Trade and Industrial Location with Heterogeneous Labor*

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Abstract

We show in the context of a new economic geography model that when labor is heterogenous trade liberalization may lead to industrial agglomeration and inter-regional trade. Labor heterogeneity gives local monopoly power to firms but also introduces variations in the quality of the job match. Matches are likely to be better when there are more firms and workers in the local market, giving rise to an agglomeration force which can offset the forces against, trade costs and the erosion of monopoly power. We derive analytically a robust agglomeration equilibrium and illustrate its properties with numerical simulations.

Keywords: agglomeration, matching, spatial mismatch, inter-regional trade

JEL Classification: F12, J41, R12, R13

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In this paper we demonstrate that when labor is heterogenous and the matching of skills with jobs below first-best, the introduction of trade may lead to industrial agglomeration and inter-regional trade. Agglomeration takes place because the average quality of matches improves when firms in the local market have a bigger pool of workers to choose from. The force against agglomeration is the existence of trade costs. At zero trade costs regions that can trade always specialize, whereas when there are positive trade costs they may or may not specialize, depending on the values taken by some other parameters.

Our model resembles other models in new trade theory, except that there are heterogeneities in the performance of tasks done by apparently similar workers. *Ex ante* workers appear identical and no worker is more productive than another across the whole range of jobs. But some workers are more productive in some jobs and other workers are more productive in other jobs. Labor productivity in the model depends on technology, training and the other factors of conventional production theory, but also on the quality of the match between the job and the worker. We postulate that even when workers are allocated to the jobs where they are most productive, companies that have specialized skill requirements can recruit better-matched workers if they recruit in larger markets. We show that when this feature is combined with features commonly assumed in new trade theory, in particular increasing returns, differentiated goods and transport costs, it alone can explain agglomeration by industries that use specialized skills.

Anecdotal evidence in favor of our hypothesis is easy to find. To give two examples, one by an employer and one by an agent looking for employers, the general manager of Sony UK explained as follows why his company remains in high-wage Britain: "What keeps us here is the quality of the staff and the research and development capacity" (*Financial Times*, January 19, 2002). On the other side of the market, Gavin Clarkson, the owner of a software company but speaking as a member of the Choktaw Nation of Oklahoma, was reported as planning to set up a technical training centre in Oklahoma to attract companies because "Having a critical mass of people who are highly skilled and resourced is what attracts business to any location or community." (*Australian Financial Review,*11 April 1999). The writers of the *Financial Times* article went further to argue that “The survival of struggling volume producers may or may not prove directly vital to the economy as a whole but their role in providing skilled staff and components infrastructure for higher-margin niche manufacturers is hard to ignore.” The contribution of “components infrastructure” to industrial agglomeration was the theme of Krugman and Venables (1995). Our focus is the role of “skilled staff.”

Formal econometric evidence in favor of labor pooling was provided by Dumais et al. (1997). Making use of the LRD manufacturing data base for the United States, they examined the relative importance of Marshall’s three reasons for agglomeration for the location of manufacturing plants; proximity to suppliers and customers, labor pooling and information spillovers. They found that labor pooling was by far the most important force for agglomeration, at least at the metropolitan area level. New entrants tended to locate in areas where existing firms had similar labor requirements to their own.

The Dumais et al. (1997) research gives support to labor pooling as an agglomeration force but does not differentiate between different reasons that might make it important. Indirect evidence, however, supports our matching reasons. Match differentiation is likely to be more important for more advanced skills. The routine tasks that dominate production in less advanced economies do not afford much scope for differentiation or creativity. But with the invention of more complicated tasks, the routine nature of agricultural and industrial work gives way to work situations which allow different and varied types of performance. If we are correct in claiming that this is a reason for agglomeration, then agglomeration should increase with economic growth and should be more prevalent in industries that require more high-tech labor. Dumais et al. (1997) find that labor pooling is especially strong as an agglomeration force in high technology industries.[1]

[1] The main industries they list are fabricated metals, industrial machinery, electronic and electrical equipment and instruments. Knowledge spillovers are also relatively more important for these industries but not as important as labor pooling. In accordance with our argument, Amiti and Cameron (2004) found that benefits of labor pooling were significant in a developing country (Indonesia), but were not as high as other agglomeration forces, such as inter-firm

Our paper is related to three strands of literature. First is the "new economic geography" literature, which shows that agglomeration of manufacturing industries can arise when combined with inter-regional labor mobility (Krugman, 1991); or when combined with input/output linkages between vertically-linked firms (Krugman and Venables, 1995). The labor market is perfectly competitive in both these models, so they do not share our reasons for agglomeration, and we do not have either of their agglomeration forces in our model. We instead introduce endogenous acquisition of labor skills and provide a richer model of the labor market.

The second strand is the "labor pooling" literature. Krugman (1991) formalizes Marshall's reasons for emphasizing labor pooling. He claims that labor pooling is a way of achieving more efficiency when firms are exposed to idiosyncratic risk, because when the number of firms in a region is large the law of large numbers ensures that on average idiosyncratic shocks wash out.[2] Rotemberg and Saloner (2000) examine the case where skilled labor has to get trained, and identify a hold-up problem in the absence of labor pooling. A worker is more likely to pay the up-front cost of training if she knows that there are many firms in her town that will compete for her services. Competition ensures that she will recoup the cost of her training through higher wages.

Although our reason for agglomeration is also labor pooling, it is very different both from Krugman's and from Rotemberg and Saloner's. It is more closely related to a third strand of literature, the external economies discussed by Henderson (1988), and the agglomeration reasons invoked by Helsley and Strange (1990) in a Henderson-type model of city size. Trade is absent from this literature strand. Helsley and Strange (1990) show that the existence of matching externalities is a force pushing for the agglomeration of production but the scarcity of land prohibits the agglomeration of all industry in a single location. In contrast, land in our model is a free commodity, workers are of two types, and location decisions are made by differentiated firms that can move from one region to another.

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linkages.

[2]Dumais et al. (1997) also find some evidence supporting this hypothesis. Labor pooling appears to be a more important agglomeration force in industries with more volatile employment.

Section 1 describes our model and derives the choices of firms and workers. Sub-section 1.1 is the core of our paper: it introduces our formal definition of heterogeneity and its connection with labor skills and derives the supply of labor to firms. Section 2 derives the labor market equilibrium and section 3 the aggregate equilibrium. It shows that there are at least two equilibria, a symmetric one with identical distribution of firms in each region and an agglomeration one with specialization in production and trade. In section 4 we define a robustness criterion based on optimal deviations from equilibrium and show that at low trade costs the agglomeration equilibrium is the only robust equilibrium whereas at high trade costs the symmetric equilibrium is the robust one.

1. The model

The model is static and we look for a Nash equilibrium. It has two regions (or countries), home and abroad, two sectors in each region (one of which may be empty), agriculture and manufacturing, and two skill levels, skilled and unskilled. Skilled workers work in manufacturing and unskilled in agriculture. We make three critical assumptions about the mobility of factors: (a) there is perfect inter-sectoral and inter-regional mobility of firms, (b) there is perfect inter-sectoral mobility of labor, although if the direction of movement is from the agricultural to the manufacturing sector it requires some fixed "training" cost, (c) there is no inter-regional mobility of labor. Decisions are made as follows. Firms take as a given constraint the demand for output function and the distribution of workers across sectors and choose: first, their region and sector, and second, conditional on their location, their wage rate and output price. Workers take as given the firms' locations, prices and wages and decide: first whether to train or not, which determines their supply of labor, and conditional on their employment, how much to consume.

We begin the derivation from the last decision facing workers, the choice of demand functions, and the last decision facing firms, the choice of wage and output price, and work backwards to the initial decisions of each. The last-stage decisions follow conventional new trade theory and we describe them briefly. The new contribution of our paper is in the labor market model and we describe it in more detail later in this section.

The agricultural sector produces a single good but there are many differentiated manufacturing goods, each produced by one firm only. Consumers have Dixit-Stiglitz preferences over the manufacturing goods, which are aggregated into a composite denoted by C_x. Consumption of the agricultural good is denoted by C_a. The utility function of individual k is

$$U_k = v_k C_{xk}^{\mu} C_{ak}^{1-\mu}, \qquad 0 < \mu < 1. \tag{1.1}$$

The parameter $v_k > 0$ depends on the sector in which the individual works, and is specified later.

All goods can be traded and we introduce imports and exports in preparation for the later analysis. Foreign-country variables are distinguished by a star. Let N be the number of domestically-produced manufacturing goods (which equals the number of domestic manufacturing firms) and N^* the number of foreign firms. The sub-utility function for manufacturing goods is

$$C_{xk} = \left[\sum_{i=1}^{N} c_{ik}^{\frac{\sigma-1}{\sigma}} + \sum_{j=1}^{N^*} \left(\frac{m_{jk}}{\tau} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \qquad \sigma > 1, \tau \geq 1, \tag{1.2}$$

where c_{ik} is the consumption of the domestically-produced manufacturing good i, m_{jk} is the demand for the imported manufacturing good j, τ are iceberg transportation costs and σ is the elasticity of substitution between varieties.

Let y_k represent the income of individual k. The budget constraint for individual k is

$$\sum_{i=1}^{N} p_i c_{ik} + \sum_{j=1}^{N^*} p_j^* m_{jk} + P_a C_{ak} = y_k, \tag{1.3}$$

where p_i is the price of good i, p_j^* is the price of the imported good and P_a is the price of the agricultural good. Define the price index of the manufacturing composite good by

$$P_x = \left[\sum_{i=1}^{N} p_i^{1-\sigma} + \sum_{j=1}^{N^*} \left(\tau p_j^* \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{1.4}$$

Maximization of the utility function for a given income level gives the following demand functions for the manufacturing composite and the agricultural good:

$$P_x C_{xk} = \mu y_k, \tag{1.5}$$

$$P_a C_{ak} = (1-\mu) y_k, \tag{1.6}$$

and for each manufacturing good:

$$c_{ik} = \mu \left(\frac{p_i}{P_x}\right)^{-\sigma} \frac{y_k}{P_x}, \tag{1.7}$$

$$m_{jk} = \mu \left(\frac{\tau p_j^*}{P_x}\right)^{-\sigma} \frac{\tau y_k}{P_x}. \tag{1.8}$$

To derive the demand constraint facing a domestic firm we aggregate the demand for good i in the domestic and foreign country over all individuals to yield, from (1.7) and (1.8),

$$\begin{aligned} x_i^d &= c_i + m_i^* \\ &= \mu p_i^{-\sigma} (P_x^{\sigma-1} Y + \tau^{1-\sigma} P_x^{*\sigma-1} Y^*), \end{aligned} \tag{1.9}$$

where x_i^d is the total demand for good i, c_i and m_i^* are respectively home and export demand and Y and Y^* are respectively aggregate income at home and abroad.

The agricultural good is produced with a linear technology, one worker producing one unit of output, and used as the numeraire, so both price and the wage in agriculture are equal to 1. Manufacturing firms are monopolistic competitors and set prices for their products by maximizing a profit function subject to the demand constraint (1.9). Let the employment level that the firm attracts at posted wage w_i be some function $L_{si}^E(w_i)$, which is differentiable and has finite elasticity $\eta_{si}^E \geq 0$. The labor market model will give a closed-form solution for this elasticity, which plays a critical role in our agglomeration argument. Labor input is measured in "effective" (or "efficiency") units (distinguished from the number of workers by superscript E) and the firm posts a wage for each effective unit of labor supplied by a worker. Manufacturing firms employ only skilled labor, distinguished by subscript s.

We assume no firm entry costs but a fixed cost of production measured in labor units. The (inverted) production function for manufacturing firm i is:

$$L^E_{si}(w_i) = \alpha + \beta x_i, \qquad \alpha, \beta > 0, \tag{1.10}$$

where x_i is the firm's output. Firms maximize profit,

$$\pi_i = p_i x_i - w_i L^E_{si}(w_i), \tag{1.11}$$

with respect to the controls w_i, p_i, x_i. The first-order conditions satisfy the equality $x_i = x^d_i$, the production function (1.10) and and the mark-up equation,

$$p_i = \frac{\sigma\beta}{\sigma - 1}(1 + \frac{1}{\eta^E_{si}})w_i. \tag{1.12}$$

For a given supply of labor function, substitution of p_i from (1.12) into (1.9) gives the firm's equilibrium output level, which, when substituted into (1.10) gives the wage rate that equates the demand for labor with the given supply. We now derive the given supply of labor.

1.1. Skill differentiation and the supply of labor to the firm

Workers have the choice of training to become skilled and enter the manufacturing sector or remain unskilled and enter the agricultural sector. Skilled workers are horizontally differentiated because of idiosyncratic characteristics that reveal themselves after their training, and which become public information. They cannot alter these characteristics or choose them before training. Their characteristics determine the quality of their match with firms.

We model the quality of a match as a unidimensional measure which we call the "distance" of the worker from the firm.[3] Skills are distributed along a circle, whose circumference is of length $2H$. H is a measure of the heterogeneity of skills, of how far match-specific productivities can vary from each other. If $H = 0$ there

[3] Helsley and Strange (1990) and Thisse and Zenou (2000) use a similar measure of heterogeneity. It goes back at least to Salop (1979).

is no heterogeneity and all skilled workers have the same productivity in all firms. Once workers acquire their skill they are allocated randomly on this circle, with all locations equally likely, so the density of workers on the circle is uniform. A worker cannot change her location but she can quit the skilled sector and move to agriculture, where her output is the same as that of an unskilled worker.

Unlike workers, firms can choose their location on the skills circle. The closer a firm and a worker are on the circle the better the quality of their match. In symmetric equilibrium, each firm and worker will be matched to the agent from the other side located closest to them. We measure the quality of the job match by the effective units of labor. We assume that the number of effective units of labor supplied by a worker to some firm i is a linear function of the distance between firm i and the worker on the skills circle. The input of a worker who has a firm located at exactly the same point as herself is the maximum number of effective units of labor that a match can yield, which is normalized to unity. If the worker is located distance d away, her input is either $1 - d$ or zero units. We ignore negative or zero inputs because they will never be equilibrium outcomes.[4]

Firms choose first their location and then post a wage for each effective unit of labor supplied by a worker. The labor input is both observable and verifiable and we assume that there is full employment. When choosing their location, firms are more likely to get workers the further away they are from other firms, unless they locate exactly at the same point as another firm and beat its wage offer. But in the latter case, Bertrand competition will lead to the equality between the posted wage and the value of marginal product of labor, and so to the exit of firms because of the fixed cost of production. Once firms deviate, Cournot-Nash competition leads to symmetric locations along the circle.[5]

[4] The coefficient on d can be set equal to unity without loss of generality by appropriate choice of units of measuring H. For example, because maximum distance is H, we can ensure that the productivity of a match is never negative by restricting the range of H to the $[0, 1]$ interval.

[5] The formal structure of the location decision in our paper is similar to the one analyzed by D'Aspremont et al. (1979) and Economides (1989), who show that firms will want to differentiate.

We therefore assume (without derivation, which is straightforward) that all firms locate symmetrically along the skills circle. It follows that if there are N manufacturing firms in the market, in equilibrium the distance between any two firms is $2H/N$ and so the worst case of mismatch is half the distance, H/N. We denote this ratio by m, for mismatch.[6] Firms post a wage w for each effective unit of labor supplied by workers who come to them, so the highest wage in the market is w and the lowest the one offered to the most distant worker, $w(1-m)$. Because skilled workers can quit to join the agricultural sector and earn a wage equal to 1, the lowest wage in manufacturing cannot be less than 1. We avoid the possibility that there are segments on the skills circle that are so far from the nearest firm that workers who are allocated there return to agriculture by assuming that the parameters are such that in equilibrium $w(1-m) \geq 1$.

To derive the wage rate we first derive the supply of labor function that constrains firms. Let a firm i be located halfway between two other firms that post wage w. By the symmetry assumptions, the distance of firm i from either neighboring firm is $2m$. A worker located to the left of i, at some distance $d \leq 2m$ away, can join firm i and supply $1-d$ effective units of labor, or she can join the firm to the left of i and supply $1-(2m-d)$ effective units. If firm i posts wage w_i the worker will go to firm i if

$$w_i(1-d) \geq w(1-(2m-d)). \tag{1.13}$$

Therefore, firm i attracts all workers located to its left up to distance d_i away, with d_i derived from (1.13) as the maximum d that satisfies inequality (1.13):

$$d_i = \frac{w_i - w(1-2m)}{w_i + w}. \tag{1.14}$$

By symmetry, (1.14) is also the maximum distance over which the firm attracts workers from its right. With workers uniformly distributed on the circle, the fraction of the number of skilled workers attracted to the firm is d_i/H and the average number of effective units of labor supplied by each is $1 - d_i/2$. So, if the

[6] Although we also use m to denote imports no confusion should arise, as imports are always qualified by subscripts.

total number of skilled workers is L_s, the total number of effective units of labor supplied to the firm that posts wage w_i is

$$\begin{aligned} L_{si}^E &= \frac{d_i L_s}{H}(1 - \frac{d_i}{2}) \\ &= \frac{L_s}{2H} \frac{(w_i - w + 2mw)(w_i + 3w - 2mw)}{(w_i + w)^2}. \end{aligned} \tag{1.15}$$

The partial derivative of (1.15) with respect to the own wage w_i is positive, confirming that the firm faces an upward-sloping labor supply curve. The elasticity of the supply of labor in symmetric equilibrium is:

$$\eta_{si}^E \equiv \frac{\partial L_{si}^E}{\partial w_i} \frac{w_i}{L_{si}^E} |_{w_i = w} = \frac{(1-m)^2}{(2-m)m} > 0. \tag{1.16}$$

Recalling that m is half the distance between two firms in this economy, we find that the elasticity of the supply of labor increases as firms move closer together (the partial of η_{si}^E with respect to m is negative). As $m \to 0, \eta_{si}^E \to \infty$ and as $m \to 1, \eta_{si}^E \to 0$. Thus, competition intensifies as firms move closer together, a feature that is essential for the existence of dispersed locational equilibrium (see Economides, 1989). The supply of labor to each firm in symmetric equilibrium is derived from (1.15):

$$L_{si}^E = \frac{L_s}{N}(1 - \frac{m}{2}). \tag{1.17}$$

The firm gets its share of workers L_s/N and the average supply of effective units from each worker is $1 - m/2$.

1.2. Occupational choice

Workers supply one unit of labor each, either in the manufacturing sector or in the agricultural sector. If they decide to enter agriculture, they know with certainty that the wage rate will be 1. If they decide to enter manufacturing, they have to train first for a cost and then discover their location on the skills circle, and hence their realized wage.

The cost of training is a proportional utility cost, shown by the parameter v_k in the utility function (1.1). We assume that $v_k = 1/t$ if the individual decides to train and $v_k = 1$ otherwise. $t > 1$ is a parameter that measures the utility cost of training. Higher t implies more expensive training. Our formulation makes training costs akin to "iceberg" costs. The skilled worker loses a fraction of her utility when transporting herself to the skilled labor pool, from which she is recruited by high-wage firms.

Workers choose their sector to maximize their utility function (1.1). By substitution from the demand functions into (1.1) we derive the indirect utility function of individual k,

$$U_k = v_k \mu^\mu (1-\mu)^{1-\mu} P_x^{-\mu} P_a^{-(1-\mu)} y_k. \tag{1.18}$$

Individuals who join the agricultural sector are characterized by $v_k = 1$ and $y_k = 1$ and those who train to join the manufacturing sector are characterized by $v_k = 1/t$ and their income is a random draw from the uniform distribution of wages. Profits in equilibrium are zero in both sectors.

Wages in manufacturing are uniformly distributed between $w(1-m) \geq 1$ and w, so the expected utility of a worker who chooses to get trained is

$$\bar{U}_k = t^{-1} \mu^\mu (1-\mu)^{1-\mu} P_x^{-\mu} P_a^{-(1-\mu)} w(1-m/2). \tag{1.19}$$

Inter-sectoral mobility of labor requires that the expected utility of those who train themselves and join manufacturing be equal to the utility of those who join agriculture, at least when both sectors are active. From (1.18) and (1.19), and given that in agriculture $v_k = y_k = 1$, we derive the condition implied by equality of the utility levels:

$$w(1-m/2) = t > 1. \tag{1.20}$$

With both sectors active, condition (1.20) gives a negative "compensating differentials" relation between the wage rate for each effective unit of labor and the density of firms. The intuition behind it is that for a given effective wage rate, the wage distribution in manufacturing with many firms dominates one with fewer firms, because of better matches. In smaller markets the effective wage rate then has to be higher to compensate for the reduced attractiveness of entering manufacturing.

With (1.20) in place, we can derive the feasible range for mismatch, which satisfies our assumption of no gaps on the skills circle. We require simultaneous satisfaction of (1.20) and of the inequality $w(1-m) \geq 1$, which, eliminating w, yields

$$m \leq \frac{2t-2}{2t-1}, \tag{1.21}$$

which is strictly less than 1 for all finite t.

2. Labor-market equilibrium

We characterize the labor market equilibrium for a single economy for a given allocation of labor to the manufacturing sector, L_s. The labor market equilibrium is defined by a price-wage markup, output and employment per firm and the number of firms.

We substitute from (1.16) into (1.12) and impose symmetry to obtain

$$p_i = \frac{w}{(1-m)^2} \frac{\sigma\beta}{\sigma - 1}. \tag{2.1}$$

All firms set the same price, therefore have the same demand for output and so employ the same number of workers. This confirms the existence of the symmetric equilibrium. Each firm posts the same wage rate to attract the same number of workers and produce the same output. Higher m implies that firms are located further away from each other, competition is less intense and so the markup of prices over wages is higher.

In long-run equilibrium freedom of entry and exit of manufacturing firms eliminates profits, which, when applied to (1.11) gives, by virtue of (2.1) and (1.10),

$$x = \frac{\alpha(\sigma-1)}{\beta} \frac{(1-m)^2}{\sigma - (\sigma-1)(1-m)^2}. \tag{2.2}$$

Unlike the typical Dixit-Stiglitz symmetric equilibrium found in trade models, each firm's output here is not constant (but converges to the usual case as mismatch vanishes, $m \to 0$). By substituting (2.2) into (1.10) we obtain the demand for effective labor units by each manufacturing firm,

$$L_{si}^{ED}(m) = \frac{\alpha\sigma}{\sigma - (\sigma - 1)(1 - m)^2}. \tag{2.3}$$

There is a link between the number of firms and a single firm's output and demand for labor. A larger number of firms (i.e., lower m) erodes the firm's monopoly power and so yields higher output per firm and more demand for labor.

The market in effective units of labor clears. We find the equilibrium equating the demand for effective labor units by each firm with the supply of effective units to the firm. The demand for labor is given by (2.3) and the supply by (1.17), which we re-write in terms of m :

$$L_{si}^{ES}(m) = \frac{L_s}{H} m(1 - \frac{m}{2}). \tag{2.4}$$

By differentiation $L_{si}^{ED\prime}(m) < 0$ and $L_{si}^{ES\prime}(m) > 0$ and at the lowest m, $L_{si}^{ED}(0) = \alpha\sigma > 0$, $L_{si}^{ES}(0) = 0$. Therefore, a necessary and sufficient condition for a unique equilibrium m is that at the maximum feasible m as defined in (1.21), $L_{si}^{ES}(m)$ exceeds $L_{si}^{ED}(m)$. This is always satisfied if the density of skilled workers on the skills circle, L_s/H, is sufficiently high, because the density of workers increases supply, at given m, but does not alter demand. Later we derive the equilibrium condition for L_s and show that it monotonically increases in μ, the fraction of manufacturing expenditure. Therefore we interpret the conditions for the existence of an equilibrium m as a condition on the minimum size of the manufacturing sector. At $H = 0$, $m = 0$ and it is straightforward to show that the equilibrium number of firms is

$$N = \frac{L_s}{\alpha\sigma}. \tag{2.5}$$

We note that the equilibrium m is monotonically increasing in H and decreasing in L_s. The intuition is that lower worker density increases the distance between firms (mismatch) and so leads to firm exit. This is a complementarity that is important in our agglomeration argument: when more skilled workers enter a market, in equilibrium more manufacturing firms also want to enter.

Formally, the unique interior equilibrium m can be obtained as the only non-trivial solution to the equation $L_{si}^{ES}(m) = L_{si}^{ED}(m)$, which can more easily be solved by defining the transformed unknown $M \equiv m(1 - m/2)$ and so obtain the quadratic:

$$2(\sigma - 1) M^2 + M - \frac{\alpha\sigma H}{L_s} = 0. \tag{2.6}$$

The single positive root of this quadratic gives the solution for equilibrium M. Given M, there are two positive roots for m, but one of them is greater than unity and so outside the feasible range. The root below unity is the unique equilibrium m.

3. Aggregate equilibrium

An aggregate equilibrium is an allocation of workers across sectors and an allocation of firms across sectors and regions that satisfies utility and profit maximization and market clearing. We will obtain an explicit analytical solution for the equilibrium allocations under the simplifying assumptions that $\mu < 0.5$ and that the two regions are of equal size. We also obtain simulation results for other parameter values. The restriction $\mu < 0.5$ is a sufficient condition for an active agricultural sector in both economies in all equilibria. It is a convenient one because it implies that (1.20) always holds in aggregate equilibrium.[7] We re-write (1.20) as an equation for the equilibrium mean manufacturing wage

$$W_s = t, \tag{3.1}$$

where $W_s = w(1 - m/2)$ is the mean wage received by skilled workers.

We saw that (2.6) gives a unique and monotonic relation between the equilibrium m and L_s, which we write implicitly as

$$m = m(L_s), \qquad m'(.) < 0. \tag{3.2}$$

[7] Although we are restricting μ to facilitate the exposition, it is a priori a reasonable assumption in the context of our model. The agglomeration arguments that we put forward to motivate our analysis are more relevant to individual sectors than to the whole of manufacturing or services. See also Fujita et al. (1999, ch.14) where a similar restriction is imposed for analytical convenience.

With (3.1) and (3.2) we obtain from (2.1) the equilibrium pricing equation for manufacturing goods

$$p = \frac{W_s}{(1-m/2)\,(1-m)^2} \frac{\sigma\beta}{\sigma-1}, \tag{3.3}$$

which is also an implicit function of L_s.

To derive the allocation of workers across sectors we need to derive an expression for national income. Because of zero profits, national income is defined by

$$Y = W_s L_s + W_a L_a. \tag{3.4}$$

With the normalization $W_a \equiv 1$ and noting that $L_a = L - L_s$, national income becomes

$$Y = L + (W_s - 1)L_s. \tag{3.5}$$

With trade, the output of each manufacturing firm has to satisfy both domestic consumption and exports. Substitution of aggregate income for each country from (3.5), and of the aggregate price indices from (1.4), into the demand functions (1.9), yields equations for the demand for manufacturing output:

$$x = \mu p^{-\sigma} \left[\frac{(W_s-1)L_s + L}{Np^{1-\sigma} + N^*(\tau p^*)^{1-\sigma}} + \frac{\tau^{1-\sigma}[(W_s^*-1)L_s^* + L^*]}{N(\tau p)^{1-\sigma} + N^* p^{*1-\sigma}} \right], \tag{3.6}$$

$$x^* = \mu p^{*-\sigma} \left[\frac{\tau^{1-\sigma}[(W_s-1)L_s + L]}{Np^{1-\sigma} + N^*(\tau p^*)^{1-\sigma}} + \frac{(W_s^*-1)L_s^* + L^*}{N(\tau p)^{1-\sigma} + N^* p^{*1-\sigma}} \right]. \tag{3.7}$$

Focusing on the home country, substitution of x from (3.6) and L_{is}^E from (1.17) into the profit expression (1.11), noting that $w = W_s(1-m/2)$, and setting the result equal to zero yields:

$$\frac{(W_s-1)L_s + L}{1+\tau^{1-\sigma}\phi} + \frac{\tau^{1-\sigma}[(W_s^*-1)L_s^* + L^*]}{\tau^{1-\sigma}+\phi} = \frac{W_s L_s}{\mu}, \tag{3.8}$$

where $\phi = \phi(L_s, L_s^*)$ is a new variable defined by

$$\phi \equiv \frac{m}{m^*}\left(\frac{p}{p^*}\right)^{\sigma-1}. \tag{3.9}$$

Of course, a similar expression holds for the foreign country:

$$\frac{\tau^{1-\sigma}[(W_s-1)L_s+L]}{\tau^{1-\sigma}+\phi}+\frac{(W_s^*-1)L_s^*+L^*}{1+\tau^{1-\sigma}\phi}=\frac{W_s^*L_s^*}{\mu}. \tag{3.10}$$

A symmetric equilibrium with identical properties in each country exists, and we now characterize it for the home country. By symmetry and given that $\mu < 0.5$ and $L = L^*$, we obtain, $L_s^* = L_s$, $W_s^* = t$ and $\phi = 1$. Substitutions into (3.8) yield

$$L_s = \frac{\mu}{t-(t-1)\mu}L \equiv \tilde{\mu}L. \tag{3.11}$$

With (3.11) in hand equilibrium allocations in symmetric equilibrium follow immediately.

All symmetric equilibrium solutions can be expressed in terms of the equilibrium m, obtained from (3.2) for $L_s = \tilde{\mu}L$. The wage per unit is given by $w = t/(1-m/2)$, the firm's price by

$$p = \frac{t\sigma\beta}{(1-m)^2(1-m/2)(\sigma-1)}, \tag{3.12}$$

and output is obtained from (3.6) after substituting all the solutions already obtained:

$$x = \frac{\tilde{\mu}Lt^2\sigma\beta m}{H(\sigma-1)(1-m)^2(1-m/2)} \tag{3.13}$$

Our characterization of trade costs as proportional "iceberg" costs implies that in the symmetric equilibrium trade costs are absorbed entirely by the household sector in their consumption allocations, and do not affect the firm's output, employment and price. In the limit, as $\tau \to \infty$, countries will not trade (because noting is left of exports after transportation) and the equilibrium is "autarkic". Again, factor allocations satisfy the same equations, as substitution of $\tau = \infty$ in (3.8) and (3.10) immediately yields.

Our assumption that firms are mobile, however, implies that with trade there is at least one other equilibrium, an agglomeration one with all firms locating in one country and selling their goods to both countries. We characterize the agglomeration equilibrium by assuming that all manufacturing output is concentrated in the home country, i.e., we let $N^* = L_s^* = 0$. Substitution of these values and $W_s = t$ into (3.8) yields a solution for L_s that is very similar to the symmetric solution, except that now manufacturing firms employ twice as much labor and supply both economies:

$$L_s = \frac{\mu(L + L^*)}{(1-\mu)t + \mu} = 2\tilde{\mu}L. \tag{3.14}$$

Clearly, our assumption $\mu < 1/2$ is sufficient to guarantee that there is a unique solution for L_s and all the other unknowns. In the agglomeration equilibrium the foreign country produces only agricultural goods but in the home country a fraction $2\tilde{\mu}$ of workers are in manufacturing and earn a wage t. The rate of mismatch is given as the unique solution to (3.2) with argument $2\tilde{\mu}L$. Given this m, the wage per unit and price are still given by the same expressions (1.20) and (3.12) but output per firm is given by an expression that is twice the value in the right-hand side of (3.13).

4. Selection of equilibrium

Having shown that with trade both a symmetric and an agglomeration equilibrium exists, how can we choose between them? The important question for us is whether our matching complementarities are strong enough to give an incentive to firms to deviate from the symmetric equilibrium when there is trade. We adapt the robustness (or "stability") criterion suggested by Fujita et al. (1999) in a related context to answer this question.

We ignore for the moment the labor allocation condition (3.1) but assume that if at a given initial condition $W_s > t$, workers will want to move into manufacturing, and if $W_s < t$ they will want to move out of it. Beginning with the symmetric equilibrium with trade, we increase the number of manufacturing workers in the home country and decrease the number of workers in the foreign country by the same amount, and look for the level of trade costs (if any) at which this increases the wage rate in the home country and reduces it in the foreign country. At this point the initial equilibrium is not robust to deviations, because more workers will want to make the same switch. In the terminology of Fujita et al. (1999) this level of trade costs is the "break point".

If we establish that a break point for the symmetric equilibrium exists, and assume that this leads all manufacturing firms to agglomerate in the home country, we then need to show that the agglomeration equilibrium is sustainable. Namely, that it is robust to a similar deviation argument. The maximum level of trade costs at which the agglomeration equilibrium becomes unsustainable is termed the "sustain point". Our claim that our matching complementarities are strong enough to lead to agglomeration require that at low enough trade costs there is a robust agglomeration equilibrium and a non-robust symmetric equilibrium.

4.1. Deviations from symmetric equilibrium

To check if there is a break point we let the number of manufacturing workers increase by a small amount dL_s in the home country and fall by a similar amount $dL_s^* = -dL_s^*$ in the foreign country. By symmetry, if the impact on home wages is dW_s, on foreign wages it will be $dW_s^* = -dW_s$ and so on for all other variables. We therefore evaluate dW_s/dL_s under these symmetry restrictions by making use of equation (3.8) and conclude that if $dW_s/dL_s < 0$, then symmetry is a robust equilibrium, but if $dW_s/dL_s > 0$ the symmetric equilibrium breaks.

Totally differentiating (3.8) and evaluating all unknowns and their displacements at the symmetric equilibrium we obtain:

$$\frac{\tilde{\mu}L}{\mu}dW_s + \frac{t}{\mu}dL_s = \frac{1-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}\left[(t-1)\,dL_s + \tilde{\mu}LdW_s\right] - \frac{2\tau^{1-\sigma}t\tilde{\mu}L}{\mu\left(1+\tau^{1-\sigma}\right)^2}d\phi \tag{4.1}$$

We evaluate $d\phi$ by differentiating equation (3.9), dp by differentiating equation (3.3), and dm by differentiating equation (2.6). The Appendix goes through the derivations and shows that the sign of dW_s/dL_s is the same as the sign of the expression

$$\begin{aligned} & -\frac{t}{\mu} + \frac{1-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}(t-1) \\ & - \frac{4t\tilde{\mu}L\tau^{1-\sigma}}{\mu\left(\tau^{1-\sigma}+1\right)^2}\left[\frac{1}{m} + (\sigma-1)\frac{1-m+2(2-m)}{(2-m)(1-m)}\right]\frac{dm}{dL_s}. \end{aligned} \tag{4.2}$$

The first two terms in the first line of this expression are independent of any complementarities and their sum gives an overall negative effect, so these effects work against agglomeration. The complementarities of our model are in the third term, the second line of (4.2), and the overall agglomeration argument hinges on whether this term is positive and strong enough to overturn the negative impact of the first two terms. Denote this term by $S(\tau)$.

The Appendix (equation (6.5)) shows that dm/dL_s is strictly negative and independent of τ, so $S(\tau) \geq 0$. By differentiation we find that $S'(\tau) < 0$ and $\lim S(\tau)_{\tau\to\infty} = 0$. So at very high trade costs symmetry is not broken. The maximum value reached by $S(\tau)$ is in free trade, $\tau = 1$. The Appendix shows that $S(1)$ is sufficiently large to make the entire expression in (4.2) positive, so in free trade our externalities are strong enough to break the symmetric equilibrium. Therefore, by monotonicity, there is a critical value of $\tau > 1$ which makes the differential $dW_s/dL_s = 0$. This critical value of τ is the break point and is denoted by $\tau(B)$. Our symmetric equilibrium is robust to deviations if $\tau \geq \tau(B)$ but breaks when $\tau < \tau(B)$.

We use numerical simulations to illustrate how this critical level $\tau(B)$ depends on the following key parameters: the level of heterogeneity, training costs, the size of the manufacturing sector and country size. The results are reported in Table 1. We consider an initial condition with sufficiently high trade costs and characterize a symmetric equilibrium. We then reduce trade costs in small steps and look for the critical value that switches the equilibrium from symmetry to agglomeration. We find that in our benchmark case the break point is $\tau(B) = 1.19$, i.e., when trade costs fall below 19% of the producer price, firms could break the symmetric equilibrium by relocating from the foreign country to the home country.[8]

The level of trade costs necessary to break the symmetric equilibrium is lower for lower levels of heterogeneity in labor skills and lower levels of training costs. Lower heterogeneity reduces the potential matching benefits from labor pooling, so the benefits from agglomeration are weaker. Lower training costs gives more

[8] The benchmark parameter values for all of the simulations are as follows: $H = 1$, $t = 1.1$, $L = L^* = 100$, $\sigma = 4$, $\alpha = 1/\sigma$, $\beta = (\sigma - 1)/\sigma$. The initial number of firms is set at $N = N^* = 40$ and we check whether one firm can profitably deviate from the foreign to the home country.

incentives to workers to enter the skilled labor market and so the benefits from agglomeration need to be stronger to make workers give up manufacturing in the foreign market. The elasticity with which the critical trade cost responds to changes in training costs is, however, small. If the manufacturing sector is larger the break point is lower. In the benchmark case, increasing the manufacturing sector from $\mu = 0.4$ to $\mu = 0.6$ reduces the break point from 1.19 to 1.16.[9]

A surprising result is that the break point is very sensitive to country size. If one country is 20 per cent bigger than the other the break point in the small country is close to free trade in virtually all cases but the break point in the larger country increases substantially, to 1.31 in the benchmark case. This implies that larger economies should be characterized by a lot more agglomeration than smaller economies. Interestingly, however, there is also a robust equilibrium where the agglomeration locates in the small country. This is because the source of agglomeration arises in the skilled labor market and not in the product market, so the benefit of locating in a country with a larger pool of skilled labor could outweigh the benefit of locating in a large market for goods. From Table 1, we see that in the benchmark case at trade costs at or below $\tau(B) \leq 1.01$, the agglomeration can locate in the smaller home country.

4.2. Is agglomeration sustainable?

We have demonstrated that our matching complementarities can break the symmetric equilibrium when trade costs are sufficiently small. In order to complete our argument we need to show that the agglomeration equilibrium is robust to small deviations at trade costs below the break point. If it is not robust to such deviations our argument would lead to the absence of a robust equilibrium altogether at low trade costs.

We suppose that all firms are located in the home country, then we allow a single manufacturing firm to establish itself in the foreign country. We then

[9] Recall that if $\mu > 0.5$ in the agglomeration equilibrium there is no agricultural sector in the home country and so the manufacturing wage is not tied to the productivity in the agricultural sector but it depends on the value of marginal product of labor.

calculate the optimal number of workers that this firm will want to recruit and the optimal price it will want to charge. We check whether when the firm makes zero profit the wage that it offers is above t. If it is, more workers will want to come in to work in manufacturing and more firms will be created to recruit them, rendering the agglomeration in the home country unsustainable.

Let $W_s^*(\tau)$ be the maximum (zero-profit) wage rate that the single firm in the foreign country can offer. The agglomeration equilibrium is sustainable at some level of trade costs τ if at this level $W_s^*(\tau) < t$. But at higher trade costs the zero-profit wage should rise until a wage is found such that $W_s^*(\tau(S)) = t$. The level of trade costs that gives that wage, $\tau(S)$, is the sustain point for the agglomeration.

Employment in the agglomeration equilibrium is given by (3.14) for the home country and by $N^* = L_s^* = 0$ for the foreign country. Substituting these values into (3.7) we obtain the foreign firm's demand at some price p^*:

$$x^* = p^{*-\sigma} \frac{\mu L}{N p^{1-\sigma}} Z(\tau), \text{ where } Z(\tau) = \tau^{1-\sigma} \left[2\left(t-1\right)\tilde{\mu} + 1\right] + \tau^{\sigma-1}. \tag{4.3}$$

Let $L_s^* > 0$ be the number of workers that the firm wants to recruit. Since the firm is the only one located on the skills circle, the number of efficiency units of labor supplied to the firm is $L_s^{*E} = L_s^*(1 - H/2)$.[10] The firm chooses p^* to maximize profit, given that its employment level is optimally chosen and that it pays w^* for each efficiency unit of labour that it gets. For any w^* the profit maximizing price is given by (see the Appendix)

$$p^* = \frac{\beta\sigma}{\sigma - 1} w^*. \tag{4.4}$$

Setting profits equal to zero we solve for w^*, which defines the maximum wage per efficiency unit that the foreign firm can pay:

[10] It is assumed that all workers who move from agriculture to manufacturing are employed by the single firm that enters. More generally the skills circle may be sufficiently big that workers located a long way from the firm are so badly matched that the firm cannot pay them above unity, and they return to agriculture. This possibility can easily be incorporated into our proof with no essential new insights. We ignore it for the sake of brevity.

$$w^* = \frac{\sigma-1}{\beta\sigma}\left(\frac{\alpha(\sigma-1)}{\beta}\frac{Np^{1-\sigma}}{\mu LZ(\tau)}\right)^{-\frac{1}{\sigma}}. \tag{4.5}$$

Note that w^* is increasing in $Z(\tau)$, and $Z'(\tau)$ has the following sign:

$$Z'(\tau) = \tau^{2(\sigma-1)} - 2\frac{(t-1)\mu}{(1-\mu)t+\mu} - 1. \tag{4.6}$$

At $\tau = 1$ this is negative, but $Z(\tau)$ has a single minimum at some $\tau > 1$ and becomes positive and increasing in τ above this value.

From equation (1.20) we know that workers are willing to accept to move into manufacturing if and only if

$$w^* \geq \frac{t}{(1-H/2)}, \tag{4.7}$$

making our agglomeration equilibrium unsustainable. The sustain point $\tau(S)$ is the level of trade costs that satisfies (4.7) with equality, given (4.5). However, because w^* in (4.5) first falls and then rises in τ, there is a unique and well-behaved sustain point only if the free trade equilibrium is sustainable. If this is so, w^* for $\tau = 1$ and values of τ that give a lower w^* violate (4.7), but at higher values of τ, where $w^{*\prime}(\tau) > 0$, (4.7) may not be violated. We therefore need to show that $Z(1)$ violates (4.7) and that there is a sufficiently high value of τ that satisfies (4.7).

The second requirement is trivial. As $\tau \to \infty$, $Z(\tau) \to \infty$, therefore the feasible w^* in (4.5) increases indefinitely and (4.7) is eventually satisfied. It therefore remains to show that at $\tau = 1$, (4.7) is not satisfied. The Appendix shows that at small feasible values of m this is indeed the case, establishing that there is a range of trade costs, which include free trade, that guarantee the robustness of our agglomeration equilibrium. The Appendix, however, also shows that there may be high values of m that do not make the agglomeration equilibrium robust to small deviations, even at zero trade costs. To understand this claim, recall that as firms agglomerate there is a tension between more competition between them and more proximity to workers, which reduces mismatch and increases productivity. m measures the distance between firms in the agglomeration equilibrium. If it is small the equilibrium is sustainable. But if it is large, it may be profitable for a firm to forego the benefits of agglomeration and establish in the foreign country where there is no competition from other firms. This result also leads to the intuitive proposition that if manufacturing is very small there may not be sufficient benefits from agglomeration to make it a sustainable equilibrium.[11]

Numerical simulations indicate that once an agglomeration is established it is sustainable over a large range of trade costs, giving rise to persistence in location patterns. We check whether 5% of all firms located in the home country would find it profitable to relocate in the foreign country. Of course, if we were to allow a larger group of firms to deviate as a group the critical level of trade costs would be lower. The results are shown in Table 1. In our benchmark case, we see that the critical value of trade costs that induces relocation is 2.04. When trade costs exceed this value, the agglomeration equilibrium is not sustainable but as we showed earlier the symmetric equilibrium is robust to deviations at even lower trade costs. As with the break point, the results indicate that reducing either t or H reduces the critical level of trade costs. The sustain point appears to be generally more responsive to parameter changes than the break point is. In general, the sustain point is higher than the break point, so there are levels of trade costs which make both the symmetric and the agglomeration equilibrium robust to small deviations. At such levels of trade costs, between $\tau(B)$ and $\tau(S)$, initial conditions determine which is the equilibrium that is adopted, at least when the selection criteria are the deviations that we proposed in this paper. In our benchmark case both equilibria are robust at trade costs between 1.19 and 2.04.

[11] The Appendix shows that in free trade the agglomeration equilibrium is not sustainable when m exceeds a critical value $\tilde{m}$ which is below H, but not necessarily below the maximum feasible value of m when there is more than one firm in the home country. It is also not possible to show whether or not $\tilde{m}$ is above the equilibrium m, in which case the free trade equilibrium is always sustainable, whatever the size of the manufacturing sector. See the Appendix for more details. Note that our condition for the existence of a feasible unique equilibrium m also required a minimum size manufacturing sector, for reasons that are unrelated to the ones here.

5. Conclusions

In this paper we showed that heterogeneity of skills could be a force for agglomeration of economic activity, even when the heterogeneity gives monopoly power to firms. The key to our model is that firms prefer to enter a market that already has a large pool of workers and firms from one that is isolated, even though in the former they forego some of their monopoly power. The reason is that in the larger market they can choose the most suitable employees, whereas in the smaller market they may have to train their own workers and rely on luck to find a good match. Agglomeration fails in our model only if trade costs are sufficiently high to make it more profitable for firms to locate in the market that they supply, rather than in the market that their labor productivity is highest.

We cited anecdotal and econometric evidence by others on the advantages of labor pooling. Our analysis has further testable implications and a necessary next step is to look for these in the data. Two forces in particular appear to be consistent with casual observation: that there should be more agglomeration of firms in small high-tech industries than in the larger and heavier type of industry, and that agglomeration should increase as trade costs come down and as the complexity of tasks increases. Testing whether this is true and whether the reasons are related to the matching of skills is a theme for future research.

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6. Appendix

6.1. Break Point: Derivations

We differentiate (3.9) to get:

$$d\phi = \phi(.)\left[\left(\frac{1}{m}+\frac{1}{m^*}\right)dm + (\sigma-1)\left(\frac{1}{p}+\frac{1}{p^*}\right)dp\right]. \tag{6.1}$$

To find dp we differentiate equation (3.3):

$$dp = \frac{p}{W_s} dW_s + \frac{1-m+2(2-m)}{(2-m)(1-m)} p dm$$

Substitute this dp into (6.1):

$$\begin{aligned} d\phi = {} & \phi(.) \left[\left(\frac{1}{m} + \frac{1}{m^*} \right) + p(\sigma-1) \left(\frac{1}{p} + \frac{1}{p^*} \right) \frac{1-m+2(2-m)}{(2-m)(1-m)} \right] dm \\ & + \phi(.)(\sigma-1) \left(\frac{1}{p} + \frac{1}{p^*} \right) \frac{p}{W_s} dW_s \end{aligned} \tag{6.2}$$

Imposing symmetry we obtain,

$$d\phi = (\sigma-1) \frac{2}{t} dW_s + 2 \left[\frac{1}{m} + (\sigma-1) \frac{1-m+2(2-m)}{(2-m)(1-m)} \right] dm \tag{6.3}$$

To find dm totally differentiate equation (2.6)

$$dm = -\frac{\alpha\sigma H}{L_s^2} \frac{1}{(1-m)[2(\sigma-1)m(2-m)+1]} dL_s \tag{6.4}$$

Making further use of (2.6) to substitute out $\alpha\sigma H/L_s$, and given $L_s = \tilde{\mu}L$, yields,

$$dm = -\frac{1}{2\tilde{\mu}L} \frac{m(2-m)[(\sigma-1)m(2-m)+1]}{(1-m)[2(\sigma-1)m(2-m)+1]} dL_s. \tag{6.5}$$

We now substitute dm from (6.5) into (6.3) and the resulting $d\phi$ into (4.1). We collect all terms that contain dW_s to the left-hand side and the terms that contain dL_s to the right-hand side. The coefficient multiplying dL_s in the right-hand side is shown in (4.2). The coefficient multiplying dW_s in the left-hand side is:

$$\tilde{\mu}L \left(\frac{1}{\mu} - \frac{1-\tau^{1-\sigma}}{1+\tau^{1-\sigma}} \right) + \frac{2\tau^{1-\sigma}[(t-1)\tilde{\mu}+1](\sigma-1)2L}{(1+\tau^{1-\sigma})^2 t}. \tag{6.6}$$

Since the first bracketed term is positive (recall $\mu < 1$, $\tau \geq 1$, $\sigma > 1$), the coefficient on dW_s is positive, so the sign of dW_s/dL_s will depend on the sign of the coefficient on dL_s, as shown in (4.2).

In order to show that $S(1)$ in (4.2) is sufficiently large to make the entire expression positive write $S(\tau)$ as the product of two terms, one that contains τ

and one that contains m, i.e., let $S(\tau) = A(\tau)B(m)$. We substitute dm from (6.5) into (4.2) to obtain:

$$A = \frac{2t\tau^{1-\sigma}}{\mu\left(\tau^{1-\sigma}+1\right)^2} \tag{6.7}$$

$$B = \frac{(2-m)(1-m) + m(1-m)(\sigma-1) + 2(\sigma-1)m(2-m)}{(1-m)^2\left[1+2(\sigma-1)m(2-m)\right]} \\ \left[m(2-m)(\sigma-1)+1\right] \tag{6.8}$$

In free trade $A(1) = t/\mu$, as is the sum of the first two terms of (4.2), and so the sign of the entire expression in (4.2) is the same as the sign of

$$\frac{t}{\mu}(B-1). \tag{6.9}$$

The long fraction in the first line of B can be shown by straightforward checking that it is a number bigger than 1 for all feasible m, so, since the second line is also a number bigger than 1, $B > 1$ and from (4.2) $dW_s/dL_s > 0$.

6.2. Sustain point: Derivations

Given demand in (4.3), the firm's revenue is given by

$$p^*x^* = p^{*1-\sigma}\frac{\mu L}{Np^{1-\sigma}}Z(\tau) \tag{6.10}$$

The production function is $L_s^{*E} = \alpha + \beta x^*$, so if the firm pays w^* for each efficiency unit of labor that it gets, its costs are $w^*(\alpha+\beta x^*)$. The firm chooses p^* to maximize

$$\pi^* = p^*x^* - w^*(\alpha+\beta x^*). \tag{6.11}$$

By differentiation we get the profit maximizing price as in (4.4). Setting profits equal to zero and substituting in for p^* from (4.4) and for x^* from (4.3) in equation (6.11), we get:

$$\left(\frac{\beta\sigma}{\sigma-1}\right)^{1-\sigma} w^{*^{-\sigma}}\frac{\mu L}{Np^{1-\sigma}}Z(\tau) - \alpha - \beta\left(\frac{\beta\sigma}{\sigma-1}\right)^{-\sigma} w^{*^{-\sigma}}\frac{\mu L}{Np^{1-\sigma}}Z(\tau) = 0 \tag{6.12}$$

Solving for w^*, we have the zero profit wage (4.5).

To show now that (4.7) is not satisfied at $\tau = 1$, note that

$$Z(1) = 2\left[(t-1)\,\tilde{\mu} + 1\right] = \frac{2t}{\mu}. \tag{6.13}$$

Denote the zero-profit wage at $\tau = 1$ by $w^*(1)$. From (4.5) and noting that $N = H/m$,

$$w^*(1) = \frac{\sigma - 1}{\beta\sigma}\left(\frac{\alpha\,(\sigma-1)\,Hp^{1-\sigma}/m}{4\beta t L}\right)^{-\frac{1}{\sigma}}. \tag{6.14}$$

Given now the equilibrium p in (3.3) and the equilibrium condition $W_s = t$, (6.14) becomes

$$w^*(1) = t\left(\frac{\alpha\sigma H}{2\tilde{\mu}L}\right)^{-\frac{1}{\sigma}} m^{-\frac{1}{\sigma}}\left(1-\frac{m}{2}\right)^{\frac{1-\sigma}{\sigma}}(1-m)^{\frac{2(1-\sigma)}{\sigma}}. \tag{6.15}$$

But $2\tilde{\mu}L = L_s$ and from (2.6),

$$\frac{\alpha\sigma H}{L_s} = m\left(1-\frac{m}{2}\right)\left[(\sigma-1)m\left(1-\frac{m}{2}\right)-1\right], \tag{6.16}$$

(noting the definition of $M = m(1-m/2)$), so $w^*(1)$ becomes

$$w^*(1) = t\left(2(\sigma-1)m\left(1-\frac{m}{2}\right)+1\right)^{-\frac{1}{\sigma}}\left(1-\frac{m}{2}\right)^{-1}(1-m)^{\frac{2(1-\sigma)}{\sigma}}. \tag{6.17}$$

Therefore, the free trade equilibrium is not sustainable if - from (4.7) -

$$\left(2(\sigma-1)m\left(1-\frac{m}{2}\right)+1\right)^{-\frac{1}{\sigma}}(1-m)^{\frac{2(1-\sigma)}{\sigma}} \geq \frac{1-\frac{m}{2}}{1-\frac{H}{2}}. \tag{6.18}$$

We aim to show that (6.18) yields a contradiction. Let the left-hand side be some function $f(m)$. Then by direct calculation, $f(0) = 1, f'(m) > 0, f'(0) = 0$. At $m = 0$ the right-hand side of (6.18) is strictly greater than 1, so there is a contradiction. But the right hand side is monotonically decreasing in m and at $m = H$, it is equal to 1. So at $m = H$ (6.18) is not contradicted. By monotonicity of both the left-hand and right-hand sides of (6.18) there is a unique $\tilde{m}$, such that the agglomeration equilibrium is sustainable at zero trade costs at $m < \tilde{m}$ and unsustainable at $m \geq \tilde{m}$. Note that $m = H$ is the value of m

when the manufacturing sector is so small that there is only one firm in the agglomeration equilibrium, so it is not surprising that it is not sustainable when one firm establishes itself in the foreign country. The maximum feasible value of m is given by (1.21) and its equilibrium by the solution to (2.6). Unfortunately it is not possible to establish analytically whether $\tilde{m}$ is below or above either of these values, but our simulations show that it is well above these values. Given that the slope of $f(0)$ is zero, and that at $m = 0$ the right-hand side of (6.18) is well above unity, it is perhaps not surprising to find that $\tilde{m}$ is a fairly large number within its feasible range.

Table 1

"Break" and "sustain" points for trade costs at different values of parameters

		$L = L^* = 100$				$L = 100 < L^* = 120$			
		$\mu = 0.4$		$\mu = 0.6$		$\mu = 0.4$			
		Both countries		Both countries		Home	Foreign	Home	Foreign
H	t	$\tau(B)$	$\tau(S)$	$\tau(B)$	$\tau(S)$	$\tau(B)$	$\tau(B)$	$\tau(S)$	$\tau(S)$
1.5	1.15	1.25	3.05	1.21	2.26	1.01	1.38	2.93	3.12
1.5	1.10	1.24	3.00	1.19	2.22	1.01	1.37	2.89	3.08
1.5	1.05	1.23	2.96	1.18	2.19	1.01	1.35	2.84	3.03
1.0	1.15	1.20	2.07	1.17	1.71	1.01	1.32	1.99	2.12
1.0	1.10	1.19	2.04	1.16	1.69	1.01	1.31	1.96	2.09
1.0	1.05	1.18	2.01	1.14	1.66	1.00	1.30	1.93	2.06
0.5	1.15	1.14	1.53	1.12	1.34	1.00	1.25	1.46	1.56
0.5	1.10	1.13	1.50	1.11	1.32	1.00	1.23	1.44	1.53
0.5	1.05	1.12	1.48	1.09	1.29	1.00	1.22	1.41	1.51

Notes. The values taken by the other parameters are $\sigma = 4$, $\alpha = 1/\sigma$ and $\beta = (\sigma - 1)/\sigma$. H measures the degree of heterogeneity and t the training cost. At trade costs below $\tau(B)$ the symmetric equilibrium can be broken and at points above $\tau(S)$ the agglomeration equilibrium becomes unsustainable.