Liquidity, Debt Denomination, and Currency Dominance

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Motivation

**Currency dominance**: world features US dollar dominance

- Historical precedents: Dutch florin (17th–18th c.), British pound sterling (19th–20th c.)

**This paper**: liquidity-based theory for currency dominance in debt issuance

- Debt obligations are denominated in the unit required to be delivered at settlement
- Obtaining unit for settlement is less costly in more liquid money markets

US $ is attractive for issuance because of a large, liquid $ stock of instruments for settlement

**Key mechanism**: complementarity in liquidity supply (issuance) & demand (settlement)

$$\Rightarrow$$ Endogenous positive feedback: $ issuance begets more debt market liquidity for settlement
Related Literature

International monetary system:


Safe asset shortages:


US dollar dominance:


Search frictions in financial markets:

Historical Example:
The First Global Currency
International payments made in **illiquid metallic coin** for much of history

- Hundreds of types; costly to verify, insure, and transport; **uncertain supply** at any given time/place

**Bank of Amsterdam** (1609) overcame fractions with **florin (ledger currency)**

- Standardized **unit of account**: obtainable with coin deposits for payments via account transfers

**Florin was liquid** $\implies$ florin-denominated “bill on Amsterdam” used internationally

- At any given time, florins available in Amsterdam; **yield premium** for florin-denominated assets
International payments made in *illiquid metallic coin* for much of history

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Contrast with *illiquid Spanish “pieces of eight”* as a potential alternative global currency

- Spain bigger, wealthier, $6\times$ trade volumes, but serial defaulter
Model: Within-Country Setup
Debt Market: Firms and Investors

Debt suppliers & demanders at \( t_0 \):
- Entrepreneur-owned Firms (mass \( F \)) and Government (mass \( G \)) issue bonds at \( t_0 \)
  - Entrepreneurs borrow to finance project which costs \( \beta^2 \), and generates profits \( \pi = 1 \)
- Investors (mass \( I \)) buy bonds, have endowments \( w \); each investor can invest in 1 bond

Preferences (risk neutral):

\[
u_{i}^{F,I} = c_0 + \beta c_1 + \beta^2 c_2, \quad c_t \geq 0
\]

Bonds:
- Face value 1, mature at \( t_2 \), indivisible
- Zero default risk, perfect substitutes \( \implies \) same endogenous price \( P_0 \)

Total bonds mass: \( m_I = F + G \leq I \)
Timing Mismatch Generates Liquidity Demand at $t_1$

$F + G$ issue bonds  
$I$ (mass $m_I$) buys bonds

Central element: potential for timing mismatch generates liquidity demand

- Firms receive profits $\pi = 1$ at either $t_1$ or $t_2$
- Probability of early profits $\phi \rightarrow$ mass $m_F = \phi F$ of mismatched firms
Timing Mismatch Generates Liquidity Demand at $t_1$

$t_0$  |  $t_1$: Trading frictions  |  $t_2$

$F + G$ issue bonds  |  $\phi F$ (mass $m_F$) with early profits can match with $m_I$ bond investors  |  Bonds mature

$I$ (mass $m_I$) buys bonds

Central element: potential for timing mismatch generates liquidity demand
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Gains from asset trade $(1 - \beta)$ possible in the market at $t_1$ if firm is early:

- Investors supply liquidity
  $$m_I = F + G$$
- Firms need liquidity
  $$m_F = \phi F$$

Meeting probability $\alpha_I$  |  Meeting probability $\alpha_F$

Surplus $\eta(1 - \beta)$  |  Surplus $(1 - \eta)(1 - \beta)$
Asset Market Equilibrium and Issuance Benefits

$F + G$ issue bonds at price $P_0$

$I$ (mass $m_I$) buys bonds

$\phi F$ (mass $m_F$) with early profits can match with $m_I$ bond investors

Bonds mature

**Solving for $P_0$:** market at $t_0$ is Walrasian, so investor bids result in price

$$P_0 = \frac{\alpha_I \beta (\beta + (1 - \eta)(1 - \beta))}{P(\text{Matched}) \times \text{PV of Sale Price}} + \frac{(1 - \alpha_I)\beta^2}{P(\text{Not Matched}) \times \text{PV of 1}}$$

**Convenience yield** at $t_0$ captured by $P_0 - \beta^2 = \beta(1 - \beta)(1 - \eta) \times \alpha_I$

- A fully illiquid bond ($\alpha_I = 0$) would be priced at $\beta^2$

**Expected utility** from debt issuance for firm $i$ is increasing $\alpha_I$ and $\alpha_F$:

$$\mathbb{E}[u_i^F] = \beta(1 - \beta) \times \left[ \frac{(1 - \eta)\alpha_I}{\text{Convenience yield at } t_0} + \frac{\eta\phi\alpha_F}{\text{Benefit of liquidity at } t_1} \right]$$
Matching function at \( t_1 \): number of meetings between firms (demanders) and investors (suppliers) is
\[
n = \lambda m_F^\theta m_I^\theta, \quad \lambda > 0, \quad \theta > 1/2
\]

- Increasing returns


Meeting probabilities:
\[
\alpha_F = \frac{n}{m_F} = \lambda m_I^\theta m_F^{\theta-1}, \quad P(\text{Firm finds a bond seller})
\]
\[
\alpha_I = \frac{n}{m_I} = \lambda m_F^\theta m_I^{\theta-1}, \quad P(\text{Bond seller finds a firm})
\]

Expected firm utility given equilibrium prices and probabilities (taking \( \theta = 1 \) case):
\[
\mathbb{E}[u_i^F] = \lambda \beta (1 - \beta) \times [ (1 - \eta) m_F + \eta \phi m_I ]
\]
- Convenience yield at \( t_0 \), increasing in liquidity demand \( m_F \)
- Benefit of liquidity at \( t_1 \), increasing in liquidity supply \( m_I \)
Model: Two-Country Environment
Debt Denomination Choice

Two countries $j = A, B$ with fundamentals $\{G_j, F_j, \lambda_j\}$

Currency denomination choice for firms $i$ in each country

- Fixed cost $\propto K_i$ of foreign issuance
  - Ex: expected costs of balance sheet currency mismatch, underwriting, risk aversion (hedging), ...
Debt Denomination Choice

Two countries \( j = A, B \) with fundamentals \( \{ G_j, F_j, \lambda_j \} \)

**Currency denomination choice** for firms \( i \) in each country

- Fixed cost \( \propto K_i \) of foreign issuance
  - Ex: expected costs of balance sheet currency mismatch, underwriting, risk aversion (hedging), ...

**Endogenous masses** \( M = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B}) \)

**Four denomination possibilities** with expected utility denoted:

\[
\begin{align*}
U_{A \rightarrow A}(M) & \quad U_{A \rightarrow B}(M, K_i) \\
U_{B \rightarrow B}(M) & \quad U_{B \rightarrow A}(M, K_i)
\end{align*}
\]
Debt Denomination Choice

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**Endogenous masses** $\mathcal{M} = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B})$

**Four denomination possibilities** with expected utility denoted:

$$U_{A\rightarrow A}(\mathcal{M}) \quad U_{A\rightarrow B}(\mathcal{M}, K_i)$$
$$U_{B\rightarrow B}(\mathcal{M}) \quad U_{B\rightarrow A}(\mathcal{M}, K_i)$$

Firm optimality requires **threshold strategy**: firms issue in foreign currency iff $K_i \leq \bar{K}$

- $H(K_i)$ is the (Pareto) CDF of $K_i \in [K, \infty)$ → share $H(\bar{K})$ issues in foreign currency
Debt Denomination Choice

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Four denomination possibilities with expected utility denoted:

$$\bar{U}_{A \rightarrow A}(M(\bar{K})) \quad \bar{U}_{A \rightarrow B}(M(\bar{K}), \bar{K})$$

$$\bar{U}_{B \rightarrow B}(M(\bar{K})) \quad \bar{U}_{B \rightarrow A}(M(\bar{K}), \bar{K})$$

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Firm optimality requires threshold strategy: firms issue in foreign currency iff $K_i \leq \bar{K}$

- $H(K_i)$ is the (Pareto) CDF of $K_i \in [K, \infty)$ → share $H(\bar{K})$ issues in foreign currency
- Class BA (focus today) and class AB (symmetric analysis) equilibria can arise
Define $\hat{K}$ as the equilibrium value of $\bar{K}$, equilibrium characterized by:

1. **Firm optimality**: the marginal firm ($K_i = \bar{K}$) has $K_i = \hat{K}$ in equilibrium and satisfies

   $$\bar{U}_{j' \rightarrow j}(\hat{K}) = \bar{U}_{j' \rightarrow j'}(\hat{K})$$

2. **Market clearing**: given $\hat{K}$, masses $M$ satisfy

   $$m_{l,j} = G_j + F_j + H(\hat{K})F_{j'} \quad m_{l,j'} = G_{j'} + [1 - H(\hat{K})] F_{j'}$$

   $$m_{F,j} = \phi [F_j + H(\hat{K})F_{j'}] \quad m_{F,j'} = \phi [1 - H(\hat{K})] F_{j'}$$
Multiple Equilibria With Symmetric Fundamentals

Class BA equilibria: B firms switch to currency A

\[ \tilde{U}_{B \rightarrow A} = \lambda_A [m_{F,A}(\tilde{K}) + \phi m_{I,A}(\tilde{K})] - \tilde{K} \]  
Expected utility of foreign denomination

\[ \tilde{U}_{B \rightarrow B} = \lambda_B [m_{F,B}(\tilde{K}) + \phi m_{I,B}(\tilde{K})] \]  
Expected utility of home denomination
Multiple Equilibria With Symmetric Fundamentals

Class BA equilibria: B firms switch to currency A

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$$\bar{U}_{B\to A} = \lambda_A [m_{F,A}(\tilde{K}) + \phi m_{I,A}(\tilde{K})] - \tilde{K}$$

Expected utility of home denomination

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Multiple Equilibria With Symmetric Fundamentals

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Expected utility of home denomination
Multiple Equilibria With Symmetric Fundamentals

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Expected utility of home denomination
Liquidity and Dominance
Throughout History
**Result 1: Understanding Historical Transitions - Fundamental Asymmetries Generate Dominance**

**Italian city-states** (15th – 16th c.) also prominent in trade and finance, but no dominant currency:

- Symmetry $\rightarrow$ stable multipolar arrangement

**Amsterdam** disrupted multipolarity:

- Govt commitment and financial technology generated asymmetrically large $G$

**Transition to British pound** had similar features:

- Bank of Amsterdam collapses in 1791 ($\downarrow G_A$)
- Britain wins Napoleonic Wars ($\uparrow G_B$)

\[ \implies \text{In paper: } \uparrow F \text{ not sufficient for eq. transition} \]

**Increasing** $G_A$ sufficiently leads to unique equilibrium selection:

\[ \bar{U}_{B\rightarrow A} = \phi \lambda_A [G_A + 2F_A + 2H(\bar{K})F_B] - \bar{K} \]
Specify the government’s objective as

\[ W_j = F_j \int u_{i,j}^F(K_i) \, dH(K_i) + G_j \left( P_{0,j} - \beta^2 \right) \]

- **Domestic firm utility**
- **Seignorage conv. yield**
Result 2: Complementarities Between Dominance and Sovereign Liquidity Provision Incentives

Specify the government’s objective as

\[ W_j = F_j \int u_{ij}^F(K_i) \, dH(K_i) + G_j (P_{0,j} - \beta^2) \]

- Domestic firm utility
- Seignorage conv. yield

Consider: \( B \rightarrow A \) equilibrium with \( G_A > G_B \), \( \lambda_A = \lambda_B \), \( F_A = F_B \)

1. Bigger incentive to create liquidity (\( G \)) for the leader (\( A \)): \( \partial W_A / \partial G_A > \partial W_B / \partial G_B \)

2. Complementarity: investment incentive reinforced by endogenous rise in entry (\( \hat{K} \)):

\[ \frac{\partial^2 W_A}{\partial G_A \, \partial \hat{K}} > 0, \quad \frac{\partial \hat{K}}{\partial G_A} > 0 \]

Incentives manifested in history of Bank of England: LoLR, backstopping of private credit market

\[ \Rightarrow \] More in paper: analogous complementarity in incentives to facilitate private liquidity creation
Result 3: Additional Complementarity Arises from International Trade Invoicing

International trade and finance are highly related

- Ex: bills of exchange in Amsterdam both settlement instruments for trade and source of credit

Trade invoicing is complementary to currency dominance in debt denomination

- If revenues in dominant currency, lower FX mismatch reduces $K_i$ (as in Gopinath Stein 2021)
- Shifting $H(K)$ to the left $\rightarrow$ more entry with $\dot{K}_1 > \dot{K}_0$:

\[
\lambda_A \phi \left[ 2F_A + G_A + F_B H(\dot{K}_0) \right] - \dot{K}_0 = \lambda_B \phi \left[ 2F_B + G_B + F_B (1 - H(\dot{K}_0)) \right]
\]

- If firms choose invoicing currency, generate trade dominance as by-product of financial dominance

$\Rightarrow$ Additional complementarity that reinforces dominant equilibrium
Welfare, Aggregate Risk, and International Cooperation
Global planner has objective:

\[ W = W_A + W_B \]

**Socially optimal entry** > **competitive equilibrium** because entry carries positive *liquidity externality*

\[ K^* > \hat{K}_{\text{max}} \]

- First best \( (K^*) \) is a Pareto improvement over competitive equilibrium (with transfers)
Global planner has objective:

$$\mathcal{W} = W_A + W_B$$

**Socially optimal entry > competitive equilibrium** because entry carries positive *liquidity externality*

![Socially optimal entry > Competitive Equilibrium]

- First best \((K^*)\) is a Pareto improvement over competitive equilibrium (with transfers)

**Country A underprovisions** \(G_A\) relative to global planner if

$$\frac{\partial \mathcal{W}}{\partial G_A} > \frac{\partial W_A}{\partial G_A}$$

- In this case, there are **gains from international cooperation in liquidity supply**
  - Historical analog: **Bretton Woods** → major economies coordinated on US-provided liquidity
  - This case occurs in the model if \(F_B\) is sufficiently larger than \(G_B\)
Aggregate risk:

- State at $t_1$ is $\omega \in \Omega$ with probability $q_\omega \to$ aggregate liquidity demand shock: $\phi_\omega$
- State-contingent liquidity supply $G^A_\omega$ chosen in advance at $t_0$

Equilibrium indifference condition now features moments of the $(\phi_\omega, G^A_\omega)$ distribution:

$$\lambda_A \left( \mathbb{E}[\phi_\omega] \left( 2F_A + H(\hat{K})F_B + \mathbb{E}[G^A_\omega] \right) + \text{Cov}[\phi_\omega, G^A_\omega] \right) - \hat{K} = \lambda_B \mathbb{E}[\phi_\omega] \left( 2(1 - H(\hat{K}))F_B + G_B \right)$$

- State-contingent liquidity provision (positive covariance) induces entry

Policy tool: Central bank swap lines that provide liquidity when it is most demanded
Conclusion: Dollar Dominance Today

Sources of dominance we highlight appear in many features of the dollar:

- Base for USD-denominated money markets is T-Bills (large, liquid, safe stock)
- Financial technologies make private assets liquid (repo, securitization, banking)
- Fed swap lines: contingent expansion of US $-denominated liquidity
- Complementarities in dollar issuance by wide spectrum of entities:
  - Safe liquidity suppliers taking advantage of US $ convenience yields (e.g., KFW)
  - Other lower-rated global corporates also issue US $ drawn in by liquidity benefit

Renminbi dominance question: current Chinese financial system lacks these elements
Model equilibrium:
- Equilibrium lemmas
- Formal firm problem
- Class $AB$ equilibria
- Increasing $F_A$

Theoretical extensions:
- Issuance complementarities
- The $\theta < 1$ case
- Limited pledgeability
- Sovereign denomination choice

History:
- Bank of Amsterdam mandate
- Florin quantities
- The florin *agio*
- Bank of England evolution

Empirics:
- Debt quantities
- British dominance
- Finance and trade
Extra Slides
The Denomination Choices of Safe and Risky Private Borrowers Are Complementary

\[ \hat{K}^+ = \phi \left[ \lambda_A F_A - \lambda_B F_B^- + (\lambda_A + \lambda_B) H(\hat{K}^-) F_B^- \right] \]

\[ \hat{K}^- = \phi \left[ \lambda_A (F_A + G_A) - \lambda_B (G_B + F_B^+) + (\lambda_A + \lambda_B) H(\hat{K}^+) F_B^+ \right] \]
Convenience Yields and Sovereign Debt Supply

\[
P_{0,j} - \beta^2 = \frac{\lambda_j \beta (1 - \beta)}{2} m_{\theta,j} m_{\theta-1,j}
\]

(a) Case 1: Convenience yield decreasing in \( G_A \)

(b) Case 2: Convenience yield increasing in \( G_A \)
Crowding In and Crowding Out of Heterogeneous Private Borrowers

- In general case ($\theta < 1$), can generate negative impact of sovereign debt supply on conv. yields

- As a result, more government debt *crowds out* safe borrowers while *crowding in* risky borrowers
Improving capacity of private sector to issue safe money-like assets also part of financial development

Extend model to include country-specific pledgeability parameter $\rho_j$

- After currency choice, firms find out if revenues are fully pledgeable (probability $\rho_j$) or not

*Ex ante* expectation of pledgeability is $\rho_j$, so equilibrium condition becomes:

$$\rho_A \left[ \lambda_A (m_{F,A} + \phi m_{I,A}) - \hat{K} \right] = \rho_B \left[ \lambda_B (m_{F,B} + \phi m_{I,B}) \right]$$

As in previous case, sovereign incentives to invest in firm pledgeability complementary to dominance:

$$\frac{\partial W_A}{\partial \rho_A} > \frac{\partial W_B}{\partial \rho_B}, \quad \frac{\partial^2 W_A}{\partial \rho_A \partial \hat{K}} > 0, \quad \frac{\partial \hat{K}}{\partial \rho_A} > 0$$
Mandate from the Bank's founding decree:

“To check all agio of the current money and confusion of coin, and to be of use to all persons who are in need of any kind of coin in business.”
Entrepreneur chooses whether to issue \((D_i)\) at \(t_0\) and whether to trade \((T_i)\) at \(t_1\):

\[
\max_{D_i, T_i} E[c_0 + \beta c_1 + \beta^2 c_2]
\]

subject to

\[
c_0 = D_i(P_0 - \beta^2),
\]

\[
c_1 = \begin{cases} 
0, & \text{late;} \\
0, & \text{early, but not matched;} \\
D_i T_i \eta(1 - \beta), & \text{early, and matched}
\end{cases}
\]

\[
c_2 = 0.
\]

Since \(P_0 \geq \beta^2\) and \(\beta < 1\), solution is to set \(D_i = 1\) and \(T_i = 1\).
Lemma 1
Consider firms $\hat{i}$ and $i$ in country $j$, where $K_i < K_{\hat{i}}$. If it is optimal for firm $\hat{i}$ to issue in foreign currency $j' \neq j$, then it is optimal for firm $i$ to issue in foreign currency $j'$.

Lemma 2
Suppose that there is a positive mass of firms in $j$ that find it optimal to issue in currency $j'$. Then, no firms in $j'$ will issue in currency $j$.

Lemma 3
A necessary condition for a collection of firm denominations choices $D_{i,j}$ to be consistent with firm optimality is that it must take the following threshold form:

$$D_{i,j'} = \begin{cases} 1 & \text{if } K_i < \bar{K}, \\ 0 & \text{if } K_i \geq \bar{K} \end{cases}, \quad D_{i,j} = 0.$$
Consider the choice for firms in $A$ and define $\hat{K}$ as the equilibrium value of $\bar{K}$.

The threshold firm ($K_i = \bar{K}$) has $K_i = \hat{K}$ in equilibrium and satisfies:

$$\lambda_A \left[ m_{F,A} + \phi m_{I,A} \right] = \lambda_B \left[ m_{F,B} + \phi m_{I,B} \right] - \hat{K}$$

$\bar{U}_{A \rightarrow A}(\lambda m)$: Utility from issuing in home currency

$\bar{U}_{A \rightarrow B}(\lambda m, \hat{K})$: Utility from issuing in foreign currency

Given $\hat{K}$, masses are:

$$m_{I,A} = G_A + \left[ 1 - H(\hat{K}) \right] F_A$$

$$m_{I,B} = G_B + F_B + H(\hat{K}) F_A$$

$$m_{F,A} = \phi \left[ 1 - H(\hat{K}) \right] F_A$$

$$m_{F,B} = \phi \left[ F_B + H(\hat{K}) F_A \right]$$
Financial Innovation Driving Florin Success

Monthly bank balances (1666 – 1703); Source: Quinn and Roberds (2014)
End of Dutch Dominance

Agio: percent premium of bank florin over current guilders (1736 – 1792)

Source: Quinn and Roberds (2019)
Short-Term Government Debt Supply Vastly Higher in the United States
Allowing Denomination Choice for G Entrenches Dominance

Allow government in $B$ to choose amount of denomination in currency $A$: $G_B^* \in [0, G_B]$

**Government’s objective:**

\[
W_j = G_j (P_{0,j} - \beta^2) + \int F_j u_{i,j}^F(K_i) dH(K_i)
\]

Seignorage conv. yield

Domestic firm utility

In equilibrium in the baseline model, the follower’s objective ($B$) is

\[
W_B = G_B \times \lambda_B m_{F,B} + F_B (1 - H(\hat{K})) \times \lambda_B (m_{F,B} + \phi m_{I,B}) + U_{B \rightarrow A}
\]

Conv. Yield in B for govt debt

Conv. Yield + Liquidity Benefit in B for firm debt

Switchers

With the choice, $B$ trades off **better convenience yields** in govt debt with **lower liquidity benefit** to private firms
Allowing Denomination Choice for $G$ Entrenches Dominance

Allow government in $B$ to choose amount of denomination in currency $A$: $G_B^* \in [0, G_B]$

Government’s objective:

$$W_j = G_j (P_0 - \beta^2) + F_j \int u_{i,j}^F(K_i) \, dH(K_i)$$

Seignorage conv. yield

Domestic firm utility

In equilibrium in the baseline model, the follower’s objective ($B$) is

$$W_B = G_B \times \lambda_B m_{F,B} + F_B (1 - H(\hat{K})) \times \lambda_B (m_{F,B} + \phi m_{I,B}) + U_{B \rightarrow A}$$

Conv. Yield in $B$

for govt debt

Conv. Yield +

Liquidity Benefit in $B$

for firm debt

Switchers

With the choice, $B$ trades off better convenience yields in govt debt with lower liquidity benefit to private firms

$$W_B = G_B^* \times \lambda_A m_{F,A} + (1 - G_B^*) \lambda_B m_{F,B} + F_B (1 - H(\hat{K})) \times \lambda_B (m_{F,B} + \phi (1 - G_B^*) + F_B H(\hat{K}))) + \ldots$$

Higher conv. yield

for own debt

Lower liquidity benefits

for domestic firm debt

$\implies G_B^*$ will issue more in $A$ if convenience yields are much better and private firms are small

[Back]
Allowing Denomination Choice for $G$ Entrenches Dominance

Allow government in $B$ to choose amount of denomination in currency $A$: $G_B^\ast \in [0, G_B]$

Government’s objective:

$$W_j = G_j (P_0 j - \beta^2) + F_j \int u_{i,j}^F (K_i) dH(K_i)$$

Seignorage conv. yield

Domestic firm utility

In equilibrium in the baseline model, the follower’s objective ($B$) is

$$W_B = G_B \times \lambda_B \lambda_B m_{F,B} + F_B (1 - H(\hat{K})) \times \lambda_B (m_{F,B} + \phi m_{L,B}) + U_B \rightarrow A$$

Conv. Yield in B for govt debt

Conv. Yield + Liquidity Benefit in B for firm debt

Switchers

With the choice, $B$ trades off better convenience yields in govt debt with lower liquidity benefit to private firms

$$W_B = G_B^\ast \times \lambda_A m_{F,A} + (1 - G_B^\ast) \lambda_B m_{F,B} + F_B (1 - H(\hat{K})) \times \lambda_B (m_{F,B} + \phi (1 - G_B^\ast)) + F_B H(\hat{K})) + \ldots$$

Higher conv. yield for own debt

Lower liquidity benefits for domestic firm debt

$\implies G_B^\ast$ will issue more in $A$ if convenience yields are much better and private firms are small
Increasing Private Sector Size Has Ambiguous Equilibrium Impact

(a) Class BA Equilibria

(b) Class AB Equilibria
The Evolution of British Pound Dominance

[Diagram showing bond value by GDP from 1820 to 1920, with two lines representing Foreign Government Bonds and Domestic & Corporate Bonds.]