# A Theory of Collateral for the Lender of Last Resort

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We build a model to analyze the optimal lending and collateral policy for the lender of last resort. Key to our theory is the idea that the central bank's policy can impose an externality on private markets. On the one hand, while lending against high-quality collateral protects the central bank from potential losses, it can adversely affect the pool of collateral in funding markets and thus impair their efficient functioning since the much needed high-quality collateral gets tied up with the central bank. On the other hand, while lending against low-quality collateral exposes the central bank to counterparty risk, it improves the pool of collateral in funding markets and can unlock frozen markets. We characterize the optimal policy for a central bank by taking account of these trade-offs. We show that, contrary to what is generally accepted, it may be optimal for the lender of last resort to lend against low-quality collateral.

**Keywords:** Central bank, liquidity, Bagehot, interbank market, lending facilities

# 1 Introduction

In his famous 1873 book *Lombard Street*, Walter Bagehot advocates four principles for central banks to follow when they act as the lender of last resort (LoLR): lend only to illiquid but solvent banks, lend at a penalty rate, lend against good collateral valued at pre-panic prices, and make clear in advance the readiness to lend any amount to any institution that meets the conditions for solvency and collateral. Despite having shaped central bank policies around the world for more than a hundred years, these principles continue to be the subject of intense debate, in part because they still lack a rigorous theoretical foundation. Moreover, the institutional environment has significantly changed since Bagehot's time. For one thing, unlike today's central banks, which are public institutions, the Bank of England was then privately held (with some privileges from the government). Also, the financial system today is much more connected and complex than ever before, making financial stability a bigger concern for authorities and, in some countries, an explicit goal of central banks.

In this paper, we attempt to contribute to the literature on the LoLR by developing a theory of collateral for the LoLR. Key to our approach is the idea that central banks' policies affect the pool of collateral and liquidity creation in private markets. On the one hand, lending against high quality collateral protects the central bank from potential losses, but it can adversely affect the pool of collateral in private markets and impair their efficient functioning because high-quality collateral gets tied up with the central bank. On the other hand, lending against low-quality collateral exposes the central bank to counterparty risk, but it improves the collateral pool in money markets and unlocks these markets when they freeze up. We characterize the optimal policy for the central bank by taking account of these trade-offs. We show that, contrary to what is generally accepted, it may be optimal for the lender of last resort to lend against low-quality collateral.

Our model involves maturity transformation and collateral circulation using three dates and three groups of banks. Banks in the first group, denoted "borrowers," have access to long-term projects at the initial date. The projects are risky but they have a positive net present value (NPV). Borrowers do not have funds at the initial date, but they have collateral, which can be of high or low quality.[1] Borrowers can use their

[1] Alternatively, we can think of high-quality collateral as being easy to value, and therefore liquid; and low-quality collateral as being difficult to value and therefore illiquid, a condition that requires a

collateral to raise funding from our second set of banks, denoted as "lenders". Lenders do not have access to the new projects at that initial date, but they have funds that they can lend against borrowers' collateral, as well as some assets in place. Lending and borrowing at the initial date is short term, which creates a maturity mismatch.

At the interim date, lenders are subject to a liquidity risk; their assets in place will generate long-term returns at the final date only if lenders meet the liquidity shock. Since the returns from their projects at the final date are non-verifiable, lenders can meet their liquidity needs only by raising funds from the third group of banks using the collateral they obtained from borrowers. When lenders have high-quality collateral, they can always raise enough funding to meet their liquidity needs; however, when they have low-quality collateral, they may not be able to do so depending on the scale of the liquidity shock. In that case, lenders will not renew the loans they provided to borrowers, which leads to a costly early liquidation of the borrowers' long-term projects. Therefore, in our setting, high-quality collateral can be circulated as cash, whereas low-quality collateral generates less liquidity, exposing both lenders and borrowers to liquidity risk.

We first analyze banks' decisions in the absence of an LoLR. Lenders' liquidity demand at the interim date creates funding risk for borrowers if the lender cannot meet its liquidity needs by recycling the collateral it received at the initial date. Hence, borrowers that pledge low-quality collateral will be exposed to funding risk, while those that pledge high-quality collateral will not. Borrowers will invest at the initial date only when the expected return from investing—taking into account the potential risk of an early liquidation—is positive.

In the presence of symmetric information on the quality of collateral, borrowers with high-quality collateral invest without experiencing any liquidity risk. In contrast, borrowers with low-quality collateral may forego investment since—if the likelihood of them not getting their loan renewed is high—the expected cost of liquidation may outweigh the return from the project. Under asymmetry of information, lenders are unable to verify the collateral quality pledged by borrowers, and the resulting liquidity risk can lead to a total breakdown of investment—even banks with high-quality collateral may refrain from investing because their collateral would be pooled with low-quality collateral and therefore may fail to generate sufficient liquidity.

Note that it is socially inefficient for banks not to invest because all projects

significant haircut when such collateral is borrowed against. See Section 6.4.

have a positive NPV. By imposing private losses on banks, early liquidation risk can lead banks to bypass valuable projects, resulting in real effects. This creates a potential role for the LoLR. Furthermore, when liquidations lead to real losses, the LoLR could improve welfare by insuring banks against liquidity risk to prevent early liquidations and thus facilitate valuable investment. In particular, when the cost of borrowing from the central bank is lower than the cost associated with liquidity risk, banks would borrow from the central bank and insure themselves against liquidity shocks.

After we establish a potential role for the LoLR, we analyze the optimal LoLR policy. One potential policy for the central bank would be to lend only against high-quality collateral, which would protect the central bank against potential losses. With symmetric information about the quality of collateral, banks with high-quality collateral are not exposed to liquidity risk. Therefore, this policy would be welfare *neutral*—it is basically an exchange of perfectly liquid collateral for central bank liquidity. However, in the presence of asymmetric information, that policy could have *negative* effects. That is because the central bank would lower the quality of the pool of collateral in the market, which would eventually impose an externality on those not borrowing from the central bank, affecting the functioning of private markets.

Consider the situation in which all banks invest but the liquidity risk is sufficient to drive banks with high-quality collateral to borrow from the central bank to avoid being pooled with banks with low-quality collateral in private markets. In this case, central bank lending against high-quality collateral has two mutually opposing effects—while it directly increases output by insuring borrowers against liquidity risk, it also imposes a negative externality on the rest of the banks because it lowers the overall collateral quality in the private market, which can more than offset the direct effect. Hence, welfare would be enhanced if the high-quality collateral remained in the private market rather than being removed by the central bank. In the extreme, this LoLR policy can increase liquidity risk in money markets to the point that they cease functioning. In short, by lending only to banks with high-quality collateral, the central bank can lead to a breakdown of markets that would otherwise function despite liquidity risk.

Despite these potential unintended consequences, lending only against high-quality collateral can improve welfare in certain situations. Consider the case in which asymmetry of information precludes all banks from investing when the liquidity risk is significantly high. In this case, a central bank that lends against high-quality collateral can provide insurance against liquidity risk, facilitate investment for those banks accessing the LoLR, and, in doing so, protect itself from any potential losses.

In the alternative policy, the central bank could lend against low-quality collateral. Although that would expose the central bank to potential losses, the policy could have positive effects that outweigh those losses. The net gain would arise if lending against low-quality collateral improved the quality of collateral in the private market, which could revive an otherwise frozen market.

We take these trade-offs into account to characterize the optimal lending and collateral policy for the central bank. We show that, indeed, in some circumstances Bagehot's policy—allowing the LoLR to lend only against high-quality collateral—is optimal. However, we also identify circumstances under which this policy is inferior to a policy in which the LoLR lends against low-quality collateral. Hence, our results have potentially important policy implications.

Our paper adds to the literature by providing a theory of collateral for the LoLR. The literature on the LoLR, which dates back to Thornton (1802) and Bagehot (1873) and is already vast.[2] Its main focus has been to provide a rationale for the LoLR, usually linked to a failure in the interbank market due to frictions.[3] Researchers have also covered other important topics related to the LoLR. Repullo (2000), Kahn and Santos (2005, 2006) and Ponce (2010), for example, investigate who should act as the LoLR LoLR in the presence of asymmetrical information and differences in policy objectives among regulators. Goodfriend and Lacker (1999) and Freixas (1999), in turn, investigate the importance of "constructive ambiguity" when central banks are unable to commit to limit lending to troubled institutions.

However, to date, researchers have not paid much attention to the optimal design of the contract used by the LoLR nor, more specifically, to the role of collateral. This is rather surprising since central banks around the world typically demand collateral when they extend liquidity support to banks. It is also surprising given the large literature on the importance of collateral for bank lending.[4] The only exceptions appear to be

[2] For surveys, see Bordo (1990), Santos (2006), and Freixas et al. (1999).

[3] The frictions include asymmetric information about banks assets (Flannery 1996; Freixas and Jorge 2007); market power of banks in the interbank market (Acharya, Gromb, and Yorulmazer 2012); banks free-riding on each others liquidity (Bhattacharya and Gale 1987); central banks liquidity (Repullo 2005) or banks incentives to hoard liquidity for strategic and precautionary incentives (Diamond and Rajan 2010; Gale and Yorulmazer 2012).

[4] Collateral can be useful in alleviating credit rationing problems (Stiglitz and Weiss 1981), signaling

Choi, Eisenbach, and Yorulmazer (2016), who compare various liquidity facilities; and Koulischer and Struyven (2014), who, like us, examine the role of central bank collateral policies. Koulischer and Struyven (2014) show that, during systemic crises, the central bank can improve welfare by also lending against low-quality collateral when all high-quality collateral is already exhausted. However, we show (1) the potential unintended consequences of lending against high-quality collateral and (2) that lending only against low-quality collateral when high-quality collateral is still available can improve welfare, a result similar to the effect of purchasing certain types of assets in Tirole (2012). Finally, Gorton and Ordoñez (2016) analyze the role of "information externality" that an LoLR generates, as we do in this paper.

The rest of the paper is organized as follows. Section 2 introduces our model. Section 3 solves for the banks behavior, both in the absence and in the presence of an LoLR. Section 4 investigates the effect of LoLR lending in stimulating liquidity in the money markets. Section 5 discusses the optimal LoLR policy. Section 6 presents some extensions to our model. Section 7 concludes with final remarks.

# 2 Model Setup

We begin by presenting a model of collateral in which agents (banks) trade excess liquidity. Our key objective is to highlight the liquidity risks that emerge with the circulation of collateral among banks.

## 2.1 Banks and liquidity shocks

Our model has three dates $t = 0, 1, 2$. There are three groups of agents, which we call banks and denote by $i = A, B, C$. We assume that all banks have access to a storage technology, with a rate of return equal to 1, and consume only at $t = 2$. All banks are risk neutral and have a discount factor of 1.

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the quality of the borrowers investment project (Chan and Kanatas 1985), mitigating underinvestment problems (Stulz and Johnson 1985), improving banks monitoring incentives (Rajan and Winton 1995; Gorton and Kahn 2000), reducing borrowers risk-shifting incentives (Boot, Thakor, and Udell 1991), improving efficiency of the renegotiation that occurs in bankruptcy (Bester 1994; and Hege and MellABarral 1999), and altering the priority position of a creditor that would otherwise be subordinated to a senior creditor (Longhofer and Santos 2003).

Banks in group $A$ ($A$-banks) are endowed with an investment opportunity that requires one unit of cash funding at $t = 0$. The investment generates a random return at $t = 2$ : with probability $p$ it generates $\bar{R} > 1$, and with probability $(1-p)$ it generates $\underline{R} < 1$. We denote the expected return by $R = p\bar{R} + (1-p)\underline{R}$ and assume the investment has a positive net present value, $R > 1$. The project is illiquid and can be liquidated at $t = 1$ in a lump sum fashion, in which case it will only generate $r$, where $\underline{R} < r < 1$. $A$ banks start out with no cash but are endowed with collateral worth $c$ at $t = 1$. Collateral, however, can be of high or low quality such that $c \in \{c_H, c_L\}$ with $0 \leq c_L < c_H \leq 1$. Further, we assume that a fraction $\alpha$ of $A$ banks own high-quality collateral, $c_H$, and a fraction $(1-\alpha)$ own low-quality collateral, $c_L$. Since $A$ banks do not have any cash at the outset, they need to borrow from other banks to finance the investment.

Group $B$ banks are endowed with one unit of cash at $t = 0$ and have no investment opportunity at that date. However, they have assets in place that produce a cash flow $e \in [e_{min}, e_{max}]$ at $t = 1$, and the long-term output worth $R^B > 1$ at $t = 2$. All agents in the economy learn the realization of $e$ at $t = 0$. We think of $e$ as a measure of "systemic" (or "fundamental") shocks since a low (high) $e$ corresponds to a low (high) cash inflow affecting all $B$ banks at $t = 1$. In addition to this fundamental shock, we assume $B$ banks receive an idiosyncratic liquidity shock at $t = 1$, which requires a cash injection $l$, with $l \sim U[0, 1]$ and *i.i.d.* across the $B$ banks. Their long-term output $R^B$ is realized only if they are able to fully fund their interim liquidity shock $l$ at $t = 1$; otherwise it is worth zero. For reasons that we will explain later, we assume that group $B$ banks' long-term return is non-verifiable à la Hart and Moore (1994, 1998).

Finally, group $C$ banks have no investment opportunities but they do receive one unit of cash at $t = 1$ (regardless of the state of the financial system), which they can lend in the money market.

For simplicity, we make a set of assumptions about the parameters of our model. To start with, we assume that $c_L + r > 1$. This implies that a lender who grants a short-term loan to $A$ banks at time $t = 0$ is not exposed to credit risk. Next, we assume that $c_H$ is high enough such that $c_H + e_{min} \geq 1$, while $c_L$ is low enough so that $c_L + e_{max} < 1$. The former condition indicates that a lender who grants a short-term loan to $A$ banks with high-quality collateral at time $t = 0$ will not be exposed to liquidity risk, while the latter condition indicates that lending to $A$ banks with low-quality collateral exposes the lender to liquidity risk. Finally, we assume that lending to an $A$ bank with high-quality collateral does not expose the lender to credit losses when the investment matures at

$t = 2$, $c_H + \underline{R} \geq 1$, while lending to an $A$ bank with low-quality collateral exposes the lender to credit losses when the investment matures at $t = 2$, $c_L + \underline{R} < 1$. We summarize these conditions in the following assumption:

**Assumption:** $c_L + e_{max} < 1 \leqslant c_H - e_{min}$, $c_L < 1 - \underline{R} \leqslant c_H$, and $c_L + r > 1$.

## 2.2 Market for liquidity

Money markets are competitive such that lenders expect to receive an expected rate of return equal to 1. At $t = 0$, an $A$ bank can take out a short-term loan of 1 unit from a $B$ bank, pledging its collateral if necessary. Recall that since we assume $c_L + r > 1$, granting a short-term loan to an $A$ bank does not expose the lender (a $B$ bank) to default risk.

At $t = 1$, the $B$ bank can either roll over the loan or refuse to renew it. If the $B$ bank refuses to renew the loan, the $A$ bank needs to liquidate its project early. However, if the $B$ bank agrees to roll over the loan, it may need to borrow some funds at $t = 1$. Recall that $B$ banks receive a cash flow $e \in [e_{min}, e_{max}]$ at $t = 1$, but they will need to meet their idiosyncratic liquidity needs $l \sim U[0, 1]$. $B$ banks that need funding will borrow from a $C$ bank, pledging the collateral they received from $A$ banks at $t = 0$. Since the return of a $B$ bank's project at $t = 2$, $R^B > 1$, is non-verifiable, then that bank can borrow only up to the value of its collateral (Hart and Moore 1994, 1998). For simplicity, we assume that $A$ banks cannot directly borrow from $C$ banks at $t = 1$, when their collateral is already pledged with $B$ banks.[5]

## 2.3 Lender of last resort

The central bank can act as the LoLR and lend to $A$ banks at $t = 0$. We focus on the LoLR lending at $t = 0$ because we want to understand how the LoLR's collateral policy can affect the liquidity in the money market *ex post* by changing the pool of collateral available for the private agents. We assume that the LoLR can verify the quality of

[5]For instance, $C$ banks are "outsiders" who cannot verify the output by $A$ and $B$ banks, while no such friction exists between $A$ and $B$ banks. It could be due to relationship lending between $A$ and $B$ banks, whereas no relationship (hence search friction) exists between $A$ and $C$ banks. See Section 6 for more discussion on the relationship lending in the interbank market.

assets that banks may post as collateral for loans from the LoLR.[6]

At $t = 0$, the LoLR will announce its policy, which may include a collateral requirement, $j \in \{H, L\}$ (i.e. high-quality, low-quality, or both) it will accept against the loan, the interest rate it demands on its loans, $\gamma_j$, and finally the maximum amount it is willing to lend, $x_j$. Both $x_j$ and $\gamma_j$ can be fixed or dependent on the borrower's collateral.[7] When the LoLR pre-specifies the amount it is willing to lend, $x_j$, , and there is excess demand, we assume that it allocates $x_j$ randomly among banks seeking loans.[8]

## 2.4 Timeline

Figure 1 summarizes the timeline of our model. At $t = 0$, the fundamental state of the economy, $e$, is revealed and the LoLR announces its lending policy. Eligible $A$ banks that choose to borrow from the LoLR do so, while the remaining $A$ banks choose whether to borrow from $B$ banks in the private market or not borrow (and thus not invest) at all. At $t = 1$, the idiosyncratic liquidity shock $l$ is realized. After they learn their liquidity shock, $B$ banks decide to either call back their loan to $A$ banks (in case they have one) or borrow from $C$ banks.[9] Output gets realized at $t = 2$, and agents consume.

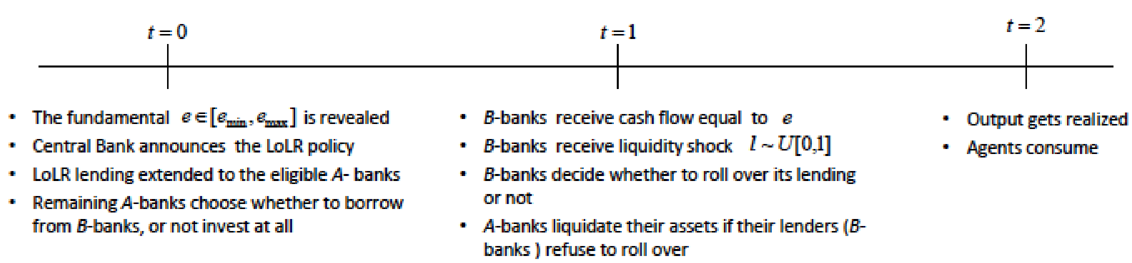


**Figure 1:** Timeline of the model

[6]If the LoLR could not evaluate the quality of collateral that banks pledge at $t = 0$, it could examine it ex post and impose a penalty on banks that do not report truthfully. This would induce banks to report the true quality ex ante. See Section 6.4 for an alternative interpretation of the collateral quality.

[7]Note that $\gamma_j$, or part of it, could arise from the stigma of borrowing from the central bank, instead of being the policy variable entirely.

[8]We discuss in Section 6.3 alternative arrangements, including auctions, that the LoLR could consider to address banks excess demand for liquidity.

[9]We assume that, when indifferent, $B$ banks choose to borrow from $C$ banks instead of calling the loan back.

# 3 Individual bank behavior

In this section, we analyze banks choices, investigating optimal bank decisions in the absence, and then in the presence, of the LoLR. On the basis of these results, we will compare the welfare implications of different LoLR policies.

## 3.1 Banks' decisions in the absence of LoLR

We start by investigating banks' decisions when there is no LoLR. We focus on $A$ banks' decisions because, as we will see, the $B$ banks' problem is trivial. At $t = 0$, after the state of the economy $e$ is realized, $A$ banks decide whether to borrow from $B$ banks and invest, or to forgo their investment opportunity.

### 3.1.1 Symmetric information

We first assume that the collateral types are common knowledge and analyze $A$ banks' optimization problem at $t = 0$, illustrating the role of collateral in mitigating liquidity risks. We begin with the $A$ bank holding high-quality collateral at $t = 0$. Recall that $c_H + e_{min} \geq 1$, so that, when an $A$ bank pledges high-quality collateral to a $B$ bank, the $B$ bank always rolls over its loan at $t = 1$. That is because $B$ banks will be able to meet their liquidity needs at $t = 1$ by borrowing from $C$ banks, pledging the high-quality collateral they received from $A$ banks and their own cash, $e$, regardless of the scale of the liquidity shock, $l$ ($\leq 1$), they experience. Therefore, there is no rollover or liquidity risk for $A$ banks that borrow against high-quality collateral, and their expected payoff can be written as

$$\Pi_H = R - 1 + c_H,$$

because they start with an investment project with an NPV equal to $R - 1$ and with collateral valued at $c_H$.

We now analyze the $A$ banks with low-quality collateral at $t = 0$. $B$ banks are willing to lend to these banks at $t = 0$ since $c_L + r > 1$. That is, they can always recover their loan by refusing to renew it at $t = 1$. However, when a $B$ bank does not renew its loan, this will lead to the early liquidation of the $A$ bank's investment, resulting in a loss of $R - r$. $B$ banks can borrow at $t = 1$ from $C$ banks and fully meet their liquidity shock only if the sum of their cash, $e$, and the collateral they received from $A$ banks, $c_L$, is greater than their liquidity needs, that is, $e + c_L \geq l$. This occurs with probability

$(c_L + e)$ since $l \sim U[0,1]$ and $e + c_L < 1$ for all $e$ under our assumption of $e_{max} + c_L < 1$. Otherwise, $B$ banks decide not to renew their loans. Therefore, since the $A$ bank always pays the expected competitive lending rate of 1, given the fundamental shock $e$, the expected payoff of an $A$ bank with low-quality collateral can be written as

$$\Pi_L = (c_L + e)\, R + (1 - c_L - e)\, r - 1 + c_L,$$

since with probability $1 - c_L - e$ its lender cannot meet its liquidity needs and refuses to renew the loan, forcing the $A$ bank to liquidate its project and generating a return of $r < R$ at $t = 1$. Otherwise, the loan is rolled over and the $A$ bank generates a return of $R$ at $t = 2$. Analogous to $\Pi_H$, this can be written as:

$$\Pi_L = R - 1 + c_L - \underbrace{(1 - c_L - e)\,(R - r)}_{\text{liquidity risk}},$$

where the last term represents liquidity risk, i.e., the liquidation cost of $R - r$, which occurs with probability $(1 - c_L - e)$.

If the $A$ bank does not borrow, it will simply end up with its collateral $c_L$. Thus, the $A$ bank will borrow and invest when $\Pi_L \geq c_L$, that is, when

$$R - 1 \geq (1 - c_L - e)\,(R - r).$$

Intuitively, if the net return $R - 1$ from the investment is less than the expected liquidation cost of $(1 - c_L - e)\,(R - r)$, then an $A$ bank with low-quality collateral will choose not to invest. The condition for the $A$ bank borrowing/investment decision can be written as

$$e \geq 1 - c_L - \frac{R - 1}{R - r} \equiv e_L. \tag{1}$$

Analogously, since $A$ banks with high-quality collateral will always choose to invest, we can define their investment threshold as $e_H \equiv e_{min}$. This gives us the following proposition on the fundamental threshold for investment.

**Proposition 1** *At $t = 0$, an $A$ bank with high-quality collateral always invests. An $A$ bank with low-quality collateral invests if and only if $e \geq e_L$, where $e_L$ is given in (1).*

Note that $e_L$ is decreasing in $c_L$ so that loans are rolled over more often for higher values of the collateral, which in turn induces more investment ex ante. Hence, better-quality collateral reduces liquidity risk for both lenders ($B$ banks) and borrowers ($A$

banks) and facilitates investment. Collateral of very high-quality (i.e. $c_H$ in our setup) is so liquid—in essence equivalent to cash—that it can circulate in the system without generating any liquidity risk.

### 3.1.2 Asymmetric information

We now assume that money market lenders cannot distinguish between high- and low-quality collateral of $A$ banks; they observe only the average quality of the collateral in the market, i.e., $\bar{c} = \alpha c_H + (1-\alpha)c_L$. Throughout the paper, we consider the case in which $\bar{c} + e_{max} \leq 1$ so that liquidity risk exists for all realizations of $e$ with the average quality of collateral $\bar{c}$ in the market. Since $B$ banks that received collateral pledged by an $A$ bank can only borrow up to the average collateral value $\bar{c}$, they can meet their liquidity needs only if $\bar{c} + e \geq l$. When the average quality of collateral $\bar{c}$ is low, or alternatively when $e$ is low compared with the liquidity shock, $B$ banks will not be able to meet their liquidity needs in the market and consequently will not renew their loans to $A$ banks at $t = 1$.

Given the fundamental $e$, the expected payoff for an $A$ bank with collateral value $c_j$, where $j \in \{H, L\}$, when $A$ banks with high- and low-quality collateral are pooled together, can be written as follows:

$$\bar{\Pi}_j = R - 1 + c_j - (1 - \bar{c} - e)(R - r).$$

As before, the liquidity risk is captured by $(1 - \bar{c} - e)(R - r)$, which is decreasing in both the fundamental $e$ and the average collateral quality in the market $\bar{c}$. $A$ banks will not invest if liquidity risk is sufficiently high compared with the return from their investment, that is, when

$$R - 1 < (1 - \bar{c} - e)(R - r).$$

Note that when $A$ banks with high and low-quality collateral are pooled in the market, this condition holds for all $A$ banks. Consequently, $A$ banks will not invest when

$$e < 1 - \bar{c} - \frac{R-1}{R-r} \equiv \bar{e}, \tag{2}$$

which gives the following proposition.

**Proposition 2 (No investment):** *For $e < \bar{e}$, no investment takes place when $A$ banks with high- and low-quality collateral are pooled in the market.*

Note that $\bar{e} < e_L$ because $\bar{c} > c_L$, which implies that low-quality collateral can generate more liquidity when pooled with high-quality collateral in the market.

## 3.2 Banks' decisions with LoLR

We now expand the previous analysis to consider the implications for $A$ banks' investment and $B$ banks' lending decisions when there is an LoLR. As we will see, banks' incentives to access the LoLR will depend on the central bank's lending policy, including its collateral and interest rate policies, and on banks' benefits from the LoLR access, that is, their liquidity risk exposures. As with the analysis in the previous subsection, we distinguish between the cases where investors can and cannot distinguish the quality of collateral pledged by $A$ banks.

### 3.2.1 Symmetric information

In the presence of an LoLR, banks could, in principle, eliminate the liquidity risk entirely by borrowing from the LoLR. They will do so, however, only if the liquidity risk is high enough to offset the cost of going to the central bank. Recall that $A$ banks with high-quality collateral are not exposed to liquidity risk, so, for $\gamma_H > 0$, they will prefer to borrow in the private market rather than going to the central bank. Therefore, with symmetric information, the decision to borrow from the central bank is only relevant for $A$ banks with low-quality collateral, which we analyze next.

We consider two cases. For $e \geq e_L$, $A$ banks with low-quality collateral can invest by borrowing in the private market, so they will reach out to the LoLR only if

$$(1 - c_L - e)(R - r) \geq \gamma_L,$$

where the LHS is the liquidity risk they face when they borrow from the market. This implies:

$$e \leq 1 - c_L - \frac{\gamma_L}{R - r} \equiv e_L^{CB}. \tag{3}$$

When $e \leq e_L$, $A$ banks with low-quality collateral do not invest if they have to borrow in the private market, forgoing the long-term return of $R - 1$. Consequently, they will borrow from the LoLR if

$$\gamma_L \leq R - 1,$$

that is, if the LoLR penalty rate is less than the return from investing. We summarize the previous results in the following proposition:

**Proposition 3** *A banks with high-quality collateral never borrow from the LoLR for $\gamma_H > 0$. For $e < e_L$, A banks with low-quality collateral borrow from the LoLR only if $\gamma_L \leq R-1$; for $e \geqslant e_L$, they borrow from the LoLR when $e \leq e_L^{CB}$, where $e_L^{CB}$, given in (3), is decreasing in $\gamma_L$.*

When there is perfect information about the quality of banks' collateral, banks with high-quality collateral are not exposed to any liquidity risk and thus will not rely on the LoLR if charged any penalty interest; they will prefer to meet their liquidity needs by borrowing in the market. In contrast, banks with low-quality collateral may prefer to meet their liquidity needs by borrowing from the LoLR, particularly if the LoLR does not charge a high penalty rate, provided the LoLR is willing to lend against low-quality collateral.

#### 3.2.2 Asymmetric information

We now turn to the case in which market participants cannot distinguish the type of collateral each $A$ bank has. An $A$ bank with collateral type $j \in \{H, L\}$ will choose to borrow from the LoLR if the benefit from lowering liquidity risk is greater than $\gamma_j$, which is the penalty rate it will have to pay when borrowing from the LoLR. Again, we distinguish two cases.

For $e \geq \bar{e}$, where all $A$ banks can borrow in the private market and invest, $A$ banks with collateral type $j$ will go to the central bank only when

$$(1 - \bar{c} - e)(R - r) \geq \gamma_j,$$

which can also be written as

$$e < 1 - \bar{c} - \frac{\gamma_j}{R - r} \equiv \bar{e}_j^{CB}. \tag{4}$$

For $e < \bar{e}$, where the liquidity risk is sufficiently high and $A$ banks do not borrow in the private market, they would borrow from the LoLR only if $R - 1 \geq \gamma_j$. We summarize these results in the following proposition:

**Proposition 4 (LoLR):** *For $e < \bar{e}$, an A bank with collateral type $j \in \{H, L\}$ borrows* *from the LoLR only if* $\gamma_j \leq R-1$*; for* $e > \bar{e}$*, it borrows from the LoLR only if* $e \leq \bar{e}_j^{CB}$*, where,* $\bar{e}_j^{CB}$*, given in (4), is decreasing in* $\gamma_j$*.*

In sum, given that access to the LoLR insures banks against liquidity risk, then the maximum rate they are willing to pay for an LoLR loan will depend on the expected loss they will incur from early liquidation of their investment project. The expected loss from early liquidation is increasing in the loss induced by early liquidation, $R-r$, and the likelihood of early liquidation, which is increasing on the likelihood that lending banks in the interbank market are unable to roll over their loans. This likelihood, in turn, is higher when the banking sector is adversely affected by a negative shock, $e$, and/or the average collateral quality of borrowing banks, $\bar{c}$, is low. While the liquidity shock is exogenous, the average quality of collateral of borrowing banks is endogenous and in particular, can be affected through the LoLR's collateral policy. In the next section, we investigate the effect of LoLR collateral policies on liquidity creation and the output in the economy.

# 4 LoLR liquidity provision and aggregate output

We investigate in this section how different collateral policies by the LoLR affect aggregate liquidity and thereby output in the economy. As we described in Section 2, the central bank lends to eligible $A$ banks at $t = 0$ after the fundamental $e$ gets realized. Also, recall that $A$ banks will lose $R - r$ on their investment if it gets liquidated early at $t = 1$. We assume that out of this *private loss*, a fraction $\Delta \in [0, 1]$ is attributed to the actual *real output loss*; that is, $\Delta(R - r)$ represents output losses from liquidations, and the rest $(1 - \Delta)(R - r)$ is a mere *transfer* within the economy.[10]

Given that information, the output in our economy, $Y$, with no asymmetry of information and no LoLR, is defined as

$$Y = Y^A + Y^B = [\alpha(R-1) + (1-\alpha)\{\{R-1-(1-c_L-e)(R-r)\Delta\} \times \mathbb{1}_{e>e_L}\}] + R^B,$$

where $Y^i$ is the output from the investments by $i$ banks. Note that $Y^A = \alpha(R-1) + (1-\alpha)\left[\{R-1-(1-c_L-e)(R-r)\Delta\} \times \mathbb{1}_{e>e_L}\right]$, where the first term is the net return from the investments by $A$ banks with high-quality collateral, and the second term is the

[10] For instance, the transfer could include legal costs, brokers' fees, or a lower price paid by the secondary market buyers due to their rent-seeking behaviors.

net return from the investments by $A$ banks with low-quality collateral, with $\mathbb{1}_{e>e_L}$ being an indicator function equal to 1 when $e > e_L$ and 0 otherwise. Note that for $e > e_L$, $A$ banks with low-quality collateral invest and are exposed to liquidity risk that arises with probability $1 - c_L - e$, while for $e < e_L$ they do not invest at all. The long-term return of $B$ banks' investments $Y^B$ is trivial because $B$ banks can always meet their liquidity shock, either by borrowing from the money markets or by calling their loans to $A$ banks, which is equal to $R^B$.

Under asymmetric information, the output of the economy, $\bar{Y}$, can be written as

$$\bar{Y} = \{R - 1 - (1 - \bar{c} - e)(R - r)\Delta\} \times \mathbb{1}_{e>\bar{e}} + R^B,$$

since all $A$ banks invest for $e \geqslant \bar{e}$, while no $A$ bank invests for $e < \bar{e}$.

## 4.1 Symmetric information

We start our analysis with the symmetric information case since this provides us with a simple benchmark to illustrate some of our main results. Output will be lost either when investment gets liquidated early or when some banks choose not to invest at all to begin with. Recall that we can write the output in this case as

$$Y = \alpha(R - 1) + (1 - \alpha)\{(R - 1 - (1 - c_L - e)(R - r)\Delta) \times \mathbb{1}_{e>e_L}\} + R^B,$$

where losses from liquidations arise when $e \geqslant e_L$, and there is no investment when $e < e_L$.

Recall from Section 3 that with no asymmetry of information, $A$ banks with high-quality collateral always invest and have no exposure to liquidity risk. Thus, LoLR lending to $A$ banks with high-quality collateral has no effect on output; in essence, this is an exchange of central bank liquidity for perfectly liquid private liquidity. $A$ banks with low-quality collateral will invest and become exposed to liquidity risk if $e \geqslant e_L$, while they will not invest at all if $e < e_L$.

Note that for $e \geqslant e_L$, output $Y$ is increasing in $c_L$ in a continuous fashion because, as the value of collateral increases, the probability that the loans are rolled over at $t = 1$ increases. In this case, lending to banks with liquidity risk exposure is beneficial: it insures them against liquidity risk since this would prevent liquidations and increase output from those projects that might otherwise be liquidated. This increase in output would be greater for a lower fundamental $e$, given that the liquidity risk exposure decreases with $e$.

For $e < e_L$, $A$ banks with low-quality collateral do not invest, and the central bank can increase investment by directly lending to these banks. We summarize these results in the following proposition.

**Proposition 5** *Lending to $A$ banks with high-quality collateral does not affect output, that is, $\partial Y/\partial x_H = 0$. Lending to $A$ banks with low-quality collateral has the following effect on output:*

$$\frac{\partial Y}{\partial x_L} = \begin{cases} R-1 & for \quad e < e_L \\ (1-c_L-e)(R-r)\Delta & for \quad e \geqslant e_L \end{cases}$$

Note that $\partial Y/\partial x_L$ is decreasing in $c_L$ and $e$. LoLR lending is more effective in increasing output when the quality of collateral $c_L$ that the central bank accepts is lower, or the fundamental $e$ is lower.

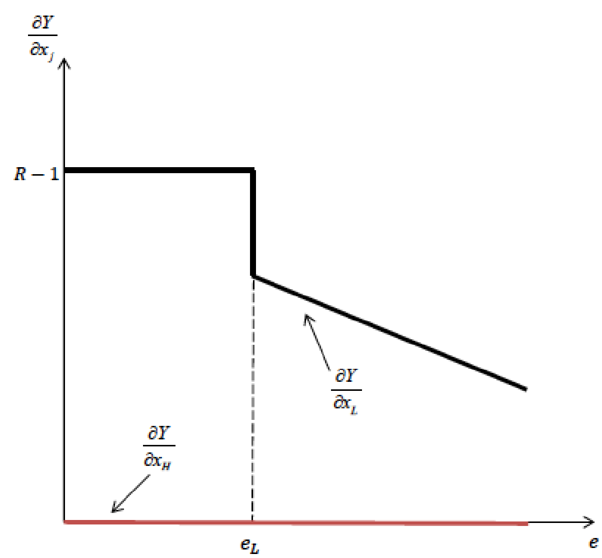


**Figure 2: The effect of LoLR on output, by type of collateral.** Lending against high-quality collateral has no effect on the output $Y$; Lending against low-quality collateral has a positive effect, which is greater when $e$ is lower and liquidity risk is higher.

Figure 2 illustrates the effect of LoLR lending on output, depending on the fundamental $e$ and the types of collateral that the central bank demands on its LoLR operations. Note that $\frac{\partial Y}{\partial x_H} = 0$ (red line) since lending against high-quality collateral has no effect on output. The effect of lending against low-quality collateral on output is illustrated by the black line. Note that $\frac{\partial Y}{\partial x_L}$ (weakly) decreases in $e$, and has a discontinuity around $e_L$. For $e < e_L$, banks with low-quality collateral do not invest, so that LoLR lending increases output by $R-1$ per unit. For $e \geq e_L$, banks with low-quality collateral already invest, but LoLR lending can insure them against liquidity risk, increasing output by $(1-c_L-e)(R-r)\Delta$ per unit, which is decreasing in $e$. At the threshold $e = e_L$, there is a discrete decline in the effect of LoLR on output by $(R-1)-(1-c_L-e_L)(R-r)\Delta$, which is decreasing in $\Delta$.[11]

As the simplest case, suppose that $\Delta = 0$, so that early liquidations entail only transfers but no output loss. In this case, output is maximized when all banks invest ex ante, and we get the following corollary to our previous proposition.

**Corollary 1** *Suppose that $\Delta = 0$ so that early liquidation does not entail any welfare loss. Lending to $A$ banks with high-quality collateral does not affect output, that is, $\partial Y/\partial x_H = 0$. Lending to $A$ banks with low-quality collateral has the following effect on output:*

$$\frac{\partial Y}{\partial x_L} = \begin{cases} R-1 & for \quad e < e_L \\ 0 & for \quad e \geqslant e_L \end{cases}.$$

Again, $A$ banks with high-quality collateral do not need to borrow from the LoLR because they can always raise enough funding on their own to finance their investment projects. Similarly, $A$ banks with low-quality collateral do not need to borrow from the LoLR when $e > e_L$. However, when $e < e_L$, banks with low-quality collateral are not able to raise funding in the market. In this case, a LoLR that is willing to lend against low-quality collateral can affect the output. Hence, even in a setup with (i) symmetric information in which private agents know the quality of collateral in the market, and

[11] $\frac{\partial Y}{\partial x_L}$ is continuous if $\Delta = 1$. As the individual loss from early liquidation (i.e., $R-r$) is greater than the actual output loss (i.e., $\Delta(R-r)$), private agents tend to not invest even if the output would be greater if they did invest. At the switching threshold $e = e_L$, $A$ banks' expected loss from liquidity risk is equal to the opportunity cost of not investing, such that $(1-c_L-e_L)(R-r) = R-1$ holds; however, the expected output loss from the liquidity risk $(1-c_L-e_L)(R-r)\Delta$, which LoLR could eliminate, is less than this if $\Delta < 1$.

(ii) liquidations lead only to transfers, the LoLR can improve output. The reason is that, even though liquidations do not result in real losses, they lead to private losses for banks, which in turn can lead them to bypass valuable investments. The LoLR is valuable because, by insuring banks against liquidity risk, it facilitates investment.

## 4.2 Asymmetric information

We now investigate the case with asymmetric information, again evaluating the effect of lending by the LoLR under differing collateral policies. Recall that under symmetric information about the quality of banks' collateral, LoLR lending against high-quality collateral does not affect output. Further, LoLR lending against low-quality collateral only induces investment and mitigates the liquidity risk of those banks to which it directly lends. That is, LoLR lending activity does not trigger any externality on other banks. Under asymmetric information, the LoLR lending policy, by affecting the quality of collateral that can be pledged in the market, can have a profound impact on the functioning of private markets. In particular, under asymmetry, the LoLR's decision on the quality of collateral that it is willing to accept can lead to regime switches between investing and not investing. That is, accepting low-quality collateral can unlock a frozen interbank market so that banks can start investing, while accepting only high-quality collateral can cause the market to freeze up.

Recall that with asymmetry of information, we can write the output of our economy as

$$\bar{Y} = (R - 1 - (1 - \bar{c} - e)(R - r)\Delta) \times \mathbb{1}_{e > \bar{e}} + R^B,$$

because all $A$ banks invest when $e \geqslant \bar{e}$, while no $A$ bank invests when $e < \bar{e}$. In each of these cases, we first analyze the (marginal) effect of LoLR lending policy on output, without entailing a regime change; we then analyze the effects of the LoLR lending when it does triggers a regime change—that is, restores a frozen market or cause a well-functioning market to freeze up.

In the first case, all $A$ banks invest, so that $e \geqslant \bar{e}$. Suppose that the LoLR lends to a measure $x_H$ of $A$ banks with high-quality collateral. Suppose also, for now, that $x_H$ is small enough so that we can focus on the marginal effect. The LoLR, by lending against collateral, will affect the mix of collateral in the money market. In particular, the average collateral quality after the LoLR provides liquidity support to banks will be

$$c_H^{CB} = \frac{(\alpha - x_H)c_H + (1-\alpha)c_L}{1-x_H}.$$

The output will be

$$\bar{Y}_H = R - 1 - (1-x_H)\left((1-c_H^{CB}-e)(R-r)\Delta\right) + R^B,$$

because banks receiving funding from the LoLR are not exposed to any liquidity risk, while those remaining in the private market experience changes in their liquidity risk exposure. Hence, the change in output can be written as

$$\begin{aligned}\bar{Y}_H - \bar{Y} &= x_H(1-\bar{c}-e)(R-r)\Delta + (1-x_H)(c_H^{CB}-\bar{c})(R-r)\Delta \\ &= x_H(1-c_H-e)(R-r)\Delta.\end{aligned} \quad (5)$$

LoLR loans have two effects that go in opposite directions. On the one hand, they eliminate the liquidity risk of $x_H$ banks, increasing output as captured by the first term in equation (5). On the other hand, LoLR loans impair the quality of collateral in the market. Given that $c_H^{CB} < \bar{c}$, this in turn increases liquidity risk for the remaining banks in the market and indirectly decreases output, as captured by the second term in (5). Note that $\bar{Y}_H - \bar{Y} \leq 0$ because our assumption of $c_H + e_{min} \geq 1$ implies $c_H + e \geq 1$ for all $e$— even though lending against high-quality collateral directly increases output, it imposes a negative information externality on the rest of the banks because it leads to a deterioration in the collateral pool in the private market, which more than offsets the direct effect. In other words, high-quality collateral would stimulate output more if it remained in the private market and was pooled with low-quality collateral to generate more liquidity than if it was taken out of the private pool by the central bank.

However, when the LoLR lends against low-quality collateral to a measure $x_L$ of banks, the average quality of collateral in the market improves to

$$c_L^{CB} = \frac{\alpha c_H + (1-\alpha-x_L)c_L}{1-x_L},$$

and assuming $x_L$ is small enough for now, the change in output can be written as

$$\begin{aligned}\bar{Y}_L - \bar{Y} &= x_L(1-\bar{c}-e)(R-r)\Delta + (1-x_L)(c_L^{CB}-\bar{c})(R-r)\Delta \\ &= x_L(1-c_L-e)(R-r)\Delta.\end{aligned}$$

which is positive because $c_L + e_{max} < 1$. Hence, when $e \geq \bar{e}$, all banks invest, and lending against high-quality collateral decreases output, whereas lending against low-quality collateral increases it.

We now analyze the second case, where $e < \bar{e}$. No $A$ bank invests and the LoLR stimulates only the investment of banks to which it directly lends, whether it is a bank with high- or low-quality collateral, that is, $\frac{\partial Y}{\partial x_H} = \frac{\partial Y}{\partial x_L} = R - 1$. Assuming that the LoLR lending is small enough and does not lead to a regime switch, we can summarize our results in the following proposition.

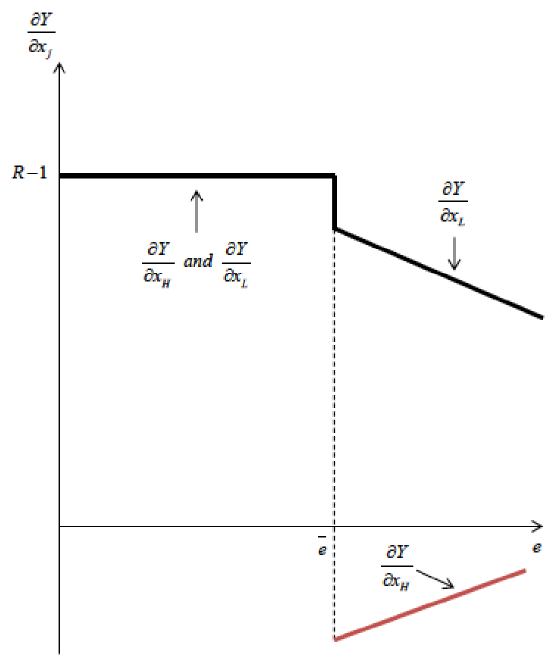


**Figure 3: The effect of LoLR on output, by type of collateral.** When $e < \bar{e}$, no $A$ bank borrows and invests, and the LoLR collateral policy has the equivalent positive effect on output. When $e > \bar{e}$, LoLR collateral policy has a positive (negative) effect on output when it lends against low- (high-) quality collateral.

**Proposition 6** *For $e < \bar{e}$, LoLR lending against high- and low-quality collateral has the same effect on output, $\frac{\partial Y}{\partial x_H} = \frac{\partial Y}{\partial x_L} = R - 1$. For $e > \bar{e}$, LoLR lending against high- and low-quality collateral has the following effects on output: $\frac{\partial Y}{\partial x_H} = (1 - c_H - e)(R - r)\Delta \leq 0$ and $\frac{\partial Y}{\partial x_L} = (1 - c_L - e)(R - r)\Delta > 0$, respectively.*

Thus, unlike the symmetric information setup, LoLR lending against high-quality collateral is no longer neutral and would have an effect on output. In particular, the effect can be negative due to the deterioration of collateral quality in private markets, imposing a negative externality on banks that borrow in private markets, which, in turn, *decreases* output.

So far, we have focused on the effect of "marginal" lending, where the LoLR lending does not lead to a regime switch between investing and non-investing. Next, we analyze how LoLR lending can lead to a regime shift. First, we show that improvements in the average quality of collateral can restore an impaired interbank market. Suppose $e < \bar{e}$, so that no $A$ bank invests. If the average quality of collateral in the private market increases sufficiently, liquidity risk for $A$ banks is mitigated, possibly leading some of these banks to start investing.

In particular, let $\bar{c}$ be the average quality of collateral at the outset. Liquidity risk is sufficiently mitigated (all banks will choose to invest) when the average quality of collateral increases to $\bar{c}'$ so that $(1 - \bar{c}' - e)(R - r) \leqslant R - 1$, that is, when

$$\bar{c}' \geqslant c_I \equiv 1 - e - \frac{R-1}{R-r}, \tag{6}$$

where $c_I$ represents the threshold level of the average quality of collateral above which banks borrow and invest. The central bank can achieve this result by lending to (and taking out of the market) a measure $x'_L$ (or more) of $A$ banks with low-quality collateral, where

$$x'_L = \frac{(R-1) - (1 - \bar{c} - e)(R-r)}{(R-1) - (1 - c_L - e)(R-r)}. \tag{7}$$

Hence, when the central bank lends to a sufficiently large proportion of banks with low-quality collateral, the economy switches from a no-investment state, where banks bypass investment opportunities and private markets are impaired, to one where banks take on investment by borrowing in restored interbank markets.

Figure 4 presents this result. As the central bank increases its lending to banks with low-quality collateral, it increases output by inducing the investment of the direct borrowers, but at the same time the quality of the collateral remaining in the private pool improves. If the central bank takes out $x'_L$ of the low-quality collateral, the quality of the remaining collateral improves sufficiently to trigger the investment of all the remaining $A$ banks in the private market. At that point, the output jumps upward by $(1 - x'_L)(1 - c_I - e)(R - r)(1 - \Delta)$.

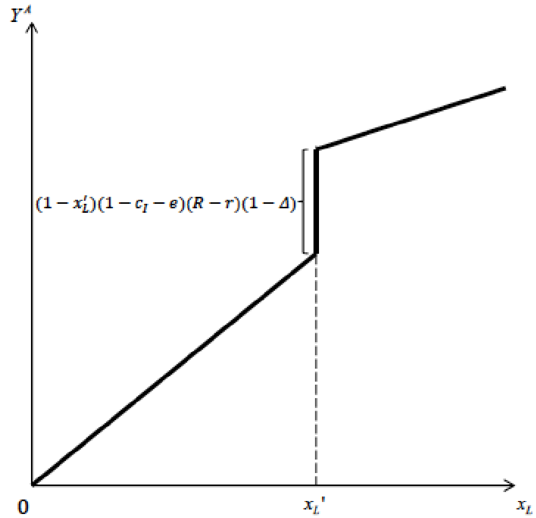


**Figure 4: Market jump-start by taking out the low-quality collateral** All banks start to borrow and invest if the central bank lends to more than $x'_L$ banks with low-quality collateral. $A$ banks' output $Y^A$ jumps upward.

Next, we show that an LoLR collateral policy that impairs the quality of collateral in the private markets can in turn lead to a breakdown of markets. For $e > \bar{e}$, all banks would borrow and invest. If the average quality of collateral in the market deteriorates, borrowers would face greater liquidity risk and could even choose not to borrow and thus make no investment. In particular, when the average quality of collateral falls below $\bar{c}' < c_I$, the economy switches to a state in which banks stop borrowing and investing. LoLR collateral policy can lead to this result when it lends to a measure $x'_H$ (or more) of $A$ banks with high-quality collateral, where

$$x'_H = \frac{(R-1) - (1-\bar{c}-e)(R-r)}{(R-1) - (1-c_H-e)(R-r)}. \tag{8}$$

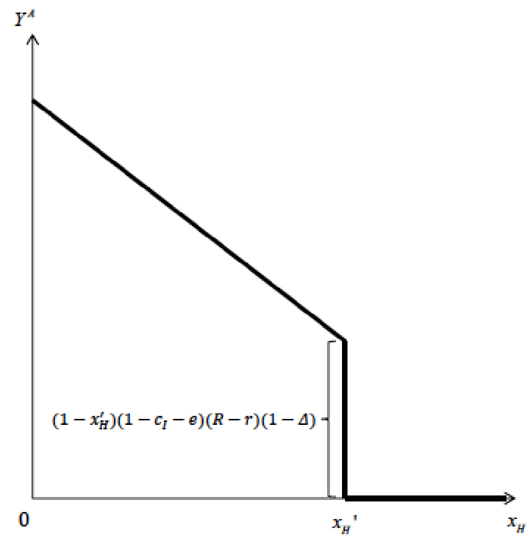


**Figure 5: Market freeze by taking out the high-quality collateral** All remaining banks stop investing if the central bank lends to more than $x'_H$ banks with high-quality collateral. $A$ banks' output $Y^A$ drops to 0.

Figure 5 presents this result. As the central bank takes out the high quality collateral, output decreases due to the negative externality on borrowers in the market by increasing their liquidity risk. At the threshold $x_H = x'_H$, the liquidity risk becomes significant enough that all $A$ banks simply opt out from investing, and the output of $A$ banks drops to 0. We summarize these results in the following proposition.

**Proposition 7** *Two types of change in the average quality of collateral can have the following effects on banks' investment decision: (i) when the average quality increases to a level above $c_I$, given in (6), it can jump-start investment; and (ii) when the average quality decreases to a level below $c_I$, banks stop borrowing in private markets and stop investing.*

# 5 Optimal LoLR policy

We now analyze the optimal LoLR policy. In the previous section, we only analyzed how the central bank's actions would affect output. However, while lending against low-quality collateral can improve output, it exposes the central bank to counterparty risk, and the optimal policy should take into account both effects. We first define the welfare function of the central bank to discuss the optimal policies that maximize welfare.

Besides affecting output, LoLR actions can impose costs on the central bank depending on the collateral it receives. Banks that borrow from the central bank pay at least the "fair" value for the loan, that is, they pay 1 *in expectation*. However, the central bank may suffer some losses when the projects to which it lends have a low return. This may arise from a loss of reputation for the central bank that can impair its independence and efficient functioning or from the cost of immediacy, to cite a few.[12] Below we discuss these costs briefly.

The central bank faces on counterparty risk when it lends against high-quality collateral because the collateral value is high enough for the loan to be always paid in full from $c_H + \underline{R} \geq 1$. Losses can arise only when the central bank lends against low-quality collateral, in which case, it becomes exposed to default risk with probability $1-p$. The loss given default is $(1 - c_L - \underline{R})$ because the central bank recovers only the low return $\underline{R}$ from the bank's project and from the collateral the bank had pledged, $c_L$. Hence, when the central bank lends to a measure $x_L$ of banks against low-quality collateral, the resulting counterparty risk exposes it to losses with an expected value of $z = x_L(1-p)(1 - c_L - \underline{R})$, which gives $\frac{dz}{dx_L} = (1-p)(1 - c_L - \underline{R})$. We denote the central banks cost of incurring these losses by $f(z)$ and assume this function is convex and increasing in $z$ with $f(0) = 0$. In principle, the cost function $f$ will be different for different central banks. For example, more conservative central banks and central banks with more restrictions on their ability to provide liquidity to banks will have a steep cost function $f(z)$. See Section 6 for further discussion.

[12]The provision of immediate funds can entail fiscal costs, which can be linked to a variety of sources, such as (i) distortionary effects of tax increases required to fund losses; and (ii) the likely effect of huge government deficits on the country's exchange rate, manifested in the fact that banking crises and currency crises have often occurred as twins in many countries. Furthermore, while government expenditures and inflows during the regular course of events are smooth, we observe a rapid growth of off-balance-sheet contingent liabilities such as deposit-insurance funds, costs of bank bailouts during crisis periods, where governments need to come up with funds in a short period of time.

Using the central bank's cost function, we can define the social welfare function as:

$$W = Y - f(z), \tag{9}$$

where the first term is the aggregate output and the second term is the cost to the central bank from its LoLR policy due to counterparty risk exposures. We assume that the LoLR access cost $\gamma_j$ is a mere transfer. We now discuss the welfare maximizing LoLR policy. As before, we begin with the symmetric information setup and then introduce asymmetric information.

## 5.1 Optimal policy under symmetric information

When collateral types are public information, we show in Proposition 5 that LoLR lending against high-quality collateral has no effect on output, and there is room for stimulating output only when lending against low-quality collateral. However, lending against low-quality collateral exposes the central bank to counterparty risk. The central bank maximizing welfare would thus lend to $A$ banks with low-quality collateral as long as the marginal increase in output $\partial Y/\partial x_L$, which is presented in Proposition 5, is greater than or equal to the marginal cost $f'(z)\frac{dz}{dx_L}$ from counterparty risk. Hence, the central bank should lend to a measure $x_L$ of $A$ banks with low-quality collateral, such that

$$\partial Y/\partial x_L = f'(z)(1-p)(1-c_L-\underline{R})$$

where $z = x_L(1-p)(1-c_L-\underline{R})$, or to all $A$ banks with low quality collateral, that is, $x_L = 1-\alpha$ if

$$\partial Y/\partial x_L > f'(z)(1-p)(1-c_L-\underline{R}).$$

Note that the central bank should lend to fewer banks when the fundamental $e$ is higher, since LoLR lending has a weaker stimulus effect when the fundamental $e$ is more robust and liquidity risk is lower, as discussed in Proposition 5. The following proposition summarizes the optimal central bank policy.

**Proposition 8** *Lending to A-banks with high-quality collateral does not affect welfare. Optimal LoLR policy can be characterized by lending to a measure $x_L$ of $A$ banks with low-quality collateral, where $z = x_L(1-p)(1-c_L-\underline{R})$ and* *i) For $e < e_L$, the central bank lends to a measure $x_L$ of A banks with low-quality collateral such that $R-1 = f'(z)(1-p)(1-c_L-\underline{R})$ or to all banks with low-quality collateral, that is, $x_L = 1-\alpha$, if $R-1 > f'(z)(1-p)(1-c_L-\underline{R})$ for $x_L = 1-\alpha$.*

*ii) For $e \geq e_L$, the central bank lends to a measure $x_L$ of A banks with low-quality collateral such that $(1-c_L-e)(R-r)\Delta = f'(z)(1-p)(1-c_L-\underline{R})$ or to all banks with low-quality collateral, that is, $x_L = 1-\alpha$, if $(1-c_L-e)\Delta(R-r) > f'(z)(1-p)(1-c_L-\underline{R})$ for $x_L = 1-\alpha$. $x_L$ is decreasing in $e$ and increasing in $p$ and $R-r$.*

In sum, when there is no asymmetry of information, central bank lending to banks with high-quality collateral is neutral; although this lending is not costly for the central bank, it also has no stimulus effect, and thus does not affect welfare. In contrast, central bank lending to banks with low-quality collateral, while exposing the central bank to counterparty risks, can be welfare improving, particularly when liquidity risk is critical. The optimal capacity of the lending facility in this case will depend on the cost to the central bank of such intervention.

A special simple case is when $\Delta = 0$ such that early liquidation entails no welfare loss but transfers. Corollary 1 suggests that (1) lending against high-quality collateral has no effect on output and (2) there is room for stimulating output through lending only when $e < e_L$ and the LoLR lends against low-quality collateral. Again, lending against low-quality collateral exposes the central bank to counterparty risk, and the central bank chooses the optimal lending by comparing the marginal stimulus effect $\partial Y/\partial x_L = R-1$ and the marginal cost of such lending $f'(z)\frac{dz}{dx_L}$ as in the previous case. Hence, we have the following corollary.

**Corollary 2** *Lending to A banks with high-quality collateral does not affect welfare. For $e \geqslant e_L$, lending to A-banks with low-quality collateral can only decrease welfare. For $e < e_L$, the central bank optimally lends to a measure $x_L$ of A banks with low-quality collateral, where $R-1 = f'(z)(1-p)(1-c_L-\underline{R})$ or to all A banks with low-quality collateral, that is, $x_L = 1-\alpha$, if $R-1 > f'(z)(1-p)(1-c_L-\underline{R})$.*

As discussed in Corollary 1, $A$ banks with low-quality collateral do not invest for $e < e_L$ although their investment should increase output and welfare. Even in this simple setup where there is no asymmetry of information and liquidations do not lead to real losses, the LoLR can improve welfare by insuring banks against liquidity risk that otherwise precludes them from undertaking valuable investments.

## 5.2 Optimal policy under asymmetric information

We now discuss the optimal LoLR policy when there is asymmetry of information about the quality of banks' collateral in the market. We continue to assume, however, that the central bank is able to distinguish low-quality collateral from high-quality collateral. Beginning with $e \geqslant \bar{e}$, from Proposition 2, all banks already invest. In that case, while lending against high-quality collateral does not expose the central bank to any counterparty risk, it is never optimal because it will either decrease output or lead to underinvestment and a freeze in the market (Propositions 6 and 7). Hence, the central bank would be better off refraining from lending against high-quality collateral and just leave the high-quality collateral in the market.

When lending against low-quality collateral, the central bank should weigh the benefit of increased output against the costs arising from counterparty risk. Hence, the optimal policy is similar to that in the case of symmetric information, where the LoLR lends to a measure $x_L$ of $A$ banks with low-quality collateral, where the first order condition

$$\frac{\partial Y}{\partial x_L} = f'(z)(1-p)(1-c_L-\underline{R}),$$

is satisfied for $x_L < 1-\alpha$ or the central bank lends to all $A$ banks with low-quality collateral, that is, $x_L = 1-\alpha$ if

$$\frac{\partial Y}{\partial x_L} \geq f'(z)(1-p)(1-c_L-\underline{R}) \text{for } x_L = 1-\alpha.$$

Next, we consider the case when $e < \bar{e}$, and none of the $A$ banks invests (from Proposition 2). Lending against high-quality collateral increases output by $R-1$ per unit lent (from Proposition 6) and does not expose the central bank to counterparty risk, hence, the net increase in welfare is $R-1$ per unit lent. Lending against low-quality collateral also increases output by $R-1$ per unit lent (from Proposition 6) but exposes the central bank to counterparty risk entailing in the cost $f(z)$. However, lending against low-quality collateral has the benefit of improving the quality of collateral in the market, which can lead to a switch from a regime in which the interbank market and investment are impaired to one where they are restored (Proposition 7). When analyzing the optimal central bank policy, we need to take into account these discontinuous effects.

A feasible policy for the central bank, Policy $H$, is to (1) lend to all $A$ banks with high-quality collateral (i.e., $x_H = \alpha$), which would increase output and welfare by $\alpha(R-1)$, and then (2) lend to the $A$ banks with low-quality collateral according to the marginal cost-benefit comparison as we did before. Note that when the central bank lends to all $A$ banks with high-quality collateral, only $A$ banks with low-quality collateral are left in the market, and since $e < \bar{e}$ and $\bar{e} < e_L$ imply $e < e_L$, the latter will invest only if they can borrow from the central bank.

When the cost of counterparty risk increases very steeply, that is, when $f'(0) > R-1$, the benefit from increased output falls short of the cost from counterparty risk, and the central bank under Policy $H$ should not lend against low quality collateral (i.e. $x_L = 0$). Otherwise, as before, the optimal policy is for the central bank to lend to a measure $x_L$ of $A$ banks with low quality collateral, where the first order condition

$$\frac{\partial Y}{\partial x_L} = R - 1 = f'(z)(1-p)(1-c_L-\underline{R}),$$

is satisfied for $x_L < 1-\alpha$; or the central bank lends to all $A$ banks with low-quality collateral, that is, $x_L = 1-\alpha$ if $\frac{\partial Y}{\partial x_L} > f'(z)(1-p)(1-c_L-\underline{R})$ for $x_L = 1-\alpha$. This results in a welfare of

$$\bar{W}_H = (\alpha + x_L)(R-1) + R^B - f(z),$$

since only the banks that borrow from the central bank, a measure $(\alpha + x_L)$, invest. Note that, as proposed by Bagehot, the central bank lends "freely" to all banks with high-quality collateral.

Under an alternative policy, Policy $L$, the central bank lends against low-quality collateral. This would be optimal only if it leads to a regime switch by restoring investment and the interbank market since, otherwise, lending against high-quality collateral generates the same increase, $R-1$, in output per unit lent without exposing the central bank to any counterparty risk. For the central bank to induce a regime shift, it should lend to at least to a measure $x'_L$, given in equation (7), of banks with low-quality collateral. When the economy switches to the investment regime by lending $x'_L$, output jumps upward by $(1-x'_L)(1-c_I-e)(R-r)(1-\Delta)$, and further lending to $A$ banks with low-quality collateral beyond that point will generate a continuous effect, as in Figure 4.

The optimal capacity of this lending facility $x_L$ depends on the cost to the LoLR. Suppose that the central bank just achieved the jump-start investment with $x_L = x'_L$. If the additional lending to $A$ banks with low-quality collateral at that point is too costly that is,

$$\frac{\partial Y}{\partial x_L}\big(=(1-c_L-e)(R-r)\Delta\big) < f'(z)(1-p)(1-c_L-\underline{R}) \tag{10}$$

for $x_L = x'_L$, then the central bank should just trigger the jump-start by lending to $x'_L$ of $A$ banks with low-quality collateral but should not lend beyond that. Otherwise, the central bank will lend to a measure $x_L > x'_L$ of $A$ banks with low quality collateral, where the first order condition

$$\frac{\partial Y}{\partial x_L}\big(=(1-c_L-e)(R-r)\Delta\big) = f'(z)(1-p)(1-c_L-\underline{R}), \tag{11}$$

is satisfied for $x_L < 1-\alpha$; or it will lend to all $A$ banks with low-quality collateral, that is, $x_L = 1-\alpha$ if

$$\frac{\partial Y}{\partial x_L}\big(=(1-c_L-e)(R-r)\Delta\big) > f'(z)(1-p)(1-c_L-\underline{R}) \tag{12}$$

for $x_L = 1-\alpha$.[13] Policy $L$ results in welfare of

$$\bar{W}_L = (R-1) - (1-x_L)\left((1-c_L^{CB}-e)(R-r)\Delta\right) + R^B - f(z),$$

where the first term is potential output and the second term represents the expected output loss, due to liquidity risk, for banks that borrow in the interbank market.

For $\bar{W}_L > \bar{W}_H$, the optimal policy would be Policy $L$, that is, to lend against low-quality collateral and induce a shift in the economy to an investment regime. For $\bar{W}_L < \bar{W}_H$, the optimal policy would be Policy $H$, that is, lending to all $A$ banks with high-quality collateral, and then to (some of the) $A$ banks with low-quality collateral. Here, the interbank market is entirely replaced by the central bank, and only banks that borrow from the central bank invest.

Let $x_L^*$ and $x_L^{**}$ be the optimal levels of lending to $A$ banks with low-quality collateral under Policy $H$ and Policy $L$, respectively; and let $f(z^*)$ and $f(z^{**})$ represent the resulting costs from counterparty risk under Policy $H$ and Policy $L$, respectively. We have $\bar{W}_L > \bar{W}_H$ when

$$(1-\alpha-x_L^*)(R-1) > (1-x_L^{**})\left((1-c_L^{CB}-e)(R-r)\Delta\right) + \left(f(z^{**})-f(z^*)\right), \tag{13}$$

[13] Recall from Proposition 6 that the high-quality collateral is better left in the private market than with the central bank, unless the interbank market is frozen. Hence, once the economy has switched to an investment regime, any lending against high-quality collateral is suboptimal.

where the LHS is the loss in output arising from the inability of banks that cannot borrow from the central bank to invest under Policy $H$; the first term on the RHS represents the output loss from liquidity risk for banks that borrow in the market under Policy $L$; and the second term represents the difference between the costs of counterparty risk under Policy $L$ and Policy $H$. We summarize these results in the following proposition.

**Proposition 9** *The optimal LoLR policy can be characterized as follows:*

*i) For $e > \bar{e}$, it is never optimal to lend against high-quality collateral, which decreases welfare. The optimal level of lending against low-quality collateral $x_L$ satisfies $R - 1 = f'(z)\frac{dz}{dx_L}$; or $x_L = 1 - \alpha$ for $R - 1 > f'(z)\frac{dz}{dx_L}$.*

*ii) For $e < \bar{e}$, when $\bar{W}_L > \bar{W}_H$, that is, when (13) holds, the central bank optimally lends against low-quality collateral and induces a switch to an investment regime. Otherwise, the central bank lends to all A banks with high-quality collateral, that is, $x_H = \alpha$, and to (some of) the A banks with low-quality collateral characterized by the FOCs.*

A unique case is depicted in Figure 6, which corresponds to Policy $L$ satisfying (10). In this case, the central bank should *increase* its lending to banks with low-quality collateral even if the marginal benefit $\frac{\partial Y}{\partial x_L}$ is *lower than* the marginal cost from the counterparty risk exposures $f'(z)\frac{dz}{dx_L}$. This is because by doing so, it can eventually lead to a regime switch with a discrete increase in welfare.

In Figure 6, the first order condition $\frac{\partial Y}{\partial x_L} = f'(z)\frac{dz}{dx_L}$ holds at $x_L = x_L^{FOC}$, and beyond that point welfare decreases with greater $x_L$. That is, the welfare function attains its local maximum at $x_L = x_L^{FOC}$. However, if the central bank expands its lending to A banks with low-quality collateral so that $x_L = x_L^{'}$, a jump-start in investment by all A banks arises, at which point the output and welfare jumps upward by $(1 - x_L')(1 - c_I - e)(R - r)(1 - \Delta)$. If the welfare under this jump-start investment (i.e. $\bar{W}_L$) is greater than the local maximum (i.e. $W^{FOC}$), then the central bank should expose itself to more counterparty risk than the marginal cost-benefit analysis suggests, so as to jump-start the private market.

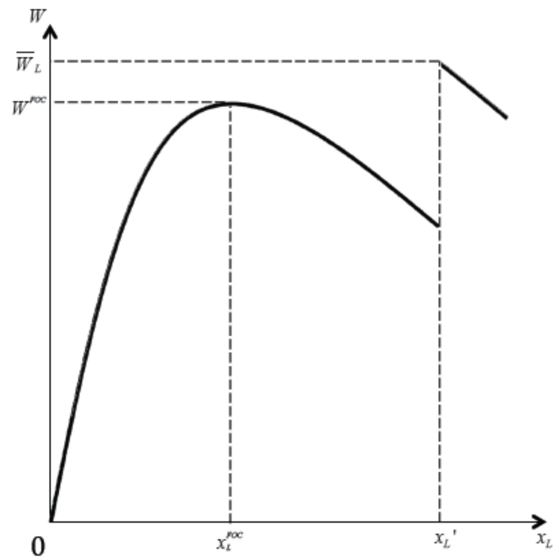


**Figure 6: Optimal jump-start with a corner solution** The marginal increase in output is less than the marginal cost to the central bank beyond $x_L = x_L^{FOC}$. However, a regime switch from no-investment to investment arises when $x_L = x'_L$, at which point the output and welfare increase in a discrete fashion. If $\bar{W}_L > W^{FOC}$, the central bank should bear greater counterparty risks to induce a jump-start than the marginal cost-benefit analysis suggests.

# 6 Discussion and extensions

## 6.1 Cash-in-the-market pricing

So far, we have assumed that the liquidation value, $r$, is fixed. However, the liquidation value can be a function of the number of projects that are being liquidated. In particular, a large proportion of projects being liquidated can lead to firesales in secondary markets when the buyers are financially constrained (Shleifer and Vishny (1992)) and the liquidation value can be determined by the available cash in the market resulting in cash-in-the-market pricing (Allen and Gale (1994, 1998)).

Next, we analyze the effects of cash-in-the-market pricing in our model. Suppose that the liquidation value decreases as more projects get liquidated, and in turn, the liquidation value is increasing in the measure $x_j$ of banks to which the central bank lends, that is, $dr/dx_j > 0$, since the banks that borrow from the central bank do not experience any liquidity risk and those projects can be held until maturity at $t = 2$.

Suppose we are in the symmetric information setup. Recall that $A$ banks with high-quality collateral do not experience any liquidity risk and always invest. Also, suppose that $e \geqslant e_L$ so that the $A$ banks with low quality collateral also invest, but the central bank can insure them against liquidity risk. Suppose that the central bank lends to a measure $x_L$ of $A$-banks with low-quality collateral. In that case, we can write the output as

$$Y_L = (R-1) - (1-\alpha-x_L)(1-c_L-e)(R-r(x_L)) + R^B,$$

so that

$$\frac{\partial Y_L}{\partial x_L} = (1-c_L-e)\left(R - r(x) + \underbrace{(1-\alpha-x)\frac{dr}{dx_L}}_{\text{price effect}}\right).$$

Note that the second term in the parentheses result from the positive effect central bank lending has on the liquidation value, since fewer projects need to be liquidated in the secondary market. Hence, central bank lending has an additional boosting effect on output through improved prices in the secondary market.

Furthermore, the investment threshold $e_L$ for the $A$ banks with low-quality collateral is as follows:

$$e_L = 1 - c_L - \frac{R-1}{R-r},$$

so that

$$\frac{de_L}{dx_L} = -\frac{(R-r)^2}{R-1}\frac{dr}{dx_L} < 0.$$

This way, we also have an effect on $e_L$. Hence, we can have a similar amplification effect when $e < e_L$. In addition to the output $R-1$ generated by the $A$ banks with low-quality collateral that borrow from the central banks, the investment threshold decreases, which makes investment more likely. We can obtain similar results in the asymmetric information setup as well.

We can model the secondary market in the following way. Let $C$ denote the available liquidity in the secondary market and $\ell$ denote the number of projects being liquidated. With symmetric information, $\ell = (1-c_L-e)I_L$, where $I_L$ is the initial investment by banks with low-quality collateral. Hence, we obtain the liquidation value as $r = \frac{C}{\ell} = \frac{C}{(1-c_L-e)I_L}$. Note that for (some of) the A banks with low-quality collateral to invest, we need $R - 1 = (1 - c_L - e)(R - r)$. This gives us the equilibrium level of investment by the $A$ banks with low-quality collateral as $I_L^* = \frac{C}{1-R(c_L+e)}$. With the central bank lending, we have $\ell_{CB} = (1 - c_L - e)I_L - x_j$, which, in turn, would increase equilibrium investment.

## 6.2 Relations in the interbank market

In our model, we assume that $A$ banks cannot borrow directly from $C$ banks at $t = 1$ if their collateral is already pledged with $B$ banks. This would be the case if relations in the interbank market matter. Relationships in the interbank market can arise from "tiering," whereby some banks ("tier-1" banks) access central bank liquidity and act as clearing banks for other banks ("tier-2" banks). Even in large interbank markets, such tiering exists, and hence issues of market power remain important. For example, in the US federal funds market, JPMC and Bank of America are much bigger borrowers than others, and State Street and JPMC are much bigger lenders. Furthermore, many banks are connected with only one or two banks, and the average number of connections is between three and four (Bech and Atalay 2010). The UK also has a tiered system, and the volatility induced in interbank lending rates due to the cornering of collateral and liquidity by some of the large settlement banks during 2001–2005 was one of the main rationales for the Sterling Money Market Reform in 2006. Post-reform, the Bank of England increased the number of banks allowed to participate in open market operations from 10 to more than 35 (Bank of England 2005 and Tucker 2004).

Another important feature is that lending and borrowing in the interbank market make peer monitoring among banks important (for theory, see Rochet and Tirole 1996 and Freixas and Holthausen 2005). Such monitoring can create information monopolies in interbank markets. Cocco, Gomes, and Martins (2005) report evidence of strong relationships in the Portuguese interbank market, suggesting that some banks are more important lenders and pivotal, even in normal times. Furthermore, smaller banks, with limited access to foreign interbank markets, concentrate all their borrowing in only a few large banks in the domestic interbank markets. They also highlight the essentially bilateral nature of interbank lending: most of the lending volume is accounted for by "direct" loans in which loan amount and interest rate are agreed to on a one-to-one basis between borrower and lender, where other banks do not necessarily have access to the same terms. Also, such relationships become more important during stress periods, which is the main focus of our paper.

### 6.3 Penalty rate and LoLR capacity

In our model, we assume that the central bank could arbitrarily limit the capacity of LoLR lending to $x_j$. Note that all $A$ banks are homogeneous in our setup, thus given the penalty rate $\gamma_j$, demand for LoLR funding should always be a corner solution. We dont provide a microfoundation for the allocation mechanism for the LoLR funds in the case of excess demand; we simply assume that it would be assigned randomly or on a first-come-first-served basis. Nonetheless, the central bank can achieve the desired allocation through a market clearing rate or via an auction mechanism if banks are heterogeneous.

Suppose that the long-term return of the $A$ banks, denoted by $R$, is heterogeneous, such that $R \sim [R-\epsilon, R+\epsilon]$ for all $A$ banks with high and low-quality collateral. As there is no adverse selection with respect to the LoLR lending,[14] given the penalty rate $\gamma_j$ and the fundamental $e$, the marginal borrowers from the LoLR, whose investment return is denoted by $R^*$, will be given by Proposition 3 and 4 under symmetric and asymmetric information, respectively. As discussed in Section 3.2, this marginal borrower is the bank whose benefit from the LoLR access is just equal to the penalty rate. Any banks with $R > R^*$ will borrow from the central bank since the benefit from borrowing from the LoLR, or the value of insurance against liquidation risk, is greater for more productive banks with greater loss given early liquidation given as $R - r$.

This implies that starting from the very high penalty rate, the LoLR will attract only the most productive banks, and it would attract the next set of productive banks by lowering the penalty rate slightly. Suppose that the central bank can maximize welfare by lending to $x_j$ of (most productive) $A$ banks with collateral type $j = H, L$. Hence, the central bank would like to limit its lending capacity to $x_j$, and also would aim at clearing the demand for LoLR funding by setting $\gamma_j$ so as to make the LoLR demand equal to $x_j$. This is possible by choosing the right penalty rate, as the LoLR demand decreases monotonically in $\gamma_j$. In other words, as the cost of accessing the LoLR goes up, only the productive banks whose benefits from liquidity insurance are large enough will borrow from the central bank, and the central bank can achieve the desired allocation

[14]Note that all the borrowers would need to pay $1 + \gamma_j$ in expectation, unlike in the credit rationing model in Stiglitz and Weiss (1981).

by choosing the penalty rate to clear the market.

Note that with heterogeneous productivity of $A$ banks, the same allocation could be achieved through an auction even without the adjustment of the penalty rate. Again, the more productive banks would bid more for the LoLR funding, and given the facility limit $x_j$, the marginal bank should bid the same rate $\gamma_j$, as in the penalty rate setup.

## 6.4 Moral hazard

One potential downside of central bank lending against low-quality collateral is that it can create incentives for banks to generate and hold low-quality collateral when it is more costly for them to hold high-quality collateral (Nyborg 2017). This, in turn, can expose the central bank to counterparty risk, especially during systemic crises, when the central bank feels compelled to lend to banks. One central bank policy that could prevent such moral hazard without curbing valuable investments by banks would be to charge a penalty rate when it lends against low-quality collateral.

Consider the following addition to our existing setup. Suppose that the bank chooses the collateral it holds at an earlier state, say at date $t = -1$. Let $\beta_j$ denote the cost of holding collateral of quality $j = H, L$ for the bank with $\beta_H > \beta_L$. Also, let $\gamma_j$ be the penalty rate the central bank charges when it lends against collateral of quality $j = H, L$. For simplicity, let's assume that $\gamma_H = 0$ so that the central bank does not charge a penalty rate when it lends against high-quality collateral. We denote by $\bar{\gamma}_L$ the expected penalty rate the bank pays when it borrows from the central bank against low-quality collateral, which takes into account the probability of borrowing from the central bank. Furthermore, let $E(\Pi_H)$ and $E(\Pi_L)$ be the expected profit for the bank when it holds high- and low- quality collateral, respectively, excluding the cost of holding collateral and the cost of borrowing from the central bank. Hence, the central bank can use the rate $\gamma_L$ to induce banks to hold high-quality collateral, where, for

$$E(\Pi_L) - \bar{\gamma}_L - \beta_L \leqslant E(\Pi_H) - \beta_H,$$

the bank would have a higher expected profit when it holds high-quality collateral. Thus, the central bank can facilitate valuable investment by insuring banks against liquidity risk; and at the same time it can prevent moral hazard using a penalty rate when it lends against low-quality collateral.

## 6.5 Information on collateral quality

In our model, we assumed either symmetric or asymmetric information on the types of collateral for the lenders. We also assumed that the central bank is not subject to this information asymmetry. Below, we provide an interpretation for this information structure as well as an extension of our model with endogenous information generation.

Suppose that some collateral are simple and can easily be valued by all market participants, while others are complex and difficult to value so that the market participants would apply a certain haircut. We denote the first type of collateral as "high quality", and the second type as "low quality". However, the true quality of collateral would be revealed only if information is generated as in Gorton and Ordoñez (2014, 2016); otherwise lenders would simply attribute the average quality $\bar{c}$ to any collateral. In particular, at $t = 0$, the owners of the collateral, i.e. A banks, can generate information about their collateral type by incurring a cost of $\delta_I$ ($< R - 1$).[15] Information generation takes time and can't be done immediately when facing a run at $t = 1$. Following Gorton and Ordoñez (2014), a loan is "information insensitive" if lent against collateral without any information being generated and is "information sensitive" if information is generated and the loan is against collateral with certain quality.

In this case, an $A$ bank compares the cost and the benefit of information generation and chooses to generate information to reveal its collateral quality if the benefit outweighs the cost. An $A$ bank with low-quality collateral will never produce information since it would be only worse off by doing so, and an $A$ bank with high-quality collateral will incur the cost of $\delta_I$ to separate itself from the A banks with low-quality collateral if the benefit from lowering liquidity risk is greater than the cost of information generation. Once perceived as a borrower with high-quality collateral, that bank can eliminate liquidity risk altogether. Therefore, $A$ banks with high-quality collateral will choose to generate information to separate themselves out when

$$(1 - \bar{c} - e)(R - r) \geq \delta_I,$$

which can also be written as

$$e < 1 - \bar{c} - \frac{\delta_I}{R - r} \equiv e_s. \tag{14}$$

[15] Note that $B$ banks don't have any incentive to generate information since they can always satisfy their liquidity need by calling back the loan from $A$ banks.

Note that $e_s > \bar{e}$ with $R - 1 > \delta_I$, which gives us the following result.

**Proposition 10 (Separation):** *For $e > e_s$, no information is generated and A-banks with high and low-quality collateral are pooled and invest. For $e < e_s$, A-banks with high-quality collateral will generate information to separate themselves.*

Hence, information generation arises as the fundamental $e$ deteriorates. For $e < e_s$, $A$ banks with high-quality collateral separate themselves out and always invest in equilibrium. Note that upon information generation, the liquidity risks for the $A$ banks with low-quality collateral *increase* as their collateral is not pooled with high-quality collateral any more, and whether these banks will invest or not depend on the position of $e$ relative $e_L$. If $e_L > e_s$, for $e \leq e_s$, A banks with high-quality collateral separate themselves and invest, but $A$ banks with low-quality collateral do not invest.

This result implies that there is a market breakdown when information is generated endogenously as the fundamental deteriorates. Unless information is generated, all of our previous results under the asymmetric information setup go through. Our results on the regime switches driven by the central bank's lending policy (Proposition 7) would also go through in a similar way. The information generation threshold is decreasing in $\bar{c}$, hence lending against high-quality collateral, which decreases $\bar{c}$ and increases $e_s$, could trigger information generation and induce a (partial) freeze of the otherwise functioning market if the $A$ banks with low-quality collateral stop investing. On the other hand, lending against low-quality collateral can prevent the market freeze by improving the overall quality of collateral in the market and reducing the incentive for information generation, making loans information insensitive.

# 7 Conclusion

We analyze the optimal lending and collateral policy for the lender of last resort, a policy issue that dates back to Thornton (1802) and Bagehot (1873). The main idea in our setup is that the actions of the central bank can impose externalities on private markets by affecting the pool of collateral and liquidity creation in these markets. Lending against high-quality collateral has the advantage of protecting the central bank against potential losses associated with its LoLR activities; however, under conditions of symmetric information about the quality of collateral, it only shas a neutral effect on welfare.

In the presence of asymmetric information and functioning markets, furthermore, that policy (lending against high-quality collateral) can have a negative effect and lead to a market breakdown. The breakdown occurs because the central bank's lending removes high-quality collateral from the market, which reduces the average quality of collateral remaining in the market and impairs its capacity to create liquidity for liquidity creation. However, through LoLR activity under asymmetric information, central banks can unlock freezes in private markets by lending against low-quality collateral, which improves the pool of collateral remaining in the market. Although lending against low-quality collateral exposes the central bank to potential losses, the resulting improvement in the average quality of privately held collateral can mitigate liquidity risk and restore private markets.

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