Cross-sectional inflation risk in menu cost models with heterogeneous firms

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Abstract

We show that firms in models with menu costs, when calibrated to have the empirically observed frequency and size of individual-goods price adjustments, have stock returns that are always positively correlated with inflation. The cross-sectional dispersion in this correlation is almost negligible, even though firms have very diverse micro-level pricing behavior. Because in this class of models positive nominal shocks are good states of nature and the correlation between stock returns and inflation is positive, agents are willing to pay a premium to hold assets whose returns covary negatively with inflation. In contrast, we empirically find that the dispersion in the correlation between stock returns and inflation is about 100 times larger than in the model, and that correlations are negative about half the time. Furthermore, and also at odds with sticky-price models, investors require a premium to hedge against states of high inflation. Because firms' heterogeneity is the key mechanism that generates a high degree of monetary non-neutrality in the models, our results suggest that we do not yet have a full account of the monetary transmission mechanism, and that asset prices can provide important information about it.

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1 Introduction

Detailed microeconomic datasets have revealed substantial heterogeneity in the frequency and size of price adjustments across different categories of goods, firms, industries and countries. This heterogeneity can have profound effects in aggregate macroeconomic variables and the degree of monetary non-neutrality of the economy, as demonstrated by Carvalho (2006) in the context of Calvo-style pricing and Nakamura and Steinsson (2010) in the context of menu-cost models. Both authors show that models calibrated to match firm-level evidence of price adjustments deliver time-series properties and impulse response functions with respect to nominal shocks of output, inflation and other macroeconomic quantities that are in close agreement with their empirical counterparts, and that they cannot be obtained using a single representative firm. Their results represent a great advance in monetary economics, since they reconcile, in a general equilibrium context, the sluggish dynamics of the aggregate price level and the large real effects of nominal shocks with the relatively high frequency and large size of price adjustments at the individual firm level.

In this paper, we show that the asset pricing implications of the heterogeneous firms in these type of models are at odds with what is empirically observed. To do so, we write the expected stock returns of firms (in excess of the risk-free rate) as the product of the market-wide price of inflation risk and the firm-specific quantity of risk:

$$E\left[R_i\right] - R^f = \beta_i \times \lambda \tag{1}$$

where $E\left[R_i\right] - R^f$ are expected excess returns for some firm i, β_i is the quantity of risk for firm i and λ is the price of risk. Equation (1) is analogous to the standard CAPM equation, with the difference that instead of using the returns of the aggregate market as a risk factor, we use inflation. We consider conditional and unconditional versions of equation (1), which deliver the same insights. In the sticky-price models we are considering, firms are heterogeneous in two dimensions: they have different menu costs and different volatilities of idiosyncratic productivity. Differences across firms can produce different levels of expected returns and exposures to inflation β_i. However, we find that differences in returns and "inflation betas" across firms are almost zero when the model is calibrated to match micro-level pricing data. The reason is that to match the relatively high frequency and large size of individual goods price adjustments, firms, even in sectors with the stickiest prices, must be calibrated to have a volatility of idiosyncratic productivity that is many times larger than the volatility of inflation, and menu costs that are many times larger than if we were calibrating a model without idiosyncratic productivity shocks. Therefore, from the point of view of a single firm, inflation shocks are so small that it is usually not worth paying the menu-cost to adjust to them — the firm can wait until a large productivity shock occurs and adjust prices to reflect changes in both productivity and inflation in one swoop. The inflation betas are also positive for all firms because higher inflation, everything else being equal, is associated with higher aggregate demand, and thus higher profits and higher returns. In brief, the model predicts a positive correlation between returns and inflation, and not much variation in this correlation across firms.

The time-series of the market price of risk λ also offers important insights into the asset pricing properties of the model. We find that the model produces a small, positive and almost constant inflation price of risk λ. The fact that it is small is nothing but the equity premium puzzle in disguise: Even though nominal shocks affect real consumption substantially, because the representative agent has power utility with a low relative risk-aversion coefficient, the changes in consumption produced by inflation shocks only command a small equity premium. There is a large literature addressing possible solutions to this puzzle, and therefore we do not consider this issue here. For the very same reason, we also do not consider the magnitude of the equity premium (and hence the inflation price of risk λ) as a defining characteristic of asset pricing in this class of models. However, the fact that the inflation price of risk is positive is a direct implication of the fact that a positive shock to inflation increases real aggregate consumption, which is a robust property of sticky price models. A positive market price of inflation risk means that inflation is a good state of nature; the representative agent must be compensated with high mean returns to hold assets that covary positively with inflation. The price of risk is also time varying, stemming from the fact that it is proportional to the variance of inflation, which is itself time varying. However, inflation is not very volatile in the model, simlpy because nominal shocks are exogenous and calibrated to match the relatively low volatility of inflation observed in the data, and the market price of inflation risk inherits this property.

To compare the model with the data, we estimate equation (1) using a standard two-pass Fama and MacBeth (1973) procedure. In the first pass, β_i is estimated by running time-series regressions of returns on inflation. In the second pass, we estimate λ by running a cross-sectional regression of returns on the $\hat{\beta}_i$'s found in the first stage. We find that inflation betas are both positive and negative, and display a cross-sectional dispersion that is about 100 times larger than the one in the model. In the data, about half of firms' returns covary positively and half negatively with inflation, unlike the model, in which all returns are positively correlated with inflation. The distribution of inflation betas estimated from the conditional version of equation (1) also varies considerably over time. The mean of the distribution is unconditionally negative, but can change signs in different periods and is negatively correlated with inflation (so is its standard deviation). In contrast, the distribution of inflation betas estimated from the unconditional version of equation (1) has a positive mean and significantly smaller tails. The difference in behavior between conditional and unconditional inflation betas imply that betas and the price of risk are correlated over time. In the model, conditional and unconditional betas behave almost identically.

The empirically estimated market price of inflation risk is, on average, -0.23, which means that a hypothetical portfolio of stocks whose excess returns move one-for-one with inflation would have an annual Sharpe ratio of -0.23. This Sharpe ratio is about half as large as the Sharpe ratio of the aggregate market (but of opposite sign). Perhaps even more at odds with the model is the standard deviation of the price of risk, which is 1.3, more than five times its mean and more than 10^5 times the one observed in the model. Even if we increased the risk-aversion of the representative agent by a factor of 100 (and thus increased the aggregate equity premium to reasonable levels), the variation in the price of risk would still be 10 times smaller than what is observed in the data. Unlike the model, the estimated market price of inflation risk also changes signs over time: it is negative in the mid-1970s and 1980s (inflation is a bad state of nature) and has been increasing since the early 2000, becoming positive in 2005 and staying mostly in positive territory until the present (inflation has become a good state of nature).

The failure of the model to match the empirically observed conditional and unconditional properties of inflation betas and the inflation price of risk implies that the underlying cross-sectional assumptions about how firms behave in the model is, at least, incomplete. Because firms' heterogeneity in price adjustment behavior is the key mechanism that generates a high degree of monetary non-neutrality in this class of models, our results suggest that it is premature to conclude that the distribution of frequency and size of individual price changes is the driving force behind the real effects of nominal shocks.

This paper is organized as follows. Section 2 sets up the menu-cost model and derives its asset pricing implications. Section 3 compares the model's asset pricing implications with what is empirically observed. Section 4 concludes.

2 Model

2.1 Setup

We use the multisector menu cost model with heterogeneous firms and intermediate inputs put forward in Nakamura and Steinsson (2010)[1]. Households maximize expected lifetime utility, given by:

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\frac{1}{1-\gamma} C_{t+\tau}^{1-\gamma} - \frac{\omega}{\psi+1} L_{t+\tau}^{\psi+1} \right], \tag{2}$$

where $C_t = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$ is consumption of a composite basket composed of a continuum of differentiated goods indexed by z, and L_t is labor supplied by the household ; β is the discount factor, γ is the coefficient of relative risk aversion, ω is the level parameter on disutility of labor, ψ is the inverse of the Frisch elasticity of labor supply, and θ is the elasticity of subsitution between differentiated goods.

Households earn real wages W_t from supplying labor, receive real profits $\Pi_t^R(z)$ from firms, and invest in a portfolio of full state-contigent claims with random payoff X_t (markets are complete). Therefore, their budget constraint expressed in nominal terms is:

$$P_t C_t + \mathbb{E}_t \left[D_{t,t+1} X_{t+1} \right] \leq X_t + W_t L_t + \int_0^1 P_t \Pi_t^R(z)\, dz, \tag{3}$$

where $P_t = \left[\int_0^1 p_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$ is the aggregate price level, $p_t(z)$ the price of good z, and $D_{t,T}$ is the nominal stochastic discount factor between periods t and T.

Given the aggregate price level P_t, household optimization yields the first-order conditions:

$$D_{t,T} = \beta \frac{C_t}{C_T} \frac{P_t}{P_T}, \tag{4}$$

[1] For a more detailed exposition of the model, a thorough discussion of the calibration used and a discussion of the excelent performance of the model in matching macroeconomic aggregates and impulse response functions, please consult their paper.

$$\frac{W_t}{P_t} = \omega C_t, \tag{5}$$

As previously alluded to, there is a continuum of firms in the economy, each of which produces a single variety of good z, so we also index firms by z. Frim z produces $y_t(z)$ units of its differentiated good according to:

$$y_t(z) = A_t(z) L_t(z)^{1-s_m} M_t(z)^{s_m}, \tag{6}$$

where $A_t(z)$ is the firm's productivity, $L_t(z)$ is the number of units of labor employed and $m_t(z, z')$ is a basket of intermediate inputs given by $M_t(z) = \left[\int_0^1 m_t(z, z')^{\frac{\theta-1}{\theta}} dz'\right]^{\frac{\theta}{\theta-1}}$. The materials share in production is given by s_m, which is the same for all firms.

Each firm z belongs to one of J sectors indexed by j. Firms in different sectors have different menu costs and different variances of stochastic productivity $A_t(z)$, which follow the exogenous process:

$$\log A_t(z) = \rho \log A_{t-1}(z) + \epsilon_t(z), \tag{7}$$

with $\epsilon_t(z) \sim N\left(0, \sigma^2_{\epsilon,j}\right)$. The firm's problem is to maximize the sum of expected discounted future real profits:

$$V_t(z) = \max_{p_t(z)} \mathbb{E}_t \sum_{\tau=0}^{\infty} D^R_{t,T} \Pi^R_{t+\tau}(z) \tag{8}$$

where real profits in period t are given by

$$\Pi^R_t(z) = \frac{p_t(z)}{P_t} y_t - \frac{W_t}{P_t}\left(L_t(z) + \chi_j I_t(z)\right) - M_t(z) \tag{9}$$

and are discounted using the household's (real) stochastic discount factor

$$D^R_{t,T} = \beta \frac{C_t}{C_T} \tag{10}$$

Profits are given by total firm production minus labor and intermediate input costs, and minus a menu cost should the firm decide to change the price p_t of its good. The indicator function $I_t(z)$ equals one if the firm decides to change the price of its good in period t, and zero otherwise. If the firm decides to change the price, it pays $\frac{W_t}{P_t}\chi_j I_t(z)$ in menu costs, which can be thought of as hiring an additional x_j units of labor. Note that the amount paid χ_j is sector-specific. We close the model by assuming that nominal aggregate demand $S_t = P_t C_t$ follows the exogenous process given by:

$$\log S_t = \mu + \log S_{t-1} + \eta_t, \tag{11}$$

where $\eta_t(z) \sim N\left(0, \sigma_\eta^2\right)$ and is independent from ϵ_t.

To maximize the discounted sum of profits (8), each firm should in general set prices based on the evolution of the prices and productivity of all the firms in the economy. To avoid the curse of dimensionality in a dynamic program with an infinite number of state variables we follow Krusell and Smith (1998), and assume that firms percieve that the evolution of the aggregate price level P_t is based on the aggregate variables S_t and P_{t-1} only:

$$\frac{P_t}{P_{t-1}} = \Gamma\left(\frac{S_t}{P_{t-1}}\right). \tag{12}$$

Nakamura and Steinsson (2010) show that the perceived law of motion for prices (12) is almost identical to the one that is realized in the equilibrium in which all agents assume it. By log-linearizing the aggregate labor supply, aggregate intermediate product output, aggregate output and aggregate consumption, around the steady state, we can write the value function $V_t(z)$ of firm z as Bellman equation in three state variables:

$$V\left(A_t(z), \frac{p_{t-1}(z)}{P_t}, \frac{S_t}{P_t}\right) = \max_{p_t(z)}\left\{\Pi_t^R(z) + \mathbb{E}_t\left[D_{t,t+1}^R V\left(A_{t+1}(z), \frac{p_t(z)}{P_{t+1}}, \frac{S_{t+1}}{P_{t+1}}\right)\right]\right\}. \tag{13}$$

An equilibrium in this economy is a set of stochastic processes for the endogenous price and quantity variables that satisfy household utility naximization, firm profit maximization, and market clearing, and are consistent with the exogenous variables $A_t(z)$, S_t, and the percieved law of motion (12).

2.2 Asset pricing

The menu cost model we consider has implications for the cross-section of stock returns. The gross real return of firm z is given by:

$$R_{t+1}(z) = \frac{V\left(A_{t+1}(z), \frac{p_t(z)}{P_{t+1}}, \frac{S_{t+1}}{P_{t+1}}\right)}{V\left(A_t(z), \frac{p_{t-1}(z)}{P_t}, \frac{S_t}{P_t}\right) - \Pi_t^R(z)} \tag{14}$$

Note that we subtract profits in the denominator because V is the value of the firm "cum dividend" (with dividends). Because of (13) the returns satisfy the conditional Euler equation

$$1 = \mathbb{E}_t\left[D_{t,t+1}^R R_{t+1}(z)\right]. \tag{15}$$

We can rewrite the dynamics of the aggregate price level given by (12) as

$$\pi_t = \Gamma\left(C_t\left(1+\pi_t\right)\right) - 1, \tag{16}$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation rate between period $t-1$ and t. The contemporanous relationship (16) between real aggregate demand and inflation allows us to rewrite (15) in terms of inflation as:

$$\mathbb{E}_t\left[R_{t+1}(z)\right] - R_t^f = \lambda_t^\pi \beta_t^\pi(z), \tag{17}$$

where $R_t^f = \frac{1}{\mathbb{E}_t\left[D_{t,t+1}^R\right]}$ is the real risk-free rate, λ_t^π is the price of inflation risk, and $\beta_t^\pi(z) = \frac{Cov_t(\pi_{t+1}, R_{t+1}(z))}{Var_t(\pi_{t+1})}$ is the exposure of firm z to inflation risk at time t. The pricing equation (17) is analogous to the familiar conditional CAPM with the difference that we use inflation as risk factor instead of aggregate market returns. Equation (17), the conditional "inflation

CAPM", determines the expected excess returns of firms based on time-t information about their exposures to inflation and the aggregate market price of inflation risk. In parallel fashion with the traditional CAPM, we refer to $\beta_t^{\pi}(z)$ as the "inflation betas". Note also that the definition of betas imply that they are the coefficient of a linear univariate regression of realized excess returns on inflation. As a consequence, an inflation beta equal to 1 means that when inflation goes up by one percentage point, so do excess returns. To understand *expected* returns, we can use the inflation CAPM equation (17). If $\lambda_t^{\pi} < 0$, households at time t require high expected returns to hold assets that have low returns when inflation is high. That is, housholds dislike future inflation and would accept lower expected returns on assets that hedge better against inflation risk. On the other hand, if $\lambda_t^{\pi} > 0$, households would benefit from future inflation, requiring higher risk premia on assets with low returns when inflation is low.

The gross real return of firm z also satisfies the Euler equation unconditionally. There is a subtle difference between the conditional and unconditional inflation CAPMs. In the conditional version, the stochastic discount factor expressed in terms of inflation depends on both current and lagged inflation (to see this, combine equations (10), (15), and (16)). Because lagged inflation is known at time t, it does not influence the covariance between the stochastic discount factor and returns, and hence lagged inflation is not priced. On the other hand, when we think about the unconditional Euler equation, we cannot condition on lagged inflation, and hence both inflation and lagged inflation are priced through inflation differences $\triangle\pi_t = \pi_t - \pi_{t-1}$. The unconditional inflation CAPM is given by:

$$E\left[R_{t+1}(z)\right] - R^f = \lambda^{\triangle\pi}\beta^{\triangle\pi} \tag{18}$$

with $R^f = \frac{1}{\mathbb{E}\left[D_{t,t+1}^R\right]}$ as real mean risk-free, $\lambda^{\triangle\pi}$ as the unconditional price of inflation risk, and $\beta^{\triangle\pi}(z) = \frac{Cov(\triangle\pi_t, R_t(z))}{Var(\triangle\pi_t)}$ as the unconditional inflation beta. Note also that while (17) could be written in terms of inflation differences as well, the static (18) is in general not simply the unconditional mean of (17), because of the possibly nonzero covariance between λ_t^π and $\beta_t^\pi(z)$.

Finally, we compare the firm's return with the market return. The market real gross return is defined as an asset that pays aggregate profits each period:

$$R_{t+1}^m = \frac{V_{t+1}^m}{V_t^m - \int_0^1 \Pi_t^R(z)\,dz}, \tag{19}$$

where because of (8) the market price V_t^m is the aggregated value of firms

$$V_t^m = \mathbb{E}_t\left[\sum_{s=t}^{\infty} D_{s,t}^R \int_0^1 \Pi_s^R(z)\,dz\right] = \int_0^1 V_s(z)\,dz \tag{20}$$

The market return also satisfies the Euler equations (17) and (18), and because of (20) can be written as a value-weighted portfolio of individual firm returns:

$$R_{t+1}^m = \int_0^1 w_t(z)\,R_{t+1}(z)\,dz, \tag{21}$$

where the time-varying value weights are given by $w_t(z) = \frac{V_t(z) - \Pi_t^R(z)}{\int_0^1 \left[V_t(z') - \Pi_t^R(z')\right]dz'}$. We denote the exposure of the market to inflation as $\beta_t^{\pi,m}$ and the unconditional market exposure to inflation differences as $\beta^{\triangle\pi,m}$.

2.3 Results

To obtain quantitative results, we use the calibration propopsed by Nakamura and Steinsson (2010), which is displayed in Table 1. In Table 2, we report the aggregate variables that are important for asset pricing. The price of inflation risk, λ_t, is normally distributed with annualized mean of $3.1 \cdot 10^{-3}\%$ and standard deviation $4.9 \cdot 10^{-6}\%$. This essentially nonnegative price of inflation risk means that households benefit from future inflation in all time periods, accepting lower expected excess returns on assets that pay well when inflation is low. However, the effect is very small.

[TABLE 1 HERE]

[TABLE 2 HERE]

Figure 1 shows that the price of inflation risk λ_t is a linear function of the conditional variance of inflation $Var_t(\pi_{t+1})$, which is on the same order of magnitude as λ_t. The reason that the price of inflation risk λ_t is small is that given the aggregate state at period t, future inflation is not expected to vary greatly, i.e. $Var_t(\pi_{t+1})$ has an annualized mean of 10^{-3} percent. Expecting only small variation in inflation, households are not willing to pay much for "protection" against low inflation.

[FIGURE 1 HERE]

As in all models in which the consumption CAPM holds and risk aversion is small, the market return is comparable to the risk-free rate (see Table 2). In Table 4 we show the sector-specific results of the menu cost model. As we expect from (21) the sector returns are spread around the market return.

[TABLE 4 HERE]

Interestingly, the sector returns are very close to the point where they are indistinguishable from one another. To build intution, consider that in (8) only the stochastic discount factor and profits affect the value of the firm and hence its returns. Figure 2 plots the impulse response function of those two building blocks after a shock in nominal aggregate demand S_t. While some sectors respond with larger profits than others (middle left plot), we see that this differentiation is dwarfed by the 10^3 times bigger response of the stochastic discount factor (upper left plot). After the shock, the realization of a larger firm value be-comes the denominator in (14), causing all expected returns to move down in unison (lower left plot). This explains why all sector returns are affected equally by aggregate shocks. The only other possible source of differentiation among firm returns are the heterogeneous idiosyncratic shocks. However, idiosyncratic risk is completely diversifiable and thus not priced.

[FIGURE 2 HERE]

The variation in inflation exposures $\beta_t^\pi(z)$ is also small both across sectors and through time (columns seven and eight in Table 4). Since all expected returns satisfy (17), a similar picture for the impulse response function of inflation exposures arises as for expected returns. All inflation exposures move down in unison due to the nominal shock (lower left plot in figure 2), and the spread between them is very small. To build more intuition about the cross-sectional and time variation of inflation exposure of firm z, $\beta_t^\pi(z)$, note that up to first order we can write

$$\beta_t^\pi(z) \approx \frac{1}{V_t(z) - \Pi_t^R} \frac{dV_t(z)}{d\pi_t}. \tag{22}$$

We can then write the beta of the market $\beta_t^{\pi,m}$ as a value-weighted average of the betas of the the firms:

$$\beta_t^{\pi,m} \approx \int_0^1 w_t(z)\, \beta_t^\pi(z)\, dz, \tag{23}$$

which explains why the market exposure to inflation $\beta_t^{\pi,m}$ is behaving similarly to the sectoral exposures to inflation due to the nominal shock.

[FIGURE 3 HERE]

Figure 3 verifies that the first order approximation (22) is accurate. We plot the contemporaneous relationship between the scaled derivative of the value function with respect to inflation (the right-hand side of equation (22)) and the firm exposures to inflation $\beta_t(z)$ (the left-hand side of (22)) for multiple time periods for each sector. This relationship shows that inflation exposures across sectors cannot vary more than the sensitivity of the value functions with respect to inflation. The histogram on the left-hand side shows that the sectors in this model are not easily distinguishable by their inflation exposures.

Figure 4 shows the origins of the small variation in the sensitivity of the value functions with respect to inflation. We plot the value function $V\left(A_t(z), \frac{p_{t-1}(z)}{P_t}, \frac{S_t}{P_t}\right)$ and its partial derivatives against the log of the second and third arguments: the log of last period's relative price $x_t(z) = \log\frac{p_{t-1}(z)}{P_t}$, and the log of real aggregate demand $c_t = \log\frac{S_t}{P_t}$. The first feature to note in the upper left corner of figure 4 is that firms only adjust their relative price x_t whenever it gets far from the desired price, because it is not worth paying the menu cost if the benefits are not large enough. Consequently, the value function plotted against $x_t(z)$ has two regions. The first is for values of x_t around zero, where the value function is a concave one. This region corresponds to the firm deciding to optimally not change prices, and therefore the firm's value changes with its relative price. The second region is on the edges of the figure, and corresponds to the firm deciding to pay the menu cost and change its relative price to the optimal level. Correspondingly, the partial derivative is downward sloping when the firm does not change prices and zero otherwise (middle left plot). The highest point of the value function occurs when last period's relative price happens to equal the firm's optimal relative price, so the firm reaps the most benefits from its pricing without having to pay the menu cost. The greater the displacement from this point, the smaller the value of the firm, until the relative price is far enough from optimum that it is worth paying the menu cost, at which point the value function is flat. Sector 6 has the largest menu costs and the most pronounced concavity, while sector 1 has almost neglagible menu costs and therefore changes prices much more frequently than other sectors. With respect to the log of real aggregate demand c_t, the value function is linear in all sectors (upper right plot) with similar positive slope (middle right plot). Sector 6 has the smallest slope, while sector 3, the sector with the largest idiosyncratic shocks has the largest slope. How large are the differences when it comes to the sensitivity of the value function with respect to inflation? By the chain rule

$$\frac{dV_t(z)}{d\pi_t} = -\frac{\partial V_t(z)}{\partial x_t(z)} + 3\frac{\partial V_t(z)}{\partial c_t} \tag{24}$$

where the factor 3 comes from inverting (16) and plugging in the corresponding parameters. In the lower left graph of figure 4, we plot two sectors whose difference in their sensitivity to inflation is the largest. Their difference is still very small. In fact, most of the difference is due to how the value function responds to real aggregate demand c_t, as its partial derivative dominates the derivative with respect to the second argument by a factor of 30. If the partial derivative of the value function with respect to the relative price of the firm were more significant, differences in the adjustment cost across sectors would interact more strongly with inflation. One of the reasons why the derivative is small is that, with the exception of sector 1, the size of the price-changing domain is too large for inflation to induce repricing directly. Rather re-pricing occurs mostly due to idiosyncratic productivity shocks with inflation only possibly impacting the size of repricing. The idiosyncratic shocks have a standard deviation that is at least 10 times as large as the standard deviation of inflation. In the case of sector 1, inflation does induce re-pricing. However, its menu costs are too small to experience a significant drop in the value function due to inflation. We conclude that inflation in this model is not volatile enough to generate costly price adjustments by itself, but only in conjunction with the idiosyncratic shocks.

[FIGURE 4 HERE]

Finally, the results on the unconditional inflation-CAPM (18) are in close agreement with all preceeding results. Table 3 shows that the price of inflation-difference risk is $3.5 \cdot 10^{-3}\%$.

The unconditional inflation-difference exposures of market and sector are also very similar in magintude and spread to the results of the condtional inflation-CAPM.

[TABLE 3 HERE]

3 Empirical Results

3.1 Data description

We use quarterly data from January 1959 to July 2012. To construct inflation, we use the price indices corresponding to personal consumption expenditures (PCE) of non-durables and services from the BEA. We use PCE inflation because, as emphasized in Piazzesi and Schneider (2006), it is better than alternatives such as the CPI to match theory to data. We obtain individual stock returns from CRSP. Fama-French factors, the momentum factor and the risk-free rate are from Prof. French's website.

Our sample spans $T = 206$ time periods and $I = 16,358$ individual firms. Not all firms have observations for all time periods (it is an unbalanced panel), with more firms available later in the sample. We will denote the subset of firms that have observations in period t by I_t. We eliminate the smallest 1% of firms each period ("microcaps") to avoid illiquid stocks. In total, we end up with $887,562$ month-firm observations for stock returns.

3.2 Inflation betas

In the model, the inflation betas from the unconditional inflation-CAPM correspond to the coefficient in a univariate regression of excess returns on inflation changes. Therefore, following Duarte (2011) and Briere, Ang, and Signori (2012), it is natural to estimate these unconditional inflation betas by running the regression:

$$R_{it} - R_t^f = \alpha_i + \beta_i^{\Delta\pi} \Delta\pi_t + c_i X_t + \varepsilon_{it}, \tag{25}$$

where we use all available observations for firm i. In equation (**??**), R_{it} is the return of firm i at time t, R_t^f is the risk-free rate at time t, $\Delta\pi_t = \pi_t - \pi_{t-1}$ and X_t are additional controls that capture systematic variation in excess returns that are not present in the model. We will use as controls X_t the Fama and French (1993) factors and the momentum factor of Jegadeesh and Titman (2012). Regressions without including the controls X_t yield similar results, showing that inflation is a pricing factor that is not captured by the Fama-French factors or momentum.

The model shows that the inflation betas in the conditional inflation-CAPM are also regression coefficients, but using inflation levels instead of changes in inflation. In addition, the betas are time-varying and depend on time-t information only. Thus, they can not be estimated by a full-sample regression like (**??**). Instead, we estimate the regression

$$\begin{aligned} R_{is} - R_s^f &= \alpha_{it} + \beta_{it}^{\pi}\pi_s + c_{it}X_s + \varepsilon_{is} \\ s &= 1, 2, ..., t-1 \end{aligned} \tag{26}$$

in which we use observations only up to time t and we let the coefficients α_{it}, β_{it} and c_{it} depend on t. An easy and popular way to estimate (**??**) is to run a rolling OLS regression, usually using a 5-year window, as in Briere et al. (2012). This is a special case of the nonparametric kernel estimator developed by Ang and Kristensen (2012) and used by Duarte (2011) in the context of inflation. We will use this estimator using an exponential kernel instead of a flat window. The main advantage of this choice is that it produces smoother estimates for β_{it} (i.e. smaller standard errors) while still preserving all the consistency and robustness properties of the rolling-window OLS estimator. Ang and Kristensen (2012) and Duarte (2011) show that estimates of this regressions are robust to including many controls, restricting the sample, double-sorting by size and other firm characteristics and forming portfolios. In order to implement the kernel estimator, we run a weighted least-squares regression:

$$\left(\hat{\alpha}_{i,t}, \hat{\beta}_{i,t}^{\pi}, \hat{c}_{i,t}\right) = \arg\min_{\alpha,\beta,\beta_l,c,d} \sum_{s=1}^{t-1} K_{i,t}\left(t-s\right)\left(R_{i,s} - R_s^f - \alpha - \beta\pi_s - cX_s\right)^2 \tag{27}$$

where

$$K_{i,t}(t-s) = K\left(\frac{t-s}{h_i T_i}\right)\frac{1}{h_i T_i}, \tag{28}$$

h_i is a stock-specific bandwidth, T_i is the number of total observations for stock i and the exponential kernel is given by

$$K(x) = \begin{cases} e^{-x} & , \quad x \leq 0 \\ 0 & , \quad x > 0. \end{cases} \tag{29}$$

We pick the bandwidth h_i so that the half-life of the exponential kernel $K_{i,t}$ is 5 years, which makes the estimates comparable to those obtained by a 5-year rolling window regression.

3.3 Market price of inflation risk

Armed with estimates for inflation betas, we use cross-sectional regressions to estimate the conditional and unconditional market prices of inflation risk λ_t^{π} and $\lambda^{\Delta\pi}$. To estimate $\lambda^{\Delta\pi}$, we run a regression of mean returns on betas:

$$\begin{aligned} \bar{R}_i - \bar{R}^f &= \lambda^{\Delta\pi}\hat{\beta}_i^{\Delta\pi} + \lambda_i^x \hat{c}_i + \varepsilon_{is} \\ i &= 1, 2, ..., I \end{aligned} \tag{30}$$

where $\hat{\beta}_i^{\Delta\pi}$ and $\hat{c}_i$ are the estimates obtained from regression (**??**) and mean returns are given by:

$$\bar{R}_i - \bar{R}^f = \frac{1}{T}\sum_{t=1}^{T}\left(R_{it} - R_t^f\right) \tag{31}$$

In equation (**??**), we have imposed that the constant term is zero to be consistent with the model's pricing equation (18). Unreported results confirm that the estimates for $\lambda^{\Delta\pi}$ are almost identical if we do include a constant. Note that in equation (**??**), the regressor $\hat{\beta}_i^{\Delta\pi}$ is itself a regression estimate, while $\lambda^{\Delta\pi}$ is the slope coefficient to be estimated.

To estimate the conditional price of risk λ_t^{π} we use a standard Fama and MacBeth (1973) procedure. We run T cross-sectional regressions, each of which consists of I_t observations:

$$R_{it} - R_t^f = \lambda_t^{\pi} \hat{\beta}_{it}^{\pi} + \varepsilon_{is} \tag{32}$$
$$i = 1, 2, ..., I_t$$

Again, we decide to run this regression without a constant as dictated by the model equation (17), although results are not sensitive to including one.

3.4 Results

Figure 5 shows the distribution of unconditional inflation betas $\beta_i^{\Delta\pi}$ estimated using equation (**??**). The same distribution in the model is centered around 3.03 with standard deviation of 1%, while it is centered at 0.88 in the data, and has a standard deviation of 10%. In the model, just as in the data, we find that stock returns are, on average, unconditionally positively correlated with inflation shocks. This means that the long-standing intuition that stocks are good protection against high inflation is true if we think about long periods of time and the mean (or median) firm. While the means of the two distributions are different by a factor of 3, we cannot reject the hypothesis that they are different because of the high dispersion of the empirical distribution. On the other hand, precisely because of the difference in dispersion of the two distributions, it is easy to statistically reject the null hypothesis that both distributions are equal.

[FIGURE 5 HERE]

Figure 6 plots the time series of the cross-sectional mean of conditional inflation betas $\hat{\beta}_{it}^{\pi}$ and realized inflation. The figure shows that the conditional covariance between inflation and stock returns is time-varying and changes signs quite often. In contrast, the model's (market) inflation betas are always positive and have a standard deviation across time that is 100 times smaller than in the data. In addition, the time-average of the cross-sectional mean of $\hat{\beta}^{\pi}_{it}$ is -0.15, while in the model it is 3.03. Thus, on average, equation (**??**) shows that a one percentage point increase in inflation during the current month is associated with a decrease in realized excess returns of 15 basis points in the data and an increase of 3.03 basis points in the model. Figure 6 also shows the correlation between inflation and betas, which is -12%, while in the model it is -100%. Figure 7 displays the histogram of the time-average of individual conditional inflation betas. The mean of this distribution is negative, which compares to the model's mean of 3.03 displayed in figure 3.

[FIGURE 6 HERE]

[FIGURE 7 HERE]

The unconditional estimate of the market price of inflation risk using (**??**) gives $\hat{\lambda}^{\Delta\pi} = -0.18$, while in the previous section we had found that the model has $\lambda^{\Delta\pi} = 2.53 \times 10^{-3}$. The estimate of $\hat{\lambda}^{\Delta\pi} = -0.18$ means that a portfolio of stocks that moves one-for-one with inflation has a Sharpe ratio of -0.18. For comparison, the aggregate market return has a Sharpe ratio between 0.3 and 0.5 depending on how it is estimated. A negative price of risk means that investors are willing to accept negative mean returns in order to be unconditionally hedged against inflation.

The unconditional mean, however, is not all there is – time variation in the conditional price of risk is an equally important concept to understand how investors evaluate inflation risk and provides a stronger rejection of the model. Recall that in the model, the conditional price of risk λ^{π}_t was always positive, almost constant over time and very small in magnitude. In contrast, figure 8 shows that the price of risk obtained from the cross-sectional regression (**??**) is negative on average, time-varying and large in magnitude (both positive and negative). We see from figure 8 that there are sustained periods of large and negative price of inflation risk, like in the mid-1970's and the mid-1980's. Since 2000, we can see an upward trend in the price of inflation risk: high inflation can become a better state of nature in a recessionary environment. Not only the mean, but also the amount of time variation in the conditional price of risk is at odds with the model. The standard deviation of $\hat{\lambda}_t^\pi$ is 1.3, more than five time its mean of -0.23 and more than 10^5 times the one observed in the model.

[FIGURE 8 HERE]

In summary, the empirically observed price of inflation risk reveals that investors are very sensitive to inflation shocks, and that whether high or low inflation is a concern depends acutely on economic conditions. In addition, the quantity of inflation risk that firms face, measured by their inflation betas, is very heterogeneous across firms and changes over time. In contrast, in the model, firms of all sectors are almost completely immune to inflation shocks, which makes the cross-section of their returns move in unison and almost not at all as a result of inflation shocks. And precisely because inflation does not alter the cross-sectional distribution of returns or output of firms, the market price of inflation risk is small.

4 Concluding remarks

We have presented the asset pricing implications of a menu cost model with heterogenous firms. Firms are assumed to be heterogeneous in two dimensions: they have different menu costs and different volatilities of idiosyncratic productivity. When these two degrees of freedom are used to match the empirically observed frequency and size of price adjustments for firms, we find that the model fails to reproduce basic asset pricing patterns observed in the data.

The first pattern we analyze is the market price of inflation risk, that is, the compensation that investors require to hold inflation risk. We find that the unconditional inflation price of risk implied by the cross-section of stocks is empirically negative and quite large, while it is positive and small in the model. More importantly, the conditional price of risk inferred from the cross-section of stock returns exhibits large time-series variation in the data, and can even change signs. This means that investors percieve inflation to be a good state of nature in certain economic times and a bad state of nature in others. This result is also consistent with what is observed in bond markets as demonstrated, for example, in Campbell, Sunderam, and Viceira (2009). The menu cost model, however, implies that the conditional price of inflation risk is barely time varying and almost identical to its unconditional counterpart. Not only is it not costly to insure against inflation in the model, the price of insurance is almost constant.

The second pattern we study is how firms' stock returns co-vary with inflation. The model predicts that all stock returns should covary positively with inflation, and that the covariance is almost identical across firms. In contrast, the data shows considerable cross-sectional heterogeneity in the covariance, with half of firms covarying negatively with inflation. As a consequence, there is a considerable spread in mean returns between firms that are exposed to inflation and those that are not, a feature absent in the model.

If we accept that stock returnrs are informative about investors' preferences and firm behavior, then our results demonstrate that the canonical menu cost model does not yet capture some important features of how economic agents perceive and react to inflation. Because the model is so successful at reproducing the empirically observed impulse response functions of macroeconomic quantities with respect to nominal shocks, and there is ample evidence supporting the existence of heterogeneous menu costs, our work by no means suggests that we should abandon such models. On the contrary, our hope is that our work will encourage further research into how to reconcile the observed behavior of asset prices with the otherwise sound menu cost models.

References

Ang, A. and D. Kristensen (2012). Testing conditional factor models. *Journal of Financial Economics 106*(1), 132 – 156.

Briere, M., A. Ang, and O. Signori (2012). Inflation and individual equities. Technical Report 4.

Campbell, J., A. Sunderam, and L. Viceira (2009). Inflation bets or deflation hedges? the changing risks of nominal bonds. Technical report, National Bureau of Economic Research.

Carvalho, C. (2006). Heterogeneity in price stickiness and the real effects of monetary shocks. *Frontiers in Macroeconomics 6*(1).

Duarte, F. (2011). Inflation risk and the cross section of stock returns. *Federal Reserve Bank of New York working paper*.

Fama, E. and K. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics 33*(1), 3–56.

Fama, E. and J. MacBeth (1973). Risk, return, and equilibrium: Empirical tests. *The Journal of Political Economy*, 607–636.

Jegadeesh, N. and S. Titman (2012). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance 48*(1), 65–91.

Krusell, P. and A. Smith, Jr (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy 106*(5), 867–896.

Nakamura, E. and J. Steinsson (2010). Monetary non-neutrality in a multisector menu cost model. *The Quarterly Journal of Economics 125*(3), 961–1013.

Piazzesi, M. and M. Schneider (2006, October). Equilibrium yield curves. (12609).

5 Tables and Figures

Table 1: Model parameters

Discount factor	$\beta = 0.96^{1/12}$
Coefficient of relative risk aversion	$\gamma = 1$
Inverse of Frisch elasticity of labor supply	$\psi = 0$
level parameter on disutility of labor	$\omega = 1.4211$
Elasticity of demand	$\theta = 4$
Intermediate inputs share in production	$s_m = 0.7$
Speed of mean reversion in idiosyncratic productivity	$\rho = 0.7$
Mean growth rate of nominal aggregate demand	$\mu = 0.002$
Std. deviation of the growth rate of nominal aggregate demand	$\sigma_\eta = 0.0037$

This table reports the Nakamura and Steinsson (2010) calibration of the multisector menu-cost model with intermediate inputs.

Table 2: Conditional inflation-CAPM for the model

	mean	std
inflation π_t	2.16	0.51
price of inflation risk λ_t^π	3.10E-03	4.91E-06
risk-free rate R_t^f	4.27	0.38
expected market-return $E_t[R_{t+1}^m]$	4.27	0.38
market inflation exposure $\beta_t^{\pi,m}$	3.03	0.66

This table presents the aggregate variables of the multisector menu cost model with intermediate inputs for the conditional inflation CAPM given by equation (17). Results are from a 400-period simulation. Mean and standard deviation are annualized and reported as percentage points.

Table 3: Unconditional inflation-CAPM for the Model

	mean	std
inflation difference $\triangle\pi_t$	0.00	0.36
market-return R_t^m	4.27	1.10
unconditional risk-free rate R_t^f	4.27	N/A
price of inflation-difference risk $\lambda^{\triangle\pi}$	3.52E-03	N/A
market inflation difference exposure $\beta^{\triangle\pi,m}$	3.03	N/A

This table presents the aggregate variables of the multisector menu cost model with intermediate inputs for the unconditional inflation-CAPM given by equation (18). Results are from a 400-period simulation. Mean and standard deviation are annualized and reported as percentage points.

Table 4: Sector variables of conditional and unconditional inflation-CAPM for the model

sector	K * 10^(-2)	$\sigma_\epsilon \times 10^{-2}$	weight	return		beta		uncond. beta
				mean	std	mean	std	
Vehicle fuelused cars	0.002	5.1	7.66	4.27	0.38	3.03	0.65	3.03
Transp. goods/utilities/travel	0.325	6.85	19.09	4.27	0.38	3.04	0.65	3.03
Unprocesed food	1.02	9.2	5.92	4.27	0.38255	3.04	0.65	3.02
Processed food other goods	1.02	5.7	13.68	4.27	0.38	3.03	0.67	3.03
Services (excl. travel)	0.69	4.05	38.53	4.27	0.38	3.02	0.73	3.03
Hh. furn./apparel/rec. goods	1.8	5.4	15.12	4.27	0.38	3.02	0.70	3.01

This table presents sector level variables of the multisector menu cost model with intermediate inputs. The classifaction and calibration of menu costs K, standard deviation of productivity shock σ_ϵ, and weight in the economy are from Nakamura and Steinsson (2010). We report the mean and standard deviation of returns and inflation exposures on the sector level for the conditional inflation-CAPM in columns 5-8. In column 9 we also report the exposure to inflation-differences in the unconditional inflation-CAPM. We simulate over (x) periods and average over 100 firms for each sector. Mean and standard deviation are annnualized and reported as percentage points.

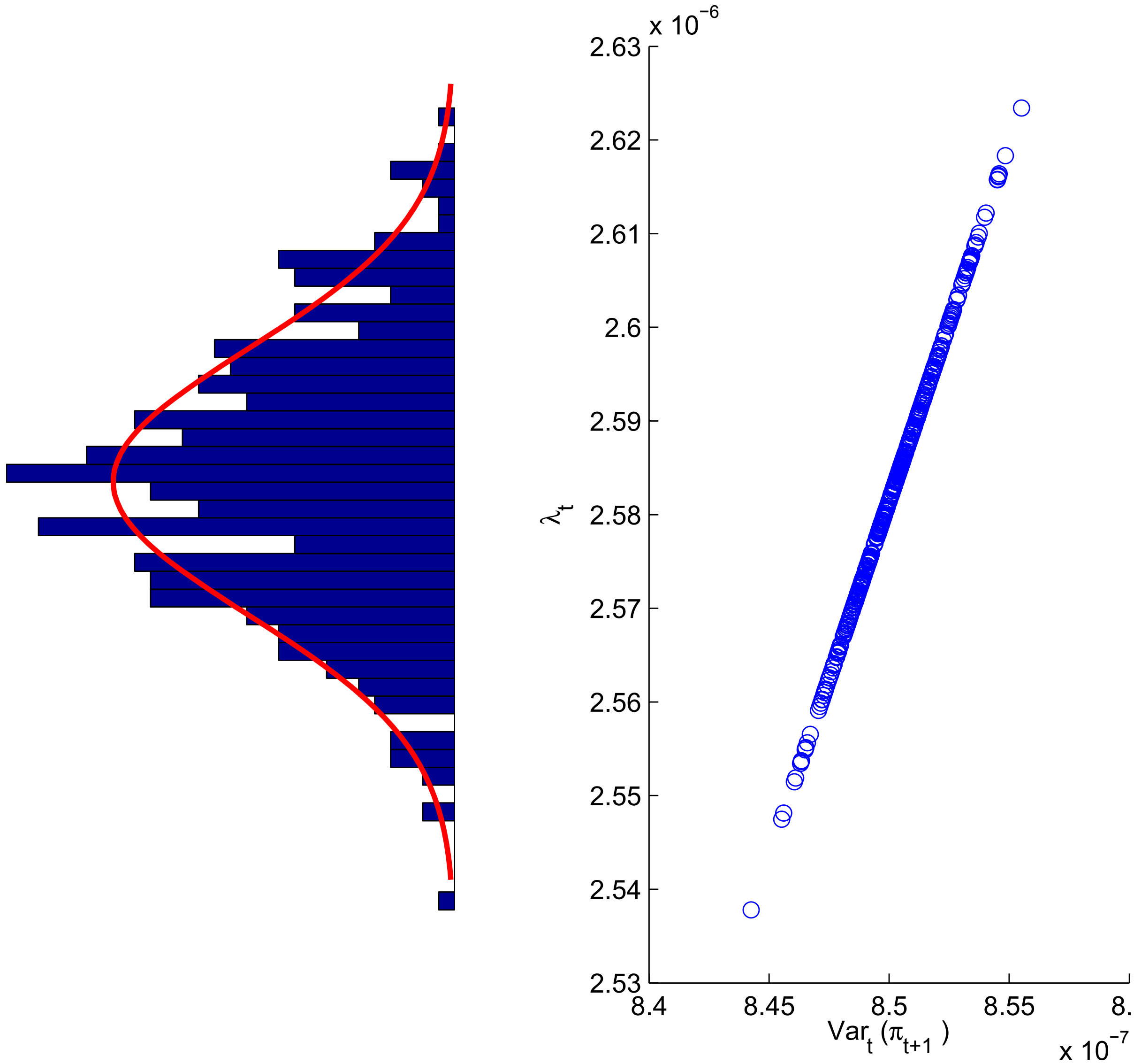

Figure 1: Price of inflation risk in the model

This figure shows the contemporaneous linear relationship between the conditional variance of inflation $Var_t(\pi_t + 1)$ and the price of inflation risk λ_t for the multisector menu cost model with intermediate inputs. Results are from a 400-period simluation. On the left hand side we plot the resulting distribution for the price of inflation risk the price with a normal fit of mean $2.58E^{-6}$ and standard deviation $1.42E^{-8}$.

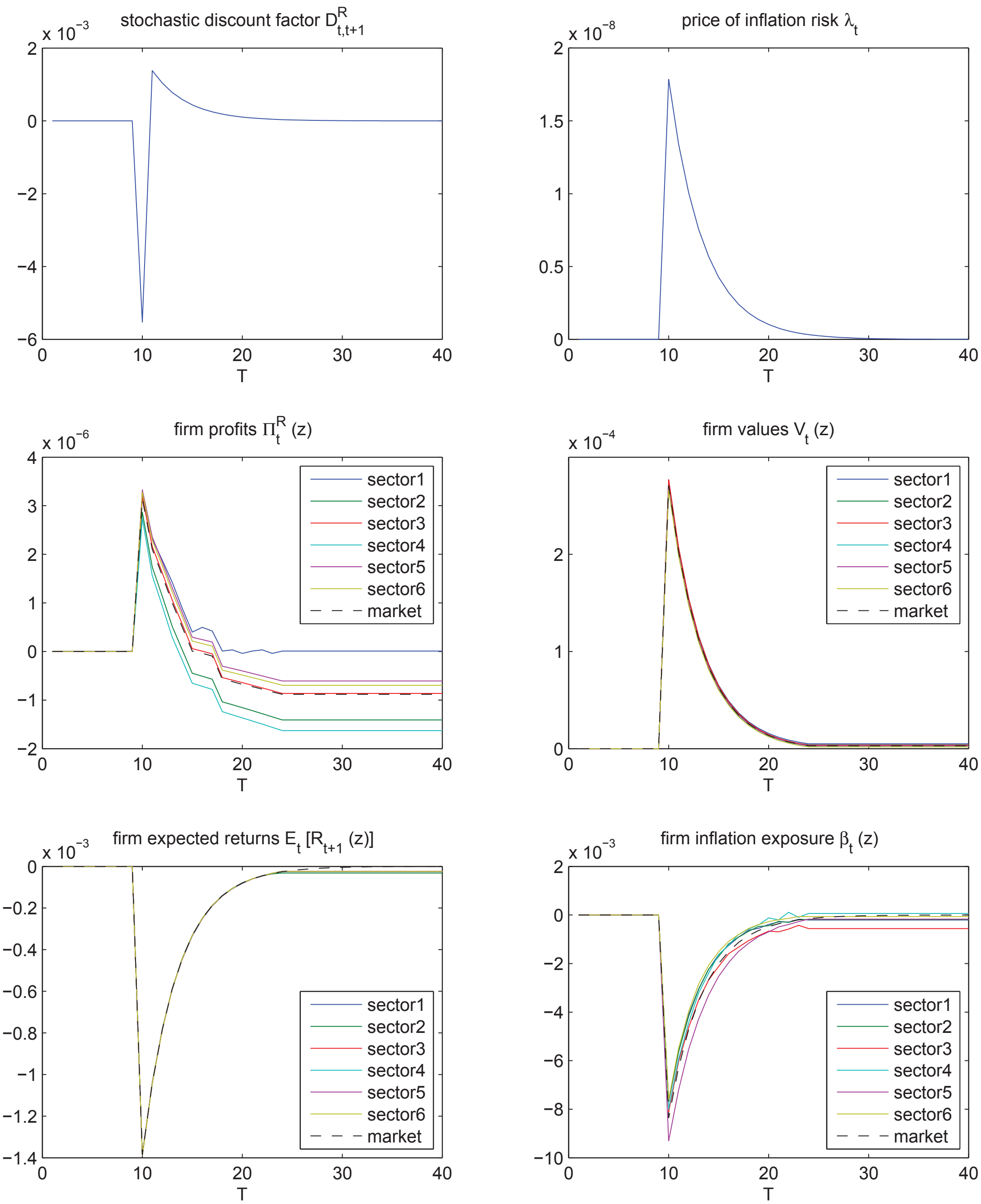

Figure 2: Impulse response functions in the model

This figure plots the impulse response function of several variables of interest due to a two standard deviation positive shock in the growth rate of nominal aggregate demand S_t at time period 10. The upper left and upper right corner shows the real stochastic discount factor $D^R_{t,t+1}$ and the price of inflation risk λ_t. The middle and bottom rows show sector variables and the corresponding variable for the market (dashed): profits $\Pi^R_t(z)$, values $V_t(z)$, expected returns $E_t[R_{t+1}]$, and exposures to inflation $\beta_t(z)$. We choose one representative firm for each sector with zero productivity shocks. For minimal distraction we carry over sector variables 15 periods after the shock.

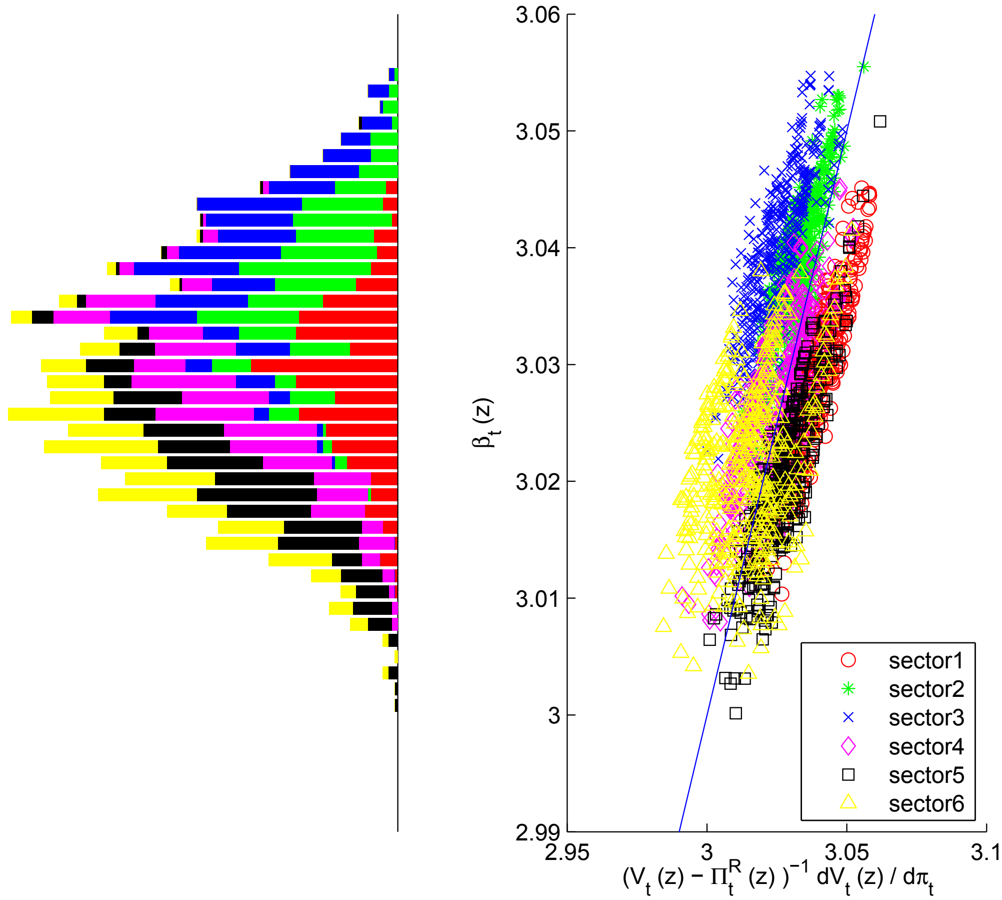

Figure 3: Exposures to inflation risk in the model

This figure shows the contemporaneous relationship between the scaled derivative of the value function with respect to inflation $(V_t(z) - \Pi_t^R(z))^{-1} dV_t(z)/d\pi_t$ and the firm exposures to inflation $\beta_t(z)$ for the multisector menu cost model with intermediate inputs. Each point represents the exposure to inflation for a specific sector, obtained from an average over 100 firms within that sector. The model is simulated for 400 periods. This relationship is approximately linear as suggested in Equation (22). The 45° line guides the eye. On the left side we plot the corresponding distribution for the inflation exposures, color-differentiated by sector.

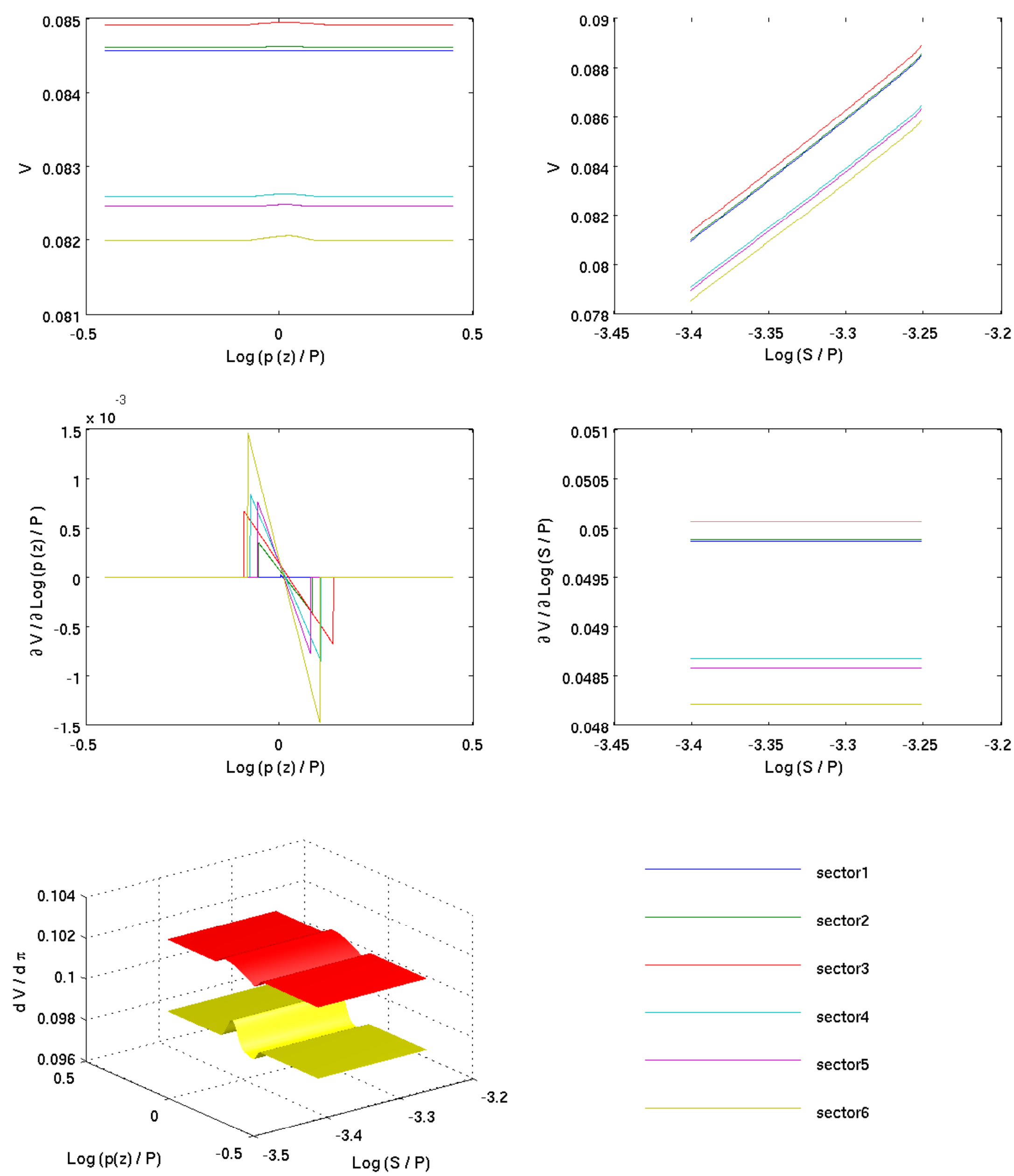

Figure 4: Value Function in the model

This figure shows in the upper left corner the Value function $V(A_t(z), p_{t-1}/P_t, S_t/P_t)$ plotted against the log of its second argument, the log of last periods price measured by today's aggregate price level p_{t-1}/P_t, for each sector; in the upper right corner plotted against the log of its third argument, real aggregate demand S_t/P_t. In the middle we show the partial derivatives of the value function with respect to these arguments. In the lower left corner, we show the total derivative of the value function with respect to inflation obtained from applying the chain rule (24) for the two sectors with the biggest spread in derivatives. All plots are at the middle of the grid with respect to the idiosyncratic shock $A_t(z)$.

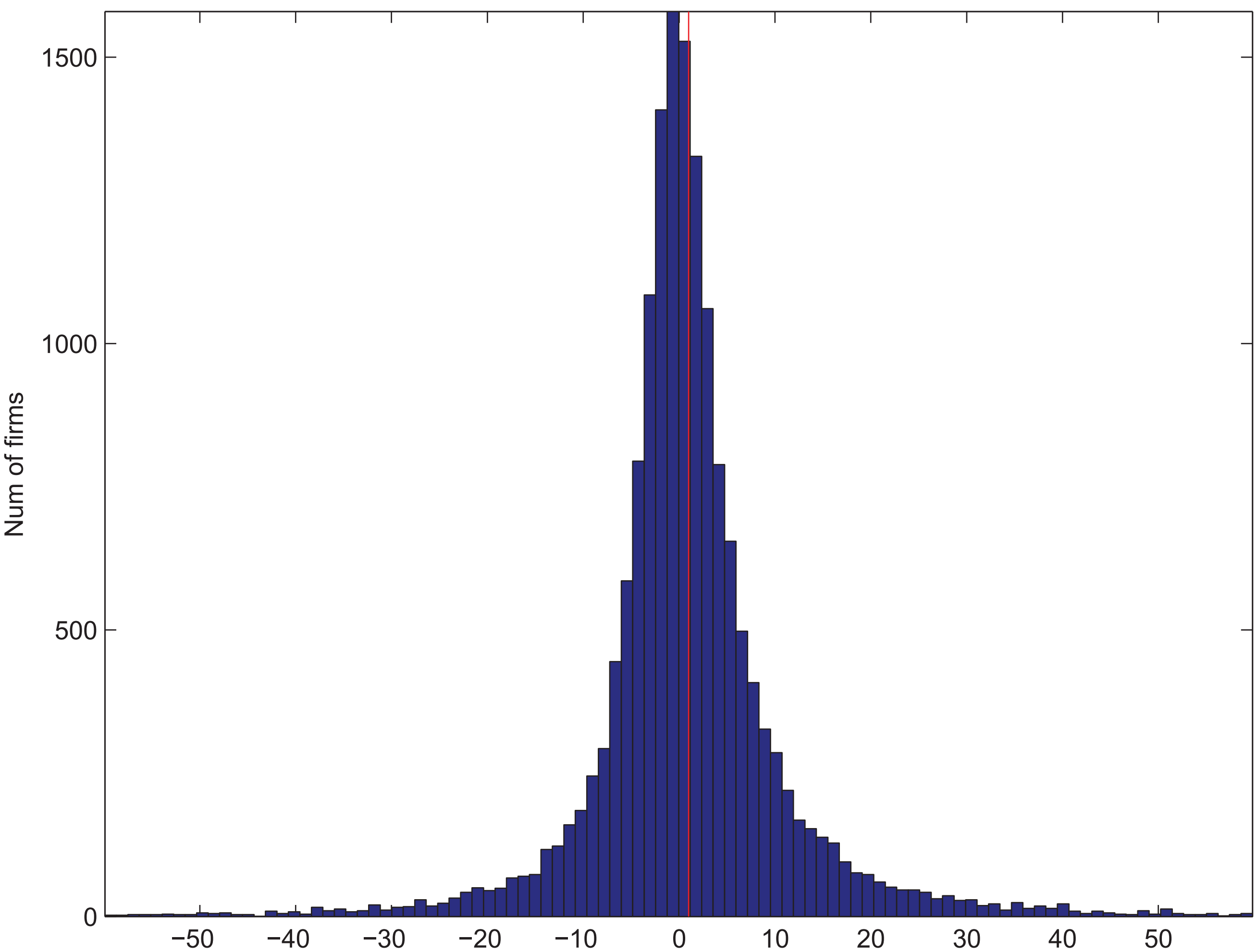

Figure 5: Empirical distribution of unconditional (full-sample) inflation betas

Unconditional inflation betas are the coefficients in full-sample, time-series regressions of excess stock returns on changes in inflation. Stock returns are from CRSP and inflation is from PCE of non-durables and services. The sample is quarterly from January of 1959 to July of 2012.

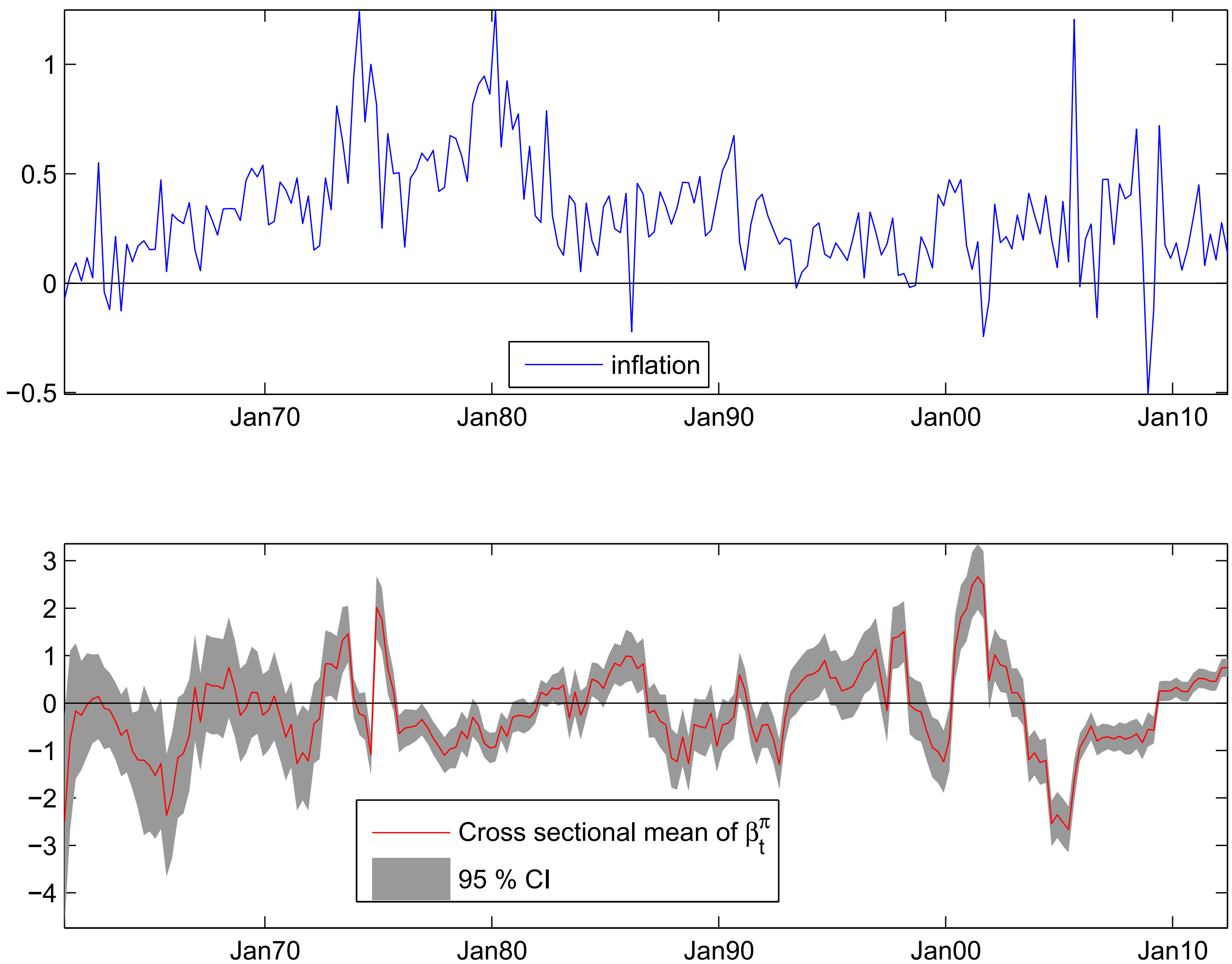

Figure 6: Empirical time evolution of inflation betas

Conditional inflation betas β_t^π are the coefficients in a real-time (i.e. backward looking) rolling regression of excess stock returns on changes in inflation. We use an exponential kernel to downweight observations that are further from the current date. The figure shows the cross-sectional mean (over all firms) of the distribution of inflation betas for each time period in the sample, together with PCE inflation of non-durables and services. Stock returns are from CRSP. The sample is quarterly from January of 1959 to July of 2012 (with the first five years used as a burn-in for the rolling regressions). The cross-section of betas gives the confidence interval.

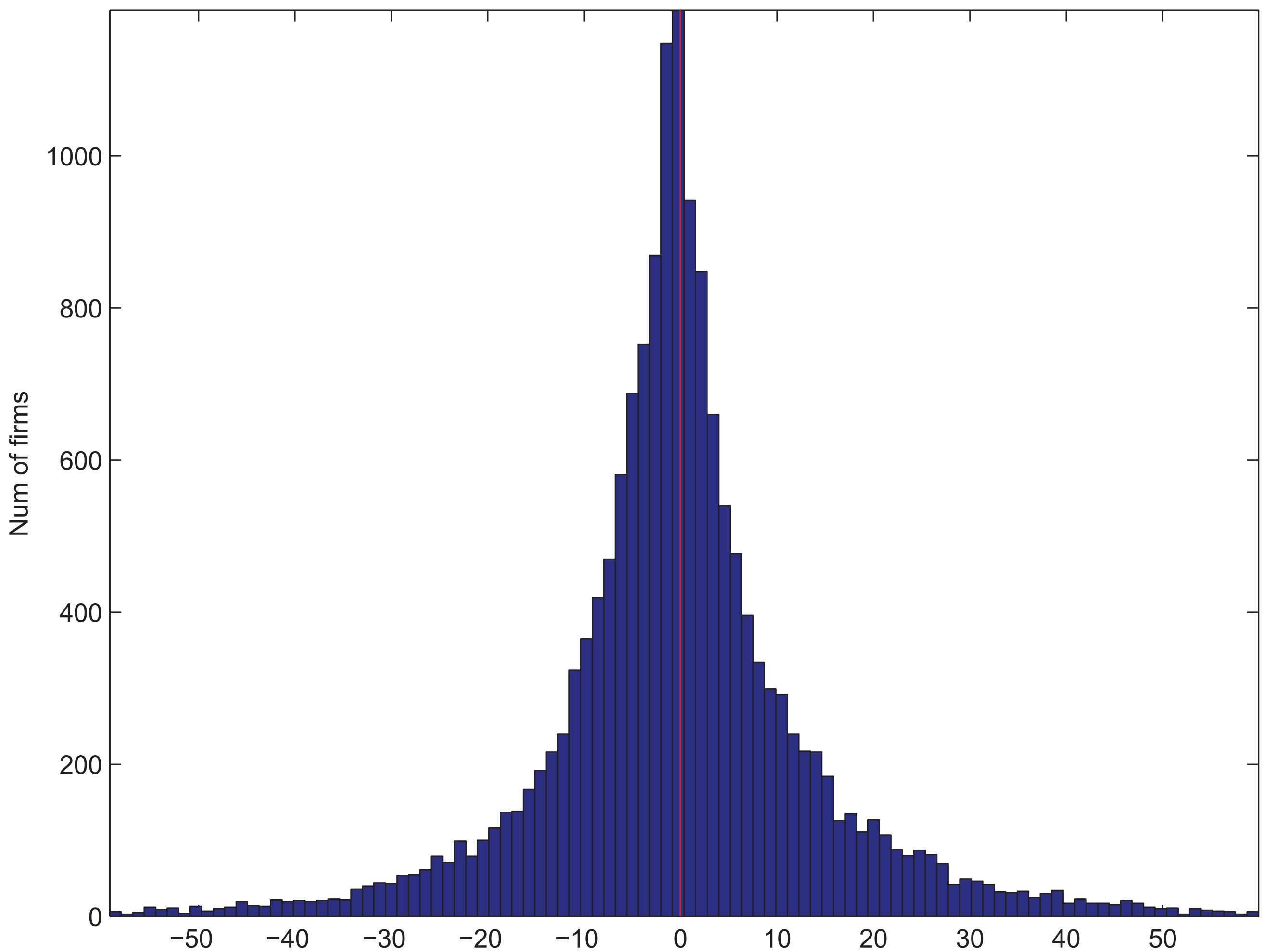

Figure 7: Empirical distribution of conditional (real-time) inflation betas

Conditional inflation betas are the coefficients in a real-time (i.e. backward looking) rolling regression of excess stock returns on changes in inflation. We use an exponential kernel to downweight observations that are further from the current date. The histogram shows the distribution of the time-average of inflation betas (there is one observation per firm). Stock returns are from CRSP and inflation is from PCE of non-durables and services. The sample is quarterly from January of 1959 to July of 2012.

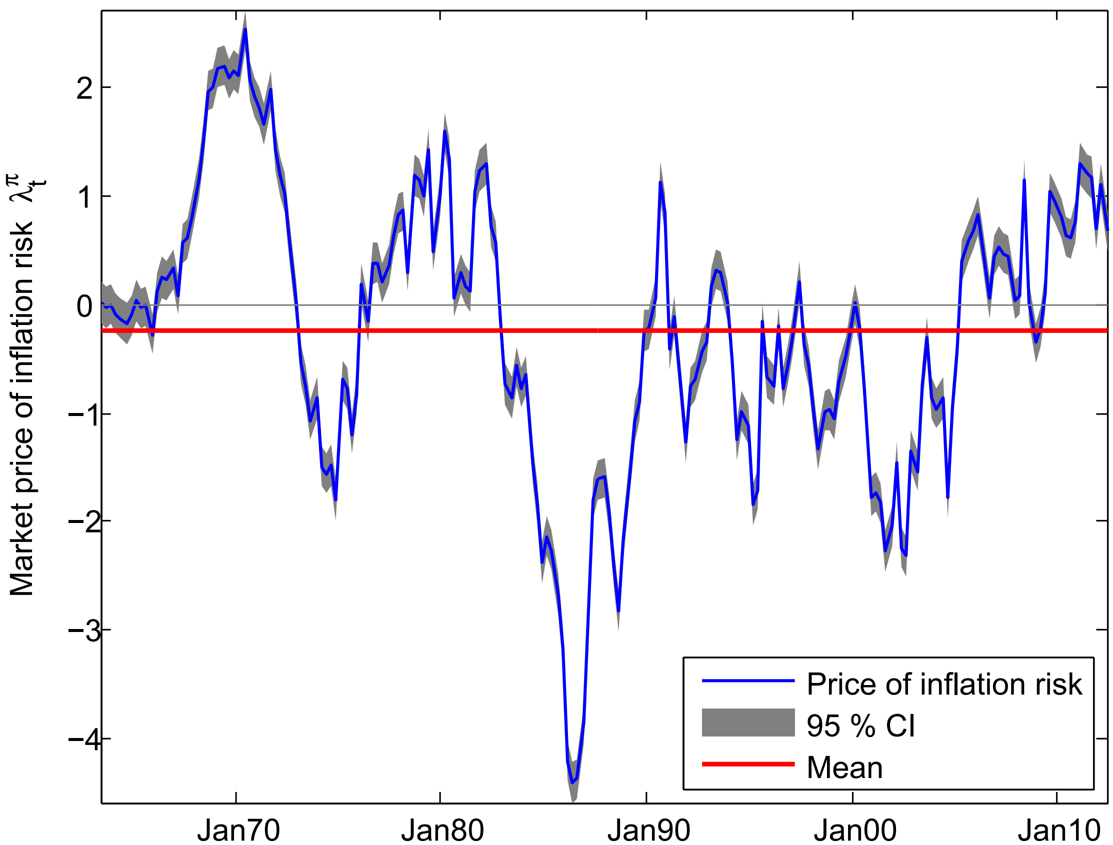

Figure 8: Empirical time-series of the market price of inflation risk

The market price of inflation risk is the coefficienet in a cross-sectional regression of excess stock returns on their inflation betas. We run one such regression per time period and plot the resulting annualized time series, together with its mean. Inflation betas are the coefficients in a real-time (i.e. backward looking) rolling regression of excess stock returns on inflation. We use an exponential kernel to downweight observations that are further from the current date. The price of risk can be interpreted as the Sharpe ratio of a portfolio whose excess returns move one-for-one with inflation. Inflation is from PCE of non-durables and services and stock returns are from CRSP. The sample is quarterly from January of 1959 to July of 2012 (with the first five years used as a burn-in for the rolling regressions). The time-series of λ_t gives the confidence interval.