INTERNATIONAL DIMENSIONS OF OPTIMAL MONETARY POLICY*

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Abstract

This paper provides a baseline general equilibrium model of optimal monetary policy among interdependent economies with monopolistic firms and nominal rigidities. An inward-looking policy of domestic price stabilization is not optimal when firms’ markups are exposed to currency fluctuations. Such a policy raises exchange rate volatility, leading foreign exporters to charge higher prices vis-à-vis increased uncertainty in the export market. As higher import prices reduce the purchasing power of domestic consumers, optimal monetary rules trade off a larger domestic output gap against lower consumer prices. Optimal rules in a world Nash equilibrium lead to less exchange rate volatility relative to both inward-looking rules and discretionary policies, even when the latter do not suffer from any inflationary (or deflationary) bias. Gains from international monetary cooperation are related in a non-monotonic way to the degree of exchange rate pass-through.

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1 Introduction

In the long-standing debate on optimal monetary policy, an open question is whether rules designed to fit the specific features of closed economies may be successfully adopted by open, interdependent, trade-oriented countries, or, rather, there exist policy trade-offs that have a specific international dimension. Some view exchange rate fluctuations as instrumental to the adjustment of relative prices. In this case, ‘inward-looking’ policies attempting to stabilize domestic prices and close the output gap are deemed to be desirable regardless of the degree of trade openness: no attempt should be made to offset currency movements and reduce the variability of import prices. But if terms of trade fluctuations are destabilizing sources of uncertainty, domestic monetary policy should be reactive to the exchange rate. A related set of questions concerns what welfare gains (if any) can be achieved through international monetary cooperation.

In this paper, we assess these issues by building a baseline general-equilibrium model of optimal monetary policy among interdependent economies with nominal rigidities, imperfect competition in production, and forward-looking price-setting. The main conclusion of our analysis is that, in an open-economy context, policies exclusively focused on stabilizing internal prices and output gap may actually result, on average, in inefficiently high consumer prices of imports, and therefore suboptimal welfare levels for domestic consumers.

The intuition underlying this result is that monetary policies aimed at internal stabilization can raise the volatility of world demand and the exchange rate. Foreign firms whose

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1 This paper shares a unifying research agenda with a number of recent contributions on optimal monetary policy in closed and open economy. For an introduction to the literature see for instance Gali [2002], Goodfriend and King [2001] and especially Woodford [2003]. A far from complete list of references on open-economy policy rules includes Ball [1999], Benigno and Benigno [2003], Carlstrom and Fuerst [1999], Clarida, Gali and Gertler [2001], Devereux and Engel [2003], Gali and Monacelli [2002], Ghironi and Rebucci [2002], Laxton and Pesenti [2003], McCallum and Nelson [1999], Obstfeld and Rogoff [2000, 2002], Parrado and Velasco [2002], Sutherland [2001], Svensson [2000], and Walsh [1999].
export revenue is exposed to such volatility will attempt to reduce the sensitivity of their profits to exchange rate fluctuations. In our economy, they can do so by charging higher average prices in the domestic market. Ultimately, higher average prices of imports amount to a reduction of domestic consumers’ real wealth. When this is the case, domestic policies that ignore their spillover effects on the markups and profits of foreign exporters can only be inefficient.

In economies with these characteristics, domestic policymakers can improve welfare by trading off, at the margin, output gap stability against lower consumer prices. More specifically, we show that, consistent with the goal of targeting domestic inflation, optimal monetary rules should stabilize a CPI-weighted average of the markups of all firms selling in the domestic market.

Our conclusions by no means rule out the possibility that optimal monetary policies in open economies be similar to the ones derived in a closed-economy context. Indeed, in the cases analyzed by Clarida, Gali and Gertler [2001] or Obstfeld [2002], internal inflation targeting is efficient exactly because firms’ markups are unaffected by exchange rate movements. Consistent with the argument in favor of flexible exchange rates made by Friedman [1953], in such an economy monetary authorities can engineer the right adjustment in relative prices through exchange rate movements, despite the presence of nominal rigidities.

Furthermore, our conclusions on the suboptimality of ‘inward-looking’ policies need not mechanically imply that there is a case for international policy cooperation. In our model there are no welfare gains from entering internationally binding agreements not only when there are no deviations from the law of one price and exporters’ revenues are independent of exchange rate movements (as in Obstfeld and Rogoff [2002]), but also when local prices are completely insulated from exchange rate fluctuations, so that exporters’ revenues move proportionally to the exchange rate (as in Devereux and Engel [2003]). That is, under the assumptions of either complete or zero pass-through, monetary policies are strategically
independent and there are no policy spillovers in equilibrium. However, in our model cooperation is beneficial for economies between the two cases described above. Under a more general specification of preferences, Benigno and Benigno [2003] show that gains from cooperation can materialize, and inward-looking policies are suboptimal, even if there is full exchange rate pass-through worldwide.

Using the same logic as above, we finally show that commitment is superior to discretion even when discretionary policies do not suffer from any inflationary (or deflationary) bias. This is because, given preset prices by firms, a discretionary policymaker has an incentive to over-stabilize the domestic economy and use monetary policy to tilt terms of trade in favor of domestic agents. But any systematic attempt to exploit this opportunity is doomed to lower domestic welfare on average, as foreign exporters react by presetting higher prices in the domestic market.

Building on Corsetti and Pesenti [2001], we set up a model that can be solved in closed form, without resorting to loglinear approximations. Different from many other contributions in the new open-economy macroeconomic literature, all our welfare results are derived without specifying a particular distribution of the stochastic disturbances underlying the economy. The other side of the coin is that our framework is deliberately stylized, and obviously abstracts from many crucial considerations such as the role of international capital flows or cost-push shocks. In fact, the spirit of our exercise is to provide a first step toward a welfare-based exploration of which dimensions of monetary policy can be regarded, specifically, as international.

The paper is organized as follows. Section 2 introduces the model. Section 3 analyzes optimal policy. We first derive a choice-theoretic policy loss function and consider three equivalent representations. Next, we characterize optimal rules and discuss their implications. In Section 4 we consider some extensions and corollaries of the baseline model, assessing the case for international monetary cooperation, revisiting the ‘rules-vs.-discretion’
debate for an open economy, and modifying price-setting. Section 5 concludes.

2 The model

2.1 Preferences

We model a world economy with two countries, \( H \) (Home) and \( F \) (Foreign), each specialized in one type of tradable good. Each good is produced in a number of brands defined over a continuum of unit mass. Brands are indexed by \( h \) in the Home country and \( f \) in the Foreign country. Each country is populated by households that are immobile across borders. Households are defined over a continuum of unit mass. They are indexed by \( j \) in the Home country and \( j^* \) in the Foreign country.

Home agent \( j \)'s lifetime expected utility \( U \) is defined as:

\[
U_t(j) \equiv E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \ln C_{\tau}(j) + \chi \ln \frac{M_{\tau}(j)}{P_{\tau}} - \kappa \ell_{\tau}(j) \right] \quad \chi, \kappa > 0
\]

where \( \beta < 1 \) is the discount rate, \( C(j) \) is consumption, \( M(j)/P \) real balances, and \( \ell(j) \) labor effort.\(^2\) Below, we will refer to \( W_t \) as the component of \( U_t \) that does not depend on real balances. Foreign agents' preferences are similarly defined: the discount rate \( \beta \) is the same as in the Home country, while \( \chi^* \) and \( \kappa^* \) in the Foreign country need not coincide with \( \chi \) and \( \kappa \) in the Home country.

For each household \( j \) in the Home country, the consumption indices of Home and Foreign brands are defined as:

\[
C_{H, t}(j) \equiv \left[ \int_0^1 C_t(h, j)^{\theta-1} dh \right]^\frac{1}{\theta-1}, \quad C_{F, t}(j) \equiv \left[ \int_0^1 C_t(f, j)^{\theta^*-1} df \right]^\frac{1}{\theta^*-1} \quad \theta, \theta^* > 1
\]

where \( C_t(h, j) \) and \( C_t(f, j) \) are respectively consumption of Home brand \( h \) and Foreign brand \( f \) by Home agent \( j \) at time \( t \). Each Home brand is an imperfect substitute for all

\(^2\)Algebraic details aside, all the results of the paper go through when a non-linear specification of labor disutility is introduced.
other Home brands, with constant elasticity of substitution $\theta$. Similarly the elasticity of substitution among Foreign brands is $\theta^*$. The consumption indices in the Foreign country, $C^*_H,t(j^*)$ and $C^*_F,t(j^*)$, are analogously defined.

Consistent with the idea that each country specializes in the production of a single type of good, the elasticity of substitution among goods produced in one country should not be lower than the elasticity of substitution across goods produced in different countries. Specifically, while both $\theta$ and $\theta^*$ are greater than one, we assume that the elasticity of substitution between Home and Foreign types is one. Under this assumption the consumption baskets of individuals $j$ and $j^*$ can be written as Cobb-Douglas functions of the Home and Foreign consumption indices:

$$C_t(j) \equiv C_{H,t}(j)^\gamma C_{F,t}(j)^{1-\gamma}, \quad C^*_t(j^*) \equiv C^*_{H,t}(j^*)^\gamma C^*_{F,t}(j^*)^{1-\gamma} \quad 0 < \gamma < 1 \quad (3)$$

where the weights $\gamma$ and $1-\gamma$ are identical across countries.

We denote the prices of brands $h$ and $f$ in the Home market (thus expressed in the Home currency) as $p_t(h)$ and $p_t(f)$, and the prices of brands $h$ and $f$ in the Foreign market (in Foreign currency) as $p^*_t(h)$ and $p^*_t(f)$. The utility-based price of a consumption bundle of domestically produced goods, denoted $P_{H,t}$, is derived as:

$$P_{H,t} = \left[\int_0^1 p_t(h)^{1-\theta} dh\right]^{1-\gamma}. \quad (4)$$

Similarly we derive the other price indices $P_{F,t}$, $P^*_H,t$, and $P^*_F,t$, as well as the utility-based CPIs:

$$P_t = \frac{P_{H,t}^{1-\gamma}}{\gamma_W}, \quad P^*_t = \frac{(P^*_H,t)^\gamma (P^*_F,t)^{1-\gamma}}{\gamma_W} \quad \gamma_W \equiv \gamma^\gamma (1-\gamma)^{1-\gamma}. \quad (5)$$

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3 The utility based price index $P_{H,t}$ is defined as the minimum expenditure required to buy one unit of the composite good $C_H$, given the prices of the brands.
2.2 Technology and resource constraints

Each brand $h$ is produced by a single Home firm and sold in both countries under conditions of monopolistic competition. Output $Y$ is produced with the following linear technology:

$$Y_t(h) = \frac{\ell_t(h)}{\alpha_t}$$  \hspace{1cm} (6)

where $\ell_t(h)$ is firm $h$’s labor demand and $\alpha_t$ a country-specific productivity shock. The resource constraint for brand $h$ is:

$$Y_t(h) \geq \int_0^1 C_t(h, j) dj + \int_0^1 C_t^*(h, j^*) dj^*.$$  \hspace{1cm} (7)

Home firms take the nominal price of labor, $W_t$, as given. The nominal marginal cost, $MC_t$, is identical across firms:

$$MC_t(h) = MC_t = \alpha_t W_t$$  \hspace{1cm} (8)

and Home firms’ nominal profits $\Pi_t$ are defined as:

$$\Pi_t(h) = (p_t(h) - MC_t) \int_0^1 C_t(h, j) dj + (E_t p_t^*(h) - MC_t) \int_0^1 C_t^*(h, j^*) dj^*$$  \hspace{1cm} (9)

where $E$ is the nominal exchange rate, expressed as Home currency per unit of Foreign currency. In the Home country’s labor market we have:

$$\int_0^1 \ell_t(j) dj \geq \int_0^1 \ell_t(h) dh$$  \hspace{1cm} (10)

Foreign variables are similarly defined.

2.3 Budget constraints and consumer optimization

Home households hold the portfolio of Home firms, the Home currency, $M$, and two international bonds, $B$ and $B^*$, denominated in Home and Foreign currency, respectively. They receive wages and profits from the firms and pay non-distortionary (lump-sum) net taxes $T$,
denominated in Home currency. The individual flow budget constraint\(^4\) for agent \(j\) in the Home country is:

\[
M_t(j) + B_{t+1}(j) + E_tB^*_t(j) \leq M_{t-1}(j) + (1 + i_t)B_t(j) + (1 + i^*_t)E_tB^*_t(j)
\]

\[
+ W_t\ell_t(j) + \int_0^1 \Pi_t(h)dh - T_t(j) - \int_0^1 p_t(h)C_t(h,j)dh - \int_0^1 p_t(f)C_t(f,j)df
\]

(11)

In the expression above, the nominal yields \(i_t\) and \(i^*_t\) are paid at the beginning of period \(t\) and are known at time \(t - 1\).

Taking prices and wages as given, Home agent \(j\) maximizes (1) subject to (11) with respect to consumption, labor effort, and asset holdings. Two optimality conditions play a key role in what follows and are worth highlighting here. First, agent \(j\)’s demand for brands \(h\) and \(f\) is a function of the relative price and total consumption of Home and Foreign goods, respectively:

\[
C_t(h,j) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}(j), \quad C_t(f,j) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\theta^*} C_{F,t}(j)
\]

(12)

and, similarly, the demand for Home and Foreign consumption goods is a constant fraction of agent \(j\)’s total consumption expenditure:

\[
P_tC_t(j) = \frac{1}{\gamma} P_{H,t}C_{H,t}(j) = \frac{1}{1 - \gamma} P_{F,t}C_{F,t}(j).
\]

(13)

Second, the intertemporal allocation is determined according to the Euler equation:

\[
1 = (1 + i_{t+1}) E_tQ_{t,t+1}(j)
\]

(14)

where \(Q_{t,t+1}(j)\) is agent \(j\)’s stochastic discount rate:

\[
Q_{t,t+1}(j) = \beta \frac{P_tC_t(j)}{P_{t+1}C_{t+1}(j)}.
\]

(15)

Note that the condition for optimal labor effort requires that:

\[
W_t = \kappa P_tC_t(j)
\]

(16)

\(^4\)We adopt the notation of Obstfeld and Rogoff [1996, ch.10]. Specifically, our timing convention has \(M_t(j)\) as agent \(j\)’s nominal balances accumulated during period \(t\) and carried over into period \(t + 1\), while \(B_t(j)\) and \(B^*_t(j)\) denote agent \(j\)’s bonds accumulated during period \(t - 1\) and carried over into period \(t\).
implying that consumption and discount rates are equalized across agents:

\[ Q_{t,t+1}(j) = Q_{t,t+1}. \quad (17) \]

Since only Home residents hold Home money and there is no government spending,\(^5\) the public budget constraint in the Home country implies that seigniorage revenue is rebated to domestic households in a lump-sum fashion:

\[ \int_0^1 (M_t(j) - M_{t-1}(j)) \, dj + \int_0^1 T_t(j) \, dj = 0. \quad (18) \]

Similar expressions hold in the Foreign country. Finally, international bonds are in zero net supply:

\[ \int_0^1 B_t(j) \, dj + \int_0^1 B_t(j^*) \, dj^* = \int_0^1 B_t^*(j) \, dj + \int_0^1 B_t^*(j^*) \, dj^* = 0. \quad (19) \]

### 2.4 Monetary policy

The Home government controls the path of short-term rates \(i\), providing a nominal anchor for market expectations. In our model, it is analytically convenient to introduce a forward-looking measure of monetary stance, \(\mu_t\), such that:

\[ \frac{1}{\mu_t} = \beta(1 + i_{t+1}) E_t \left( \frac{1}{\mu_{t+1}} \right) \quad (20) \]

or, integrating forward:

\[ \frac{1}{\mu_t} = E_t \lim_{N \to \infty} \beta^N \frac{1}{\mu_{t+N}} \prod_{\tau=0}^{N-1} (1 + i_{t+\tau+1}). \quad (21) \]

In a non stochastic steady state, \(\mu_{t+1}/\mu_t\) determines the (gross) inflation target, say \(\pi\), and the steady-state nominal interest rate is \(1 + i = \pi/\beta\). Note that under price level targeting, \(\pi = 1\). Home monetary easing at time \(t\) (\(\mu_t\) temporarily above trend) reflects either a temporary interest rate cut at time \(t\) (i.e., \(1 + i_{t+1} < \pi/\beta\)), or expectations of temporary interest rates cuts sometime in the future. Similar expressions hold for the Foreign country.

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\(^5\)For an extension of the model dealing with fiscal interdependence and the interaction between monetary and fiscal policies see Corsetti and Pesenti [2001].
2.5 Producer optimization and price setting

Individual firms set nominal prices one period in advance, and stand ready to meet (domestic and foreign) demand at given prices for one period. The number of producers is large enough so that firms ignore the impact of their pricing decisions on the aggregate price indices.

Consider first internal price-setting. Home firms selling in the Home market choose \( p_t(h) \) at time \( t-1 \) by maximizing the present discounted value of profits, that is:

\[
\max_{p_t(h)} E_{t-1} Q_{t-1, t} \Pi_t(h),
\]

accounting for (12). Domestic firms optimally set prices equal to expected nominal marginal cost, appropriately discounted and augmented by the equilibrium markup \( \theta / (\theta - 1) \):

\[
p_t(h) = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left( Q_{t-1, t} p_t(h)^{-\theta} P^\theta H_t C_{H, t} M C_t \right)}{E_{t-1} \left( Q_{t-1, t} p_t(h)^{-\theta} P^\theta H_t C_{H, t} \right)}.
\]

Accounting for (13), (15), and (17) the previous expression can be rewritten as:

\[
p_t(h) = P_{H, t} = \frac{\theta}{\theta - 1} E_{t-1} M C_t = \frac{1}{\Phi} E_{t-1} (\alpha_t P_t C_t)
\]

where we define \( \Phi \equiv (\theta - 1) / \theta \). As we will see below, the constant \( \Phi \) measures the expected level of labor effort in the Home country.

Price setting in the export markets is more complex, as it raises the issue of how prices abroad react to exchange rate movements. Empirically we know little about exchange rate pass-through on import (export) prices. We know that it is on average below 1, that it varies across sectors and countries, and is different for consumer goods and wholesale prices.\(^6\) Considered as a decision variable of the exporter, the determinants of exchange rate pass-through may clearly include some of the variables considered in our model — such as the volatility of monetary and real shocks, as suggested by Taylor [2000] and analyzed by Corsetti and Pesenti [2002] in the context of optimal monetary rules, and by Devereux, \(^6\)

\[^6\]See the discussion in Goldberg and Knetter [1997] and Obstfeld and Rogoff [2000]. Campa and Goldberg [2002] provide updated estimates of exchange rate pass-through across countries.
Engel and Storgaard [2003] in non-optimizing models. But it may reasonably depend on many other factors outside the scope of our contribution — such as the exporter-importer working relationship as stressed by the relationship-marketing literature, the presence of distribution costs as in Corsetti and Dedola [2002] and Laxton and Pesenti [2003], the size of the market share as in Bacchetta and Van Wincoop [2002], or the availability of financial strategies to limit exposure of exporters’ profits to exchange rate fluctuations as in Friberg [1998].

Many contributions link the degree of pass-through to the invoice currency in the presence of nominal rigidities. If exports are invoiced in the producer’s currency, the Foreign-currency price of brand $h$ is determined by the law of one price and exchange rate pass-through is 100 percent (a scenario usually referred to as ‘Producer Currency Pricing’ or PCP). If instead exports are invoiced in the importer’s currency, Foreign-currency prices are sticky regardless of exchange rate movements and pass-through is zero (‘Local Currency Pricing’ or LCP).  

This paper follows an approach similar to the latter group of studies, assuming that the degree of export price indexation to the exchange rate is constant within the period (albeit potentially time-varying from period to period) and across producers. In our model, however, we adopt a more flexible approach and, as a descriptive device, we let such elasticity vary between 0 and 1. We are therefore able to obtain PCP and LCP as particular cases of a

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unified parameterization.\textsuperscript{8} Our approach should be viewed only as a crude approximation to the actual determinants of markup exposure to currency movements, whose endogenization should be eventually incorporated in a self-contained model. It remains unclear, however, whether a model that accounts for a large number of complications from which we abstract here would add much to our qualitative conclusions on monetary policy interdependence.

Formally, by definition of pass-through elasticity $\eta^* \equiv \partial \ln p^*_t(h)/\partial \ln (1/E_t)$, Foreign-currency prices of Home brands are:

$$p^*_t(h) = \frac{\tilde{p}_t(h)}{\xi_t^{\eta^*}} \quad 0 \leq \eta^* \leq 1 \quad (25)$$

where $\tilde{p}_t(h)$ is the predetermined component of the Foreign-currency price of good $h$ that is not adjusted to variations of the exchange rate during period $t$.\textsuperscript{9} Home firms choose $\tilde{p}_t(h)$ one period in advance at time $t-1$ in order to maximize $E_{t-1}Q_{t-1}\Pi_t(h)$, while the actual $p^*_t(h)$ depends on the realization of the exchange rate at time $t$.\textsuperscript{10} The solution is:

$$p^*_t(h) = \frac{\theta}{\theta - 1} \frac{1}{\xi_t^{\eta^*}} E_{t-1} \left( Q_{t-1,1}p^*_t(h)^{-\theta} P_{t,t}^\alpha C_{t,t}^\alpha MC_t \right) \frac{\theta}{\theta - 1} \frac{1}{\xi_t^{\eta^*}} E_{t-1} \left( Q_{t-1,1}p^*_t(h)^{-\theta} P_{t,t}^\alpha C_{t,t}^\alpha \xi_t^{1-\eta^*} \right). \quad (26)$$

Accounting for (13), (15), and (17), and letting $\Theta_t \equiv \gamma/(1 - \gamma) (P^*_t C_t / \xi_t^{1-\eta^*})$, we can also write:

$$P^*_{t,t} = \frac{\theta}{\theta - 1} \frac{1}{\xi_t^{\eta^*}} E_{t-1} \left( \Theta_t MC_t / \xi_t^{1-\eta^*} \right) = \frac{1}{\Phi} \left( \frac{\theta}{\theta - 1} \frac{1}{\xi_t^{\eta^*}} E_{t-1} \left( \Theta_t \alpha_t P_t C_t / \xi_t^{1-\eta^*} \right) \right). \quad (27)$$

Analogous expressions can be derived for the prices set by Foreign firms in the Foreign and the Home market (where the pass-through is denoted $\eta$).

\textsuperscript{8}In Section 4 below we account for alternative specifications of nominal rigidities.

\textsuperscript{9}For instance, if $\eta^* = 1$, pass-through in the Foreign country is complete — as in the PCP case. If $\eta^* = 0$, we have $p^*_t(h) = \tilde{p}_t(h)$ which coincides with the price chosen by the Home producer in the LCP case.

\textsuperscript{10}Although in our model the pass-through is constant, it could vary over time in a more general specification. Also, the model could be easily extended to encompass the case in which the pass-through elasticity is a non-linear function of the exchange rate (e.g., $\eta^*$ is close to zero for small changes of the exchange rate $E$ but close to one for large exchange rate fluctuations). The key results of our analysis would remain unchanged.
Clearly, Home firms are willing to supply goods at given prices as long as their \textit{ex-post} markup does not fall below one:

\[ P_{H,t} \geq MC_t, \quad P_{H,t}^* \geq \frac{MC_t}{\mathcal{E}_t}. \]  

(28)

Otherwise, agents would be better off by not accommodating shocks to demand. In what follows, we restrict the set of shocks so that the ‘participation constraint’ (28) and its Foreign analog are never violated.

### 2.6 The closed-form solution of the model

In what follows we drop the indices \( j \) and \( j^* \) and interpret all variables in per-capita (or aggregate) terms. We solve the model under the assumption that, at some initial time \( t_0 \), net non-monetary wealth is zero in each country, that is, \( B_{t_0} = B^*_{t_0} = 0 \). Then, in equilibrium, non-monetary wealth is zero at any subsequent point in time: Home imports are always equal in value to Foreign imports.\(^{11}\)

Table 1 presents the general solution of the model. All endogenous variables (29) through (41) are expressed in closed form as functions of real shocks \((\alpha_t, \alpha^*_t)\) and the Home and Foreign monetary stances \(\mu_t\) and \(\mu^*_t\). This solution does not depend on specific assumptions or restrictions on the nature of the shocks, as long as the participation constraints (28) are not violated.

Crucial to interpreting Table 1 is the fact that, in equilibrium, \(\mu_t\) is equal to \(W_t/\kappa\) and \(P_tC_t\) (and \(\mu^*_t = W^*_t/\kappa^*\) is equal to \(P^*_tC^*_t\)): a monetary expansion raises the wage rate and nominal spending.\(^{12}\) Since the current account is always balanced in equilibrium, and the consumption of imports is proportional to nominal spending, the nominal exchange rate \(\mathcal{E}_t\)

\(^{11}\)This stems from the combination of three hypotheses: (i) Cobb-Douglas consumption indexes; (ii) logarithmic consumption preferences; (iii) zero initial non-monetary wealth. As shown in Corsetti and Pesenti [2001], under PCP the result also holds when assumption (ii) is relaxed.

\(^{12}\)This result can be obtained by comparing the Home Euler equation under logarithmic utility, \(1 = \beta (1 + i_{t+1}) E_t (P_tC_t/P_{t+1}C_{t+1})\), with (20).
Table 1: The closed-form solution of the model

\[ \mathcal{E}_t = \frac{1 - \gamma}{\gamma} \frac{\mu_t}{\mu_t^*} \]  

(29)

\[ MC_t = \kappa \alpha_t \mu_t \]  

(30)

\[ MC_t^* = \kappa^* \alpha_t^* \mu_t^* \]  

(31)

\[ M_t = \frac{\chi_t \mu_t}{1 - \beta \mu_t \mathcal{E}_t (\mu_t + 1)} \]  

(32)

\[ M_t^* = \frac{\chi_t^* \mu_t^*}{1 - \beta \mu_t^* \mathcal{E}_t (\mu_t^* + 1)} \]  

(33)

\[ P_{H,t} = \frac{\theta}{\theta - 1} E_{t-1} (MC_t) \]  

(34)

\[ P_{F,t} = \frac{\theta^*}{\theta^* - 1} E_{t-1} \left( \frac{MC_t}{\mathcal{E}_{t-1}^1} \right) \]  

(35)

\[ P_{H,t}^* = \frac{\theta}{\theta - 1} \frac{\mathcal{E}_{t-1}^*}{E_{t-1}} \left( \frac{MC_t}{\mathcal{E}_{t-1}^1} \right) \]  

(36)

\[ P_{F,t}^* = \frac{\theta^*}{\theta^* - 1} E_{t-1} \left( \frac{MC_t}{\mathcal{E}_{t-1}^1} \right) \]  

(37)

\[ C_t = \gamma W \left( \frac{\theta - 1}{\theta} \right)^{\gamma} \left( \frac{\theta^* - 1}{\theta^*} \right)^{1-\gamma} \mu_t \mathcal{E}_t^{1-\eta} \]  

(38)

\[ C_t^* = \gamma W \left( \frac{\theta - 1}{\theta} \right)^{\gamma} \left( \frac{\theta^* - 1}{\theta^*} \right)^{1-\gamma} \mu_t^* \mathcal{E}_t^{\eta \gamma} \]  

(39)

\[ \ell_t = \Phi \left[ \gamma \frac{MC_t}{E_{t-1} (MC_t)} + (1 - \gamma) \frac{MC_t / \mathcal{E}_{t-1}^{1-\eta}}{E_{t-1} (MC_t / \mathcal{E}_{t-1}^{1-\eta})} \right] \]  

(40)

\[ \ell_t^* = \Phi^* \left[ (1 - \gamma) \frac{MC_t}{E_{t-1} (MC_t^*)} + \gamma \frac{MC_t^* / \mathcal{E}_{t-1}^{1-\eta}}{E_{t-1} (MC_t^* / \mathcal{E}_{t-1}^{1-\eta})} \right] \]  

(41)
in (29) is proportional to $P_t C_t / P_t^* C_t^*$, i.e. a function of the relative monetary stance.\textsuperscript{13} This implies $\Theta_t = 1$ in (27).

Domestic prices of domestic goods are predetermined according to (34) and (37), while import prices vary with the exchange rate, depending on the degree of pass-through according to (35) and (36). Given interest rates, money is determined residually according to (32) and (33). Equilibrium consumption is displayed in (38) and (39). Finally, employment levels are determined according to (40) and (41), whereas output levels are obtained as $\ell_t / \alpha_t$ and $\ell_t^* / \alpha_t^*$.

In a nutshell, in our model agents optimally predetermine prices so as to stabilize expected employment at the level $\Phi$ or $\Phi^*$ — that is, the equilibrium employment when prices are fully flexible:

$$ E_{t-1} \ell_t = \Phi, \quad E_{t-1} \ell_t^* = \Phi^*. $$

(42)

Consider the effects of an unanticipated rise in $\mu$. When the pass-through is high, consumption increases in both countries but the depreciation of the Home currency worsens the Home terms of trade, so that more units of Home labor are required to buy one unit of Home consumption. With a low degree of pass-through, instead, local prices are invariant to the exchange rate. For each unit of exports, Home exporters' sales revenue in domestic currency rises in proportion to the rate of depreciation. This additional revenue finances higher Home consumption, thus leading to higher production and imports. Foreign exporters now have to supply more goods while suffering a fall in revenue in terms of their own currency. Foreign consumption falls relative to Home consumption.\textsuperscript{14}

\textsuperscript{13}Note that the exchange rate is a forward-looking variable, as it depends on current and future expected changes in short-term nominal interest rates worldwide.

\textsuperscript{14}On the ‘beggar-thyself’ vs. ‘beggar-thy-neighbor’ effects of exchange rate depreciations see Corsetti and Pesenti [2001] and Tille [2001].
3 A framework for the analysis of monetary policy in an open economy

3.1 The goals of monetary policy

In this section, we consider the problem faced by a policymaker who seeks to maximize Home agents’ expected utility. In our analysis, we focus on the non-monetary components of utility — i.e. we assume that $\chi$ is arbitrarily small — adopting $E_{t-1} W_t$ as a measure of national welfare.\footnote{If real balance effects were considered, monetary policy could affect consumption and output even under flexible prices. However, these effects are unlikely to be very large from an empirical viewpoint. See for instance Cooley and Hansen [1989].}

As a first step, we show that in our framework the flex-price allocation provides a welfare benchmark. With flexible prices, the pricing equations (34) through (37) hold in any state of nature, not in expectation, so that pass-through is complete, the law of one price is valid and purchasing power parity holds. It is easy to verify that the flex-price level of employment is a constant depending on the sensitivity of labor effort disutility and the magnitude of domestic monopolistic distortions, $\Phi$ and $\Phi^*$, while consumption at Home and abroad is a function of global shocks and global monopolistic distortions.

Using the expressions in Table 1 under flexible and sticky prices, the gap between expected utility with flexible prices, indexed $W^{flex}$, and expected utility with predetermined prices is:

$$
E_{t-1} \left( W^{flex}_t - W_t \right) = E_{t-1} \left[ \gamma \ln \left( \frac{E_{t-1} (\alpha_t \mu_t)}{\alpha_t \mu_t} \right) + 
+ (1 - \gamma) \ln \left( \frac{E_{t-1} (\alpha_t^* (\mu_t)^{\eta} \mu_t^{1-\eta})}{\alpha_t^* (\mu_t^{\eta})^\eta \mu_t^{1-\eta}} \right) \right] \geq 0.
$$

(43)

By Jensen’s inequality, the above expression cannot be negative: expected utility with price rigidities is never above expected utility with flexible prices. It follows that, at best,
what monetary policy rules can do is to bridge the gap between the two. The expression
\[ E_{t-1} \left( W_{t}^{\text{flex}} - W_{t} \right) \]
thus provides a choice-theoretic policy loss function.

There are three equivalent representations of (43), each highlighting a different aspect
of the policy problem. First, it is straightforward to show that the policy loss function (43)
is equal to the expected value of the log average markup in the Home market, minus a
constant independent of monetary policy:

\[ E_{t-1} \left( W_{t}^{\text{flex}} - W_{t} \right) = E_{t-1} \left( \gamma \ln \frac{P_{H,t}}{MC_{t}} + (1 - \gamma) \ln \frac{P_{F,t}}{E_{t}MC_{t}} \right) - \text{const.} \]  \hspace{1cm} (44)

Since the logarithmic function is a concave transformation of its argument, Home welfare is
negatively related to the markups’ volatilities. Under the constraint of rational expectations,
Home policymakers cannot systematically resort to monetary surprises to push markups
below their flex-price equilibrium levels. They can, however, design policy rules aimed at
stabilizing (a CPI-weighthed average of) markups’ volatilities, by affecting either domestic
marginal costs \( MC \) or the exchange rate \( E \).

In general equilibrium, consumer prices will respond to such policy rules. Intuitively,
producers optimally set their monopoly prices given their expectations about monetary and
real fundamentals. The higher these prices, the lower the representative consumer’s welfare.
A second way to write the policy loss function is therefore in terms of consumer prices, with
and without nominal rigidities:

\[ E_{t-1} \left( W_{t}^{\text{flex}} - W_{t} \right) = E_{t-1} \ln \frac{P_{t}}{P_{t}^{\text{flex}}} \geq 0. \]  \hspace{1cm} (45)

For any given monetary stance, the consumer price index under sticky prices cannot be
expected to be lower than the CPI under flexible prices. This expression underlies the case
for price stability in an open economy: optimal monetary policy rules minimize the expected
deviation of the (log) CPI from a target given by the (log) equilibrium CPI under flexible
prices.

Finally, (43) can be expressed as a function of output gaps in the Home and Foreign
economy — defined as the distance between actual and desired employment levels, i.e. \( \ell/\Phi \)
and \( \ell^*/\Phi^* \) — and deviations from the law of one price in each market:

\[
E_{t-1} \left( W_{t}^{\text{flex}} - W_{t} \right) = E_{t-1} \left\{ \gamma \ln \frac{\ell_t}{\Phi} + (1 - \gamma) \ln \frac{\ell_t^*}{\Phi^*} + \gamma \left[ \ln \left( \frac{\gamma - (1 - \gamma) P_{H,t}}{\eta} \right) \right] + (1 - \gamma) \left[ \ln \left( \frac{\gamma + (1 - \gamma) P_{F,t}}{\eta^*} \right) \right] \right\}. \tag{46}
\]

If the law of one price holds, i.e. if \( P_H = E P_H^* \), and \( P_F = E P_F^* \) because \( \eta = \eta^* = 1 \), the two terms in square brackets disappear, and from (41) the Foreign output gap is independent of the exchange rate, thus unaffected by Home monetary policy. In this case the policy objective in open economy does not substantially differ from the closed-economy one, as stressed by Clarida, Gali and Gertler [2001]: Home welfare is maximized when Home optimal policy closes the domestic output gap and Home policymakers are only concerned with stabilizing the markups of domestic producers. As soon as the law of one price fails to hold exactly, however, the policy loss function for an interdependent economy is a function of domestic output gap, international cyclical conditions and exchange rate.

### 3.2 Monetary rules in a world Nash equilibrium

We now derive the optimal monetary stances in a world Nash equilibrium, where national policymakers are able to commit to preannounced rules. Assuming that policymakers can observe and respond to productivity shocks within the period, the policy problem faced by the Home monetary authority is to minimize the policy loss \( E_{t-1} \left( W_{t}^{\text{flex}} - W_{t} \right) \) in (43) with respect to \( \{\mu_t\}_{T=t}^\infty \), taking \( \{\mu^*_t, \alpha_t, \alpha^*_t\}_{T=t}^\infty \) as given. The Foreign authority faces a similar problem. Table 2 presents the reaction functions, respectively (47) for Home and (48) for Foreign, whose solution is the global Nash equilibrium.16 The reaction functions are expressed in three ways: as explicit functions of shocks and monetary policies; as implicit functions of shocks only; and as functions of monetary policies only. As is well known, in this class of models optimal monetary rules such as the reaction functions of Table 2 do not provide a nominal anchor to pin down nominal expectations. The issue can be addressed by assuming that governments set national nominal anchors (such as target paths for \( P_H \) and \( P_F^* \)) and credibly threaten to tighten monetary policy if the price of domestic goods deviates from the target. See e.g. Woodford [2003].
Table 2: Policy reaction functions under commitment

\[
1 - \eta(1 - \gamma) = \frac{\gamma \alpha_t \mu_t}{E_{t-1} (\alpha_t \mu_t)} + \frac{(1 - \gamma) (1 - \eta) \alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta}}{E_{t-1} \left( \alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right)}
\]

\[
= \frac{\gamma \ell_t}{\Phi} + \frac{(1 - \gamma) (1 - \eta) \ell_t}{\Phi^*} \gamma + (1 - \gamma) \frac{P_{H,t}}{E_{t}P_{H,t}} \gamma + (1 - \gamma) \frac{P_{F,t}}{E_{t}P_{F,t}}
\]

\[
= \gamma \frac{\theta}{\theta - 1} \frac{MC_t}{P_{H,t}} + (1 - \gamma) (1 - \eta) \frac{\theta}{\theta - 1} \frac{MC_t}{P_{F,t}}
\]

(47)

\[
1 - \eta^* \gamma = \frac{(1 - \gamma) \alpha_t^* \mu_t^*}{E_{t-1} (\alpha_t^* \mu_t^*)} + \frac{(1 - \eta^*) \alpha_t \mu_t^* (\mu_t^*)^{1-\eta^*}}{E_{t-1} \left( \alpha_t \mu_t^* (\mu_t^*)^{1-\eta^*} \right)}
\]

\[
= \frac{(1 - \gamma) \ell_t^*}{\Phi^*} + \frac{(1 - \eta^*) \ell_t}{\Phi} \frac{\gamma}{1 - \gamma} \frac{MC_t^*}{P_{H,t}} + \frac{\theta}{\theta - 1} \frac{MC_t^*}{E_{t}P_{H,t}}
\]

(48)

functions of output gaps and deviations from the law of one price; and as implicit functions of the markups.

An optimal policy requires that the Home monetary stance be eased (an increase in \(\mu_t\)) in response to a positive domestic productivity shock (a fall in \(\alpha_t\)). Monetary policy thus leans against the wind and moves to close the employment and output gaps opened by productivity improvements.\(^{17}\) But in general the optimal response of Home interest rates

\(^{17}\)In the absence of a policy reaction, a productivity shock would create both an output and an employment gap, lowering employment below \(\Phi\). Actual output would not change, but potential output, defined as the flex-price equilibrium output, would increase. Note that optimal monetary policy rules accommodate
to a Home productivity shock does not aim at closing the output gap completely and does not stabilize the domestic markup. Unless $\eta = 1$, at an optimum $\ell_t$ will not be equal to $\Phi$ and $MC_t$ will differ from $E_{t-1}MC_t$.

To understand this result, consider the expected real profits accruing to Foreign firms, $E_{t-1}Q_{t-1,t}^*\Pi_t^*(f)$. In general equilibrium, Home monetary policy $\mu_t$ affects this expression both through the exchange rate channel (29) and the demand side ($C_F$ in equation (13)). Substituting for the equilibrium expressions we obtain:

$$E_{t-1} \left( Q_{t-1,t}^* \Pi_t^*(f) \right) = E_{t-1} \left[ \Omega(.) - \Lambda(.) \frac{\alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta}}{\tilde{p}_t^*} \right]$$

(49)

where $\Omega(.) > 0$ and $\Lambda(.) > 0$ are expressions that do not depend on $\alpha_t$, $\mu_t$ or $\tilde{p}_t^*$. Suppose the Home authorities follow the inward-looking policy $\mu_t = 1/\alpha_t$, stabilizing domestic markups. For any $\eta \neq 1$, the above expression in square brackets is now a concave function of $\alpha$: other thing being equal, Home shocks lower Foreign exporters’ expected profits. In other words, with $\eta \neq 1$, Home stabilization policy induces fluctuations in the exchange rate and in world demand that add uncertainty to Foreign firms’ sales revenue from the Home market.

To see what prompts Foreign firms to charge higher average export prices in the Home market, consider the elasticity of Foreign real profits relative to the Home monetary stance $\mu$:

$$- \frac{\partial}{\partial \mu_t} \frac{\mu_t}{Q_{t-1,t}^* \Pi_t^*(f)} \bigg|_{\mu_t = 1/\alpha_t} = (1 - \eta) \frac{\Lambda \alpha_t^* (\mu_t^*)^\eta \alpha_t^{1-\eta}}{\Omega \tilde{p}_t^* - \Lambda \alpha_t^* (\mu_t^*)^\eta \alpha_t^{1-1}}.$$

(50)

When $\eta < 1$, this elasticity is decreasing in $\tilde{p}_t^*(f)$. Intuitively, charging a higher average price $\tilde{p}_t^*(f)$ is a way to reduce the sensitivity of profits to external shocks. But the higher average export prices charged by Foreign firms translate into higher average import prices in the Home country, reducing Home residents’ purchasing power and welfare. If Home

domestic productivity disturbances.

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18To wit, the expressions are $\Omega \equiv Q_t^* (\tilde{p}_t^*(f) - MC_t^*) C_{F,t}^* + \beta \gamma \mu_t^{1-1}$ and $\Lambda = \beta \gamma^\eta (1-\gamma)^{1-\eta} \kappa^* \mu_t^{1-1}$.

19If $\eta^* = 1$, this policy also closes the Home output gap.
authorities followed a different monetary policy, they could temper the negative impact of Home shocks on the expected profits of Foreign exporters, who would in turn lower their prices.

This is why a Home monetary stance designed without considering the impact of nominal demand and exchange rate volatility on the profits of Foreign exporters is not optimal. Relative to such a stance, domestic policymakers can improve utility by adopting a policy that equates, at the margin, the benefit from keeping domestic output close to its potential level with the loss from lower consumption, as prescribed by equation (47). Rather than stabilize only the markup of Home firms selling at Home, optimal monetary policy stabilizes a CPI-weighted average of the markups of all firms selling in the Home market.\footnote{Relative to inward-looking policies, the optimal policy rules translate into higher domestic prices, and therefore improve the domestic terms of trade. See Broda [2003] for an empirical assessment of the impact of suboptimal monetary rules on prices.}

As long as $\eta$ is below one, the optimal Home monetary stance tightens when productivity worsens abroad, loosens otherwise. Fluctuations in the costs of production abroad — reflecting changes in $\alpha^*$ — translate into uncertain profits for Foreign producers. If neither Foreign nor Home policymakers were expected to stabilize their markups, Foreign firms would charge higher average prices. It follows that low degrees of pass-through induce policymakers to react to country-specific shocks no matter where they originate, since these shocks have global repercussions.

By the same token, Home monetary authorities also react to shocks to $\mu^*$: an exogenous Foreign expansion lowers the markup on Foreign exports, requiring a Home monetary response to sustain Foreign firms' profits by appreciating the exchange rate. Thus, in general, there will be strategic interdependence among policymakers. The only cases in which Home monetary policy does not react to Foreign policy are either when $\eta = 0$ or when $\eta = 1$. In the case $\eta = 1$, there is nothing the Home authorities can do to reduce the volatility of Foreign firms' profits, since $P_F/E$ does not respond to Home interest rates. In the case $\eta = 0$,
instead, Foreign policy shocks do not affect Foreign exporters' markups since their marginal costs and sales revenue move in tandem \( (P_F \text{ does not respond to the current exchange rate}). \)

Once again, there is nothing for the Home authorities to stabilize, and Home interest rates do not respond to Foreign policy shocks.

### 3.3 Two important cases: PCP and LCP

Can monetary rules close the world utility gap (43) completely? In the presence of asymmetric shocks the answer is negative in all but one important case, PCP. With complete pass-through in both countries, the Nash equilibrium policy is:

\[
1 = \frac{\alpha_t \mu_t}{E_{t-1} (\alpha_t \mu_t)} \quad 1 = \frac{\alpha^*_t \mu^*_t}{E_{t-1} (\alpha^*_t \mu^*_t)}.
\]

Monetary authorities optimally stabilize marginal costs and markups, keeping employment at its flex-price level \( \Phi \) and \( \Phi^* \).

Domestic and global consumption optimally co-move with productivity shocks and the optimal monetary policies support the same allocation as with flexible prices, that is constrained Pareto-efficient as discussed in Obstfeld and Rogoff [2000].

This result provides an extreme version of the case for flexible exchange rates made by Friedman [1953]: even without price flexibility, monetary authorities can engineer the right adjustment in relative prices through exchange rate movements. In our model with PCP, expenditure-switching effects make exchange rate and price movements perfect substitutes.

The equivalence between Nash equilibrium and flex-price allocation need not go through under more general conditions, for instance with less restrictive preference specifications as shown by Benigno and Benigno [2003]. In the context of our model, the Nash equilibrium

\[21\] Note that, because of our utility parameterization, domestic monetary policy only responds to domestic shocks. This will not be true, for instance, with a power utility of consumption. Yet, even with power utility the Nash allocation under PCP coincides with the flex-price allocation as long as the consumption basket of Home and Foreign goods is Cobb-Douglas.
will not coincide with a flex-price equilibrium when the pass-through is less than perfect in either market. In the case of LCP, for instance, neither country’s optimal monetary policy will be inward-looking; in fact, they must be perfectly symmetric in their response to shocks anywhere in the world economy. With zero pass-through worldwide the system (47)-(48) can be written as:

\[ \mu_t = \left[ \gamma \frac{\alpha_t}{E_{t-1} (\alpha_t \mu_t)} + (1 - \gamma) \frac{\alpha^*_t}{E_{t-1} (\alpha^*_t \mu_t)} \right] - 1 \]  
\[ \mu^*_t = \left[ \gamma \frac{\alpha_t}{E_{t-1} (\alpha_t \mu^*_t)} + (1 - \gamma) \frac{\alpha^*_t}{E_{t-1} (\alpha^*_t \mu^*_t)} \right] - 1 \]  

For any given shock, consumption increases by the same percentage everywhere in the world economy. Labor effort instead responds asymmetrically to the shocks in the two countries, so that post-shock welfare at Home is not identical to welfare abroad, as is the case under PCP.

Since the exchange rate remains constant as long as \( \mu_t = \mu^*_t \), our analysis also suggests that — provided monetary authorities in the two countries adopt optimal stabilization rules — exchange rate volatility will be higher in a world economy close to purchasing power parity, and lower in a world economy where deviations from the law of one price are large. In fact, if the exposure of firms’ revenue to exchange rate fluctuations is limited, welfare-optimizing policymakers assign high priority to stabilizing domestic output and prices, regardless of the impact on exchange rate movements. Otherwise, they must take into account the repercussions of exchange rate volatility on import prices; hence, the monetary stances in the world economy become more symmetric, reducing currency fluctuations.

4 Extensions

In this section we extend our model in three directions. First, we revisit the case for international monetary cooperation, and show that the magnitude of gains from cooperation are related in a non-linear way to the degree of pass-through. Next, we show that com-
mitment is superior to discretion even when discretionary policies do not suffer from any inflationary (or deflationary) bias. Finally, we show that our main conclusions generalize to economies with different price adjustment mechanisms — therefore checking the robustness of our results.

4.1 International monetary cooperation

Are there welfare gains from cooperating in the design and implementation of optimal monetary rules? To address these issues, consider a binding cooperative agreement between the two countries such that the policymakers jointly minimize the objective function

$$E_{t-1} \left[ \xi \left( W^f_{t} - W_t \right) + (1 - \xi) \left( W^*_{f} - W^*_t \right) \right],$$

where $\xi \in (0, 1)$ is a constant parameter indexing the bargaining power of the Home country.\(^{22}\) The first order conditions with respect to $\mu_t$ and $\mu^*_t$ are reported in Table 3, expressed in terms of markups for notational simplicity.

To verify whether there are gains from cooperation, we can compare the optimal cooperative policy defined by the system (54)-(55) in Table 3 with the Nash equilibrium (47)-(48) in Table 2. It is straightforward to verify that the two systems are identical — and therefore there are no welfare gains from cooperation — in three cases.

The first — and well known — case is when all shocks are global, i.e. $\alpha_t = \alpha^*_t$ for any $t$. The optimal Nash policies can be written as in (51) with $\alpha_t = \alpha^*_t$, implying a constant exchange rate. Consumption and employment coincide with the flex-price allocation.

The other two cases are PCP, as discussed in Obstfeld and Rogoff [2002], and LCP, as emphasized by Devereux and Engel [2003]. Under the extreme assumptions of either complete or zero pass-through, the allocation remains the same whether or not monetary authorities cooperate, and regardless of the value of $\xi$. We have established above that these are precisely the cases in which monetary policies are strategically independent of

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\(^{22}\) A country’s bargaining power is not necessarily related to its economic size, so that $\xi$ need not be equal to $\gamma$. 

23
Table 3: Policy rules under cooperation

\[
\begin{align*}
\xi [1 - \eta (1 - \gamma)] + (1 - \xi) \gamma \eta^* &= \xi \gamma \frac{\theta}{\theta - 1} \frac{MC_t}{P_{H,t}} \\
+ \xi (1 - \gamma) (1 - \eta) \frac{\theta^*}{\theta^* - 1} \frac{MC_t^*}{P_{E,t}} + (1 - \xi) \gamma \eta^* \frac{\theta}{\theta - 1} \frac{MC_t^*}{P_{H,t}} \\
(1 - \xi) (1 - \eta^* \gamma) + \xi (1 - \gamma) \eta &= (1 - \xi) (1 - \gamma) \frac{\theta^*}{\theta^* - 1} \frac{MC_t^*}{P_{E,t}} \\
+ (1 - \xi) \gamma (1 - \eta^*) \frac{\theta}{\theta - 1} \frac{MC_t^*}{P_{H,t}} + \xi (1 - \gamma) \eta \frac{\theta^*}{\theta^* - 1} \frac{MC_t^*}{P_{E,t}}
\end{align*}
\] (54)

(55)

each other: in either case, there are no policy spillovers in equilibrium.

In the case of PCP, the Nash equilibrium coincides with the flex-price allocation. This implies that the distortions induced by nominal rigidities and the monopoly power of a country on its terms of trade have no welfare consequences: the utility level is the same in each country, both \textit{ex-ante} and \textit{ex-post}. As mentioned above, this result need not go through under more general preference specifications (a point developed in Benigno and Benigno [2003]).

The absence of gains from cooperation in the LCP case is less straightforward. Crucial to this result is the fact that, whether or not monetary authorities cooperate, the optimal monetary policy in each country responds symmetrically to the same average of real shocks, implying that the equilibrium exchange rate is constant. Since exchange rate fluctuations are the only source of international spillover, there cannot be gains from cooperation under LCP, the case in which non-cooperative monetary rules already imply stable exchange rates!
Besides such extreme cases, a country can in general do better than simply ‘keeping its own house in order’ by engaging in binding international agreements. Whether these gains are sizable, however, remains an open issue.

4.2 Commitment vs. discretion in open economy

So far we have assumed that policymakers worldwide are able to commit to optimal policy rules. In this section, we compare the policy problem under commitment with the policy problem under discretion. We do not attempt to characterize the equilibrium under discretion. Instead, we limit our analysis to a discussion of inflationary bias in open economy, identifying the determinants of its magnitude and sign. We show that the optimal policy is not time-consistent, even in the absence of an inflationary or deflationary bias.

Under discretion, the Home policymaker minimizes the loss function \( W^f_{t} - W_t \) with respect to \( \mu_t \) after observing the shocks \( \alpha_t \) and \( \alpha^*_t \), taking firms’ prices and Foreign policy as given. The Foreign policymaker solves a similar problem. The first order conditions are reported in Table 4.

In a closed economy, monopolistic distortions lend an inflationary bias to discretionary monetary policy. This can be seen by evaluating (56) at \( \gamma = 1 \):

\[
\frac{\theta}{\theta - 1} = \frac{\alpha_t \mu_t}{E_{t-1} (\alpha_t \mu_t)}.
\]

(58)

Given expectations \( E_{t-1} (\alpha_t \mu_t) \), the Home policymaker has an ex-post incentive to increase the monetary stance \( \mu_t \) for any realized level of \( \alpha_t \). A surprise stimulus to the demand for Home products pushes Home labor effort and Home prices above their expected levels.

This need not be true in an open economy, as analyzed in Corsetti and Pesenti [2001]. From (56) it is clear that a discretionary policy will be subject to either inflationary or deflationary bias in the Home country depending on the sign of the inequality:

\[
1 - \eta (1 - \gamma) \geq \left[ \gamma + (1 - \gamma) \eta^* \right] \frac{\theta - 1}{\theta}.
\]

(59)
Table 4: Policy reaction functions under discretion

\[
\frac{1 - \eta(1 - \gamma)}{(\theta - 1)/\theta} = \frac{\gamma \alpha_t \mu_t}{E_{t-1}(\alpha_t \mu_t)} + \frac{(1 - \gamma) \eta^\ast \alpha_t (\mu_t)^{1 - \eta^\ast} \mu_t^\ast}{E_{t-1}(\alpha_t (\mu_t)^{1 - \eta} \mu_t^\ast)}
\]
\[
= \gamma \frac{\theta}{\theta - 1} \frac{MC_t}{P_{H,t}} + (1 - \gamma) \eta^\ast \frac{\theta}{\theta - 1} \frac{MC_t}{E_t P_{H,t}^\ast}
\] (56)

\[
\frac{1 - \eta^\ast \gamma}{(\theta^\ast - 1)/\theta^\ast} = \frac{(1 - \gamma) \alpha_t^\ast \mu_t^\ast}{E_{t-1}(\alpha_t^\ast \mu_t^\ast)} + \frac{\gamma \eta^\ast \alpha_t^\ast (\mu_t^\ast)^{1 - \eta} (\mu_t^\ast)^{\eta}}{E_{t-1}(\alpha_t^\ast \mu_t^\ast)}
\]
\[
= (1 - \gamma) \frac{\theta^\ast}{\theta^\ast - 1} \frac{MC_t^\ast}{P_{F,t}^\ast} + \gamma \eta^\ast \frac{\theta^\ast}{\theta^\ast - 1} \frac{E_t MC_t^\ast}{P_{F,t}}
\] (57)

Under discretion, there is an inflationary bias in Home monetary policy if the economy is sufficiently ‘large’, in the sense that a large share of world consumption falls on Home products. Since the exchange rate affects the price of a relatively small share of consumption goods, policymakers are less concerned with adverse import price movements than with the distortions associated with monopoly power in production.

The reverse is true when \( \gamma \) is sufficiently small and \( \eta \) and \( \eta^\ast \) sufficiently large. A monetary expansion, while raising output and employment, also increases the price of a substantial proportion of consumption goods. The terms of trade movement becomes the dominant concern in discretionary policy making, leading to a deflationary bias. Reducing the degree of pass-through blunts the adverse terms of trade effects of monetary expansion. For given \( \gamma, \theta, \) and \( \eta^\ast \), a lower pass-through in the domestic market (i.e. a lower \( \eta \)) is associated with a stronger inflationary bias.\(^{23}\)

\(^{23}\)Recent literature provides evidence of a negative relationship between openness and inflation in a large
Of course, if the government were able to use an appropriate fiscal instrument to offset the impact of monopolistic distortions, any inflationary/deflationary bias in policy making could be appropriately corrected and eq. (59) — appropriately modified to account for the subsidy or tax — would hold as an equality. One might expect that the first order conditions of the policy problems under discretion would then coincide with the first order conditions under commitment. Indeed, in the PCP case \( \eta = \eta^* = 1 \), the optimal policy under discretion coincides with the optimal policy under commitment, as can be verified by comparing (47) with (56) adjusted for the fiscal instrument. But this property holds only in this very special case, as firms’ profits are not exposed at all to exchange rate variability. In all other cases, discretionary policies do not coincide with policies under commitment.

Let us examine the reason why. Under discretion, for given domestic and import prices, Home policymakers respond to shocks without taking into account the effects of domestic monetary policy on the markup of producers abroad. As analyzed before, the variability of the Foreign exporters’ markup will be reflected into a higher average level of Home import prices. Under commitment, instead, Home policymakers take into account the impact of their actions on Foreign markups and prices. Thus, they adjust the Home monetary response to shocks so as to contain the effects of exchange rate variability on the income of Foreign producers, giving up to some extent the stabilization of Home producers’ income. It is worth emphasizing that, under discretion, Home monetary policy attempts to tilt \textit{ex-post} the external terms of trade in favor of domestic producers, and the discretionary stance is a cross-section of countries (Romer [1993], Campillo and Miron [1997], Lane [1997], and Cavallari [2001]). In light of our contribution these empirical findings can be interpreted as evidence that the incentive to adopt expansionary monetary policies is lower in economies that are more open, and therefore more likely to suffer from the price effects of a depreciating currency, after controlling for differences in national degrees of pass-through.

The result of over-stabilization under discretion complements and extends a similar result discussed in the context of closed-economy models with staggered price adjustments by firms and inflation dynamics, as summarized in Gali [2002].
function of \( \eta^* \), the degree of pass-through in the Foreign market that is relevant for Home exporters. Instead, under commitment Home policymakers do not attempt to manipulate the terms of trade, and the monetary rule is a function of \( \eta \), the degree of pass-through in the Home market that is relevant for Home consumers.

4.3 Partial adjustment of prices

In our baseline model there is an explicit asymmetry in price adjustment. While domestic prices are pre-set one period in advance, export prices can be altered to a certain extent in response to exchange rate fluctuations. This descriptive device is obviously open to the criticism that if prices can be altered for the export market they ought to be altered in the domestic market as well. In this section we revisit our model under a different price-setting setup, and assume that all prices are initially set in the consumer’s currency, but they can be partially adjusted in response to shocks. In other words, we now emphasize the implications of imperfect pass-through of marginal costs into prices in general, rather than imperfect pass-through of exchange rates into export prices.

To facilitate the comparison with the above framework we assume that, contingent on the realization of the shocks, Home firms can adjust their domestic prices by a given fraction \( \phi \) and their export prices by a fraction \( \phi^* \). Formally, this implies that domestic prices are partially indexed to their flexible levels:

\[
p_t(h) = P_{H,t} = p_t'(h)^{1-\phi} p_t^{fex}(h)\phi
\]

where \( p_t'(h) \) is the sticky component of the price.\(^{25}\) Similarly, exports are invoiced in the

\(^{25}\)This equation is formally different from the standard Calvo adjustment. According to the latter, only a fraction \( \phi \) of firms are able to adjust their prices, implying that, in the aggregate, \( P_{H,t} = \left[ (1-\phi)p_t^{1-\theta} + \phi(p_t^{fex})^{1-\theta} \right]^{1/(1-\theta)} \). Note however that, for sufficiently small levels of \( \theta \) or for small shocks, the two formalizations are isomorphic.
currency of the importer but prices can be partially modified:

\[ p^*_i(h) = P^*_{H,t} = p^*_t(h)^{1-\phi^*}p^{*\text{flex}}_t(h)^{\phi^*}. \] (61)

It is then straightforward to show that, in equilibrium:

\[ P^*_{H,t} = \frac{\theta}{\theta - 1} MC^*_{i}E_{t-1}MC^*_i E_{t}^{1-\phi} \] (62)

and:

\[ P^*_{H,t} = \frac{\theta}{\theta - 1} \left( \frac{MC_i}{E_t} \right)^{\phi^*} E_{t-1} \left( \frac{MC_i}{E_t} \right)^{1-\phi^*} \] (63)

which are two self-explanatory equations. Similar expressions characterize Foreign firms’ prices, after denoting \( \zeta^* \) the degree of adjustment of \( P^*_F \) and \( \zeta \) the degree of adjustment of \( P^*_E \).

Under the new price setting assumptions, the optimal monetary stance in the Home country is implicitly defined by:

\[ 1 - \gamma \phi - (1 - \gamma) \zeta = \gamma (1 - \phi) \frac{MC^1_{i}}{E_{t-1}} \left( \frac{MC^1_{i}}{E_{t}} \right)^{\phi^*} + (1 - \gamma) (1 - \zeta) \frac{(MC^*_{i}E_t)^{1-\zeta}}{E_{t-1}} \left( \frac{(MC^*_{i}E_t)^{1-\zeta}}{E_{t}} \right). \] (64)

It is useful to compare the previous expression with two ‘inward-looking’ policies. A policy of domestic price stabilization would require:

\[ 1 = \frac{MC^1_{i}}{E_{t-1}} \left( \frac{MC^1_{i}}{E_{t}} \right)^{\phi^*} \] (65)

while a policy of domestic output stabilization would imply:

\[ 1 = \gamma \frac{MC^1_{i}}{E_{t-1}} \left( \frac{MC^1_{i}}{E_{t}} \right)^{\phi^*} + (1 - \gamma) \frac{(MC^*_{i}E_t)^{1-\zeta}}{E_{t-1}} \left( \frac{(MC^*_{i}E_t)^{1-\zeta}}{E_{t}} \right). \] (66)

In general the three expressions above are different, which is a restatement of our result on the suboptimality of inward-looking policies. When \( \zeta = 1 \) optimal monetary policy implies domestic price stability but does not close the output gap. This result is similar to the case \( \eta = 1 \) in the model of Section 3. Only when \( \phi^* = \zeta = 1 \) do the three expressions coincide, and optimal monetary policy has no international dimension.
5 Conclusion

The key message of our contribution is that standard policy objectives for a closed economy setting may not be appropriate for the design of optimal monetary policy in open economies. Inward-looking goals in policy making — such as the complete stabilization of domestic output and domestic producers’ markups — are not optimal in interdependent economies in which firms’ profits are exposed to currency fluctuations. Intuitively, in our economy producers set higher prices in response to higher profit volatility. Unless markups are insulated from exchange rate movements, inward-looking policies make the profits of foreign exporters suboptimally volatile. At an optimum, the welfare costs from higher consumer prices must equate the benefits of bringing domestic output towards its potential, flex-price level.

In our study, the degree of pass-through and exchange rate exposure in domestic and foreign markets emerges as a key parameter in the design of optimal monetary rules, as well as in the welfare analysis of alternative monetary arrangements. If the exposure of firms’ revenue to exchange rate fluctuations is limited, inward-looking policymakers assign high priority to stabilizing domestic output and prices, with ‘benign neglect’ of exchange rate movements. Otherwise, optimal policymakers ‘think globally,’ taking into account the repercussions of exchange rate volatility on import prices; hence, the monetary stances in the world economy come to mimic each other, reducing currency volatility. Thus, a world economy closer to purchasing power parity will be characterized by higher exchange rate volatility and monetary stances more closely focused on internal conditions, and vice-versa.

These considerations cast new light on the case for international monetary cooperation. Interdependent economies gain little from cooperating in the design of policy rules when exchange rate fluctuations do not impinge on exporters’ profits, so that inward-looking policies are optimal. We have shown that the same can be true even when firms’ profits are highly exposed to the exchange rate. This is because, even without international agreement, domestic policies would optimally respond symmetrically to worldwide cyclical developments.
For intermediate cases (in our model corresponding to intermediate levels of pass-through) such as those observed for a wide range of countries, there will be gains from cooperation.

References


Appendix

Intratemporal allocation. For given consumption indices, utility-based price indices are derived as follows. $P_H$ is the price of a consumption bundle of domestically produced goods that solves:

$$\min_{C(h,j)} \int_0^1 p(h)C(h,j)dh$$  \hspace{1cm} (A.1)

subject to $C_H(j) = 1$. This constraint can be rewritten as:

$$[\theta/ (\theta - 1)] \ln \left[ \int_0^1 C(h, j)^{-\theta} dh \right] = 0.$$  \hspace{1cm} (A.2)

The solution is equation (4) in the main text.

Consider now the problem of allocating a given level of nominal expenditure $\bar{Z}$ among domestically produced goods:

$$\max_{C(h,j)} C_H(j) \quad s.t. \quad \int_0^1 p(h)C(h,j)dh = \bar{Z}$$  \hspace{1cm} (A.3)

Clearly, across any pair of goods $h$ and $h'$, it must be true that:

$$\frac{C(h', j)}{C(h, j)} = \left( \frac{p(h')}{p(h)} \right)^{-\theta}$$  \hspace{1cm} (A.4)

or, rearranging:

$$C(h, j)^{\frac{\theta-1}{\theta}} p(h')^{1-\theta} = C(h', j)^{\frac{\theta-1}{\theta}} p(h)^{1-\theta}.$$  \hspace{1cm} (A.5)

Integrating both sides of the previous expression we obtain:

$$\left( \int_0^1 C(h, j)^{\frac{\theta-1}{\theta}} p(h')^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}} = \left( \int_0^1 C(h', j)^{\frac{\theta-1}{\theta}} p(h)^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}}$$  \hspace{1cm} (A.6)

so that:

$$C(h, j) \left( \int_0^1 p(h')^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}} = p(h)^{-\theta} \left( \int_0^1 C(h', j)^{\frac{\theta-1}{\theta}} dh' \right)^{\frac{\theta}{\theta-1}}$$  \hspace{1cm} (A.7)

which can be rewritten as expression (12) in the main text. Note that:

$$\int_0^1 p(h)C(h, j)dz = P_HC_H(j).$$  \hspace{1cm} (A.8)
**Intertemporal allocation.** Consider the optimal allocation by Home agent \( j \). The maximization problem can be written in terms of the following Lagrangian:

\[
L_t(j) \equiv E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \{ \ln C_\tau(j) + \chi \ln M_\tau(j) / P_\tau - \kappa \ell_\tau(j) \\
+ \lambda_\tau(j)[-B_{\tau+1}(j) + (1 + i_\tau)B_\tau(j) - \mathcal{E}_\tau B_{\tau+1}^*(j) + (1 + i_\tau^*)\mathcal{E}_\tau B_\tau(j) \\
- M_\tau(j) + M_{\tau-1}(j) + W_\tau \ell_\tau(j) + \int_0^1 \Pi_\tau(h)dh - T_\tau(j) \\
- P_{H,\tau} C_{H,\tau}(j) - P_{F,\tau} C_{F,\tau}(j)] \}.
\]  

(A.9)

The first order conditions with respect to \( C_{H,t}(j), C_{F,t}(j), \ell_t(j), B_{t+1}(j), B_{t+1}^*(j) \) and \( M_t(j) \) are, respectively:

\[
\frac{\gamma}{C_{H,t}(j)} = \lambda_t(j) P_{H,t} 
\]  

(A.10)

\[
\frac{1 - \gamma}{C_{F,t}(j)} = \lambda_t(j) P_{F,t} 
\]  

(A.11)

\[
\lambda_t(j)W_t = \kappa 
\]  

(A.12)

\[
\lambda_t(j) = \beta (1 + i_{t+1}) E_t \lambda_{t+1}(j) 
\]  

(A.13)

\[
\mathcal{E}_t \lambda_t(j) = \beta (1 + i_{t+1}^*) E_t \mathcal{E}_{t+1} \lambda_{t+1}(j) 
\]  

(A.14)

\[
\frac{\chi}{M_t(j)} = E_t [\lambda_t(j) - \beta \lambda_{t+1}(j)].
\]  

(A.15)

First we solve for the multiplier \( \lambda_t(j) \). Take a geometric average of (A.10) and (A.11) with weights \( \gamma \) and \( 1 - \gamma \), respectively:

\[
\gamma (1 - \gamma)^{1-\gamma} = \lambda_t(j)P_{H,t}^\gamma P_{F,t}^{1-\gamma} C_{H,t}^\gamma(j) C_{F,t}^{1-\gamma}(j).
\]  

(A.16)

This yields:

\[
\lambda_t(j) = \frac{1}{P_tC_t(j)}
\]  

(A.17)

so that, at the optimum, agent \( j \)’s demand for Home and Foreign consumption goods are a constant fraction of agent \( j \)’s total consumption expenditure:

\[
P_tC_t(j) = \frac{1}{\gamma} P_{H,t} C_{H,t}(j) = \frac{1}{1 - \gamma} P_{F,t} C_{F,t}(j).
\]  

(A.18)
Using these expressions, it is easy to verify that:

\[ P_tC_t(j) = P_{H,t}C_{H,t}(j) + P_{F,t}C_{F,t}(j). \]  

(A.19)

Condition (A.12) can be written as:

\[ W_t = \kappa P_tC_t(j) \]  

(A.20)

implying that consumption is equalized across Home agents.

Combining (A.10), (A.11) and (A.13) the intertemporal allocation of consumption is determined according to the Euler equation:

\[ \frac{1}{C_t(j)} = \beta (1 + i_{t+1}) E_t \left( \frac{1}{\pi_{t+1} C_{t+1}(j)} \right) \]  

(A.21)

where \( \pi_t \) denotes the gross rate of inflation:

\[ \pi_t = \frac{P_t}{P_{t-1}}. \]  

(A.22)

Finally, condition (A.15) can be written as the money demand function:

\[ \frac{M_t(j)}{P_t} = \chi \frac{1 + i_{t+1}}{i_{t+1}} C_t(j). \]  

(A.23)

Define now the variable \( Q_{t,t+\tau}(j) \) as:

\[ Q_{t,t+\tau}(j) \equiv \beta^\tau \frac{\lambda_{t+\tau}(j)}{\lambda_t(j)}. \]  

(A.24)

The previous expression can be interpreted as agent \( j \)'s stochastic discount rate. Since consumption is symmetric across agents, the stochastic discount rate is identical across individuals:

\[ Q_{t,t+1}(j) = Q_{t,t+1}. \]  

(A.25)

Comparing (A.24) with (A.13) and (A.14) we obtain:

\[ E_t Q_{t,t+1} = \frac{1}{1 + i_{t+1}}, \quad E_t [Q_{t,t+1} \Delta_{t+1}] = \frac{1}{1 + i^*_{t+1}} \]  

(A.26)

where \( \Delta_t \) denotes the expected rate of exchange rate depreciation:

\[ \Delta_t = \frac{\xi_t}{\xi_{t-1}}. \]  

(A.27)
Note that in the absence of uncertainty the previous condition implies the familiar uncovered interest parity expression $1 + i_{t+1} = \left(1 + i_{t+1}^*\right) \mathcal{E}_{t+1}/\mathcal{E}_t$.

Using (A.26) we can write:

$$M_t(j) + B_{t+1}(j) = \frac{i_{t+1} M_t(j)}{1 + i_{t+1}} + E_t \left\{ Q_{t,t+1} \left[ M_t(j) + (1 + i_{t+1}) B_{t+1}(j) \right] \right\} \quad (A.28)$$

and:

$$\mathcal{E}_t B_{t+1}^*(j) = E_t \left\{ Q_{t,t+1}(1 + i_{t+1}^*) \mathcal{E}_{t+1} B_{t+1}^*(j) \right\}. \quad (A.29)$$

It follows that the flow budget constraint (11) can also be written as:

$$\frac{i_{t+1} M_t(j)}{1 + i_{t+1}} + E_t \left\{ Q_{t,t+1} R_{t+1}(j) \right\} \leq R_t(j) + W_t \ell_t(j) + \int_0^1 \Pi_t(h) dh - T_t(j) - P_t C_t(j) \quad (A.30)$$

where $R_{t+1}$ is wealth at the beginning of period $t+1$, defined as:

$$R_{t+1}(j) \equiv M_t(j) + (1 + i_{t+1}) B_{t+1}(j) + (1 + i_{t+1}^*) \mathcal{E}_{t+1} B_{t+1}^*(j). \quad (A.31)$$

Optimization implies that households exhaust their intertemporal budget constraint. The flow budget constraint hold as equality and the transversality condition:

$$\lim_{N \to \infty} E_t [Q_{t,N} R_N(j)] = 0 \quad (A.32)$$

is satisfied, where $Q_{t,N} \equiv \prod_{s=t+1}^N Q_{s-1,s}$.

**Foreign constraints and optimization conditions.** Similar results characterize the optimization problem of Foreign agent $j^*$. For convenience, we report here the key equations. The resource constraint for agent $f$ is:

$$\frac{\ell^*(f)}{\alpha_t^*} \geq p_t(f)^{-\theta} P^\theta_{F,f} C_{F,t} + p_t^*(f)^{-\theta} (P_{F,t}^*)^{-\theta} C_{F,t}^*. \quad (A.33)$$

The flow budget constraint is:

$$\frac{B_{t+1}(j^*)}{\mathcal{E}_t} + B_{t+1}^*(j^*) + M_t^*(j^*) \leq (1 + i_t) \frac{B_t(j^*)}{\mathcal{E}_t} + (1 + i_t^*) B_t^*(j^*) + M_{t-1}^*(j^*)$$

$$+ W_t^* \ell_t^*(j^*) + \int_0^1 \Pi_t^*(f) df - T_t^*(j^*) - P_{H,t}^* C_{H,t}^*(j^*) - P_{F,t}^* C_{F,t}^*(j^*). \quad (A.34)$$
First order conditions yield:

\[ P_t^* C_t^*(j^*) = \frac{1}{\gamma} P_{H,t}^* C_{H,t}^*(j^*) = \frac{1}{1 - \gamma} P_{F,t}^* C_{F,t}^*(j^*) \] (A.35)

\[ W_t^* = \kappa^* P_t^* C_t^*(j^*) \] (A.36)

\[ \frac{1}{C_t^*(j^*)} = \beta (1 + i_{t+1}^*) E_t \left( \frac{1}{\pi_{t+1}^*} \frac{1}{C_{t+1}^*(j^*)} \right) \] (A.37)

\[ \frac{M_t^*(j^*)}{P_t^*} = \chi^* \frac{1 + i_{t+1}^*}{i_{t+1}^*} C_t^*(j^*) \] (A.38)

\[ Q_{t,t+1}^*(j^*) = Q_{t,t+1}^* = \beta \frac{C_{t}^*(j^*)}{\pi_{t+1}^* C_{t+1}^*(j^*)} = \beta \frac{\lambda_{t,t+1}^*(j^*)}{\lambda_t^*(j^*)} \] (A.39)

and:

\[ E_t Q_{t,t+1}^* = \frac{1}{1 + i_{t+1}^*}, \quad E_t \left[ Q_{t,t+1}^* \frac{1}{\Delta_{t+1}} \right] = \frac{1}{1 + i_{t+1}^*}. \] (A.40)

**Equilibrium.** Given initial wealth levels \( R_{t_0}^*(j) = M_{t_0}^* \) and \( R_{t_0}^*(j^*) = M_{t_0}^* \), and the processes for \( \alpha_t, \alpha_t^*, T_t, T_t^*, i_{t+1}^* \) and \( i_{t+1}^* \) for all \( t \geq t_0 \), the fix-price equilibrium is a set of processes for \( C_t(h,j), C_t(f,j), C_{H,t}(j), C_{F,t}(j), C_t(j), \) \( Q_{t,t+1}(j), B_{t+1}(j), B_{t+1}^*(j), M_t(j), \ell_t(j), R_t(j), P_t, C_t^*(h,j^*), C_t^*(f,j^*), C_{H,t}^*(j^*), C_{F,t}^*(j^*), C_t^*(j^*), Q_{t,t+1}^*(j^*), B_{t+1}(j^*), B_{t+1}^*(j^*), \) \( M_t^*(j^*), \ell_t^*(j^*), R_t^*(j), P_t^*, \pi_t, \mu_t, \) \( \mu_t^*, Y_t(h), \ell_t(h), M_t(c), \Pi_t(c), p_t(h), p_t(f), P_{H,t}, P_{F,t}, W_t, Y_t(f), \ell_t^*(f), M_t^*(f), \Pi_t^*(f), p_t^*(h), p_t^*(f), P_{H,t}, P_{F,t}, W_t^*, \lambda_t(j), \lambda_t^*(j^*), \Delta_t \) and \( \mathcal{E}_t \), such that, for all \( t \geq t_0 \) and \( \tau > t_0 \), (i) Home households’ equations (12), (2), (3), (A.24), (A.13), (A.14), (A.15), (A.12), (A.31), (A.22) and their Foreign analogs hold as equalities; (ii) the Home government’s equations (18) and (20) and their Foreign analogs hold as equalities; (iii) Home firms’ equations (6), (7), (8), (9), (23), (26), (4), (10) and their Foreign analogs hold as equalities; (iv) the Home transversality condition (A.32) and its Foreign analog hold; (v) the markets for the international bonds clear, that is conditions (19) and (A.27) hold.

A flex-price equilibrium is similarly defined, after imposing that for all \( t \geq t_0 \) Home conditions (23), (26), and their Foreign analogs hold in any state of nature at time \( t \) rather than in expectation at time \( t - 1 \).
Equilibrium balanced current account. Aggregating the individual budget constraints and using the government budget constraint we obtain an expression for the Home current account:

\[ E_t \{ Q_{t+1} A_{t+1} \} = A_t - (1 - \gamma) P_t C_t + \gamma \varepsilon_t P_t^* C_t^* \]  

(A.41)

where \( A \) is defined as wealth net of money balances, or

\[ A_{t+1} \equiv R_{t+1} - M_t. \]  

(A.42)

Since at time \( t_0 \) \( A_{t_0} = 0 \), it is easy to show that, for all \( t \geq t_0 \), the equilibrium conditions above are solved by the allocation:

\[ (1 - \gamma) P_t C_t + \gamma \varepsilon_t P_t^* C_t^* = 0, \quad A_t = -\varepsilon_t A_t^* = 0 \quad t \geq t_0. \]  

(A.43)

The utility gap. To derive (43) and to show its sign observe that:

\[ C_t^{\text{flex}} = \gamma \frac{\Phi(W)}{\alpha_t^*(\alpha_t^*)^{1-\gamma}} \]  

(A.44)

and consider the following steps:

\[
E_{t-1} W_t^{\text{flex}} - E_{t-1} W_t = E_{t-1} \left( \ln C_t^{\text{flex}} - \ln C_t \right) + \kappa \Phi - \kappa \Phi \\
= E_{t-1} \ln \frac{C_t^{\text{flex}}}{C_t} = E_{t-1} \ln \left\{ E_{t-1} [\alpha_t \mu_t] \right\}^{\gamma} \left\{ \frac{E_{t-1} \left[ \alpha_t^* (\mu_t)^{\eta} \mu_t^{1-\eta} \right]}{\alpha_t^* (\alpha_t^*)^{1-\gamma} \mu_t^{1-\eta (1-\gamma)} (\mu_t^*)^{\eta (1-\gamma)} \right\}^{1-\gamma} \\
= E_{t-1} \left[ \gamma \ln \frac{E_{t-1} [\alpha_t \mu_t]}{\alpha_t \mu_t} + (1 - \gamma) \ln \frac{E_{t-1} \left[ \alpha_t^* (\mu_t)^{\eta} \mu_t^{1-\eta} \right]}{\alpha_t^* (\mu_t^{1-\eta})} \right] \\
= E_{t-1} \left[ \gamma (\ln E_{t-1} [\alpha_t \mu_t] - \ln \alpha_t \mu_t) + (1 - \gamma) \left( \ln E_{t-1} \left[ \alpha_t^* (\mu_t)^{\eta} \mu_t^{1-\eta} \right] - \ln \alpha_t^* (\mu_t)^{\eta} \mu_t^{1-\eta} \right) \right] \\
\geq E_{t-1} \left[ \gamma (E_{t-1} [\ln \alpha_t \mu_t] - \ln \alpha_t \mu_t) + (1 - \gamma) \left( E_{t-1} \ln \alpha_t^* (\mu_t)^{\eta} \mu_t^{1-\eta} - \ln \alpha_t^* (\mu_t)^{\eta} \mu_t^{1-\eta} \right) \right] \\
= \gamma \left( E_{t-1} \ln \alpha_t \mu_t - E_{t-1} \ln \alpha_t \mu_t + (1 - \gamma) \left( E_{t-1} \ln \alpha_t^* (\mu_t)^{\eta} \mu_t^{1-\eta} - E_{t-1} \ln \alpha_t^* (\mu_t)^{\eta} \mu_t^{1-\eta} \right) \right) \\
= 0. \]  

(A.45)

Optimal monetary rules. Recalling that:

\[
\frac{\partial f [E_{t-1} (x_t \mu_t^*)]}{\partial \mu_t} = f' [E_{t-1} (x_t \mu_t^*)] x_t \sigma \mu_t^{\sigma-1}, \]  

(A.46)
the first order condition for maximizing (A.45) with respect to $\mu_t$ is:

$$\frac{\gamma \alpha_t}{E_{t-1}[\alpha_t \mu_t]} - \frac{\gamma}{\mu_t} + \frac{(1 - \gamma)(1 - \eta) \alpha_t^* (\mu_t^*)^\eta \mu_t^{-\eta}}{E_{t-1}[\alpha_t^* (\mu_t^*)^\eta \mu_t^{-\eta}]} - \frac{(1 - \gamma)(1 - \eta)}{\mu_t} = 0 \quad (A.47)$$

which can be written as (47) in the main text. The other rules are similarly obtained.