subscript i in this discussion. I consider the case where one contract is debt paying the amount d in the three successor products. The other contract is a claim to qy' units of the successor products, where y' is the realized amount of the endowment. If these securities could replicate the Arrow-Debreu allocation, the values of d and q would need to solve the following equations, written in an obvious notation:

$$c_- = qy_- - d = q(y - \Delta_-) - d, \quad (24)$$

$$c_0 = qy_0 - d = qy - d, \quad (25)$$

and

$$c_+ = qy_+ - d = q(y + \Delta_+) - d. \quad (26)$$

The sign convention here presumes that the risk-tolerant investors will borrow from the risk-averse in the earlier period and pay back in the later period. These three equations have only two unknowns, d and q , so only by coincidence would the three equations have a solution. Replication of the Arrow-Debreu allocation would require a third security, such as one that paid off only case of the bad outcome $y' = y - \Delta_-$. This finding is no surprise, as it is well known in finance that, with binary increments (two analogous equations rather than three), two securities can replicate the Arrow-Debreu allocation.

As a measure of the role of safe debt implied by the full Arrow-Debreu pattern of trade and resulting consumption allocations, I use the amount of debt, d , that, along with the loading, q , on the endowment, yields the best fit of the consumption allocation implied by d and q in the equations above to the Arrow-Debreu allocation. The best least-squares fit is revealed by the regression of the three values of c' on a constant and the three values of y' .

		<i>Base value</i>	<i>Alternative value</i>
<i>Parameters of the physical environment</i>			
Δ_+	Increment to endowment	0.04	
Δ_-	Decrement to endowment	0.6	
α	Share of endowment owned by risk-tolerant consumers	0.5	0.25
<i>Parameters of preferences and beliefs of consumers</i>			
		<i>Risk tolerant</i>	<i>Risk averse</i>
		<i>Base value</i>	<i>Base value Alternative value</i>
β	Utility discount ratio	0.93	0.93
γ	Coefficient of relative risk aversion	2	2 2.5
π_-	Belief about probability of bad decrement to endowment	0.01	0.01 0.02
π_+	Belief about probability of good increment to endowment	0.15	0.15 0.30

Table 1: Parameter Values

The constant is the best-fitting value of d and the slope is the best-fitting value of q . The residuals in the equation measure the incompleteness of the two-security setup relative to the complete-market Arrow-Debreu allocation.

4 Parameters

Table 1 shows the parameter values for the calculations that follow. The upper panel contains the parameters of the physical environment. In all calculations, the upward jump in the joint endowment y is $\Delta_+ = 0.04$ and the downward jump is $\Delta_- = 0.6$. Most of the calculations set the share of the endowment owned by the risk-tolerant investors to $\alpha = 0.5$ but an alternative uses 0.25.

The lower panel of Table 1 describes the preferences of the investors, including their beliefs about the probability distribution of the endowments. In all cases, the impatience parameter is $\beta = 0.93$ and the coefficient of relative risk aversion of risk-tolerant investors is $\gamma = 2$. Some cases set the coefficient of relative risk aversion of the risk-averse to 2 as well, but an alternative case has $\gamma^* = 2.5$. The beliefs about the probability of a sharp contraction in the endowment, π_- and π_-^* , are 0.01 (once in a hundred years), except for the alternative

value for the risk-averse, $\pi_-^* = 0.02$. The beliefs of both types about the probability of an improvement in the endowment are the values described earlier needed to ensure that both types of investors believe that the expected change in the endowment is zero. These are $\pi_+ = \pi_-^* = 0.15$ in the base case and $\pi_-^* = 0.30$ in the alternative case.

5 Equilibrium in the Model

Trade in the model takes the form of non-zero values of contracted transfers, x_i , from the risk-tolerant investors to the risk-averse investors. A negative value of x_1 is typical of the equilibria and represents an insurance payment from the risk-averse for positive transfers of products for which the endowments will be unusually small. Positive values of x_i arise for products where the endowment is small, because the risk-averse find their small endowments of these products to be more painful than do the risk-tolerant. This motivation for trade exists even when the two types of investors agree on the probabilities of the endowment process. Positive values also arise for products where the risk-averse perceive higher probabilities than do the risk-tolerant. This motivation for trade would exist even if the two types of investors were equally risk-averse. Thus trade arises from a mixture of two kinds of heterogeneity among investors.

The numerical results in this section are from the model with $T = 2$, that is, one date when trades are contracted for a single future date. Calculations for versions with many dates give closely similar results.

5.1 No-trade equilibrium

Absent both sources of heterogeneity, the equilibrium involves no trade. Lucas (1978) describes the equilibrium. Latent Arrow-Debreu product prices are the marginal rates of substitution between product 1 and the other products. Table 2 shows the prices, along with the values of the endowment, and the corresponding probabilities perceived by both types of investors. Parameter values are the base values shown in Table 1. The upper panel describes the situation at the time that prices are set. The middle panel shows the three possible realizations a year later. The top line describes the worst realization, with an endowment of 0.4. The price of that product conditional on its realization is $0.058/0.01=5.81$. Output is quite valuable when the endowment is so low. For the other two realizations, with

	<i>Endow- ment, y</i>	<i>Trade, x</i>	<i>Consump- tion of risk tolerant</i>	<i>Consump- tion of risk averse</i>	<i>A-D price, p</i>	<i>Proba- bility, π</i>	<i>Condi- tional price, p/π</i>
Initial	1	0.0000	0.5	0.5	1	1	1
	0.4	0.0000	0.2	0.2	0.058	0.01	5.81
	1	0.0000	0.5	0.5	0.781	0.84	0.93
After one year	1.04	0.0000	0.52	0.52	0.129	0.15	0.86
Price with certainty					0.968		
Annual interest					3.27		
Debt					0.00		

Table 2: Latent Prices with No Trade

endowments of 1 and 1.04, output is not particularly valuable, with prices conditional on realization of 0.93 and 0.86.

The bottom panel of Table 2 shows the price of the package of products that delivers one unit of output with certainty—the sum of the numbers in the corresponding column above: $P = 0.968$. The corresponding annual interest rate or yield is 3.27 percent. It is representative of the historical real yield on Treasury debt in recent decades, though not in the current decade, as shown in Figure 1. The volume of debt implicit in the zero-trade allocation is, of course, zero.

5.2 Equilibrium with trade resulting from heterogeneous risk aversion

Table 3 shows the the equilibrium, with the same parameter values as in Table 2, except that the risk-tolerant investors have coefficients of relative risk aversion of 2 and the risk-averse have coefficients of 2.5. Extensive trade occurs in this economy. Initially, the risk-averse investors pay the risk-tolerant ones 0.001 units of output as an insurance premium. In the most common outcome a year later—with probability 0.84, as before—the risk-averse provide 0.001 units of output to the risk-tolerant. In the bad outcome, with a joint endowment of only 0.4, the risk-tolerant provide 0.02 units of output to the risk-averse that cushions them against the low value of the endowment. In that outcome, the risk tolerant consume 0.18 units of the endowment while the risk-averse consume 0.22 units. The Arrow-Debreu price

	<i>Endow- ment, y</i>	<i>Trade, x</i>	<i>Consump- tion of risk tolerant</i>	<i>Consump- tion of risk averse</i>	<i>A-D price, p</i>	<i>Proba- bility, π</i>	<i>Condi- tional price, p/π</i>
Initial	1.000	-0.001	0.501	0.499	1.000	1.000	1
	0.400	0.020	0.180	0.220	0.072	0.010	7.21
	1.000	-0.001	0.501	0.499	0.781	0.840	0.93
After one year	1.040	-0.003	0.523	0.517	0.128	0.150	0.85
	Price with certainty				0.981		
	Annual interest				1.92		
	Debt				0.034		

Table 3: Equilibrium with Heterogeneity in Risk Aversion

of the product is 0.072, but the conditional price is $0.072/0.014 = 7.21$. This is, in effect, the price that the risk-averse pay the risk-tolerant per unit of that product. In the best outcome, with an endowment of 1.04 units, the risk-averse provide the risk-tolerant 0.003 units. Output is cheap in that contingency—the conditional price is only 0.85. The effect of trade is to compress the range of consumption levels for the risk-averse, relative to the no-trade case and relative to the consumption of the risk-tolerant. In effect, the risk-averse hold 0.034 units of safe debt claims on the risk-tolerant. Of great interest in light of the message of this paper, the interest rate that the risk-averse receive on the debt is only 1.92 percent, compared to 3.27 percent without trade. A fairly small amount of heterogeneity in risk aversion generates a considerably lower safe real rate. The lower interest rate is not the result of a rise in the average coefficient of relative risk aversion among investors—the rate is somewhat above the no-trade rate if both types of investors have coefficients of 2.5.

5.3 Equilibrium with trade resulting from heterogeneity in beliefs about probabilities

Table 4 illustrates the depression of the safe interest rate that occurs in an economy where some investors are risk-tolerant in the sense that they believe that low realizations of the endowments are unlikely and others are risk-averse in the sense that they believe that those endowment realizations are more likely. The parameter values are the same here as in Table 2 except that $\pi_-^* = 0.02$, so the risk-averse investors believe that adverse shocks to the endowment are twice as likely as risk-tolerant investors believe. The corresponding values of

	<i>Endow- ment, y</i>	<i>Trade, x</i>	<i>Consump- tion of risk tolerant</i>	<i>Consump- tion of risk averse</i>	<i>A-D price, p</i>	<i>Proba- bility, π</i>	<i>Proba- bility, π^*</i>
Initial	1.000	-0.001	0.501	0.499	1.000	1.000	1.000
	0.400	0.034	0.166	0.234	0.085	0.010	0.020
	1.000	-0.027	0.527	0.473	0.705	0.840	0.680
After one year	1.040	0.089	0.431	0.609	0.188	0.150	0.300
			Price with certainty		0.977		
			Annual interest		2.31		
			Debt		0.029		

Table 4: Equilibrium with Heterogeneity in Beliefs about Probabilities

π_0^* and π_+^* , dictated by the assumption of no expected change in the endowment, are shown in the middle panel.

Table 4 shows more trade than occurs in Table 3. As in the earlier case, trade cushions the risk-averse against the bad outcome—the risk-tolerant provide 0.034 units to the risk averse in that contingency. The driving force of trade is powerful because the risk-averse believe that the bad outcome is twice as likely as the risk-tolerant believe. But the risk-averse also believe that the best outcome, on the bottom line of the middle panel, is twice as likely as the risk-tolerant believe. As a result, the risk-tolerant also provide a large amount of output, 0.089 units, in that contingency, the opposite of what happens with heterogeneity in the coefficient of relative risk aversion. The risk averse pay for these two contingent transfers by providing the risk-averse 0.001 units up front and 0.027 in the most like contingency, an endowment of 1.

Although there is more trade in this example of heterogeneous beliefs than in the earlier example of heterogeneous risk aversion coefficients, somewhat less of the trade has the form of debt—only 0.029, as shown in the bottom panel of the table. Heterogeneous beliefs result in a substantially lower interest rate, 2.31 percent, about one percentage point below the no-trade benchmark of 3.27 percent shown in Table 2.

Table 5 combines the two forms of heterogeneity, with $\gamma^* = 2.5$ and $\pi_-^* = 0.02$. Trade is even greater than in the two previous tables. Debt is 0.059 and the safe real interest rate is 0.44 percent, barely positive. The combination of the two forces that increase the demand for safe investments generates an interest rate in the range of current safe real rates.

	<i>Endow- ment, y</i>	<i>Trade, x</i>	<i>Consump- tion of risk tolerant</i>	<i>Consump- tion of risk averse</i>	<i>A-D price, p</i>	<i>Proba- bility, π</i>	<i>Proba- bility, π^*</i>
Initial	1.000	-0.001	0.501	0.499	1.000	1.000	1.000
	0.400	0.050	0.150	0.250	0.104	0.010	0.020
	1.000	-0.025	0.525	0.475	0.713	0.840	0.680
After one year	1.040	0.077	0.443	0.597	0.178	0.150	0.300
			Price with certainty		0.996		
			Annual interest		0.44		
			Debt		0.059		

Table 5: Heterogeneity in Both Probability Beliefs and Risk Aversion Coefficients

5.4 How heterogeneity affects debt and the interest rate over ranges of determinants

Figure 2 traces out the safe real interest rate and the amount of debt as functions of the coefficient of relative risk aversion of the risk-averse investors. It holds the other determinants constant at the base values in Table 1. The left ends of the two curves correspond to the no-trade equilibrium, with a high interest rate and zero debt. In the extreme case of $\gamma^* = 4$, at the right ends, the interest rate is minus 3 percent and debt is 0.10.

Figure 3 shows the two variables as functions of the belief π_-^* of risk-averse investors about the probability of a bad outcome. Again, other determinants are at their base values. Starting from the no-trade case at the left, the interest rate declines with rising heterogeneity of beliefs until $\pi_-^* = 0.02$, then rises again. Debt rises continuously.

Figure 4 shows the implications of different values of α , the fraction of the endowment owned by the risk-tolerant investors. Parameters are at their base values except that $\gamma^* = 2.5$, so the figure shows the implications in the case of mild heterogeneity of the risk-aversion coefficient. In the middle, at $\alpha = 0.5$, the interest rate and debt level are at the values in Table 3. Debt turns out to be at its maximal level at that value. Lower values of the endowment fraction held by the risk-tolerant imply lower interest rates. This conclusion is central for the paper because there are reasons to believe that growth of wealth in countries with large appetites for low-risk investments, notably China, is higher than in the U.S. and other risk-tolerant countries.

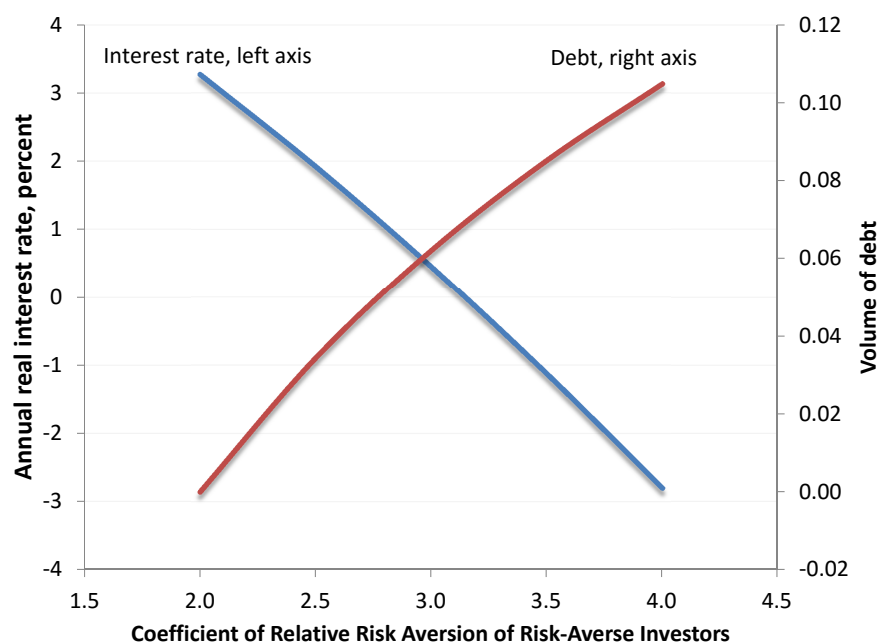


Figure 2: Relation between Real Interest Rate and Debt, and Coefficient of Relative Risk Aversion of Risk-Averse Investors

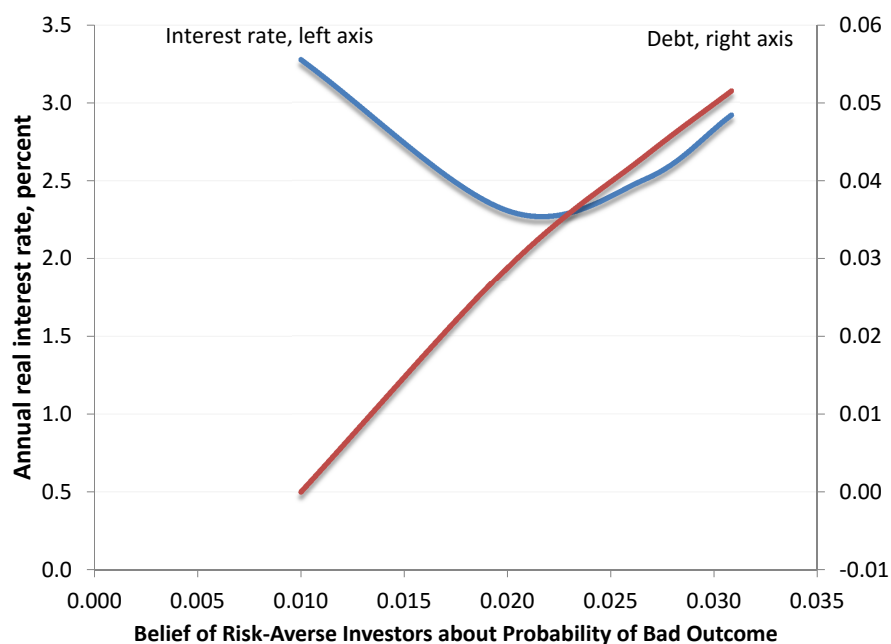


Figure 3: Relation between Real Interest Rate and Debt, and Belief of Risk-Averse Investors about the Probability of a Bad Outcome

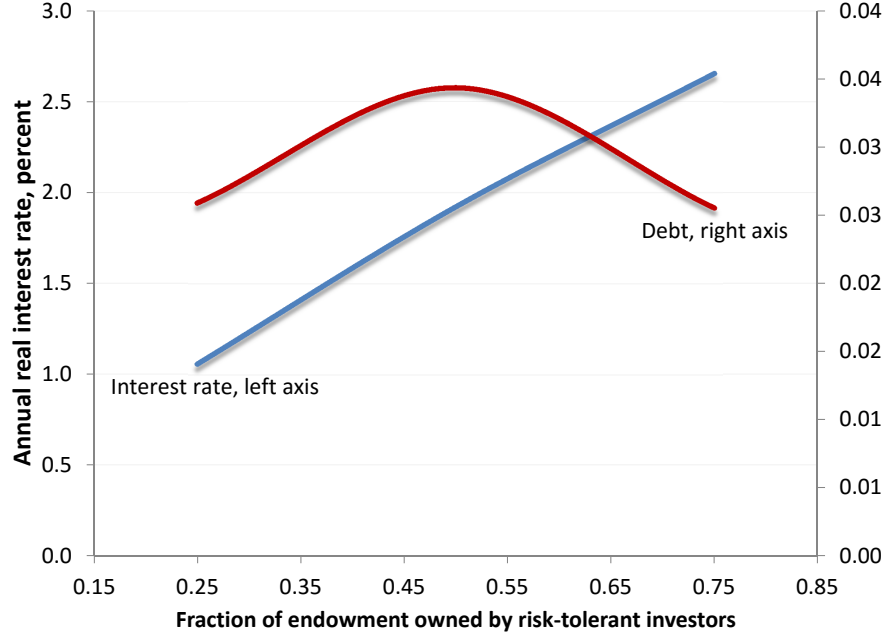


Figure 4: Relation between Real Interest Rate and Debt, and Fraction of Endowment Owned by Risk-Averse Investors

5.5 Importance of improbable highly adverse outcomes

All the results in this section are based on $\Delta_- = 0.6$, which implies that investors believe that every 100 or 50 years, the endowment drops by 60 percent. Volatility of this character is essential to the large effects on interest rates found here. An extensive literature emphasizes the importance of large unfavorable events with low probabilities in explaining the equity premium. Barro and Mollerus (2014) survey that literature and discuss the issue in connection with the demand of risk-averse investors for safe debt-type investments.

6 Evidence

This paper's main points are (1) that risk-splitting is socially efficient and will arise naturally in a market economy where risk-averse investors can hold debt claims on risk-tolerant investors, and (2) as the fraction of wealth in the hands of the risk-averse investors rises, the terms of trade shift in favor of the risk-tolerant investors, and the safe real interest rate will decline. This section identifies financial institutions that facilitate risk-splitting. It provides data on the current volume of debt issuance of the institutions and on the trend, generally but not always upward, in the volume.

	<i>\$ trillions</i>	<i>Ratio to GDP</i>
Federal government debt	15.2	0.85
Federally guaranteed GSE debt and guaranteed mortgages	8.1	0.45
State and local government debt	3.0	0.17
Non-financial business, bonds and loans	12.8	0.71
Non-guaranteed household mortgages	1.4	0.08
Other debt of households	4.7	0.26
Total	45.1	2.52

Table 6: Debt of U.S. Investors in 2015, in Trillions of Dollars and as Ratios to GDP

6.1 Debt

A reasonably safe debt instrument has the effect of concentrating the risk facing the borrower and reducing the risk facing the holder. The amount of debt issued by the agents in an economy is a good metric of general importance of the risk-splitting within the economy, or, in an open economy, of the importance of providing debt to domestic and foreign holders. Table 6 shows debt in trillions of dollars and as a ratio to GDP in 2015. To avoid double-counting debt held on both sides of the balance sheets of financial institutions, the table omits the financial sector. The items in the table are in rough order of safety, inferred from the relative yields of the debt. Federal debt stands at the top, in both safety and quantity. Just below are the obligations that are as safe as federal debt, the debt of the government-sponsored enterprises (Fannie Mae and Freddie Mac) and guarantees on mortgages securitized by the GSEs. Debt claims on households are at the bottom of the table.

The table shows that government has a large role in creating safe debt. It's obvious that U.S. government debt provides safe investments to investors around the world who want to shed risk. It's somewhat less obvious that they do so by moving that risk to risk-tolerant investors. Taxpayers are the effective equity holders in the government. A substantial fraction of government revenue, apart from the federal payroll tax earmarked for retirement benefits, arises from high-income taxpayers, who are presumably more risk-tolerant. The top 3.2 percent of personal income-tax returns arise from taxpayers with at least \$200,000 in adjusted gross income who, as a group, pay 48 percent of all tax. Thus, by issuing debt, the

government facilitates value-enhancing trade between risk-tolerant taxpayers and risk-averse investors around the world. Government guarantees of mortgage and other types of debt have the same effect—the safe guaranteed bonds suit risk-averse investors and the risk falls on risk-tolerant taxpayers.

Non-financial businesses issues bonds and borrows in the loan market in large volumes. Some businesses participate actively in risk-splitting by taking on debt and thus concentrating risks on their shareholders. Others, including some of the most valuable corporations, do the opposite, by accumulating debt on the asset side of their balance sheets. It is beyond the scope of this paper to explain the heterogeneity in corporate leverage. The important fact for this paper is that non-financial businesses as a group are major participants in splitting risks between debt held by risk-averse investors and equity held by risk-tolerant ones.

Households play a role in the creation of debt. They rank at the bottom in terms of the safety of their debt obligations and thus highest in yields paid. The biggest element is federally guaranteed mortgages, accounted for at the top. Guaranteed mortgages are claims on households with moderate incomes who are not likely to be risk-tolerant. They take on debt in spite of the resulting undesirable leverage because of the benefits of home mortgages. Much of the other debt, \$4.7 trillion or about a quarter of a year's flow of GDP, does fit the paradigm of this paper, as Figure 5 shows. The figure plots the ratio of holdings of equity to net worth. A value less than one indicate that a portfolio contains some debt held as an asset, as in the portfolio of a risk-averse investor. A value greater than one indicates that the investor has borrowed to hold additional equity and thus has a levered position in the stock market. The figure uses data on individual investors in the Survey of Consumer Finances ranked by the ratio of equity to net worth and collected into deciles. Each bar corresponds to a set of investors who hold 10 percent of the total net worth in the survey. On the left, it shows that the households with the lowest ratio of equity to net worth hold little equity, and therefore a large amount of debt on the asset side. These are the most risk-averse households, according to standard portfolio theory. The two bars at the right are the only ones that rise above the horizontal line marking pure-equity portfolios. The ninth percentile borrows only a little to participate in the process of risk-splitting. But the tenth percentile, on the far right, hold almost \$5 equity for each dollar of net worth. They borrow \$4 to fund the holding of the extra equity. Thus there is a minority, holding 10 percent of total net worth but accounting for essentially all of the non-mortgage, non-credit card borrowing in

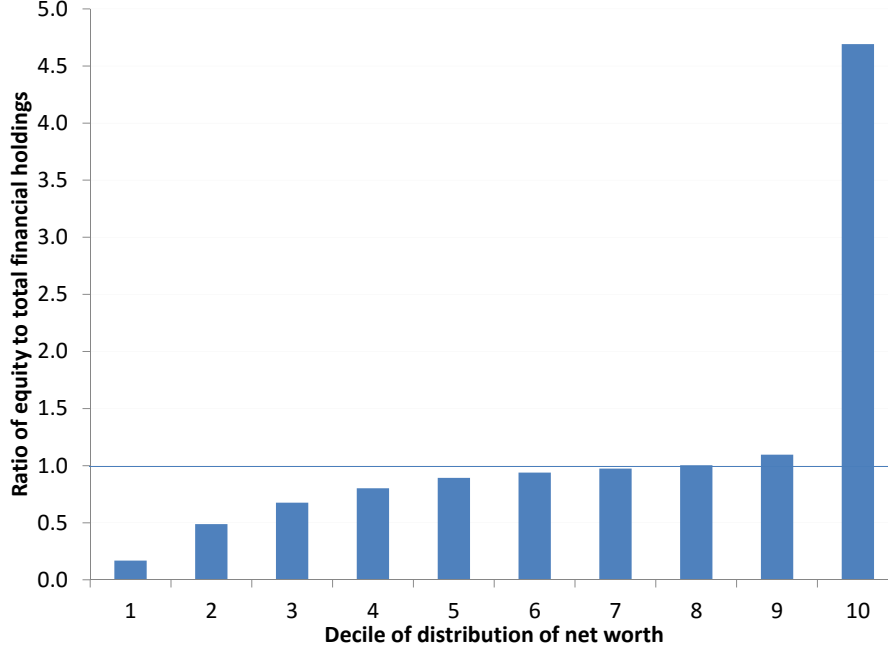


Figure 5: Equity as a Ratio to Net Financial Assets, SCF 2013

the households, who participate massively in risk-splitting. In effect, they create personal levered hedge funds to increase their exposure to risk and receive the market’s high return to bearing that risk.

6.2 Growth of institutions that facilitate trade between risk-tolerant and risk-averse investors

The canonical institution whose existence is rationalized by heterogeneity in risk aversion is the bank. See Landvoigt (2015) for a discussion along these lines. A traditional bank holds risky assets, funded by equity supplied by risk-tolerant investors and by risk-averse depositors. The latter generally consider deposits as risk-free debt, because the depositors have a primary claim over all of the assets, and the value of the assets is well above the promised value of the debt. The debt is over-collateralized. The equity holders face not just the risk of the assets, but the risk magnified by the prior claim of the depositors. It is difficult to analyze the growth of banks in the framework of this paper, for two reasons. One is that banks hold many debt-type claims as assets. The other is that big banks take large positions in derivatives that are not reported to the public in normal accounting disclosures in a way that helps understand banks’ role in splitting risk.

<i>Government</i>				<i>Private</i>				
<i>Decade</i>	<i>Consolidated government debt</i>	<i>GSE debt</i>	<i>GSE guaranteed debt</i>	<i>Private equity funds</i>	<i>Securitizations</i>	<i>Non-financial corporate debt</i>	<i>Repos</i>	<i>Non-mortgage household debt</i>
1980s	0.469	0.061	0.091		0.012	0.163	0.103	0.186
1990s	0.611	0.101	0.204		0.086	0.211	0.166	0.204
2000s	0.574	0.203	0.293	0.058	0.233	0.238	0.237	0.239
2010s	0.936	0.126	0.347	0.140	0.109	0.275	0.221	0.251

Table 7: Examples of the Scale of Risk-Splitting Institutions

Table 7 gives some major examples of risk-splitting institutions and the level and growth of their volumes, stated as in the previous table as ratios to U.S. GDP. The left panel describes government institutions and the right panel private ones. The most important way the consolidated government meets the needs of risk-averse investors around the world is the issuance of debt. The volume of debt was a bit below half a year's GDP averaged over the 1980s. In the 1990s the ratio reached over 0.6, then fell slightly in the 2000s (as deficits fell), then jumped upward in period 2010 through 2015, thanks to the financial crisis, recession, and poor growth thereafter. Changes in income tax rates starting in the 1990s shifted the burden of paying for government upward in the wealth distribution and presumably onto more risk-tolerant investors. A substantial widening of the wealth distribution compounded the tendency for taxpayers weighted by wealth to be increasingly risk-tolerant. The other two columns of the table relating to the government quantify programs that create safe debt instruments through government guarantees, that is, to the debt of the GSEs and to the debt guaranteed by the GSEs. Guarantees transfer the risk to the taxpayers in the same way as direct issuance of debt. GSE debt grew as a fraction of GDP until the crisis, then fell in the 2010s. Guarantees have grown monotonically.

On the private side of Table 7, the first column refers to private-equity funds (data are available only starting in 2000; these funds were small in prior decades). These funds sell limited-partnership equity interests to risk-tolerant investors, including high-wealth families and endowments. The funds sell debt or obtain loans from financial institutions, which directly or indirectly serve the needs of risk-averse investors. Private-equity funds are a good example of the principle of creating safe debt through over-collateralization. By contract, the debt-holders have rights to the value of the risky assets of the fund, up to the face value

of the debt. The debt is over-collateralized as long as the value of the assets exceeds the face value. At issuance, that margin is substantial, so default on the debt is unlikely. The value of the assets in private-equity funds grew rapidly after 2000, averaging 0.14 years of GDP in the years 2010-2014.

The next column shows private securitizations—private-label mortgage-backed bonds and bonds backed by other assets, such as car loans and credit-card balances. Securitizations accomplish risk-splitting by offering different tranches to investors with different risk aversions. Typically a large tranche has sufficient over-collateralization to qualify as almost fully safe. The remaining tranches are under-collateralized and thus magnify risk to suit risk-tolerant investors. Private securitizations became an important component of risk-splitting in the 2000s, but shrank after the crisis, as private-label mortgage securitizations virtually disappeared. The types of mortgages previously securitized in the private market are now held directly by banks, who accomplish risk-splitting in the traditional bank mode.

The column headed *Non-financial corporate debt* combines debt and loans that are claims on corporate business. Corporations in certain industries, notably transportation, are active in risk-splitting. Such a corporation offers reasonably safe bonds and loans that are over-collateralized by the value of the corporation's equity. Its equity is correspondingly riskier. Many corporations with high equity values, especially in high tech, do not participate in risk-splitting and often hold more debt on the asset side than they borrow.

The next column shows repurchase contracts or repos. These contracts are loans secured by claims on assets that over-collateralize the amount of the loan by an amount called the haircut. The size of the haircut depends on the volatility of the collateral asset. Repos are generally designed with enough haircut to make the loan quite safe—eligible for holding by risk-averse investors. Hedge funds and other institutions that borrow in the repo market on risky collateral, such as equity, can magnify the risks of their portfolios substantially, accomplishing a great deal of risk-splitting. Not all the repos included in the table have that role, however. Some are collateralized by safe assets and thus involve little risk-splitting. The volume of repos as a ratio to GDP roughly doubled from the 1980s to the 2000s, then fell a bit after the crisis.

The last column in Table 7 shows the volume of non-mortgage household debt. Most of this is incurred by high-wealth households to fund purchases of business assets or equity holdings. These households magnify their own risk by offering reasonably safe over-collateralized

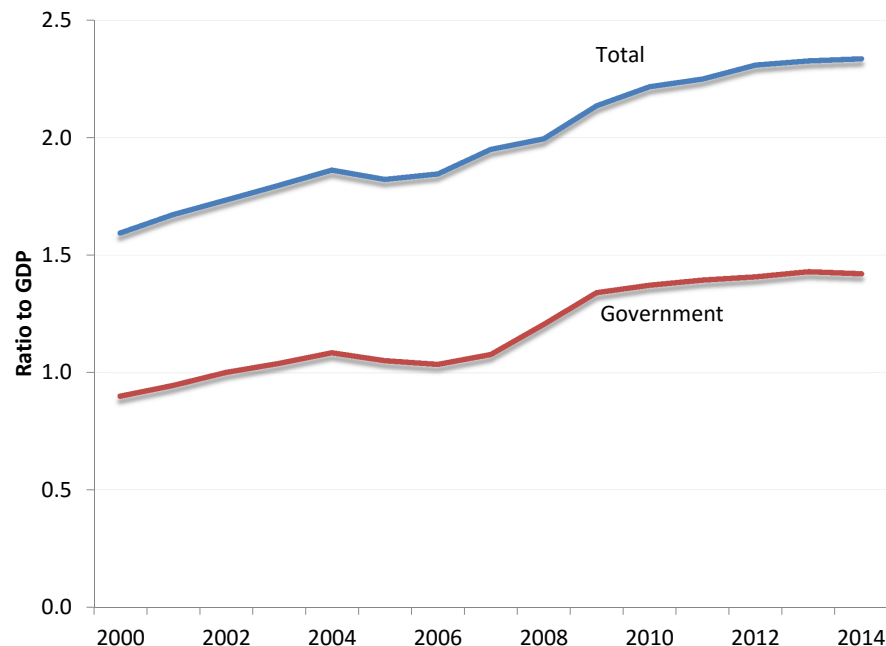


Figure 6: Scale of Risk-Splitting Institutions Relative to GDP

debt. In the 1980s, this form of risk-splitting was the second-largest entry in the table. It has grown slowly since then.

Figure 6 plots the annual sums of the scale variables in Table 7 since 2000. Both the total and the government component have growth smoothly over the period. The only declines occurred during the real-estate boom in the middle of the 2000s. There is a hint of a slowdown in the total starting in 2011. In general, the figure gives strong support to the hypothesis that risk-splitting is growing in the U.S. economy. The point of this paper is that capital markets respond to global shifts that place a growing fraction of wealth in the hands of risk-averse investors by doing more risk-splitting. Lower world interest rates are a side-effect of that trend.

6.3 The global saving glut

Table 8 shows data calculated from the main table in Bernanke et al. (2011). It compares worldwide holdings of securities that are claims on U.S. entities in 2007 and 2003. It breaks down the holders into foreign and U.S., and further breaks out designated countries as particularly important in the rising holdings of safe debt claims on the U.S. The designated countries are China and others in east Asia, leaving out Japan. The top line in each panel shows total U.S. issuances and the lines below shows the percentage distribution among the

		<i>Government issued or guaranteed</i>	<i>Corporate debt</i>	<i>Securiti- zations</i>	<i>Total debt</i>	<i>Equity</i>
	Total US issued, \$ trillion	9.3	4.5	1.7	15.5	14.3
	Percents:					
2003	Foreign holdings	22.0	25.9	11.3	21.9	12.9
	Designated countries	6.9	0.8	0.8	4.5	1.2
	US holdings	78.0	74.1	89.0	78.1	87.1
	Total US issued, \$ trillion	10.9	5.7	3.6	20.2	19.9
	Percents:					
2007	Foreign holdings	34.6	33.1	25.0	32.5	16.2
	Designated countries	14.3	1.4	1.4	8.4	2.0
	US holdings	65.4	66.8	75.0	67.5	83.8

Table 8: Holdings of U.S. Securities, 2003 and 2007, from Bernanke, *et al.*, “Global Savings Glut”

countries. In the framework of this paper, the first observation is that foreign investors are more risk averse than are American ones—foreigners in general hold almost twice as much U.S. debt as U.S. equity, whereas U.S. investors hold more equity than debt, in both years. And the designated countries are even more risk averse, holding about four times as much debt as equity in both years. Second, foreign holdings of U.S. securities in general grew rapidly over the four years, supporting the hypothesis that risk-averse portfolios are growing faster than risk-tolerant ones.

7 Concluding Remarks

The modeling and data in this paper support the hypothesis that growth in the wealth in the portfolios of risk-averse investors relative to the wealth of risk-tolerant investors is a source of the downward trend in the worldwide real interest rate. Continuation of the higher growth rates in China and other countries with high propensities to hold debt rather than equity suggests that real rates may continue to decline, or at least not rise back to earlier levels. The extraordinarily low rates for real long-term U.S. Treasury bonds confirms this hypothesis—the 30-year TIPS rate at this writing is 0.89 percent.

The federal government in recent years has contributed immensely to the risk-splitting that is the efficient market response to growing wealth among risk-averse investors. Current predictions show the national debt rising moderately in relation to GDP in the next decade,

followed by a return to rapid growth in later decades. The feasibility of this path depends on real rates on that debt at current rates or below. The ideas in this paper point in the direction of feasibility, as does the world’s continuing willingness to treat federal debt as safe.

Most discussions of monetary policy, on the other hand, foresee normalization of short-term real rates substantially higher than current rates. For example, at the FOMC’s meeting on March 16, 2016, the median in the Economic Projections document (containing the “dot plot”) for the federal funds rate in the longer run, was 3.3 percent, 3 full percentage points above the then-current actual rate. Under the FOMC’s forecast, monetary policy would have that many points of headroom to stimulate the economy in the face of a mildly recessionary shock. In a world economy with continuing declines in the fundamental determinants of the safe interest rate, nowhere near that much headroom would exist. The equilibrium real rate could be minus one percent, corresponding to a nominal rate of plus one percent. The lower bound on the nominal rate would become a serious obstacle to effective monetary policy even in the face of small adverse shocks.

8 Related Literature

Furceri and Pescatori (2014) is a comprehensive review of real safe interest rates in recent decades and in many countries, with results similar to those in Figure 1.

The paper closest to this one is Barro and Mollerus (2014), which studies heterogeneity in risk aversion as a property of an economy that explains substantial trade in risk-free assets. Their paper focuses on explaining the observed volume of outstanding risk-free debt based on heterogeneity in two populations of investors, one risk averse and the other risk tolerant. The paper contains cites for a number of earlier papers on heterogeneity in risk aversion. It notes the relation between heterogeneity and the safe interest rate, but focuses on the effects of disasters on debt and interest rates rather than trends in heterogeneity, the topic of this paper.

For recent surveys of the finance literature on heterogeneous beliefs, see Basak (2005) and Xiong (2013).

Wang (1996) derives closed-form solutions for safe yields in an economy with heterogeneous risk aversion and homogeneous beliefs. The coefficient of relative risk aversion of risk-averse investors is an integer multiple of the coefficient for risk-tolerant investors. The paper does not explore the relation between heterogeneity of risk aversion and the level of

safe interest rates. The paper surveys earlier contributions to the study of heterogeneous risk aversion.

Duffie (2010), chapter 1, section E, discusses heterogeneous risk aversion and the existence of a representative-agent economy that mimics the complete-markets equilibrium of the economy with heterogeneity.

Chen, Joslin and Tran (2012) considers an endowment economy with investors who are heterogeneous with respect to beliefs about the probability of a disaster, an event that cuts consumption by 40 percent. The focus is on the effect of heterogeneity on the equity premium and on the nonlinear response of the premium to the share of investors who are optimistic about the probability of a disaster. Their economy achieves complete markets with a safe interest rate, a market for claims on the aggregate endowment, and a continuum of disaster-insurance contracts.

Ilut, Krivenko and Schneider (2016) is a recent contribution to the literature on heterogeneity in beliefs using the ambiguity-aversion framework.

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