Technical Appendix for "Central Bank Communication and Expectations Stabilization"

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August 20, 2008

Abstract

This technical appendix provides some calculations underlying the model used in Eusepi and Preston (2008).

Contents

1 A Simple Model

The following details a simple model of output gap and inflation determination that is similar in spirit to Goodfriend and King (1997), Rotemberg and Woodford (1999) and Svensson and Woodford (2005). A continuum of households face a canonical consumption allocation problem and decide how much to consume of available differentiated goods and how much labor to supply to firms for the production of such goods. A continuum of monopolistically competitive firms produce differentiated goods using labor as the only input and face a price setting problem of the kind proposed by Rotemberg (1982).[1] The major difference is the incorporation of non-rational beliefs, delivering an anticipated utility model. The analysis follows Marcet and Sargent (1989) and Preston (2005), solving for optimal decisions conditional on current beliefs. Various mechanisms of persistence, such as habit formation, price indexation and inertial monetary policy are abstracted from. This provides sharp, perspicuous analytical results.[2] An earlier version of this paper, Eusepi and Preston (2007a), demonstrates that our conclusions regarding the value of communication in policy design remain pertinent in models with such modifications.

1.1 Microfoundations

Households. Households maximize their intertemporal utility derived from consumption and leisure

$$\hat{E}^i_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[\ln C^i_T - h^i_T \right]$$

subject to the flow budget constraint

$$B^i_t \leq R_{t-1} B^i_{t-1} + W_t h^i_t + P_t \Pi_t - P_t C^i_t - T^i_t$$

where B^i_t denotes holdings of the one period riskless bond, R_t denotes the gross interest paid on the bond, W_t the nominal wage, h^i_t labor supplied by household i and T^i_t lump-sum taxes

[1] An analysis of price setting of the kind proposed by Calvo (1983), as implemented by Yun (1996), would lead to similar conclusions.

[2] It is also motivated by Milani (2006) and Eusepi and Preston (2008a) which suggest that purely forward looking business cycle models with learning dynamics provide a superior characterization of various U.S. macroeconomic time series than do rational expectations models with various persistence mechanisms.

and transfers for household i. Financial markets are assumed to be incomplete and Π_t denotes profits from holding shares in an equal part of each firm. Nominal income in any period t is $P_t Y_t^i = W_t h_t^i + P_t \Pi_t$ and P_t is the aggregate price level defined below. $\hat{E}_t^i$ denote the beliefs at time t held by each household i, which satisfy standard probability laws. Section 3 describes the precise form of these beliefs and the information set available to agents in forming expectations. However, two points are worth noting. First, in forming expectations, households and firms observe only their own objectives, constraints and realizations of aggregate variables that are exogenous to their decision problems and beyond their control. They have no knowledge of the beliefs, constraints and objectives of other agents in the economy: in consequence agents are heterogeneous in their information sets in the sense that even though their decision problems are identical, they do not know this to be true. Second, given the assumed conditioning information for expectations formation, consumption plans are made one period in advance and therefore predetermined.[3] Labor supply decisions are not predetermined and are conditioned on period t information.[4]

Each household consumes a composite good

$$C_t^i = \left[\int_0^1 c_t^i (j)^{\frac{\theta_t - 1}{\theta_t}} \, dj \right]^{\frac{\theta_t}{\theta_t - 1}}$$

which is made of a continuum of differentiated goods, $c_t^i (j)$, each produced by a monopolistically competitive firm j. The elasticity of substitution among differentiated goods, θ_t, is time-varying, with $E[\theta_t] = \theta > 1$. This is a simple way of modeling time-varying mark-ups, introducing a trade-off between inflation and output stabilization relevant to optimal policy design.

A log-linear approximation to the first order conditions of the household problem provides the household Euler equation

$$\hat{C}_t^i = \hat{E}_{t-1} \left[\hat{C}_{t+1}^i - (\hat{\imath}_t - \pi_{t+1}) \right] \tag{1}$$

[3]We consider a model with pricing and spending decisions determined one period in advance so as to put households, firms and policymakers on an identical informational footing. This could similarly be achieved by the alternative assumption that the central bank has a policy reaction function that responds to one period ahead expectations of inflation and agents condition decisions on period t information. All results continue to hold.

[4]This assumption ensures markets clear in equilibrium.

and the intertemporal budget constraint

$$\hat{E}_{t-1}\sum_{T=t}^{\infty}\beta^{T-t}\hat{C}_T^i = \omega_{t-1}^i + \hat{E}_{t-1}\sum_{T=t}^{\infty}\beta^{T-t}\hat{Y}_T^i \tag{2}$$

where

$$\hat{Y}_t \equiv \ln(Y_t/\bar{Y});\ \hat{C}_t \equiv \ln(C_t/\bar{C});\ \hat{\imath}_t \equiv \ln(R_t/\bar{R});\ \pi_t = \ln\left(P_t/P_{t-1}\right);\ \omega_t^i = B_t^i/\bar{Y};$$

and $\bar{z}$ denotes the steady state value of any variable z.

Solving the Euler equation recursively backwards, taking expectations at time $t-1$ and substituting into the intertemporal budget constraint gives

$$\hat{C}_t^i = (1-\beta)\,\omega_{t-1}^i + \hat{E}_{t-1}\sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)\hat{Y}_T^i - \beta(i_T - \pi_{T+1})\right]. \tag{3}$$

Optimal consumption decisions depend on current wealth at the beginning of the period, ω_{t-1}^i, and on the expected future path of income and the real interest rate.[5] The optimal allocation rule is analogous to permanent income theory, with differences emerging from allowing variations in the real rate of interest, which can occur either due to variations in the nominal interest rate or inflation. Nominal interest rates affect consumption demand only through expectations. Moreover, consumption decisions depend on the *entire expected future path of the nominal interest rate*, in contrast with Bullard Mitra (2002) and Orphanides and Williams (2005), among others, where only the current interest rate matters for output determination. This property underscores the role of managing expectations in policy design. Note also, that as households become more patient, current consumption demand is more sensitive to expectations about future macroeconomic conditions.

Firms. There is a continuum of monopolistically competitive firms. Each differentiated consumption good is produced according to the linear production function

$$Y_{j,t} = A_t h_{j,t}$$

[5] Using the fact that total household income is the sum of dividend and wage income, combined with the first order conditions for labor supply and consumption, delivers a decision rule for consumption that depends only on forecasts of prices: that is, goods prices, nominal interest rates, wages and dividends. However, we make the simplifying assumption that households forecast total income, the sum of dividend payments and wages received.

where A_t denotes an aggregate technology shock. Each firm chooses a price $P_{j,t}$ in order to maximize its expected discounted value of profits

$$\hat{E}^j_{t-1} \sum_{T=t}^{\infty} Q_{t,T} P_T \Pi_{j,T}$$

where

$$\Pi_{j,t} = (1-\tau) \frac{P_{j,t}}{P_t} Y_{j,t} - \frac{W_t}{P_t} h_{jt} - \frac{\psi}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2$$

denotes period profits and the quadratic term the cost of adjusting prices as in Rotemberg (1982).[6] The tax, τ, on revenues is chosen to eliminate the steady state distortion arising from monopolistic competition. Given the incomplete markets assumption it is assumed that firms value future profits according to the marginal rate of substitution evaluated at aggregate income

$$Q_{t,T} = \beta^{T-t} \frac{P_t Y_t}{P_T Y_T}$$

for $T \geq t$.[7]

The intratemporal consumer problem implies aggregate demand for each differentiated good is

$$Y_{jt} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_t} Y_t$$

where Y_t denotes aggregate output and

$$P_t = \left[\int_0^1 (P_{j,t})^{1-\theta_t} \, dj \right]^{\frac{1}{1-\theta_t}}$$

is the associated price index. Summing up, the firm chooses a sequence for $P_{j,t}$ to maximize profits, given the constraint that demand should be satisfied at the posted price, taking as given P_t, Y_t, and W_t. Again, given the information upon which expectations are conditioned, prices are determined one period in advance.

[6] The results are similar to the case of a Calvo pricing model.

[7] The precise details of this assumption are not important to the ensuing analysis so long as in the log linear approximation future profits are discounted at the rate β^{T-t}.

The first-order condition to the firm's problem is

$$\begin{aligned}\hat{E}_{t-1}\left[\psi\left(\frac{P_{j,t}}{P_{j,t-1}}-1\right)\frac{P_t}{P_{j,t-1}}\right] &= \hat{E}_{t-1}\left[Q_{t,t+1}\psi\left(\frac{P_{j,t+1}}{P_{j,t}}-1\right)\frac{P_{j,t+1}}{{P_{j,t}}^2}P_t\right] \\ &+\hat{E}_{t-1}\left[\theta_t Y_t\left(\frac{P_{j,t}}{P_t}\right)^{-\theta_t}\left(\left(\frac{P_{j,t}}{P_t}\right)^{-1}S_t-\frac{(\theta_t-1)}{\theta_t}\right)\right].\end{aligned}$$

A log-linear approximation provides

$$\hat{P}_{j,t}-\hat{P}_{j,t-1}=\beta\hat{E}_{t-1}\left[\hat{P}_{j,t+1}-\hat{P}_{j,t}\right]+\xi\hat{E}_{t-1}\left[\hat{s}_t+\hat{\mu}_t+\hat{P}_t-\hat{P}_{j,t}\right]$$

where $\hat{P}_t=\log P_t$; $\hat{P}_{j,t}=\log P_{j,t}$; $\xi\equiv(1-\theta)\bar{Y}/\psi$; $\mu_t=\theta_t(\theta_t-1)^{-1}$ denotes the mark-up and satisfies $\hat{\mu}_t=\ln(\mu_t/\bar{\mu})$; and $\hat{s}_t\equiv\ln\left(S_t/\bar{S}\right)$ is marginal costs (defined below) in deviations from steady state. Collecting terms in the price of firm j provides

$$\left[1-\left(\frac{\xi}{\beta}+\frac{1}{\beta}+1\right)L+\frac{1}{\beta}\right]\hat{E}_{t-1}\hat{P}_{j,t+1}=-\frac{\xi}{\beta}L\hat{E}_{t-1}\left[\hat{s}_{t+1}+\hat{\mu}_{t+1}+\hat{P}_{t+1}\right]$$

where L denotes the lag operator. Factoring the polynomial and solving the unstable root forward determines the optimal price of the firm as

$$\hat{P}_t(j)=\gamma_1\hat{P}_{t-1}(j)+\xi\gamma_1\hat{E}_{t-1}\sum_{T=t}^{\infty}(\gamma_1\beta)^{T-t}\left[\hat{s}_T+\hat{\mu}_T+\hat{P}_T\right] \tag{4}$$

where the roots γ_1 and γ_2 satisfy

$$0<\gamma_1<1,\quad \gamma_2>1,\quad \gamma_1\gamma_2=\beta^{-1}\ \text{ and }\ \gamma_1+\gamma_2=\beta^{-1}(\xi+1+\beta).$$

The latter two properties combined imply $\xi=(1-\gamma_1)(1-\gamma_1\beta)\gamma_1^{-1}$. Noting that

$$\hat{E}_{t-1}\sum_{T=t}^{\infty}\left(\frac{1}{\gamma_2}\right)^{T-t}\pi_T=-\hat{P}_{t-1}+\left(1-\frac{1}{\gamma_2}\right)\hat{E}_{t-1}\sum_{T=t}^{\infty}\left(\frac{1}{\gamma_2}\right)^{T-t}\hat{P}_T$$

permits the optimal price decision to be written in terms of aggregate inflation as

$$\hat{P}_{j,t}=\gamma_1\hat{P}_{j,t-1}+\left(1-\frac{1}{\gamma_2}\right)^{-1}\frac{\xi}{\gamma_2\beta}\left\{P_{t-1}+\hat{E}_{t-1}\sum_{T=t}^{\infty}\left(\frac{1}{\gamma_2}\right)^{T-t}\left[\left(1-\frac{1}{\gamma_2}\right)(\hat{s}_T+\hat{\mu}_T)+\pi_T\right]\right\}.$$

This condition states that each firm's current price depends on the expected future path of real marginal costs, the aggregate price level and cost-push shocks.[8]

[8]In an earlier version of this paper, Eusepi and Preston (2007a), the firm's decision problem was simplified by making certain assumptions about the information available to firms when setting prices. Mike Woodford and an anonymous referee are thanked for encouraging the authors to characterize the more general case presented here. The general tenor of results is unchanged.

The real marginal cost function is

$$S_t = \frac{w_t}{A_t} = \frac{C_t}{A_t}$$

where the second equality comes from the household's labor supply decision. Log-linearizing we obtain

$$\hat{s}_t = \hat{C}_t - \hat{a}_t,$$

so that current prices depend on expected future demand and technology. The responsiveness of current prices to changes in expected demand depends on the degree of nominal rigidity. A low degree of nominal rigidity implies a high value of ξ (corresponding to a low value of the cost ψ): in this case firms respond aggressively to changes in perceived demand because price changes are less costly. The opposite occurs in the case of higher costs of price adjustment. The degree of price rigidity plays a key role in the stability analysis.

1.2 Market clearing, efficient output and aggregate dynamics

The model is closed with assumptions on monetary and fiscal policy. The fiscal authority, aside from levying taxes to eliminate the steady state distortion from monopolistic competition, is assumed to follow a zero debt policy in every period t and this is understood to be true by agents.[9] Monetary policy is discussed in detail in the subsequent section. For now it suffices to note that a nominal interest rate rule is implemented. For a more general treatment of the interactions of fiscal and monetary policy under learning dynamics see Eusepi and Preston (2008b) and Evans and Honkapohja (2007).

General equilibrium requires that the goods market clears, so that

$$A_t h_t - \frac{\psi}{2}\left(\Pi_t - 1\right)^2 = \int C_t dj = C_t. \tag{5}$$

This condition states that output net of adjustment costs is equal to aggregate consumption, determining the equilibrium demand for labor h_t at the wage $w_t = C_t$. This relation satisfies the log-linear approximation

$$\hat{h}_t + \hat{a}_t = \hat{C}_t = \hat{Y}_t.$$

[9] This implies agents do not need to forecast future tax obligations as in the analyses of Eusepi and Preston (2007b, 2007d).

It is useful to characterize the *efficient* level of output that would occur absent nominal rigidities and distortionary shocks *under rational expectations*. Under these assumptions, optimal price setting implies the log-linear approximation $E_{t-1}\hat{Y}_t^e = E_{t-1}\hat{a}_t$. Hence predictable movements in the efficient rate of output are entirely determined by the aggregate technology shock. Nominal bonds are also in zero net supply requiring

$$\int_0^1 B_t^i di = 0.$$

Aggregating firm and household decisions, using (3) and (4), provides

$$x_t = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)x_T - \beta(i_T - \pi_{T+1}) + \beta\hat{r}_t^e\right] \tag{6}$$

and

$$\pi_t = \frac{\gamma_1 \xi}{(1-\gamma_1\beta)} \hat{E}_{t-1} \sum_{T=t}^{\infty} (\gamma_1\beta)^{T-t} \left[(1-\gamma_1\beta)(x_T + \hat{\mu}_T) + \pi_T\right] \tag{7}$$

where $\int_0^1 \hat{E}_t^i di = \hat{E}_t$ gives average expectations; $x_t = \hat{Y}_t - E_{t-1}\hat{Y}_t^e$ denotes the log-deviation of output from its expected efficient level; and $\hat{r}_t^e = \left(\hat{Y}_{t+1}^e - \hat{Y}_t^e\right)$ the corresponding efficient rate of interest. The average expectations operator does not satisfy the law of iterated expectations due to the assumption of completely imperfect common knowledge on the part of all households and firms. Because agents do not know the beliefs, objectives and constraints of others in the economy, they cannot infer aggregate probability laws.

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