Financial Vulnerability and Monetary Policy

Tobias Adrian and Fernando Duarte

International Monetary Fund and Brown University

October 2022

The views expressed here are the authors’ and are not necessarily representative of the views of the International Monetary Fund, its Management, or its Executive Directors.
Introduction

What is the Nexus Between Monetary Policy and Financial Vulnerability?

Financial vulnerability: Amplification mechanisms in the financial sector

Two questions are hotly debated

1. Does monetary policy impact the degree of financial vulnerability?
2. Should monetary policy take financial vulnerability into account?
Traditional View:
Financial Vulnerability not Crucial for Monetary Policy

- Inflation targeting literature largely dismisses relevance of financial stability
  - Bernanke Gertler (1999), Curdia Woodford (2016)

- Cost-benefit analysis argues never to use monetary policy for financial stability
  - Svensson (2014, 2016)

- Monetary policy too blunt an instrument, use macro-prudential tools instead
Our Contributions

- Framework that captures joint behavior of inflation, output, and financial vulnerability
  - Realistic and empirically relevant based on GDP-at-Risk
  - Tractable and parsimonious
  - Can be expanded to larger scale DSGEs

- New Keynesian (NK) model with financial vulnerability
  - Intermediation sector with frictions: Value-at-Risk (VaR) constraint
  - VaR constraint creates vulnerability through asset prices

- NKV
Preview of Conclusions

1. Monetary policy should always take financial vulnerability into account

2. Quantitatively large tradeoff between financial vulnerability and dual mandate
   - Through the risk-taking channel of monetary policy

3. Optimal policy can be implemented with flexible inflation targeting
Financial Variables Predict Tail of Output Gap Distribution

Based on “Vulnerable Growth” by Adrian, Boyarchenko and Giannone (AER, 2018)
Motivation

High-Mean Low-Vol for Conditional Output Gap Growth

Cond Mean = 1.12 - 1.11 x Cond Vol + \( \varepsilon \)
Output Gap Local Projections Show Intertemporal Tradeoff

 Conditioning on financial conditions reveals “Volatility Paradox”

 IRF from LP equivalent to VAR, Plagborg-Møller and Wolf (Econometrica, 2021)
Conditional Inflation Quantiles Are Symmetric
No Conditional Mean-Vol Correlation for Inflation

\[ \text{Cond Mean} = 0.02 - 0.07 \times \text{Cond Vol} + \varepsilon \]
Inflation Local Projections Give No Volatility Paradox
Similar Patterns Hold in Panel of Countries
Based on Adrian, Duarte, Grinberg and Mancini-Griffoli (IMF volume, 2018)
Overview of Microfounded Non-Linear Model

- Firm optimization gives standard New Keynesian Phillips Curve
- Households as in New Keynesian model but
  - Cannot finance firms directly
  - Can trade any financial assets (stocks, riskless deposits, etc.) with banks
- Banks
  - Finance firms
  - Trade financial assets with households and among themselves
  - Have a preference (risk aversion) shock
  - Subject to Value-at-Risk constraint
- Financial markets are complete but prices are distorted
Price of Risk and No Arbitrage

- Single source of risk $Z_t$
- Stochastic discount factor $SDF_{s|t} = Q_s / Q_t$ with

\[
dQ_t \equiv -Q_t R_t dt - Q_t \eta_t dZ_t \quad \text{and} \quad Q_0 \equiv 1
\]

such that for all assets with payoffs $D_s$ the price is

\[
Q_t S_t = E_t \left[ \int_t^\infty Q_s D_s ds \right]
\]

- $\eta_t$ is the market price of risk
- $R_t$ is real risk-free rate, $i_t = R_t - \pi_t$ is nominal risk-free rate
- With volatility $\sigma_t$ expected excess returns $\mu_t = \eta_t \sigma_t$
The Intermediation Sector Setup

- “Banks” solve portfolio problem with VaR constraint and preference shocks

\[
\max_{\{\theta_t, \delta_t\}} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta(s-t)} e^{\zeta_s} \log (\delta_s X_s) \, ds \right]
\]

subject to

Budget constraint:

\[
\frac{dX_t}{X_t} = (i_t - \pi_t - \delta_t + \theta_t \mu_t) \, dt + \theta_t \sigma_t \, dZ_t
\]

Value-at-Risk constraint:

\[
\text{VaR}_{\tau, \alpha}(X_t) \leq a \sqrt{\text{Var}(X_t)}
\]

Exogenous processes:

\[
\begin{cases}
  d\zeta_t = -\frac{1}{2} s_t^2 \, dt - s_t \, dZ_t \\
  ds_t = -\kappa (s_t - \bar{s}) + \sigma_s \, dZ_t
\end{cases}
\]
The Intermediation Sector Setup

▶ “Banks” solve portfolio problem with VaR constraint and preference shocks

\[
\max_{\{\theta_t, \delta_t\}} \mathbb{E}_t^{bank} \left[ \int_t^\infty e^{-\beta(s-t)} \log (\delta_s X_s) \, ds \right]
\]

subject to

Budget constraint: \[
\frac{dX_t}{X_t} = (i_t - \pi_t - \delta_t + \theta_t \mu_t - \theta_t \sigma_t s_t) \, dt + \theta_t \sigma_t dZ_t^{bank}
\]

Value-at-Risk constraint: \[
\text{VaR}_{\tau, \alpha}^{bank} (X_t) \leq a \sqrt{X_t}
\]

Exogenous process: \[
ds_t = -\kappa (s_t - \bar{s}) + \sigma_s dZ_t^{bank}
\]
The VaR Constraint Limits Tail Risk

Let $\hat{X}_t$ be projected wealth with fixed portfolio weights from $t$ to $t + \tau$.

$VaR_{\tau, \alpha}(X_t)$ is the negative of the $\alpha^{th}$ quantile of the distribution of $\hat{X}_{t+\tau}$ conditional on time-$t$ information.
The VaR Constraint Creates Vulnerability

Intermediation Sector

Increase in value of securities
Increase in equity

Initial balance sheet
After shock

Final balance sheet
New purchase of securities
New borrowing
Optimal Portfolio and Dividends

Portfolio of risky assets (leverage): \[ \theta_t = \frac{1}{\gamma_t} \left( \frac{\mu_t}{\sigma_t^2} - \frac{s_t}{\sigma_t} \right) \]

Dividend distribution: \[ \delta_t = u \left( \gamma_t, \eta_t - s_t \right) \beta \]

\( \gamma_t \in (1, \infty) \) such that VaR binds with equality

or \( \gamma_t = 1 \) if VaR constraint does not bind

- Changes in “effective risk aversion” \( \gamma_t \) amplify leverage response
- Lower \( \delta_t \) when \( \gamma_t, \lambda_t, \eta_t \) are higher
Stochastic Discount Factor of Intermediaries

- Lagrange multiplier of VaR increasing in $\eta_t$ and $\gamma_t$

$$
\lambda_{VaR,t} = \frac{1}{\beta \tau} \left( \frac{1}{u(\gamma_t, \eta_t - s_t)} - 1 \right)
$$

- Marginal value of one unit of wealth is

$$
Q_{\text{bank}}^t = \frac{e^{-\beta t} e^{\zeta_t}}{\beta X_t} (1 + 2 \beta \tau \lambda_{VaR,t})
$$
Representative Household

- Household solves

\[
\max \left\{ C_t, N_t, \omega_t \right\} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta (s-t)} \left( \frac{C_s^{1-\gamma}}{1-\gamma} - \frac{N_s^{1+\xi}}{1+\xi} \right) ds \right]
\]

subject to

\[
\frac{dF_t}{F_t} = \left( i_t - \pi_t + \omega_t \mu_{\text{banks},t} - \frac{1}{F_t} \left( C_t - (1-s_t) \frac{W_t}{P_t} N_t + T_t \right) \right) dt + \omega_t \sigma_{\text{banks},t} dB_t
\]

- Households face complete markets
- Portfolio of bonds and stock of banks
Household’s FOC Give IS Equation

The household’s stochastic discount factor is

\[ Q_{\text{house}}^t = e^{-\beta t} C_t^{-\gamma} \]

Household’s Euler equation and market clearing \((C_t = Y_t)\) give IS curve

\[ dy_t = \frac{1}{\gamma} \left( i_t - \pi_t - \beta + \frac{1}{2} \eta^2_t \right) dt + \frac{\eta_t}{\gamma} dZ_t \]
Household and Intermediaries Equalize Marginal Valuation

- Banks and household trade in complete markets implies \( Q_{t}^{house} = Q_{t}^{bank} \)

- Matching the volatility of \( Q_{t}^{house} \) and \( Q_{t}^{bank} \)

\[
\frac{\eta_t}{\gamma} = \frac{\eta_t - g_t}{\gamma} + g_t - \text{stoch} \left( d \log \left( \frac{1}{\beta} + 2\tau \lambda_t \right) \right)
\]

we find a function \( G \) such that

\[
\eta_t = G \left( i_t - \pi_t, s_t \right)
\]

- Monetary policy impacts price of risk \( \eta_t \) via changes in \( i_t \)
Risk-Taking Channel of Monetary Policy
Change of Variables to Growth-at-Risk To Match Data

To link to empirical findings, define “Growth-at-Risk”

\[ GaR_t \equiv \text{VaR}_{\tau,\alpha}(Y_t) = -\tau \mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} \text{Vol}_t(dy_t/dt) \]

From the IS equation

\[ \mathbb{E}_t[dy_t/dt] = \frac{1}{\gamma} \left( i_t - \pi_t - \beta + \frac{1}{2} \eta_t^2 \right) \]

\[ \text{Vol}_t(dy_t/dt) = \frac{\eta_t}{\gamma} \]
Risk-Taking Channel of Monetary Policy

Plugging into $GaR_t$

\[
GaR_t = -\frac{\tau}{\gamma} \left( i_t - \pi_t - \beta + \frac{\mathcal{N}^{-1}(\alpha)}{\sqrt{\tau}} |\eta_t| + \frac{1}{2} \eta_t^2 \right)
\]

\[
= -\frac{\tau}{\gamma} \left( i_t - \pi_t - \beta + \frac{\mathcal{N}^{-1}(\alpha)}{\sqrt{\tau}} \left| G(i_t - \pi_t, s_t) \right| + \frac{1}{2} G(i_t - \pi_t, s_t)^2 \right)
\]

$GaR_t$ and $i_t$ are one-to-one: The risk-taking channel of monetary policy
Equilibrium price of risk

Risk-Taking Channel of Monetary Policy

\[ \text{GaR}_t \]

\[ i_t \]

Tobias Adrian and Fernando Duarte
Financial Vulnerability and Monetary Policy
October 2022
Power of Continuous Time

- Can solve banks' VaR problem in closed form (even if markets were incomplete)
- Linearizing drift and stochastic parts retains time variation in risk premium

\[ dy_t = \frac{1}{\gamma} (i_t - i^* - \pi_t) \, dt + \xi (GaR_t - s_t) \, dZ_t \]

\[ GaR_t = -\frac{1}{\gamma} (i_t - i^* - \pi_t) \tau - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} \xi (GaR_t - s_t) \]

where \( \xi \) is a linearization constant

- Need at least 3rd order approximation in discrete time
Equilibrium price of risk

NKV

Dynamic IS: \[ dy_t = \frac{1}{\gamma} (i_t - i^* - \pi_t) \, dt + \xi (GaR_t - s_t) \, dZ_t \]

Growth-at-Risk: \[ GaR_t = -\tau \mathbb{E}_t [dy_t / dt] - N^{-1}(\alpha) \sqrt{\tau} \text{Vol}_t(dy_t/dt) \]

Bank shocks: \[ ds_t = -\kappa (s_t - \bar{s}) + \sigma_s dZ_t \]

NKPC: \[ d\pi_t = (\beta \pi_t - \kappa y_t) \, dt \]
Optimal Monetary Policy Problem

Central bank solves

\[ L = \min_{\{y_s, \pi_s, i_s\}} \mathbb{E}_t \int_t^{\infty} e^{-s\beta} (y_s^2 + \lambda \pi_s^2) ds \]

subject to

Dynamic IS: \[ dy_t = \frac{1}{\gamma} (i_t - i^* - \pi_t) \, dt + \xi (GaR_t - s_t) dZ_t \]

Growth-at-Risk: \[ GaR_t = -\tau \mathbb{E}_t [dy_t/dt] - N^{-1}(\alpha) \sqrt{\tau} Vol_t (dy_t/dt) \]

Bank shocks: \[ ds_t = -\kappa (s_t - \bar{s}) + \sigma_s dZ_t \]

NKPC: \[ d\pi_t = (\beta \pi_t - \kappa y_t) \, dt \]
Optimal Monetary Policy

- Optimal $i_t$ satisfies augmented Taylor

$$i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \phi_v GaR_t$$

- Or flexible inflation targeting

$$\pi_t = \psi_0 + \psi_y y_t + \psi_v GaR_t + \psi_s s_t$$

- Coefficients $\phi$ and $\psi$ are a function of structural vulnerability parameters
Output Gap Mean-Volatility Tradeoff

- Eliminating $i_t$, dynamics of the economy are

$$
dy_t = \xi \left( M \cdot \text{GaR}_t + \frac{N^{-1}(\alpha)}{\sqrt{T}} s_t \right) dt + \xi (\text{GaR}_t - s_t) dZ_t
$$

where

$$
M \equiv -\frac{\xi + N^{-1}(\alpha) \sqrt{T}}{T \xi}
$$

is the slope of the mean-volatility line for output gap

$$
\mathbb{E}_t \left[ \frac{dy_t}{dt} \right] = M \cdot \text{Vol}_t \left( \frac{dy_t}{dt} \right) - \frac{1}{T} s_t
$$
Recall Mean-Vol Line for Output Gap Growth

\[ \text{Cond Mean} = 1.12 - 1.11 \times \text{Cond Vol} + \varepsilon \]
Mean-Vol Line Moments Pin Down New Parameters

- Use standard New Keynesian parameters when possible

- For parameters relating to vulnerability, match empirical and model-based regression of the conditional mean on the conditional vol of output gap growth

- Intercept, slope, standard deviation and AR(1) coefficient of residuals identify all new parameters
Dynamics After a Shock

Conditional Mean vs. Conditional Volatility Graph

Steady-state
Dynamics After a Shock

![Graph showing the dynamics after a shock in terms of conditional volatility and conditional mean. The steady-state is marked by a horizontal line, and the after shock is indicated by a vertical line. The graph illustrates the movement from the initial state to the new steady-state following a shock.]
Dynamics After a Shock

The graph illustrates the dynamics of Conditional Volatility and Conditional Mean after a shock. The steady-state is indicated by the point where the optimal path intersects the x-axis. The change after the shock is depicted by the arrows, showing how the system moves from the initial steady-state to a new equilibrium. The optimal path is marked by a blue line, while the after shock trajectory is represented by a blue arrow.
Dynamics After a Shock

Optimal Monetary Policy Dynamics After a Shock

Conditional Volatility
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4

Conditional Mean
Steady-state
After shock
Optimal
Taylor

Optimal Monetary Policy

Tobias Adrian and Fernando Duarte
Financial Vulnerability and Monetary Policy
October 2022
Welfare Gains: Steady State Distribution of Output Gap

The diagram illustrates the steady state distribution of output gap with two curves:
- Red curve: Optimal
- Blue curve: Best Taylor Rule

The x-axis represents the output gap, while the y-axis represents the welfare gains.
Conclusion

▶ We augment the NK model with a financial sector with a Value-at-Risk constraint

▶ Model matches key empirical GaR patterns

▶ Mathematically tractable

▶ Optimal monetary policy always conditions on vulnerability
  ▶ Vulnerability responds to monetary policy
  ▶ LAW or clean up after crisis depending on vulnerability
  ▶ Magnitudes are quantitatively large