Pricing Government Credit: A New Method For Determining Government Risk Exposure

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The federal government administers over 100 direct loan and guarantee programs that exceeded $18 trillion in 2013 (Lucas, 2014b).

Controversy surrounds the methods used to reflect these programs on the federal budget and balance sheet.

We apply a method for estimating the government’s required capital allocation that explicitly recognizes the interdependence associated with the likelihood of joint risk in loan and guarantee programs.

- Builds on the insight of Hanson et al. (2016) that program social and fiscal risks depend on how they covary with each other.
The Problem

- How do we report the risk associated with government insurance and guarantee programs?
- Current practice is to report the “Economic Value” of the program (i.e. present value of expected losses).
- A debate exists over implementation with respect to discount rates:
  - Federal Credit Reform Act of 1990 (FCRA) versus “Fair Value Approach”
    - (see, for example, Congressional Budget Office, 2012; Lucas, 2016, 2012, 2014a,b; Hanson et al., 2016; Kelly et al., 2016; Heaton et al., 2010).

- Current practice also fails to recognize:
  1. the potential for dependence in losses (and “tail dependence” in particular),
  2. and the risk margin over and above “economic value”.

Examples

- Consider the government’s housing and education loan guarantee programs.
  - Research on student loans and mortgages documents that default rates are positively correlated with periods of economic stress (for example, see IFE, 2007; Ambrose et al., 2016; Looney and Yannelis, 2015).
  - Ambrose et al. (2016) show that individuals living in areas with higher house price growth were less reliant on student debt to fund education because of the ability to tap into growing home equity.
  - Thus, dependence in losses exists across programs.

- Consider the FHA Mutual Mortgage Insurance Fund (MMI).
  - MMI consists of variety of different mortgage types (30-yr FRM, 15-yr FRM, ARMs, 30-yr SR, 15-yr SR, SR ARMs, HECMs) and LTV levels.
  - Thus, tail dependence likely exists across the mortgage types.
An Example: Weak versus Strong Tail Dependence

**Figure:** 5,000 samples of two losses: $X_1$ and $X_2$, both following $\text{Exp}(\text{mean} = 1)$. The two losses have the Gaussian copula with parameter $\rho = 0.87$ (left) or the Gumbel copula with parameter $\theta = 3$ (right).
Suppose government has $n$ loan guarantee programs.

Each program has a random loss of $X_i$ at the end of the period, $i = 1, \ldots, n$, where $X_i \geq 0$.

The government holds risk capital that is determined through the Tail Value at Risk (TVaR) of the program loss: $E [X_i | X_i \geq \text{VaR}_{z\%}[X_i]]$.

The risk capital needed for assuming program $i$’s risk is

$$TVaR_{99\%} [X_i] - E [X_i],$$

The premium charged for the guarantee is:

$$\text{prem}_i = \frac{E [X_i] + \delta \times (TVaR_{99\%}[X_i] - E [X_i])}{1 + r_f}. \quad (1)$$
Now recognize potential for holding aggregate risk capital for all programs as a whole.

Total cost to the government is: \( S = X_1 + \cdots + X_n \).

The total amount of capital is: \( E[S | S \geq \text{VaR}_{99\%}[S]] - E[S] \).

Assume capital allocated to each program based on its marginal contribution to total risk: \( CA_i = E[X_i | S \geq \text{VaR}_{99\%}[S]] - E[X_i] \).

The resulting premium charged for the guarantee is:

\[
prem_i = \frac{E[X_i] + \delta \times (CA_i)}{1 + r_f}.
\] (2)
Four cases that illustrate the determination of the risk associated with each program:

1. Price the programs as stand alone entities using each line’s capital reserve;
2. Assume independence across programs and use their marginal risk contribution to allocate risk capital and set the premiums;
3. Assume losses are dependent through a Gaussian copula and use their marginal risk contribution to allocate risk capital;
4. Assume losses are dependent through a Gumbel copula and use their marginal risk contribution to allocate risk capital.
Simulation

Percentage Difference in Premiums Over Current Practice (Unequal Means)

Line 1
Line 2
Line 3

Stand Alone
Independent
Gaussian Dependence
Gumbel Dependence
Simulation

Percentage Difference in Premiums Over Current Practice (Equal Means)

- Stand Alone
- Independent
- Gaussian Dependence

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Simulation

Estimated Risk (TVaRs)

- Equal Expected Losses
- Unequal Expected Losses

- Stand Alone
- Independent
- Gaussian Dependence
- Gumbel Dependence

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Simulation

Marginal Contribution of Risk Assuming Unequal Means Across Lines

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<tr>
<th>Line</th>
<th>Independent</th>
<th>Gaussian Dependence</th>
<th>Gumbel Dependence</th>
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Future Work

- Estimate the structure of the dependence of losses across programs.
- Estimate the strength of tail dependence in loan guarantee programs.
- Continue research on appropriate discount rate for determining present value of expected losses.
- Policy question: determine confidence level ($z\%$) for TVaR
Thank You!


